A Group-Based Approach to Measuring Polarization

Online Replication Supplement

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R1 Additional Simulation Plots

This section presents additional plots of simulation results, analogous to Figures S2 and S3 in SI section S3.2, for the cases of three and four components.

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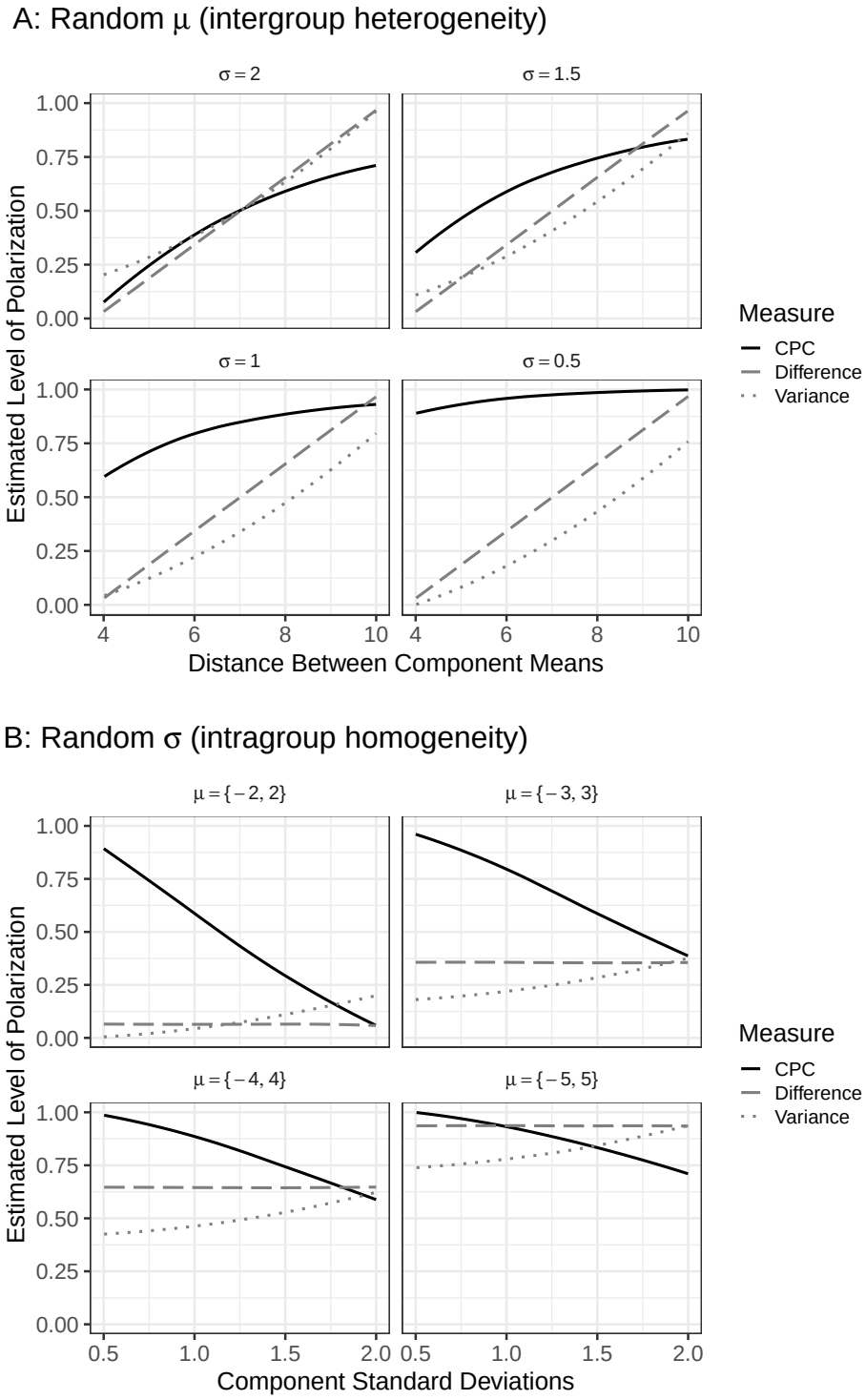

Figure R1: Univariate Polarization Estimates with Three Components Results from univariate simulations of polarization measures with three components, showing estimated level of polarization for a randomly varying distribution parameter, holding the other parameter constant. All measures scaled to $[0, 1]$ to enable comparison and plotted using LOESS with a span of 0.75.

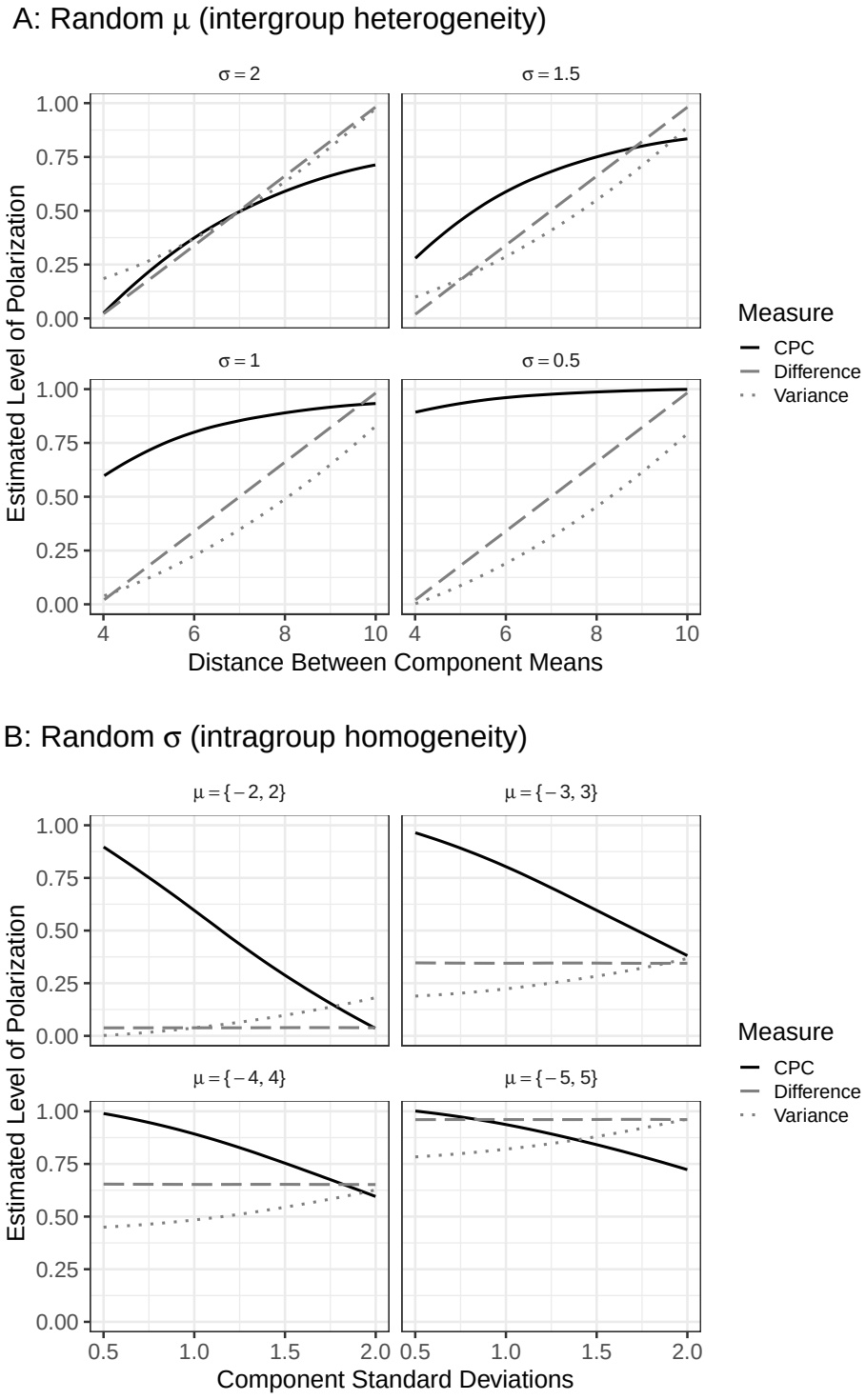

Figure R2: Bivariate Polarization Estimates with Three Components. Results from bivariate simulations of polarization measures with three components, showing estimated level of polarization for a randomly varying distribution parameter, holding the other parameter constant. All measures scaled to $[0, 1]$ to enable comparison and plotted using LOESS with a span of 0.75.

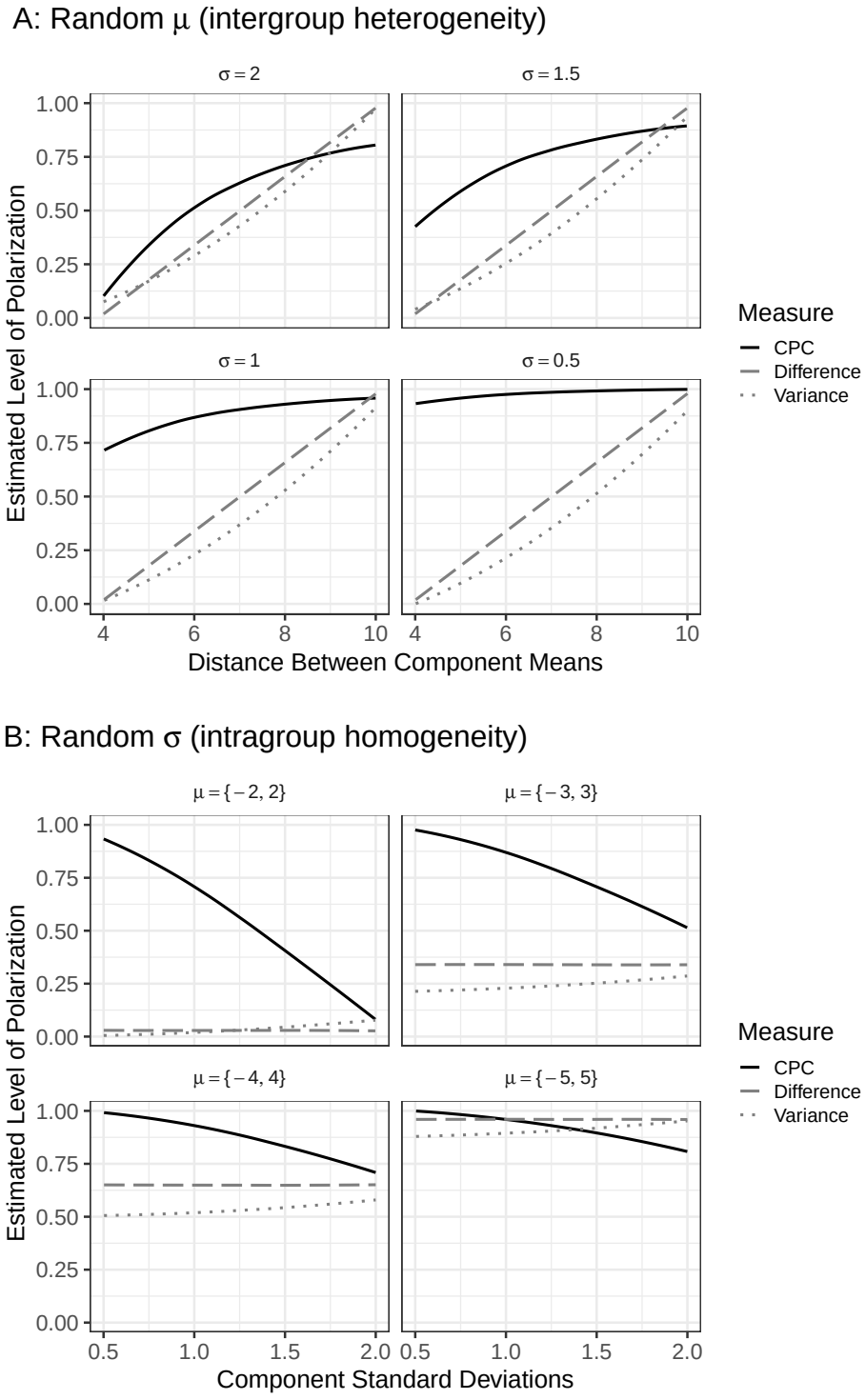

Figure R3: Univariate Polarization Estimates with Four Components. Results from univariate simulations of polarization measures with four components, showing estimated level of polarization for a randomly varying distribution parameter, holding the other parameter constant. All measures scaled to $[0, 1]$ to enable comparison and plotted using LOESS with a span of 0.75.

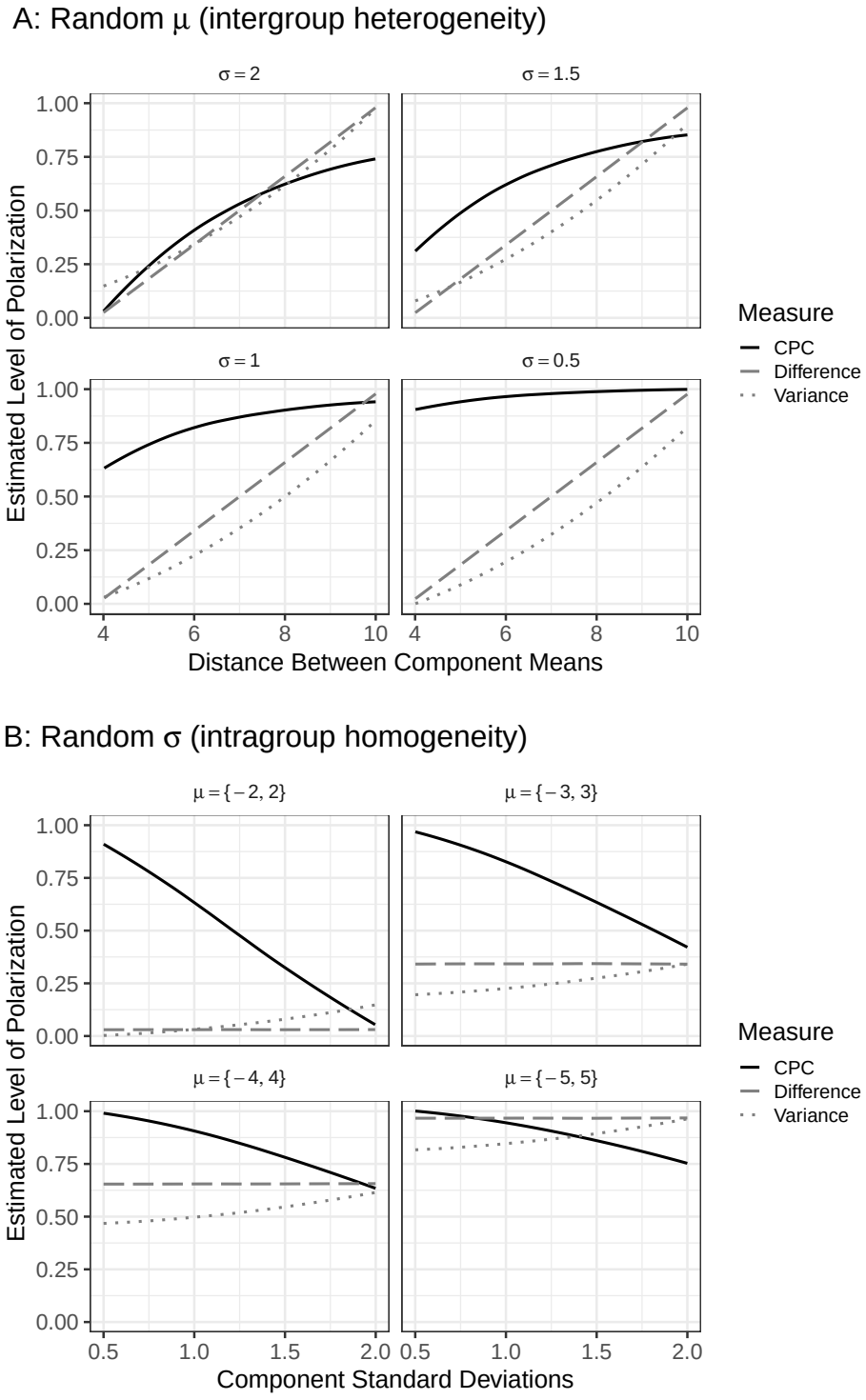

Figure R4: Bivariate Polarization Estimates with Four Components. Results from univariate simulations of polarization measures with four components, showing estimated level of polarization for a randomly varying distribution parameter, holding the other parameter constant. All measures scaled to $[0, 1]$ to enable comparison and plotted using LOESS with a span of 0.75.

R2 Cross-National Polarization Application

R2.1 Selecting Number of Clusters

As noted in the main text, one challenge in using clustering methods is the need to specify the number of clusters n_k. Although Figure 3A in the main text makes this selection relatively clear, I use silhouette scores to confirm the appropriate number of clusters in each distribution. This method involves repeatedly running the clustering algorithm with an increasingly large number of clusters. For each n_k, a "silhouette score" is then calculated. Silhouette scores use an arbitrary distance metric to determine how similar each point is to its own cluster and how different it is from other clusters. These individual scores are then aggregated into a value in the range $[-1, 1]$, where -1 implies the worst possible fit and 1 implies the best possible fit. The number of clusters which produces the maximum silhouette score is the most appropriate value for n_k. Figure R5 displays silhouette plots for each country. There is a clear maximum value for each country, corresponding to a two-cluster specification in Germany, Italy, the United States, and the United Kingdom, and three-cluster specifications in the Netherlands and Spain.

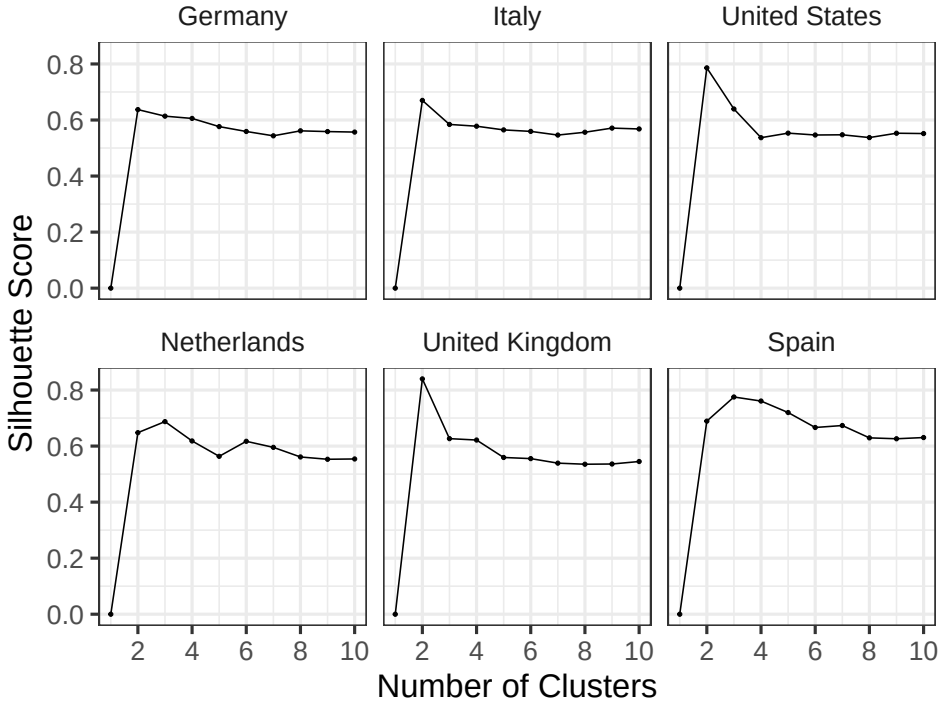

Figure R5: Silhouette Scores of Ideology Estimates by Country. Table R1 presents results in tabular form.

	Country					
K	Germany	Italy	Netherlands	Spain	United Kingdom	United States
1	0.000	0.000	0.000	0.000	0.000	0.000
2	0.637	0.670	0.648	0.689	0.840	0.786
3	0.614	0.584	0.687	0.775	0.627	0.639
4	0.606	0.578	0.618	0.761	0.622	0.538
5	0.576	0.565	0.563	0.720	0.559	0.553
6	0.559	0.560	0.617	0.666	0.555	0.546
7	0.544	0.546	0.595	0.673	0.539	0.547
8	0.561	0.556	0.582	0.629	0.535	0.537
9	0.559	0.571	0.561	0.626	0.536	0.553
10	0.557	0.568	0.546	0.633	0.545	0.552

Table R1: Silhouette Scores of Ideology Estimates by Country. Bolded values denote maximum within each country. Corresponds to Figure R5.

R2.2 Robustness to Clustering Method

The CPC and difference-in-means measures require knowledge of the group to which each observation belongs, so I use three different unsupervised clustering methods to assign observations to clusters within each country. The first method is k-means clustering, which iteratively assigns observations to the nearest cluster centroid (Hartigan and Wong 1979). The results reported in the main text use cluster memberships recovered from this method.

To ensure polarization results are not merely artifacts of this particular approach to recovering cluster memberships, I also implement both agglomerative and divisive hierarchical clustering algorithms. Agglomerative clustering with complete linkage begins by assigning each observation to its own cluster. At each iteration, the algorithm then merges the two clusters with the shortest distance between each other. The distance $D(A, B)$ between clusters A and B is determined by $D(A, B) = \max_{x \in X, y \in Y} d(x, y)$, where $d(x, y)$ is the distance between x and y (Defays 1977).

Divisive clustering, on the other hand, begins by assigning all observations to the same group. At each iteration, the algorithm selects the cluster with the largest distance between its members and finds the observation in that cluster with the largest average distance to all other observations in that cluster. This observation becomes the first observation in the "splinter group." The algorithm then divides the selected cluster into two clusters by assigning observations either to the existing cluster or the splinter group based, again, on average distance to all observations in each cluster (Kaufman and Rousseeuw 2005).

Figure R6 displays polarization estimates generated by the CPC (top row) and difference-in-means (bottom row) with cluster memberships recovered from k-means, complete-linkage agglomerative clustering, and divisive clustering. Countries are ordered by CPC estimates and error bars give 95% confidence intervals. Other than wider confidence intervals in the case of agglomerative clustering, results are consistent across all three clustering methods. In each case, the CPC recovers the same ordering presented in the main text, showing Germany and Italy with the lowest levels of polarization, the United States, Netherlands, and United Kingdom with higher levels, and Spain with the highest level overall. Difference-in-means estimates are also consistent, with the United Kingdom appearing by far the most polarized, followed by Italy, the Netherlands, Spain, Germany, and the United States as the least polarized. These consistent results reassure me that the

polarization levels reported in the main text are not merely a mechanical result due to the specific clustering algorithm used.

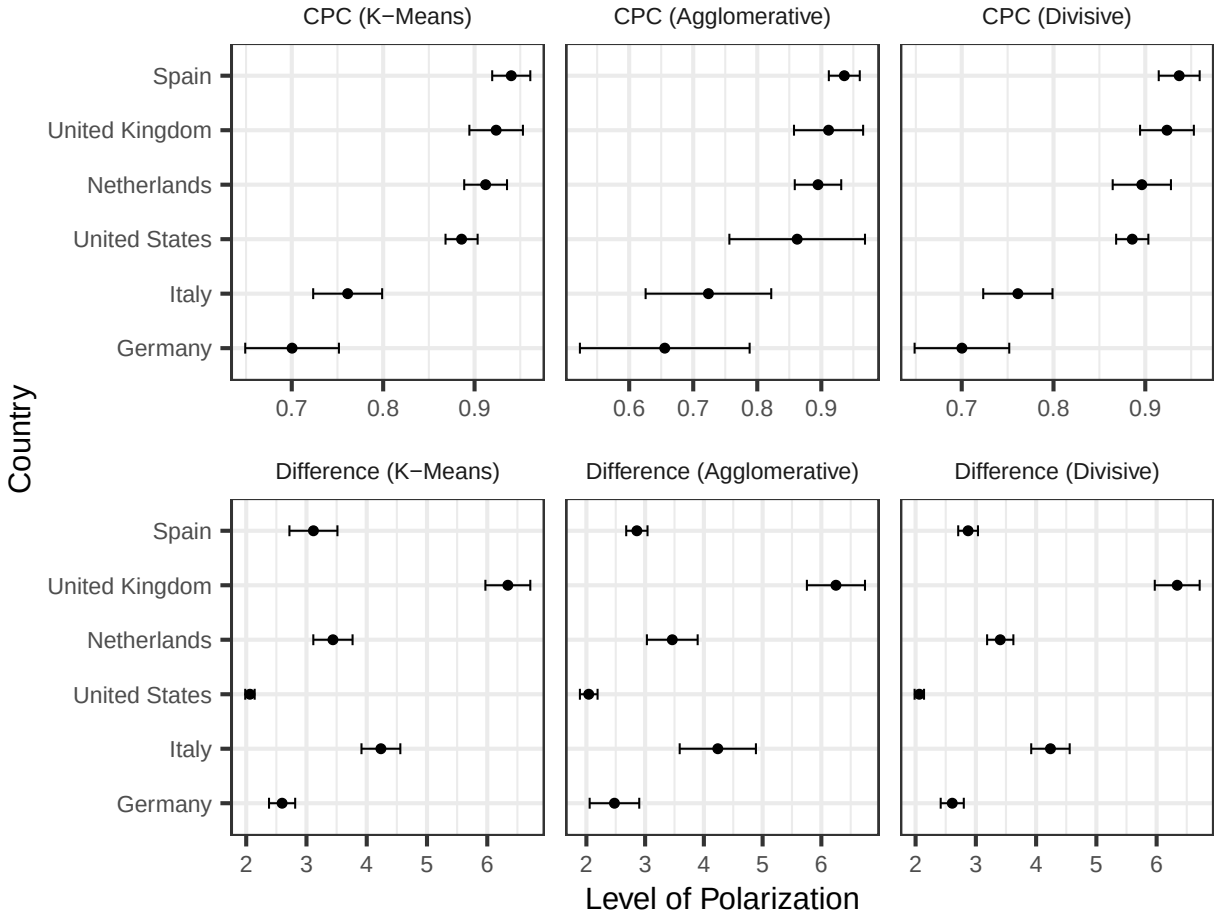

Figure R6: CPC and Difference-in-Means Estimates Across Clustering Methods. Error bars give 95% confidence intervals, calculated using a case-resampling bootstrap. Table R2 presents point estimates and standard errors.

R2.3 Full Polarization Estimates

Table R2 displays the polarization estimates and associated standard errors for each country and measure presented in the analysis of elite ideological ideal points in the main text. The estimates in Table R2 correspond to Figure 3B in the main text and Figure R6 above.

	Measure			
	Difference (K-Means)	Difference (Agglomerative)	Difference (Divisive)	Variance
Germany	2.595	2.478	2.608	1.935
	(0.111)	(0.215)	(0.098)	(0.127)
Italy	4.235	4.237	4.238	5.078
	(0.164)	(0.331)	(0.163)	(0.355)
Netherlands	3.439	3.462	3.405	3.991
	(0.166)	(0.220)	(0.111)	(0.338)
Spain	3.115	2.859	2.871	2.337
	(0.203)	(0.093)	(0.084)	(0.222)
United Kingdom	6.342	6.249	6.342	9.675
	(0.190)	(0.252)	(0.190)	(0.730)
United States	2.062	2.040	2.062	1.177
	(0.040)	(0.077)	(0.040)	(0.046)
	CPC (K-Means)	CPC (Agglomerative)	CPC (Divisive)	Expert-Coded
Germany	0.700	0.656	0.700	0.337
	(0.026)	(0.068)	(0.026)	
Italy	0.761	0.724	0.761	0.319
	(0.019)	(0.050)	(0.019)	
Netherlands	0.912	0.895	0.896	0.382
	(0.012)	(0.019)	(0.016)	
Spain	0.940	0.936	0.937	0.474
	(0.011)	(0.012)	(0.011)	
United Kingdom	0.924	0.911	0.924	0.380
	(0.015)	(0.028)	(0.015)	
United States	0.886	0.862	0.886	0.376
	(0.009)	(0.054)	(0.009)	

Table R2: Polarization Estimates and Associated Standard Errors from Analysis of Cross-National Elite Ideal Points. Corresponds to main text Figure 3B and SI Figure R6. Bootstrapped standard errors are in parentheses (N = 100).

R3 Affective Polarization Application

R3.1 Polarization Estimates

Figure R7 displays estimated levels of affective polarization across all twenty-eight countries in the sample, according to each measure. Countries are ordered by their estimated level of polarization as measured by the CPC.

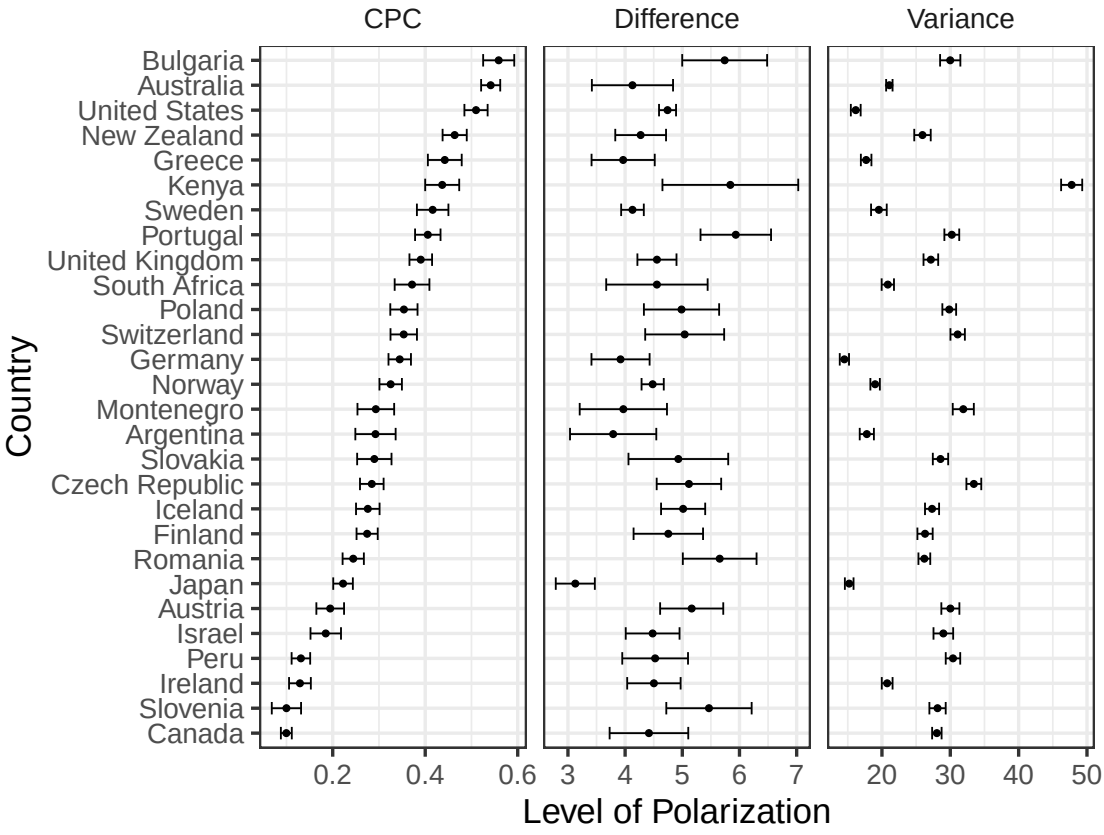

Figure R7: Affective Polarization Levels Across Countries. Error bars give 95% confidence intervals, calculated using a case-resampling bootstrap. Table R3 presents point estimates and standard errors

	Measure				*Measure*		
	Difference	Variance	CPC		Difference	Variance	CPC
Argentina	3.791	17.787	0.292	Montenegro	3.969	31.901	0.293
	(0.385)	(0.530)	(0.022)		(0.389)	(0.787)	(0.020)
Australia	4.128	21.084	0.542	New Zealand	4.272	25.932	0.464
	(0.362)	(0.239)	(0.011)		(0.227)	(0.614)	(0.013)
Austria	5.164	30.012	0.195	Norway	4.482	19.001	0.325
	(0.282)	(0.673)	(0.015)		(0.099)	(0.353)	(0.012)
Bulgaria	5.741	29.992	0.559	Peru	4.525	30.394	0.131
	(0.379)	(0.757)	(0.017)		(0.293)	(0.541)	(0.010)
Canada	4.417	28.035	0.100	Poland	4.986	29.848	0.354
	(0.351)	(0.354)	(0.006)		(0.336)	(0.514)	(0.015)
Czech Republic	5.116	33.447	0.285	Portugal	5.935	30.222	0.406
	(0.288)	(0.555)	(0.013)		(0.315)	(0.555)	(0.014)
Finland	4.755	26.308	0.274	Romania	5.654	26.204	0.244
	(0.310)	(0.570)	(0.012)		(0.329)	(0.442)	(0.012)
Germany	3.920	14.504	0.345	Slovakia	4.931	28.567	0.290
	(0.259)	(0.345)	(0.012)		(0.445)	(0.573)	(0.019)
Greece	3.965	17.688	0.442	Slovenia	5.466	28.135	0.100
	(0.282)	(0.394)	(0.019)		(0.381)	(0.608)	(0.016)
Iceland	5.014	27.331	0.276	South Africa	4.556	20.859	0.371
	(0.197)	(0.526)	(0.013)		(0.453)	(0.460)	(0.019)
Ireland	4.503	20.765	0.129	Sweden	4.129	19.554	0.416
	(0.238)	(0.403)	(0.012)		(0.101)	(0.585)	(0.017)
Israel	4.481	28.982	0.185	Switzerland	5.042	31.078	0.354
	(0.240)	(0.735)	(0.017)		(0.352)	(0.539)	(0.015)
Japan	3.129	15.210	0.222	United Kingdom	4.557	27.156	0.390
	(0.174)	(0.323)	(0.011)		(0.174)	(0.548)	(0.012)
Kenya	5.840	47.760	0.437	United States	4.742	16.167	0.510
	(0.606)	(0.788)	(0.019)		(0.076)	(0.367)	(0.013)

Table R3: Estimates of Affective Polarization. Corresponds to Figure R7. Bootstrapped standard errors are in parentheses (N = 100).

R3.2 Full Correlation Results

Table R4 displays estimated correlations between affective polarization estimates and the additional variables discussed in the main text, along with the standard errors of those correlations. The estimates in Table R4 correspond to Figure 4 in the main text.

	Measure		
	Difference	Variance	CPC
Who in Power Makes Difference	0.118	-0.077	0.311
	(0.045)	(0.026)	(0.012)
Vote Makes Difference	-0.048	-0.254	0.318
	(0.045)	(0.034)	(0.021)
Very Close to Party	-0.294	-0.521	0.335
	(0.055)	(0.027)	(0.036)
Ideological Extremity	0.399	0.467	0.252
	(0.036)	(0.035)	(0.037)

Table R4: Correlates of Affective Polarization. Corresponds to main text Figure 4. Bootstrapped standard errors are in parentheses.

References

Defays, D. 1977. "An Efficient Algorithm for a Complete Link Method." *The Computer Journal* 20, no. 4 (January): 364–366.

Hartigan, J. A., and M. A. Wong. 1979. "Algorithm AS 136: A K-Means Clustering Algorithm." *Journal of the Royal Statistical Society, Series C (Applied Statistics)* 28 (1): 100–108.

Kaufman, Leonard, and Peter J. Rousseeuw. 2005. *Finding Groups in Data: An Introduction to Cluster Analysis.* Wiley Series in Probability and Mathematical Statistics. Hoboken, NJ: Wiley.