Supporting Information for

On Spatio-temporal Dependence of Bird's Migration Path Direction on the change of the Earth's Magnetic field, Sun and the Moon Positions (American Region Study)

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Supporting Information Text

1. Details about the Bird Migration Datasets

A. Dataset 1: American Region Bird path. We have compiled a data set of 620 observations from online resources (1), where we specifically focused on the. Birds of the American region. The dataset contains six bird species' names, migration path latitude, longitude, and observation date. The dataset contains different bird species' names, migration path latitudes, longitude, and observation dates.

- **Swainson's hawk**: The Swainson's hawk is a raptor with a thin body and narrow wings that migrates impressive distances. Swainson's hawks rely heavily on thermals and typically avoid crossing water from which thermals rarely rise. The entire population of Swainson's hawks must migrate through the narrow land bridge between North and South America, often in massive flocks of hundreds of thousands of birds. During their journey from North America to South America.

- **Black-Bellied Plover**: The Black-bellied Plover is Washington's largest plover. Their breeding habitat is Arctic islands and coastal areas throughout the northern coasts of Alaska, Canada, and Russia. They nest on the ground in a dry, open tundra with good visibility; the nest is a shallow gravel scrape. They migrate to winter in coastal areas all through the world. In the New World, they winter from southwest British Columbia and Massachusetts south to Argentina and Chile.

- **Black-crowned night heron**: Black-crowned Night-Herons often spend their days perched on tree limbs or concealed among foliage and branches. These social birds roost and nest in groups, although they typically forage independently. Look for them in most wetland habitats across North America, including estuaries, marshes, streams, lakes, and reservoirs.

- **Brown pelican**: Brown Pelicans live along southern and western sea coasts and are rarely seen inland (except at the Salton Sea in California, where they are regular in large numbers). They nest in colonies, often on isolated islands free of land predators. On the Pacific Coast, Brown Pelican adults have red skin on their throats in the breeding season. On the Atlantic and Gulf Coasts, Brown Pelicans are slightly smaller, and their throat skin is greenish-black.

- **Long-billed curlew**: The long-billed curlew (Numenius americanus) is a large North American shorebird of the family Scolopacidae. This species was called the "sickle-bird" and the "candlestick bird." The species breeds in central and western North America, migrating southward and coastward for the winter.

- **Pacific loon**: Pacific Loons Nests in freshwater lakes in arctic and subarctic tundra and taiga. Winters in coastal ocean waters, bays, and estuaries. Occasionally, migrants appear in inland bodies of water, especially large reservoirs.

The dataset is taken from https://nationalzoo.si.edu/migratory-birds/migratory-birds-tracking-map

2. Methods

A. Haversine formula and Computation of Directional Change in Bird Path on Earth Surface (Column θ). We require the Haversine formula (see, for example, (2)) to compute the Column θ, a Directional change in the bird's migration path.

The Haversine formula is an exact way to compute distances between two points on the surface of a sphere. This formula is essential for navigation. It is a particular case of a more general formula in spherical trigonometry. The law of haversines relates the sides and angles of spherical triangles. The formula uses the latitude and longitude of the two points. The haversine formula is a re-formulation of the spherical law of cosines, but the formulation in terms of haversines is more beneficial for small angles and distances. For details, we refer to (3), (4). Applying the Haversine formula, we can accurately compute the direction between two points on a sphere. The mathematical form is

$$\cos\theta = \sin\phi_A \sin\phi_B + \cos\phi_A \cos\phi_B \cos\Delta L \quad [1]$$

where $\Delta L = L_A - L_B$. Point A will have latitude ϕ_A and longitude L_A. Similarly, point B will have latitude ϕ_B and longitude L_B.

B. World Magnetic Model. The World Magnetic Model (WMM) provides a comprehensive representation of the Earth's magnetic field on a global scale (5). This model utilizes a spherical harmonic expansion up to the 12^{12} degree and order, capturing the magnetic scalar potential generated by the Econtributingh contributes to the geomagnetic main field. Alongside the 168 spherical-harmonic "Gauss" coefficients, the model incorporates an equal number of spherical-harmonic Secular-Variation (S.V.) coefficients. All quantities in this section adhere to the following measurement conventions: angles are in radians, lengths are in meters, magnetic intensities are in nanoteslas (nT, where one tesla is one weber per square meter or one kg.s^{-2}.A^{-1}), and times are in years (6) (7).

The primary magnetic field B_m is a potential field and can, therefore, be expressed in geocentric spherical coordinates (longitude λ, latitude ϕ', radius r) as the negative spatial gradient of a scalar potential.

$$B_m(\lambda, \phi', r, t) = - \bigtriangledown V(\lambda, phi', r, t) \quad [2]$$

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where t represents the time. The potential can be expressed as a series expansion in spherical harmonics: (8).

$$V(\lambda,\phi',r,t) = a(\sum_{n=1}^{N}\sum_{m=1}^{M}(\frac{a}{r})^{n+1}(g_n^m(t)\cos(m\lambda) + h_n^m(t)\sin(m\lambda)\breve{p}_n^m(t)(\sin\phi')) \quad [3]$$

We selected N = 36 as the truncation level for the internal expansion of the World Magnetic Model. Here, 'a' (6371200 m) represents the geomagnetic reference radius, closely approximating the mean Earth radius. The variables (λ, ϕ', r) denote the longitude, latitude, and radius in a spherical geocentric reference frame, respectively. Additionally, $g_n^m(t)$ and $h_n^m(t)$ represent the time-dependent Gauss coefficients of degree n and order m, describing the Earth's main magnetic field. The parameters are defined as follows: (9)

$$g_n^m(t) = g_n^m + \dot{g}_n^m(t-t_0) + \ddot{g}_n^m(t-t_0)^2 \quad [4]$$

$$h_n^m(t) = h_n^m + \dot{h}_n^m(t-t_0) + \ddot{h}_n^m(t-t_0)^2 \quad [5]$$

In this equation, $g_n^m, h_n^m, \dot{g}_n^m, \dot{h}_n^m, \ddot{g}_n^m, \ddot{h}_n^m$ from Equation 4 are considered constants. For any real number μ, $\breve{p}_n^m(\mu)$ represents the Schmidt semi-normalized associated Legendre functions and is defined as:

$$\breve{p}_n^m(\mu) = \sqrt{2\frac{(n-m)!}{(n+m)!}}p_{n,m}(\mu) \quad ,if \quad m>0 \quad [6]$$

$$\breve{p}_n^m(\mu) = p_{n,m}(\mu) \quad ,if \quad m=0 \quad [7]$$

Finally, in the case of a data set containing hourly mean observatory data, offsets must be incorporated at each observatory to consider the local magnetic field, primarily induced in the Earth's crust, which is beyond the model's scope. Consequently, at a given observatory, the magnetic field (B) is described as follows:

$$B_m(\lambda,phi',r,t) = -\bigtriangledown V(\lambda,phi',r,t) + O(\lambda,phi',r,t). \quad [8]$$

The offset vector $O(\lambda,\phi',r,t)$, commonly known as the crustal bias, remains constant over time. The parameterization above is employed to model datasets selected from satellite measurements and hourly mean values observed at the observatory. The equations governing the internal component of the field are:

$$X'(\lambda,\phi',r,t) = -\frac{\partial V}{r\partial\phi'} = -\sum_{n=1}^{N}(\frac{a}{r})^{n+2}\sum_{m=0}^{M}(g_n^m(t)\cos(m\lambda) + h_n^m(t)\sin(m\lambda))\frac{d\breve{p}_n^m(\sin\phi')}{d\phi'} \quad [9]$$

$$Y'(\lambda,\phi',r,t) = -\frac{\partial V}{r\cos\phi'\partial\lambda} = \frac{1}{r\cos\phi'}\sum_{n=1}^{N}(\frac{a}{r})^{n+2}\sum_{m=0}^{M}m(g_n^m(t)\cos(m\lambda) + h_n^m(t)\sin(m\lambda))\breve{p}_n^m(\sin\phi') \quad [10]$$

$$Z'(\lambda,\phi',r,t) = \frac{\partial V}{\partial r} = -\sum_{n=1}^{N}(n+1)(\frac{a}{r})^{n+2}\sum_{m=0}^{M}(g_n^m(t)\cos(m\lambda) + h_n^m(t)\sin(m\lambda))\breve{p}_n^m(\sin\phi') \quad [11]$$

The equations above, with the magnetic field vector observations on the left-hand side, constitute the system of condition equations. Consequently, if there are d data points, the system comprises d linear equations involving the p parameters of the parent model:

$$y = Am, \quad [12]$$

Where $\mathbf{y}$ is the column vector $(d \times 1)$ of observations, $\mathbf{A}$ is the matrix $(d \times p)$ of coefficients corresponding to the unknowns, which are functions of position, and $\mathbf{m}$ is the column vector $(p \times 1)$ of unknowns, representing the Gauss coefficients of the model. Since there are more observations than unknowns $(d > p)$, the system is over-determined and, thus, does not have an exact solution. The final main-field coefficients for 2005.0 were obtained by polynomial extrapolation of the main-field Gauss coefficients from the parent model to this date, using Equation 4. Equations 9, with the time-varying Gauss coefficients g_n^m, h_n^m replaced by their time derivatives $\dot{g}_n^m, \dot{h}_n^m$, were then utilized to determine the final secular-variation coefficients.(10)

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C. Spatio-temporal Calculation of the Position of Sun and Moon. The mentioned research hypothesis, if valid, gives us a new model used to understand Migrating bird paths depend on the sun's position as birds use the sun's position in the sky as a compass to navigate their way during their migratory journeys. The results indicate that the proposed model can estimate the Position of the Sun concerning the change of longitude, latitude, and day of the year.
(11)

D. Calculation for the zenith and azimuth angles for Sun and Moon. Suppose the observer's coordinates, or latitude and longitude, are (ϕ_0, λ_0), and the subsolar point's coordinates are (ϕ_s, λ_s), then the x-, y- and z-components of the unit vector, S, pointing from the observer to the center of the Sun.

- $\phi_s = \delta$
- λ_s = -15(T_{GMT} - 12 + E_{min}/60)
- $S_x = \cos\phi_s \sin(\lambda_s - \lambda_0)$
- $S_y = \cos\phi_0 \sin\phi_s$ - $\sin\phi_0 \cos\phi_s \cos(\lambda_s - \lambda_0)$
- $S_z = \cos\phi_0 \sin\phi_s$ - $\cos\phi_0 \cos\phi_s \cos(\lambda_s - \lambda_0)$

In this context, the influence of parallax is disregarded, effectively assuming an infinite Earth-Sun distance. It can be demonstrated that this assumption holds, and as such, there exists. $S_x^2 + S_y^2 + S_z^2 = 1$
Here T_{GMT} is the Greenwich Mean Time or UTC

Remark 1. *The derivation of S_x, S_y, and S_z becomes straightforward when working within the Earth-Centered Earth-Fixed (ECEF) (12) coordinate system. The ECEF system is a geocentric right-handed Cartesian system, and the process can be outlined as follows:*

Begin at the subsolar point in the ECEF system and construct a unit vector pointing upward. At the observer's coordinates in the ECEF system, create three unit vectors pointing east, north, and upward, respectively. Compute the dot product of each unit vector with the vector from Step 1 to obtain the right-hand sides of S_x, S_y, and S_z. Please note that this procedure disregards the influence of parallax, assuming an infinite Earth-Sun distance for simplicity and ease of calculation.

The solar zenith angle is now simply

$$Z = \cos^{-1} S_z \qquad [13]$$

and the solar azimuth angle following the South-Clockwise convention is

$$\gamma_s = a\tan 2(-s_x, -s_y) \qquad [14]$$

Remark 2. *The azimuth angle of the sun and moon provides valuable directional information relative to an observer, measured in degrees clockwise from the north. It reveals their lateral positions in the sky. The sun's azimuth angle continuously changes throughout the day as it moves across the sky. It starts at 90 degrees (east) during sunrise, increases until solar noon (varies by location and time of year), and sets at 270 degrees (west) during sunset. At solar noon, the sun is due south in the northern hemisphere and north in the southern hemisphere. Similarly, the azimuth angle of the moon changes as it orbits the Earth. During a full moon, it rises around 90 degrees, reaches its highest point at approximately 180 degrees, and sets at 270 degrees. During a new moon, the moon's azimuth at rising and setting aligns more closely with the sun's position. In conjunction with the altitude angle (vertical angle above the horizon), the azimuth angle allows observers to precisely determine the sun and moon's positions in the sky at any time and location. This knowledge is crucial for various applications, including astronomy, navigation, and aligning solar panels for maximum sunlight exposure.*

Remark 3. *The Sun's zenith angle is the angle between the Sun and the vertical point directly above an observer on Earth. At solar noon, it is 0 degrees when the Sun is directly overhead. During sunrise and sunset, it is 90 degrees when the Sun is at the horizon. The zenith angle changes throughout the day due to Earth's rotation and varies with the observer's location, time of year, and time of day. It is crucial for solar energy applications, affecting the amount of solar and lunar position change.*

To compute the column Z and γ_s of dataset 2, we use the position of the Sun for each species. For a given bird species, we sort the dataset concerning latitude. Then, we took the first difference of magnitude of the Sun Position(computed using the formulae discussed before).

3. Detail of Von Mises Fisher Mixture Model Fit for the Two Datasets

A mixture model is a probabilistic model for representing the presence of sub-populations within an overall population without requiring that an observed data set identify the sub-population to which an individual observation belongs. However, while problems associated with "mixture distributions" relate to deriving the properties of the overall population from those of the sub-populations, "mixture models" are used to make statistical inferences about the properties of the sub-populations given only observations on the pooled population, without sub-population identity information.

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the density for directional parametric mixture distribution $p(x|\boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_K)$ represent appropriate parameter set, as follows.

$$p(\boldsymbol{\theta}) = \sum_{i=1}^{K} \phi_i f(\vec{x}; \vec{\mu}, \kappa) \quad [15]$$

$$\begin{array}{lcl} K & = & \text{number of mixture components} \\ N & = & \text{number of observations} \\ \theta_{i=1\ldots K} & = & \text{parameter of distribution of observation associated with component i} \\ \phi_{i=1\ldots K} & = & \text{mixture weight, i.e., prior probability of a particular component i} \\ \boldsymbol{\phi} & = & K\text{-dimensional vector set } \phi_{1\ldots K}\text{; sum to 1 ,} \\ z_{i=1\ldots N} & = & \text{component of observation } i \\ x_{i=1\ldots N} & = & \text{observation } i \\ F(x|\theta) & = & \text{probability distribution of an observation, parametrized on } \theta \\ z_{i=1\ldots N} & \sim & \text{Categorical}(\boldsymbol{\phi}) \\ x_{i=1\ldots N}|z_{i=1\ldots N} & \sim & F(x_{i=1\ldots N}|\theta_{z_i}) \end{array} \quad [16]$$

In our paper, the i th vector component is characterized by $f(\vec{x}; \vec{\mu}, \kappa) = C_3(\kappa) \exp(\kappa \vec{x} \cdot \vec{\mu})$ as von Mises-Fisher distribution with weights ϕ_i, parameter θ_i as means $\boldsymbol{\mu_i}$ and concentration matrices $\boldsymbol{\kappa_i}$. Here, $C_3(\kappa)$ is the normalization coefficient, which can be shown to be dependent only on κ and the dimension (3, in the case of the sphere). For details, we refer to (13), (14).

For fitting the mixture Von Mises Fisher distribution to both datasets, we take the help of R package **movMF** of (15). We treat the circular uniform distribution as Von Mises Fisher with large means $\boldsymbol{\mu}$ and large (diluted) concentration matrices $\boldsymbol{\kappa}$. In the "Statistical Analysis for Directional Mixture Distribution Model Fitting for Dataset 1" section in the original paper, we used those tables and gave an interpretation of how they are used.

The codes in the table below give us α value, the weights used in the below parameter results. The vMF (Von Mises Fisher) distribution is commonly parameterized as

$$\theta = \kappa\mu \quad [17]$$

where $\kappa = ||\theta||$ and we have μ are the concentration and mean direction parameters. respectively (if θ not equal to 0, μ is uniquely determined as $\frac{\theta}{||\theta||}$ https://cran.r-project.org/web/packages/movMF/vignettes/movMF.pdf. According to the notation used in the package movMF $\alpha = \phi$ and $\theta = \mu, \kappa$

Sl No.	Lat range	Est μ	Est κ	Dist
1	(33.71 to 70.27)	-2.978336	0.2750625	VonMises
2	(64.86 to 64.87)	2.032647	-	Uniform
3	(54.23 to 69.94)	2.825971	0.3877652	VonMises
4	(59.62 to 70.90)	-1.76427	1.506198	VonMises
5	(54.20 to 61.53)	-1.700339	3.581648	VonMises
6	(57.84 to 61.07)	3.08695	1.768419	VonMises
7	(56.89 to 62.96)	-0.4778354	0.4880568	VonMises
8	(54.60 to 70.37)	-0.08920665	0.7903575	VonMises
9	(54.80 to 71.36)	1.681203	0.8349311	VonMises
10	(35.20 to 39.23)	-2.319494	0.9116722	VonMises
11	(26.71 to 70.68)	0.7958838	-	Uniform
12	(61.20 to 61.22)	-1.626239	32752.33	VonMises
13	(26.73 to 51.14)	2.695495	1.500461	VonMises
14	(23.21 to 32.93)	1.236262	1.268279	VonMises
15	(38.89 to 38.94)	1.222303	3708.063	VonMises
16	(26.67 to 26.74)	1.582261	1380.315	VonMises
17	(26.73 to 51.14)	2.695495	1.500461	VonMises
18	(28.11 to 52.67)	1.933752	1.259675	VonMises
19	(38.88 to 38.93)	1.209689	1903.807	VonMises
20	(25.69 to 26.99)	1.159925	3.330345	VonMises
21	(-36.172 to 41.991)	1.256473	0.3443209	VonMises

Table S1. Latitude of all the Bird Species for Dataset 1. This table contains a serial number which is similar to the Longitude table S2. Together(According to the Serial Number) they give us the location of the bird parameter similar to our Dataset. In this table we estimated the circular mean and variance of each partitions we considered.

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Sl No.	Lon range	Est μ	Est κ	Dist
1	(-236.3 to -166.3)	-2.766708	0.7512237	VonMises
2	(-166.3 to -166.3)	-2.925382	-	Uniform
3	(-166.3 to -165.9)	-2.744784	45.86884	VonMises
4	(-165.9 to -165.8)	-2.499111	738.0808	VonMises
5	(-165.8 to -165.5)	-2.29021	118.4591	VonMises
6	(-165.4 to -165.2)	-1.935756	174.6475	VonMises
7	(-165.1 to -164.4)	-1.401699	19.20709	VonMises
8	(-164.2 to -163.1)	-0.3995937	7.54067	VonMises
9	(-163.1 to -151.0)	0.06193641	1.021821	VonMises
10	(-75.76 to -75.43)	-0.2345965	68.17028	VonMises
11	(-151.0 to -113.2)	-0.03902648	-	Uniform
12	(-165.1 to -165.12)	-1.704596	3625.177	VonMises
13	(-113.1 to -111.6)	0.3206552	6.151091	VonMises
14	(-81.00 to -80.12)	0.9164418	15.45086	VonMises
15	(-77.15 to -77.05)	-1.714681	912.4339	VonMises
16	(-113.2 to -113.1)	-0.04765007	8712.64	VonMises
17	(-113.1 to -11.6)	0.3206552	6.151091	VonMises
18	(-111.6 to -111.1)	1.736291	58.35508	VonMises
19	(-77.05 to -77.00)	-1.641978	3131.77	VonMises
20	(-109.9 to - 109.3)	-2.692523	17.03175	VonMises
21	(-121.96 to -61.23)	-0.2878401	0.3737553	VonMises

Table S2. Longitude of all the Bird Species for Dataset 1. This table contains a serial number which is similar to the Latitude table S1. Together(According to the Serial Number) they give us the location of the bird parameter similar to our Dataset. In this table we estimated the circular mean and variance of each partitions we considered.

K	BIC Values	K	BIC Values
1	-2700.392	11	-3561.971
2	-3234.192	12	-3566.762
3	-3398.578	13	-3540.655
4	-3440.240	14	-3544.801
5	-3461.325	15	-3525.539
6	-3437.471	16	-3519.325
7	-3609.032	17	-3491.873
8	-3594.971	18	-3488.005
9	-3577.902	19	-3464.621
10	-3401.628	20	-3504.006

Table S3. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked 20 values of k from 1 to 20, where the optimal number of partitions is 7. We get that this will give us the weights(ϕ) or α values as 0.17285200, 0.03501382, 0.08647874, 0.29227203, 0.11655564, 0.20562294, 0.09120483. This is done for the Volcano part of the dataset.

4. *Swainson's hawk*

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Species	Parameters	Test Statistics	Critical Value	Distribution
Swainson's hawk	θ(Birds angular directional Change)	0.081	0.0244	vonMises Distribution
	Solar Azimuth Change	0.2011	0.11	Not vonMises Distribution
	Solar Zenith Change	0.0701	0.09	vonMises Distribution
	Lunar Azimuth Change	0.1245	0.09	Not vonMises Distribution
	Magnetic field Change	0.1426	0.11	Not vonMises Distribution
Pacific loon	θ(Birds angular directional Change)	0.0141	0.081	vonMises Distribution
	Solar Azimuth Change	0.0399	0.09	vonMises Distribution
	Solar Zenith Change	0.0409	0.09	vonMises Distribution
	Lunar Azimuth Change	0.0851	0.09	vonMises Distribution
	Magnetic field Change	0.7983	0.128	Not vonMises Distribution
Long-billed curlew	θ(Birds angular directional Change)	0.0285	0.081	vonMises Distribution
	Solar Azimuth Change	0.1394	0.11	Not vonMises Distribution
	Solar Zenith Change	0.1005	0.05	Not vonMises Distribution
	Lunar Azimuth Change	0.0902	0.081	Not vonMises Distribution
	Magnetic field Change	4.9538	0.142	Not vonMises Distribution
Brown pelican	θ(Birds angular directional Change)	0.0224	0.081	vonMises Distribution
	Solar Azimuth Change	0.2175	0.128	Not vonMises Distribution
	Solar Zenith Change	0.1967	0.128	Not vonMises Distribution
	Lunar Azimuth Change	0.1304	0.09	Not vonMises Distribution
	Magnetic field Change	0.1055	0.164	vonMises Distribution
Black-crowned night heron	θ(Birds angular directional Change)	0.0488	0.081	vonMises Distribution
	Solar Azimuth Change	0.0499	0.09	vonMises Distribution
	Solar Zenith Change	0.1014	0.09	Not vonMises Distribution
	Lunar Azimuth Change	0.1081	0.081	Not vonMises Distribution
	Magnetic field Change	0.7223	0.142	Not vonMises Distribution
Black-Bellied Plover	θ(Birds angular directional Change)	0.0286	0.081	vonMises Distribution
	Solar Azimuth Change	0.529	0.128	Not vonMises Distribution
	Solar Zenith Change	0.2878	0.128	Not vonMises Distribution
	Lunar Azimuth Change	0.3329	0.09	Not vonMises Distribution
	Magnetic field Change	4.0786	0.142	Not vonMises Distribution

Table S4. We used the Watson test to get a circular distributional overview of how the change of direction(θ), change of sun and moon position, and change of magnetic field change are distributed. We checked whether those angular or directional parameters follow a von Mises distribution. We took a 0.01 level of significance to test. The table is created concerning the species.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0127	0.09	vonMises
Longitude	0.0307	0.081	vonMises
Latitude	0.0284	0.081	vonMises
Lunar azimuth difference	0.0619	0.09	vonMises
Solar Azimuth difference	0.077	0.11	vonMises
Solar Zenith difference	0.0335	0.09	vonMises
Magnetic Declination	0.0763	0.11	vonMises

Table S5. Used Watson test on all the directional parameters of the bird species *Swainson's hawk* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 75 observations.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0307	0.11	vonMises
Longitude	0.0647	0.164	vonMises
Latitude	0.0418	0.164	vonMises
Lunar azimuth difference	0.0697	0.11	vonMises
Solar Azimuth difference	0.0749	0.142	vonMises
Solar Zenith difference	0.0578	0.09	vonMises
Magnetic Declination	0.1035	0.158	vonMises

Table S6. Used Watson test on all the directional parameters of the bird species *Swainson's Hawk* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 76 to 78th observations.

5. *Black Crowned Night Heron*

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Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0683	0.09	vonMises
Longitude	0.0612	0.158	vonMises
Latitude	0.127	0.164	vonMises
Lunar azimuth difference	0.1197	0.142	vonMises
Solar Azimuth difference	0.0751	0.164	vonMises
Solar Zenith difference	0.0481	0.158	vonMises
Magnetic Declination	0.1024	0.142	vonMises

Table S7. Used Watson test on all the directional parameters of the bird species *Swainson's Hawk* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 79 to 82th observations.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.046	0.158	vonMises
Longitude	0.0626	0.164	vonMises
Latitude	0.0983	0.158	vonMises
Lunar azimuth difference	0.0827	0.128	vonMises
Solar Azimuth difference	0.0435	0.164	vonMises
Solar Zenith difference	0.045	0.164	vonMises
Magnetic Declination	0.0502	0.164	vonMises

Table S8. Used Watson test on all the directional parameters of the bird species *Swainson's Hawk* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 83 to 87th observations.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0241	0.09	vonMises
Longitude	0.0841	0.11	vonMises
Latitude	0.0722	0.09	vonMises
Lunar azimuth difference	0.0683	0.09	vonMises
Solar Azimuth difference	0.1099	0.128	vonMises
Solar Zenith difference	0.0224	0.09	vonMises
Magnetic Declination	0.071	0.11	vonMises

Table S9. Used Watson test on all the directional parameters of the bird species *Swainson's Hawk* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 88 to 99th observations.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.032	0.081	vonMises
Longitude	0.03123	0.09	vonMises
Latitude	0.0473	0.09	vonMises
Lunar azimuth difference	0.0427	0.09	vonMises
Solar Azimuth difference	0.108	0.11	vonMises
Solar Zenith difference	0.0258	0.081	vonMises
Magnetic Declination	0.0407	0.09	vonMises

Table S10. Used Watson test on all the directional parameters of the bird species *Swainson's Hawk* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 100 to 167th observations.

K	BIC Values	K	BIC Values
1	-563.6	6	-625.0477
2	-613.82	7	-620.4164
3	-652.94	8	-601.6789
4	-646.1603	9	-593.9517
5	-638.8709	10	-610.7275

Table S11. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 3. This will give us the weights(ϕ) or α values as 0.430, 0.148, and 0.4125. This is done for the bird species *Swainson's Hawk*

6. *Brown Pelican*

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Part	Parameters used for regression with θ	ρ values	Correlation value	P-value
Part-1	Magnetic declination	0.299	0.09	0.46
	Solar Azimuth Change	0.229	0.082	0.46
	Solar Zenith Change	0.242	0.05	0.64
	Lunar Azimuth Change	0.225	-0.09	0.39
Part-2	Magnetic declination	0.75930	-0.64	0.189
	Solar Azimuth Change	0.7316	0.1739	0.72
	Solar Zenith Change	0.42	0.288	0.560
	Lunar Azimuth Change	0.71	0.608	0.21
Part-3	Magnetic declination	0.759	-0.64	0.189
	Solar Azimuth Change	0.73	0.1739	0.72
	Solar Zenith Change	0.424	0.2887	0.5603
	Lunar Azimuth Change	0.712	0.6	0.213
Part-4	Magnetic declination	0.75930	1	0.157
	Solar Azimuth Change	0.732	-1	0.155
	Solar Zenith Change	0.424	-1	0.1577
	Lunar Azimuth Change	0.712	-1	0.15
Part-5	Magnetic declination	0.279	0.1492	0.425
	Solar Azimuth Change	0.5804732	0.2377	0.379
	Solar Zenith Change	0.255	0.175	0.5622
	Lunar Azimuth Change	0.425	-0.363	0.21
Part-6	Magnetic declination	0.225	-0.22	0.0903
	Solar Azimuth Change	0.219	-0.06	0.614
	Solar Zenith Change	0.17	-0.144	0.188
	Lunar Azimuth Change	0.229	-0.1558	0.202

Table S12. Based on our supervised partitions where all the parameters follow von Mises distribution, we perfomed the circular regression and the correlation test in the above table.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.1032	0.11	vonMises
Longitude	0.0966	0.164	vonMises
Latitude	0.0641	0.09	vonMises
Lunar azimuth difference	0.1079	0.11	vonMises
Solar Azimuth difference	0.0513	0.142	vonMises
Solar Zenith difference	0.1262	0.128	vonMises
Magnetic Declination	0.1546	0.164	vonMises

Table S13. Used Watson test on all the directional parameters of the bird species *Black Crowned Night Heron* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the first 11 observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0283	0.09	vonMises
Longitude	0.1091	0.128	vonMises
Latitude	0.0993	0.11	vonMises
Lunar azimuth difference	0.0626	0.158	vonMises
Solar Azimuth difference	0.0614	0.142	vonMises
Solar Zenith difference	0.602	0.11	vonMises
Magnetic Declination	0.1399	0.164	vonMises

Table S14. Used Watson test on all the directional parameters of the bird species *Black Crowned Night Heron* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 12 to 15th observations .

7. *Black Bellied Plover*

Draft

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0438	0.09	vonMises
Longitude	0.1383	0.128	Not vonMises
Latitude	0.1757	0.09	Not vonMises
Lunar azimuth difference	0.0324	0.09	vonMises
Solar Azimuth difference	0.0373	0.09	vonMises
Solar Zenith difference	0.039	0.11	vonMises
Magnetic Declination	0.1225	0.142	vonMises

Table S15. Used Watson test on all the directional parameters of the bird species *Black Crowned Night Heron* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 16 to 25th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0641	0.09	vonMises
Longitude	0.088	0.142	vonMises
Latitude	0.1044	0.164	vonMises
Lunar azimuth difference	0.0685	0.09	vonMises
Solar Azimuth difference	0.0327	0.09	vonMises
Solar Zenith difference	0.0272	0.09	vonMises
Magnetic Declination	0.1314	0.164	vonMises

Table S16. Used Watson test on all the directional parameters of the bird species *Black Crowned Night Heron* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 26 to 36th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.026	0.09	vonMises
Longitude	0.1356	0.11	Not vonMises
Latitude	0.2401	0.158	Not vonMises
Lunar azimuth difference	0.0312	0.081	vonMises
Solar Azimuth difference	0.0362	0.11	vonMises
Solar Zenith difference	0.0354	0.09	vonMises
Magnetic Declination	0.1323	0.142	vonMises

Table S17. Used Watson test on all the directional parameters of the bird species *Black Crowned Night Heron* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 37 to 45th observations .

K	BIC Values	K	BIC Values
1	-257.7290	6	-432.4833
2	-341.3141	7	-423.0890
3	-360.4782	8	-437.2051
4	-394.1849	9	-427.2747
5	-422.8772	10	-409.4395

Table S18. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 8. This will give us the weights(ϕ) or α values as 0.04347739, 0.32491631, 0.17270467, 0.19104898, 0.02173913, 0.04347739, 0.04465062, 0.15798551. This is done for the bird species *Black Crowned Night Heron*

8. *Pacific Loon*

Draft

Part	Parameters used for regression with θ	ρ values	Correlation value	P-value
Part-1	Magnetic declination	0.5049	-0.1469	0.584
	Solar Azimuth Change	0.64	0.2079	0.530
	Solar Zenith Change	0.4698	0.0307316	0.398
	Lunar Azimuth Change	0.4957	0.3895	0.235
Part-2	Magnetic declination	0.7717	-0.4823	0.2545
	Solar Azimuth Change	0.942	0.36150	0.4687
	Solar Zenith Change	0.782	-0.225	0.660
	Lunar Azimuth Change	0.887	-0.9266	0.04881
Part-3	Magnetic declination	0.565	-0.1823	0.557
	Solar Azimuth Change	0.498	-0.37044	0.322
	Solar Zenith Change	0.414	-0.2708	0.1525
	Lunar Azimuth Change	0.7255	0.0204	0.929
Part-4	Magnetic declination	0.4265	-0.42313	0.3256
	Solar Azimuth Change	0.6325	0.3679	0.2108
	Solar Zenith Change	0.5539	0.0173	0.9548
	Lunar Azimuth Change	0.5435	-0.5491	0.0731
Part-5	Magnetic declination	0.5927	-0.6819	0.1358
	Solar Azimuth Change	0.4927	0.1373	0.7261
	Solar Zenith Change	0.5194	0.1344	0.7234
	Lunar Azimuth Change	0.5369	0.2791	0.3608

Table S19. Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Black Crowned Night Heron*

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0507	0.081	vonMises
Longitude	0.0664	0.164	vonMises
Latitude	0.454	0.128	Not vonMises
Lunar azimuth difference	0.0722	0.081	vonMises
Solar Azimuth difference	0.1013	0.11	vonMises
Solar Zenith difference	0.1232	0.128	vonMises
Magnetic Declination	0.0414	0.164	vonMises

Table S20. Used Watson test on all the directional parameters of the bird species *Brown Pelican* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 1 to 25th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0189	0.09	vonMises
Longitude	0.0801	0.164	vonMises
Latitude	0.1333	0.164	vonMises
Lunar azimuth difference	0.0718	0.011	vonMises
Solar Azimuth difference	0.0486	0.142	vonMises
Solar Zenith difference	0.1218	0.128	vonMises
Magnetic Declination	0.0239	0.164	vonMises

Table S21. Used Watson test on all the directional parameters of the bird species *Brown Pelican* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 26 to 45th observations.

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0308	0.09	vonMises
Longitude	0.0517	0.164	vonMises
Latitude	0.6167	0.142	Not vonMises
Lunar azimuth difference	0.0794	0.09	vonMises
Solar Azimuth difference	0.1112	0.128	vonMises
Solar Zenith difference	0.0678	0.128	vonMises
Magnetic Declination	0.0906	0.164	vonMises

Table S22. Used Watson test on all the directional parameters of the bird species *Brown Pelican* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 46 to 71th observations .

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Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0229	0.11	vonMises
Longitude	0.0762	0.164	vonMises
Latitude	0.1213	0.164	vonMises
Lunar azimuth difference	0.0393	0.128	vonMises
Solar Azimuth difference	0.0834	0.128	vonMises
Solar Zenith difference	0.0561	0.158	vonMises
Magnetic Declination	0.0316	0.164	vonMises

Table S23. Used Watson test on all the directional parameters of the bird species *Brown Pelican* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 72 to 76th observations .

K	BIC Values	K	BIC Values
1	-776.002	6	-944.846
2	-932.995	7	-942.795
3	-942.365	8	-936.181
4	-940.017	9	-925.996
5	-935.657	10	-910.717

Table S24. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 6. This will give us the weights(ϕ) or α values as 0.06361403, 0.69665602, 0.07828312, 0.07740401, 0.04473266, 0.03931016 This is done for the bird species *Brown Pelican*

Part	Parameters used for regression with θ	ρ values	Correlation value	P-value
Part-1	Magnetic declination	0.2177	-0.1491	0.386
	Solar Azimuth Change	0.239	-0.3765	0.081
	Solar Zenith Change	0.3254	-0.0938	0.5455
	Lunar Azimuth Change	0.3254	0.4462	0.0184
Part-2	Magnetic declination	0.35	0.3062	0.1539
	Solar Azimuth Change	0.3737	-0.1933	0.3586
	Solar Zenith Change	0.4222	0.0019	0.9929
	Lunar Azimuth Change	0.486	0.2116	0.3319
Part-3	Magnetic declination	0.2209	0.0829	0.573
	Solar Azimuth Change	0.3532	0.1541	0.4331
	Solar Zenith Change	0.2806	0.2446	0.1783
	Lunar Azimuth Change	0.4088	-0.3682	0.1207
Part-4	Magnetic declination	0.4265	-0.42313	0.3256
	Solar Azimuth Change	0.6325	0.3679	0.2108
	Solar Zenith Change	0.5539	0.0173	0.9548
	Lunar Azimuth Change	0.5435	-0.5491	0.0731

Table S25. Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Brown Pelican*

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0364	0.09	vonMises
Longitude	0.0734	0.09	vonMises
Latitude	0.102	0.111	Not vonMises
Lunar azimuth difference	0.0785	0.09	vonMises
Solar Azimuth difference	0.0733	0.128	vonMises
Solar Zenith difference	0.0511	0.11	vonMises
Magnetic Declination	0.1181	0.128	vonMises

Table S26. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 99 to 109th observations .

9. *Long Bellied curlew*

10. Grid and Canonical Theory

A. Utilizing Grid-Based Methodology for Comprehensive Dependency Analysis. Our approach involved a meticulous partitioning of the dataset into discrete grids, yielding a transformative reduction from all initial data points to a mere 20 data points, each meticulously organized based on grid-wise delineations.

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Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0263	0.09	vonMises
Longitude	0.0474	0.164	vonMises
Latitude	0.645	0.158	Not vonMises
Lunar azimuth difference	0.0317	0.09	vonMises
Solar Azimuth difference	0.0609	0.11	vonMises
Solar Zenith difference	0.0247	0.158	vonMises
Magnetic Declination	0.0439	0.164	vonMises

Table S27. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 110 to 121th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0422	0.142	vonMises
Longitude	0.1143	0.164	vonMises
Latitude	0.0843	0.081	Not vonMises
Lunar azimuth difference	0.092	0.142	vonMises
Solar Azimuth difference	0.1381	0.158	vonMises
Solar Zenith difference	0.096	0.158	vonMises
Magnetic Declination	0.1582	0.164	vonMises

Table S28. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 1 to 7th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0434	0.11	vonMises
Longitude	0.055	0.164	vonMises
Latitude	0.2014	0.164	Not vonMises
Lunar azimuth difference	0.0552	0.081	vonMises
Solar Azimuth difference	0.1158	0.164	vonMises
Solar Zenith difference	0.1195	0.142	vonMises
Magnetic Declination	0.1115	0.142	vonMises

Table S29. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 8 to 17th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.1032	0.158	vonMises
Longitude	0.787	0.09	vonMises
Latitude	0.1133	0.128	Not vonMises
Lunar azimuth difference	0.0464	0.09	vonMises
Solar Azimuth difference	0.0187	0.09	vonMises
Solar Zenith difference	0.0712	0.09	vonMises
Magnetic Declination	0.097	0.128	vonMises

Table S30. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 18 to 22th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0756	0.09	vonMises
Longitude	0.1301	0.164	vonMises
Latitude	0.0902	0.128	vonMises
Lunar azimuth difference	0.1381	0.142	vonMises
Solar Azimuth difference	0.714	0.164	vonMises
Solar Zenith difference	0.0452	0.142	vonMises
Magnetic Declination	0.1069	0.164	vonMises

Table S31. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 50 to 56th observations .

Following this restructuring, we embarked on an exhaustive analysis of the dataset, employing the Circular Regression model.

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Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0245	0.09	vonMises
Longitude	0.381	0.11	Not vonMises
Latitude	0.3738	0.164	Not vonMises
Lunar azimuth difference	0.0249	0.09	vonMises
Solar Azimuth difference	0.0632	0.128	vonMises
Solar Zenith difference	0.1299	0.128	Not vonMises
Magnetic Declination	0.7383	0.164	Not vonMises

Table S32. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 23 to 35th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0272	0.09	vonMises
Longitude	0.0815	0.158	vonMises
Latitude	0.2396	0.09	Not vonMises
Lunar azimuth difference	0.1831	0.128	Not vonMises
Solar Azimuth difference	0.3026	0.142	Not vonMises
Solar Zenith difference	0.0739	0.158	vonMises
Magnetic Declination	0.2959	0.164	Not vonMises

Table S33. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 36 to 49th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0354	0.081	vonMises
Longitude	0.2541	0.09	Not vonMises
Latitude	0.1547	0.11	Not vonMises
Lunar azimuth difference	0.0829	0.09	vonMises
Solar Azimuth difference	0.0568	0.11	vonMises
Solar Zenith difference	0.0263	0.11	vonMises
Magnetic Declination	1.4186	0.142	Not vonMises

Table S34. Used Watson test on all the directional parameters of the bird species *Black Bellied Plover* where all of the mentioned parameters follow von Mises distribution for 0.01 level of significance. This table is done for the first supervised partition of the species where we took the 57 to 98th observations .

K	BIC Values	K	BIC Values
1	-731.4458	6	-969.6409
2	-850.7185	7	-985.7637
3	-844.6696	8	-1000.2251
4	-917.6586	9	-992.1987
5	-954.7474	10	-987.5260

Table S35. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 8. This will give us the weights(ϕ) or α values as 0.154732262, 0.148760330, 0.033057697, 0.051891329, 0.363624344, 0.156100218, 0.083569358, 0.008264462 This is done for the bird species *Black Bellied Plover*

Draft

This sophisticated statistical methodology empowered us to delve into the intricate dependencies and patterns ingrained within the directional data, adeptly accommodating the circular nature inherent in our measurements. Through the application of Circular Regression, we unearthed nuanced trends and associations that might have otherwise eluded detection. We took the median value for each grid for each species.

The adoption of grid-wise partitioning offers a myriad of advantages in the realm of data analysis:

1. **Streamlined Complexity:** Partitioning the dataset into discrete grids serves to streamline the analytical process by significantly reducing the complexity of the dataset. Rather than grappling with a multitude of individual data points, our focus narrows to aggregated data within each grid, facilitating enhanced manageability and interpretability.

2. **Enhanced Visualization:** Grid-based partitioning affords enhanced visualization capabilities, allowing for the elucidation of spatial patterns and trends with heightened clarity. By condensing data within each grid, we can craft visually compelling representations, such as heatmaps or contour plots, offering profound insights into the distribution and variation of data across the study area.

3. **Augmented Statistical Analysis:** The aggregation of data within grids fosters a more robust statistical analysis

Part	Parameters used for regression with θ	ρ values	Correlation value	P-value
Part-1	Magnetic declination	0.2508	-0.0913	0.7779
	Solar Azimuth Change	0.4116	0.1157	0.729
	Solar Zenith Change	0.4934	0.4534	0.165
	Lunar Azimuth Change	0.4929	-0.033	0.9219
Part-2	Magnetic declination	0.5277	-0.1346	0.6662
	Solar Azimuth Change	0.6118	-0.447	0.1915
	Solar Zenith Change	0.4321	0.0608	0.8149
	Lunar Azimuth Change	0.5254	-0.3265	0.2916
Part-3	Magnetic declination	0.8963	0.7869	0.208
	Solar Azimuth Change	0.8731	-0.763	0.2143
	Solar Zenith Change	0.8673	-0.8103	0.1926
	Lunar Azimuth Change	0.8517	0.04703	0.8941
Part-4	Magnetic declination	0.4866	0.5317	0.2875
	Solar Azimuth Change	0.5175	-0.2232	0.6243
	Solar Zenith Change	0.3875	-0.1928	0.6035
	Lunar Azimuth Change	0.6314	0.0439	0.8409
Part-5	Magnetic declination	0.87005	-0.0631	0.8447
	Solar Azimuth Change	0.8405	0.177	0.6429
	Solar Zenith Change	0.87422	0.4063	0.2912
	Lunar Azimuth Change	0.8918	-0.4963	0.2723
Part-6	Magnetic declination	0.87005	-0.0631	0.8447
	Solar Azimuth Change	0.8405	0.177	0.6429
	Solar Zenith Change	0.9397	0.729	0.0801
	Lunar Azimuth Change	0.977	-0.9741	0.042
Part-7	Magnetic declination	0.5524	-0.0447	0.5867
	Solar Azimuth Change	0.4667	0.2425	0.276
	Solar Zenith Change	0.5208	0.1198	0.3707
	Lunar Azimuth Change	0.4202	0.0704	0.8116
Part-8	Magnetic declination	0.404	-0.2999	0.3429
	Solar Azimuth Change	0.386	0.0703	0.7730
	Solar Zenith Change	0.4216	0.3132	0.3138
	Lunar Azimuth Change	0.456	0.085	0.762
Part-9	Magnetic declination	0.2351	0.18	0.2666
	Solar Azimuth Change	0.2015	0.2051	0.1664
	Solar Zenith Change	0.1676	0.01403	0.9294
	Lunar Azimuth Change	0.1936	0.0765	0.615

Table S36. Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Black Bellied Plover*

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.022	0.09	vonMises
Longitude	0.1401	0.128	Not vonMises
Latitude	0.0972	0.09	Not vonMises
Lunar azimuth difference	0.054	0.11	vonMises
Solar Azimuth difference	0.0281	0.09	vonMises
Solar Zenith difference	0.0291	0.09	vonMises
Magnetic Declination	0.1343	0.142	vonMises

Table S37. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 1 to 17th observations .

framework. This approach serves to mitigate the impact of spatial autocorrelation and spatial heterogeneity, thereby bolstering the reliability of statistical tests and ensuring that resultant findings are more reflective of the underlying data patterns.

4. **Enhanced Interpretability:** Grid-based partitioning facilitates enhanced interpretability by furnishing a structured framework for the analysis of spatial relationships. This enables researchers to hone in on specific regions of interest and systematically evaluate the variability of different factors across the study area, culminating in the extraction of more meaningful insights and actionable conclusions.

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Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0294	0.09	vonMises
Longitude	0.0925	0.09	Not vonMises
Latitude	0.0525	0.142	vonMises
Lunar azimuth difference	0.0335	0.09	vonMises
Solar Azimuth difference	0.0331	0.09	vonMises
Solar Zenith difference	0.0178	0.09	vonMises
Magnetic Declination	0.1209	0.128	vonMises

Table S38. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 18 to 36th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0361	0.09	vonMises
Longitude	0.1054	0.142	vonMises
Latitude	0.0311	0.11	vonMises
Lunar azimuth difference	0.0339	0.11	vonMises
Solar Azimuth difference	0.0229	0.09	vonMises
Solar Zenith difference	0.0684	0.128	vonMises
Magnetic Declination	0.1376	0.142	vonMises

Table S39. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 37 to 47th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0317	0.081	vonMises
Longitude	0.0701	0.081	vonMises
Latitude	0.0792	0.09	vonMises
Lunar azimuth difference	0.0367	0.11	vonMises
Solar Azimuth difference	0.029	0.11	vonMises
Solar Zenith difference	0.0196	0.09	vonMises
Magnetic Declination	0.098	0.11	vonMises

Table S40. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 48 to 78th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0365	0.09	vonMises
Longitude	0.1193	0.142	vonMises
Latitude	0.2487	0.128	Not vonMises
Lunar azimuth difference	0.0189	0.11	vonMises
Solar Azimuth difference	0.0746	0.11	vonMises
Solar Zenith difference	0.0349	0.09	vonMises
Magnetic Declination	0.1247	0.128	vonMises

Table S41. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 79 to 89th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.04	0.09	vonMises
Longitude	0.0904	0.11	vonMises
Latitude	0.2113	0.11	Not vonMises
Lunar azimuth difference	0.033	0.081	vonMises
Solar Azimuth difference	0.0537	0.11	vonMises
Solar Zenith difference	0.0531	0.09	vonMises
Magnetic Declination	0.1567	0.158	vonMises

Table S42. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 90 to 115th observations .

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Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0244	0.09	vonMises
Longitude	0.1483	0.11	Not vonMises
Latitude	0.119	0.09	Not vonMises
Lunar azimuth difference	0.0595	0.09	vonMises
Solar Azimuth difference	0.0309	0.09	vonMises
Solar Zenith difference	0.0383	0.09	vonMises
Magnetic Declination	0.1304	0.142	Not vonMises

Table S43. Used Watson test on all the directional parameters of the bird species *Pacific Loon* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 116 to 161th observations .

K	BIC Values	K	BIC Values
1	-147.4662	6	-1076.1542
2	-937.2215	7	-1075.0842
3	-1030.6938	8	-1080.8440
4	-1064.73	9	-1070.9659
5	-1091.3939	10	-1055.7243

Table S44. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 5. This will give us the weights(ϕ) or α values as 0.4694, 0.19875, 0.15090, 0.12301, 0.0579. This is done for the bird species *Pacific Loon*

Part	Parameters used for regression with θ	ρ values	Correlation value	P-value
Part-1	Magnetic declination	0.3758	0.07835	0.7497
	Solar Azimuth Change	0.463	-0.3257	0.1574
	Solar Zenith Change	0.3774	0.01732	0.938
	Lunar Azimuth Change	0.4273	0.0978	0.6588
Part-2	Magnetic declination	0.3428	0.24	0.314
	Solar Azimuth Change	0.3751	0.1859	0.4444
	Solar Zenith Change	0.4474	0.0064	0.9759
	Lunar Azimuth Change	0.3451	0.2036	0.4005
Part-3	Magnetic declination	0.5883	-0.4531	0.1568
	Solar Azimuth Change	0.5705	0.1037	0.730061
	Solar Zenith Change	0.6476	-0.1761	0.6033
	Lunar Azimuth Change	0.4775	-0.1854	0.5615
Part-4	Magnetic declination	0.3431	0.0568	0.7563
	Solar Azimuth Change	0.1098	0.0567	0.7305
	Solar Zenith Change	0.2118	0.1411	0.4714
	Lunar Azimuth Change	0.2009	-0.1758	0.321
Part-5	Magnetic declination	0.4967	0.5196	0.1307
	Solar Azimuth Change	0.5634	-0.4682	0.17411
	Solar Zenith Change	0.3599	0.0222	0.9464
	Lunar Azimuth Change	0.4823	-0.1969	0.4013
Part-6	Magnetic declination	0.4507	-0.1207	0.5928
	Solar Azimuth Change	0.3025	-0.09322	0.6415
	Solar Zenith Change	0.3523	0.09644	0.644
	Lunar Azimuth Change	0.4405	-0.2826	0.194
Part-7	Magnetic declination	0.1967	0.02418	0.8721
	Solar Azimuth Change	0.2524	0.174313	0.2494
	Solar Zenith Change	0.2225	0.1149	0.4529
	Lunar Azimuth Change	0.3457	0.1183	0.4298

Table S45. Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular regression and the correlation test in the above table for *Pacific loon*

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B. Canonical Correlation. Canonical correlation analysis (CCA) is a statistical method that explores the linear relationship between two sets of variables. In our analysis X vector representing variables in the first set which are the Magnetic, Solar and Lunar changes, and Y be a vector representing variables in the second set which is the change of bird position.

The objective of CCA is to find linear combinations of X and Y, denoted as $U = Xa$ and $V = Yb$, such that the correlation between U and V is maximized.

Mathematically, this can be expressed as:

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.033	0.081	vonMises
Longitude	1.352	0.142	Not vonMises
Latitude	1.008	0.11	Not vonMises
Lunar azimuth difference	0.0383	0.09	vonMises
Solar Azimuth difference	0.1481	0.11	Not vonMises
Solar Zenith difference	0.0534	0.09	vonMises
Magnetic Declination	1.7933	0.164	Not vonMises

Table S46. Used Watson test on all the directional parameters of the bird species *Long Bellied curlew* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 1 to 50th observations .

Parameter	Test Statistics	Critical Value	Distribution
θ(Bird directional change)	0.0194	0.081	vonMises
Longitude	0.3383	0.09	Not vonMises
Latitude	0.3524	0.09	Not vonMises
Lunar azimuth difference	0.0673	0.081	vonMises
Solar Azimuth difference	0.0388	0.11	vonMises
Solar Zenith difference	0.0723	0.11	vonMises
Magnetic Declination	2.2371	0.142	Not vonMises

Table S47. Used Watson test on all the directional parameters of the bird species *Long Bellied curlew* where all of the mentioned parameters follow von Mises distribution for 0.01 significance level. This table is done for the first supervised partition of the species where we took the 1 to 50th observations .

K	BIC Values	K	BIC Values
1	-829.2649	11	-1267.5782
2	-1054.1596	12	-1296.8095
3	-1189.9874	13	-1291.6324
4	-1194.7899	14	-1265.8217
5	-1223.4830	15	-1276.4488
6	-1233.1482	16	-1229.4502
7	-1243.2841	17	-1276.0840
8	-1238.6233	18	-1278.2504
9	-1266.8802	19	-1256.7196
10	-1268.8967	20	-1184.2690

Table S48. Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 8. This will give us the weights(ϕ) or α values as -829.2649, -1054.1596, -1189.9874, -1194.7899, -1223.4830, -1233.1482, -1243.2841, -1238.6233, -1266.8802, -1268.8967, -1267.5782 , -1296.8095, -1291.6324, -1265.8217, -1276.4488, -1229.4502, -1276.0840, -1278.2504, -1256.7196, -1184.2690 This is done for the bird species *Long Bellied Curlew*

Part	Parameters used for regression with θ	ρ values	Correlation value	P-value
Part-1	Magnetic declination	0.2308	-0.07	0.4659
	Solar Azimuth Change	0.1808	0.003971	0.9774
	Solar Zenith Change	0.1674	0.00088	0.994
	Lunar Azimuth Change	0.24776	0.0376	0.79
Part-2	Magnetic declination	0.1744	0.0461	0.4699
	Solar Azimuth Change	0.189	0.0349	0.7418
	Solar Zenith Change	0.2015	-0.17102	0.1162
	Lunar Azimuth Change	0.1619	0.1504	0.1952

Table S49. Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Long Bellied curlew*

$$\text{maximize } \rho(U,V) = \text{cor}(U,V) = \frac{\text{cov}(U,V)}{\sqrt{\text{var}(U)\cdot\text{var}(V)}}$$

where ρ denotes the canonical correlation coefficient, cov denotes covariance, and var denotes variance.

The canonical correlation coefficients $(r_1, r_2, \ldots, r_k)$ are obtained as the square roots of the canonical eigenvalues of the matrix $\text{cov}(U,V)$ (16).

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CCA aims to find k pairs of canonical variates (U_1, V_1), (U_2, V_2), ..., (U_k, V_k) that maximize the correlation, where k is the minimum of p and q, the dimensions of X and Y (17).

The adoption of grid-based partitioning emerges as a pivotal technique in data analysis, particularly for spatial datasets. By simplifying the intricate complexities inherent in spatial data, this methodology serves as a catalyst for more effective exploration and interpretation of spatial patterns and relationships, thereby advancing the frontiers of knowledge and understanding.

11. Black Bellied Plovers

In this table S50 we can clearly see that the correlation between Solar positional change and the *Black Bellied Plovers* directional change is the highest with the ρ value of 0.44 and the correlation test suggest inverse relation between majority of the parameters(Magnetic, Solar and Lunar positional changes). So clearly we can see a clear evidence that the physical parameters influence *Black Bellied Plovers* directional change.

Parameter	ρ Value	r Value
Solar Azimuth Change	0.3507626	-0.2236839
Magnetic declination	0.3212564	-0.2383581
Magnetic Inclination	0.322164	0.2546108
Solar Zenith Change	0.4441466	-0.2265752
Lunar Azimuth Change	0.2994615	-0.2736092

Table S50. $gridwise_{bbpl}$

12. Black Crowned Night Heron

In this table S51 we can clearly see that the correlation between Lunar positional change and the *Black Crowned Night Heron* directional change is the highest with the ρ value of 0.60 and the correlation test suggest inverse relation between majority of the parameters(Magnetic, Solar and Lunar positional changes). So clearly we can see a clear evidence that the physical parameters influence *Black Crowned Night Heron* directional change. The magnetic field is also a important factor with ρ value of 0.53.

Parameter	ρ Value	r Value
Solar Azimuth Change	0.4256448	0.1947791
Magnetic declination	0.5393819	-0.4151493
Magnetic Inclination	0.5026821	-0.1813082
Solar Zenith Change	0.2419564	0.07140608
Lunar Azimuth Change	0.6092881	-0.6615668

Table S51. $gridwise_{bcnh}$

13. Pacific Loon

In this table S52 we can clearly see that the correlation between Geomagnetic change and the *Pacific Loon* directional change is the highest with the ρ value of 0.39 and the correlation test suggest inverse relation between majority of the parameters(Magnetic, Solar and Lunar positional changes). So clearly we can see a clear evidence that the physical parameters influence *Pacific Loon* directional change. The Solar positional Change is also a important factor with ρ value of 0.37.

Parameter	ρ Value	r Value
Solar Azimuth Change	0.3775842	0.3568713
Magnetic declination	0.3326586	-0.0634063
Magnetic Inclination	0.3952472	0.1996955
Solar Zenith Change	0.2518959	-0.05576555
Lunar Azimuth Change	0.3060159	-0.2035507

Table S52. $gridwise_{pl}$

14. Swainson's Hawk

In this table S53 we can clearly see that the correlation between Geomagnetic change and the *Swainson's Hawk* directional change is the highest with the ρ value of 0.26 and the correlation test suggest inverse relation between the parameters(Magnetic declination, and Lunar positional changes). So clearly we can see a clear evidence that the physical parameters influence *Swainson's Hawk* directional change. The Solar positional Change is also a important factor with ρ value of 0.232.

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Parameter	ρ Value	r Value
Solar Azimuth Change	0.2329224	0.2667975
Magnetic declination	0.2604017	-0.2532493
Magnetic Inclination	0.1840154	0.1014968
Solar Zenith Change	0.2209601	0.01040154
Lunar Azimuth Change	0.2159096	-0.1199818

Table S53. $gridwise_{sh}$

15. Brown Pelican

In this table S54 we can clearly see that the correlation between Geomagnetic change and the *Brown Pelican* directional change is the highest with the ρ value of 0.35 and the correlation test suggest inverse relation between the parameters(Solar and Lunar positional changes). So clearly we can see a clear evidence that the physical parameters influence *Brown Pelican* directional change. The Lunar positional Change is also a important factor with ρ value of 0.31.

Parameter	ρ Value	r Value
Solar Azimuth Change	0.226116	0.01833741
Magnetic declination	0.2265675	0.1731488
Magnetic Inclination	0.3541475	0.09170218
Solar Zenith Change	0.2476157	-0.1655759
Lunar Azimuth Change	0.3190035	-0.3316322

Table S54. $gridwise_{bp}$

16. Long Bellied Curlew

In this table S55 we can clearly see that the correlation between Lunar position change and the *Long Bellied Curlew* directional change is the highest with the ρ value of 0.61 and the correlation test suggest inverse relation between the parameters(Magnetic Field change and Lunar positional changes). So clearly we can see a clear evidence that the physical parameters influence *Long Bellied Curlew* directional change. The Geomagnetic Change is also a important factor with ρ value of 0.53.

Parameter	ρ Value	r Value
Solar Zenith Change	0.2419564	0.07140608
Magnetic declination	0.5393819	-0.4151493
Magnetic Inclination	0.5026821	-0.1813082
Solar Azimuth Change	0.4256448	0.1947791
Lunar Azimuth Change	0.6092881	-0.6615668

Table S55. $gridwise_{lbq}$

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