Calculations

for
Wage Dispersion, Over-Qualification, and Reder Competition
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economics - The Open-Access, Open-Assessment E-Journal, 2007-13.
http://www.economics-ejournal.org/economics/journalarticles/2007-13

■ Paper

■ Clear all symbols

```
Quit[];
```

■ Definitions (1) - (3)

```
In[1]:= profit = p α - w - c;
        ρ = q (1 + μ Log[w / W]);
        α = ρ x + (1 - ρ) y;
```

■ Zero profit price (6)

```
In[4]:= W = w;
        Solve[{profit == 0}, p]
```

$$\text{Out[5]}= \left\{\left\{p \to \frac{c + w}{q\,x + y - q\,y}\right\}\right\}$$

■ Clear the assignment W=w

```
In[11]:= Clear[W]
```

■ Check equation (6)

```
In[7]:= Simplify[(c + W) / (q x + y - q y) == (W + c) / (q (x - y) + y)]
Out[7]= True
```

■ Selection wage curve (7) - (9)

```
In[12]:= Dprofit = D[profit, w];
         D2profit = D[profit, {w, 2}];
         W = w;
```

```
In[15]:= Simplify[Dprofit == p q μ (x - y) 1/w - 1]
```

```
Out[15]= True
```

```
In[16]:= Simplify[D2profit == -p q μ (x - y) 1/w^2]
```

```
Out[16]= True
```

```
In[17]:= Solve[Dprofit == 0, p]
```

$$\text{Out[17]= } \left\{\left\{p \to \frac{w}{q\,(x-y)\,\mu}\right\}\right\}$$

■ Equilibrium (10), (11)

```
In[18]:= Solve[{profit == 0, Dprofit == 0}, {w, p}]
```

$$\text{Out[18]= } \left\{\left\{w \to -\frac{c\,(q\,x\,\mu - q\,y\,\mu)}{-q\,x - y + q\,y + q\,x\,\mu - q\,y\,\mu},\ p \to -\frac{c}{-q\,x - y + q\,y + q\,x\,\mu - q\,y\,\mu}\right\}\right\}$$

```
In[19]:= Simplify[-c (q x μ - q y μ)/(-q x - y + q y + q x μ - q y μ) == μ q (x - y)/(q (x - y) (1 - μ) + y) c]
```

```
Out[19]= True
```

```
In[24]:= Simplify[-c/(-q x - y + q y + q x μ - q y μ) == c/(q (x - y) (1 - μ) + y)]
```

```
Out[24]= True
```

■ Stability (12)

```
In[30]:= Clear[W];
```

```
In[35]:= Solve[p q μ (x - y) 1/w - 1 == 0, w]
```

```
Out[35]= {{w → p q x μ - p q y μ}}
```

```
In[48]:= p = (W + c)/(q (x - y) + y);
```

```
In[43]:= w = p q x μ - p q y μ
```

$$\text{Out[43]= } \frac{q\,(c+W)\,x\,\mu}{q\,(x-y)+y} - \frac{q\,(c+W)\,y\,\mu}{q\,(x-y)+y}$$

```
In[44]:= Simplify[(q (c + W) x μ)/(q (x - y) + y) - (q (c + W) y μ)/(q (x - y) + y) == μ (x - y) q/((x - y) q + y) (W + c)]
```

```
Out[44]= True
```

$$\bar{W} = \frac{\mu\, q\, (x - y)}{q\, (x - y)\, (1 - \mu) + y}\, c;$$

In[46]:=
$$\text{Simplify}\left[(w - W) == -\frac{(1 - \mu)\, (x - y)\, q + y}{(x - y)\, q + y}\, (W - \bar{W})\right]$$

Out[46]= True

■ Inequality (13)

```
Clear[W, p];
```

Condition is given wrongly in the manuscript. It should relate to mediocre productivity y rather than prolific productivity x and read

$$p\, y > W$$

and inequalities (13) should read

$$y > \frac{q\, x\, \mu}{1 + q\, \mu} \text{ and } \mu\, q < \frac{y}{x - y}$$

Replacing the inequality with equality, I check whether these inequalities are equivalent to p y > W.

In[65]:=
$$\bar{p} = \frac{c}{y + q\, (x - y)\, (1 - \mu)};$$

In[67]:=
$$\text{Solve}[p\, y == \bar{W}\, ,\, y]$$

Out[67]=
$$\left\{\left\{y \to \frac{q\, x\, \mu}{1 + q\, \mu}\right\}\right\}$$

In[68]:=
$$\text{Solve}[p\, y == \bar{W}\, ,\, q]$$

Out[68]=
$$\left\{\left\{q \to -\frac{y}{(-x + y)\, \mu}\right\}\right\}$$

■ Quality mix (14) and (15)

In[69]:=
$$D[\bar{W}, q]$$

Out[69]=
$$\frac{c\, (x - y)\, \mu}{y + q\, (x - y)\, (1 - \mu)} - \frac{c\, q\, (x - y)^2\, (1 - \mu)\, \mu}{(y + q\, (x - y)\, (1 - \mu))^2}$$

In[71]:=
$$\text{Simplify}\left[\left(\frac{c\, (x - y)\, \mu}{y + q\, (x - y)\, (1 - \mu)} - \frac{c\, q\, (x - y)^2\, (1 - \mu)\, \mu}{(y + q\, (x - y)\, (1 - \mu))^2}\right) == \left(\frac{1}{q\, (x - y)\, (1 - \mu) + y}\right)^2 c\, (x - y)\, y\, \mu\right]$$

Out[71]= True

In[72]:=
$$D[p, q]$$

Out[72]=
$$-\frac{c\, (x - y)\, (1 - \mu)}{(y + q\, (x - y)\, (1 - \mu))^2}$$

```
In[75]:= Simplify[-(c (x - y) (1 - μ))/(y + q (x - y) (1 - μ))^2 == -(1/(q (x - y) (1 - μ) + y))^2 c (x - y) (1 - μ)]
```

```
Out[75]= True
```

```
In[50]:= Simplify[(c q (x - y) (q (x - y) + y) μ)/((c + W) (y + q (x - y) (1 - μ)))]
```

```
Out[50]= (c q (x - y) (q (x - y) + y) μ)/((c + W) (y - q (x - y) (-1 + μ)))
```

■ Appendix: Performance Pay

```
In[86]:= Clear[p, W, w];
```

■ The profit function (17) is

```
In[87]:= prof = p (ρ x + (1 - ρ) y) - ρ w - (1 - ρ) y/x w - c;
         ρ = q (1 + μ Log[w/W]);
```

■ Its derivative with respect to w is

```
In[89]:= Dprof = D[prof, w];
         W = w;
```

```
In[91]:= solution = Solve[{Dprof == 0, prof == 0}, {w, p}]
```

```
Out[91]= {{w → -(-c q x^2 μ + c q x y μ)/(q x + y - q y)^2, p → -(-c q x - c y + c q y - c q x μ + c q y μ)/(q x + y - q y)^2}}
```

■ Check that expression (18) is correct

```
In[92]:= Simplify[-(-c q x^2 μ + c q x y μ)/(q x + y - q y)^2 == c q x μ (x - y)/(q x + (1 - q) y)^2]
```

```
Out[92]= True
```

■ Compare flat pay with performance pay

```
Clear[W]
```

```
In[98]:= W̃ = c q x μ (x - y)/(q x + (1 - q) y)^2;
         Ṽ = y/x W̃;
         W̄ = (μ q (x - y))/(q (x - y) (1 - μ) + y) c;
```

In[103]:=

```
Simplify[(W̃ - (q W̃ + (1 - q) Ṽ)) == c q^2 (x - y)^2 μ^2 / ((q x + (1 - q) y) (q (x - y) (1 - μ) + y))]
```

Out[103]=

True

■ Wage Compression

In[142]:=

```
Clear[p, W, V];
```

■ The Lagrangean is

In[143]:=

```
L = q (1 + μ Log[w/W]) (p x - w) +
    (1 - q) (1 + μ Log[v/V]) (p y - v) - c - λ (q Log[w/W] + (1 - q) Log[v/V]);
```

■ The derivatives at w=W are

In[144]:=

```
DLw = D[L, w];
DLv = D[L, v];
```

In[146]:=

```
W = w;
Simplify[DLw == q/w ((p x - w) μ - λ - w)]
```

Out[147]=

True

In[148]:=

```
V = v;
Simplify[DLv == (1 - q)/v ((p y - v) μ - λ - v)]
```

Out[149]=

True

In[150]:=

```
Solve[{DLw == 0, DLv == 0}, {v, w}]
```

Out[150]=

$$\left\{\left\{w \to \frac{-\lambda + p\,x\,\mu}{1+\mu},\ v \to \frac{-\lambda + p\,y\,\mu}{1+\mu}\right\}\right\}$$

In[151]:=

```
Simplify[(-λ + p x μ)/(1 + μ) - (-λ + p y μ)/(1 + μ)]
```

Out[151]=

$$\frac{p\,(x-y)\,\mu}{1+\mu}$$