Calculations for
"Selection Wages and Discrimination"
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Mathematica 6 Notebook

Definition of Auxiliary Function ϕ and its Derivatives

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ϕ[w_, W_, s_, n_] := Integrate[f[θ, w, W], {θ, s, θ1}] - n
ϕ[w, W, s, n]
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$$-n+\int_{s}^{\theta_1} f[\theta, w, W]\, d\theta$$

```
ϕw[w_, W_, s_, n_] := D[ϕ[w, W, s, n], w]
ϕw[w, W, s, n]
```

$$\int_{s}^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta$$

```
ϕW[w_, W_, s_, n_] := D[ϕ[w, W, s, n], W]
ϕW[w, W, s, n]
```

$$\int_{s}^{\theta_1} f^{(0,0,1)}[\theta, w, W]\, d\theta$$

```
ϕs[w_, W_, s_, n_] := D[ϕ[w, W, s, n], s]
ϕs[w, W, s, n]
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$$-f[s, w, W]$$

```
ϕn[w_, W_, s_, n_] := D[ϕ[w, W, s, n], n]
ϕn[w, W, s, n]
```

$$-1$$

Derivatives (5)

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sw = -ϕw[w, W, s, n] / ϕs[w, W, s, n]
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$$\frac{\int_{s}^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta}{f[s, w, W]}$$

$$s_W = -\frac{\phi_W[w, W, s, n]}{\phi_s[w, W, s, n]}$$

$$\frac{\int_s^{\theta_1} f^{(0,0,1)}[\theta, w, W]\, d\theta}{f[s, w, W]}$$

$$s_n = -\frac{\phi_n[w, W, s, n]}{\phi_s[w, W, s, n]}$$

$$-\frac{1}{f[s, w, W]}$$

Derivatives of Auxiliary Function A (Average Productivity) and Derivatives

```
A[s_, w_, W_] := Integrate[θ f[θ, w, W], {θ, s, θ1}] / Integrate[f[θ, w, W], {θ, s, θ1}]
A[s, w, W]
```

$$\frac{\int_s^{\theta_1} \theta\, f[\theta, w, W]\, d\theta}{\int_s^{\theta_1} f[\theta, w, W]\, d\theta}$$

```
As[s_, w_, W_] := D[A[s, w, W], s]
As[s, w, W]
```

$$-\frac{s\, f[s, w, W]}{\int_s^{\theta_1} f[\theta, w, W]\, d\theta} + \frac{f[s, w, W] \int_s^{\theta_1} \theta\, f[\theta, w, W]\, d\theta}{\left(\int_s^{\theta_1} f[\theta, w, W]\, d\theta\right)^2}$$

```
Aw[s_, w_, W_] := D[A[s, w, W], w]
Aw[s, w, W]
```

$$-\frac{\left(\int_s^{\theta_1} \theta\, f[\theta, w, W]\, d\theta\right) \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta}{\left(\int_s^{\theta_1} f[\theta, w, W]\, d\theta\right)^2} + \frac{\int_s^{\theta_1} \theta\, f^{(0,1,0)}[\theta, w, W]\, d\theta}{\int_s^{\theta_1} f[\theta, w, W]\, d\theta}$$

```
AW[s_, w_, W_] := D[A[s, w, W], W]
AW[s, w, W]
```

$$-\frac{\left(\int_s^{\theta_1} \theta\, f[\theta, w, W]\, d\theta\right) \int_s^{\theta_1} f^{(0,0,1)}[\theta, w, W]\, d\theta}{\left(\int_s^{\theta_1} f[\theta, w, W]\, d\theta\right)^2} + \frac{\int_s^{\theta_1} \theta\, f^{(0,0,1)}[\theta, w, W]\, d\theta}{\int_s^{\theta_1} f[\theta, w, W]\, d\theta}$$

Derivatives (7)

```
aw = Simplify[Aw[s, w, W] + As[s, w, W] sw /. ∫_s^θ1 f[θ, w, W] dθ → n]
```

$$\frac{-s \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,1,0)}[\theta, w, W]\, d\theta}{n}$$

$$a_w = \text{Simplify}\left[A_w[s, w, W] + A_s[s, w, W]\, s_w \;/.\; \int_s^{\theta_1} f[\theta, w, W]\, d\theta \to n\right]$$

$$\frac{-s \int_s^{\theta_1} f^{(0,0,1)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,0,1)}[\theta, w, W]\, d\theta}{n}$$

$$a_n = \text{Simplify}[A_s[s, w, W]\, s_n]$$

$$\frac{s \int_s^{\theta_1} f[\theta, w, W]\, d\theta - \int_s^{\theta_1} \theta\, f[\theta, w, W]\, d\theta}{\left(\int_s^{\theta_1} f[\theta, w, W]\, d\theta\right)^2}$$

$$a_{ww} = D\left[\frac{-s \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,1,0)}[\theta, w, W]\, d\theta}{n}, w\right] +$$
$$D\left[\frac{-s \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,1,0)}[\theta, w, W]\, d\theta}{n}, s\right] s_w$$
$$-\frac{\left(\int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta\right)^2}{n\, f[s, w, W]} + \frac{-s \int_s^{\theta_1} f^{(0,2,0)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,2,0)}[\theta, w, W]\, d\theta}{n}$$

$$a_{wW} = D\left[\frac{-s \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,1,0)}[\theta, w, W]\, d\theta}{n}, W\right] +$$
$$D\left[\frac{-s \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,1,0)}[\theta, w, W]\, d\theta}{n}, s\right] s_W$$

$$-\frac{\left(\int_s^{\theta_1} f^{(0,0,1)}[\theta, w, W]\, d\theta\right) \int_s^{\theta_1} f^{(0,1,0)}[\theta, w, W]\, d\theta}{n\, f[s, w, W]} +$$
$$\frac{-s \int_s^{\theta_1} f^{(0,1,1)}[\theta, w, W]\, d\theta + \int_s^{\theta_1} \theta\, f^{(0,1,1)}[\theta, w, W]\, d\theta}{n}$$

Elasticity of Productivity (10)

$$\eta = -a_n \frac{n}{a}$$

$$-\frac{-a + s}{a}$$

$$\text{Simplify}[\eta == 1 - s / a]$$

True

Lagrangian (13) and Derivatives (14), (15)

$$L = p\left(\frac{n_f}{n} a_f[w_f, W_f, n_f] + \frac{n_m}{n} a_m[w_m, W_m, n_m]\right) - \left(\frac{n_f}{n} w_f + \frac{n_m}{n} w_m\right) + \lambda\,(n - n_f - n_m)$$

$$\lambda\,(n - n_f - n_m) - \frac{n_f\, w_f}{n} - \frac{n_m\, w_m}{n} + p\left(\frac{n_f\, a_f[w_f, W_f, n_f]}{n} + \frac{n_m\, a_m[w_m, W_m, n_m]}{n}\right)$$

D[L, n_f]

$$-\lambda - \frac{w_f}{n} + p\left(\frac{a_f[w_f, W_f, n_f]}{n} + \frac{n_f\, a_f^{(0,0,1)}[w_f, W_f, n_f]}{n}\right)$$

D[L, n_m]

$$-\lambda - \frac{w_m}{n} + p\left(\frac{a_m[w_m, W_m, n_m]}{n} + \frac{n_m\, a_m^{(0,0,1)}[w_m, W_m, n_m]}{n}\right)$$

D[L, w_f]

$$-\frac{n_f}{n} + \frac{p\, n_f\, a_f^{(1,0,0)}[w_f, W_f, n_f]}{n}$$

D[L, w_m]

$$-\frac{n_m}{n} + \frac{p\, n_m\, a_m^{(1,0,0)}[w_m, W_m, n_m]}{n}$$

D[L, λ]

$$n - n_f - n_m$$