Introduction to Basis Instruments Contracts (BICs) for Mathematics, Finance, and Economics

Grounding the world of finance, economics, probabilities and risk management on more solid foundations

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Abstract

This article provides a definition for the concept of Basis Instrument Contracts (BICs) and explains why such a concept is needed and useful in Finance, Economics, Mathematics and risk management in Decision sciences in general. It explains how BICs are practical and represent both a prophylactic and a therapeutic structural tool for a crisis such as the Sept 2008 financial crisis. BICs help mitigate market volatility and facilitate more robust risk management.

It is complemented by peer-reviewed Wolfram Demonstration that uses BICs in the framework of the baseball 2001 and 2009 World Series and includes illustrating free software code.

This material is intended for an educated audience with basic awareness of financial economics.
Familiarity with the concept of Arrow Debreu Security(ADS) would facilitate speedy reading. However tutorial material on ADS is provided in references and links

"But if thought corrupts language,

language can also corrupt thought."

George Orwell

Laws Of Nature – 3 – BICs

Introduction: Refining & Extending

Humans have achieved structural progress throughout history by *extending* or *refining* their understanding of the various concepts they must handle. For example, with numbers, our understanding extended from only positive numbers; then necessity led to the introduction of negative numbers, then there were fractions, rational numbers, real numbers, complex numbers[1] and more. Likewise, our understanding of matter refined over time from the macroscopic to the microscopic molecular, atomic, nuclear and so on to ever smaller particles; this lead to the capacity to unleash previously unthinkable power or manufacture in economically viable ways various types of material components. Generally, each successful stage of these extensions or refinements led to greater power in handling more complex issues in ever more efficient ways.

These processes of refinement and extension are often dual and related approaches to address the same needs. That duality can be expressed as a *refinement* of the physical or material object of study which in turn lead to an *extension* of the analytical or mathematical framework needed to handle the physical refinements.

In modern finance, for a variety of reasons that will be discussed at length, the growth in the complexity, scale and sophistication of contracts generally known as derivatives contracts leads to a need for refinement of contracts into more granular and more manageable units. This in turn leads to an extension of the traditional

mathematical framework within which derivatives risk management is handled.

Successful extensions and refinements need to integrate a set of requirements rich and important enough to make the undertaking worthwhile. At the same time, to remain practical the refinements and extensions must not go beyond what is deemed necessary: "*entia non sunt multiplicanda praeter necessitatem*"[2]

BICs were developed starting in 2001[3] by the author to enable better risk management. It is the topic of the book BICs 4 Derivatives Volume I: Theory[4]. One of the key requirements was to help prevent or mitigate a crisis such as the economic and financial shock of 2008.

Indeed, the 2008 crisis originated in the failure of risk managers to accurately price and hedge the complex mortgage backed derivatives securities in their portfolio. BICs' current relevance is therefore even more acute.

Why BICs: Requirements

Fully appreciating the need for BICs, the mechanisms of operation, the value of the structures as proposed requires first clarifying what a derivative contract actually is.

Definition of Derivatives

According to the the US Commodity Futures Trading Commission (CFTC)'s glossary definition, a Derivative is a financial instrument, traded on or off an exchange, the price of which is directly dependent upon (i.e., "derived from") the value of one or more underlying securities, equity indices, debt instruments, commodities, other derivative instruments, or any agreed upon pricing index or arrangement (e.g., the movement over time of the Consumer Price Index or freight rates). Derivatives involve the trading of rights or obligations based on the underlying product, but do not directly transfer property. They are used to hedge risk or to exchange a floating rate of return for fixed rate of return. Derivatives include futures, options, and swaps. For example, futures contracts are derivatives of the physical contract and options on futures are derivatives of futures contracts.

Such a definition is intelligible enough, yet translating it into precise terms that can be unambiguously decided upon may not be trivial. The struggles of the Financial Accounting Standard Board in FAS 133 & FAS 138[5] in coming up with an operational definition that retains basic intelligibility are instructive.

For the purposes of our investigation in this note, it is enough to identify a derivatives contract by its payout which must be a function f of the realized values of underlying(s) S from a contract date t_0 to a maturity date t_n . As such it may simply be noted as $f(S_0,\ldots,S_n)$[6].

For example a call option which grants the right but not the obligation to buy at a maturity time t_n an underlying S at a strike price K will be denoted as the greater of S_n-K and zero or $Max(S_n\text{-}K,0)$ or simply $(S_n\text{-}K)^+$.

BICs must be designed to provide immunity against model misspecification

Traditional derivatives hedging strategies are based upon frequently or dynamically adjusting the positions held in hedging instruments or underlyings to offset or minimize the effect of incremental changes in the value of the contract to be hedged. For non trivial derivatives, such practices depend on models that make assumptions of stochastic processes governing the movements in the value of the hedging instrument., leading to parametric formula for the price of derivatives contracts and "Greeks" based dynamic hedging. Very often the actual movements of underlyings are inconsistent with the assumed stochastic processes predictions. The consequences of such modeling mismatches can range from mild in normal times to severe in times of market stress. BICs are designed to immunize against this type of problems.

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- An important category of such model mis-specification is the assumption of infinite liquidity of trades on the underlying. In times of market stress, volatility increases, spreads on instruments' price quotes widen or liquidity altogether dries up. Furthermore regulatory or political intervention further dramatically changes the nature of what can be done; these often take the forms of trading interruptions, bans on (naked) short selling for equities as was the case during the 2008 crisis in the US; or currency trading restrictions, change of currency regime for foreign exchange as was the case during the 1997 Asian Crisis. As a result, dynamic hedging and various program trading strategies can no longer be executed. BICs must be designed such that such events do not alter the execution of hedging strategies based on their prescription.
- At the core of mathematical modeling failure in times of stress is the built-in continuous space, continuous time orthodoxy that is the standard assumption in stochastic process description. BICs must be designed in a discrete time and discrete space representation representative of reality that may be extended to the limit continuous time/continuous case only as a computational method where they provide for quicker approximate estimates, but not as the a priori postulated expression of base reality.

BICs must be designed to be tractable

One of the most natural ways of replicating a derivatives contract statically in a manner which insulates against model misspecification is to extend state securities also known as Arrow-Debreu Securities[7] (or ADS[8]) along the underlying(s) ***state and time span*** as Basis Instrument Contract.

For example, if a general derivatives contract payout $f(S_1,S_2)$ is a function of an underlying S that can take only 2 values x and y at two future times t_1 and t_2, that derivatives contract $f(S_1,S_2)$ can be decomposed along the underlying(s) state and time span as the sum of 4 elementary basis functions representing basis derivatives contracts:

$$f(S_1,S_2)=f(x,x)*1_{\{S1=x,S2=x\}}(S_1,S_2)+ f(x,y)* 1_{\{S1=x,S2=y\}}(S_1,S_2)+ f(y,x)*1_{\{S1=y,S2=x\}}(S_1,S_2) + f(y,y)*1_{\{S1=y,S2=y\}}(S_1,S_2)$$

where for example in the notations above $1_{\{S1=x,S2=x\}}$ represents the ADS identified as a function of S_1 and S_2 that returns zero for all values of S_1 and S_2 except for S_1=x and S_2=x, in which case it returns the value 1.

Using such a basis decomposition is natural and straightforward. However it is not very tractable; if there are s states the underlying S can reach at each time step, and n time steps, that yields s^n basis instrument contracts. A simple one month derivatives contract which depends on the closing values of each of 22 trading dates and can reach 50 possible states would use $\mathbf{2.38418579 \times 10^{37}}$basis contracts! This exponential explosion leading to practical intractability is a new instance of what is generally known as the curse of dimensionality. Except perhaps in a one (or a limited few) future state(s) world, its interest is merely theoretical. Our approach to designing BICs will avoid this pitfall.

Defining BICs

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In defining BICs, we assume **space is discrete, time is discrete**, **finite.**

BICs Structure

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In order to make the definition more accessible, we first stress characteristic features of BICs and the rationale for those:

1) A Basis Instrument Contract (BIC) is a Derivatives contract and involves and identifies two parties: one or more buyer(s) named B and one or more seller(s) named A;
2) The definition of each contract comprises *three dates*:

· i) The contract agreement date t_0, which is the date at which the binding rights and obligations on both sides of the contract are agreed upon;

· ii) The premium payment date t_i , with $t_i>=t_0$,which is the date at which the party identified in the contract as the buyer B complies with its part of the agreement by paying the seller A an amount in units of basis currency known as the premium of the contract or the premium payment amount. ***The premium payment amount may be expressed as a function of the values of the fluctuating variables between time t_0 and time t_i*** ;

· iii) The contract expiry date also known as maturity or payout payment date t_j, with $t_j>t_i$, which is the date at which the party(ies) identified in the contract as the seller(s) A complies with its (their) part of the agreement by paying the buyer(s) B an amount in units of basis currency defined in the *BIC set format* further detailed below*. When used to replicate a derivatives contract, we have* $t_j = t_{i+1}$;

The distinction of these three dates is meant to draw a contrast with traditional derivatives contracts features where there are effectively only two dates as the contract agreement date and the premium payment date are essentially the same since the premium payment amount is a ***constant*** function of the values of the fluctuating variables between time t_0 and time t_i. While it is true that in most derivatives contracts there is a contract agreement date or value date and a settlement date different from the value date, the difference, assuming – as is done for modeling purposes – that the value date and the premium payment date are the same, is unconsequential and it is straightforward to make the adjustment. So, each of the dates t_i and t_j may actually be coupled with settlement dates a few days later.

3) A BIC further admits notional amounts that may be ***functions of the realized values of underlyings and related derivatives contracts whose realized value shall be known by premium payment date t_i*** at the latest.

4) A BIC is defined in reference to a **BIC set** whose properties determine whether its elements are the payout of BICs. A BIC set is a set of functions who represent a linear basis of the space of functions taking values in the space spanned by the underlying(s).

For example if the underlying S may span *s* states $x_1,...x_s$, a natural **BIC set** may simply be the *s* Arrow Debreu indexing functions which take the value 1 for one particular state x_i and equal zero for any other value. Another example BIC set that would be more used in actual markets may be the set of functions indexing the payout of calls, puts, plus a forward contract and a zero coupon bond. A BIC set is more precisely defined below.

With their pre-agreement features, BICs may be somewhat reminiscent of known simpler BICs such as Forward Rate Agreements (FRAs) and Forward contracts. However, unlike forward rate agreement (FRAs) or Forward contracts or Futures contracts, because of their general functional nature, we do not usually know the numerical value of the notional of a typical BIC at contract agreement time.

Sample trivial BICs

In a one future state world with merely two 2 states, BICs reduce to Arrow Debreu Securities (ADS).

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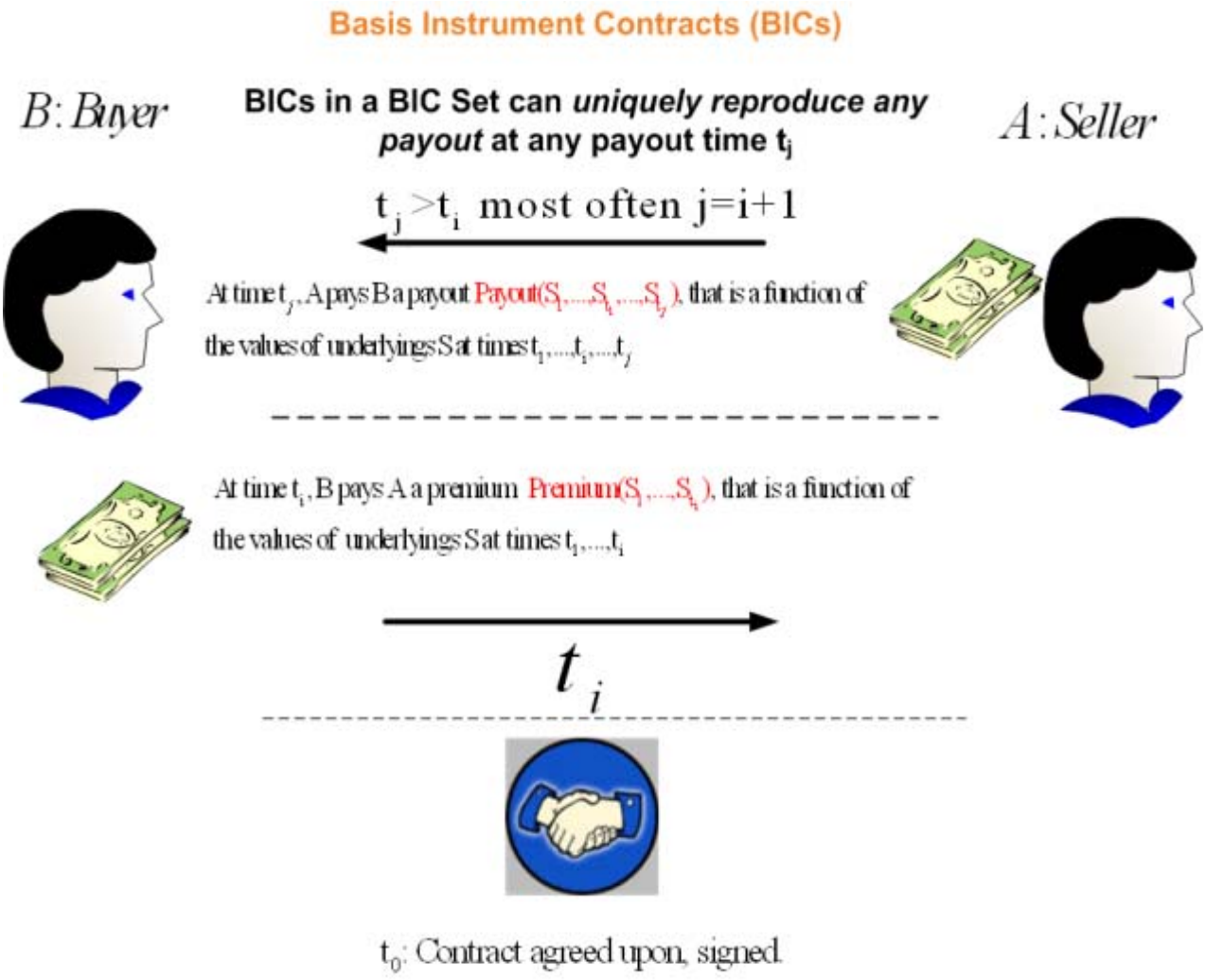
In a multiple states, one future period world, BICs reduce to ADS or any equivalent set of derivatives identified by any isomorphic space spanning family of functions such as state spanning call/put with a zero coupon bond and a forward contract on the underlying.

BICs, BIC Sets and BIC sets formats



Let's consider a derivatives contracts market M.

(i) A derivatives contracts set $B_{t0,tk,tk+1}$ is said to be a (t_0,t_k,t_{k+1}) Basis Instrument Contract set or **(t_0,t_k,t_{k+1})-BIC set** in M if and only if the payout functions of its elements are independent and changes in the value of any derivative contract in M between t_k and t_{k+1} may be replicated by a linear combination of derivatives contracts in $B_{t0,tk,tk+1}$ contracted at time t_0.

(ii) A Basis Instrument Contract or BIC is a special type of Derivatives contract that is an element of an identified **(t_0,t_k,t_{k+1})-BIC set**.

(iii) The set of payout payment functions of a BIC set $B_{t0,tk,tk+1}$ is called its **BIC set format** and noted $B^f_{t0,tk,tk+1}$.

In general BIC set formats will be defined for a given payout payment date and assumed to be the same for all other payout payment dates. However, one can change the format for two different payout payment dates in the market M.

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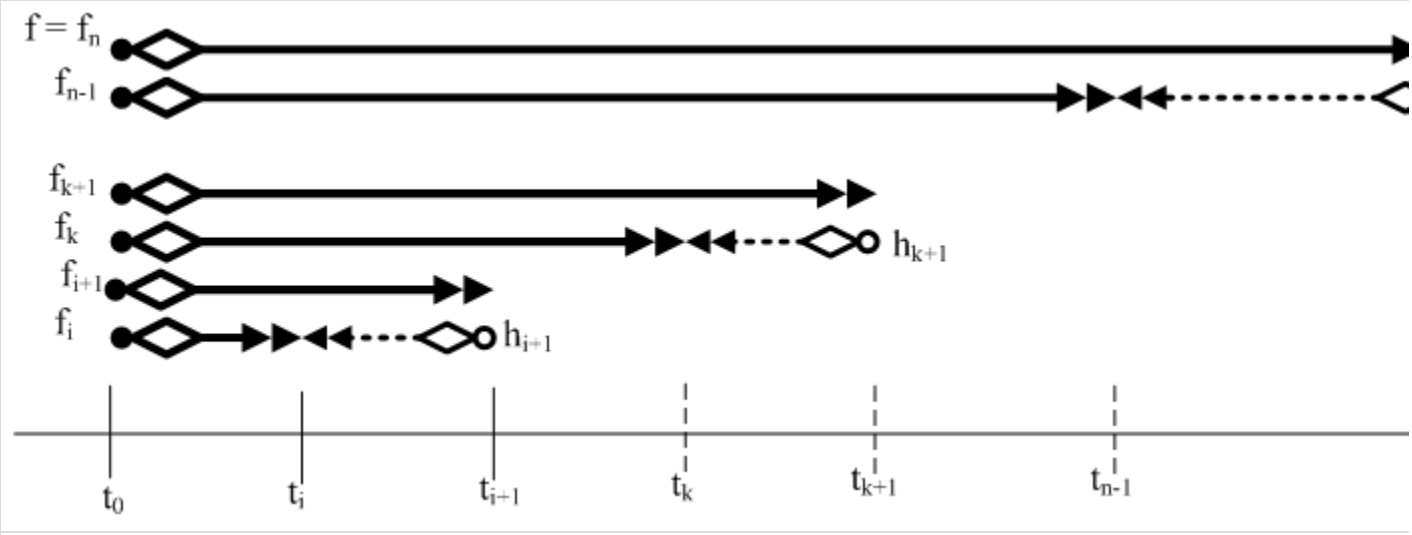
The BICs Derivatives Static Replication Strategy



Derivatives Contract Static Backward Replication Process

The recursive backward replication process starts with the terminal payout $f=f_n$ and works from k = n-1 to k = 0 as follows:

For time t_{k+1} $0<k+1<n+1$, we equate the premium f_{k+1} to be paid at time t_{k+1} to be the payout payment of a derivatives contract. We reach an agreement at time t_0 with one or more counterparties that at time t_k, we will pay (resp. receive) a premium payment amount f_k for a combination of BICs or otherwise exact or approximate replicating derivatives contracts with functional notionals; In return, we will receive (resp. pay) at time t_{k+1}, the corresponding payout payment f_{k+1}. The combination of offsetting BICs or otherwise exact or approximate replicating derivatives contracts between t_k and t_{k+1} is noted as h_{k+1}.

We thus see that by paying (resp. receiving) at time t_0 the premium payment amount f_0, at each intermediary trading date $t_1,\ldots,t_{n-1}$, the payout payment to be received (resp. paid) will coincide with the premium payment to be paid out (resp. received), i.e. the sum of all debit positions is exactly equal to the sum of all credit positions. We can therefore even automate the netting process, so that by paying (resp. receiving) the premium payment amount f_i at time t_i, we guarantee, without further action or intervention, receipt (resp. payment) of the payout payment f_n at time t_n. This means any derivative security would be statically replicated at t_i with the selected BICs.

In the end, having paid (resp. received) at time t_i the premium payment f_i, only at time t_n will we receive (resp. pay) the payout payment amount $f = f_n$, exactly what we set out to accomplish in the first place!

The Larger Scope of the BICs Hedging Approach

- Note that the backward decomposition process outlined above does not pre-suppose the existence of BICs, rather it merely relies on the existence of replicating instruments with functional notionals, the combination of which may be determined by an error minimizing algorithm at each step. For example, merely using underlying and bonds with functional notionals may be the basis of a multi-step static replication strategy where one seeks to minimize an expected quadratic difference between contract to be hedged and hedging instruments.

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- Note also the synthetic nature of hedging contracts independent of underlyings which clarifies that hedging a derivatives contract does not require taking positions in the underlying as traditional derivatives dynamic hedging strategy teaches.

These two concepts are of critical importance to achieve surgically precise and effective risk management on an even larger scale.

Properties of BICs

BICs as outlined satisfy the the necessary requirements formulated above.

How BICs are immune to model specification and would have helped mitigate the effects of the 2008 Crisis

Since the agreements on the hedging BICs would have been contracted before liquidity dried up, their would be no risk that the position would not end up dynamically hedged.

Furthermore a BICs market would help structurally mitigate market volatility in markets and reduce transaction costs because hedging activity is substantially reduced to the settlement of agreements contracted beforehand.

On January, 28th, 2010, Risk.net reported that losses and increasing hedging costs have led many smaller firms to leave the Power-Reverse Dual Currency (PRDC) business, leaving a rump of major banks operating in the market. This represents yet another example where had a BICs market on the currencies and interest rates involved existed, there would have been no such issue. Not only hedging activity would have been contracted beforehand, in addition the atomic structure of replicating BICs would have made it nearly impossible for counterparties to identify what the contract held is in order to attempt a 'squeeze'. Furthermore most of the individual BICs would be used for other hedging purposes unrelated to PRDCs, ensuring the liquidity of the market on those contracts.

How BICs Solve the Curse of Dimensionality Problem

BICs solve the curse of dimensionality problem associated with a naive Arrow Debreu decomposition by:

- First, transforming the exponentially growing number of replicating contracts with numerical notionals into a linearly growing number of replicating contracts with functional notionals.
- Then, noting that efficient functional representation can be effectively addressed in practice using a number of clever representations:
  - In exact terms; the resolution in this case takes advantage of the empirical observation that the description of virtually all contracts enables, via the use of summary variables, a reduction of the dimensionality to merely a few, often 2 or 3 dimensions.
  - In approximate terms, there are many basis for functional approximation that require using merely a few coefficients. For example in signal theory functional representation using Fourier series or wavelets are effective means of addressing the problem that can be reused here.

There are many explanations for this phenomenon. Some call it the law of the few; in information theory terms, it can be seen as the result of inherent limited information entropy. A more mathematically grounded argument is the

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concentration of measure phenomenon. The Concentration of measure phenomenon[9] that says in short that a Lipschitz function, which refine a continuous function, is nearly constant is one one of the most powerful theoretical argument to justify the resolution of the curse of dimensionality in functional representation.

Example: BICs & New York Yankees Baseball World Series of 2001 & 2009

This example is reviewed as Wolfram Demonstration that uses the multi-period (seven time periods for seven games) setting of the Baseball World Series of 2001 and 2009. In such a series, the first two games are played at the original host team's city and the third game is played at the original guest team's city. The original host and original guest teams were the Arizona Diamondbacks (AZD) versus the New York Yankees (NYY) in 2001 and the New York Yankees (NYY) versus the Philadelphia Phillies (PHI) in 2009.

We use these series to illustrate an implementation of BICs insights for odds pricing purposes in a manner that reduces the curse of dimensionality and yields flexibility in the choice of dynamics of underlyings (i.e., outcome of a game), and help identify potential mispricings or arbitrage opportunities. The example also serves as a ready-made template for the analysis of odds in seven-game contests in baseball or other sports, such as basketball.
This example is presented in further details in the linked peer-reviewed Wolfram demonstration below (click the image):

Conclusion

The range of applications of BICs is wide and well beyond the scope discussed in this article. Some of them are reviewed in subsequent articles.

The knol article titled "Estimating Asset Costs for PPIP Assets in a Market Making Framework & BICs" illustrates one of the many applications of BICs to solve a non trivial problem.

The knol article titled: "The wider scope of BICs" further expands on the scope of BICs.

The article in the 2009 summer edition of The Investment Professional uses BICs to analyze the flaws in the Treasury's PPIP for Banks' troubled assets.Check it out http://www.theinvestmentprofessional.com/vol_2_no_3/abstract-bics.html

For more on the theoretical foundations and ramifications, buy the book: BICs 4 Derivatives Volume I : Theory. Chapter I, which makes the original Case for BICs is freely availble here as a scribd publication.

Indeed an immediate practical application is in the development of BICs exchanges:
YouTube Video
For the use of BICs that shows their superior power in addressing Economic, Financial & Current Issues in the news, check the blog, http://kongtcheu.blogspot.com/

Check your basic knowledge of BICs by filling the BICs Trivia Questionnaire

This knol tiny url: **http://tinyurl.com/cyxhpa**

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What did you think of this article? Do you have any feedback, any criticisms, praise or suggestions? If so then we would love to hear from you and appreciate any comments that you make.

References

1. These links on negative, rational and complex numbers refer to lighthearted but incisive articles/blogs in the NY Times by Steven Strogatz. See my blog comments at:
Reference Link

2. Philosophical principle also known as "Occam's Razor" and which roughly translates "entities must not be multiplied beyond necessity."
Reference Link

3. BICs International patent disclosure:
Reference Link

4.
Reference Link

5. in FAS 133 pp3-7, paragraph 6-11 and its 10(b) amendment in FAS 138
Reference Link

6. BICs 4 Derivatives Volume I: Theory Chapter III provides the most inclusive and operational mathematical definition
Reference Link

7. Introduction Financial Economics class at Berkeley.This example is very propedeutic to digest in order to appreciate well the value of BICs
Reference Link

8. This course by William Sharpe is also very propedeutic to digest the value of BICs
Reference Link

9. This article that I am just discovering (Feb 16, 2009) via a quick Google search is a very clear articulation of the ideas I have advanced for the practical usability of BICs. It explains the mathematical reasons at the root of the resolution of the curse of dimensionality problem in the form of the concentration of measure phenomenon. Note: If my memory serves me right I think I attended the 1994 Saint Flour Summer School on Probabilities with him where one of the 3 main Lecturers Michel Ledoux delivered a course on Isoperimetry and Gaussian analysis relating to the concentration of measure phenomenon. See:http://math.univ-bpclermont.fr/stflour/published.php

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Reference Link

10. I make a significant case for discreteness in Chapter I of my book "The Case for BICs" which is also provided as one of my Knols. Note that the continuous Vs. Discrete question is an issue that has also been of concern to physicists for a while. For reference, note for example the 1983 paper "Can Time Be a Discrete Dynamical Variable?";, T. D. Lee1957 Physics Nobel Prize , which led to a series of publications by Lee and collaborators on the formulation of fundamental physics in terms of difference equations.
Reference Link

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