Some Comments on Unitary Qubit Lattice Algorithms for Classical Problems

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Abstract

A qubit lattice algorithm (QLA), which consists of a set of interleaved unitary collision-streaming operators, is developed for electromagnetic wave propagation in tensor dielectric media. External potential operators are required to handle gradients in the refractive indices, and these operators are typically non-unitary. A similar problem arises in the QLA for the Korteweg-de Vries equation, as the potential operator that models the KdV nonlinear term is also non-unitary. Several QLAs are presented here that avoid the need of this non-unitary potential operator by perturbing the collision operator. These QLAs are fully unitary.

1 Introduction

We have been investigating qubit lattice algorithms (QLA) for some time [1-21]. The aim of QLA is to develop a unitary interleaved sequence of collision-streaming operators which in the continuum limit reduces perturbatively to the desired differential equations describing the system of interest. The first step is to associate a basis set of qubits for the lattice, which on taking appropriate moments will recover the classical fields of interest. Thus the QLA would be immediately encodable onto a quantum computer. However, from our earlier nonlinear studies of 2D and 3D quantum turbulence [8,9, 11], the QLA is ideally parallelized on classical supercomputers with no degradation in parallel performance as the number of cores are ramped up [e.g., to over 750 000 cores on the IBM BlueGene $Mira$ supercomputer at Argonne]. Some care is needed in the choice of the qubit basis. For example, it will be shown in Sec. 2 that the simple basis choice of $(\mathbf{E}, \mathbf{H})$ will never lead to a unitary representation of the Maxwell equations of electrodynamics. [$\mathbf{E}$ is the electric field, and $\mathbf{H}$ is the magnetic field].

At the heart of an efficient algorithm on a quantum computer is quantum entanglement of the qubits. For example, consider a 2-qubit representation [e.g., for a basis for a differential equation like the scalar nonlinear Schrodinger (NLS) equation or the Korteweg de Vries (KdV) equation]. A basis is the 2^2 elements $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)$. Now consider a unitary 2×2 collision operator

$$C = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

acting on the qubit subspace of $(|01\rangle, |10\rangle)$. One of the post-collision qubit elements is

$$\cos\theta\,|01\rangle + \sin\theta\,|10\rangle. \tag{2}$$

However, this post-collision state cannot be represented by a tensor product of the 2^2 - basis, since the most general tensor product state is

$$a_0 b_0|00\rangle + a_0 b_1|01\rangle + a_1 b_0|10\rangle + a_1 b_1|11\rangle \tag{3}$$

where one qubit state is $a_0|0\rangle + a_1|1\rangle$, and the other state is $b_0|0\rangle + b_1|1\rangle$, for some coefficients $a_0 \quad \ldots \quad b_1$. To recover the state Eq. (2) from the tenor product state Eq. (3) one would have to eliminate the $|00\rangle$-term. However one must then set either $a_0 = 0$ or $b_0 = 0$. But this would eliminate either the state $|01\rangle$ or the state $|10\rangle$ - both of which are needed to recover Eq. (2). States which cannot be represented in a tensor product basis of qubits are called entangled states. A maximally entangled state is achieved on taking $\theta = \pi/4$, and is known as a Bell state [22]

$$B_1 = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Note that the quantum entanglement is achieved here by the unitary collision operator. The streaming operator in QLA will then propagate this entanglement throughtout the lattice.

In Sec. 2 we will develop a QLA for the solution of 2D Maxwell equations in a tensor Hermitian dielectric medium. All our previous Maxwell QLA [18, 21] were restricted to scalar dielectrics. We will present a simplified discussion of the Dyson map [23] that will permit us to transform from a non-unitary to unitary basis for the representation of the two curl equations of Maxwell. From these qubit equations we will generate a QLA for tensor dielectric media that is second order accurate. The QLA that we discuss here is not fully unitary. While the collide-stream operator sequence is fully unitary, the external potential operators required to recover the Maxwell equations are not. However these non-unitary matrices are very sparse and could be amenable to some unitary approximate representation. The role of the perturbation parameter ϵ introduced in the QLA is quite subtle.

As to an understanding of the subtlety of ϵ in QLA we return to the KdV equation in Sec. 3. Our original QLA for KdV [1] consisted of maximally entanglement Bell collision operators together with a required external potential operator to model the KdV nonlinearity. This external potential was not unitary. Here we present a modified collision operator that is fully unitary and which leads to a QLA-KdV that does not require any external potential to be introduced.

Finally, in Sec. 4 we make some concluding remarks about future QLA simulations that are needed to elucidate the perturbation parameter ϵ. This parameter is required in order to move the discrete QLA into a continuum representation.

2 QLA for Maxwell Equations

2.1 Qubit-Electromagnetic field representation

Consider a simple dielectric non-magnetic medium with the constitutive equations

$$\mathbf{D} = \epsilon\mathbf{E}, \quad \mathbf{B} = \mu_0\mathbf{H}. \tag{4}$$

Treating $\mathbf{u} = (\mathbf{E}, \mathbf{H})^{\mathbf{T}}$ as the fundamental fields, and $\mathbf{d} = (\mathbf{D}, \mathbf{B})^{\mathbf{T}}$ the derived fields , Eq. (2) can be written in matrix form

$$\mathbf{d} = \mathbf{W}\mathbf{u} \tag{5}$$

where $\mathbf{W}$ is a Hermitian 6×6 matrix

$$\mathbf{W} = \begin{bmatrix} \epsilon_{3\times 3} & 0 \\ 0 & \mu_0 \mathbf{I}_{3\times 3} \end{bmatrix} \tag{6}$$

with $\mathbf{I}_{3\times 3}$ the 3×3 identity matrix. and $\mathbf{T}$ is the transpose operator. The curl-curl (source-free) Maxwell equations $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$, and $\nabla \times \mathbf{H} = \partial \mathbf{D}/\partial t$ in matrix form are just

$$i\frac{\partial \mathbf{d}}{\partial t} = \mathbf{M}\mathbf{u} \tag{7}$$

where, under standard boundary conditions, the curl-matrix operator $\mathbf{M}$ is Hermitian

$$\mathbf{M} = \begin{bmatrix} 0_{3\times 3} & i\nabla\times \\ -i\nabla\times & 0_{3\times 3} \end{bmatrix}. \tag{8}$$

Since $\mathbf{W}$ is invertible, Eq. (5) can be written in terms of the basic electromagnetic fields $\mathbf{u} = (\mathbf{E}, \mathbf{H})$

$$i\frac{\partial \mathbf{u}}{\partial t} = \mathbf{W^{-1}}\mathbf{M}\mathbf{u} \tag{9}$$

In continuum applications, one typically treats the two Maxwell divergence equations $\nabla \cdot \mathbf{B} = 0$ and $\nabla \cdot \mathbf{D} = 0$ as initial conditions. From the curl-curl equations we see that they will then be satisfied for all time.

2.1.1 homogeneous dielectric medium

If one is dealing with a homogeneous dielectric medium (e.g., a vacuum), then the constitutive matrix $\mathbf{W}$ is a constant and trivially commutes with the curl-operator $\mathbf{M}$. As a result, the product of the two Hermitian matrices, $\mathbf{W^{-1}M}$ is itself Hermitian, and Eq. (7) gives a unitary evolution of the electromagnetic fields $\mathbf{u} = (\mathbf{E}, \mathbf{H})^{\mathbf{T}}$. Thus $\mathbf{u}$ is an appropriate basis for the qubit fields and for quantum computation.

2.1.2 inhomogeneous dielectric media

However, when the matrix $\mathbf{W}$ is spatiaily dependent, then $\mathbf{W^{-1}M} \neq \mathbf{MW^{-1}}$ and $\mathbf{W^{-1}M}$ is not Hermitian. Under these conditions, the qubit representation of the electromagnetic fields $\mathbf{u} = (\mathbf{E}, \mathbf{H})^{\mathbf{T}}$ will not yield a unitary evolution of these qubits. However Koukoutsis et. al. [23] have shown how to determine the so-called Dyson map from the fields $\mathbf{u}$ to a new field representation $\mathbf{U}$ such that the resultant representation in terms of the new field $\mathbf{U}$ will result in a unitary evolution of these fields. Indeed, it can be shown [23], that the Dyson map

$$\mathrm{U} = \mathrm{W}^{1/2}\mathrm{u} \tag{10}$$

will yield a unitary evolution equation for $\mathbf{U}$ with

$$i\frac{\partial \mathbf{U}}{\partial t} = \mathbf{W}^{-1/2}\mathbf{M}\mathbf{W}^{-1/2}\mathbf{U} \tag{11}$$

as the matrix operator $\mathbf{W^{-1/2}MW^{-1/2}}$ is Hermitian.

Thus one could start to build a QLA based on the electromagnetic fields

$$\mathbf{U} = \left(\epsilon^{1/2}\mathbf{E}, \mu_0^{1/2}\mathbf{H}\right)^T \tag{12}$$

or under the rotation matrix

$$\mathbf{L} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_{3\times 3} & iI_{3\times 3} \\ I_{3\times 3} & -iI_{3\times 3} \end{bmatrix} \tag{13}$$

one could base a QLA on the field representation $\mathbf{U_{RSW}} = \mathbf{LU}$ where

$$\mathbf{U_{RSW}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \epsilon^{1/2}\mathbf{E} + i\mu_0^{1/2}\mathbf{H} \\ \epsilon^{1/2}\mathbf{E} - i\mu_0^{1/2}\mathbf{H} \end{bmatrix}. \tag{14}$$

This is nothing but the unitary evolution of the Riemann-Silberstein-Weber (RSW) vector - a representation used to represent Maxwell equations from the early 1920's [24-26].

Moreover, the theory can be readily extended to diagonal tensor dielectric media, with (assuming non-magnetic materials) the 6-qubit representation $\mathbf{Q}$ of the field

$$\mathbf{U} = \left(n_x E_x, n_y E_y, n_z E_z, \mu_0^{1/2}\mathbf{H}\right)^T = \mathbf{Q} \tag{15}$$

(n_x, n_y, n_z) is the vector (diagonal) refractive index, with $\epsilon_x = n_x^2$ ... . We work in Cartesian coordinates.

2.2 2D QLA for x − y dependent propagation of Maxwell Equations

From Eqs. (9) and (13), Maxwell equations for 2D x-y spatially dependent fields written in terms of the 6-$\mathbf{Q}$ vector components

$$\begin{gathered} \frac{\partial q_0}{\partial t} = \frac{1}{n_x}\frac{\partial q_5}{\partial y}, \qquad \frac{\partial q_1}{\partial t} = -\frac{1}{n_y}\frac{\partial q_5}{\partial x}, \qquad \frac{\partial q_2}{\partial t} = \frac{1}{n_z}\left[\frac{\partial q_4}{\partial x} - \frac{\partial q_3}{\partial y}\right] \\ \frac{\partial q_3}{\partial t} = -\frac{\partial (q_2/n_z)}{\partial y}, \qquad \frac{\partial q_4}{\partial t} = \frac{\partial (q_2/n_z)}{\partial x}, \qquad \frac{\partial q_5}{\partial t} = -\frac{\partial (q_1/n_y)}{\partial x} + \frac{\partial (q_0/n_x)}{\partial y} \end{gathered} \tag{16}$$

This representation is unitary.

Our QLA representation focusses on recovering Eq. (14) perturbatively. One can thus consider developing the representation dimension by dimension. In particular we introduce the following unitary collision operator with collision angles θ_1 and θ_2 (to be specified later):

$$C_X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & cos\,\theta_1 & 0 & 0 & 0 & -sin\,\theta_1 \\ 0 & 0 & cos\,\theta_2 & 0 & -sin\,\theta_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & sin\,\theta_2 & 0 & cos\,\theta_2 & 0 \\ 0 & sin\,\theta_1 & 0 & 0 & 0 & cos\,\theta_1 \end{bmatrix} \tag{17}$$

and the unitary collision operator

$$C_Y = \begin{bmatrix} cos\,\theta_0 & 0 & 0 & 0 & 0 & sin\,\theta_0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & cos\,\theta_2 & 0 & sin\,\theta_2 & 0 \\ 0 & 0 & -sin\,\theta_2 & cos\,\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -sin\,\theta_0 & 0 & 0 & 0 & 0 & cos\,\theta_0 \end{bmatrix} \tag{18}$$

with collision angles θ_0 and θ_2. The unitary streaming operator S_{14}^{+x} shifts qubits q_1 and q_4 one lattice unit in the $+x$ direction, while leaving the remaining 4 qubits alone. We finally need to introduce the external potential operators

$$V_X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -sin\,\beta_2 & 0 & cos\,\beta_2 & 0 \\ 0 & sin\,\beta_0 & 0 & 0 & 0 & cos\,\beta_0 \end{bmatrix} \tag{19}$$

and

$$V_Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & o \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cos\,\beta_3 & \sin\,\beta_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -sin\,\beta_1 & 0 & 0 & 0 & 0 & cos\,\beta_1 \end{bmatrix} \tag{20}$$

for particular angles β_0, β_1 and β_2. These potential operators are not unitary, but very sparse.

We now consider the following unitary sequence of interleaved collision-streaming operators:

$$\mathbf{U_X} = S_{25}^{+x}.C_X^{\dagger}.S_{25}^{-x}.C_X.S_{14}^{-x}.C_X^{\dagger}.S_{14}^{+x}.C_X.S_{25}^{-x}.C_X.S_{25}^{+x}.C_X^{\dagger}.S_{14}^{+x}.C_X.S_{14}^{-x}.C_X^{\dagger} \tag{21}$$

and

$$\mathbf{U_Y} = S_{25}^{+y}.C_Y^{\dagger}.S_{25}^{-y}.C_Y.S_{03}^{-y}.C_Y^{\dagger}.S_{03}^{+y}.C_Y.S_{25}^{-y}.C_Y.S_{25}^{+y}.C_Y^{\dagger}.S_{03}^{+y}.C_Y.S_{03}^{-y}.C_Y^{\dagger} \tag{22}$$

with the discrete time advancement of the 6-qubit $\mathbf{Q}$ given by

$$\mathbf{Q}(t+\delta t) = V_Y.V_X.\mathbf{U_Y}.\mathbf{U_X}.\mathbf{Q}(t) \tag{23}$$

To recover the desired Maxwell equations (14) perturbatively, one employ a small parameter ϵ as the spatial lattice shift unit (assuming a square $x-y$ lattice), and the unitary collision angles

$$\theta_0 = \frac{\epsilon}{4n_x} \quad , \qquad \theta_1 = \frac{\epsilon}{4n_y} \quad , \qquad \theta_2 = \frac{\epsilon}{4n_z}. \tag{24}$$

so as to recover the coefficients of the $\partial\mathbf{Q}/\partial(x,y)$ terms. Finally, the nonunitary external potential angles need to be defined as

$$\beta_0 = \epsilon^2 \frac{\partial n_y/\partial x}{n_y^2} \quad , \quad \beta_1 = \epsilon^2 \frac{\partial n_x/\partial y}{n_x^2} \quad , \quad \beta_2 = \epsilon^2 \frac{\partial n_z/\partial x}{n_z^2} \quad , \quad \beta_3 = \epsilon^2 \frac{\partial n_z/\partial y}{n_z^2} \tag{25}$$

Indeed, using Mathematica to evaluate Eq. (21), one obtains in the continuum spatial limit the desired Maxwell equations to errors of ϵ^4

$$\begin{gathered} \frac{\partial q_0}{\partial t} = \epsilon^2 \delta t \frac{1}{n_x} \frac{\partial q_5}{\partial y}, \qquad \frac{\partial q_1}{\partial t} = -\epsilon^2 \delta t \frac{1}{n_y} \frac{\partial q_5}{\partial x}, \qquad \frac{\partial q_2}{\partial t} = \epsilon^2 \delta t \frac{1}{n_z} \left[\frac{\partial q_4}{\partial x} - \frac{\partial q_3}{\partial y} \right] \\ \frac{\partial q_3}{\partial t} = -\epsilon^2 \delta t \frac{\partial (q_2/n_z)}{\partial y}, \qquad \frac{\partial q_4}{\partial t} = \epsilon^2 \delta t \frac{\partial (q_2/n_z)}{\partial x}, \qquad \frac{\partial q_5}{\partial t} = -\epsilon^2 \delta t \left(\frac{\partial (q_1/n_y)}{\partial x} + \frac{\partial (q_0/n_x)}{\partial y} \right) \end{gathered} \tag{26}$$

i.e., under diffusion ordering, $\epsilon^2 \delta t \approx O(1)$, one recovers the continuum Maxwell equations to errors $O(\epsilon^2)$. We will explore QLA simulations of 2D Maxwell equations in a subsequent paper.

3 QLA for KdV without external non-unitary potential operators

The KdV equation is an important nonlinear equation and was developed to explore the evolution of shallow water waves. Interestingly, it [27] has also been associated with the Fermi-Pasta-Ulam-Tsingou simulations of the 1950's. Fermi wanted to examine the equipartition of energy among the modes of a many body problem of weakly coupled nonlinear oscillators. Statistical mechanics indicates that the time- asymptotic state will be one in which there is equipartition of energy among all the oscillator modes. Instead, in the parameter regime they considered, Fermi et. al. found recurrence of initial conditions, but a recurrence that was not a usually extremely long Poincare recurrence time of Hamiltonian systems. Interestingly, this would turn out to be a precursor to soliton theory.

The general KdV equation for arbitrary positive constants a and b

$$\frac{\partial \psi}{\partial t} + a\psi \frac{\partial \psi}{\partial x} + b\frac{\partial^3 \psi}{\partial x^3} = 0 \tag{27}$$

is exactly integrable. One of its solutions is the right traveling soliton with speed c - a free parameter

$$\psi(x,t) = \frac{3c}{a}\,\mathrm{sech}^2\left(\frac{1}{2}\sqrt{\frac{c}{b}}[x-ct]\right) \tag{28}$$

Notice that for the KdV soliton, the amplitude and its speed are correlated (unlike the NLS soliton).

Since the KdV equation is a scalar equation for the real function $\psi(x,t)$ one need only to employ 2 qubits / lattice site. First we shall reconsider the QLA for KdV with the use of an external potential to model the nonlinear term in KdV [1]. The collision operator is nothing but Eq. (1). We denote the operator S_0^+ to be the streaming operator that translates the qubit q_0 one lattice unit in the $+x$-direction. To eliminate the 2nd order spatial derivative one must choose the interleaved sequence of collision-stream unitary operators carefully. In particular the following sequence will generate a second order QLA for the KdV equation

$$Q(t+\Delta t) = V_{pot}\cdot S_0^+ C\cdot S_1^- C^T\cdot S_0^- C\cdot S_1^+ C^T\cdot S_0^- C^T\cdot S_1^+ C\cdot S_0^+ C^T\cdot S_1^- C\cdot Q(t) \tag{29}$$

where the unitary collision operator C is nothing but the maximally entangling operator. Eq, (1), with $\theta = \pi/4$. $Q = (q_0\, q_1)^T$. The external potential $V_{\rm pot}$ is the Hermitian matrix

$$V_{\rm pot} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \quad \text{with} \quad \alpha = \epsilon^3 m[x]. \tag{30}$$

In the continuum limit, one recovers

$$\frac{\partial \psi}{\partial t} + \epsilon^3\left(m[x]\cdot\psi(x,t) + \frac{1}{2}\frac{\partial^3\psi}{\partial x^3}\right) = 0 + O\left(\epsilon^5\right) \tag{31}$$

on defining $\psi = q_0 + q_1$. With the choice of $m[x] = \partial\psi/\partial x$ we have a second order accurate QLA for KdV. Note that the QLA of Eq. (26) is not fully unitary because of the non-unitary property of the external potential operator $V_{\rm pot}$.

3.1 Fully unitary QLAs for KdV

There is a large class of unitary QLAs all of which recover the KdV equation to second order accuracy. Here, we will present two QLAs, both having the same unitary collision operator, but

with different streaming sequences on the two qubits. Indeed, using Mathematica, it can be shown that the following QLA

$$Q(t+\Delta t) = S_0^- C_1 \cdot S_0^+ C_1 \cdot S_1^+ C_1 \cdot S_1^- C_1^T \cdot S_0^- C_1^T \cdot S_0^+ C_1^T \cdot S_1^+ C_1^T \cdot S_1^- C_1 \cdot Q(t) \tag{32}$$

with unitary collision operator C_1

$$C_1 = \begin{bmatrix} \cos\alpha_1 & \sin\alpha_1 \\ -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix} \quad \text{with} \quad \alpha_1 = \frac{\pi}{4} + \epsilon^2 m_1[x]. \tag{33}$$

leads in the continuum limit to

$$\frac{\partial \psi_1}{\partial t} + \epsilon^3 \left(4m_1[x] \cdot \frac{\partial \psi_1}{\partial x} + \frac{1}{2} \frac{\partial^3 \psi_1}{\partial x^3} \right) = 0 + O\left(\epsilon^5\right) \tag{34}$$

so that the choice of $m[x] = \psi_1$ will recover KdV.

Another fully unitary QLA that recovers KdV has the following interleaved sequence of unitary collision-streaming operators:

$$Q(t+\Delta t) = C_1 S_0^- \cdot C_1 S_1^+ \cdot C_1 S_0^- \cdot C_1 S_1^+ \cdot C_1^T S_0^+ \cdot C_1^T S_1^- \cdot C_1^T S_0^+ \cdot C_1^T S_1^- \cdot Q(t) \tag{35}$$

C_1 is the same collision operator, Eq. (33). In the continuum limit, we find

$$\frac{\partial \psi_1}{\partial t} + \epsilon^3 \left(-4m_1[x] \cdot \frac{\partial \psi_1}{\partial x} + \frac{1}{2} \frac{\partial^3 \psi_1}{\partial x^3} \right) = 0 + O\left(\epsilon^5\right) \tag{36}$$

so that the choice of $m_1[x] = -\psi_1$ will recover the KdV equation.

The implementation of these fully unitary algorithms may not necessarily be straightforward as the perturbation parameter ϵ introduced into the Mathematica algorithm requires a perturbation in the collision angle of $O\left(\epsilon^2\right)$, Eq. (33), while the continuum limit has scaling proceeds as $O\left(\epsilon^3\right)$. In previous QLA for nonlinear physics, the order of the function ψ controlled the ϵ-factor.

4 SUMMARY

The development of a fully unitary QLA for plasma physics [28-31] in particular, is of considerable interest to us as these algorithms are immediately encodable on quantum computers. Of course, since they are time evolution algorithms, they will have to wait till there are error-correcting quantum computers available. In developing QLAs for plasma physics we have taken the tack of first concentrating on the Maxwell equations in a given scalar dielectric media. Then one would eventually generalize to a tensor unitary dielectric description of a cold magnetized plasma.

Here, we have shown how to generalize our QLA-scalar dielectric Maxwell equations to handle tensor Hermitian dielectric media. This was facilitated by the use of the Dyson map [23]. Indeed, the explicit determination of the Dyson map proves that there exists a unitary quantum algorithm to describe such a Maxwell system. The problem, of course, is to explicitly construct such an algorithm. We have concentrated on the QLA approach, which employs a non-trivial sequence of interleaved collide-stream unitary operators. To proceed explicitly with QLA, one must resort to perturbation theory and the introduction of a small parameter ϵ. The uinitary collide-stream operator sequence does not in all our Maxwell equation considerations recover the required full set of evolution equations. This has resulted in the need to introduce so-called potential operators in order to recover the equation of interest. At least one of these potential operators turns out to be non-unitary. As these QLAs have parallelized outstandingly on classical supercomputer architectures,

outperforming standard computational fluid dynamic codes for the study of quantum turbulence, we have proceeded with the numerical implementation of such QLAs. This seems prudent as an error-correcting quantum computer with long qubit coherence times is still on the somewhat distant horizon. Nevertheless we are also pursuing a fully unitary QLA. In particular, we are revisiting our original non-unitary QLA-KdV [1] to determine a fully unitary QLA. We have found a large class of such unitary QLA-KdV, based on changing the particular collide-stream sequences. The implementation of these fully unitary algorithms may not necessarily be straightforward as the perturbation parameter ϵ introduced into the Mathematica symbolic manipulations requires a perturbation in the collision angle of $O\left(\epsilon^2\right)$, Eq. (33), while the continuum limit has scaling proceeds as $O\left(\epsilon^3\right)$, Eq. (34) or (36). In previous QLA for nonlinear physics, the order of the function ψ was used as the ϵ-factor. We believe that understanding the role of ϵ in the QLA-KdV simulations will be pivotal in handling the role of ϵ in QLA-Maxwell in both scalar and tensor dielectric media. These simulations will be reported in a future publication.

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