Technical Supplement (proofs of propositions) to "Does Private Information Lead to Delay or War in Crisis Bargaining?"

Bahar Leventoğlu
Department of Political Science
Duke University
Durham, NC 27708
email: bahar.leventoglu@duke.edu

Ahmer Tarar
Department of Political Science
Texas A&M University
4348 TAMU
College Station, TX 77843-4348
email: ahmertarar@polisci.tamu.edu

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1 Proofs of Propositions

1.1 SPE in Powell's Model

The following proposition characterizes SPE in a generalization of Powell's (1996a, 1996b, 1999: Chapter 3) model in which D and S are allowed to have different discount factors, δ_D and δ_S. The SPE that Powell (1996a, 263) characterizes is the one in which $\delta_S = \delta_D$ and D chooses to always satisfy S's minimal demand when making a proposal.

Proposition 10 *The following are SPE when D is dissatisfied, i.e., when $q < p - c_D$.*
(a) In any period in which D makes a proposal, he (i) proposes $x^* = q(1-\delta_S) + \delta_S(p - c_D)$ if $\delta_D < \delta_S$, (ii) proposes some $x > x^*$ if $\delta_D > \delta_S$ (this proposal is rejected and agreement is reached on y^* in the next period), and (iii) is indifferent between proposing x^* and proposing some $x > x^*$ if $\delta_D = \delta_S$ (this latter proposal is rejected and agreement is reached on y^* in the next period), and hence can be choosing either (or mixing). He always accepts any offer $(y, 1-y)$ such that $y \geq p - c_D$. In any period in which he gets a lower offer than this, he fights (does not continue to the next period).
(b) S always proposes $(y^*, 1-y^*)$ where $y^* = p - c_D$, and always accepts any offer $(x, 1-x)$ such that $x \leq q(1-\delta_S) + \delta_S(p - c_D)$. In any period in which she gets a worse offer than this, she continues to the next period (does not fight).

Proof: First consider S's decisions. Given D's acceptance rule, S is strictly best off proposing $y^* = p - c_D$ whenever she makes a proposal (if S proposes a lower y, D chooses war, which is strictly worse for S). Now consider periods in which D makes an offer. We have just shown that S's optimal continuation value for moving to the next period is $(1-q) + \frac{\delta_S(1-y^*)}{1-\delta_S}$. If she goes to war instead, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. It is easy to show that the former is strictly greater than the latter, and hence S cannot credibly reject any proposal $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq (1-q) + \frac{\delta_S(1-y^*)}{1-\delta_S}$, or $x \leq q(1-\delta_S) + \delta_S(p - c_D)$, and she must move to the next period rather than fight if she gets a worse offer.

Now consider D's decisions. Given S's acceptance rule, the best (for himself) acceptable proposal that D can make in a period in which he makes a proposal is $x^* = q(1-\delta_S) + \delta_S(p - c_D)$. (Because this is strictly greater than q, if that agreement will eventually be reached, it

is strictly better to reach it now rather than in a later period.) D's other option is to propose some $x > x^*$, which is rejected, and in the next period, S proposes $y^* = p - c_D$, which D can (i) accept, (ii) reject it and fight, or (iii) reject it and make a counteroffer. In case (iii), D is back in the same scenario that he is in the current period, except that he has received q for two periods. Since $x^* > q$, offering x^* in the current period is strictly better than ending up in case (iii). Cases (i) and (ii) give D the same total payoff of $q + \frac{\delta_D(p-c_D)}{1-\delta_D}$. Therefore, D is strictly best off offering x^* in the current period rather than some $x > x^*$ if and only if $\frac{x^*}{1-\delta_D} > q + \frac{\delta_D(p-c_D)}{1-\delta_D}$, which can be shown to hold if and only if $\delta_D < \delta_S$. If $\delta_D > \delta_S$, then D strictly prefer to offer some $x > x^*$ (which is rejected and leads to agreement being reached on $y^* = p - c_D$ in the next period), and if $\delta_D = \delta_S$, then D is indifferent between proposing x^* and some $x > x^*$, and hence can be choosing either, or mixing.

Now consider a period in which S makes an offer. If D chooses to fight upon receiving a low offer, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to move to the next period instead, he either (depending on the relative values of δ_S and δ_D) (i) finds it optimal to offer x^* (if $\delta_D \leq \delta_S$), which is accepted, or (ii) finds it optimal to propose some $x > x^*$ (if $\delta_D \geq \delta_S$), which is rejected. In case (ii), D is back in the same position that he is in the current period (in which S makes an offer), except that he has received q for two periods, and he can get at best $y^* = p - c_D$ in the period that he now finds himself in. Since $p - c_D > q$ and S's strategy does not change, if case (ii) holds, D is strictly better fighting rather than saying no if gets a low offer in the current period, and therefore he cannot credibly reject any offer $(y, 1-y)$ such that $y \geq p - c_D$. If case (i) holds, then D's optimal continuation value for moving to the next period is $q + \frac{\delta_D x^*}{1-\delta_D}$. It is easy to show that $\frac{p-c_D}{1-\delta_D}$ is strictly greater than this, and hence D cannot credibly reject any proposal $(y, 1-y)$ such that $y \geq p - c_D$, and must choose to fight if he gets a lower offer. Q.E.D.

1.2 Proof of Proposition 1

First consider D's decisions. Given S's acceptance rule, D is strictly best off proposing $x^* = p + c_S$ whenever he makes a proposal, as this is the best possible payoff he can effectively get in the model (given that S can choose to fight instead of accepting a worse offer, and if

S chooses to fight, D's payoff is $p - c_D$, which is strictly worse than $p + c_S$). Now consider a period in which S makes an offer. If she makes a low offer and D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to say no instead, we have just shown that his optimal continuation value is $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that if D says no, S chooses to pass rather than fight). For the upper bound on δ_D in this equilibrium, the former is greater than the latter, and hence D cannot credibly reject any offer $(y, 1-y)$ such that $y \geq p - c_D$, and must be choosing to fight rather than say no if he gets a worse offer. Now suppose D has made an offer and S has said no. If D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to pass instead, we have just shown that his payoff is $q + \frac{\delta_D y^*}{1-\delta_D}$. The former is strictly greater than the latter, and hence D must be choosing to fight.

Now consider S's decisions. Given D's acceptance rule, S is strictly best off proposing $y^* = p - c_D$ (if S makes a lower offer, D chooses to fight, in which case S is strictly worse off). Now consider a period in which D makes an offer. Since D is choosing to fight if S says no to his offer, S cannot credibly reject any offer than gives her at least her utility from war, i.e., she cannot credibly reject any offer $(x, 1-x)$ such that $x \leq p + c_S$. If she gets a worse offer, she is indifferent between fighting and saying no, since in the latter case, D chooses to fight anyway. Therefore, she can be doing either, or she can be mixing. Now suppose S has made an offer and D has said no. If S chooses to fight, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. If she chooses to pass instead, we have just shown that her payoff is $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. The latter is strictly greater than the former, and hence S must be choosing to pass. Q.E.D.

1.3 Proof of Proposition 2

First consider D's decisions. The same argument as above shows that D is strictly best off proposing $x^* = p + c_S$ whenever he makes a proposal, given S's acceptance rule. Now consider a period in which S makes an offer. If she makes a low offer and D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to say no instead, we have just shown that his optimal continuation value is $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that if D says no, S chooses to pass rather than fight). For the lower bound on δ_D in this equilibrium, the latter is greater than the former, and hence D cannot credibly reject any offer $(y, 1-y)$ such that $\frac{y}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $y \geq q(1-\delta_D) + \delta_D(p + c_S)$,

and must choose to say no rather than fight if he gets a worse offer. Now suppose D has made an offer and S has said no. If D chooses to fight, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he chooses to pass instead, his payoff is $q + \frac{\delta_D y^*}{1-\delta_D}$. (This is based on the assumption that S proposes y^* in the next period. As we show below, depending on the size of δ_S relative to δ_D, S may or may not find it optimal to offer y^* in the next period — however, whatever S chooses to do, D's average per-period payoff in the subgame beginning in the next period is y^* (this is because y^* is the offer that makes him just indifferent between accepting it and moving to the next period and reaching agreement on x^* therein), and hence this argument is fine.) For the upper bound on δ_D in this equilibrium, the former is greater than the latter, and hence D chooses to fight rather than pass.

Now consider S's decisions. S cannot credibly reject any offer that gives her at least her utility from war, i.e., any offer $(x, 1-x)$ such that $x \leq p + c_S$. This is because D chooses to fight if S says no to his offer. If S gets a worse offer, she is indifferent between fighting and saying no (since in the latter case D fights anyway), and hence can be doing either, or mixing. Now suppose D has said no to S's offer. We have just shown that S's continuation value for passing is $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. This is strictly greater than her payoff $\frac{1-p-c_S}{1-\delta_S}$ for fighting, and hence S must be passing rather than fighting. Now consider periods in which S makes an offer. Given D's acceptance rule, the best possible (for herself) acceptable agreement that S can propose in the current period is $y^* = q(1-\delta_D) + \delta_D(p + c_S)$, for a total payoff of $\frac{1-y^*}{1-\delta_S}$. Setting this greater than her payoff for proposing a lower y that is rejected and leads to agreement being reached in the next period, $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$, and simplifying, we obtain $\delta_S > \delta_D$. Hence, if $\delta_S > \delta_D$, S is strictly best off proposing y^*. If $\delta_S < \delta_D$, S is strictly best off proposing some $y < y^*$ which is rejected and leads to agreement being reached on x^* in the next period. If $\delta_S = \delta_D$, then S is indifferent between proposing y^* and some $y < y^*$, and hence can be choosing either, or mixing. Q.E.D.

1.4 Proof of Proposition 3

Note that in this proof, we use the "one-stage-deviation principle," henceforth OSDP, for infinite horizon games with discounting of future payoffs (Fudenberg and Tirole 1991, 108-110). This principle states that, to verify that a profile of strategies comprises a SPE, one

just has to verify that, given the other players' strategies, no player can improve her payoff at any history at which it is her turn to move by deviating from her equilibrium strategy at that history and then reverting to her equilibrium strategy afterwards.

We want to look for a SPE in which D is mixing between passing and fighting, at any decision node at which S has said no to D's offer.[1] Suppose that in this (supposed) SPE, D's average per-period payoff for the subgame beginning in the next period (in which S makes an offer) is y'. Then, for mixing to be okay, it must be the case that D is indifferent between fighting and passing, i.e., $\frac{p-c_D}{1-\delta_D} = q + \frac{\delta_D y'}{1-\delta_D}$, or $y' = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$. It is easy to verify that, given S's strategy and D's strategy for the rest of the game, D's average per-period payoff for the subgame beginning in the next period is indeed y', and hence D's strategy of mixing at this stage satisfies the OSDP. Therefore, suppose that D is choosing to fight with some probability $\beta \in (0,1)$ and pass with probability $1-\beta$.

Now consider when D has to make a proposal. Given S's acceptance rule, the best D can do if he wants an agreement to be reached in the current period is to propose $x^* = \frac{(p-c_D)-q(1-\delta_D^2)}{\delta_D^2}$. If he proposes some bigger x, S says no and D's expected payoff (if he then uses his equilibrium strategy for the rest of the game) is $\beta(\frac{p-c_D}{1-\delta_D}) + (1-\beta)[q + \frac{\delta_D y'}{1-\delta_D}]$. Setting $\frac{x^*}{1-\delta_D}$ strictly greater than the latter and simplifying, we obtain $q < p - c_D$, which is true (note that β drops out of the simplification, so this is true for any value of β). Therefore, D cannot profitably deviate from proposing x^*, and then revert to his equilibrium strategy, and hence D's strategy satisfies the OSDP at histories at which D makes a proposal.

Now suppose S has just made an offer to D. If D fights, his payoff is $\frac{p-c_D}{1-\delta_D}$. If he says no instead, then, according to his equilibrium strategy for the rest of the game, his overall payoff will be $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that S's strategy is to pass if D says no, and so the next period will be reached). Setting the latter strictly greater than the former and simplifying,

[1] A natural way to establish continuity with Propositions 1 and 2 would be if, when δ_D is high, as in Proposition 3, D chooses to pass with certainty even when S rejects his offer. In Proposition 2, δ_D is high enough that D chooses to say no (rather than fight) if S makes too low an offer, but still low enough that he prefers to fight if S rejects D's offer (his continuation value for moving to the next period is higher in the former case than in the latter, since in the former case he gets to make the proposal in the next period). So, it would be natural to expect that when δ_D is even higher, as in Proposition 3, D would choose to pass with certainty even when S rejects his offer. However, it turns out that this behavior cannot be supported as part of a stationary SPE, and instead D starts passing with positive probability, and this probability begins from zero and approaches one as the players' discount factors approach one. So, Proposition 3 *is* a natural continuation of Propositions 1 and 2, but uses a mixed strategy.

we obtain $q < p - c_D$, which is true. Therefore, D is strictly better off saying no rather than fighting, if S's offer is too small. Therefore, he cannot do any better (assuming that he uses his equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(y, 1-y)$ such that $\frac{y}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $y \geq \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$, and say no (rather than fight) if he gets a lower offer.[2] We have thus verified that D's strategy satisfies the OSDP, i.e., there exists no history at which D can profitably deviate from his equilibrium strategy at that stage and then revert back to his equilibrium strategy. Now we have to verify that the same is true for S.

Suppose D has just said no to S's offer. If S fights, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. If she passes instead and follows her equilibrium strategy in the future, her payoff is $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. Setting the latter strictly greater than the former and simplifying, we obtain $\delta_D^2 > \frac{\delta_S[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our restriction in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$, and hence S's strategy satisfies the OSDP at this stage.

Now consider periods in which S makes a proposal. Given D's acceptance rule, the most favorable (for herself) acceptable agreement that S can propose is $y^* = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$, leaving her with an overall payoff of $\frac{1-y^*}{1-\delta_S}$. If she instead proposes some $y < y^*$, D rejects it and agreement is reached on x^* in the next period (assuming that S follows her equilibrium strategy), giving S an overall payoff of $(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. Setting the former strictly greater than the latter and simplifying, we obtain $\delta_S > \delta_D$. Hence, (i) when $\delta_S > \delta_D$, S is strictly better off proposing y^* rather than doing something else and then reverting to her equilibrium strategy, (ii) when $\delta_S < \delta_D$, she is strictly better off proposing some $y < y^*$ rather than doing something else and then reverting to her equilibrium strategy, and (iii) when $\delta_S = \delta_D$, she is indifferent between proposing y^* and some $y < y^*$, and hence can be choosing either, or mixing. (And, this is strictly preferred to doing something else, namely proposing some $y > y^*$, and then reverting to her equilibrium strategy, although the latter point is moot because the agreement will be accepted and the game will end.) Therefore, we have verified that S's strategy satisfies the OSDP at histories at which she makes a proposal.

[2]Note that the only acceptance rule which is as good as this one for *any* offer by S, i.e., at *any* history at which S has just made an offer to D, is to accept any proposal $(y, 1-y)$ such that $y > \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$, and say no (rather than fight) if he gets a lower offer.

Finally, we need to verify that S's acceptance rule satisfies the OSDP. We consider the three cases in turn.

Case (i): $\delta_S > \delta_D$

Consider a period in which D makes an offer. According to S and D's equilibrium strategies, in the next period, agreement would be reached on y^* (since $\delta_S > \delta_D$). Therefore, in the current period, S's continuation value for saying no (if she uses her equilibrium strategy in the future) is $\beta[\frac{1-p-c_S}{1-\delta_S}]+(1-\beta)[(1-q)+\frac{\delta_S(1-y^*)}{1-\delta_S}]$. If she fights instead, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. Setting the former strictly greater than the latter and simplifying, we obtain $\delta_D > \frac{\delta_S[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our restriction in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$. Therefore, S is strictly better off saying no rather than fighting, if she gets a low offer. Therefore, she cannot do any better (assuming she uses her equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq \beta[\frac{1-p-c_S}{1-\delta_S}]+(1-\beta)[(1-q)+\frac{\delta_S(1-y^*)}{1-\delta_S}]$, and say no (rather than fight) if she gets a worse offer. Setting this equivalent to the acceptance rule described in the statement of the proposition (namely, $x \leq \frac{(p-c_D)-q(1-\delta_D^2)}{\delta_D^2}$) and solving for β, we obtain $\beta = \frac{(1-\delta_S\delta_D)[(p-c_D)-q]}{\delta_D\{\delta_D[(p+c_S)-q]-\delta_S[(p-c_D)-q]\}} \in (0,1)$. That is, when β takes on this value, S's acceptance rule as described in the proposition satisfies the OSDP at any history at which D has just made an offer to S. Note that $\beta \to 1$ (from below) as $\delta_D \to \frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]}$ (from above). That is, this equilibrium converges to that of Proposition 2. Also note that our requirement in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$ means that $\beta > 0$ always. As $\delta_S, \delta_D \to 1$ (from below), $\beta \to 0$ (from above).

Case (ii): $\delta_S < \delta_D$

Consider a period in which D makes an offer. According to S and D's equilibrium strategies, in the next period, S will propose some $y < y^*$, which D rejects, and agreement will be reached on x^* in the following period. Therefore, in the current period, S's continuation value for saying no (if she uses her equilibrium strategy in the future) is $\beta[\frac{1-p-c_S}{1-\delta_S}]+(1-\beta)[(1-q)+\delta_S(1-q)+\frac{\delta_S^2(1-x^*)}{1-\delta_S}]$. If she fights instead, her payoff is $\frac{1-p-c_S}{1-\delta_S}$. Setting the former strictly greater than the latter and simplifying, we obtain $\delta_D^2 > \frac{\delta_S^2[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our restriction in this proposition that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$. Therefore, S is strictly better off saying no rather than fighting, if she gets a low offer. Therefore, she cannot do any better (assuming

she uses her equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq \beta[\frac{1-p-c_S}{1-\delta_S}] + (1-\beta)[(1-q) + \delta_S(1-q) + \frac{\delta_S^2(1-x^*)}{1-\delta_S}]$, and say no (rather than fight) if she gets a worse offer. Setting this equivalent to the acceptance rule described in the statement of the proposition and solving for β, we obtain $\beta = \frac{(1-\delta_S^2)[(p-c_D)-q]}{\delta_D^2[(p+c_S)-q]-\delta_S^2[(p-c_D)-q]} \in (0,1)$. Note that $\beta \to 1$ (from below) as $\delta_D \to \frac{(p-c_D)-q}{\delta_D[(p+c_S)-q]}$ (from above). That is, this equilibrium converges to that of Proposition 2. Also note that $\beta > 0$ always, since $\delta_D > \delta_S$. As $\delta_S, \delta_D \to 1$ (from below), $\beta \to 0$ (from above).

Case (iii): $\delta_S = \delta_D = \delta$

Consider a period in which D makes an offer. According to S and D's equilibrium strategies, S's continuation value for saying no in the current period (regardless of whether she chooses to propose y^* or some $y < y^*$, or mix, in the next period) is $\beta[\frac{1-p-c_S}{1-\delta}]+(1-\beta)[(1-q) + \frac{\delta(1-y^*)}{1-\delta}]$. If she fights instead, her payoff is $\frac{1-p-c_S}{1-\delta}$. Setting the former strictly greater than the latter and simplifying, we obtain $c_S + c_D > 0$, which is true. Therefore, S is strictly better off saying no rather than fighting, if she gets a low offer. Therefore, she cannot do any better (assuming she uses her equilibrium strategy in the future) than use the acceptance rule of accepting any offer $(x, 1-x)$ such that $\frac{1-x}{1-\delta} \geq \beta[\frac{1-p-c_S}{1-\delta}] + (1-\beta)[(1-q) + \frac{\delta(1-y^*)}{1-\delta}]$, or $x \leq \beta(p+c_S) + (1-\beta)(p-c_D)$, and say no (rather than fight) if she gets a worse offer. Setting this equivalent to the acceptance rule described in the statement of the proposition and solving for β, we obtain $\beta = \frac{(1-\delta^2)[(p-c_D)-q]}{\delta^2(c_D+c_S)} \in (0,1)$. Note than $\beta \to 1$ (from below) as $\delta \to \frac{(p-c_D)-q}{\delta[(p+c_S)-q]}$ (from above), and hence this equilibrium converges to that of Proposition 2. Also note that $\beta \to 0$ (from above) as $\delta \to 1$ (from below).

Therefore, we have verified that D and S's strategies satisfy the OSDP at any history at which it is their turn to move. Q.E.D.

1.5 Proof of Proposition 4

The SPE characterized in Proposition 3 are stationary, except that when $\delta_S \leq \delta_D$, S can be choosing different actions (among which she is indifferent) at different histories (but that lead to structurally identical subgames) at which it is her turn to make an offer, and this allows for non-stationarity (but D and S's payoffs are the same in all of these SPE).

It turns out that when δ_D is high, there are also SPE that are non-stationary in a more genuine sense. Suppose that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$, so that Proposition 3 holds. Consider the model in which D makes the first offer. Suppose that, in the subgame beginning in the second period, D and S use the strategies of Proposition 3, which we already know are best responses to each other. Then, in the last decision node of the first period, D is indifferent between passing and fighting, and so suppose D is choosing to fight with certainty (as opposed to fighting with probability β, as in Proposition 3). Then, in the first period, S's acceptance rule must be to accept any proposal $(x, 1-x)$ such that $x \leq p + c_S$, and so in the first period, D optimally proposes $x^* = p + c_S$. Hence, in Figure 6, there exist non-stationary SPE when δ_D is high in which x^* remains at $p + c_S$, rather than gradually decreasing to $p - c_D$, as in the stationary SPE.

In fact, we can suppose that in the last decision node of the first period, D fights with some probability λ and passes with probability $1 - \lambda$. When $\lambda = 1$, we are in the SPE described above, and when $\lambda = \beta$, we are in the stationary SPE of Proposition 3. As λ decreases, D's proposal for himself in the first period, x^*, decreases. When $\delta_S \geq \delta_D$ (so that agreement will be reached on $y^* = \frac{(p-c_D)-q(1-\delta_D)}{\delta_D}$ in the second period — see Proposition 3), then in the first period, S accepts all agreements $(x, 1-x)$ such that $\frac{1-x}{1-\delta_S} \geq \lambda[\frac{1-p-c_S}{1-\delta_S}] + (1-\lambda)[(1-q) + \frac{\delta_S(1-y^*)}{1-\delta_S}]$, and says no (rather than fight) for any worse offer.

This can be simplified to obtain that in the first period, S accepts all offers $(x, 1-x)$ such that $x \leq x^*$, where $x^* = \lambda[(p + c_S) - q] + \delta_S y^*(1-\lambda) + q(1-\delta_S) + \lambda q \delta_S$. When $\lambda = 1$, $x^* = p + c_S$, and when $\lambda = 0$, $x^* = \delta_S y^* + q(1-\delta_S)$. (And, since x^* is a continuous function of λ, any value of x^* in between these two extreme values can be obtained, for the right value of λ.) $\frac{\partial x^*}{\partial \lambda} > 0$ can be simplified to obtain $\delta_D > \frac{\delta_S[(p-c_D)-q]}{(p+c_S)-q}$, which is implied by our stipulation that $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$. Therefore, what S allows D to keep for himself in the first period is increasing in λ, which makes intuitive sense. (Also note that $\frac{\partial x^*}{\partial \delta_S} = 0$ when $\lambda = 1$ and $\frac{\partial x^*}{\partial \delta_S} > 0$ when $\lambda < 1$.)

Therefore, in the model in which D makes the first offer, there exist non-stationary SPE when $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$ (the binding condition of Proposition 3) and $\delta_S \geq \delta_D$ in which D's offer to himself (which S accepts) in the first period ranges from a minimum of $x^* = \delta_S y^* + q(1-\delta_S)$

to a maximum of $x^* = p + c_S$. Note that when $\lambda = 0$ and $\delta_S = \delta_D$, then $x^* = p - c_D$, i.e., D offers himself just his payoff from war. Of course, x^* can never be lower than $p - c_D$ in a SPE.

Now consider the model in which S makes the first offer. Suppose that, beginning in the third period and forever afterwards, both players use the strategies of Proposition 3. Then, in the last stage of the second period, D can be choosing to fight with any probability $\lambda \in [0, 1]$, and hence his payoff in the second period ranges from a minimum of $x^* = \delta_S y^* + q(1 - \delta_S)$ to a maximum of $x^* = p + c_S$. Now consider the first period. If $x^* = p + c_S$ in the second period (i.e., $\lambda = 1$), then our restriction on δ_D means that, in the first period, D must be accepting any offer $(y, 1 - y)$ such that $y \geq q(1 - \delta_D) + \delta_D(p + c_S)$, and saying no (rather than fight) for any lower y. Since we have been assuming that $\delta_S \geq \delta_D$, S chooses to offer $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ in the first period. Thus, there exist non-stationary SPE (in which $\lambda = 1$) when $\delta_S \geq \delta_D$ in which, in Figure 6, the trend continues as δ_D moves from medium to large, i.e., $x^* = p + c_S$ and $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ even when δ_D becomes very large. Note that, in these equilibria, as $\delta_D \to 1$, the agreement reached approaches $p + c_S$, i.e., D gets *all* of the gains from avoiding war, regardless of who gets to make the first proposal.

Now suppose that $\lambda = 0$ and $\delta_S = \delta_D$ (the worse possible case for D). Then, in the second period, D will propose $x^* = p - c_D$, and hence, in the first period, D's acceptance rule must be to accept any offer $(y, 1 - y)$ such that $y \geq p - c_D$, and fight (rather than say no) for any lower y. Thus, S proposes $y^* = p - c_D$, and hence S gets *all* of the gains from avoiding war.

Thus, when $\delta_S \geq \delta_D$ and S makes the first offer, the agreement reached in a non-stationary SPE can range from a minimum (for D) of $y^* = p - c_D$ (when $\lambda = 0$ and $\delta_S = \delta_D$) to a maximum of $y^* = q(1 - \delta_D) + \delta_D(p + c_S)$ (when $\lambda = 1$; as $\delta_D \to 1$, this converges to $p + c_S$).

We can generate additional non-stationary SPE. Suppose that in the model in which D makes the first proposal, the two players begin using the strategies of Proposition 3 beginning in the fourth period. Then, in the last stage of the third period, D can be choosing to fight with any probability $\lambda \in [0, 1]$. We can continue to build non-stationary SPE like this. The

key is to stipulate that the players eventually start using the strategies of Proposition 3 in some period in which S makes an offer, and forever afterwards. In the last stage of the period just before then, D can be choosing to fight with any probability $\lambda \in [0,1]$, and the choice of λ uniquely determines what has to occur previous to that stage (i.e., everything before then can be solved using backwards induction). Thus, we can create non-stationary SPE for any value of λ and any number of non-stationary initial periods. Thus, we have a folk-theorem type result when $\delta_D^2 \geq \frac{(p-c_D)-q}{(p+c_S)-q}$ (the binding condition of Proposition 3), in which a whole lot of payoff combinations can be supported as SPE (these payoffs lie between the upper and lower bounds identified earlier, since those bounds are determined by the most and least favorable agreements that D can possibly get in the first period in which he makes a proposal).

1.6 Proof of Proposition 5

We want to construct a PBE in which neither type of D rejects S's initial offer in order to make a counteroffer. Each type accepts all initial offers $(y, 1-y)$ such that y is at least as great as its expected utility from war, and fights (rather than says no) if it gets a lower offer. We also want that if the second period is reached (this is off-the-equilibrium path behavior), the strategies of the players are such that agreement is reached on $x^* = p + c_S$, i.e., D gets all of the gains from avoiding war. First note that if such an agreement were to be reached, then S would be strictly best off passing rather than fighting if D says no to S's initial offer, i.e., $\frac{1-p-c_S}{1-\delta_S} < (1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}$. Then, for type c_{D_l} to be fighting rather than saying no if he gets a low initial offer, it must be that $\frac{p-c_{D_l}}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $\delta_D \leq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$. This also ensures that type c_{D_l} cannot credibly reject any initial offer $(y, 1-y)$ such that $y \geq p - c_{D_l}$. Similarly, for type c_{D_h}'s acceptance rule to be to accept any initial offer $(y, 1-y)$ such that $y \geq p - c_{D_h}$ and go to war (rather than say no) for a lower y, it must be that $\frac{p-c_{D_h}}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$. Since $c_{D_h} > c_{D_l}$, the latter is the binding restriction on δ_D.

Now we need to construct a PBE of the subgame beginning in the second period, which is in never reached in equilibrium, in which agreement is reached on $x^* = p + c_S$. The simplest way to do this is to stipulate that if this subgame is reached, S believes that it is facing type c_{D_l} (the low-cost, or highly resolved, type) with certainty (and that this belief

never changes later on), and that S and type c_{D_l} therefore use the complete information strategies of Proposition 1, which are best responses to each other (note that we could also stipulate that S believes she is facing type c_{D_h} with certainty, and this belief never changes; the argument below would require only minor modifications). This requires the binding condition of Proposition 1 to hold (when D's cost of war is c_{D_l}), namely that $\delta_D \leq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$, which is already implied by our binding condition in the previous paragraph.

Now we need to construct a strategy (in this subgame) for type c_{D_h} that is a best response to S's strategy (which is given by Proposition 1). Given S's acceptance rule, type c_{D_h} is strictly best off proposing $x^* = p + c_S$ whenever he makes a proposal. Now suppose S has just made a low offer to type c_{D_h}. We have just shown that if type c_{D_h} says no, his optimal continuation value is $q + \frac{\delta_D x^*}{1-\delta_D}$ (note that S's strategy is to pass rather than fight if D says no, and hence the next period will be reached). Given the upper bound on δ_D that we have derived earlier, namely that $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, type c_{D_h}'s payoff from war, $\frac{p-c_{D_h}}{1-\delta_D}$, is greater than this, and hence type c_{D_h}'s acceptance rule must be to always accept any offer $(y, 1-y)$ such that $y \geq p - c_{D_h}$, and fight if he gets a lower offer. Finally, suppose S has said no to type c_{D_h}'s offer. Given S's proposal and the acceptance rule we have just derived for type c_{D_h}, the latter's continuation value for passing is $q + \frac{\delta_D(p-c_{D_l})}{1-\delta_D}$. Given the upper bound on δ_D that we have derived earlier, namely that $\delta_D \leq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, type c_{D_h}'s payoff from war, $\frac{p-c_{D_h}}{1-\delta_D}$, is strictly greater than this, and hence type c_{D_h} must always be choosing to fight rather than pass. This completes the description of type c_{D_h}'s best response to S's strategy.

All that remains is to specify the optimal offer that S makes in the first period of the game. Given the acceptance rules of types c_{D_l} and c_{D_h}, S's best response is either to make the big offer $y^* = p - c_{D_l}$, which both types accept (and so war is avoided with certainty), or to make the lower offer $y^* = p - c_{D_h}$, which only type c_{D_h} accepts. Type c_{D_l} rejects it and goes to war. It is easy to see that no other proposal can be a best response. If $0 < s < 1$ is the prior probability that D is of type c_{D_l}, then making the big offer is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[\frac{1-(p-c_{D_h})}{1-\delta_S}]$, or $s \geq \frac{c_{D_h}-c_{D_l}}{c_{D_h}+c_S} \in (0,1)$. Q.E.D.

1.7 Proof of Proposition 6

This equilibrium is similar to the previous one in that, if the second period is reached, agreement is reached on $x^* = p + c_S$. However, because we have stipulated in this proposition that $\delta_D \geq \frac{(p-c_{D_h})-q}{(p+c_S)-q}$, in the first period, if type c_{D_h} gets a low initial offer, he prefers to move to the second period and get x^* rather than fight. Thus, his acceptance rule in the first period must be to accept any offer $(y, 1-y)$ such that $\frac{y}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $y \geq q(1-\delta_D)+\delta_D(p+c_S)$, and say no (rather than fight) for any lower y. Because we have stipulated in this proposition that $\delta_D \leq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$, type c_{D_l} prefers to go to war if he gets a low initial offer rather than get x^* in the next period, and so his acceptance must be to accept all initial offers $(y, 1-y)$ such that $y \geq p - c_{D_l}$, and go to war for any lower y. Because agreement will be reached on x^* in the next period, if D says no to S's initial offer, S is strictly better off passing rather than fighting.

If the second period is reached, we stipulate that S believes with certainty that she is facing type c_{D_h}, and this belief never changes. (If the second period is reached on-the-equilibrium path, this belief follows from Bayes' rule, and if it is reached off-the-equilibrium path, we as the analyst stipulate that this is S's belief, since Bayes' rule does not apply. This off-the-equilibrium path belief is quite reasonable, because type c_{D_h}'s payoff from war is lower and hence he is more likely to say no rather than go to war than type c_{D_l}.) Since we have stipulated in this proposition that $\frac{(p-c_{D_h})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, the conditions for Proposition 2 are satisfied (when D is of type c_{D_h}), and hence we stipulate that, beginning in the second period, S and type c_{D_h} play the strategies of Proposition 2, which are best responses to each other. Because S is using the strategy of Proposition 2, agreement will indeed be reached on $x^* = p + c_S$, which we have been assuming. It is easy to construct type c_{D_l}'s best response to S's strategy, in the subgame beginning in the second period.

All that remains is to determine S's optimal offer in the first period. She can either make the large offer $y^* = p - c_{D_l}$, which both types of D accept, or make a smaller offer $y \leq q(1-\delta_D) + \delta_D(p + c_S)$, which type c_{D_l} rejects and goes to war. It is easy to see that no other offer can be a best response. We know from our proof of Proposition 2 that if $\delta_S \geq \delta_D$, then if S prefers to make the small offer, she prefers to offer exactly $y^* = q(1-\delta_D)+\delta_D(p+c_S)$,

whereas if $\delta_S \leq \delta_D$, then if S prefers to make the small offer, she prefers to offer some $y < y^*$, so that agreement is reached on x^* in the next period (if D turns out to be type c_{D_h}). We consider the two cases in turn.

If $\delta_S \geq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[\frac{1-q(1-\delta_D)-\delta_D(p+c_S)}{1-\delta_S}]$, or $s \geq \frac{(p-c_{D_l})-[q(1-\delta_D)+\delta_D(p+c_S)]}{(p+c_S)-[q(1-\delta_D)+\delta_D(p+c_S)]} \in [0,1)$.

If $\delta_S \leq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}]$, or $s \geq \frac{(p-c_{D_l})-[q(1-\delta_S)+\delta_S(p+c_S)]}{(p+c_S)-[q(1-\delta_S)+\delta_S(p+c_S)]} \in [0,1)$. Q.E.D.

1.8 Proof of Proposition 7

We want to construct a risk-return tradeoff equilibrium even when δ_D is high. That is, type c_{D_l} goes to war rather than saying no if he gets a low initial offer. We stipulate that if the second period is reached, S believes that it is facing type c_{D_h} with certainty (on-the-equilibrium path, this will follow from Bayes' rule, and off-the-equilibrium path, this is a reasonable stipulation, for the same reason as in the previous proof), and that this belief never changes. Therefore, since we want to allow δ_D to be high, we stipulate that in the subgame beginning in the second period, S and type c_{D_h} use the strategies of Proposition 3, which are best responses to each other if $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, which we therefore stipulate to hold in this proposition. Therefore, we know from the proof of Proposition 3 that type c_{D_h}'s acceptance rule in the first period is optimal, given that he expects agreement to be reached on $x^* = \frac{(p-c_{D_h})-q(1-\delta_D^2)}{\delta_D^2}$ if the second period is reached.

Now consider type c_{D_l}'s behavior in the first period. He knows that if the second period is reached, S adopts the strategy of Proposition 3, treating D as if he is of type c_{D_h} (since S believes this with certainty). What is type c_{D_l}'s best response to this (in the subgame beginning in the second period)? It is either to propose $x^* = \frac{(p-c_{D_h})-q(1-\delta_D^2)}{\delta_D^2}$, or to propose some $x > x^*$ which S rejects, and then go to war. (Since type c_{D_h} is indifferent between passing and fighting if S rejects D's offer in the second period, type c_{D_l}, whose payoff from war is strictly higher, strictly prefers to fight rather than pass.) He prefers to offer x^* if $\frac{x^*}{1-\delta_D} \geq \frac{p-c_{D_l}}{1-\delta_D}$, or $\delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]}$. On the other hand, if $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]}$, then c_{D_l} prefers

to instigate war in the second period rather than offer x^*.

Note that $\frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]} > \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$. Therefore, suppose that $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]}$, so that c_{D_l} is best off instigating war in the subgame beginning in the second period. He is strictly better off going to war in the first period than in the second period, and therefore his acceptance rule in the first period is fine. Now suppose that $\frac{(p-c_{D_h})-q}{\delta_D[(p-c_{D_l})-q]} \geq \delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, so that c_{D_l} is best off proposing x^* in the subgame beginning in the second period. Then, c_{D_l}'s acceptance rule in the first period is fine as long as $\frac{p-c_{D_l}}{1-\delta_D} \geq q + \frac{\delta_D x^*}{1-\delta_D}$, or $\delta_D \geq \frac{(p-c_{D_h})-q}{(p-c_{D_l})-q}$. Therefore, as long as $\delta_D \geq max\{\frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}, \frac{(p-c_{D_h})-q}{(p-c_{D_l})-q}\}$, type c_{D_l}'s acceptance rule in the first period is fine, and therefore we stipulate this to hold in this proposition.

Note that, if D says no to S's initial offer, the proof of Proposition 3 shows that S is strictly better off passing rather than fighting, since she expects agreement to be reached on x^* in the next period.

All that remains is to determine S's optimal offer in the first period. She can either make the large offer $y^* = p - c_{D_l}$, which both types of D accept, or make a smaller offer $y \leq \frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}$, which type c_{D_l} rejects and goes to war. It is easy to see that no other offer can be a best response. We know from the proof of Proposition 3 that if $\delta_S \geq \delta_D$, then if S prefers to make the small offer, she prefers to offer exactly $y^* = \frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}$, whereas if $\delta_S \leq \delta_D$, then if S prefers to make the small offer, she prefers to offer some $y < y^*$, so that agreement is reached on x^* in the next period (if D turns out to be type c_{D_h}). We consider the two cases in turn.

If $\delta_S \geq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[\frac{1-\frac{(p-c_{D_h})-q(1-\delta_D)}{\delta_D}}{1-\delta_S}]$, or $s \geq \frac{[q(1-\delta_D)+\delta_D(p-c_{D_l})]-(p-c_{D_h})}{[q(1-\delta_D)+\delta_D(p+c_S)]-(p-c_{D_h})} \in [0,1)$.

If $\delta_S \leq \delta_D$, then making the large offer $y^* = p - c_{D_l}$ is a best response if and only if $\frac{1-(p-c_{D_l})}{1-\delta_S} \geq s[\frac{1-p-c_S}{1-\delta_S}] + (1-s)[(1-q) + \frac{\delta_S(1-x^*)}{1-\delta_S}]$, or $s \geq \frac{\delta_D^2[(p-c_{D_l})-q]-\delta_S[(p-c_{D_h})-q]}{\delta_D^2[(p+c_S)-q]-\delta_S[(p-c_{D_h})-q]} \in [0,1)$. Q.E.D.

1.9 Proof of Proposition 8

We want to construct a PBE in which agreement is reached on $x^* = p + c_S$ if the second period is reached, and in which both types of D make counteroffers rather than go to war if

S's initial offer is too small. The natural way to do this is to use the results of Proposition 2, in which agreement is reached on $x^* = p + c_S$ whenever D makes a proposal, and D says no rather than fights, if S makes a small offer. The stipulation in this proposition that $\frac{(p-c_{D_l})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$ means that the conditions of Proposition 2 hold for both types of D, i.e., it means that $\frac{(p-c_{D_l})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_l})-q}{\delta_D[(p+c_S)-q]}$ and $\frac{(p-c_{D_h})-q}{(p+c_S)-q} \leq \delta_D \leq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$.

Thus, we simply specify that, beginning from the very first period and continuing forever after, both types of D use the strategy of Proposition 2. Note that this strategy does not depend on D's cost of war in any way, and hence both types are adopting identical strategies. Since they are adopting identical strategies, S's best response is to adopt the strategy of Proposition 2, regardless of the value of s. And, given that S is adopting the strategy of Proposition 2, the best response of both types of D is to use the strategy of Proposition 2.

If the second period is reached on-the-equilibrium path, S's belief will remain at s, by Bayes' rule. If it is reached off-the-equilibrium path, then we specify that S's belief can be anything, and that S as well as both types of D continue to use the strategies of Proposition 2. If the third period is reached (this can only happen off-the-equilibrium path, since both types of D fight if S rejects D's offer in the second period), then the belief can be anything, and everyone continues to use the strategies of Proposition 2, and so on. Q.E.D.

1.10 Proof of Proposition 9

We want to construct a PBE in which both type of D make counteroffers rather than fight, if S's initial offer is too small, and in which δ_D can be very high. Because we want to allow δ_D to be high, we use the results of Proposition 3. In particular, we stipulate that if the third period is reached (it will turn out that this can only happen off-the-equilibrium path), then S believes with certainty that she is facing type c_{D_h} (which, as we have been discussing earlier, is a sensible belief), and this belief never changes. Thus, we stipulate that, beginning in the third period, S and c_{D_h} use the strategies of Proposition 3 (with S treating D as though it is type c_{D_h} with certainty), which are best responses to each other if $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$, which we therefore stipulate to hold in this proposition. Type c_{D_l}'s best response to S's strategy

is easy to construct.

Now consider the last decision node of the second period, in which D has to decide whether to pass or fight. We know from Proposition 3 that type c_{D_h} is indifferent between fighting and passing, since he expects his payoff from Proposition 3 to be obtained in the next period. Since he is indifferent, we stipulate that he chooses to fight with certainty (as opposed to fighting with probability β, as in Proposition 3). Since c_{D_h} is indifferent, type c_{D_l}, whose payoff from war is strictly higher, strictly prefers to fight. Therefore, since both types of D are choosing to fight if S rejects D's offer in the second period, S's acceptance rule in the second period must be to accept any offer $(x, 1-x)$ such that $x \leq p + c_S$, regardless of what her belief about D's type is at that point. Therefore, both types of D are strictly best off proposing $x^* = p + c_S$ in the second period.

Now consider the first period. Our previously derived stipulation that $\delta_D \geq \frac{(p-c_{D_h})-q}{\delta_D[(p+c_S)-q]}$ ensures that type c_{D_h} strictly prefers to say no (and get x^* in the next period) rather than fight if S's initial offer is too small, and therefore c_{D_h}'s acceptance rule in the first period is fine. Type c_{D_l} prefers to say no and obtain x^* in the next period rather than fight if S's initial offer is too low as long as $\delta_D \geq \frac{(p-c_{D_l})-q}{(p+c_S)-q}$, which we therefore stipulate to hold in this proposition, and hence c_{D_l}'s acceptance rule in the first period is fine. The last thing to note is that if D says no to S's initial offer, S is strictly better of passing rather than fighting, since she expects agreement to be reached on x^* in the next period. Q.E.D.

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