Supplimentary Materials: Bird's Decision to Redirect the Migration Path Depends on the Sun and the Moon Position Change (Part 1: American Regional Study)

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ABSTRACT

We take a rigorous directional statistical approach to explore the distributional properties of migrating birds' travel paths across American continents. We start with an existing dataset containing the migratory path of six different bird species, latitude, longitude, and observation date from a bird-path tracking web resource. Based on that, a second dataset containing the directional change of bird path (in terms of angle) and the corresponding spatiotemporal Position of the Sun is compiled using the specified latitude, longitude, date, and other factor. The available bird path dataset is partitioned into 18 optimal sub-region divisions. We establish that each subdivision's latitude and longitude distribution separately follows either circular Von Mises distribution or circular uniform distribution. This finding proposes a directional mixture model comprising Von Mises distribution and uniform distribution for general bird path data. Moreover, we seek a rigorous statistical answer on whether a suitable change in the Position of the Sun at a spot on the Earth directly influences the decision of a bird flying on that spot to change its paths, and the statistical result affirms that. A directional statistical regression analysis approach is also selected, and a novel circular-circular regression model is proposed. Goodness-of-fit test results on that circular-circular regression analysis result support the hypothesis about the dependency of change in birds' migration paths with the corresponding change in the availability of Sun Position. We can also affirm that indirectly, the Milankovitch cycles of the Earth affect bird migration paths because Milankovitch cycles and solar and lunar activity have a deep connection.

1 Appendix 1

1.1 Calculation of the declination of the Sun (δ), the Earth-Sun distance (R_{ES}), and the equation of time (E)

During bird migration, the positions of the sun and moon are crucial for shaping the paths birds take. The sun acts as a compass, helping them orient their flight and keep track of time. At night, the moon and stars serve as reliable reference points for maintaining consistent flight paths. Birds also possess the ability to perceive polarized light, aiding accurate navigation, even in challenging conditions. Additionally, their inherent magnetic compass assists in maintaining a general direction during migration. While celestial cues are vital, birds also consider landmarks, geography, and wind patterns to fine-tune their routes. The study of bird migration remains a captivating field of research, unveiling the extraordinary navigational abilities of these remarkable journeys.

The provided formulas have been obtained directly from their original sources. However, a modification has been made to the time argument 'n' to represent an arbitrary year, preferably falling between 1950 and 2050. These equations have been carefully arranged in a specific order, allowing for the results obtained from earlier expressions to be used in subsequent calculations.

Species	Location(Range)
Swainson's hawk	(-121.96, -36.172) to (-61.23,41.991)
Pacific loon	(-236.3,27.62) to (-111.4, 71.36)
Long-billed curlew	(-107.00,27.43) to (-81.27,51.02)
Brown pelican	(-76.47,35.10) to (-75.51,38.88)
Blackcrowned night heron	(-81.54,22.25) to (-75.38,39.53)
Black-Bellied Plover	(-166.34,25.69) to (-91.27,70.50)

Table 1. Species Specific Partition

Only the equations that hold immediate importance have been assigned specific numbers.[1]

- n = -1.5 + (Y_{in} - 2000) . 365 + N_{leap} + Day of Year + Fraction of Day from 0:00 UT(day)
- L = 280.466 + 0.9856474n(Degree)
- g = 357.528 + 0.9856003n(Degree)
- λ = L + 1.915 $\sin g$ + 0.020 $\sin(2g)$(Degree)
- ε = 23.440 - 0.0000004n (Degree)
- $\alpha = \tan^{-1}(\cos\varepsilon \tan\lambda).180/\pi$ (Degree)
- $\delta = \sin^{-1}(\sin\varepsilon \sin\lambda)$. $180/\pi$(Degree)
- R_{ES} = 1.00014 - 0.01671$\cos g$ - 0.00014 $\cos(2g)$ (Astronomical Unit)
- E_{min} = (L - α).4 (min)

Where:

- n is the number of days of Terrestrial Time(TT) from J2000.0 UT
- Y_{in} is the input year
- L is the mean longitude of the Sun corrected for aberration
- g is the mean anomaly
- λ is the ecliptic longitude
- ε is the obliquity of the ecliptic
- α is the right ascension
- δ is the declination of the Sun
- R_{ES} is the Earth-Sun distance
- E_{min} is the equation of time

Please note that both L and g, as well as λ given above, can have either positive or negative values. However, for computational purposes, they should be constrained to the range of 0°–360°, which can be achieved by using the modulo function α. Additionally, it is essential for α to be in the same quadrant as λ, which can be ensured by using the atan2 function, which takes two arguments, as opposed to the atan function which takes only one.[2]

1.2 Calculation of Julian Day

The Julian day is a continuous count of days since the beginning of the Julian period, which started on January 1, 4713 BCE. It is a widely used system for representing dates in astronomy and other fields. The Julian day for a given date can be calculated using the following formula[3]

$$A = \left(\frac{1461 \times (Y + 4800 + \frac{M-14}{12})}{4} \right)$$

$$B = \left(\frac{367 \times (M - 2 - 12 \times (\frac{M-14}{12}))}{12} \right)$$

$$C = \left(\frac{3 \times \left(\frac{Y+4900+\frac{M-14}{12}}{100} \right)}{4} \right)$$

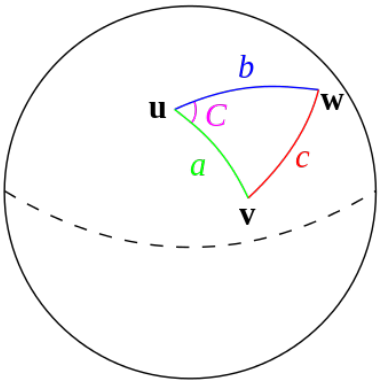

Figure 1. Haversine formula to compute the Column θ, a Directional change in the bird's migration path. u is the north pole, while v and w are the two bird path dataset points whose separation d is to be determined to calculate directional change,θ. In this case, a and b are $\pi/2-\phi_{1,2}$ (the co-latitudes), C is the longitude separation $\lambda_2-\lambda_1$, and c is the desired angle d/R, here $R=$ radius of the Earth.

$$D = (D - 32075)$$

$$J = (A + B - C - D) \tag{1}$$

Where: - J is the Julian Day - Y is the year - M is the month - D is the day of the month

1.3 Von Mises Distribution

A circular random variable θ is said to have a Von Mises distribution or Circular Normal Distribution if it has the probability density function (pdf):

$$f(\theta;\mu,k) = \frac{1}{2\pi I_0(k)} e^{k\cos\theta(\theta-\mu)}, \tag{2}$$

where θ is $[0,2\pi)$, μ is between $[0,2\pi)$ and $(k>0)$. Here $I_0(k)$ in the normalizing constant is the modified Bessel function of the first kind and order zero and is given by

$$\frac{1}{2\pi}\int_1^{2\pi} \exp(k\cos\theta)d\theta = \sum_{r=0}^{\infty}\left(\frac{k}{2}\right)^{2r}\left(\frac{1}{r!}\right)^2 \tag{3}$$

The cumulative distribution of the circular normal or the Von Mises Distribution is obtained from the integration of the pdf. The form of the cdf is

$$F(\theta) = \frac{1}{2\pi I_0(k)}\left(\theta I_0(k) + 2\sum_{p=1}^{\infty}\frac{I_p(k)\sin p(\theta-\mu)}{p}\right), \tag{4}$$

, where θ is in $[0,2\pi)$ Here we used this Von Mises distribution for distribution checking of the given data where the data is giving insight into the Migrating Birds flying over the American Region.

1.4 Wrapped Uniform Distribution

Here the total probability is spread out uniformly on the circumference of a circle. We get the Uniform circular distribution with the constant density: $f(\theta)=\frac{1}{2\pi}$. All directions are equally likely; hence, this is also known as the isotropic or random distribution.

1.5 Watson Test

[4] Provided a statistic for directional data, like the Kolmogorov-Smirnov nonparametric test statistic, to verify one sample and two sample data if the data is following uniform distribution or Von-Mises Distribution. Watson's statistic is defined by:

$${W_n}^2 = \int_0^{2\pi}\left[(F_n - F) - \int_0^{2\pi}(F_n - F)\,dF\right]^2 dF \tag{5}$$

If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are from iid F(α) then $\alpha_{(1)} \leqslant \cdots \leqslant \alpha_{(n)}$ denote the ordered observations(with any starting point and any sense rotation) corresponding to $\alpha_1, \alpha_2, \cdots, \alpha_n$, the empirical distribution function is defined by:
Empirical distribution function:

$$F_n(\alpha) = \begin{cases} 0 & \text{for } \alpha < \alpha_{(1)} \\ \frac{i}{n} & \text{for } \alpha_{(i)} \leqslant \alpha \leqslant \alpha_{(i+1)} \\ 1 & \text{for } \alpha \geqslant \alpha_{(n)} \end{cases}$$

Where F = $F_0(\alpha)$

it can also be written in the form of:

$$W_n{}^2 = \sum_{i=1}^{n} \left[\left(U_{(i)} - \frac{i - \frac{1}{2}}{n} \right) - \bar{U} - \frac{1}{2} \right]^2 + \frac{1}{12n}. \tag{6}$$

Here $U_i = F(\alpha_i)$. The Cramer-Von-Mises statistic can be thought of as the "second moment" of $(F_n - F)$. Watson's statistic is similar to the expression for "variance."

1.6 Circular correlation coefficient

In linear analysis, the correlation between two variables X and Y is defined as:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}} \tag{7}$$

The correlation coefficient has certain properties:

$$-1 \leq \rho(X,Y) \leq 1$$

$$\rho(X,Y) = \rho(Y,X)$$

$$\rho(aX+b, cY+d) = (sgn(a))(sgn(a))(sgn(c))\rho(X,Y) \tag{8}$$

To retain these properties when dealing with circular variables α and β, we introduce the circular correlation coefficient $\rho_c(\alpha,\beta)$, defined as:

$$\rho_c(\alpha,\beta) = \frac{E[\sin(\alpha-\mu)\sin(\beta-\nu)]}{\sqrt{Var[\sin(\alpha-\mu)] \cdot Var[\sin(\beta-\nu)]}} \tag{9}$$

Here, we consider the random sample of observations (α_1,β_1), (α_2,β_2), (α_3,β_3), $\cdots$ (α_n,β_n), where both attributes are measured as angles with reference to the same zero direction and the same sense of rotation. Let $f(\alpha,\beta)$ be the joint probability density function on the torus $0 \leq \alpha < 2\pi$, $0 \leq \beta < 2\pi$. Let μ and ν denote the mean direction of the two variables.

Considering that $E[\sin(\alpha-\mu)] = E[\sin(\beta-\nu)] = 0$, similar to the linear case, we can take $\sin(\alpha-\mu)$ and $\sin(\beta-\nu)$ to represent the deviations of α and β from their mean direction, leading naturally to the circular correlation coefficient ρ_c. Therefore, we can rewrite the equation as:

$$\rho_c(\alpha,\beta) = \frac{E[\cos(\alpha-\beta-\mu+\nu) - \cos(\alpha+\beta-\mu-\nu)]}{2\sqrt{E[\sin^2(\alpha-\mu)] \cdot E[\sin^2(\beta-\nu)]}} \tag{10}$$

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