1 Formal Proofs

1.1 Leader's utility for bias level β

Consider the leader's expected utility for a given level of bias. Note that from the perspective of the leader, foreign repeals a policy after the diplomat transmits a message of opposition with probability $Pr(f < c_F * \frac{F_x(-c_L)}{F_x(-\beta)-F_x(-c_L)})$ While foreign plays a pure strategy, f is not known to the leader when he selects β. For concision we will represent this probability of repeal after a message of opposition as $\gamma(\beta)$. Thus, the leader's expected utility for a level of bias β is:

$$(1 - F_x(-\beta)) * E(x|x > -\beta) + (F_x(-\beta) - F(-c_L)) * (1 - \gamma(\beta)) * E(x| - \beta > x > -c_L) + F_x(-c_L) * (1 - \gamma(\beta)) * (-c_L)$$

We now re-express the truncated expectations by definition and simplify

$$(1 - F_x(-\beta)) * \frac{\int_{-\beta}^{x_H} x f_x(x)dx}{1 - F_x(-\beta)} + (F_x(-\beta) - F_x(-c_L)) * (1 - \gamma(\beta)) * \frac{\int_{-c_L}^{-\beta} x f_x(x)dx}{F_x(-\beta) - F_x(-c_L)} + F_x(-c_L) * (1 - \gamma(\beta)) * (-c_L)$$
$$= \int_{-\beta}^{x_H} x f_x(x)dx + (1 - \gamma(\beta)) * \int_{-c_L}^{-\beta} x f_x(x)dx + (1 - \gamma(\beta)) * F_x(-c_L) * (-c_L)$$

Define β^* as the value of β that maximizes this expression. The leader will select $\beta = \beta^*$ in equilibrium. It is not possible to derive the value β^* without specific distributional assumptions for f_x and f_f, but propositions 1 and 2 are distribution-independent. In order to prove these, it will be useful to take the derivative of this expected utility with respect to β, which gives:

$$- \beta * f_x(-\beta) - (1 - \gamma(\beta)) * (-\beta) * f_x(-\beta) - \gamma'(\beta) * \int_{-c_L}^{-\beta} x f_x(x)dx + \gamma'(\beta) * F_x(-c_L) * (c_L)$$
$$= \gamma(\beta) * (-\beta) * f_x(-\beta) + \gamma'(\beta) * \left[F_x(-c_L) * c_L - \int_{-c_L}^{-\beta} x f_x(x)dx\right]$$

Here, I represent the derivative γ function for concision as γ', but it will be necessary below to expand this term:

$$\gamma'(\beta) = f_f\left(c_F * \frac{F_x(-c_L)}{F_x(-\beta) - F_x(-c_L)}\right) * \left(c_F * \frac{F_x(-c_L) * f_x(-\beta)}{[F_x(-\beta) - F_x(-c_L)]^2}\right)$$

1.2 Proof of Proposition 1

Recall that it is shown above that there is no maximum for $\beta < 0$, thus it is sufficient to show that the derivative of the leader's expected utility with respect to β is positive at $\beta = 0$, in which case $\beta = 0$ is not a maximum.

Given $\beta = 0$, the generic derivative from the section above becomes:

$$= \gamma(0) * 0 * f_x(0) + \gamma'(0) * \left[F_x(-c_L) * c_L - \int_{-c_L}^{0} x f_x(x)dx\right]$$
$$= 0 + \gamma'(0) * \left[F_x(-c_L) * c_L - \int_{-c_L}^{0} x f_x(x)dx\right]$$

Note that by assumption $c_L > 0$, so the term in the brackets is always positive. Consequently, it is sufficient to show $\gamma'(0) > 0$.

Taking the γ' function from above:

$$\gamma'(0) = f_f\left(c_F * \frac{F_x(-c_L)}{F_x(0) - F_x(-c_L)}\right) * \left(c_F * \frac{F_x(-c_L) * f_x(0)}{[F_x(0) - F_x(-c_L)]^2}\right)$$

By assumption $c_L > 0$ and $c_F > 0$ and f_x has full support over the appropriate interval, the second term here is certainly positive. Given the assumption $f_H > c_F * \frac{F_x(-c_L)}{F_x(0)-F_x(-c_L)}$ and the full support assumption for f_f, this is satisfied.

Note, however, that if $f_H < c_F * \frac{F_x(-c_L)}{F_x(0)-F_x(-c_L)}$, then $\beta = 0$ will be the equilibrium choice of diplomat. This occurs because given f_H below the threshold, a diplomat of bias 0 has automatic credibility. That is $Pr(f < c_F * \frac{F_x(-c_L)}{F_x(0)-F_x(-c_L)}) = 1$ and foreign *always* revises in response to a message of opposition.

1.3 Proof of Proposition 2

To begin the proof, define β_g as the "guaranteed credibility" level of β. Substantively, this is the value of β for which messages of opposition lead the opponent to revise its policy with certainty. Mathematically, β_g is the value such that:

$$f_H = c_F * \frac{F_x(-c_L)}{F_x(-\beta_g) - F_x(-c_L)}$$

Given $f_H > 0$ (as assumed), we know $\beta_g < c_L$.

Given β_g defined in this way, we know that, for $\beta \geq \beta_g$, $\gamma(\beta) = 1$ and $\gamma'(\beta) = 0$. As such, for all values of β above β_g (including c_L), the generic derivative above becomes:

$$\gamma(\beta) * (-\beta) * f_x(-\beta) + \gamma'(\beta) * \left[F_x(-c_L) * c_L - \int_{-c_L}^{-\beta} x f_x(x) dx\right]$$
$$= -\beta * f_x(-\beta)$$

This is certainly negative. Thus, no value above β_g can be a maximum.

Note, however, that this does not rule out the existence of a maximum at $\beta^* = \beta_g$, which will depend on the density of f_f.

Substantively, a maximum at β_g requires high density at f_H, which appears implausible. That is, it would appear reasonable to make a an assumption such as $\lim_{f \to f_H} f_f(f) = 0$, which eliminates the possibility of a maximum at β_g.

1.4 Proof of Proposition 3

Consider the game where foreign rather than the leader selects β. Recall that for any $\beta \geq c_L$, the diplomat will transmit a message of opposition given $x < -c_L$ and a message of support otherwise. For any $\beta < c_L$, the diplomat transmits a message of opposition given $x < -\beta$ and a message of opposition otherwise.

Thus,for a diplomat with bias $\beta \geq c_L$, foreign always receives a message of opposition and revises the policy given $x < -c_L$ (for a utility of zero), while for any $x > -c_L$, foreign receives a message of support and never revises (giving utility f). That is, foreign's utility is:

$$Pr(x < -c_L) * 0 + Pr(x > -c_L) * E(f)$$

For a diplomat with bias $\beta < c_L$, foreign will, as above, revise subject to $f < c_F * \frac{F_x(-c_L)}{F_x(-\beta)-F_x(-c_L)}$. For concision let η denote the quantity $c_F * \frac{F_x(-c_L)}{F_x(-\beta)-F_x(-c_L)}$. Then foreign's utility is:

$$Pr(x < -c_L) * (Pr(f < \eta) * 0 + (1 - Pr(f < \eta)) * -c_F) + Pr(-c_L < x < -\beta) * (Pr(f < \eta) * 0 + (1 - Pr(f < \eta)) * E[f|f > \eta]) + Pr(x > -\beta) * E[f]$$

Note that $Pr(x < -c_L) * (Pr(f < \eta) * 0 + (1 - Pr(f < \eta)) * c_F)$ is certainly negative. Recall that, by assumption $f > 0$, thus $Pr(x > -c_L) * E(f)$ is certainly greater than $Pr(-c_L < x < -\beta) * (Pr(f < \eta) * 0 + (1 - Pr(f < \eta)) * E[f|f > \eta]) + Pr(x > -\beta) * E[f]$ for any $\beta < c_L$. Thus, foreign always sets $\beta \geq c_L$.