* Read me : please exclude from running

*The following Mathematica code displays the various steps which we perform in order to formulate and solve the exact equations in the paper, that is System (5) at the Internet Appendix A. It creates the “numbers” which we later use in order to create our plots. Furthermore, all other model specifications, follow same reasoning and can be derived from this code, adjusting appropriately. The case where x%=y%=50% corresponds to ON/OFF while the cases where x%=100% and y%=0%corresponds to NO. We normalize the fundamental value of the asset V to be one.We define a function that depends on x% and on the tick size which later using afor-loop we can define any granular tick size as we desire.

*Step 1:
*Model formulation and solution. In this step we obtain the cut-off values

*Step 2:
*Markov state space equilibrium and probability of each state of the book appearing

*Step 3:
*We compute all types of trading rates (overall, first in line, second in line, conditional trading rates etc.)

*Step 4:
*We compute all types of investor welfare (overall, first in line, second in line,conditional investor welfare etc.)

****

*** Step 1 ****

```
ClearAll["Global`*"]
SetDirectory["C:/Users/……"];
V := 1;
BLSy00 := BLSx00
BLSy01y := BLSx01x
BLSy01x := BLSx01y

f[x_, k_] := ( y := 1 - x; A := V + k; B := V - k;
   Sol = NSolve[{(B - BLSx00 V) ==
       x/2 (BLSx00 /2 ) (A - BLSx00 V) +
        y/2 (BLSy00 /2) (A - BLSx00 V) +
        (x/2 (A/V - BLSx00)/2  (x/2  BLSx01x/2  + y/2 BLSy01x/2) +
           y/2 (A/V - BLSy00)/
             2  (x/2  BLSx01y/2  + y/2 BLSy01y/2)) (A - BLSx00 V) +
        1/2 (B/V)/2  (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx00 V) +
        (x/2 (BLSx01x /2) (x/2 BLSx00/2 + y/2 BLSy00/2) +
           y/2 (BLSy01x /2) (x/2 BLSx00/2 + y/2 BLSy00/2)) (A -
           BLSx00 V) +
```

```
     (x/2 (A/V - BLSx01x)/2  (x/2 BLSx00/2 + y/2 BLSy00 /2) +
        y/2 (A/V - BLSy01x)/2 (x/2 BLSx00/2 + 0 )) (A -
        BLSx00 V) +
     1/2 (B/V)/2  (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx00 V),

   (B - BLSx01x V) ==
    x/2 BLSx00/2 (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01x V) +
     y/2  BLSy00/2 (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01x V) +
     (x/2 (A/V - BLSx00)/2  (x/2  BLSx01x/2  + y/2 BLSy01x/2) +

        y/2 (A/V - BLSy00)/
          2  (x/2  BLSx01y/2  + y/2 BLSy01y/2)) (A - BLSx01x V) +
     1/2 (B/V)/2  (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01x V) +
     (x/2 (BLSx01x /2) (x/2 BLSx00/2 + y/2  BLSy00/2) +
        y/2 (BLSy01x /2) (x/2 BLSx00/2 + y/2 BLSy00/2)) (A -
        BLSx01x V) +
     (x/2 (A/V - BLSx01x)/2  (x/2 BLSx00/2 + y/2 BLSy00 /2) +
        y/2 (A/V - BLSy01x)/2 (x/2 BLSx00/2 + 0 )) (A -
        BLSx01x V) +
     1/2 (B/V)/2  (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01x V),

   (B - BLSx01y V) ==
    x/2 BLSx00/2 (A - BLSx01y V) +
     y/2  BLSy00/2 (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01y V) +
      (x/2 (A/V - BLSx00)/2  (x/2  BLSx01x/2  + y/2 BLSy01x/2) +
        y/2 (A/V - BLSy00)/
          2  (x/2  BLSx01y/2  + y/2 BLSy01y/2)) (A - BLSx01y V) +
     1/2 (B/V)/2  (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01y V) +
     (x/2 (BLSx01x /2) (x/2 BLSx00/2 + y/2  BLSy00/2) +
        y/2 (BLSy01x /2) (x/2 BLSx00/2 + y/2 BLSy00/2)) (A -
        BLSx01y V) +
     (x/2 (A/V - BLSx01x)/2  (x/2 BLSx00/2 + y/2 BLSy00 /2) +
        y/2 (A/V - BLSy01x)/2 (x/2 BLSx00/2 + 0 )) (A -
        BLSx01y V) +
     1/2 (B/V)/2  (x/2 BLSx00 /2 + y/2 BLSy00/2) (A - BLSx01y V),

   BLSx00 > 0, BLSy00 > 0, BLSx01x > 0, BLSy01x > 0, BLSx01y > 0,
   BLSy01y > 0,
   BLSx00 < 2, BLSy00 < 2, BLSx01x < 2, BLSy01x < 2, BLSx01y < 2,
   BLSy01y < 2},

  {BLSx00, BLSy00, BLSx01x, BLSy01x, BLSx01y, BLSy01y}, Reals,
  WorkingPrecision -> Infinity]  ;

xcs00 =  BLSx00 /. Sol[[1]];
     xcs01x = BLSx01x /. Sol[[1]];
     xcs01y =   BLSx01y /. Sol[[1]];
ycs00 =   BLSy00 /. Sol[[1]];
     ycs01y =   BLSy01y /. Sol[[1]];
     ycs01x =  BLSy01x /. Sol[[1]];
r1 = 1/4*(B/V); r2x = xcs01x/4;
     r3x = xcs01y/4;
     r4x = 1/4*(A/V - xcs00);
     r5x = xcs00/4;
```

```
      r6x = 1/4*(A/V - xcs01x);
      r7x = 1/4*(A/V - xcs01y);
 r2y = ycs01y/4;
      r3y = ycs01x/4;
      r4y = 1/4*(A/V - ycs00);
      r5y = ycs00/4;
      r6y = 1/4*(A/V - ycs01y);
      r7y = 1/4*(A/V - ycs01x);


*** Step 2 ****


 AA = {
   {2*r1 + 2*r5x*x + 2*r5y*y, r4x*x, r4y*y, r4x*x, r4y*y, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {r5x*x + r5y*y, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y, 0, 0, 0, r6x*x,
     0, r7y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 0, 1},
   {r5x*x + r5y*y, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r2y*y, 0, 0, 0,
    r7x*x, 0, r6y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 1},
   {r5x*x + r5y*y, 0, 0, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y, 0, 0, 0,
    0, 0, r6x*x, 0, r7y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 0, 1},
   {r5x*x + r5y*y, 0, 0, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r2y*y, 0, 0,
    0, 0, 0, r7x*x, 0, r6y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 1},
   {2*r1 + 2*r5x*x + 2*r5y*y, r4x*x, r4y*y, r4x*x, r4y*y, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {2*r1 + 2*r5x*x + 2*r5y*y, r4x*x, r4y*y, r4x*x, r4y*y, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {2*r1 + 2*r5x*x + 2*r5y*y, r4x*x, r4y*y, r4x*x, r4y*y, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {2*r1 + 2*r5x*x + 2*r5y*y, r4x*x, r4y*y, r4x*x, r4y*y, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1},
   {0, 0, 0, 0, 0, 2*r1 + r2x*x + r5x*x + r3y*y + r5y*y, 0, 0, 0,
    r6x*x, 0, r7y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 0,
    1},
   {r5x*x, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y + r5y*y, 0, 0, 0, r6x*x,
     0, r7y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 0, 1},
   {r5y*y, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r5x*x + r2y*y, 0, 0, 0,
    r7x*x, 0, r6y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 1},
   {0, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r5x*x + r2y*y + r5y*y, 0, 0, 0,
     r7x*x, 0, r6y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 1},
   {0, 0, 0, 0, 0, 0, 0, 2*r1 + r2x*x + r5x*x + r3y*y + r5y*y, 0, 0,
     0, 0, 0, r6x*x, 0, r7y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 0, 1},
   {r5x*x, 0, 0, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y + r5y*y, 0, 0, 0,
    0, 0, r6x*x, 0, r7y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 0,
    1},(*15*)
   {r5y*y, 0, 0, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r5x*x + r2y*y, 0, 0,
    0, 0, 0, r7x*x, 0, r6y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 1},
   {0, 0, 0, 0, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r5x*x + r2y*y + r5y*y, 0,
     0, 0, 0, 0, r7x*x, 0, r6y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 1},
   {r5x*x + r5y*y, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y, 0, 0, 0, r6x*x,
     0, r7y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 0, 1},
   {r5x*x + r5y*y, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y, 0, 0, 0, r6x*x,
     0, r7y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 0, 1},
```

```
  {r5x*x + r5y*y, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r2y*y, 0, 0, 0,
   r7x*x, 0, r6y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y,
   1},(*20*)
  {r5x*x + r5y*y, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r2y*y, 0, 0, 0,
   r7x*x, 0, r6y*y, 0, 0, 0, 0, 0, 0, 0, 0, 0, r4x*x, 0, r4y*y, 1},
  {r5x*x + r5y*y, 0, 0, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y, 0, 0, 0,
   0, 0, r6x*x, 0, r7y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 0, 1},
  {r5x*x + r5y*y, 0, 0, 0, 0, 0, 0, 2*r1 + r2x*x + r3y*y, 0, 0, 0,
   0, 0, r6x*x, 0, r7y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 0, 1},
  {r5x*x + r5y*y, 0, 0, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r2y*y, 0, 0,
   0, 0, 0, r7x*x, 0, r6y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 1},
  {r5x*x + r5y*y, 0, 0, 0, 0, 0, 0, 0, 2*r1 + r3x*x + r2y*y, 0, 0,
   0, 0, 0, r7x*x, 0, r6y*y, 0, r4x*x, 0, r4y*y, 0, 0, 0, 0, 1}
  };

X = {p1, p2, p3, p4, p5, p6, p7, p8, p9, p10, p11, p12, p13, p14,
  p15, p16, p17,
  p18, p19, p20, p21, p22, p23, p24, p25};
b1 = {p1, p2, p3, p4, p5, p6, p7, p8, p9, p10, p11, p12, p13, p14,
  p15, p16, p17,
  p18, p19, p20, p21, p22, p23, p24, p25, 1};

Sol2 =
 Solve[X.AA == b1, {p1, p2, p3, p4, p5, p6, p7, p8, p9, p10, p11,
   p12, p13, p14, p15, p16, p17,
     p18, p19, p20, p21, p22, p23, p24, p25}, Reals];


(*Trading Rates*)
Sol3 = { Sol2[[All, 1, 2]][[1]], Sol2[[All, 2, 2]][[1]],
  Sol2[[All, 3, 2]][[1]], Sol2[[All, 4, 2]][[1]],
  Sol2[[All, 5, 2]][[1]], Sol2[[All, 6, 2]][[1]],
  Sol2[[All, 7, 2]][[1]], Sol2[[All, 8, 2]][[1]],
  Sol2[[All, 9, 2]][[1]], Sol2[[All, 10, 2]][[1]],
  Sol2[[All, 11, 2]][[1]], Sol2[[All, 12, 2]][[1]],
  Sol2[[All, 13, 2]][[1]], Sol2[[All, 14, 2]][[1]],
  Sol2[[All, 15, 2]][[1]], Sol2[[All, 16, 2]][[1]],
  Sol2[[All, 17, 2]][[1]], Sol2[[All, 18, 2]][[1]],
  Sol2[[All, 19, 2]][[1]], Sol2[[All, 20, 2]][[1]],
  Sol2[[All, 21, 2]][[1]], Sol2[[All, 22, 2]][[1]],
  Sol2[[All, 23, 2]][[1]], Sol2[[All, 24, 2]][[1]] ,
  Sol2[[All, 25, 2]][[1]]};


r1 = 1/2*(B/V);
r2x = xcs01x/2;
r3x = xcs01y/2;
r4x = 1/2*(A/V - xcs00);
r5x = xcs00/2;
r6x = 1/2*(A/V - xcs01x);
r7x = 1/2*(A/V - xcs01y);
r2y = ycs01y/2;
r3y = ycs01x/2;
r4y = 1/2*(A/V - ycs00);
r5y = ycs00/2;
```

```
   r6y = 1/2*(A/V - ycs01y);
   r7y = 1/2*(A/V - ycs01x);G
   = 0.5*x*r5x + 0.5*y*r5y;
   Gx2 = 0.5*x*r2x + 0.5*y*r3y;
   Gx3 = 0.5*x*r3x + 0.5*y*r2y;


*** Step 3 ****



   xFirstinL = G + (0.5*x*r4x*Gx2 + 0.5*y*r4y*Gx3) +
     0.5*r1*G + (0.5*x*r2x*G + 0.5*y*r3y*G) + 0.5*x*r6x*G +
     0.5*y*r7y*(0.5*x*r5x + 0) + 0.5*x*r1*G + 0.5*y*r1*G;

   yFirstinL = G + (0.5*x*r4x*Gx2 + 0.5*y*r4y*Gx3) +
     0.5*r1*G + (0.5*x*r3x*G + 0.5*y*r2y*G) +
     0.5*x*r7x*(0 + 0.5*y*r5y) +
     0.5*y*r6y*G + 0.5*x*r1*G + 0.5*y*r1*G;

   xSecondinLAfterx = 0.5*x*r5x*G + 0.5*y*r5y*G +
     (0.5*x*r4x*Gx2 + 0.5*y*r4y*Gx3) +
     0.5*r1*G + (0.5*x*r2x*G + 0.5*y*r3y*G) + 0.5*x*r6x*G +
     0.5*y*r7y*(0.5*x*r5x + 0) + 0.5*x*r1*G + 0.5*y*r1*G;

   xSecondinLAftery = 0.5*x*r5x +
     0.5*y*r5y*G +
     (0.5*x*r4x*Gx2 + 0.5*y*r4y*Gx3) +
     0.5*r1*G + (0.5*x*r2x*G + 0.5*y*r3y*G) + 0.5*x*r6x*G +
     0.5*y*r7y*(0.5*x*r5x + 0) + 0.5*x*r1*G + 0.5*y*r1*G;

   ySecondinLAftery = 0.5*x*r5x*G + 0.5*y*r5y*G +
     (0.5*x*r4x*Gx2 + 0.5*y*r4y*Gx3) +
     0.5*r1*G + (0.5*x*r3x*G + 0.5*y*r2y*G) +
     0.5*x*r7x*(0 + 0.5*y*r5y) +
     0.5*y*r6y*G + 0.5*x*r1*G + 0.5*y*r1*G;

   ySecondinLAfterx = 0.5*x*r5x*G + 0.5*y*r5y +
     (0.5*x*r4x*Gx2 + 0.5*y*r4y*Gx3) +
     0.5*r1*G + (0.5*x*r3x*G + 0.5*y*r2y*G) +
     0.5*x*r7x*(0 + 0.5*y*r5y) +
     0.5*y*r6y*G + 0.5*x*r1*G + 0.5*y*r1*G;

   Rates and welfare Per first and second in line given Seller;

   (*First we will calculate the trading rate of a seller*)

   AuxVectorSel = SetPrecision[{(x*r5x + y*r5y),
      (x*r2x + y*r3y),
      (x*r3x + y*r2y),
      (x*r5x + y*r5y),
      (x*r5x + y*r5y),
      (x*r5x + y*r5y),
      (x*r5x + y*r5y),
```

```
   (x*r5x + y*r5y),
   (x*r5x + y*r5y),
   (x*r2x + y*r3y),
   (x*r2x + y*r3y),
   (x*r3x + y*r2y),
   (x*r3x + y*r2y),
   (x*r5x + y*r5y),
   (x*r5x + y*r5y),
   (x*r5x + y*r5y),
   (x*r5x + y*r5y),
   (x*r2x + y*r3y),
   (x*r2x + y*r3y),
   (x*r3x + y*r2y),
   (x*r3x + y*r2y),
   (x*r5x + y*r5y),
   (x*r5x + y*r5y),
   (x*r5x + y*r5y),
   (x*r5x + y*r5y)}, 6];

PrLOSubmission = SetPrecision[{x*r4x + y*r4y,
   x*r6x + y*r7y,
   x*r7x + y*r6y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r6x + y*r7y,
   x*r6x + y*r7y,
   x*r7x + y*r6y,
   x*r7x + y*r6y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r6x + y*r7y,
   x*r6x + y*r7y,
   x*r7x + y*r6y,
   x*r7x + y*r6y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y,
   x*r4x + y*r4y
   }, 6];

PrLOExecutionSeller =
 SetPrecision[{  x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
    (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
      y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
    1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
    (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
      y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
    (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
```

```
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 y/2  ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2  ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 + y/2 ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
 y/2 (A/V - ycs00)/2 (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,
 x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
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 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 y/2  ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2  ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 y/2  ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2  ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 + y/2 ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
  (x/2 (A/V - xcs00)/2 (x/2 xcs01x/2  + y/2 ycs01x/2) +y/2
  (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
```

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 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2  ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 + y/2 ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
 y/2 (A/V - ycs00)/2 (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

 x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

 x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

 x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
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   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 y/2  ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2  ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 y/2  ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2  ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 + y/2 ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
 y/2 (A/V - ycs00)/2 (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 xcs00/2 + y/2 ycs00/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
 y/2 (A/V - ycs00)/2 (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
 (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
   y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
 1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
 (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
   y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
 (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
   y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
 1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,
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   x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
    (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
      y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
    1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
    (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
      y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
    (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
      y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
    1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

   x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
    (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
      y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
    1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
    (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
      y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
    (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
      y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
    1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) ,

   x/2 (xcs00 /2 )  + y/2 (ycs00 /2) +
    (x/2 (A/V - xcs00)/2 (x/2  xcs01x/2  + y/2 ycs01x/2) +
      y/2 (A/V - ycs00)/2  (x/2  xcs01y/2  + y/2 ycs01y/2)) +
    1/2 (B/V)/2 (x/2 xcs00 /2 + y/2 ycs00/2) +
    (x/2 (xcs01x /2) (x/2 xcs00/2 + y/2 ycs00/2) +
      y/2 (ycs01x /2) (x/2 xcs00/2 + y/2 ycs00/2)) +
    (x/2 (A/V - xcs01x)/2  (x/2 xcs00/2 + y/2 ycs00 /2) +
      y/2 (A/V - ycs01x)/2  (x/2 xcs00/2  + 0 )) +
    1/2 (B/V)/2  (x/2 xcs00 /2 + y/2 ycs00/2) }, 6];

TrRateSeller = SetPrecision[
  Total[Sol3   AuxVectorSel] , 40];

FirstInLineSeller = { Sol2[[All, 1, 2]][[1]], 0, 0,
  Sol2[[All, 4, 2]][[1]], Sol2[[All, 5, 2]][[1]],
  Sol2[[All, 6, 2]][[1]], Sol2[[All, 7, 2]][[1]],
  Sol2[[All, 8, 2]][[1]], Sol2[[All, 9, 2]][[1]], 0, 0, 0, 0,
  Sol2[[All, 14, 2]][[1]], Sol2[[All, 15, 2]][[1]],
  Sol2[[All, 16, 2]][[1]], Sol2[[All, 17, 2]][[1]], 0, 0, 0, 0,
  Sol2[[All, 22, 2]][[1]], Sol2[[All, 23, 2]][[1]],
  Sol2[[All, 24, 2]][[1]] , Sol2[[All, 25, 2]][[1]]};


FirstInLineSellerE = { Sol2[[All, 1, 2]][[1]], 0, 0, 0, 0,
  Sol2[[All, 6, 2]][[1]], Sol2[[All, 7, 2]][[1]], 0, 0, 0, 0, 0,
  0, 0, 0, Sol2[[All, 16, 2]][[1]], 0, 0, 0, 0, 0, 0, 0, 0 , 0};

FirstInLineSellerI = {0, 0, 0, Sol2[[All, 4, 2]][[1]],
  Sol2[[All, 5, 2]][[1]], 0, 0, Sol2[[All, 8, 2]][[1]],
  Sol2[[All, 9, 2]][[1]], 0, 0, 0, 0, Sol2[[All, 14, 2]][[1]],
  Sol2[[All, 15, 2]][[1]], Sol2[[All, 16, 2]][[1]],
  Sol2[[All, 17, 2]][[1]], 0, 0, 0, 0, Sol2[[All, 22, 2]][[1]],
  Sol2[[All, 23, 2]][[1]], Sol2[[All, 24, 2]][[1]] ,
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  Sol2[[All, 25, 2]][[1]]};

SecondInLineSeller = { 0, Sol2[[All, 2, 2]][[1]],
  Sol2[[All, 3, 2]][[1]], 0, 0, 0, 0, 0, 0,
  Sol2[[All, 10, 2]][[1]], Sol2[[All, 11, 2]][[1]],
  Sol2[[All, 12, 2]][[1]], Sol2[[All, 13, 2]][[1]], 0, 0, 0, 0,
  Sol2[[All, 18, 2]][[1]], Sol2[[All, 19, 2]][[1]],
  Sol2[[All, 20, 2]][[1]], Sol2[[All, 21, 2]][[1]], 0, 0, 0 , 0};

Sol3Tr = { 0, 0, 0, Sol2[[All, 4, 2]][[1]],
  Sol2[[All, 5, 2]][[1]], 0, 0, Sol2[[All, 8, 2]][[1]],
  Sol2[[All, 9, 2]][[1]], 0, 0, 0, 0, Sol2[[All, 14, 2]][[1]],
  Sol2[[All, 15, 2]][[1]], Sol2[[All, 16, 2]][[1]],
  Sol2[[All, 17, 2]][[1]], Sol2[[All, 18, 2]][[1]],
  Sol2[[All, 19, 2]][[1]], Sol2[[All, 20, 2]][[1]],
  Sol2[[All, 21, 2]][[1]], Sol2[[All, 22, 2]][[1]],
  Sol2[[All, 23, 2]][[1]], Sol2[[All, 24, 2]][[1]] ,
  Sol2[[All, 25, 2]][[1]]};

Sol3Cr = { Sol2[[All, 1, 2]][[1]], Sol2[[All, 2, 2]][[1]],
  Sol2[[All, 3, 2]][[1]], 0, 0, Sol2[[All, 6, 2]][[1]],
  Sol2[[All, 7, 2]][[1]], 0, 0, Sol2[[All, 10, 2]][[1]],
  Sol2[[All, 11, 2]][[1]], Sol2[[All, 12, 2]][[1]],
  Sol2[[All, 13, 2]][[1]], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 , 0};

Sol3TrFiL = { 0, 0, 0, Sol2[[All, 4, 2]][[1]],
  Sol2[[All, 5, 2]][[1]], 0, 0, Sol2[[All, 8, 2]][[1]],
  Sol2[[All, 9, 2]][[1]], 0, 0, 0, 0, Sol2[[All, 14, 2]][[1]],
  Sol2[[All, 15, 2]][[1]], Sol2[[All, 16, 2]][[1]],
  Sol2[[All, 17, 2]][[1]], 0, 0, 0, 0, Sol2[[All, 22, 2]][[1]],
  Sol2[[All, 23, 2]][[1]], Sol2[[All, 24, 2]][[1]] ,
  Sol2[[All, 25, 2]][[1]]};

Sol3TrSiL = { 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  Sol2[[All, 18, 2]][[1]], Sol2[[All, 19, 2]][[1]],
  Sol2[[All, 20, 2]][[1]], Sol2[[All, 21, 2]][[1]], 0, 0, 0 , 0};

Sol3CrFiL = { Sol2[[All, 1, 2]][[1]], 0, 0, 0, 0,
  Sol2[[All, 6, 2]][[1]], Sol2[[All, 7, 2]][[1]], 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 , 0};

Sol3CrSiL = { 0, Sol2[[All, 2, 2]][[1]], Sol2[[All, 3, 2]][[1]], 0,
   0, 0, 0, 0, 0, Sol2[[All, 10, 2]][[1]],
  Sol2[[All, 11, 2]][[1]], Sol2[[All, 12, 2]][[1]],
  Sol2[[All, 13, 2]][[1]], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 , 0};

TrRate = 100*SetPrecision[
   Total[Sol3   AuxVectorSel] , 40];

TrRateTr = 100*SetPrecision[
   Total[Sol3Tr   AuxVectorSel] , 40];

TrRateCr = 100*SetPrecision[
   Total[Sol3Cr   AuxVectorSel] , 40];
```

```
   TrRateFirstSeller = 100* SetPrecision[
      Total[FirstInLineSeller   AuxVectorSel], 40];

   TrRateSecondInLineSeller = 100* SetPrecision[
      Total[SecondInLineSeller   AuxVectorSel], 40];

        TrRateFirstSellerTr = 100*  SetPrecision[
      Total[ Sol3TrFiL   AuxVectorSel], 40];

   TrRateSecondInLineSellerTr = 100*  SetPrecision[
      Total[ Sol3TrSiL   AuxVectorSel], 40];

   TrRateFirstSellerCr = 100*  SetPrecision[
      Total[ Sol3CrFiL   AuxVectorSel], 40];

   TrRateSecondInLineSellerCR = 100*  SetPrecision[
      Total[ Sol3CrSiL   AuxVectorSel], 40];



*** Step 4 ****



   (*Welfare*)
   R1FiLx =
    Piecewise[{{0.5*(B - b*V)*Total[FirstInLineSeller], 0 < b < xcs00},
      {0.5*(
         Sol2[[All, 1, 2]][[1]]*(A - b*V)*xFirstinL
          + (Sol2[[All, 2, 2]][[1]] + Sol2[[All, 3, 2]][[1]])*(B -
             b*V)*0 +
          Total[Sol2[[All, 4 ;; 9, 2]][[1]]]*(A - b*V)*xFirstinL +
          Total[Sol2[[All, 10 ;; 13, 2]][[1]]]*(B - b*V)*0 +
          Total[Sol2[[All, 14 ;; 17, 2]][[1]]]*(A - b*V)*xFirstinL +
          Total[Sol2[[All, 18 ;; 21, 2]][[1]]]*(B - b*V)*0 +
          Total[Sol2[[All, 22 ;; 25, 2]][[1]]]*(A - b*V)*xFirstinL),
       xcs00 < b < xcs01y},
      {0.5*(
         Sol2[[All, 1, 2]][[1]]*(A - b*V)*xFirstinL +
          Sol2[[All, 2, 2]][[1]]*(B - b*V)*0 +
          Sol2[[All, 3, 2]][[1]]*(A - b*V)*xSecondinLAftery*0 +
          (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*xFirstinL +
          (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(B - b*V)*0 +
          (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(A - b*V)*
           xSecondinLAftery*0 +
          (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*xFirstinL +
          (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(B - b*V)*0 +
          (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(A - b*V)*
           xSecondinLAftery*0 +
          (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*xFirstinL
```

```
      ), xcs01y < b < xcs01x},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*xFirstinL +
       Sol2[[All, 2, 2]][[1]]*(A - b*V)*xSecondinLAfterx*0 +
       Sol2[[All, 3, 2]][[1]]*(A - b*V)*xSecondinLAftery*0 +
       (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*xFirstinL +
       (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(A - b*V)*
        xSecondinLAfterx*0 +
       (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(A - b*V)*
        xSecondinLAftery*0 +
       (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*xFirstinL +
       (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(A - b*V)*
        xSecondinLAfterx*0 +
       (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(A - b*V)*
        xSecondinLAftery*0 +
       (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*xFirstinL
      ), xcs01x < b < A/V},
   {0*b, A/V < b < 2}
   }];

Int1FiLx = SetPrecision[
  Integrate[R1FiLx, {b, 0, 2}], 40] ;

R1SiLx =
 Piecewise[{{0.5*(B - b*V) Total[SecondInLineSeller],
    0 < b < xcs00},

   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*xFirstinL*0
       + (Sol2[[All, 2, 2]][[1]] + Sol2[[All, 3, 2]][[1]])*(B -
          b*V) +
       Total[Sol2[[All, 4 ;; 9, 2]][[1]]]*(A - b*V)*xFirstinL*0 +
       Total[Sol2[[All, 10 ;; 13, 2]][[1]]]*(B - b*V) +
       Total[Sol2[[All, 14 ;; 17, 2]][[1]]]*(A - b*V)*xFirstinL*0 +
       Total[Sol2[[All, 18 ;; 21, 2]][[1]]]*(B - b*V) +
       Total[Sol2[[All, 22 ;; 25, 2]][[1]]]*(A - b*V)*xFirstinL*0),
     xcs00 < b < xcs01y},

   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*xFirstinL*0 +
       Sol2[[All, 2, 2]][[1]]*(B - b*V) +
       Sol2[[All, 3, 2]][[1]]*(A - b*V)*xSecondinLAftery +
       (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*xFirstinL*0 +
       (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(B - b*V) +
       (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(A - b*V)*
        xSecondinLAftery +
       (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*
        xFirstinL*0 +
       (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(B - b*V) +
       (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(A - b*V)*
        xSecondinLAftery +
       (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*xFirstinL*0
      ), xcs01y < b < xcs01x},
```

```
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*xFirstinL*0 +
       Sol2[[All, 2, 2]][[1]]*(A - b*V)*xSecondinLAfterx +
       Sol2[[All, 3, 2]][[1]]*(A - b*V)*xSecondinLAftery +
       (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*xFirstinL*0 +
       (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(A - b*V)*
        xSecondinLAfterx +
       (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(A - b*V)*
        xSecondinLAftery +
       (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*
        xFirstinL*0 +
       (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(A - b*V)*
        xSecondinLAfterx +
       (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(A - b*V)*
        xSecondinLAftery +
       (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*xFirstinL*0
      ), xcs01x < b < A/V},

   {0*b, A/V < b < 2}
   }];

Int1SiLx = SetPrecision[
  Integrate[R1SiLx, {b, 0, 2}], 40];

R2FiLy =
 Piecewise[{{0.5*(B - b*V)*Total[FirstInLineSeller], 0 < b < ycs00},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*yFirstinL
       + (Sol2[[All, 2, 2]][[1]] + Sol2[[All, 3, 2]][[1]])*(B -
          b*V)*0 +
       Total[Sol2[[All, 4 ;; 9, 2]][[1]]] (A - b*V)*yFirstinL +
       Total[Sol2[[All, 10 ;; 13, 2]][[1]]]*(B - b*V)*0 +
       Total[Sol2[[All, 14 ;; 17, 2]][[1]]]*(A - b*V)*yFirstinL +
       Total[Sol2[[All, 18 ;; 21, 2]][[1]]]*(B - b*V)*0 +
       Total[Sol2[[All, 22 ;; 25, 2]][[1]]]*(A - b*V)*yFirstinL
      ), ycs00 < b < ycs01x},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*yFirstinL +
       Sol2[[All, 2, 2]][[1]]*(A - b*V)*ySecondinLAfterx*0 +
       Sol2[[All, 3, 2]][[1]]*(B - b*V)*0 +
       (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*yFirstinL +
       (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(A - b*V)*
        ySecondinLAfterx*0 +
       (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(B - b*V)*0 +
       (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*yFirstinL +
       (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(A - b*V)*
        ySecondinLAfterx*0 +
       (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(B - b*V)*0 +
       (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*yFirstinL
      ), ycs01x < b < ycs01y},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*yFirstinL +
       Sol2[[All, 2, 2]][[1]]*(A - b*V)*ySecondinLAfterx*0 +
```

```
      Sol2[[All, 3, 2]][[1]]*(A - b*V)*ySecondinLAftery*0 +
      (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*yFirstinL +
      (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(A - b*V)*
       ySecondinLAfterx*0 +
      (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(A - b*V)*
       ySecondinLAftery*0 +
      (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*yFirstinL +
      (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(A - b*V)*
       ySecondinLAfterx*0 +
      (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(A - b*V)*
       ySecondinLAftery*0 +
      (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*yFirstinL
     ), ycs01y < b < A/V},
   {0*b, A/V < b < 2}
   }];
Int2FiLy = SetPrecision[
  Integrate[R2FiLy, {b, 0, 2}], 40] ;

R2SiLy =
 Piecewise[{{0.5*(B - b*V) Total[SecondInLineSeller],
    0 < b < ycs00},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*yFirstinL*0
       + (Sol2[[All, 2, 2]][[1]] + Sol2[[All, 3, 2]][[1]])*(B -
          b*V) +
       Total[Sol2[[All, 4 ;; 9, 2]][[1]]] (A - b*V)*yFirstinL*0 +
       Total[Sol2[[All, 10 ;; 13, 2]][[1]]]*(B - b*V) +
       Total[Sol2[[All, 14 ;; 17, 2]][[1]]]*(A - b*V)*yFirstinL*0 +
       Total[Sol2[[All, 18 ;; 21, 2]][[1]]]*(B - b*V) +
       Total[Sol2[[All, 22 ;; 25, 2]][[1]]]*(A - b*V)*yFirstinL*0),
     ycs00 < b < ycs01x},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*yFirstinL*0 +
       Sol2[[All, 2, 2]][[1]]*(A - b*V)*ySecondinLAfterx +
       Sol2[[All, 3, 2]][[1]]*(B - b*V) +
       (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*yFirstinL*0 +
       (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(A - b*V)*
        ySecondinLAfterx +
       (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(B - b*V) +
       (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*
        yFirstinL*0 +
       (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(A - b*V)*
        ySecondinLAfterx +
       (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(B - b*V) +
       (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*yFirstinL*0
      ), ycs01x < b < ycs01y},
   {0.5*(
      Sol2[[All, 1, 2]][[1]]*(A - b*V)*yFirstinL*0 +
       Sol2[[All, 2, 2]][[1]]*(A - b*V)*ySecondinLAfterx +
       Sol2[[All, 3, 2]][[1]]*(A - b*V)*ySecondinLAftery +
       (Total[Sol2[[All, 4 ;; 9, 2]][[1]]])*(A - b*V)*yFirstinL*0 +
       (Total[Sol2[[All, 10 ;; 11, 2]][[1]]])*(A - b*V)*
        ySecondinLAfterx +
       (Total[Sol2[[All, 12 ;; 13, 2]][[1]]])*(A - b*V)*
```

```
          ySecondinLAftery +
         (Total[Sol2[[All, 14 ;; 17, 2]][[1]]])*(A - b*V)*
          yFirstinL*0 +
         (Total[Sol2[[All, 18 ;; 19, 2]][[1]]])*(A - b*V)*
          ySecondinLAfterx +
         (Total[Sol2[[All, 20 ;; 21, 2]][[1]]])*(A - b*V)*
          ySecondinLAftery +
         (Total[Sol2[[All, 22 ;; 25, 2]][[1]]])*(A - b*V)*yFirstinL*0
        ), ycs01y < b < A/V},
     {0*b, A/V < b < 2}
     }];

  Int2SiLy = SetPrecision[
     Integrate[R2SiLy, {b, 0, 2}], 40];
  TotalWel = (x*Int1FiLx + y*Int2FiLy) +
                      (x*Int1SiLx + y*Int2SiLy);
  WelFiL = (x*Int1FiLx + y*Int2FiLy);
  WelSiL = (x*Int1SiLx + y*Int2SiLy);
  WelXFiL = Int1FiLx/Total[FirstInLineSeller];
  WelXSiL = Int1SiLx/Total[SecondInLineSeller];
  WelYFiL = Int2FiLy/Total[FirstInLineSeller];
  WelYSiL = Int2SiLy/Total[SecondInLineSeller];
  WelX =
   x (WelXFiL  Total[FirstInLineSeller]   +
      WelXSiL Total[SecondInLineSeller]);
  WelY = (1 - x) (WelYFiL  Total[FirstInLineSeller]   +
      WelYSiL Total[SecondInLineSeller]);

  r = {xcs00, xcs01y, xcs01x, TrRate, TrRateFirstSeller,
    TrRateSecondInLineSeller, TotalWel, WelFiL, WelSiL,
    Total[FirstInLineSeller], Total[SecondInLineSeller], TrRateTr,
    TrRateCr, Total[FirstInLineSellerE], Total[FirstInLineSellerI],
    TrRateFirstSellerTr, TrRateSecondInLineSellerTr,
    TrRateFirstSellerCr,
    TrRateSecondInLineSellerCR}

  );

(*we can now iterate the function from k=0 to 1 with any \
infinitesimal increments of k *)
```