Title: Analytical Solutions for the Two-Level Quantum System Model in Periodontal LLLT

1. Model Overview

The two-level system (TLS) represents a biomolecular chromophore (e.g., cytochrome c oxidase) under low-level laser irradiation. The system is described by a driven, dissipative quantum model with Hamiltonian and Lindblad decoherence.

2. Hamiltonian and Master Equation

In the rotating frame at resonance **($\omega_0 = \omega_L$):**

$$\boldsymbol{H = \frac{\hbar\Omega}{2}\sigma_x}$$

The Lindblad master equation with spontaneous emission:

$$\boldsymbol{\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \gamma(\sigma_- \rho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\})}$$

3. Steady-State Solution

The excited-state population at steady-state has the exact analytical form:

$$\boldsymbol{P_e = \langle e \mid \rho_{ss} \mid e \rangle = \frac{\Omega^2}{\gamma^2 + 2\Omega^2}}$$

Where:

- **Ω:** Rabi frequency **(MHz)**, proportional to laser irradiance
- **γ:** Decoherence rate **(MHz),** representing biological noise
- **P_e:** Dimensionless excited population **($0 \leq P_e \leq 0.5$)**

4. Time-Dependent Coherence Decay

The off-diagonal coherence decays exponentially with time constant:

$$\boldsymbol{\tau = \frac{2}{\gamma}}\textbf{(in microseconds for } \gamma \textbf{ in MHz)}$$

The coherence envelope follows:

$$\boldsymbol{\mid C(t) \mid \approx \frac{\Omega}{2\gamma}\exp\left(-\frac{\gamma t}{2}\right)}$$

5. Parameter Ranges Used (Clinical LLLT Context)

- **Rabi frequencies: Ω** = 0.05, 0.10, 0.20, 0.50 MHz
  Corresponds to clinical irradiance ~1-100 mW/cm²

- **Decoherence rates:** γ = 0.05, 0.10, 0.20 MHz
  Models biological noise in periodontal tissues
- **Characteristic rate: Γ = 1 MHz**
  Sets scaling: 1 normalized time unit = 1 μs

6. Derived Quantities

From the above formulas:

Table 1 values: $\boldsymbol{P_e = \Omega^2/(\gamma^2 + 2\Omega^2)}$

Table 2 values:

- Amplitude $\boldsymbol{A = \Omega/(2\gamma)}$
- Decay time $\boldsymbol{\tau = 2/\gamma}$ **(μs)**

7. Physical Interpretation

- **$\Omega > \gamma$:** Coherent driving dominates → higher P_e
- **$\gamma > \Omega$:** Decoherence dominates → lower P_e
- **$\tau = 2/\gamma$:** Quantum coherence persists for microseconds under modeled conditions

8. MATLAB/Excel Verification

These formulas can be verified in any computational environment:

- **Excel:** =Omega^2/(Gamma^2 + 2*Omega^2)
- **MATLAB:** Pe = Omega.^2./(Gamma.^2 + 2*Omega.^2)

Analytical formulas are standard results in quantum optics [and can be verified using the provided reproduction script.

References

1. Scully MO, Zubairy MS. Quantum Optics. Cambridge: Cambridge University Press; 1997.
2. Breuer HP, Petruccione F. The Theory of Open Quantum Systems. Oxford: Oxford University Press; 2002.
3. Gardiner CW, Zoller P. Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics. 3rd ed. Berlin: Springer; 2004.
4. Carmichael HJ. Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations. Berlin: Springer; 1999.