Online Appendix 2 of Calzolari, Denicolò and Zanchettin "The demand-boost theory of exclusive dealing," 2019

The Reduced Form model with Duopoly of Section 4 (No coordination).

November 2019

Defining payoffs

In[1]:=
```
(*Buyer's utility*)
u = (q1 + q2) - 1 / 2 (q1^2 + q2^2) - γ q1 q2;
ue1 = u /. q2 → 0;
ue2 = u /. q1 → 0;

(*Parameters: c in [0,1], γ in [0,1]*)

P1 = p1 q1 + F1; (*Two part tariff of dominant firm 1*)
P2 = p2 q2 + F2; (*Two part tariff of inefficient firm 2*)
```

In[6]:=
```
(*Preparing payoffs for the different cases*)
```

In[7]:=
```
(*Common representation*)
tmpu = FullSimplify[Solve[{D[u - P1 - P2, q1] == 0, D[u - P1 - P2, q2] == 0}, {q1, q2}]];
q1NE = tmpu[[1, 1, 2]]
q2NE = tmpu[[1, 2, 2]]
```

Out[8]=
$$\frac{-1 + p1 + \gamma - p2\,\gamma}{-1 + \gamma^2}$$

Out[9]=
$$\frac{-1 + p2 + \gamma - p1\,\gamma}{-1 + \gamma^2}$$

In[10]:=
```
(*Non exclusive payoffs*)
u - p1 q1 - p2 q2 /. {q1 → q1NE, q2 → q2NE};
ProfitBNE = FullSimplify[%] - (1 + μ) (F1 + F2)

P1 /. {q1 → q1NE, q2 → q2NE};
Profit1NE = FullSimplify[%]

P2 - c q2 /. {q1 → q1NE, q2 → q2NE};
Profit2NE = FullSimplify[%]
```

Out[11]=
$$-\frac{2-2\,p1+p1^2-2\,p2+p2^2-2\,(-1+p1)\,(-1+p2)\,\gamma}{2\,(-1+\gamma^2)}-(F1+F2)\,(1+\mu)$$

Out[13]=
$$F1+\frac{p1\,(-1+p1+\gamma-p2\,\gamma)}{-1+\gamma^2}$$

Out[15]=
$$\frac{-c\,(-1+p2+\gamma-p1\,\gamma)+p2\,(-1+p2+\gamma-p1\,\gamma)+F2\,(-1+\gamma^2)}{-1+\gamma^2}$$

In[16]:=
```
(*Monopoly/Exclusive firm 1*)
q1e = FullSimplify[Solve[D[ue1 - p1 q1, q1] == 0, q1]][[1, 1, 2]]

ue1 - p1 q1 - (1 + μ) F1 /. q1 → q1e;
ProfitBE1 = FullSimplify[%]

P1 /. q1 → q1e;
Profit1E = FullSimplify[%]
```

Out[16]=
$$1-p1$$

Out[18]=
$$\frac{1}{2}\left((-1+p1)^2-2\,F1\,(1+\mu)\right)$$

Out[20]=
$$F1+p1-p1^2$$

In[21]:=
```
(*Monopoly/Exclusive firm 2*)
q2e = FullSimplify[Solve[D[ue2 - p2 q2, q2] == 0, q2]][[1, 1, 2]]

FullSimplify[ue2 - p2 q2 - (1 + μ) F2 /. q2 → q2e];
ProfitBE2 = FullSimplify[%]

P2 - c q2 /. q2 → q2e;
Profit2E = FullSimplify[%]
```

Out[21]=
$$1-p2$$

Out[23]=
$$\frac{1}{2}\left((-1+p2)^2-2\,F2\,(1+\mu)\right)$$

Out[25]=
$$F2+c\,(-1+p2)+p2-p2^2$$

Analysis: Exclusive contracts prohibited

In[26]:=
```
(*Determining the "Bertrand-Nash" marginal
 prices (indicated with the star in the paper).*)
F1tmp = FullSimplify[Solve[ProfitBNE == k, F1]][[1, 1, 2]];
F2tmp = FullSimplify[Solve[ProfitBNE == k, F2]][[1, 1, 2]];
Profit1NEtmp = Profit1NE /. F1 → F1tmp;
Profit2NEtmp = Profit2NE /. F2 → F2tmp;
tmpp =
 FullSimplify[Solve[{D[Profit1NEtmp, p1] == 0, D[Profit2NEtmp, p2] == 0}, {p1, p2}]]
p1NE = tmpp[[1, 1, 2]];
p2NE = tmpp[[1, 2, 2]];

pricesNE = {p1 → p1NE, p2 → p2NE};
```

Out[30]=
$$\left\{\left\{p1 \to \frac{\mu\,(-1-2\,\mu+\gamma\,(1+\mu+\gamma\,\mu-c\,(1+\mu)))}{-1+\mu\,(-4+(-4+\gamma^2)\,\mu)},\right.\right.$$
$$\left.\left.p2 \to \frac{-c\,(1+\mu)\,(1+2\,\mu)+(-1+\gamma)\,\mu\,(1+(2+\gamma)\,\mu)}{-1+\mu\,(-4+(-4+\gamma^2)\,\mu)}\right\}\right\}$$

In[34]:=
```
(*For future reference we determine the Limit Pricing region*)
q2NE /. pricesNE;
FullSimplify[Solve[% == 0, c]]
clim = %[[1, 1, 2]]; (*for c above clim the inefficient firm 2 cannot be active*)
```

Out[35]=
$$\left\{\left\{c \to \frac{(-1+\gamma)\,(1+(2+\gamma)\,\mu)}{-1+(-2+\gamma^2)\,\mu}\right\}\right\}$$

In[37]:=
```
(*We determine the limit pricing strategy. The
  limit pricing strategy for the inefficient firm 2 is p2=
 c and F2=0. That for firm 1 is determined as follows.*)
p1Limit = FullSimplify[Solve[0 == q2NE /. p2 → c, p1]][[1, 1, 2]]

(*The associated fixed fee is determined as follows.*)
ProfitBE2 /. {p2 → c, F2 → 0};
FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit];
F1Limit = FullSimplify[Solve[%% == %, F1]][[1, 1, 2]]

(*The associated payoffs are as follows.*)
PROFITBLimit = FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit /. F1 -> F1Limit]
PROFIT1Limit = FullSimplify[Profit1E /. q1 -> q1e /. p1 → p1Limit /. F1 -> F1Limit]
```

Out[37]=
$$\frac{-1+c+\gamma}{\gamma}$$

Out[40]=
$$-\frac{(-1+c)^2\,(-1+\gamma^2)}{2\,\gamma^2\,(1+\mu)}$$

Out[41]=
$$\frac{1}{2}\,(-1+c)^2$$

Out[42]=
$$-\frac{(-1+c)\,(-(-1+\gamma)\,(-1+\gamma-2\,\mu)+c\,(1+\gamma^2+2\,\mu))}{2\,\gamma^2\,(1+\mu)}$$

In[43]:=
```
(*We now find the "Drastic region" in which also without
 exclusivity clause the dominant can price at the monopoly price*)

(*First we need to find the unconstrained monopolist's optimal price*)
ProfitBE1 - ProfitBE2 /. {p2 → c, F2 → 0};
tmpF1m = FullSimplify[Solve[% == 0, F1]];
Profit1E /. F1 → tmpF1m[[1, 1, 2]];
p1m = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]]
F1m = FullSimplify[tmpF1m[[1, 1, 2]] /. p1 → p1m]

(*And the associated payoffs are as follows.*)
PROFIT1Em = FullSimplify[Profit1E /. p1 → p1m /. F1 → F1m]
PROFITBE1m = FullSimplify[ProfitBE1 /. p1 → p1m /. F1 → F1m]

(*Now, we determine the minimum c that allows the dominant firm to give-
 up with limit pricing and directly price as a monopolist.*)
FullSimplify[Solve[p1m == p1Limit, c]];
cdrast = %[[1, 1, 2]]
```

Out[46]=
$$\frac{\mu}{1+2\mu}$$

Out[47]=
$$-\frac{\left(c-\frac{\mu}{1+2\mu}\right)\left(-2+c+\frac{\mu}{1+2\mu}\right)}{2\,(1+\mu)}$$

Out[48]=
$$\frac{\mu^2-c^2\,(1+2\mu)+c\,(2+4\mu)}{2+6\mu+4\mu^2}$$

Out[49]=
$$\frac{1}{2}\,(-1+c)^2$$

Out[51]=
$$\frac{1+2\mu-\gamma\,(1+\mu)}{1+2\mu}$$

In[52]:=
```
(*For future reference we also define the Positive Primary Otuput boundary*)
Solve[{D[u - c q2, q1] == 0, D[u - c q2, q2] == 0}, {q1, q2}];
Solve[%[[1, 2, 2]] == 0, c];
cPPO = %[[1, 1, 2]]
```

Out[54]=
$$1-\gamma$$

In[55]:=

In[56]:=
```
(*We now derive the most cooperative equilibrium. To this end we define
 the conditions that identify the equilibrium and barrage tariffs,
here identified with notation pi and Fi.*)
```

In[57]:=
```
(*First define k:*)
kdef = FullSimplify[
  u - q1 p1 - q2 p2 - (1 + μ) (F1 + F2) /. {q1 → q1NE, q2 → q2NE} /. {p1 → p1NE, p2 → p2NE} /.
   {F1 -> F1star, F2 -> F2star}]
```

Out[57]=
$$\left((1+\mu)\left(-2c(-1+\gamma)(1+\mu)(1+(2+\gamma)\mu)^2-2(-1+\gamma)(1+(2+\gamma)\mu)^2\left(-1-\mu+\text{F1star}(1+\gamma)(-1+(-2+\gamma)\mu)^2+\text{F2star}(1+\gamma)(-1+(-2+\gamma)\mu)^2\right)+c^2(1+\mu)\left(-1+\mu\left(-4(1+\mu)+\gamma^2(2+3\mu)\right)\right)\right)\Big/\left(2\left(-1+\gamma^2\right)(-1+(-2+\gamma)\mu)^2(1+(2+\gamma)\mu)^2\right)$$

In[58]:=
```
(*Second, firms obtain in equilibrium not less that the profit as with tariff
   in the best deviation to exclusivity (here indicated with (pi,Fi)):*)
p1 q1NE + F1star /. {p1 → p1NE, p2 → p2NE};
NoDev1 = FullSimplify[% - (p1 q1e + F1)]

(p2 - c) q2NE + F2star /. {p1 → p1NE, p2 → p2NE};
NoDev2 = FullSimplify[% - ((p2 - c) q2e + F2)]
```

Out[59]=
$$-\text{F1}+\text{F1star}+(-1+\text{p1})\,\text{p1}-\frac{\mu(1+\mu)(1+2\mu-\gamma^2\mu+(-1+c)\gamma(1+\mu))^2}{(-1+\gamma^2)(1+\mu(4-(-4+\gamma^2)\mu))^2}$$

Out[61]=
$$-\text{F2}+\text{F2star}+(-1+\text{p2})(-c+\text{p2})-\frac{\mu(1+\mu)(c-c(-2+\gamma^2)\mu+(-1+\gamma)(1+(2+\gamma)\mu))^2}{(-1+\gamma^2)(1+\mu(4-(-4+\gamma^2)\mu))^2}$$

In[62]:=
```
(*Third, the best deviation contract:
   A- Buyer must accept the deviation rather
    than obtaining the candidate equilibrium payoff k:*)
ue1 - p1 q1 - (1 + μ) F1 /. q1 → q1e;
BuyerPartDev1 = FullSimplify[% - k]
ue2 - p2 q2 - (1 + μ) F2 /. q2 → q2e;
BuyerPartDev2 = FullSimplify[% - k]

(*B- Buyer must also accept the deviation rather than mixing
  the deviation contract with the candidate equilibrium contract
  of the other firm and purchase under common representation:*)

u - q1 p1 - q2 p2 - (F1 + F2star) (1 + μ) /. {q1 → q1NE, q2 → q2NE} /. {p2 → p2NE};
ue1 - p1 q1 - F1 (1 + μ) /. q1 → q1e;
BuyerNoMix1 = FullSimplify[% - %%]

u - q1 p1 - q2 p2 - (F1star + F2) (1 + μ) /. {q1 → q1NE, q2 → q2NE} /. {p1 → p1NE};
ue2 - p2 q2 - F2 (1 + μ) /. q2 → q2e;
BuyerNoMix2 = FullSimplify[% - %%]
```

Out[63]=
$$\frac{1}{2}\left(-2\,k+(-1+p1)^2-2\,F1\,(1+\mu)\right)$$

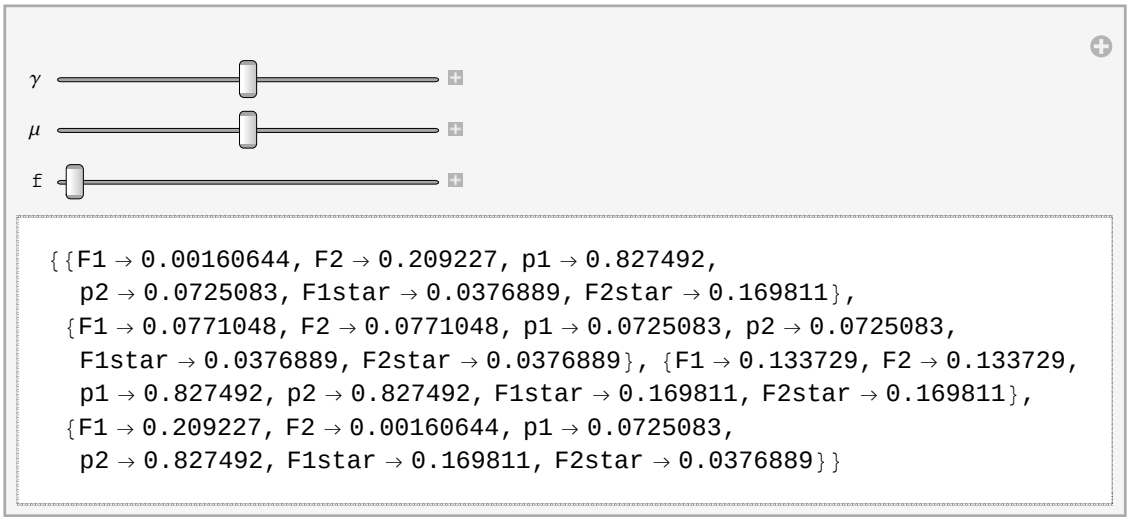
Out[65]=
$$\frac{1}{2}\left(-2\,k+(-1+p2)^2-2\,F2\,(1+\mu)\right)$$

Out[68]=
$$\Big(c^2\,(1+\mu)^2\,(1+2\,\mu)^2+2\,c\,(1+\mu)\,(1+2\,\mu)\,(1+(2+\gamma)\,\mu)\,(-1-\mu+(-1+p1)\,\gamma\,(-1+(-2+\gamma)\,\mu))+(1+(2+\gamma)\,\mu)^2\left(2\,F2star\left(-1+\gamma^2\right)(1+\mu)\,(-1+(-2+\gamma)\,\mu)^2+(1+\mu-(-1+p1)\,\gamma\,(-1+(-2+\gamma)\,\mu))^2\right)\Big)\Big/\left(2\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

Out[71]=
$$\Big(2\,F1star\left(-1+\gamma^2\right)(1+\mu)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2+\left((-1+p2)\,\gamma^3\,\mu^2-(1+\mu)\,(1+2\,\mu)+\gamma\left(1+(3+c)\,\mu\,(1+\mu)-p2\,(1+2\,\mu)^2\right)\right)^2\Big)\Big/\left(2\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

In[72]:=
```
{NoDev1 == 0, NoDev2 == 0, BuyerPartDev1 == 0,
   BuyerPartDev2 == 0, BuyerNoMix1 == 0, BuyerNoMix2 == 0} /. k -> kdef;
Solmostcooptmp = Solve[%, {F1, F2, p1, p2, F1star, F2star}];
```

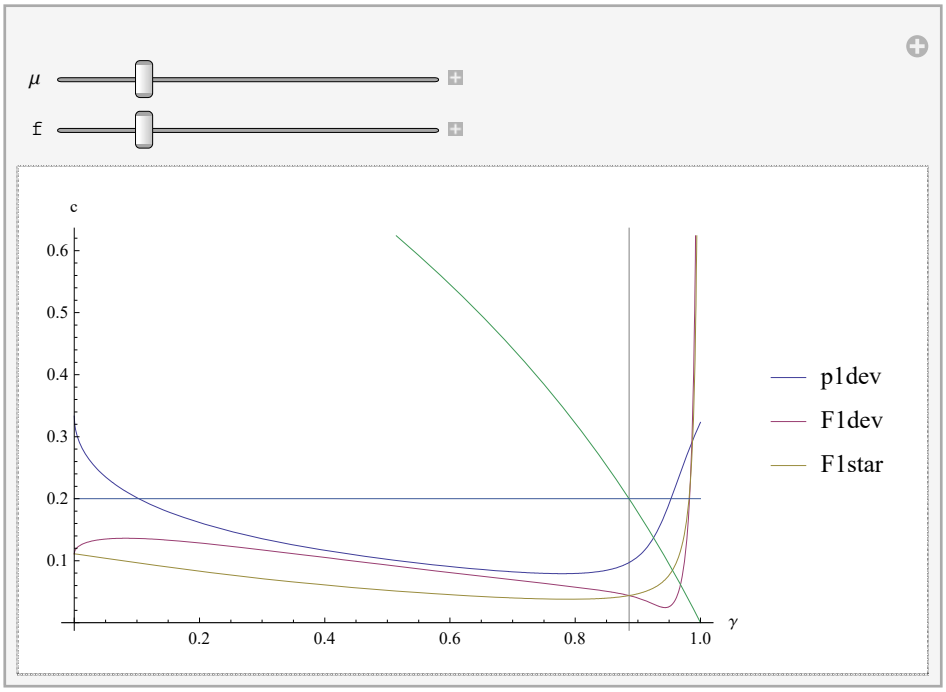
In[74]:=
```
Solmostcooptmp;
Manipulate[%, {{γ, 0.5}, 0, 1}, {{μ, 1}, 0, 2}, {c, 0, 1}]
```



Out[75]=
```
{{F1 → 0.00160644, F2 → 0.209227, p1 → 0.827492,
  p2 → 0.0725083, F1star → 0.0376889, F2star → 0.169811},
 {F1 → 0.0771048, F2 → 0.0771048, p1 → 0.0725083, p2 → 0.0725083,
  F1star → 0.0376889, F2star → 0.0376889}, {F1 → 0.133729, F2 → 0.133729,
  p1 → 0.827492, p2 → 0.827492, F1star → 0.169811, F2star → 0.169811},
 {F1 → 0.209227, F2 → 0.00160644, p1 → 0.0725083,
  p2 → 0.827492, F1star → 0.169811, F2star → 0.0376889}}
```

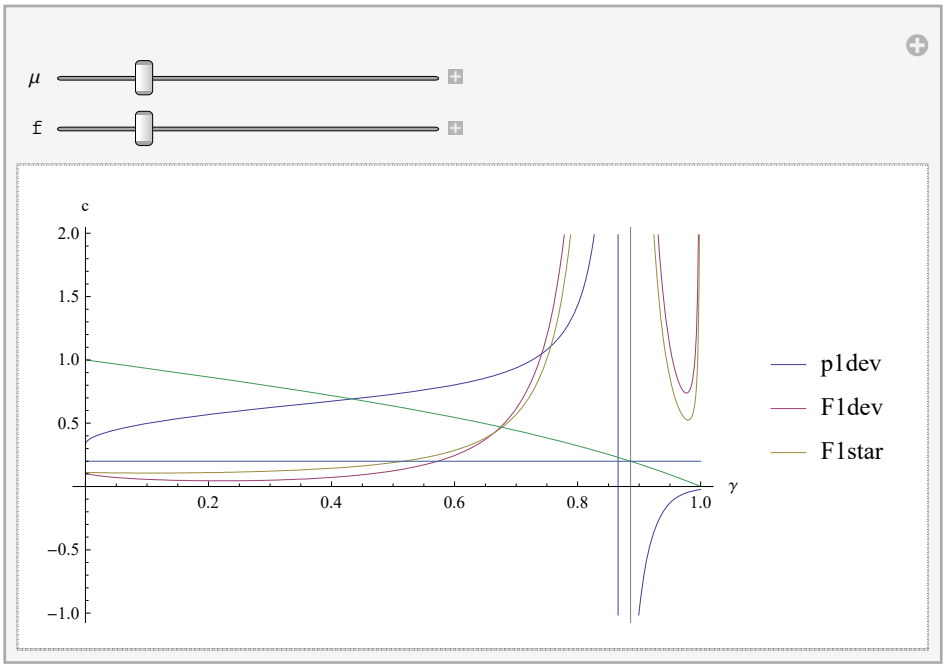
In[76]:=
```
(*The asymmetric solutions are not admissible. We plot
 the two symmetric solution, the second and the third.*)
{Solmostcooptmp[[2, 3, 2]], Solmostcooptmp[[2, 1, 2]],
  Solmostcooptmp[[2, 5, 2]], clim, c};
Manipulate[Plot[%, {γ, 0, 1},
  GridLines → {{(1 + μ - Sqrt[(1 + μ)^2 + 4 (-1 + c)^2 μ (1 + 2 μ)])/(2 (-1 + c) μ)}, {0}},
  PlotLegends → {p1dev, F1dev, F1star}, AxesLabel → {"γ", "c"}
 ], {{μ, 1}, 0, 5}, {{c, 0.2}, 0, 1}]
```

Out[77]=



In[78]:=
```
(*As it can be seen the third solution is not admissible
 as for some admissible values of γ it delivers marginal price
 above 1 and exhorbitant fixed fees that has shown below generate
 negative buyer's utility (better not participate at all).*)
{Solmostcooptmp[[3, 3, 2]], Solmostcooptmp[[3, 1, 2]],
  Solmostcooptmp[[3, 5, 2]], clim, c};
Manipulate[Plot[%, {γ, 0, 1},
  GridLines → {{(1 + μ - Sqrt[(1 + μ)^2 + 4 (-1 + c)^2 μ (1 + 2 μ)])/(2 (-1 + c) μ)}, {0}},
  PlotLegends → {p1dev, F1dev, F1star}, AxesLabel → {"γ", "c"}
 ], {{μ, 1}, 0, 5}, {{c, 0.2}, 0, 1}]
```

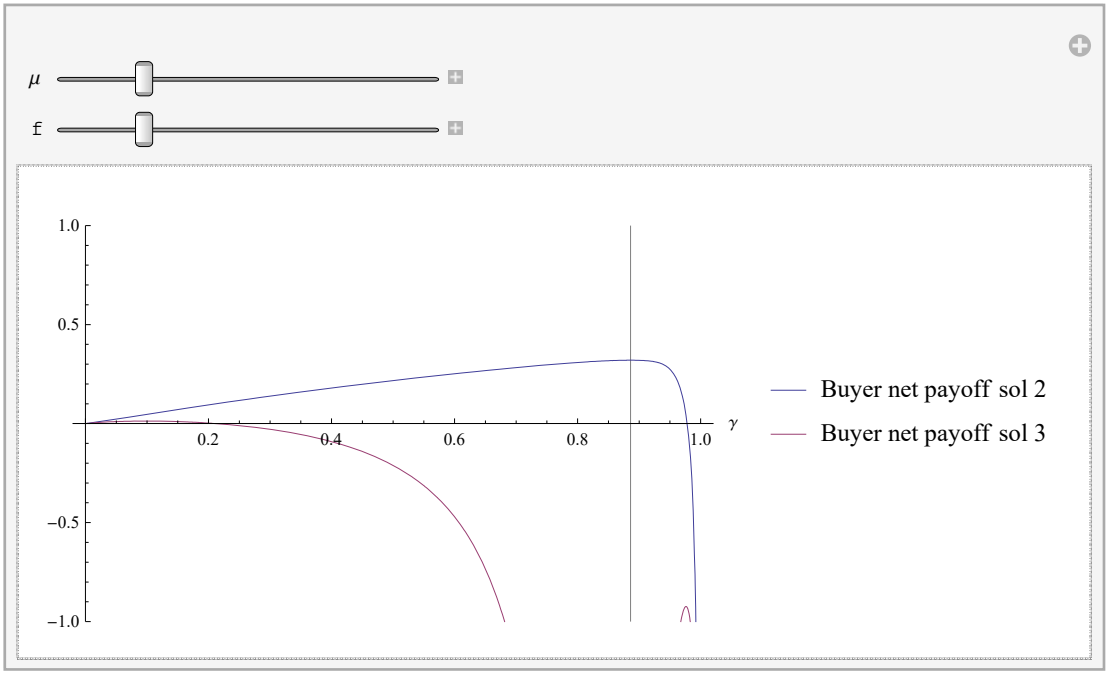
Out[79]=



In[80]:=
```
(*Defining the tariffs of the most
  cooperative equilibrium with Non-Exclusive:*)
TariffsMostCoopNE = {p1 → p1NE, F1 -> Solmostcooptmp[[2, 5, 2]],
   p1barr -> Solmostcooptmp[[2, 3, 2]], F1barr -> Solmostcooptmp[[2, 1, 2]],
   p2 → p2NE, F2 -> Solmostcooptmp[[2, 6, 2]], p2barr -> Solmostcooptmp[[2, 4, 2]],
   F2barr -> Solmostcooptmp[[2, 2, 2]]
  };
```

In[81]:=
```
(*Double check on retailer's participation with the two solutions*)
ProfitBNE /. pricesNE /. TariffsMostCoopNE;
ProfitBNE /. pricesNE /.
  {F1 -> Solmostcooptmp[[3, 5, 2]], F2 -> Solmostcooptmp[[3, 6, 2]]};
Manipulate[Plot[{%%, %}, {γ, 0, 1}, PlotRange → {-1, 1},
  GridLines → {{(1 + μ - Sqrt[(1 + μ)^2 + 4 (-1 + c)^2 μ (1 + 2 μ)])/(2 (-1 + c) μ)}, {0}},
  PlotLegends → {"Buyer net payoff sol 2", "Buyer net payoff sol 3"},
  AxesLabel → {"γ", ""}
 ], {{μ, 1}, 0, 5}, {{c, 0.2}, 0, 1}]
```

Out[83]=



In[84]:=
```
(*We next identify the region of existence of other,
less profitable for firms, equilibria parametrized
  by the (candidate) equilibrium fixed fees F1star and F2star.

   With respect to the previous analysis we proceed
  by not considering the NoDev constraints of the two firms.
   Inspection of the constraints shows that the No-
 mixing constraints allow to determine the barrage marginal prices
  (as functions of the equilibrium fixed fees Fstar only) and then,
with these prices, the buyer's no deviation constraints
 allow to determined the barrage fixed fees.*)

{BuyerNoMix1 == 0, BuyerNoMix2 == 0} /. k -> kdef;
Solmarginalptmp = FullSimplify[Solve[%, {p1, p2}]];

{Solmarginalptmp, q1e /. Solmarginalptmp} /.
 {F1star → 0.1, F2star → 0.1, γ → 0.5, μ → 0.7, c → 0}
(*This shows that the correct solution is the forth.*)
```

Out[86]=
$$\{\{\{p1 \to -1.66849, p2 \to -1.66849\}, \{p1 \to -1.66849, p2 \to 0.351414\}, \{p1 \to 0.351414, p2 \to -1.66849\}, \{p1 \to 0.351414, p2 \to 0.351414\}\}, \{2.66849, 2.66849, 0.648586, 0.648586\}\}$$

In[87]:=
```
p1barrgen = Solmarginalptmp[[4, 1, 2]];
p2barrgen = Solmarginalptmp[[4, 2, 2]];
```

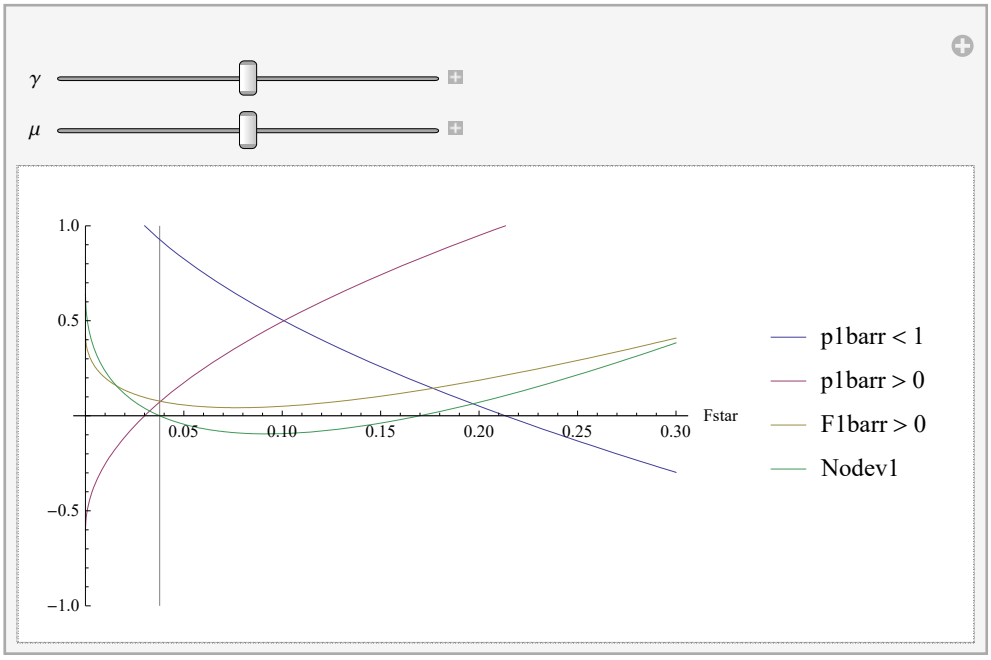
In[89]:=
```
(*Then we determine the barrage fixed fees:*)
{BuyerPartDev1 == 0, BuyerPartDev2 == 0};
FullSimplify[Solve[%, {F1, F2}]]

F1barrgen = %[[1, 1, 2]] /. k -> kdef /. p1 -> p1barrgen;
F2barrgen = %%[[1, 2, 2]] /. k -> kdef /. p2 -> p2barrgen;
```

Out[90]=
$$\left\{\left\{F1 \to \frac{-2k + (-1+p1)^2}{2(1+\mu)}, F2 \to \frac{-2k + (-1+p2)^2}{2(1+\mu)}\right\}\right\}$$

In[93]:=
```
(*Next we study other and les cooperative equilibria,
by vaying the fixed fees (e.g. leaving more rents to the buyer
   as compared with the most cooperative equilibrium determined),
without violating constraints on marginal prices and their own no-
 deviation constraint*)
```

In[94]:=
```
(*Consider first the symmetric case in which c=0*)
Solmostcooptmp[[2, 6, 2]] /. c → 0;
NoDev1 /. {F1 -> F1barrgen, F2 -> F2barrgen, p1 -> p1barrgen, p2 -> p2barrgen} /.
    F1star -> F2star /. c → 0;
{1 - p1barrgen, p1barrgen - 0, F1barrgen - 0, % - 0} /. F1star -> F2star /. c → 0;
Manipulate[Plot[%, {F2star, 0, 0.3}, PlotRange → {-1, 1}, GridLines → {{%%%}, {0}},
  PlotLegends → {p1barr < 1, p1barr > 0, F1barr > 0, Nodev1},
  AxesLabel → {"Fstar", ""}], {{γ, 0.5}, 0, 1}, {{μ, 1}, 0, 2}]
```

Out[97]=



In[98]:=
```
(*Note that the second and largest root for Nodev1
 (and the apparently admissible associated area) is not
 admissible because the associated barrage prices are not feasible
 (corresponds to the solution 3 tha delivers negative buyer's payoff).*)
```

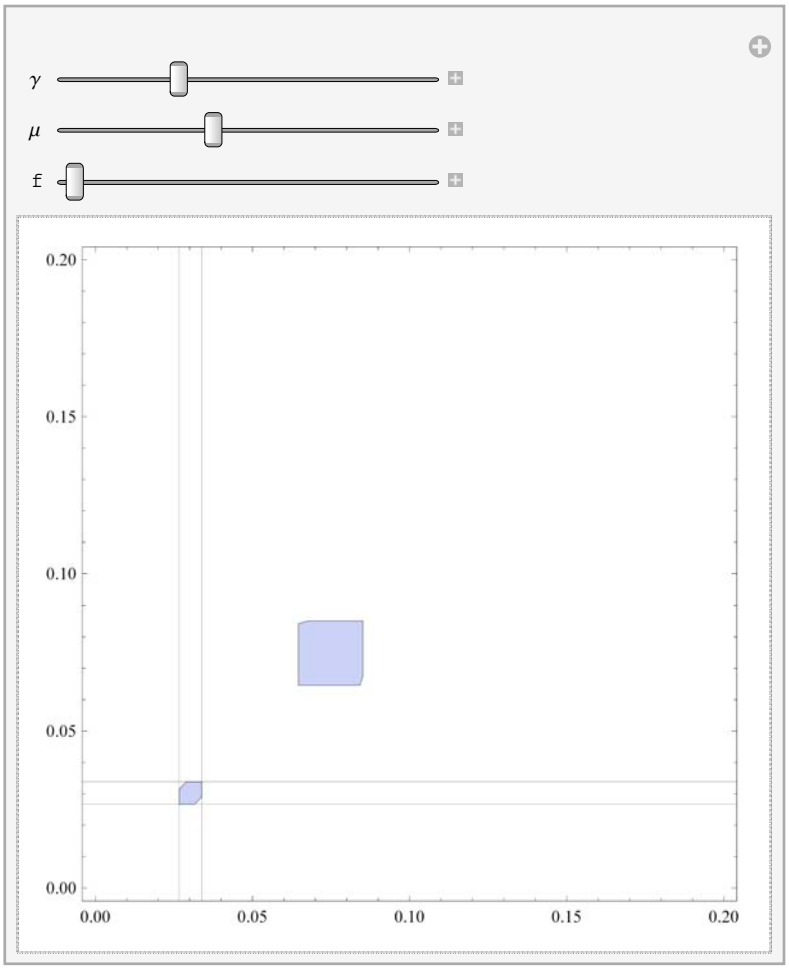
In[99]:=
```
(*As it can be seen,
the lower bound on the fixed fees (i.e. the minimally cooperative)
 are bounded by the non negativity of the marginal barrage prices:*)
{Numerator[p1barrgen] == 0, Numerator[p2barrgen] == 0};
MinFstar = FullSimplify[Solve[%, {F1star, F2star}]]
```

Out[100]=

$$\left\{\left\{\text{F1star} \to -\frac{(-1+\gamma+(-3+(3+c)\,\gamma)\,\mu+(-2+(3+c)\,\gamma-\gamma^3)\,\mu^2)^2}{2\,(-1+\gamma^2)\,(1+\mu)\,(1+\mu\,(4-(-4+\gamma^2)\,\mu))^2},\right.\right.$$
$$\left.\left.\text{F2star} \to -\frac{(-1+c+\gamma+3\,(-1+c+\gamma)\,\mu+(-2+2\,c+3\,\gamma-\gamma^3)\,\mu^2)^2}{2\,(-1+\gamma^2)\,(1+\mu)\,(1+\mu\,(4-(-4+\gamma^2)\,\mu))^2}\right\}\right\}$$

In[101]:=
```
(*Considering also non symmetric case,
all the constraints are satisfied in the colored
 area (where we also show the most cooperative fixed
   fees and the minimal (least cooeprative) fixed fees)*)

MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{NoDev1 > 0 && NoDev2 > 0 && p1barrgen > 0 && p2barrgen > 0 &&
    p1barrgen < 1 && p2barrgen < 1 && F1barrgen > 0 && F2barrgen > 0} /.
  {F1 -> F1barrgen, F2 -> F2barrgen, p1 -> p1barrgen, p2 -> p2barrgen};
Manipulate[RegionPlot[%, {F1star, 0, 0.2}, {F2star, 0, 0.2},
  GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.3}, 0, 1}, {{μ, 2}, 0, 5}, {c, 0, clim}]
```

Out[106]=



In[107]:=
```
(*Again the solutions on the morth-east area are not admissible
  because they give the buyer negative net payoff as shown above.*)
```

In[108]:=
```
(*As it can be seen the most cooperative fixed fees are the largest admissible.
    Least cooperative fixed fees are bounded away from zero for the non-
  negativity of the marginal barrage price.
    As it can be seen as γ→
 0 or γ→1 the only surviving equilibrium is the most cooperative.*)
```

In[109]:=
```
(*For future reference we define the payoffs
 associated with the most cooperative equilibrium:*)
Profit1NEMostCoop = Profit1NE /. TariffsMostCoopNE;
Profit2NEMostCoop = Profit2NE /. TariffsMostCoopNE;
ProfitBNEMostCoop = ProfitBNE /. TariffsMostCoopNE;
```

In[112]:=

In[113]:=
```
(*As a last step we check that the buyer's "no-mixing constraint" of the
 optimal deviation to exclusivity program must be binding in equilibrium,
otherwise either at the implied solution,
either it is violated or at least one of the firms deviates.

  The solution of the optimal deviation program is now either the monopolist's
 marginal price when the fixed fees is positive (case A) or when it is nil,
it is the marginal price the satisfies the buyer's
 participation constraint of the deviation program (case B).*)
```

In[114]:=
```
{NoDev1 == 0, NoDev2 == 0, BuyerPartDev1 == 0,
   BuyerPartDev2 == 0, BuyerNoMix1 == 0, BuyerNoMix2 == 0} /. k -> kdef;
```

In[115]:=
```
Profit1E
```

Out[115]=
$$F1 + p1 - p1^2$$

In[116]:=
```
(*Case A*)
{BuyerPartDev1 == 0, BuyerPartDev2 == 0};
FAtmp = FullSimplify[Solve[%, {F1, F2}]]
Profit1E /. F1 → FAtmp[[1, 1, 2]];
p1Em = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]]
Profit2E /. F2 → FAtmp[[1, 2, 2]];
p2Em = FullSimplify[Solve[D[%, p2] == 0, p2]][[1, 1, 2]]
FAsol = FAtmp /. {p1 → p1Em, p2 → p2Em} /. k → kdef;
```

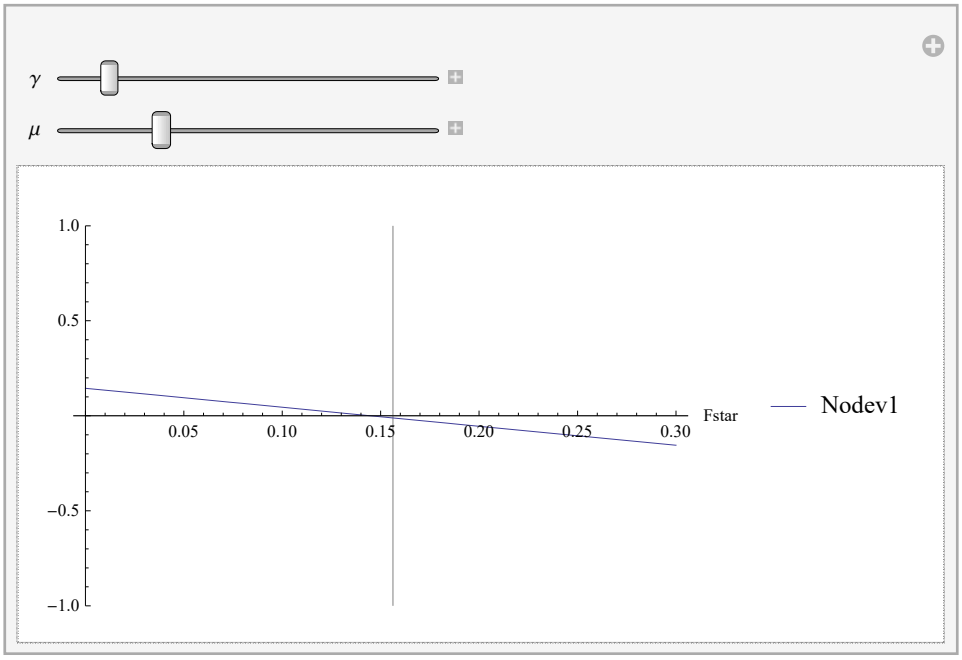
Out[117]=
$$\left\{\left\{F1 \to \frac{-2\,k + (-1 + p1)^2}{2\,(1+\mu)},\ F2 \to \frac{-2\,k + (-1 + p2)^2}{2\,(1+\mu)}\right\}\right\}$$

Out[119]=
$$\frac{\mu}{1 + 2\,\mu}$$

Out[121]=
$$\frac{c + \mu + c\,\mu}{1 + 2\,\mu}$$

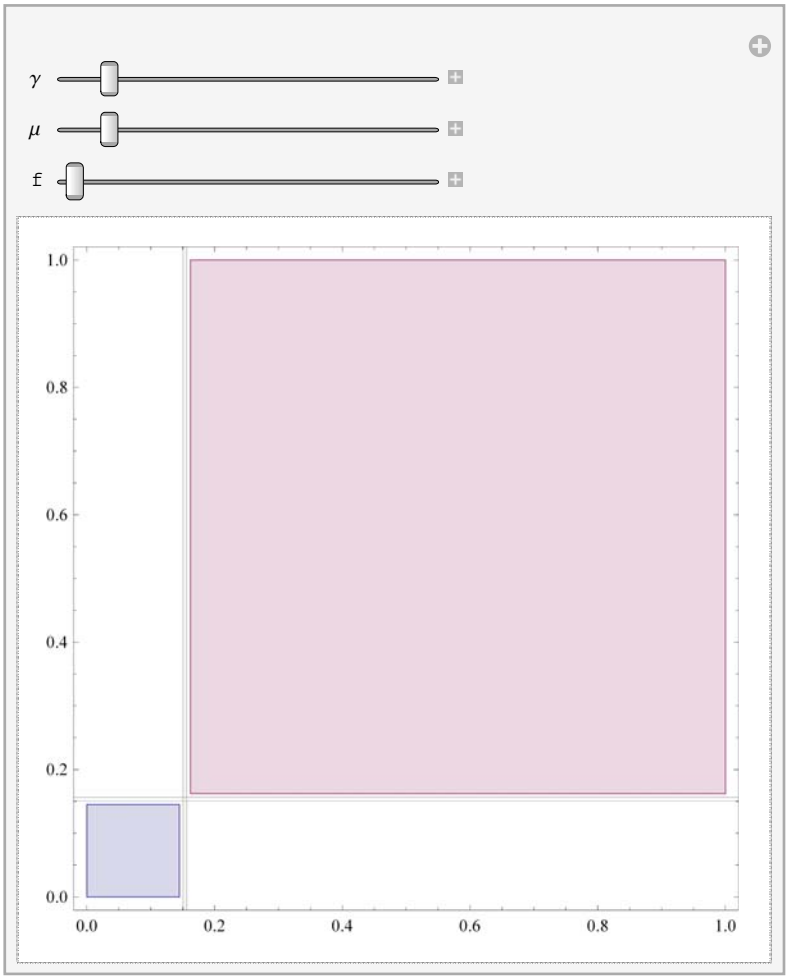
In[123]:=
```
(*Consider first the symmetric case in which c=0*)
Solmostcooptmp[[2, 6, 2]] /. c → 0;
NoDev1 /. {F1 -> FAsol[[1, 1, 2]], F2 -> FAsol[[1, 2, 2]], p1 → p1Em, p2 → p1Em} /.
   F1star -> F2star /. c → 0;
{% - 0} /. F1star -> F2star /. c → 0;
Manipulate[Plot[%, {F2star, 0, 0.3},
  PlotRange → {-1, 1}, GridLines → {{%%%}, {0}}, PlotLegends → {Nodev1},
  AxesLabel → {"Fstar", ""}], {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 2}]
```

Out[126]=



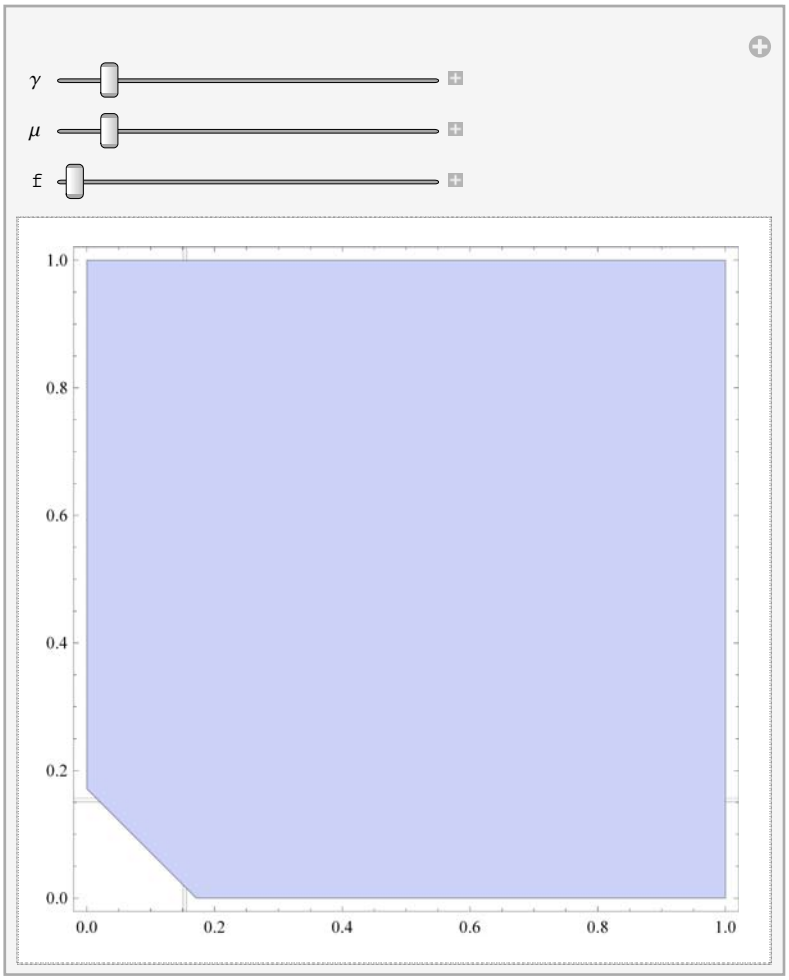
In[127]:=
```
(*Now the general case: as it can be seen there are no
   admissible Fstar that satisfy both sets of remining constraints*)
MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{NoDev1 > 0 && NoDev2 > 0, BuyerNoMix1 > 0 && BuyerNoMix2 > 0} /.
  {F1 -> FAsol[[1, 1, 2]], F2 -> FAsol[[1, 2, 2]], p1 → p1Em, p2 → p1Em};
Manipulate[RegionPlot[%, {F1star, 0, 1}, {F2star, 0, 1},
  GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 5}, {c, 0, clim}]
```

Out[132]=



In[133]:=
```
(*The fixed barrage fees studied in case A can be negative,
hence we need to consider also case B*)
MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{FAsol[[1, 1, 2]] > 0 && FAsol[[1, 2, 2]] > 0};
Manipulate[
 RegionPlot[%, {F1star, 0, 1}, {F2star, 0, 1}, GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 5}, {c, 0, clim}]
```

Out[138]=

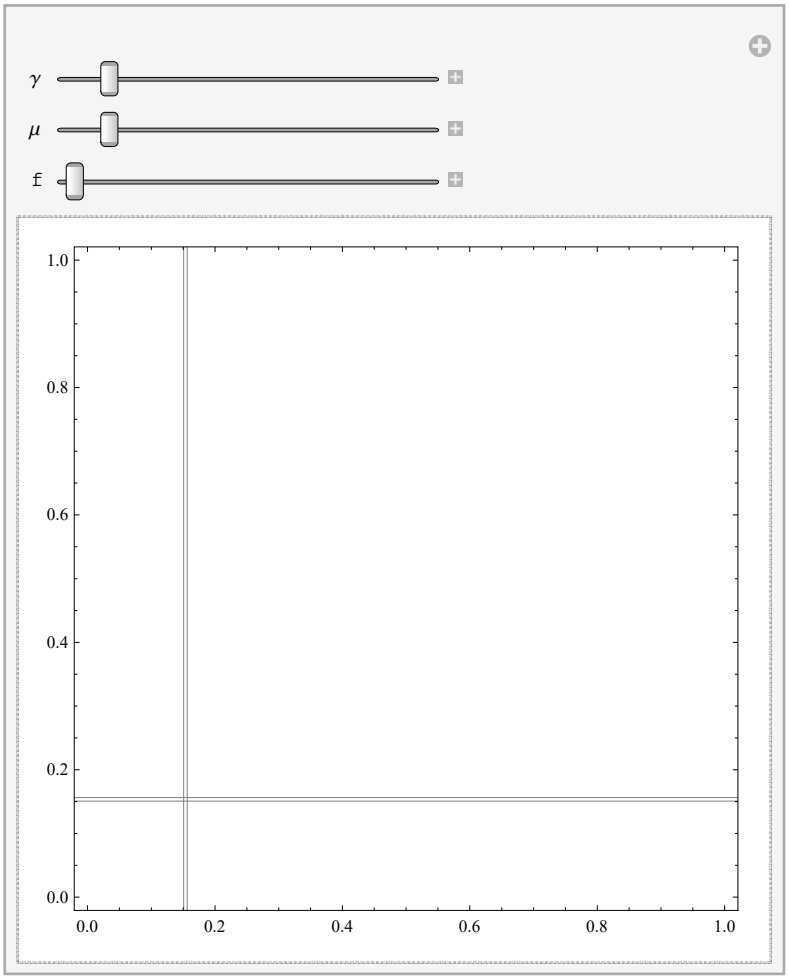


In[139]:=
```
(*Case B*)
{BuyerPartDev1 == 0, BuyerPartDev2 == 0} /. {F1 → 0, F2 → 0};
pBtmp = FullSimplify[Solve[%, {p1, p2}]]
```

Out[140]=
$$\left\{\left\{p1 \to 1-\sqrt{2}\ \sqrt{k},\ p2 \to 1-\sqrt{2}\ \sqrt{k}\right\},\ \left\{p1 \to 1-\sqrt{2}\ \sqrt{k},\ p2 \to 1+\sqrt{2}\ \sqrt{k}\right\},\ \left\{p1 \to 1+\sqrt{2}\ \sqrt{k},\ p2 \to 1-\sqrt{2}\ \sqrt{k}\right\},\ \left\{p1 \to 1+\sqrt{2}\ \sqrt{k},\ p2 \to 1+\sqrt{2}\ \sqrt{k}\right\}\right\}$$

In[141]:=
```
(*The next graph shows that in case B the buyer always prefers
 to mix the contracts hence, case B cannot be a solution.*)
MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{ BuyerNoMix1 > 0 && BuyerNoMix2 > 0} /.
    {F1 → 0, F2 → 0, p1 → pBtmp[[1, 1, 2]], p2 → pBtmp[[1, 2, 2]]} /. k → kdef;
Manipulate[RegionPlot[%, {F1star, 0, 1}, {F2star, 0, 1},
  GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 5}, {c, 0, clim}]
```

Out[146]=



In[147]:=

Analysis: Exclusive contracts permitted (equilibria with exclusive representation: no coordination)

In[148]:=
```
(*EQUILIBRIA WITH EXCLUSIVE REPRESENTATION*)
(*As discussed in the text, with exclusive contracts admissible,
any equilibrium involves exculisvity and contemplates the buyer purchasing
 from the dominant firm 1, at the following prices and associated
 payoffs and firm 2 offering the following exclusive contract.*)
p2Eequ = c; F2Eequ = 0;

(*if c≥p1m=μ/(1+2 μ) then*)
p1Eequ = p1m; F1Eequ = F1m;

PROFIT1Eequ = PROFIT1Em
PROFITBEequ = PROFITBE1m
```

Out[150]=
$$\frac{\mu^2 - c^2\,(1 + 2\,\mu) + c\,(2 + 4\,\mu)}{2 + 6\,\mu + 4\,\mu^2}$$

Out[151]=
$$\frac{1}{2}\,(-1 + c)^2$$

In[152]:=
```
(*if instead c<p1m=μ/(1+2 μ) then we are in a corner solution with*)
p1Eequcorn = c; F1Eequbcorn = 0;

q1e /. p1 -> p1Eequcorn;
PROFIT1Eequcorn = FullSimplify[% * p1Eequcorn ]

ue1 /. q1 -> q1e /. p1 -> p1Eequcorn;
 PROFITBEequcorn = FullSimplify[% - %%% p1Eequcorn]
```

Out[154]= $-(-1+c)\,c$

Out[156]= $\frac{1}{2}(-1+c)^2$

In[157]:=
```
(*The threshold for these two cases is*)
cEcorner = p1m;
```

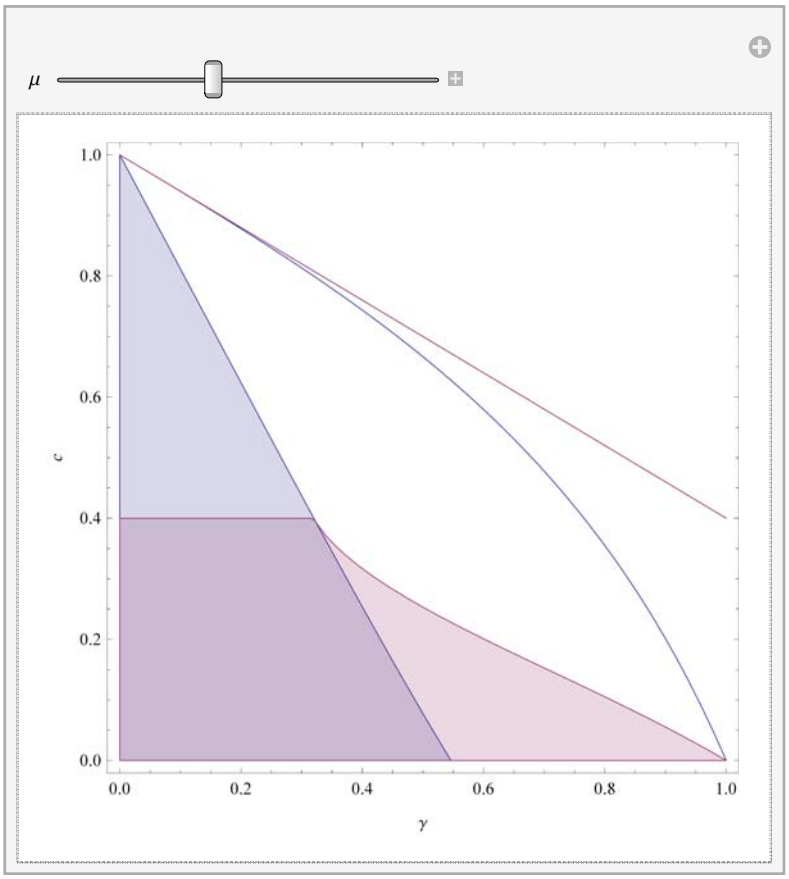
In[158]:=
```
(*The equilibria with comon representation when exclusive
 contracts are still admissible are investigated in another file.*)
```

Comparison with and without exclusive contracts (Figure 3)

In[159]:=
```
(*We first consider the most cooperative equilibrium with non-exclusivity.*)
```

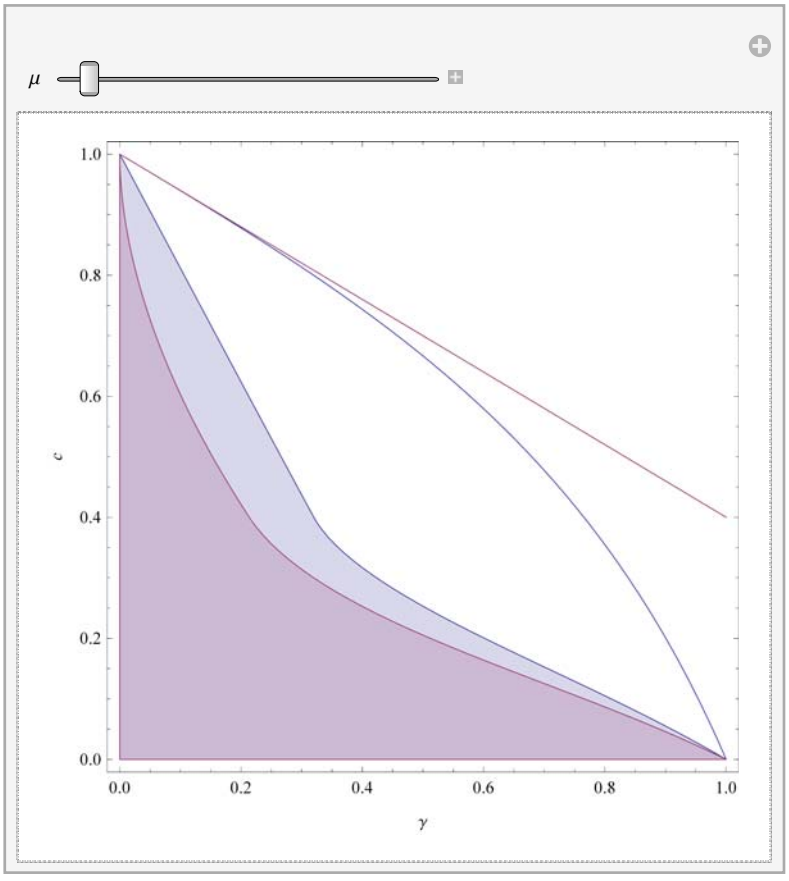
In[160]:=
```
{clim, cdrast};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO,
  Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO};
Manipulate[Show[RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c}],
  Plot[%%, {γ, 0, 1}]], {{μ, 2}, 0, 5}]
```

Out[162]=



In[163]:=
```
(*We consider here the least cooperative equilibrium with no-
 exclusivity as well*)
Profit1NELeastCoop = Profit1NE /. pricesNE /. F1 -> MinFstar[[1, 1, 2]];
{clim, cdrast};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
   Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO,
  Profit1NELeastCoop - PROFIT1Eequ > 0 && c < cPPO ||
   Profit1NELeastCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO
 };
Manipulate[Show[RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c}],
  Plot[%%, {γ, 0, 1}]], {{μ, 2}, 0, 50}]
```

Out[166]=



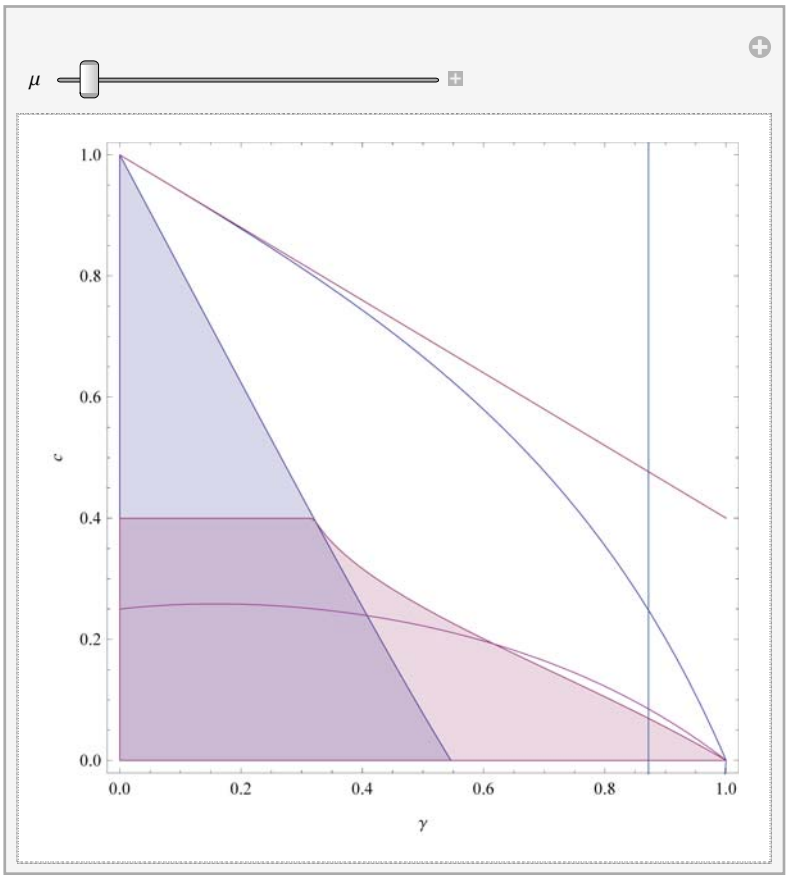
In[167]:=
```
(*As expected the two cases are qualitatively similar,
and they almost coincide when μ is large (in fact in both cases
   we converge to linear prices where fized fees are irrelevant)*)
```

In[168]:=
```
(*From now on for simplicity we will present figures
  with the most cooperative equilibrium with no-exclusivity.*)
```

In[169]:=
```
(*We verify the effect of Exclusivity
 on welfare with the same graphical analysis.
  As discussed in the text the appropriate measure to be used
 is the buyer's gross surplus (gross of the fixed fees)*)
u - p1 q1 - p2 q2 /. {q1 → q1NE, q2 → q2NE} /. pricesNE;
UgrossNE = %;

ue1 - p1 q1 /. {q1 → q1e} /. {p1 → p1Eequ};
UgrossE = %;

ue1 - p1 q1 /. {q1 → q1e} /. {p1 → p1Eequcorn};
UgrossEcorn = %;

(*The next comparison is the one relevant for c>p1m,
where p1m is the blue line in figure. Hence,
between the green lines and above the
  blue line Exclusivity dominates Non-Exclusivity*)
tmpwelf = Solve[UgrossNE - (UgrossE) == 0, c];
(*The graph will show that for this case c>p1m,
E is always detrimental for the buyer*)

(*The next comparison is the one relevant below the blue
 line: between the black lines Exclusivity dominates Non-Exclusivity*)
tempwelfcorn = Solve[UgrossNE - (UgrossEcorn) == 0, c];
```

In[177]:=
```
{clim, cdrast, tmpwelf[[1, 1, 2]], tmpwelf[[2, 1, 2]],
  tempwelfcorn[[1, 1, 2]], tempwelfcorn[[2, 1, 2]]};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO,
  Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO};
Manipulate[Show[RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c}],
  Plot[%%, {γ, 0, 1}]], {{μ, 2}, 0, 50}]
```

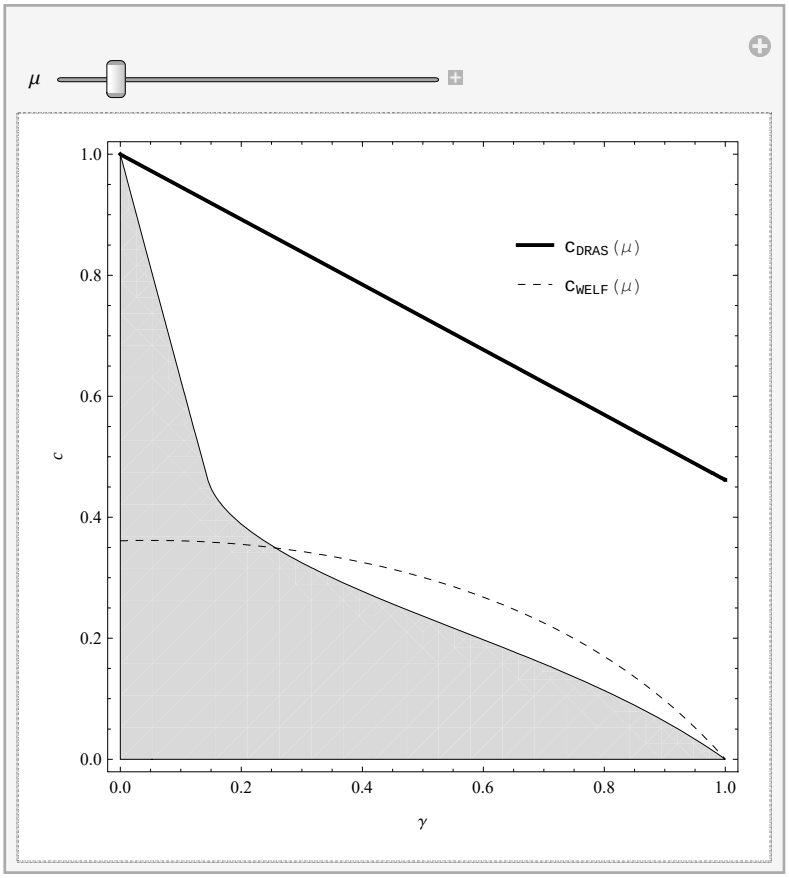
Out[179]=



In[180]:=
```
(*Hence the relevant boundary is:*)
EffectonConsSurplMostCoop = tempwelfcorn[[2, 1, 2]];
```

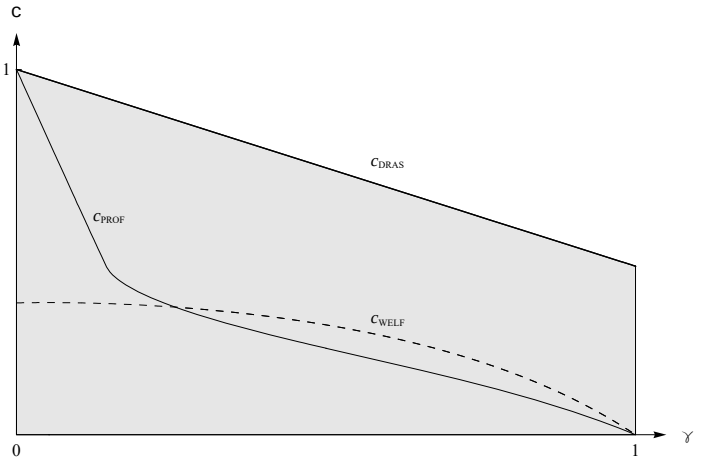
In[181]:=
```
(*Putting toghether the boundaries with welfare effect:*)
{cdrast, EffectonConsSurplMostCoop};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
   Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO};
Manipulate[Show[

  RegionPlot[%, {γ, 0, 1}, {c, 0, 1},
   FrameLabel → {γ, c}, BoundaryStyle → Black, PlotStyle → LightGray

  ],

  Plot[%%, {γ, 0, 1}, PlotStyle → {{Black, Thick}, {Black, Dashed}},
   PlotLegends → Placed[LineLegend[{"c_DRAS(μ)", "c_WELF(μ)"},
      LabelStyle → {FontFamily → "Courier"}], {0.75, 0.8}]]

 ], {{μ, 6}, 0, 50}]
```

Out[183]=



In[184]:=
```
(*We now plot the Figure in the paper.*)
μplot = 6;
vertline = {{1, 0}, {1, p1Em}} /. μ → μplot;
{cdrast, EffectonConsSurplMostCoop} /. μ → μplot;
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
    Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO} /. μ → μplot;
Figure3 = Show[

  Plot[%%, {γ, 0, 1},
   PlotStyle → {{Black, Thickness[0.002]}, {Black, Dashed}}, AxesLabel → {"γ", "c"},
   Ticks → {{0, 1}, {-1, 1}},
   PlotRange → {{0, 1.05}, {0, 1.1}},
   AxesStyle → Arrowheads[{0.0, 0.02}],
   Filling → {{1 -> Bottom}},
   FillingStyle → {Lighter[Gray, 0.8]}
  ],

  RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c},
   BoundaryStyle → Black, PlotStyle → Lighter[Gray, 0.8], Frame → False],




  Plot[%%, {γ, 0, 1},
   PlotStyle → {{Black, Thickness[0.002]}, {Black, Dashed}}, Ticks → {Null}],

  Graphics[{Line[vertline]}],
  Graphics[
   {Text["c_DRAS", {0.6, 0.75}], Text["c_PROF", {0.15, 0.6}], Text["c_WELF", {0.6, 0.32}]}]
 ]


SetDirectory[NotebookDirectory[]];
SetDirectory[ParentDirectory[]];
SetDirectory["Figures"];
Export["Figure3.pdf", Figure3];
```

Out[188]=



In[193]:=
```
(*As discussed in the main text,
for μ large enough there exist a region in which Exclusivity is both welfare
 increasing (below the black line) and optimal for the dominant firm 1.*)
```