Online Appendix 1 of Calzolari, Denicolò and Zanchettin “The demand-boost theory of exclusive dealing,” 2019

The Reduced Form model with Competitive Fringe of Section 3

November 2019

Defining payoffs and tariffs

In[1]:=
```
(*Retailer's utility*)
u = (q1 + q2) - 1 / 2 (q1^2 + q2^2) - γ q1 q2;
ue1 = u /. q2 → 0;
ue2 = u /. q1 → 0;

(*Parameters: c in [0,1], γ in [0,1]*)

P1 = p1 q1 + F1; (*Two part tariff of dominant firm 1*)
P2 = c q2; (*Competitive pricing of the Fringe*)
```

In[6]:=
```
(*Semi-indirect utility of buyer*)
q2star = Solve[D[u - P2, q2] == 0, q2][[1, 1, 2]]
v = FullSimplify[u - P2 /. q2 → q2star]
```

Out[6]= $1 - c - q1\,\gamma$

Out[7]= $\frac{1}{2}\left(1 + c^2 + 2\,c\,(-1 + q1\,\gamma) + q1\,(-1 + \gamma)\,(-2 + q1 + q1\,\gamma)\right)$

In[8]:=
```
(*Quantities*)
q1NE = Solve[D[v - P1, q1] == 0, q1][[1, 1, 2]]
q2NE = FullSimplify[q2star /. q1 → q1NE]
q1E = Solve[D[ue1 - P1, q1] == 0, q1][[1, 1, 2]]
```

Out[8]= $\frac{-1+p1+\gamma-c\,\gamma}{-1+\gamma^2}$

Out[9]= $\frac{-1+c+\gamma-p1\,\gamma}{-1+\gamma^2}$

Out[10]= $1-p1$

In[11]:=
```
(*Non exclusive payoffs*)
u - p1 q1 - c q2 /. {q1 → q1NE, q2 → q2NE};
ProfitBNE = FullSimplify[%] - (1 + μ) F1
ProfitBE1 = FullSimplify[ue1 - p1 q1 /. q1 → q1E] - (1 + μ) F1

P1 /. {q1 → q1NE, q2 → q2NE};
Profit1NE = FullSimplify[%]
Profit1E = FullSimplify[P1 /. q1 → q1E]
```

Out[12]= $-\frac{2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma}{2\,(-1+\gamma^2)}-F1\,(1+\mu)$

Out[13]= $\frac{1}{2}\,(-1+p1)^2-F1\,(1+\mu)$

Out[15]= $F1+\frac{p1\,(-1+p1+\gamma-c\,\gamma)}{-1+\gamma^2}$

Out[16]= $F1+p1-p1^2$

In[17]:=
```
v0 = Simplify[v /. q1 → 0] (*Reservation payoff for buyer,
 exclusively dealing with the fringe*)
```

Out[17]= $\frac{1}{2}\,(-1+c)^2$

Analysis: Exclusive contracts prohibited

In[18]:=
```
(*Firm 1 sets the fixed fee so as to extract the bilateral surplus*)
tmpF = FullSimplify[Solve[{v0 == ProfitBNE}, {F1}]]
F1dtmp = tmpF[[1, 1, 2]];
```

Out[18]= $\left\{\left\{F1\to-\frac{(-1+p1+\gamma-c\,\gamma)^2}{2\,(-1+\gamma^2)\,(1+\mu)}\right\}\right\}$

In[20]:=
```
(*Substituting the fixed fees in the profits,
we determine the equilibrium marginal price of firm 1*)
Profit1NEtmp = Profit1NE /. F1 → F1dtmp;
tmpp = FullSimplify[Solve[{D[Profit1NEtmp, p1] == 0}, {p1}]]
p1NE = tmpp[[1, 1, 2]];
p2NE = c;

pricesNE = {p1 → p1NE, p2 → p2NE};

(*and then the associated equilibrium payoffs*)
PROFIT1NE = FullSimplify[Profit1NEtmp /. pricesNE];
PROFITBNE = FullSimplify[ProfitBNE /. pricesNE /. F1 → F1dtmp]
WelfNE = FullSimplify[u - p1 q1 - c q2 /. {q1 → q1NE, q2 → q2NE} /. pricesNE]
```

Out[21]=
$$\left\{\left\{p1 \to \frac{(1+(-1+c)\,\gamma)\,\mu}{1+2\,\mu}\right\}\right\}$$

Out[26]=
$$\frac{-2+2\,c-c^2+(-1+p1+\gamma-c\,\gamma)^2-\frac{(1+(-1+c)\,\gamma)^2\,\mu^2}{(1+2\,\mu)^2}+\frac{2\,(1+(-1+c)\,\gamma)\,\mu}{1+2\,\mu}+2\,(-1+c)\,\gamma\left(-1+\frac{(1+(-1+c)\,\gamma)\,\mu}{1+2\,\mu}\right)}{2\,(-1+\gamma^2)}$$

Out[27]=
$$\frac{1}{2\,(-1+\gamma^2)\,(1+2\,\mu)^2}\Big(-2\,c\,(-1+\gamma)\,(1+\mu\,(4+2\,\gamma+4\,\mu+3\,\gamma\,\mu))+(-1+\gamma)\,(2+\mu\,(6+5\,\mu+\gamma\,(2+3\,\mu)))+c^2\left(-1+\mu\left(-4\,(1+\mu)+\gamma^2\,(2+3\,\mu)\right)\right)\Big)$$

In[28]:=
```
(*With μ→0 marginal prices are efficient*)
Limit[p1NE, μ → 0]
```

Out[28]= 0

In[29]:=
```
(*Here we redefine the fixed fees to account for the equilibrium prices*)
F1NE = FullSimplify[F1dtmp /. pricesNE]
```

Out[29]=
$$-\frac{(1+(-1+c)\,\gamma)^2\,(1+\mu)}{2\,(-1+\gamma^2)\,(1+2\,\mu)^2}$$

In[30]:=
```
(*With μ→Infinity fixed fee vanishes and we are left with linear pricing*)
Limit[F1NE, μ → Infinity]
```

Out[30]= 0

In[31]:=
```
(*We show that these fixed fees are always positive
 and we do not need to take care of corner solutions:*)
Reduce[{F1NE ≥ 0, 1 ≥ μ ≥ 0, 1 ≥ γ ≥ 0, 1 ≥ c ≥ 0}]
```

Out[31]= $0 \le \mu \le 1 \;\&\&\; 0 \le \gamma < 1 \;\&\&\; 0 \le c \le 1$

In[32]:=
```
(*Now we determine the Limit Pricing strategy and its region*)
q2NE /. pricesNE;
tmpqlim = FullSimplify[Solve[% == 0, c]]
clim = tmpqlim[[1, 1, 2]];
(*for c above clim the inefficient firm 2 cannot be active*)
```

Out[33]= $\left\{\left\{c \to \frac{(-1+\gamma)\ (1+(2+\gamma)\ \mu)}{-1+(-2+\gamma^2)\ \mu}\right\}\right\}$

In[35]:=
```
(*We determine the limit pricing strategy.*)
p1Limit = FullSimplify[Solve[0 == q2NE /. p2 → c, p1]][[1, 1, 2]]
```

Out[35]= $\frac{-1+c+\gamma}{\gamma}$

In[36]:=
```
(*The associated fixed fee is determined as follows.*)
FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit];
F1Limit = FullSimplify[Solve[v0 == %, F1]][[1, 1, 2]]
```

Out[37]= $-\frac{(-1+c)^2\ (-1+\gamma^2)}{2\gamma^2\ (1+\mu)}$

In[38]:=
```
(*The associated payoffs are as follows.*)
PROFITBLimit = FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit /. F1 -> F1Limit]
PROFIT1Limit = FullSimplify[Profit1E /. q1 -> q1e /. p1 → p1Limit /. F1 -> F1Limit]
WelfLimit = FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit /. F1 → 0]
```

Out[38]= $\frac{1}{2}\ (-1+c)^2$

Out[39]= $-\frac{(-1+c)\ (-(-1+\gamma)\ (-1+\gamma-2\mu)+c\ (1+\gamma^2+2\mu))}{2\gamma^2\ (1+\mu)}$

Out[40]= $\frac{(-1+c)^2}{2\gamma^2}$

In[41]:=
```
(*We now find the "Drastic region" in which also without
 exclusivity clause the dominant can price at the monopoly price*)

(*First we need to find the unconstrained monopolist's optimal price*)
ProfitBE1 - v0;
tmpF1m = FullSimplify[Solve[% == 0, F1]]
Profit1E /. F1 → tmpF1m[[1, 1, 2]];
p1m = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]]
F1m = FullSimplify[tmpF1m[[1, 1, 2]] /. p1 → p1m]
```

Out[42]= $\left\{\left\{F1 \to -\frac{(c - p1)\ (-2 + c + p1)}{2\ (1 + \mu)}\right\}\right\}$

Out[44]= $\frac{\mu}{1 + 2\ \mu}$

Out[45]= $-\frac{\left(c - \frac{\mu}{1+2\ \mu}\right)\ \left(-2 + c + \frac{\mu}{1+2\ \mu}\right)}{2\ (1 + \mu)}$

In[46]:=
```
(*And the associated payoffs are as follows.*)
PROFIT1Em = FullSimplify[Profit1E /. p1 → p1m /. F1 → F1m]
PROFITBE1m = FullSimplify[ProfitBE1 /. p1 → p1m /. F1 → F1m]
Welf1m = FullSimplify[ProfitBE1 /. p1 → p1m /. F1 → 0]
```

Out[46]= $\frac{\mu^2 - c^2\ (1 + 2\ \mu) + c\ (2 + 4\ \mu)}{2 + 6\ \mu + 4\ \mu^2}$

Out[47]= $\frac{1}{2}\ (-1 + c)^2$

Out[48]= $\frac{(1 + \mu)^2}{2\ (1 + 2\ \mu)^2}$

In[49]:=
```
(*Now, we determine the minimum c that allows the dominant firm to give-
 up with limit pricing and directly price as a monopolist.*)
FullSimplify[Solve[p1m == p1Limit, c]];
cdrast = %[[1, 1, 2]]
```

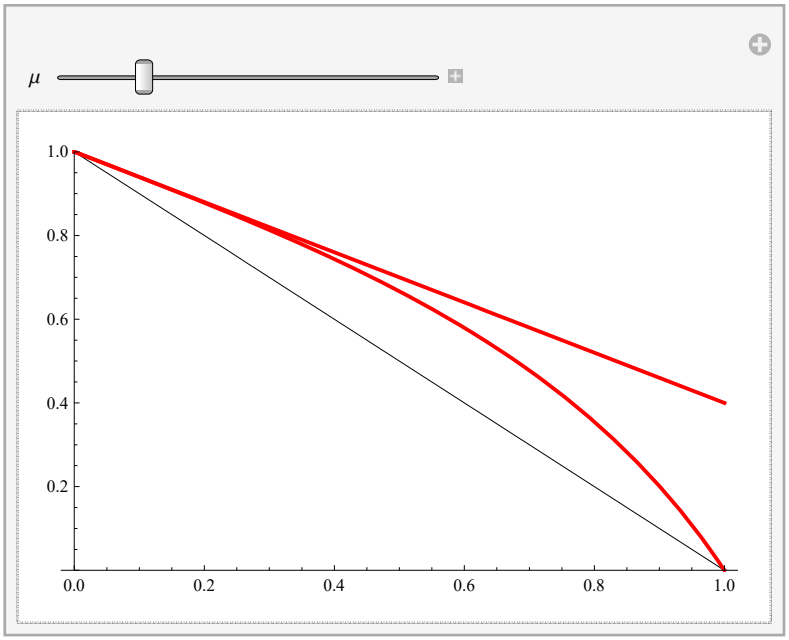
Out[50]= $\frac{1 + 2\ \mu - \gamma\ (1 + \mu)}{1 + 2\ \mu}$

In[51]:=
```
(*For future reference we also define
 a Positive Primary Otuput (PPO) boundary, cPPO*)
Solve[{D[u - c q2, q1] == 0, D[u - c q2, q2] == 0}, {q1, q2}];
Solve[%[[1, 2, 2]] == 0, c];
cPPO = %[[1, 1, 2]]
```

Out[53]= $1 - \gamma$

In[54]:=
```
(*We now plot these threshold for any Lambda, c and gamma*)
{cPPO, clim, cdrast};
Manipulate[Plot[%, {γ, 0, 1}, PlotStyle -> {{Black}, {Red, Thick}, {Red, Thick}},
  PlotRange → {0, 1}], {{μ, 2}, 0, 10}]
```

Out[55]=



Analysis: Exclusive contracts allowed

In[56]:=
```
(*EQUILIBRIA WITH EXCLUSIVE REPRESENTATION*)
(*As discussed in the text, with exclusive contracts admissible,
any equilibrium involves exculisvity and contemplates the buyer purchasing
 from the dominant firm 1, at the following prices and associated
 payoffs and firm 2 offering the following exclusive contract.*)
p2Eequ = c; F2Eequ = 0;

(*if c≥p1m=μ/(1+2 μ) then*)
p1Eequ = p1m; F1Eequ = F1m;

PROFIT1Eequ = PROFIT1Em
PROFITBEequ = PROFITBE1m
```

Out[58]=
$$\frac{\mu^2 - c^2\,(1 + 2\,\mu) + c\,(2 + 4\,\mu)}{2 + 6\,\mu + 4\,\mu^2}$$

Out[59]=
$$\frac{1}{2}\,(-1 + c)^2$$

In[60]:=
```
(*if instead c<p1m=μ/(1+2 μ) then we are in a corner solution with*)
p1Eequcorn = c; F1Eequbcorn = 0;

q1E /. p1 -> p1Eequcorn;
PROFIT1Eequcorn = FullSimplify[% * p1Eequcorn ]

ue1 /. q1 -> q1E /. p1 -> p1Eequcorn;
PROFITBEequcorn = FullSimplify[% - %%% p1Eequcorn]
```

Out[62]= $-(-1+c)\,c$

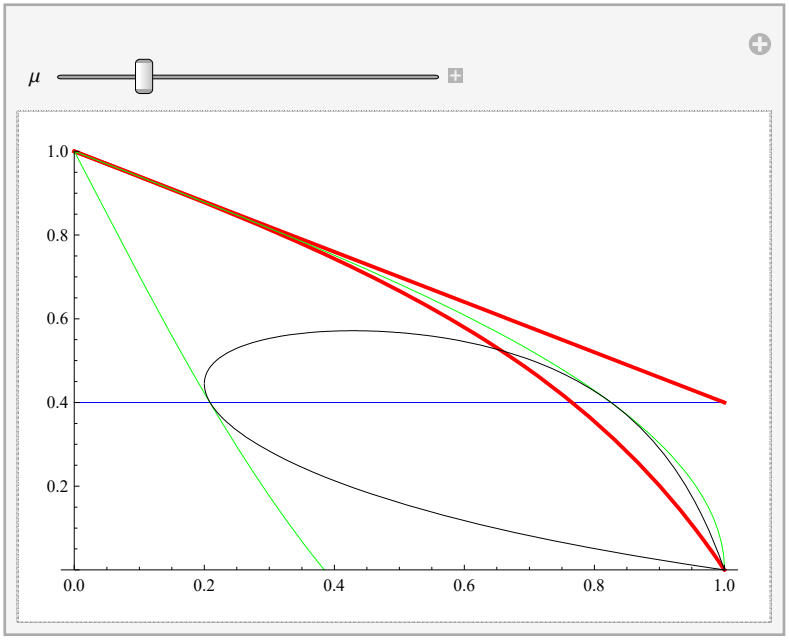
Out[64]= $\frac{1}{2}(-1+c)^2$

In[65]:=
```
(*The threshold for these two cases is*)
cEcorner = p1m;
```

Effects of Exclusive contracts, Figure 1 and Figure 2

In[66]:=
```
(*We compare the common representation equilibrium when exclusive
 contracts are forbidden with the equilibrium with exclusivity.*)

(*Profitability of Exclusivity for the dominant Firm 1:
 First notice that when with Non-Exclusivity the limit pricing
   emerges (i.e. above the red dashed curved line in the figure below)
   then Exclusivity clearly dominates for firm 1.*)

(*The next comparison is the one relevant for c>p1m,
where p1m is the blue line in figure. Hence,
between the green lines and above the
  blue line Exclusivity dominates Non-Exclusivity*)
tmpprofNE = Solve[PROFIT1NE - PROFIT1Eequ == 0, c];

(*The next comparison is the one relevant below the blue line i.e. for c<
 p1m: between the black lines Exclusivity dominates Non-Exclusivity*)
tmpprofNEcorn = Solve[PROFIT1NE - PROFIT1Eequcorn == 0, c];

(*Plotting all boundaries*)
{p1m, cdrast, clim, tmpprofNE[[1, 1, 2]], tmpprofNE[[2, 1, 2]],
  tmpprofNEcorn[[1, 1, 2]], tmpprofNEcorn[[2, 1, 2]]};
Manipulate[Plot[%, {γ, 0, 1}, PlotRange → {0, 1}, PlotStyle →
   {Blue, {Red, Thick}, {Red, Thick}, Green, Green, Black, Black}], {{μ, 2}, 0, 10}]
```

Out[69]=

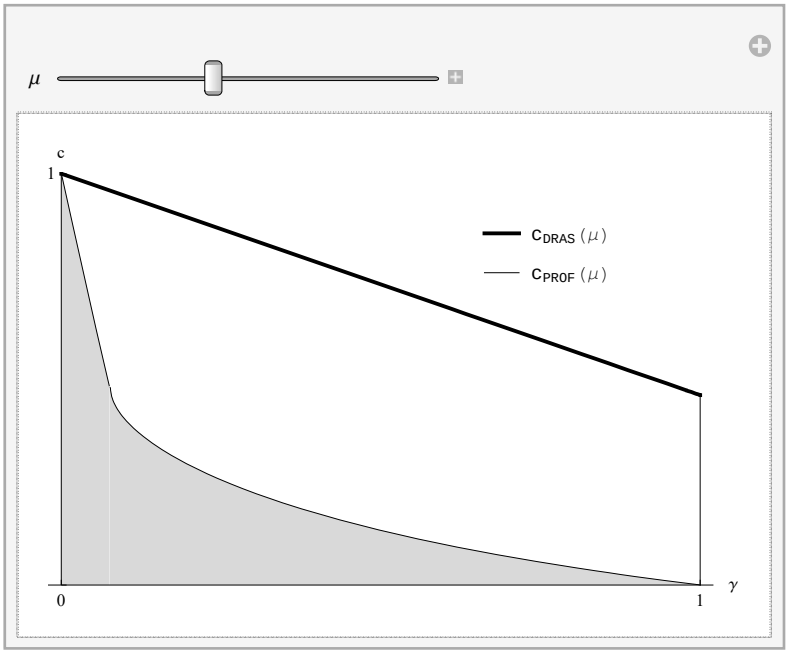


In[70]:=
```
(*Hence we define the relevant boundaries*)
```

In[71]:=
```
profitability1E = tmpprofNE[[1, 1, 2]];
profitability1Ecorn = tmpprofNEcorn[[2, 1, 2]];
```

In[73]:=
```
(*Summarizing the effect of exclusivity as in the paper (figure 1)*)
vertline = {{1, 0}, {1, p1Eequ}};
{cdrast, Max[Re[profitability1E], Re[profitability1Ecorn]]};
Manipulate[Show[
  Quiet[Plot[%, {γ, 0, 1}, PlotRange → {0, 1}, PlotStyle → {{Black, Thick}, {Black}},

    Filling → {2 -> Bottom},
    FillingStyle → {LightGray},
    AxesLabel → {"γ", "c"},

    PlotLegends → Placed[LineLegend[{"c_DRAS(μ)", "c_PROF(μ)"},
       LabelStyle → {FontFamily → "Courier"}], {0.75, 0.8}],
    Ticks → {{0, 1}, {0, 1}}

   ]],
  Graphics[{Line[%%]}]
 ], { {μ, 6}, 0.01, 15}]
```

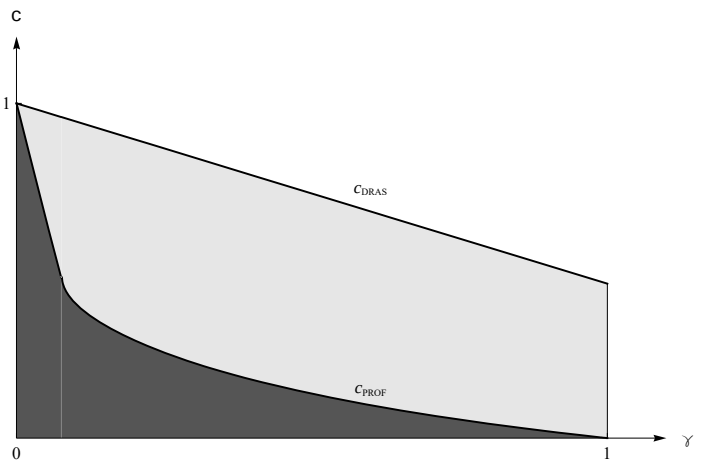
Out[75]=



In[76]:=

```
(*Summarizing the effect of exclusivity as in the paper (figure 1)*)

μplot = 6;
vertline = {{1, 0}, {1, p1Eequ}} /. μ → μplot;
{cdrast, Max[Re[profitability1E], Re[profitability1Ecorn]]} /. μ → μplot;
Figure1 = Show[Quiet[Plot[%, {γ, 0, 1}, PlotStyle → {{Black, Thickness[0.003]},
      {Black, Thickness[0.003]}},

    Filling → {{2 -> Bottom}, {2 → {1}}},
    FillingStyle → {{Lighter[Gray, 0.8]}, {Darker[Gray]}},
    AxesLabel → {"γ", "c"},

    Ticks → {{0, 1}, {-1, 1}},
    PlotRange → {{0, 1.1}, {0, 1.2}},
    AxesStyle → Arrowheads[{0.0, 0.02}]

   ]],
  Graphics[{Line[%%]}],
  Graphics[{Text["c_DRAS", {0.6, 0.75}], Text["c_PROF", {0.6, 0.15}]}]
 ]

SetDirectory[NotebookDirectory[]];
SetDirectory[ParentDirectory[]];
SetDirectory["Figures"];
Export["Figure1.pdf", Figure1];
```

Out[79]=

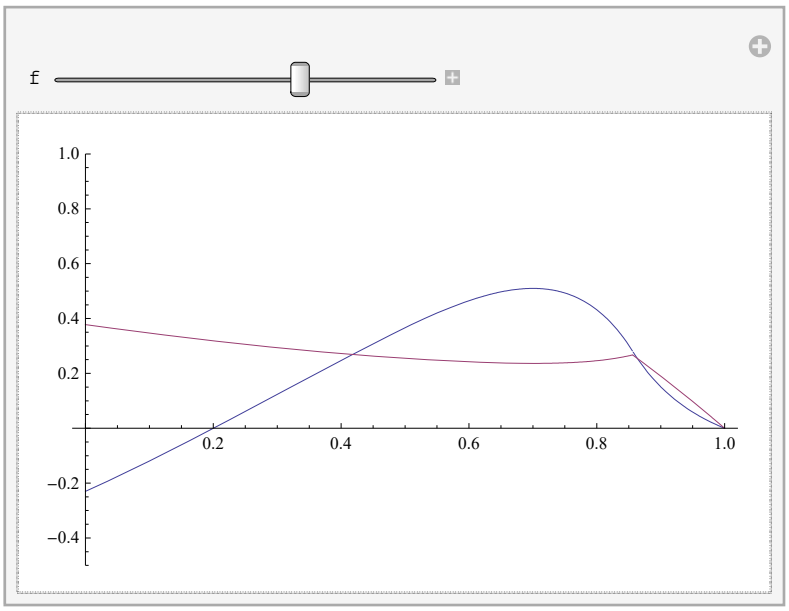


In[84]:=

In[85]:=
```
(*Changes in profitability and welfare*)
param = {μ → 5};
p1m /. param
(PROFIT1NE - PROFIT1Eequ) / PROFIT1NE /. param;
(WelfNE - PROFITBEequcorn) / WelfNE /. param;
(WelfLimit - PROFITBEequcorn) / WelfLimit /. param;
Dprofit = Piecewise[{{(PROFIT1NE - PROFIT1Eequcorn) / PROFIT1NE, c < clim}, {
      (PROFIT1Limit - PROFIT1Eequcorn) / PROFIT1Limit, c ≥ clim}}] /. param;
Manipulate[Plot[{-%, Min[%%%, %%]}, {γ, 0, 1}, PlotRange → {-0.5, 1}],
 {{c, 0.3}, 0, %%%%%}]
```

Out[86]=
$$\frac{5}{11}$$

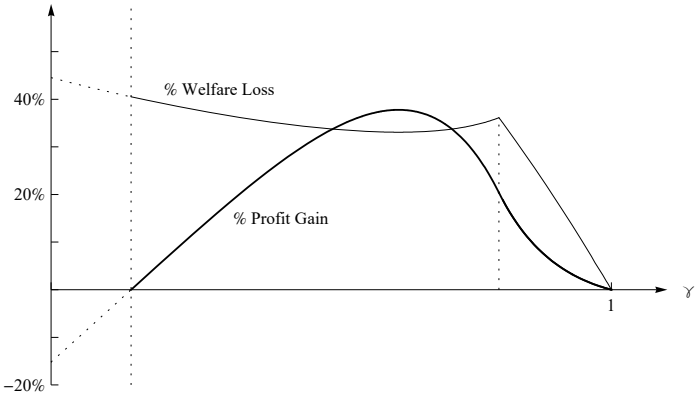
Out[91]=



In[92]:=

In[93]:=
```
(*Small program to write Ticks in figures as % automaticlaly,
used in figure 2 below*)
```

In[111]:=
```
(*Summarizing the effect of exclusivity (figure 2)*)

param = {μ → 4,  c → 0.38};
PROFIT1NE - PROFIT1Eequcorn /. param;
rootProfit = Quiet[Solve[% == 0, γ]][[1, 1, 2]];
vertlineroot = {{rootProfit, -0.2}, {rootProfit, 1}} /. param;

(WelfNE - PROFITBEequcorn) / WelfNE /. param;
rootc = Solve[c == clim, γ][[1, 1, 2]];
vertline = {{rootc, 0}, {rootc, %% /. γ -> rootc}} /. param;

(WelfNE - PROFITBEequcorn) / WelfNE;
(WelfLimit - PROFITBEequcorn) / WelfLimit;
```

```
Dprofit = Piecewise[{{(PROFIT1NE - PROFIT1Eequcorn) / PROFIT1NE, γ < rootc}, {
      (PROFIT1Limit - PROFIT1Eequcorn) / PROFIT1Limit, γ ≥ rootc}}];

{-%, Min[%%%, %%]} /. param;

Figure2 = Show[Quiet[Plot[%, {γ, rootProfit, 1},
     PlotStyle → {{Black, Thickness[0.003]}, {Black, Thickness[0.001]}},

     AxesLabel → {"γ", Null},

     Ticks → {{0, 1}, PercentTicks},
     PlotRange → {{0, 1.1}, {-0.2, 0.6}},
     AxesStyle → Arrowheads[{0.0, 0.02}]

    ]],

  Quiet[Plot[%, {γ, 0, rootProfit},
    PlotStyle → {{Black, Thickness[0.001], Dotted},
      {Black, Thickness[0.001], Dotted}},
    PlotRange → {{0, 1.1}, {-0.2, 0.6}}]
  ],

  Quiet[Plot[-(PROFIT1Limit - PROFIT1Eequcorn) / PROFIT1Limit /. param,
    {γ, rootc /. param, 1},
    PlotStyle → {{Black, Thickness[0.003]},
      {Black, Thickness[0.001], Dotted}},
    PlotRange → {{0, 1.1}, {-0.2, 0.6}}]
  ],


  Graphics[{Dotted, Line[vertline]}],
  Graphics[{Dotted, Line[vertlineroot]}],
  Graphics[{Text["% Profit Gain", {0.41, 0.15}],
    Text["% Welfare Loss", {0.3, 0.42}]}]
 ]

SetDirectory[NotebookDirectory[]];
SetDirectory[ParentDirectory[]];
SetDirectory["Figures"];
Export["Figure2.pdf", Figure2];
```

Out[122]=