Online Appendix 3 of Calzolari, Denicolò and Zanchettin "The demand-boost theory of exclusive dealing," 2019

The Reduced Form model with Duopoly of Section 4 with Coordination (same as Appendix 2 with in addition the case of coordination and Figure 4).

November 2019

Defining payoffs

In[1]:=
```
(*Buyer's utility*)
u = (q1 + q2) - 1 / 2 (q1^2 + q2^2) - γ q1 q2;
ue1 = u /. q2 → 0;
ue2 = u /. q1 → 0;

(*Parameters: c in [0,1], γ in [0,1]*)

P1 = p1 q1 + F1; (*Two part tariff of dominant firm 1*)
P2 = p2 q2 + F2; (*Two part tariff of inefficient firm 2*)
```

In[6]:=
```
(*Preparing payoffs for the different cases*)
```

In[7]:=
```
(*Common representation*)
tmpu = FullSimplify[Solve[{D[u - P1 - P2, q1] == 0, D[u - P1 - P2, q2] == 0}, {q1, q2}]];
q1NE = tmpu[[1, 1, 2]]
q2NE = tmpu[[1, 2, 2]]
```

Out[8]=
$$\frac{-1 + p1 + \gamma - p2\,\gamma}{-1 + \gamma^2}$$

Out[9]=
$$\frac{-1 + p2 + \gamma - p1\,\gamma}{-1 + \gamma^2}$$

In[10]:=
```
(*Non exclusive payoffs*)
u - p1 q1 - p2 q2 /. {q1 → q1NE, q2 → q2NE};
ProfitBNE = FullSimplify[%] - (1 + μ) (F1 + F2)

P1 /. {q1 → q1NE, q2 → q2NE};
Profit1NE = FullSimplify[%]

P2 - c q2 /. {q1 → q1NE, q2 → q2NE};
Profit2NE = FullSimplify[%]
```

Out[11]= $-\left(2-2\,p1+p1^2-2\,p2+p2^2-2\,(-1+p1)\,(-1+p2)\,\gamma\right)\Big/\left(2\left(-1+\gamma^2\right)\right)-(F1+F2)\,(1+\mu)$

Out[13]= $F1+\dfrac{p1\,(-1+p1+\gamma-p2\,\gamma)}{-1+\gamma^2}$

Out[15]= $\dfrac{-c\,(-1+p2+\gamma-p1\,\gamma)+p2\,(-1+p2+\gamma-p1\,\gamma)+F2\,(-1+\gamma^2)}{-1+\gamma^2}$

In[16]:=
```
(*Monopoly/Exclusive firm 1*)
q1e = FullSimplify[Solve[D[ue1 - p1 q1, q1] == 0, q1]][[1, 1, 2]]

ue1 - p1 q1 - (1 + μ) F1 /. q1 → q1e;
ProfitBE1 = FullSimplify[%]

P1 /. q1 → q1e;
Profit1E = FullSimplify[%]
```

Out[16]= $1-p1$

Out[18]= $\dfrac{1}{2}\left((-1+p1)^2-2\,F1\,(1+\mu)\right)$

Out[20]= $F1+p1-p1^2$

In[21]:=
```
(*Monopoly/Exclusive firm 2*)
q2e = FullSimplify[Solve[D[ue2 - p2 q2, q2] == 0, q2]][[1, 1, 2]]

FullSimplify[ue2 - p2 q2 - (1 + μ) F2 /. q2 → q2e];
ProfitBE2 = FullSimplify[%]

P2 - c q2 /. q2 → q2e;
Profit2E = FullSimplify[%]
```

Out[21]= $1-p2$

Out[23]= $\dfrac{1}{2}\left((-1+p2)^2-2\,F2\,(1+\mu)\right)$

Out[25]= $F2+c\,(-1+p2)+p2-p2^2$

Analysis: Exclusive contracts prohibited

In[26]:=
```
(*Determining the "Bertrand-Nash" marginal
 prices (indicated with the star in the paper).*)
F1tmp = FullSimplify[Solve[ProfitBNE == k, F1]][[1, 1, 2]];
F2tmp = FullSimplify[Solve[ProfitBNE == k, F2]][[1, 1, 2]];
Profit1NEtmp = Profit1NE /. F1 → F1tmp;
Profit2NEtmp = Profit2NE /. F2 → F2tmp;
tmpp =
 FullSimplify[Solve[{D[Profit1NEtmp, p1] == 0, D[Profit2NEtmp, p2] == 0}, {p1, p2}]]
p1NE = tmpp[[1, 1, 2]];
p2NE = tmpp[[1, 2, 2]];

pricesNE = {p1 → p1NE, p2 → p2NE};
```

Out[30]=
$$\left\{\left\{p1 \to \frac{\mu\,(-1-2\,\mu+\gamma\,(1+\mu+\gamma\,\mu-c\,(1+\mu)))}{-1+\mu\,(-4+(-4+\gamma^2)\,\mu)},\right.\right.$$
$$\left.\left.p2 \to \frac{-c\,(1+\mu)\,(1+2\,\mu)+(-1+\gamma)\,\mu\,(1+(2+\gamma)\,\mu)}{-1+\mu\,(-4+(-4+\gamma^2)\,\mu)}\right\}\right\}$$

In[34]:=
```
(*For future reference we determine the Limit Pricing region*)
q2NE /. pricesNE;
FullSimplify[Solve[% == 0, c]]
clim = %[[1, 1, 2]]; (*for c above clim the inefficient firm 2 cannot be active*)
```

Out[35]=
$$\left\{\left\{c \to \frac{(-1+\gamma)\,(1+(2+\gamma)\,\mu)}{-1+(-2+\gamma^2)\,\mu}\right\}\right\}$$

In[37]:=
```
(*We determine the limit pricing strategy. The
  limit pricing strategy for the inefficient firm 2 is p2=
 c and F2=0. That for firm 1 is determined as follows.*)
p1Limit = FullSimplify[Solve[0 == q2NE /. p2 → c, p1]][[1, 1, 2]]

(*The associated fixed fee is determined as follows.*)
ProfitBE2 /. {p2 → c, F2 → 0};
FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit];
F1Limit = FullSimplify[Solve[%% == %, F1]][[1, 1, 2]]

(*The associated payoffs are as follows.*)
PROFITBLimit = FullSimplify[ProfitBE1 /. q1 -> q1e /. p1 → p1Limit /. F1 -> F1Limit]
PROFIT1Limit = FullSimplify[Profit1E /. q1 -> q1e /. p1 → p1Limit /. F1 -> F1Limit]
```

Out[37]=
$$\frac{-1+c+\gamma}{\gamma}$$

Out[40]=
$$-\frac{(-1+c)^2\,(-1+\gamma^2)}{2\,\gamma^2\,(1+\mu)}$$

Out[41]=
$$\frac{1}{2}\,(-1+c)^2$$

Out[42]=
$$-\frac{(-1+c)\,(-(-1+\gamma)\,(-1+\gamma-2\,\mu)+c\,(1+\gamma^2+2\,\mu))}{2\,\gamma^2\,(1+\mu)}$$

In[43]:=
```
(*We now find the "Drastic region" in which also without
 exclusivity clause the dominant can price at the monopoly price*)

(*First we need to find the unconstrained monopolist's optimal price*)
ProfitBE1 - ProfitBE2 /. {p2 → c, F2 → 0};
tmpF1m = FullSimplify[Solve[% == 0, F1]];
Profit1E /. F1 → tmpF1m[[1, 1, 2]];
p1m = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]]
F1m = FullSimplify[tmpF1m[[1, 1, 2]] /. p1 → p1m]

(*And the associated payoffs are as follows.*)
PROFIT1Em = FullSimplify[Profit1E /. p1 → p1m /. F1 → F1m]
PROFITBE1m = FullSimplify[ProfitBE1 /. p1 → p1m /. F1 → F1m]

(*Now, we determine the minimum c that allows the dominant firm to give-
 up with limit pricing and directly price as a monopolist.*)
FullSimplify[Solve[p1m == p1Limit, c]];
cdrast = %[[1, 1, 2]]
```

Out[46]=
$$\frac{\mu}{1+2\mu}$$

Out[47]=
$$-\frac{\left(c-\frac{\mu}{1+2\mu}\right)\left(-2+c+\frac{\mu}{1+2\mu}\right)}{2\,(1+\mu)}$$

Out[48]=
$$\frac{\mu^2-c^2\,(1+2\mu)+c\,(2+4\mu)}{2+6\mu+4\mu^2}$$

Out[49]=
$$\frac{1}{2}\,(-1+c)^2$$

Out[51]=
$$\frac{1+2\mu-\gamma\,(1+\mu)}{1+2\mu}$$

In[52]:=
```
(*For future reference we also define the Positive Primary Otuput boundary*)
Solve[{D[u - c q2, q1] == 0, D[u - c q2, q2] == 0}, {q1, q2}];
Solve[%[[1, 2, 2]] == 0, c];
cPPO = %[[1, 1, 2]]
```

Out[54]=
$$1-\gamma$$

In[55]:=

In[56]:=
```
(*We now derive the most cooperative equilibrium. To this end we define
 the conditions that identify the equilibrium and barrage tariffs,
here identified with notation pi and Fi.*)
```

In[57]:=
```
(*First define k:*)
kdef = FullSimplify[
  u - q1 p1 - q2 p2 - (1 + μ) (F1 + F2) /. {q1 → q1NE, q2 → q2NE} /. {p1 → p1NE, p2 → p2NE} /.
   {F1 -> F1star, F2 -> F2star}]
```

Out[57]=
$$\left((1+\mu)\left(-2c(-1+\gamma)(1+\mu)(1+(2+\gamma)\mu)^2-2(-1+\gamma)(1+(2+\gamma)\mu)^2\right.\right.$$
$$\left.\left(-1-\mu+\text{F1star}(1+\gamma)(-1+(-2+\gamma)\mu)^2+\text{F2star}(1+\gamma)(-1+(-2+\gamma)\mu)^2\right)+$$
$$\left.c^2(1+\mu)\left(-1+\mu\left(-4(1+\mu)+\gamma^2(2+3\mu)\right)\right)\right)\Big/$$
$$\left(2\left(-1+\gamma^2\right)(-1+(-2+\gamma)\mu)^2(1+(2+\gamma)\mu)^2\right)$$

In[58]:=
```
(*Second, firms obtain in equilibrium not less that the profit as with tariff
  in the best deviation to exclusivity (here indicated with (pi,Fi)):*)
p1 q1NE + F1star /. {p1 → p1NE, p2 → p2NE};
NoDev1 = FullSimplify[% - (p1 q1e + F1)]

(p2 - c) q2NE + F2star /. {p1 → p1NE, p2 → p2NE};
NoDev2 = FullSimplify[% - ((p2 - c) q2e + F2)]
```

Out[59]=
$$-\text{F1}+\text{F1star}+(-1+\text{p1})\,\text{p1}-\left(\mu(1+\mu)\left(1+2\mu-\gamma^2\mu+(-1+c)\gamma(1+\mu)\right)^2\right)\Big/\left(\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

Out[61]=
$$-\text{F2}+\text{F2star}+(-1+\text{p2})(-c+\text{p2})-\left(\mu(1+\mu)\left(c-c\left(-2+\gamma^2\right)\mu+(-1+\gamma)(1+(2+\gamma)\mu)\right)^2\right)\Big/\left(\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

In[62]:=
```
(*Third, the best deviation contract:
   A- Buyer must accept the deviation rather
    than obtaining the candidate equilibrium payoff k:*)
ue1 - p1 q1 - (1 + μ) F1 /. q1 → q1e;
BuyerPartDev1 = FullSimplify[% - k]
ue2 - p2 q2 - (1 + μ) F2 /. q2 → q2e;
BuyerPartDev2 = FullSimplify[% - k]

(*B- Buyer must also accept the deviation rather than mixing
  the deviation contract with the candidate equilibrium contract
  of the other firm and purchase under common representation:*)

u - q1 p1 - q2 p2 - (F1 + F2star) (1 + μ) /. {q1 → q1NE, q2 → q2NE} /. {p2 → p2NE};
ue1 - p1 q1 - F1 (1 + μ) /. q1 → q1e;
BuyerNoMix1 = FullSimplify[% - %%]

u - q1 p1 - q2 p2 - (F1star + F2) (1 + μ) /. {q1 → q1NE, q2 → q2NE} /. {p1 → p1NE};
ue2 - p2 q2 - F2 (1 + μ) /. q2 → q2e;
BuyerNoMix2 = FullSimplify[% - %%]
```

Out[63]=
$$\frac{1}{2}\left(-2\,k+(-1+p1)^2-2\,F1\,(1+\mu)\right)$$

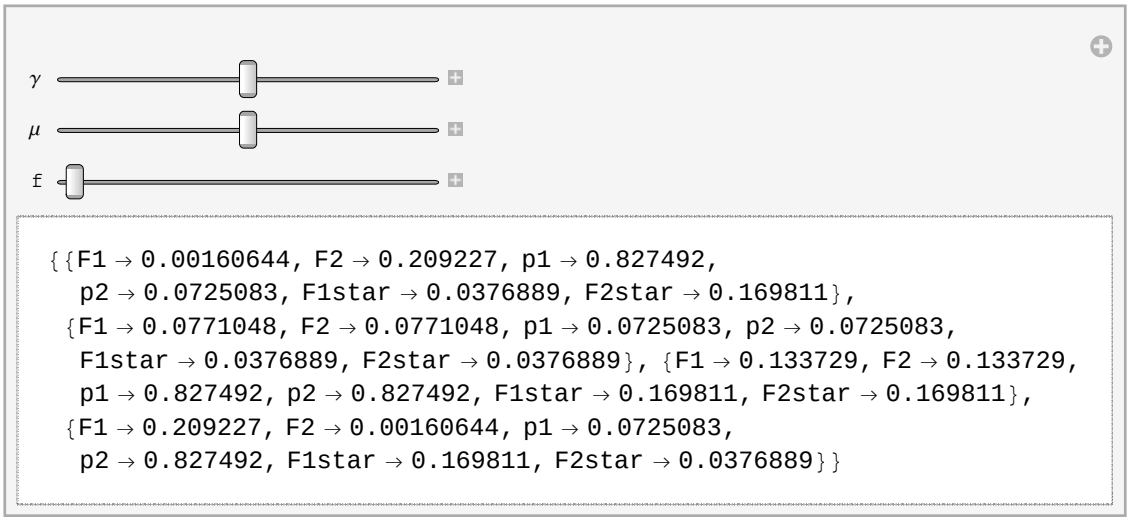
Out[65]=
$$\frac{1}{2}\left(-2\,k+(-1+p2)^2-2\,F2\,(1+\mu)\right)$$

Out[68]=
$$\Big(c^2\,(1+\mu)^2\,(1+2\,\mu)^2+2\,c\,(1+\mu)\,(1+2\,\mu)\,(1+(2+\gamma)\,\mu)\,(-1-\mu+(-1+p1)\,\gamma\,(-1+(-2+\gamma)\,\mu))+(1+(2+\gamma)\,\mu)^2\left(2\,F2star\left(-1+\gamma^2\right)(1+\mu)\,(-1+(-2+\gamma)\,\mu)^2+(1+\mu-(-1+p1)\,\gamma\,(-1+(-2+\gamma)\,\mu))^2\right)\Big)\Big/\left(2\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

Out[71]=
$$\Big(2\,F1star\left(-1+\gamma^2\right)(1+\mu)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2+\left((-1+p2)\,\gamma^3\,\mu^2-(1+\mu)\,(1+2\,\mu)+\gamma\left(1+(3+c)\,\mu\,(1+\mu)-p2\,(1+2\,\mu)^2\right)\right)^2\Big)\Big/\left(2\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

In[72]:=
```
{NoDev1 == 0, NoDev2 == 0, BuyerPartDev1 == 0,
   BuyerPartDev2 == 0, BuyerNoMix1 == 0, BuyerNoMix2 == 0} /. k -> kdef;
Solmostcooptmp = Solve[%, {F1, F2, p1, p2, F1star, F2star}];
```

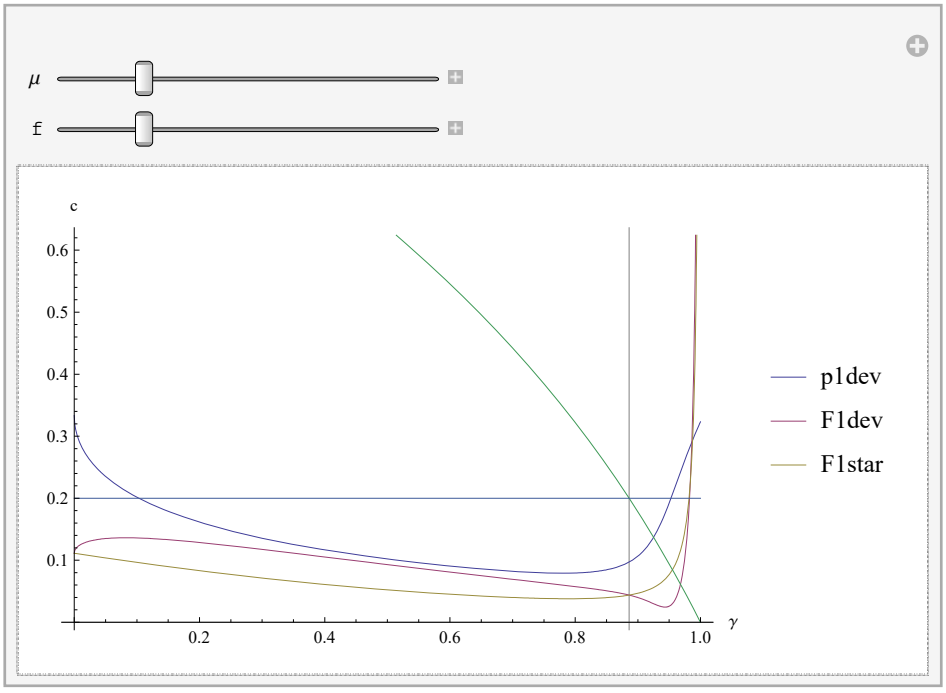
In[74]:=
```
Solmostcooptmp;
Manipulate[%, {{γ, 0.5}, 0, 1}, {{μ, 1}, 0, 2}, {c, 0, 1}]
```



Out[75]=
```
{{F1 → 0.00160644, F2 → 0.209227, p1 → 0.827492,
  p2 → 0.0725083, F1star → 0.0376889, F2star → 0.169811},
 {F1 → 0.0771048, F2 → 0.0771048, p1 → 0.0725083, p2 → 0.0725083,
  F1star → 0.0376889, F2star → 0.0376889}, {F1 → 0.133729, F2 → 0.133729,
  p1 → 0.827492, p2 → 0.827492, F1star → 0.169811, F2star → 0.169811},
 {F1 → 0.209227, F2 → 0.00160644, p1 → 0.0725083,
  p2 → 0.827492, F1star → 0.169811, F2star → 0.0376889}}
```

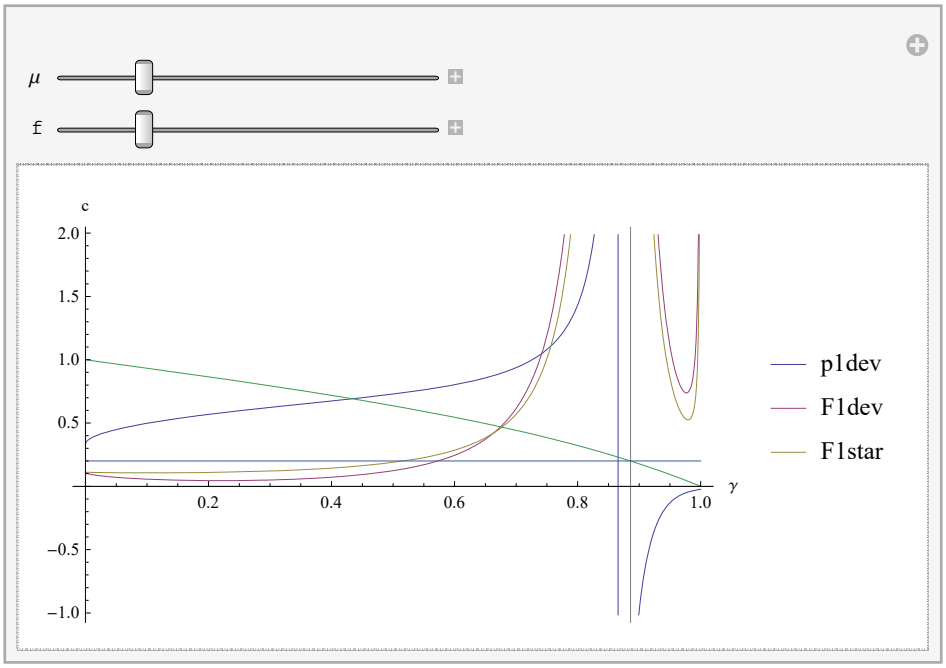
In[76]:=
```
(*The asymmetric solutions are not admissible. We plot
 the two symmetric solution, the second and the third.*)
{Solmostcooptmp[[2, 3, 2]], Solmostcooptmp[[2, 1, 2]],
  Solmostcooptmp[[2, 5, 2]], clim, c};
Manipulate[Plot[%, {γ, 0, 1}, GridLines →
    {{(1 + μ - √((1 + μ)^2 + 4 (-1 + c)^2 μ (1 + 2 μ))) / (2 (-1 + c) μ)}, {0}},
  PlotLegends → {p1dev, F1dev, F1star}, AxesLabel → {"γ", "c"}

 ], {{μ, 1}, 0, 5}, {{c, 0.2}, 0, 1}]
```

Out[77]=



In[78]:=
```
(*As it can be seen the third solution is not admissible
 as for some admissible values of γ it delivers marginal price
 above 1 and exhorbitant fixed fees that has shown below generate
 negative buyer's utility (better not participate at all).*)
{Solmostcooptmp[[3, 3, 2]], Solmostcooptmp[[3, 1, 2]],
  Solmostcooptmp[[3, 5, 2]], clim, c};
Manipulate[Plot[%, {γ, 0, 1}, GridLines →
    {{(1 + μ - √((1 + μ)^2 + 4 (-1 + c)^2 μ (1 + 2 μ))) / (2 (-1 + c) μ)}, {0}},
  PlotLegends → {p1dev, F1dev, F1star}, AxesLabel → {"γ", "c"}

 ], {{μ, 1}, 0, 5}, {{c, 0.2}, 0, 1}]
```

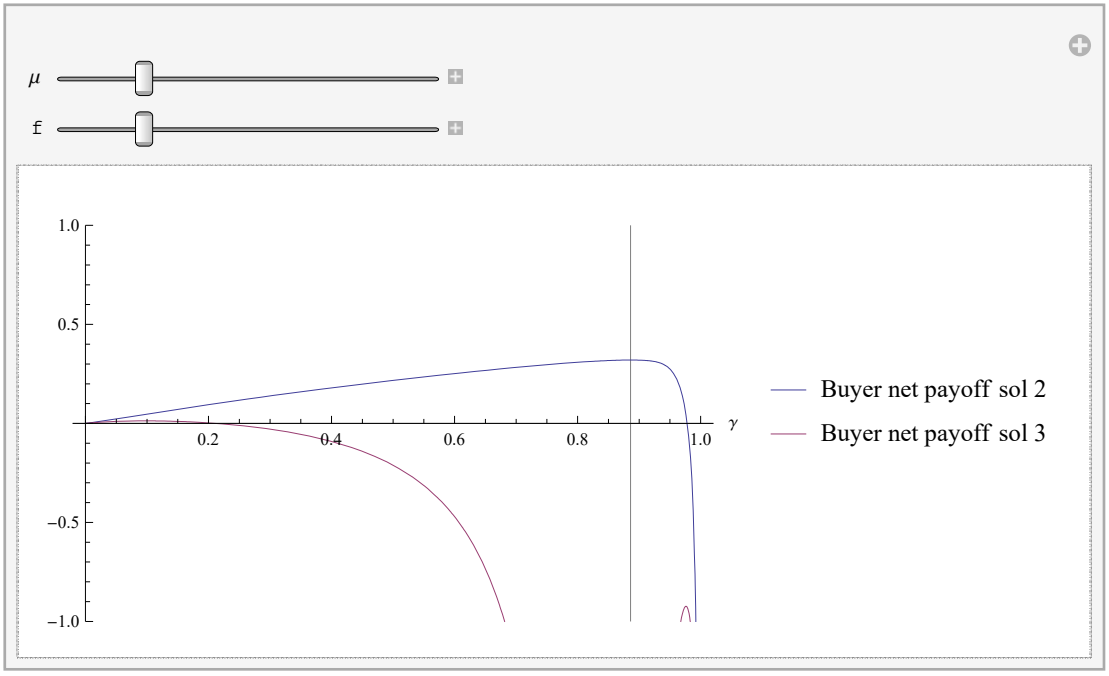
Out[79]=



In[80]:=
```
(*Defining the tariffs of the most
  cooperative equilibrium with Non-Exclusive:*)
TariffsMostCoopNE = {p1 → p1NE, F1 -> Solmostcooptmp[[2, 5, 2]],
    p1barr -> Solmostcooptmp[[2, 3, 2]], F1barr -> Solmostcooptmp[[2, 1, 2]],
    p2 → p2NE, F2 -> Solmostcooptmp[[2, 6, 2]], p2barr -> Solmostcooptmp[[2, 4, 2]],
    F2barr -> Solmostcooptmp[[2, 2, 2]]
  };
```

In[81]:=
```
(*Double check on retailer's participation with the two solutions*)
ProfitBNE /. pricesNE /. TariffsMostCoopNE;
ProfitBNE /. pricesNE /.
  {F1 -> Solmostcooptmp[[3, 5, 2]], F2 -> Solmostcooptmp[[3, 6, 2]]};
Manipulate[Plot[{%%, %}, {γ, 0, 1}, PlotRange → {-1, 1},
  GridLines → {{(1 + μ - √((1 + μ)^2 + 4 (-1 + c)^2 μ (1 + 2 μ))) / (2 (-1 + c) μ)}, {0}},
  PlotLegends → {"Buyer net payoff sol 2", "Buyer net payoff sol 3"},
  AxesLabel → {"γ", ""}

 ], {{μ, 1}, 0, 5}, {{c, 0.2}, 0, 1}]
```

Out[83]=



In[84]:=
```
(*We next identify the region of existence of other,
less profitable for firms, equilibria parametrized
  by the (candidate) equilibrium fixed fees F1star and F2star.

   With respect to the previous analysis we proceed
  by not considering the NoDev constraints of the two firms.
   Inspection of the constraints shows that the No-
 mixing constraints allow to determine the barrage marginal prices
  (as functions of the equilibrium fixed fees Fstar only) and then,
with these prices, the buyer's no deviation constraints
 allow to determined the barrage fixed fees.*)

{BuyerNoMix1 == 0, BuyerNoMix2 == 0} /. k -> kdef;
Solmarginalptmp = FullSimplify[Solve[%, {p1, p2}]];

{Solmarginalptmp, q1e /. Solmarginalptmp} /.
 {F1star → 0.1, F2star → 0.1, γ → 0.5, μ → 0.7, c → 0}
(*This shows that the correct solution is the forth.*)
```

Out[86]=
$$\{\{\{p1 \to -1.66849, p2 \to -1.66849\}, \{p1 \to -1.66849, p2 \to 0.351414\}, \{p1 \to 0.351414, p2 \to -1.66849\}, \{p1 \to 0.351414, p2 \to 0.351414\}\}, \{2.66849, 2.66849, 0.648586, 0.648586\}\}$$

In[87]:=
```
p1barrgen = Solmarginalptmp[[4, 1, 2]];
p2barrgen = Solmarginalptmp[[4, 2, 2]];
```

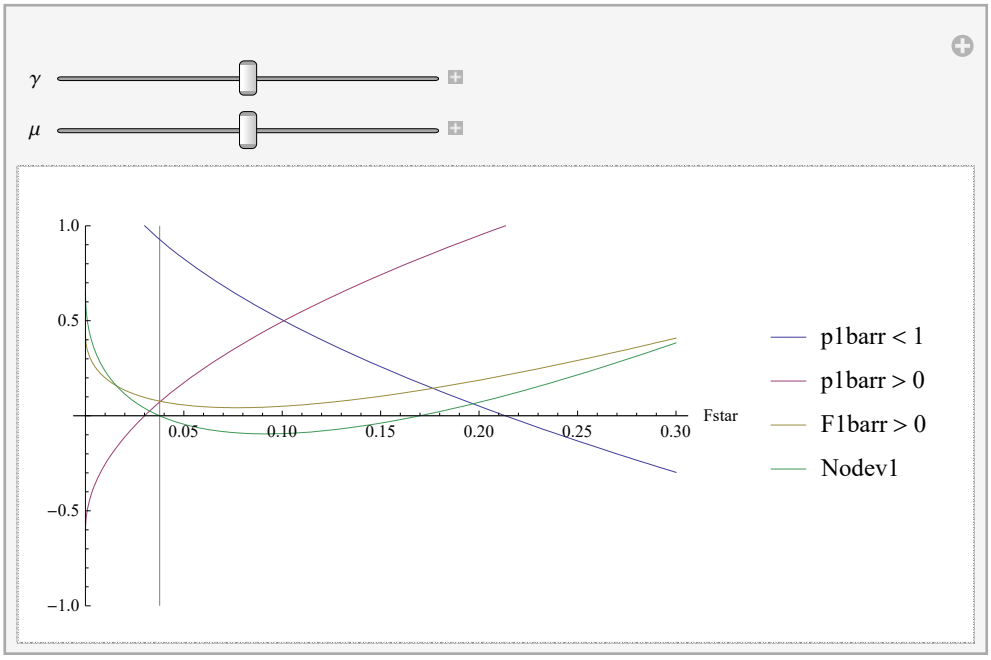
In[89]:=
```
(*Then we determine the barrage fixed fees:*)
{BuyerPartDev1 == 0, BuyerPartDev2 == 0};
FullSimplify[Solve[%, {F1, F2}]]

F1barrgen = %[[1, 1, 2]] /. k -> kdef /. p1 -> p1barrgen;
F2barrgen = %%[[1, 2, 2]] /. k -> kdef /. p2 -> p2barrgen;
```

Out[90]=
$$\left\{\left\{F1 \to \frac{-2k + (-1+p1)^2}{2(1+\mu)}, F2 \to \frac{-2k + (-1+p2)^2}{2(1+\mu)}\right\}\right\}$$

In[93]:=
```
(*Next we study other and les cooperative equilibria,
by vaying the fixed fees (e.g. leaving more rents to the buyer
   as compared with the most cooperative equilibrium determined),
without violating constraints on marginal prices and their own no-
 deviation constraint*)
```

In[94]:=
```
(*Consider first the symmetric case in which c=0*)
Solmostcooptmp[[2, 6, 2]] /. c → 0;
NoDev1 /. {F1 -> F1barrgen, F2 -> F2barrgen, p1 -> p1barrgen, p2 -> p2barrgen} /.
   F1star -> F2star /. c → 0;
{1 - p1barrgen, p1barrgen - 0, F1barrgen - 0, % - 0} /. F1star -> F2star /. c → 0;
Manipulate[Plot[%, {F2star, 0, 0.3}, PlotRange → {-1, 1}, GridLines → {{%%%}, {0}},
  PlotLegends → {p1barr < 1, p1barr > 0, F1barr > 0, Nodev1},
  AxesLabel → {"Fstar", ""}], {{γ, 0.5}, 0, 1}, {{μ, 1}, 0, 2}]
```

Out[97]=



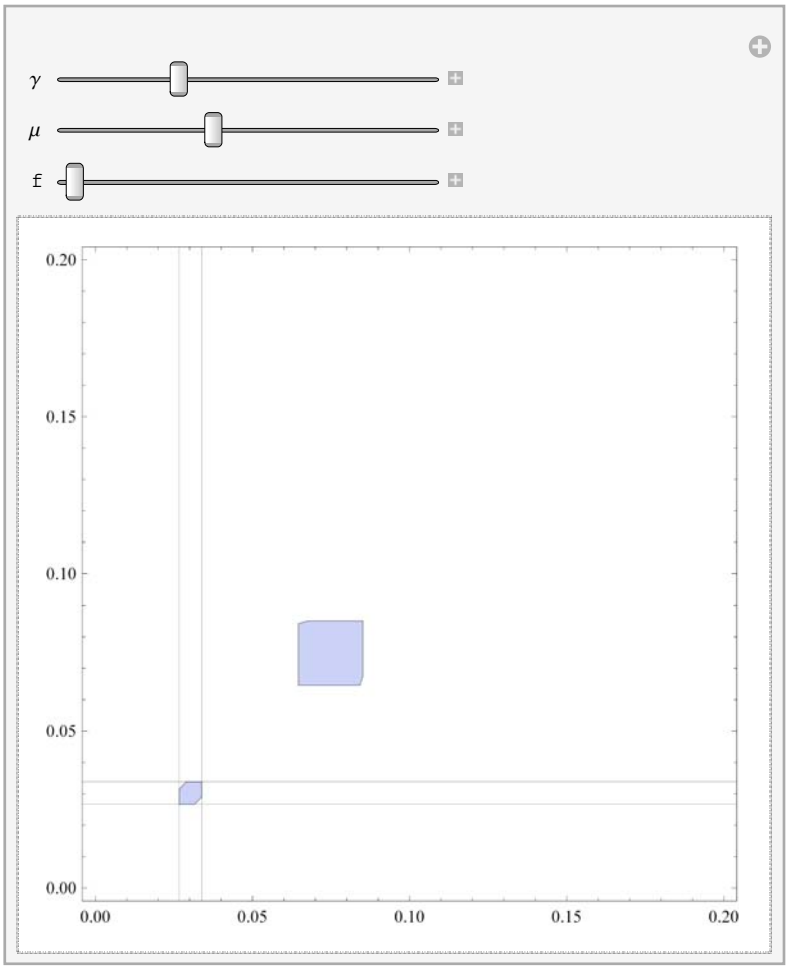
In[98]:=
```
(*Note that the second and largest root for Nodev1
 (and the apparently admissible associated area) is not
 admissible because the associated barrage prices are not feasible
 (corresponds to the solution 3 tha delivers negative buyer's payoff).*)
```

In[99]:=
```
(*As it can be seen,
the lower bound on the fixed fees (i.e. the minimally cooperative)
 are bounded by the non negativity of the marginal barrage prices:*)
{Numerator[p1barrgen] == 0, Numerator[p2barrgen] == 0};
MinFstar = FullSimplify[Solve[%, {F1star, F2star}]]
```

Out[100]=
$$\left\{\left\{\text{F1star} \to -\left(-1+\gamma+(-3+(3+c)\,\gamma)\,\mu+\left(-2+(3+c)\,\gamma-\gamma^3\right)\mu^2\right)^2\Big/\left(2\left(-1+\gamma^2\right)(1+\mu)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right),\right.$$
$$\left.\text{F2star} \to -\left(-1+c+\gamma+3\,(-1+c+\gamma)\,\mu+\left(-2+2\,c+3\,\gamma-\gamma^3\right)\mu^2\right)^2\Big/\left(2\left(-1+\gamma^2\right)(1+\mu)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)\right\}\right\}$$

In[101]:=
```
(*Considering also non symmetric case,
all the constraints are satisfied in the colored
 area (where we also show the most cooperative fixed
   fees and the minimal (least cooeprative) fixed fees)*)

MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{NoDev1 > 0 && NoDev2 > 0 && p1barrgen > 0 && p2barrgen > 0 &&
    p1barrgen < 1 && p2barrgen < 1 && F1barrgen > 0 && F2barrgen > 0} /.
  {F1 -> F1barrgen, F2 -> F2barrgen, p1 -> p1barrgen, p2 -> p2barrgen};
Manipulate[RegionPlot[%, {F1star, 0, 0.2}, {F2star, 0, 0.2},
  GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.3}, 0, 1}, {{μ, 2}, 0, 5}, {c, 0, clim}]
```

Out[106]=



In[107]:=
```
(*Again the solutions on the morth-east area are not admissible
  because they give the buyer negative net payoff as shown above.*)
```

In[108]:=
```
(*As it can be seen the most cooperative fixed fees are the largest admissible.
    Least cooperative fixed fees are bounded away from zero for the non-
  negativity of the marginal barrage price.
    As it can be seen as γ→
 0 or γ→1 the only surviving equilibrium is the most cooperative.*)
```

In[109]:=
```
(*For future reference we define the payoffs
 associated with the most cooperative equilibrium:*)
Profit1NEMostCoop = Profit1NE /. TariffsMostCoopNE;
Profit2NEMostCoop = Profit2NE /. TariffsMostCoopNE;
ProfitBNEMostCoop = ProfitBNE /. TariffsMostCoopNE;
```

In[112]:=

In[113]:=
```
(*As a last step we check that the buyer's "no-mixing constraint" of the
 optimal deviation to exclusivity program must be binding in equilibrium,
otherwise either at the implied solution,
either it is violated or at least one of the firms deviates.

  The solution of the optimal deviation program is now either the monopolist's
 marginal price when the fixed fees is positive (case A) or when it is nil,
it is the marginal price the satisfies the buyer's
 participation constraint of the deviation program (case B).*)
```

In[114]:=
```
{NoDev1 == 0, NoDev2 == 0, BuyerPartDev1 == 0,
   BuyerPartDev2 == 0, BuyerNoMix1 == 0, BuyerNoMix2 == 0} /. k -> kdef;
```

In[115]:=
```
Profit1E
```

Out[115]=
$$F1 + p1 - p1^2$$

In[116]:=
```
(*Case A*)
{BuyerPartDev1 == 0, BuyerPartDev2 == 0};
FAtmp = FullSimplify[Solve[%, {F1, F2}]]
Profit1E /. F1 → FAtmp[[1, 1, 2]];
p1Em = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]]
Profit2E /. F2 → FAtmp[[1, 2, 2]];
p2Em = FullSimplify[Solve[D[%, p2] == 0, p2]][[1, 1, 2]]
FAsol = FAtmp /. {p1 → p1Em, p2 → p2Em} /. k → kdef;
```

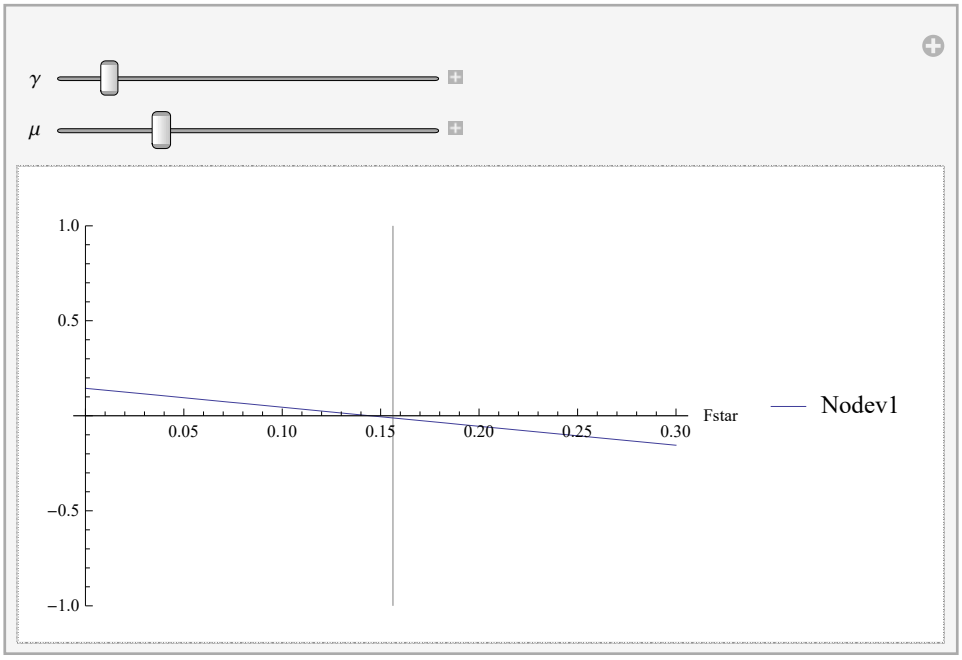
Out[117]=
$$\left\{\left\{F1 \to \frac{-2\,k + (-1+p1)^2}{2\,(1+\mu)},\ F2 \to \frac{-2\,k + (-1+p2)^2}{2\,(1+\mu)}\right\}\right\}$$

Out[119]=
$$\frac{\mu}{1+2\,\mu}$$

Out[121]=
$$\frac{c + \mu + c\,\mu}{1+2\,\mu}$$

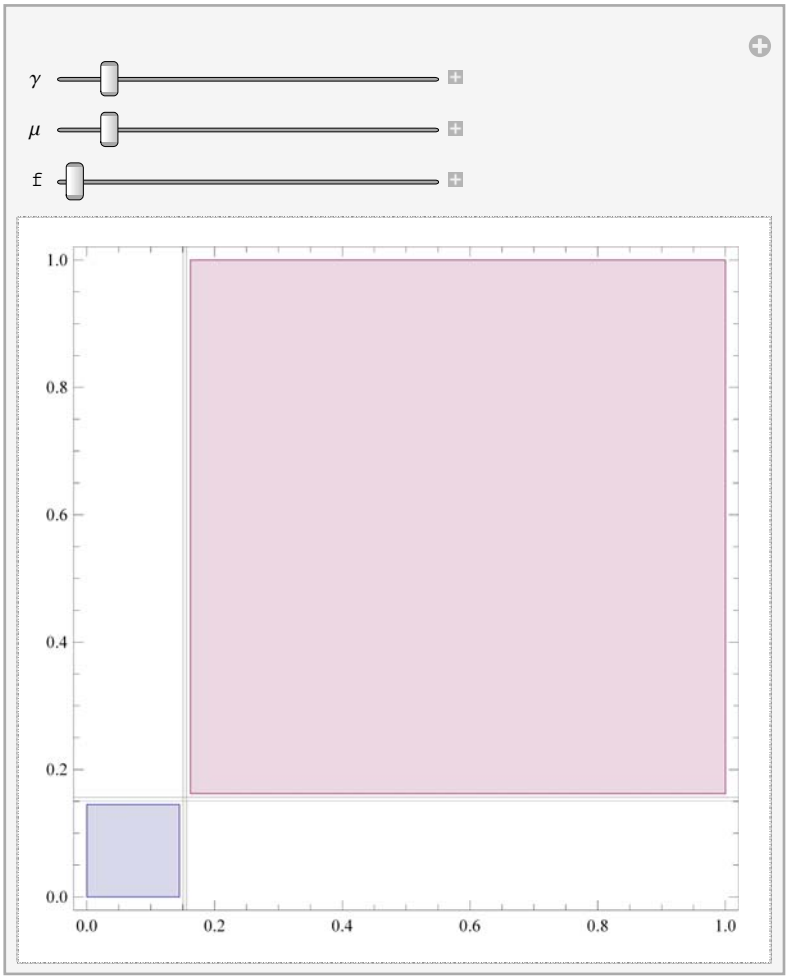
In[123]:=
```
(*Consider first the symmetric case in which c=0*)
Solmostcooptmp[[2, 6, 2]] /. c → 0;
NoDev1 /. {F1 -> FAsol[[1, 1, 2]], F2 -> FAsol[[1, 2, 2]], p1 → p1Em, p2 → p1Em} /.
   F1star -> F2star /. c → 0;
{% - 0} /. F1star -> F2star /. c → 0;
Manipulate[Plot[%, {F2star, 0, 0.3},
  PlotRange → {-1, 1}, GridLines → {{%%%}, {0}}, PlotLegends → {Nodev1},
  AxesLabel → {"Fstar", ""}], {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 2}]
```

Out[126]=



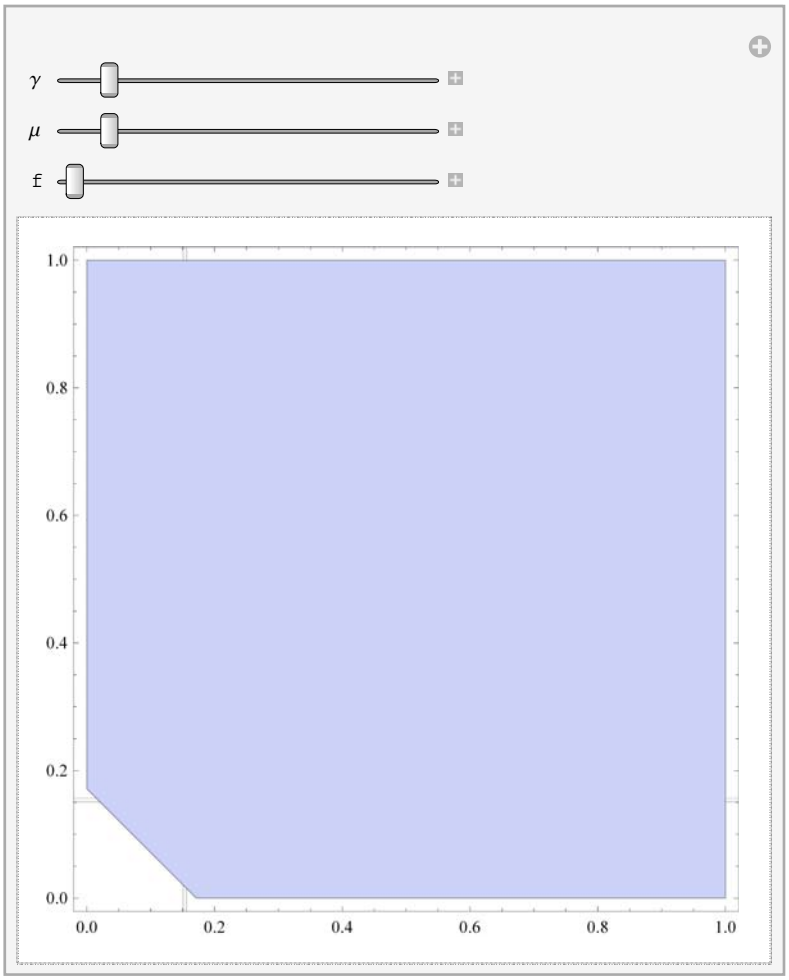
In[127]:=
```
(*Now the general case: as it can be seen there are no
   admissible Fstar that satisfy both sets of remining constraints*)
MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{NoDev1 > 0 && NoDev2 > 0, BuyerNoMix1 > 0 && BuyerNoMix2 > 0} /.
  {F1 -> FAsol[[1, 1, 2]], F2 -> FAsol[[1, 2, 2]], p1 → p1Em, p2 → p1Em};
Manipulate[RegionPlot[%, {F1star, 0, 1}, {F2star, 0, 1},
  GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 5}, {c, 0, clim}]
```

Out[132]=



In[133]:=
```
(*The fixed barrage fees studied in case A can be negative,
hence we need to consider also case B*)
MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{FAsol[[1, 1, 2]] > 0 && FAsol[[1, 2, 2]] > 0};
Manipulate[
 RegionPlot[%, {F1star, 0, 1}, {F2star, 0, 1}, GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 5}, {c, 0, clim}]
```

Out[138]=

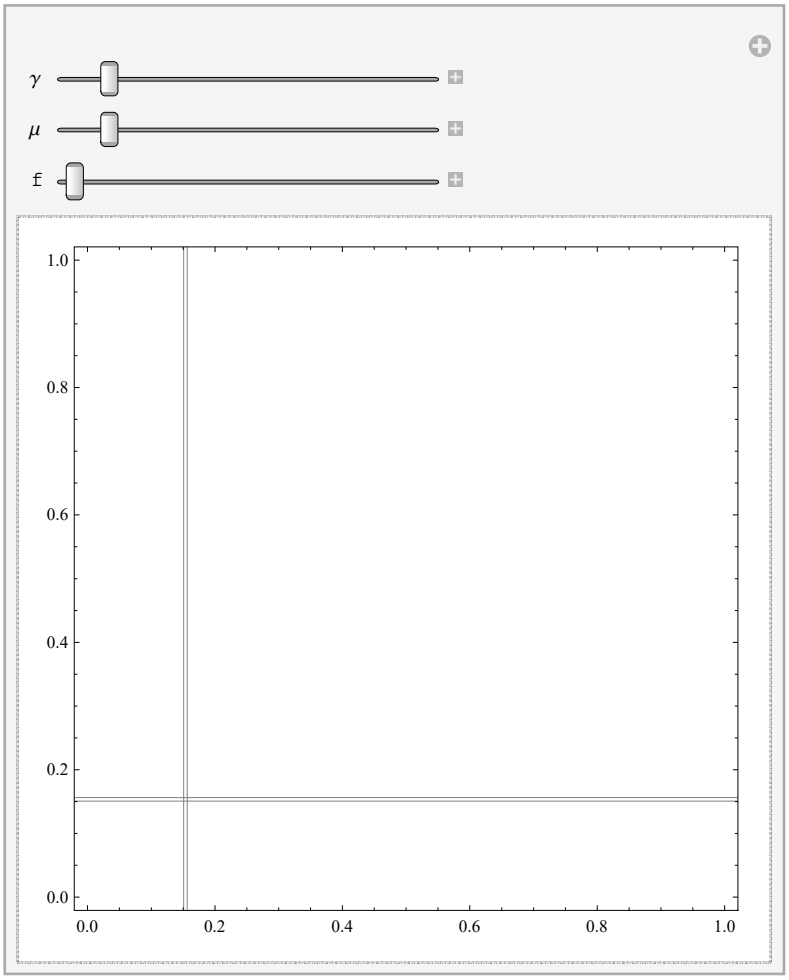


In[139]:=
```
(*Case B*)
{BuyerPartDev1 == 0, BuyerPartDev2 == 0} /. {F1 → 0, F2 → 0};
pBtmp = FullSimplify[Solve[%, {p1, p2}]]
```

Out[140]=
$$\left\{\left\{p1 \to 1-\sqrt{2}\ \sqrt{k},\ p2 \to 1-\sqrt{2}\ \sqrt{k}\right\},\ \left\{p1 \to 1-\sqrt{2}\ \sqrt{k},\ p2 \to 1+\sqrt{2}\ \sqrt{k}\right\},\ \left\{p1 \to 1+\sqrt{2}\ \sqrt{k},\ p2 \to 1-\sqrt{2}\ \sqrt{k}\right\},\ \left\{p1 \to 1+\sqrt{2}\ \sqrt{k},\ p2 \to 1+\sqrt{2}\ \sqrt{k}\right\}\right\}$$

In[141]:=
```
(*The next graph shows that in case B the buyer always prefers
 to mix the contracts hence, case B cannot be a solution.*)
MinFstar[[1, 1, 2]];
MinFstar[[1, 2, 2]];
Solmostcooptmp[[2, 5, 2]](*F1star most cooperative*);
Solmostcooptmp[[2, 6, 2]](*F2star most cooperative*);
{ BuyerNoMix1 > 0 && BuyerNoMix2 > 0} /.
    {F1 → 0, F2 → 0, p1 → pBtmp[[1, 1, 2]], p2 → pBtmp[[1, 2, 2]]} /. k → kdef;
Manipulate[RegionPlot[%, {F1star, 0, 1}, {F2star, 0, 1},
  GridLines → {{%%%, %%%%}, {%%, %%%%%}}],
 {{γ, 0.1}, 0, 1}, {{μ, 0.5}, 0, 5}, {c, 0, clim}]
```

Out[146]=



In[147]:=

Analysis: Exclusive contracts permitted (equilibria with exclusive representation: no coordination)

In[148]:=
```
(*EQUILIBRIA WITH EXCLUSIVE REPRESENTATION*)
(*As discussed in the text, with exclusive contracts admissible,
any equilibrium involves exculisvity and contemplates the buyer purchasing
 from the dominant firm 1, at the following prices and associated
 payoffs and firm 2 offering the following exclusive contract.*)
p2Eequ = c; F2Eequ = 0;

(*if c≥p1m=μ/(1+2 μ) then*)
p1Eequ = p1m; F1Eequ = F1m;

PROFIT1Eequ = PROFIT1Em
PROFITBEequ = PROFITBE1m
```

Out[150]=
$$\frac{\mu^2 - c^2\,(1 + 2\,\mu) + c\,(2 + 4\,\mu)}{2 + 6\,\mu + 4\,\mu^2}$$

Out[151]=
$$\frac{1}{2}\,(-1 + c)^2$$

In[152]:=
```
(*if instead c<p1m=μ/(1+2 μ) then we are in a corner solution with*)
p1Eequcorn = c; F1Eequbcorn = 0;

q1e /. p1 -> p1Eequcorn;
PROFIT1Eequcorn = FullSimplify[% * p1Eequcorn ]

ue1 /. q1 -> q1e /. p1 -> p1Eequcorn;
 PROFITBEequcorn = FullSimplify[% - %%% p1Eequcorn]
```

Out[154]= $-(-1+c)\,c$

Out[156]= $\frac{1}{2}(-1+c)^2$

In[157]:=
```
(*The threshold for these two cases is*)
cEcorner = p1m;
```

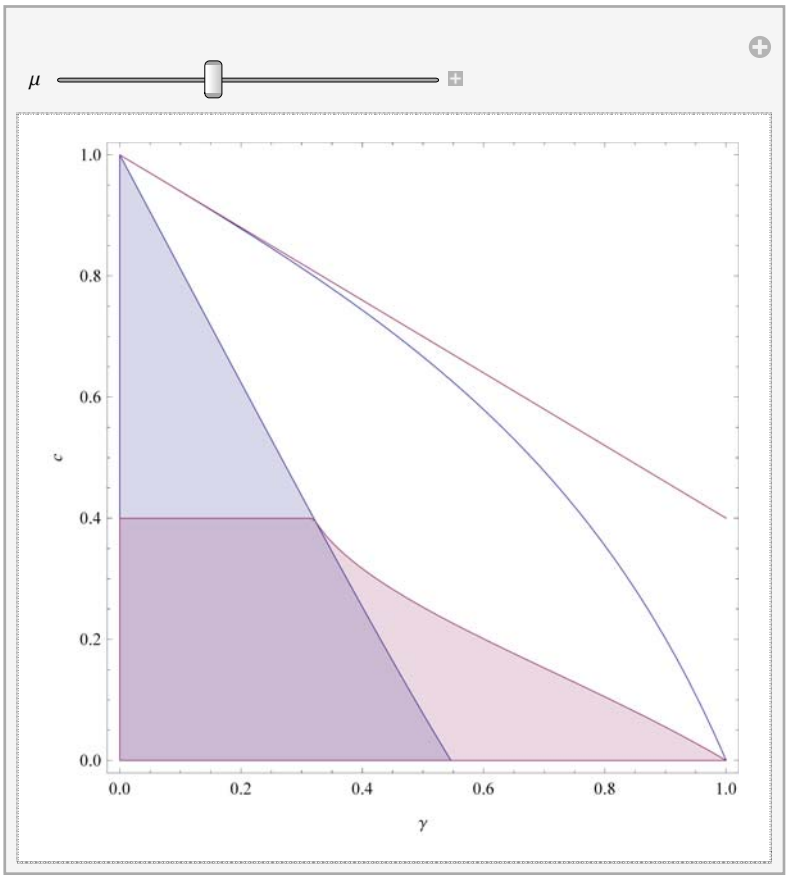
In[158]:=
```
(*The equilibria with comon representation when exclusive
 contracts are still admissible are investigated in another file.*)
```

Comparison with and without exclusive contracts

In[159]:=
```
(*We first consider the most cooperative equilibrium with non-exclusivity.*)
```

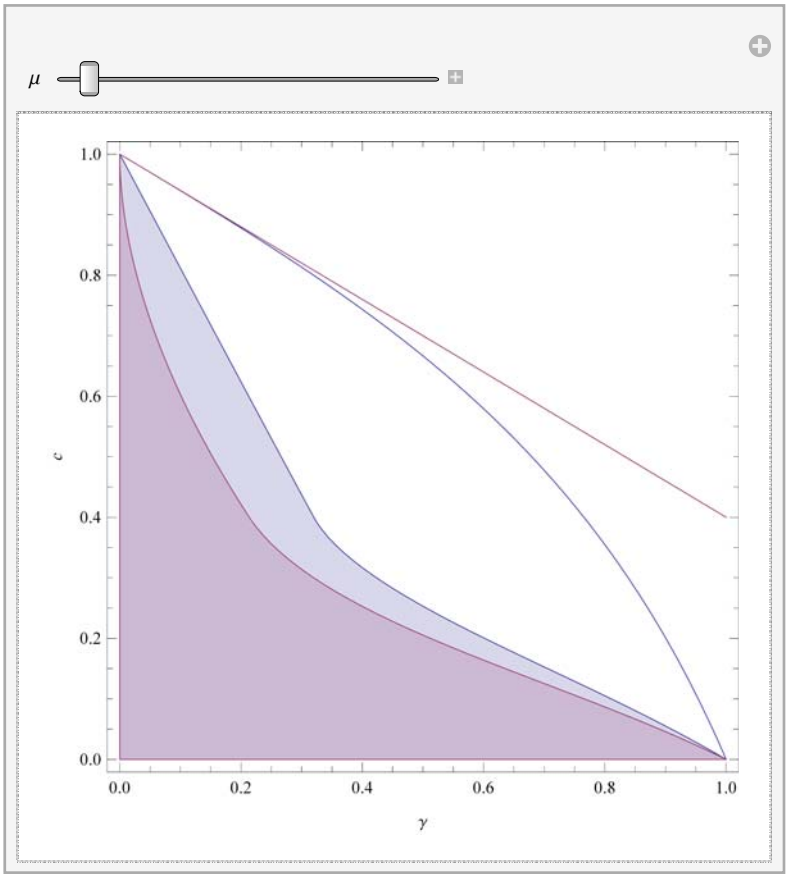
In[160]:=
```
{clim, cdrast};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO,
  Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO};
Manipulate[Show[RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c}],
  Plot[%%, {γ, 0, 1}]], {{μ, 2}, 0, 5}]
```

Out[162]=



In[163]:=
```
(*We consider here the least cooperative equilibrium with no-
 exclusivity as well*)
Profit1NELeastCoop = Profit1NE /. pricesNE /. F1 -> MinFstar[[1, 1, 2]];
{clim, cdrast};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
   Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO,
  Profit1NELeastCoop - PROFIT1Eequ > 0 && c < cPPO ||
   Profit1NELeastCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO
 };
Manipulate[Show[RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c}],
  Plot[%%, {γ, 0, 1}]], {{μ, 2}, 0, 50}]
```

Out[166]=



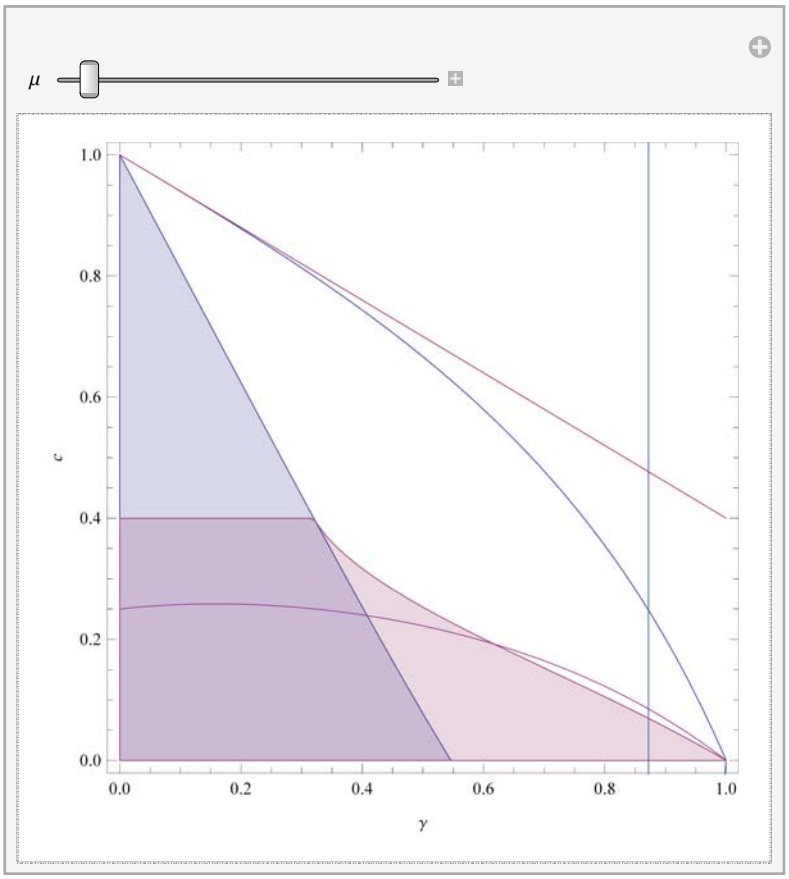
In[167]:=
```
(*As expected the two cases are qualitatively similar,
and they almost coincide when μ is large (in fact in both cases
   we converge to linear prices where fized fees are irrelevant)*)
```

In[168]:=
```
(*From now on for simplicity we will present figures
  with the most cooperative equilibrium with no-exclusivity.*)
```

In[169]:=
```
(*We verify the effect of Exclusivity
 on welfare with the same graphical analysis.
  As discussed in the text the appropriate measure to be used
 is the buyer's gross surplus (gross of the fixed fees)*)
u - p1 q1 - p2 q2 /. {q1 → q1NE, q2 → q2NE} /. pricesNE;
UgrossNE = %;

ue1 - p1 q1 /. {q1 → q1e} /. {p1 → p1Eequ};
UgrossE = %;

ue1 - p1 q1 /. {q1 → q1e} /. {p1 → p1Eequcorn};
UgrossEcorn = %;

(*The next comparison is the one relevant for c>p1m,
where p1m is the blue line in figure. Hence,
between the green lines and above the
  blue line Exclusivity dominates Non-Exclusivity*)
tmpwelf = Solve[UgrossNE - (UgrossE) == 0, c];
(*The graph will show that for this case c>p1m,
E is always detrimental for the buyer*)

(*The next comparison is the one relevant below the blue
 line: between the black lines Exclusivity dominates Non-Exclusivity*)
tempwelfcorn = Solve[UgrossNE - (UgrossEcorn) == 0, c];
```

In[177]:=
```
{clim, cdrast, tmpwelf[[1, 1, 2]], tmpwelf[[2, 1, 2]],
  tempwelfcorn[[1, 1, 2]], tempwelfcorn[[2, 1, 2]]};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO,
  Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO};
Manipulate[Show[RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c}],
  Plot[%%, {γ, 0, 1}]], {{μ, 2}, 0, 50}]
```

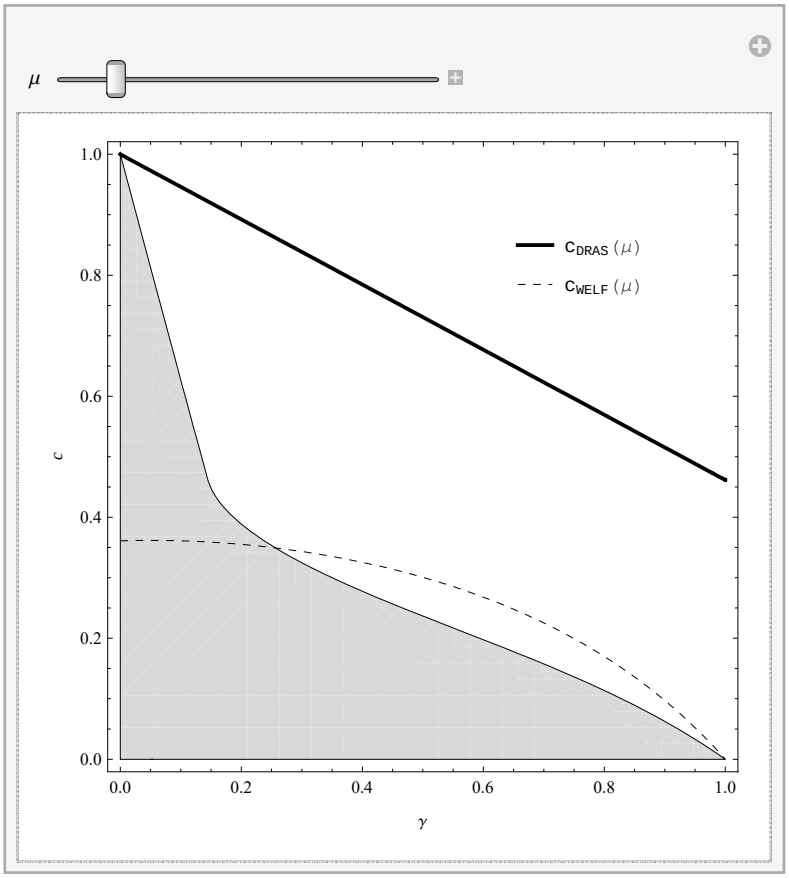
Out[179]=



In[180]:=
```
(*Hence the relevant boundary is:*)
EffectonConsSurplMostCoop = tempwelfcorn[[2, 1, 2]];
```

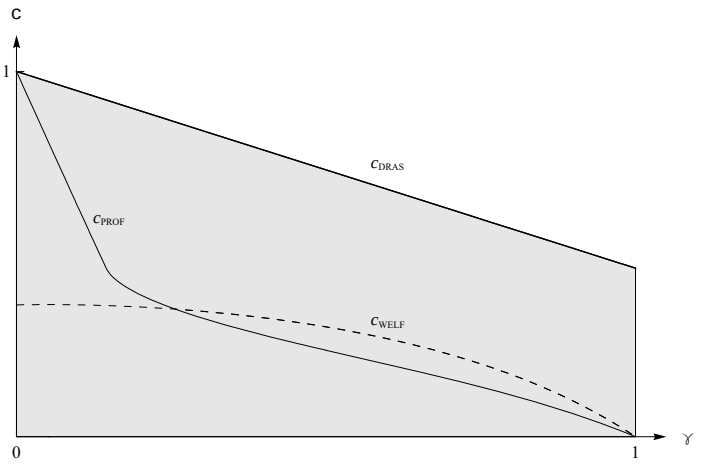
In[181]:=
```
(*Putting toghether the boundaries with welfare effect:*)
{cdrast, EffectonConsSurplMostCoop};
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
   Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO};
Manipulate[Show[

  RegionPlot[%, {γ, 0, 1}, {c, 0, 1},
   FrameLabel → {γ, c}, BoundaryStyle → Black, PlotStyle → LightGray

  ],

  Plot[%%, {γ, 0, 1}, PlotStyle → {{Black, Thick}, {Black, Dashed}},
   PlotLegends → Placed[LineLegend[{"c_DRAS(μ)", "c_WELF(μ)"},
      LabelStyle → {FontFamily → "Courier"}], {0.75, 0.8}]]

 ], {{μ, 6}, 0, 50}]
```

Out[183]=



In[184]:=
```
(*We now plot the Figure in the paper.*)
μplot = 6;
vertline = {{1, 0}, {1, p1Em}} /. μ → μplot;
{cdrast, EffectonConsSurplMostCoop} /. μ → μplot;
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
    Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO} /. μ → μplot;
Figure3 = Show[

  Plot[%%, {γ, 0, 1},
   PlotStyle → {{Black, Thickness[0.002]}, {Black, Dashed}}, AxesLabel → {"γ", "c"},
   Ticks → {{0, 1}, {-1, 1}},
   PlotRange → {{0, 1.05}, {0, 1.1}},
   AxesStyle → Arrowheads[{0.0, 0.02}],
   Filling → {{1 -> Bottom}},
   FillingStyle → {Lighter[Gray, 0.8]}
  ],

  RegionPlot[%, {γ, 0, 1}, {c, 0, 1}, FrameLabel → {γ, c},
   BoundaryStyle → Black, PlotStyle → Lighter[Gray, 0.8], Frame → False],




  Plot[%%, {γ, 0, 1},
   PlotStyle → {{Black, Thickness[0.002]}, {Black, Dashed}}, Ticks → {Null}],

  Graphics[{Line[vertline]}],
  Graphics[
   {Text["c_DRAS", {0.6, 0.75}], Text["c_PROF", {0.15, 0.6}], Text["c_WELF", {0.6, 0.32}]}]
 ]


SetDirectory[NotebookDirectory[]];
SetDirectory[ParentDirectory[]];
SetDirectory["Figures"];
Export["Figure3.pdf", Figure3];
```

Out[188]=



In[193]:=
```
(*As discussed in the main text,
for μ large enough there exist a region in which Exclusivity is both welfare
 increasing (below the black line) and optimal for the dominant firm 1.*)
```

Coordination on common representation when exclusive contracts are allowed (Figure 4)

In[194]:=
```
(*To account for different levels of coordination by the two firms
 (and associated multiple equilibria), now identify the Exclusive
 contracts that guarantee the buyer a given level of payoff k,
which will then be used as outside-option for the analysis of Common
  representation equilibria with different levels of coordination.
   The exclusive contracts will support common representation
  equilibria in which the buyer gets a payof k and the firms
  obtain the associated profit in case they deviate to exclusivity.

   There are several cases to analyse because the
  efficient exclusive contracts may involve corner solutions.*)
```

In[195]:=
```
(*Before moving on we define the firms' best responses:*)
FullSimplify[Solve[ProfitBNE == k, F1]];
Profit1NE /. F1 → %[[1, 1, 2]];

BR1 = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]];

FullSimplify[Solve[ProfitBNE == k, F2]];
Profit2NE /. F2 → %[[1, 1, 2]];

BR2 = FullSimplify[Solve[D[%, p2] == 0, p2]][[1, 1, 2]];
```

In[201]:=
```
(*Firm 1: Consider first the case with F1E>0.*)
FE1ktmp = FullSimplify[Solve[ProfitBE1 == k, F1]][[1, 1, 2]];
Profit1E /. F1 → FE1ktmp;
pE1k = FullSimplify[Solve[D[%, p1] == 0, p1]][[1, 1, 2]]
FE1k = FullSimplify[FE1ktmp /. p1 -> pE1k]
Profit1Ek = FullSimplify[Profit1E /. F1 → FE1k /. p1 -> pE1k]
```

Out[203]= $\frac{\mu}{1+2\mu}$

Out[204]= $\frac{-2k+\frac{(1+\mu)^2}{(1+2\mu)^2}}{2(1+\mu)}$

Out[205]= $\frac{1}{4}\left(1-\frac{4k}{1+\mu}+\frac{1}{1+2\mu}\right)$

In[206]:=
```
(*Indeed FE1 is positive if:*)
FullSimplify[FE1k]
FullSimplify[Reduce[{% ≥ 0, μ ≥ 0, k ≥ 0}]]
k1 = %[[2, 5]]
```

Out[206]= $\frac{-2k+\frac{(1+\mu)^2}{(1+2\mu)^2}}{2(1+\mu)}$

Out[207]= $\mu \geq 0 \&\& 0 \leq k \leq \frac{(1+\mu)^2}{2(1+2\mu)^2}$

Out[208]= $\frac{(1+\mu)^2}{2(1+2\mu)^2}$

In[209]:=
```
(*Hence for k≥
  k1 then the exclusive contract of firm 1 is at the corner solution with FE1=
 0 and the marginal price is determined as follows*)
ProfitBE1 /. F1 → 0;
pE1kcorn = Solve[% == k, p1][[1, 1, 2]]
Profit1Econstr = FullSimplify[Profit1E /. F1 → 0 /. p1 -> pE1kcorn]
```

Out[210]= $1-\sqrt{2}\ \sqrt{k}$

Out[211]= $\sqrt{2}\ \sqrt{k}-2k$

In[212]:=
```
(*The following shows that the upper limit for k is in this case*)
FullSimplify[Reduce[{pE1kcorn ≥ 0, μ ≥ 0, k ≥ 0}]]
```

Out[212]= $\mu \geq 0 \&\& 0 \leq k \leq \frac{1}{2}$

In[213]:=
```
(* Notice that this limit at 1/2 for k is also the largest possible
 profit in exclusive the buyer can obtain with efficient pricing*)
PROFITBEequ /. c → 0
```

Out[213]= $\frac{1}{2}$

In[214]:=
```
(*Clearly, this limit is always admissible:*)
FullSimplify[Reduce[{1/2 ≥ k1, μ ≥ 0, k ≥ 0}]]
```

Out[214]= $\mu \geq 0 \&\& k \geq 0$

In[215]:=
```
(* The analysis of Exclusivity for Firm 2
 and a given payoff k for the buyer is similar.*)
FE2ktmp = FullSimplify[Solve[ProfitBE2 == k, F2]][[1, 1, 2]];
Profit2E /. F2 → FE2ktmp;
pE2k = FullSimplify[Solve[D[%, p2] == 0, p2]][[1, 1, 2]]
FE2k = FullSimplify[FE2ktmp /. p2 → pE2k]
FullSimplify[Reduce[{% ≥ 0, μ ≥ 0, k ≥ 0, 1 > c ≥ 0}]]
k2 = %[[3, 5]]
Profit2Ek = FullSimplify[Profit2E /. F2 → FE2k /. p2 -> pE2k]
```

Out[217]= $\frac{c + \mu + c\,\mu}{1 + 2\,\mu}$

Out[218]= $\frac{-2\,k + \frac{(-1+c)^2\,(1+\mu)^2}{(1+2\,\mu)^2}}{2\,(1+\mu)}$

Out[219]= $0 \leq c < 1 \&\& \mu \geq 0 \&\& 0 \leq k \leq \frac{(-1+c)^2\,(1+\mu)^2}{2\,(1+2\,\mu)^2}$

Out[220]= $\frac{(-1+c)^2\,(1+\mu)^2}{2\,(1+2\,\mu)^2}$

Out[221]= $\frac{1}{4}\left((-1+c)^2 - \frac{4\,k}{1+\mu} + \frac{(-1+c)^2}{1+2\,\mu}\right)$

In[222]:=
```
(*Hence for k≥k2 the exclusive contract of firm 2 is*)
ProfitBE2 /. F2 → 0;
pE2kcorn = Solve[% == k, p2][[1, 1, 2]]
Profit2Ekcorn = FullSimplify[Profit2E /. F2 → 0 /. p2 -> pE2kcorn]
```

Out[223]= $1 - \sqrt{2}\,\sqrt{k}$

Out[224]= $\left(\sqrt{2} - \sqrt{2}\,c - 2\sqrt{k}\right)\sqrt{k}$

In[225]:=
```
(*As expected there is the same logic for the limit for k as seen for firm
 1 with the profit the buyer can obtain now with excluivity and firm 2*)
FullSimplify[Reduce[{pE2kcorn ≥ c, μ ≥ 0, k ≥ 0, 1 > c ≥ 0}]]
```

Out[225]= $\mu \geq 0 \&\& 0 \leq c < 1 \&\& 0 \leq k \leq \frac{1}{2} (-1+c)^2$

In[226]:=
```
(*We now define the last relevant boundary for k*)
kmax = PROFITBEequ;
```

In[227]:=
```
(*Now we rank the k1,k2,PROFITBEequ: *)
Reduce[{k2 <= 1/2 (-1+c)^2, 1 ≥ c ≥ 0, μ ≥ 0}]
```

Out[227]= $\mu \geq 0 \&\& 0 \leq c \leq 1$

In[228]:=
```
Reduce[{k1 ≥ k2, 1 ≥ c ≥ 0, μ ≥ 0}]
```

Out[228]= $\mu \geq 0 \&\& 0 \leq c \leq 1$

In[229]:=
```
Reduce[{k1 ≤ PROFITBEequ, 1 ≥ c ≥ 0, μ ≥ 0}]
```

Out[229]= $(\mu == 0 \&\& c == 0) \,||\, \left(\mu > 0 \&\& 0 \leq c \leq \frac{\mu}{1+2\mu}\right)$

In[230]:=
```
(*Hence, the ranking between the boundaries for k is:
   k2≤min{k1,PROFITBEequ}≤max{k1,PROFITBEequ} with min{k1,PROFITBEequ}=
 k1 iff c≤μ/(1+2 μ)=p1Eequ*)
```

In[231]:=
```
(*Hence we define the relevant regions for k:
  - "RegimeEInt":for k≤k2 both FE1 and FE2 are strictly positive
    and the two firms can implement k with the previous exclusive
    "interior" exclusive contracts seen above and the associated profits;
- "RegimeEMix": for k2≤k≤min{k1,PROFITBEequ} the contract for
    firm 2 is constrained with F2E=0, that of firm 1 not;
- "RegimeEConstr": this exists only if c≤μ/(1+2 μ)=p1Eequ so that k1<PROFITBEequ,
in which case the last regime is defined as k1<k≤
  PROFITBEequ where both contracts are constrained with F1E=F2E=0.

   Finally, we notice that k>
 PROFITBEequ is irrelevant because firms are here coordinating and trying
  to reduce the payoff the buyer can obtain in exclusivity with respect to
  that she can obtain in the exclusive equilibrium (with firm 1, clearly),
i.e. PROFITBEequ. Notice also that since PROFITBEequ≤1/2,
we always have pE1kcorn≥
 0 and do not need to account for the constraint k≤1/2 identified above.
*)
```

In[232]:=
```
(*As next step of the analysis,
here we define the constraints for the Non-Exclusive set.*)

BuyerIndiff = ProfitBNE - k (*Notice that this constraint must be binding in
  equilibrium if at least one fixed fee F1 and F2 is strictly positive*);

Profit1E /. {p1 → p1E, F1 → F1E};
E1 = Profit1NE - % (*E1≥0 no-deviation constraint to exclusivity for firm 1*);

Profit2E /. {p2 → p2E, F2 → F2E} ;
E2 = Profit2NE - % (*E2≥0 no-deviation constraint to exclusivity for firm 2*);

NE1 = BR1 - p1 (*NE1≥0 Best response constraint of firm 1*);
NE2 = BR2 - p2 (*NE1≥0 Best response constraint of firm 1*);

F1 (*F1≥0 non-negativity of the fixed fee*);
F2 (*F2≥0 non-negativity of the fixed fee*);

CS1 = NE1 * F1 (*CS1==0, this complementary slackness condition must be nill*);
CS2 = NE2 * F2 (*CS2==0, this complementary slackness condition must be nill*);

(*The full set of constraints is then:*)
ConstraintsE = {BuyerIndiff == 0 && E1 ≥ 0 && E2 ≥ 0 && NE1 ≥ 0 &&
    NE2 ≥ 0 && F1 ≥ 0 && F2 ≥ 0 && CS1 == 0 && CS2 == 0 && p1 ≥ 0 && p2 ≥ c};
```

In[244]:=
```
(*We now study when the coordination among
            the firms on "Bertrand" maringal-prices is possible.
             a) Firm 1 is the one with the strongest incentive to deviate
        to Exclusivity thus we will set F2=0 with the idea that is almost zero,
      so that the Best response constraint of firm 2 is still binding;
      b) Buyer's participation must be binding and determines F1;
    c) we are then left with E1 constraint to be checked positive: for point b)
 this does not depend on k (it cancels out), but we will still have to
 check in which of the three exclusive cases illustrated above we are.
*)
```

In[245]:=
```
(*Now notice that, since we are looking for the possibility
 that the firms are able to coordinate on teh "Bertrand" marginal
 prices (p1NE,p2NE) that would apply without exclusivity,
we will consider F1 and F2 that relax the incentives to
 deviate. Since the incentive to deviate are stronger for firm 1,
we will consider F2 (almost) equal to zero (almost because for the marginal
    price to be  (p1NE,p2NE) both fixed fees must be strictly positive)
  and check that firm 1 has no incentive to deviate and that F=F1+F2>0*)
```

In[246]:=
```
F1extrm = FullSimplify[Solve[BuyerIndiff == 0, F1] /. F2 → 0][[1, 1, 2]]
 (*(Almost) all fixed fee to dominant firm 1*);
(*No deviation constraint of firm 1*)
E1M = E1 /. F1 -> F1extrm /. pricesNE;
```

In[248]:=
```
FullSimplify[Solve[BuyerIndiff == 0, F1] /. F2 → 0 /. pricesNE];
(*Non negative sum of fixed fees
 (in fact, non negative F1 since here F2 is almost nil)*)
FPositive = FullSimplify[%[[1, 1, 2]]];
```

Regime 1) of exclusivity, “RegimeEInt”, k≤k2

In[250]:=
```
ExclRegimeEInt = {F1E -> FE1k, p1E -> pE1k, F2E -> FE2k, p2E -> pE2k};
```

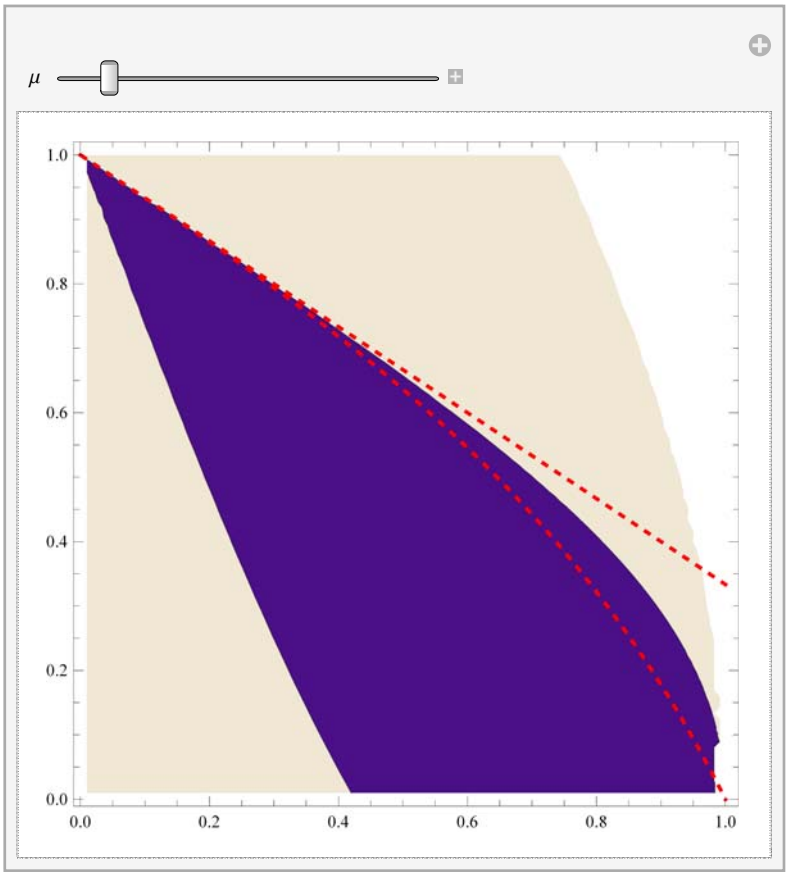
In[251]:=
```
(*In this regime E1 not depend on k*)
E1M /. ExclRegimeEInt;
FullSimplify[D[%, k]]
```

Out[252]=
```
0
```

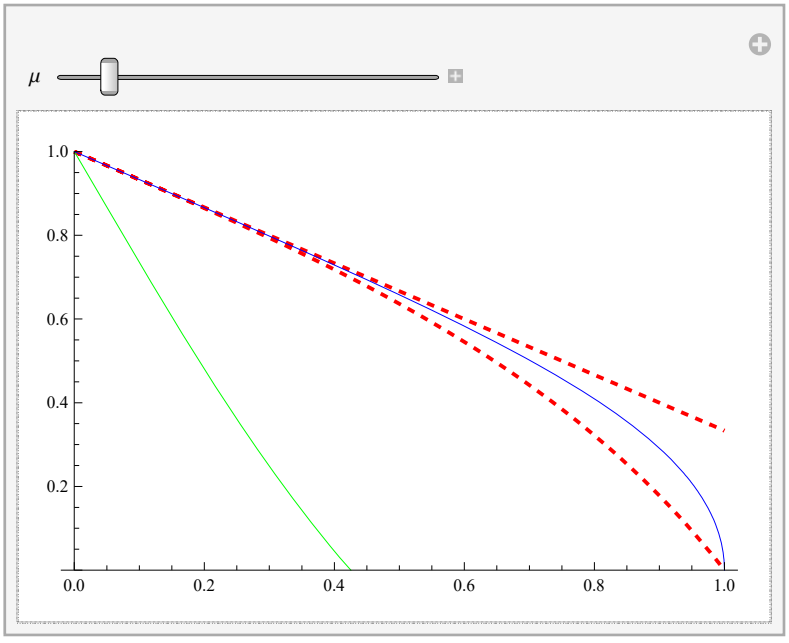
In[253]:=
```
(*Hence, any k is compatible for E1M and E2M ,
but we still have to check the combinations
  of (c,γ,μ) that guarantee that E1M≥ 0*)
```

In[254]:=
```
FullSimplify[E1M /. ExclRegimeEInt];
tmpE1 = FullSimplify[Solve[% == 0, c]];

clim;
cdrast;
Manipulate[Show[ContourPlot[%%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}],
  Plot[{%, %%}, {γ, 0, 1}, PlotStyle → {{Red, Thick, Dashed}, {Red, Thick, Dashed}}]],
 {{μ, 1}, 0.01, 10}]

Manipulate[Plot[{%%, %%%, %%%%[[1, 1, 2]], %%%%[[2, 1, 2]]}, {γ, 0, 1},
  PlotStyle → {{Red, Thick, Dashed}, {Red, Thick, Dashed}, Green, Blue},
  PlotRange → {0, 1}], {{μ, 1}, 0.01, 10}]
```

Out[258]=



Out[259]=



In[260]:=
```
(*Hence, firm 1 does not deviate (i.e. E1M≥0) for c lower than the
 following threshold, i.e. below the gree boundary in the previous graph*)
cE1 = tmpE1[[1, 1, 2]]
```

Out[260]=
$$\left(\gamma^2 \mu^2 (1+2\mu)^2 + \gamma^3 \mu^2 (1+2\mu)^2 + (1+2\mu)^3 - \gamma (1+2\mu)^2 (1+2\mu(1+\mu)) - \sqrt{\left(-\gamma^2\left(-1+\gamma^2\right)\mu^2 (1+2\mu)^2 \left(-1+\mu\left(-4+\left(-4+\gamma^2\right)\mu\right)\right)^2\right)}\right) \Big/ \left((1+2\mu)^2\left(1+\mu\left(2+\gamma^2\mu\right)\right)\right)$$

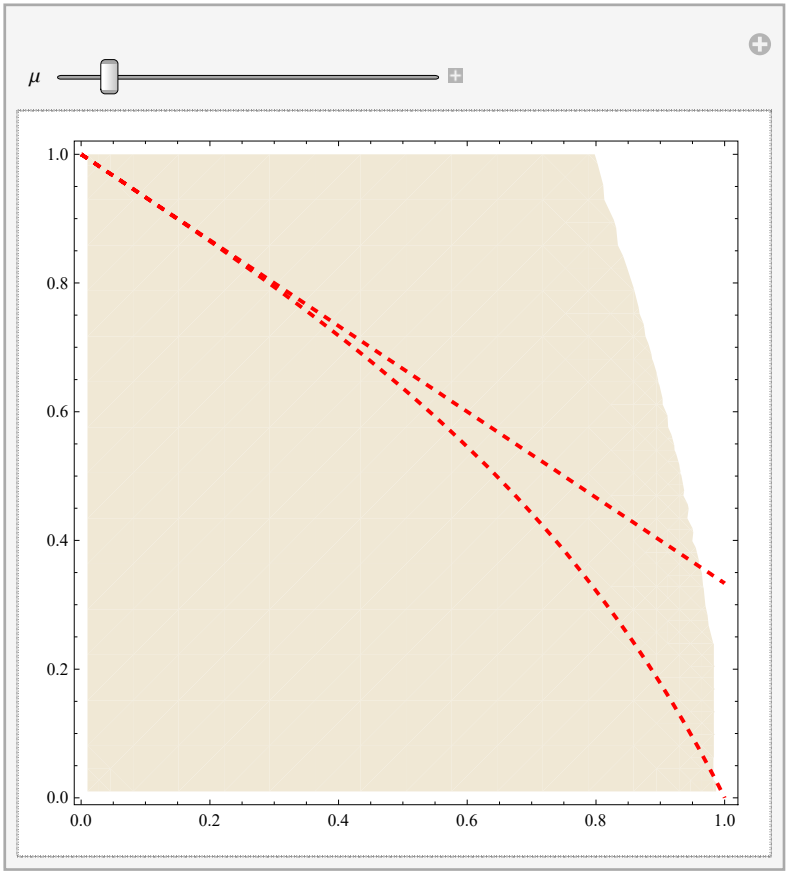
In[261]:=
```
(*Hence, we now check that F=F1+F2 is positive*)
```

In[262]:=
```
FullSimplify[D[FPositive, k]]
```

Out[262]=
$$-\frac{1}{1+\mu}$$

In[263]:=
```
(*Since F1+F2 is decreasing in k we study this sum for the
  maximal value of this regime and show it is always positive:*)
FPositive /. k → k2 (*At the maximum value of k in this regime*);
clim;
cdrast;
Manipulate[Show[
  ContourPlot[%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}], Plot[{%, %%}, {γ, 0, 1},
   PlotStyle → {{Red, Thick, Dashed}, {Red, Thick, Dashed}}]], {{μ, 1}, 0.01, 10}]
```

Out[266]=



Regime 2) of exclusivity, “RegimeEMix”, for k2≤k≤min{k1,PROFITBEequ} where the contract for firm 2 is constrained with F2E=0, that of firm 1 is not not

In[267]:=
```
(*Recall that if c≤μ/(1+2 μ)=p1Eequ then k1<PROFITBEequ.

     Hence we consider first the  case c≤μ/(1+2 μ)=
  p1Eequ so that the relevant constraints for k are k2≤k≤k1
*)
RegimeEMix = {F1E -> FE1k, p1E -> pE1k, F2E → 0, p2E -> pE2kcorn};
```

In[268]:=
```
(*Since the contract of firm 1 in exclusivity is the same
 as in the previous regime, the analysis for E1 is unchanged.*)
```

In[269]:=
```
(*Finally, we check that the sum of
 the fxied fees (or equivalently F1) is positive.
  First we show that is clearly decreasing in k,
so that it sufficies to study it for the largest value of admissible k.*)
```

In[270]:=
```
FullSimplify[D[FPositive, k]]
```

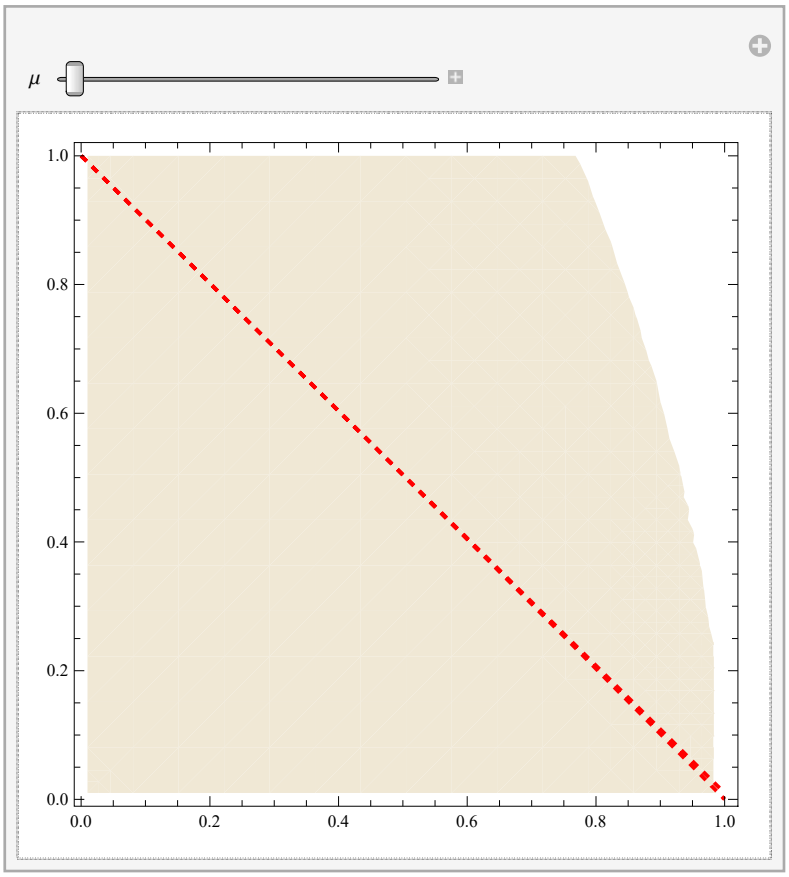
Out[270]=
$$-\frac{1}{1+\mu}$$

$$-\frac{1}{1+\mu}$$

$$-\frac{1}{1+\mu}$$

In[271]:=
```
(*As before we study it at the maximal admissible value of k:*)
FPositive /. k → k1 (*At the maximum value of k in this regime*);
clim;
cdrast;
Manipulate[Show[
  ContourPlot[%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}], Plot[{%, %%}, {γ, 0, 1},
   PlotStyle → {{Red, Thick, Dashed}, {Red, Thick, Dashed}}]], {μ, 0.01, 10}]
```

Out[274]=

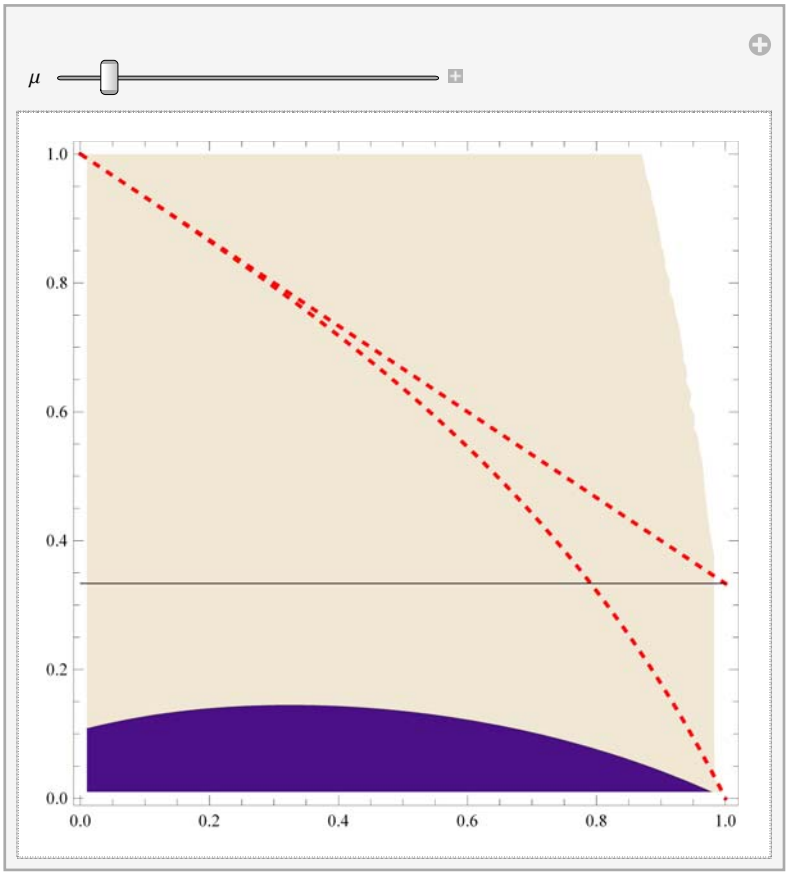


In[275]:=
```
(*Hence, the sum of fixed fees is always positive.*)
```

In[276]:=
```
(*Hence, as for constraint E2M≥0 this regime is admissible as long as E2M≥
  0 is satisfied for the largest admissible k i.e. k=k1*)
```

In[277]:=
```
FPositive /. k → PROFITBEequ
 (*At the maximum value of k in this regime and for this case c>μ/(1+2 μ)*);
clim;
cdrast;
Manipulate[Show[ContourPlot[%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}],
  Plot[{%, %%, μ/(1 + 2 μ)}, {γ, 0, 1}, PlotStyle →
    {{Red, Thick, Dashed}, {Red, Thick, Dashed}, Black}]], {{μ, 1}, 0.01, 10}]
```

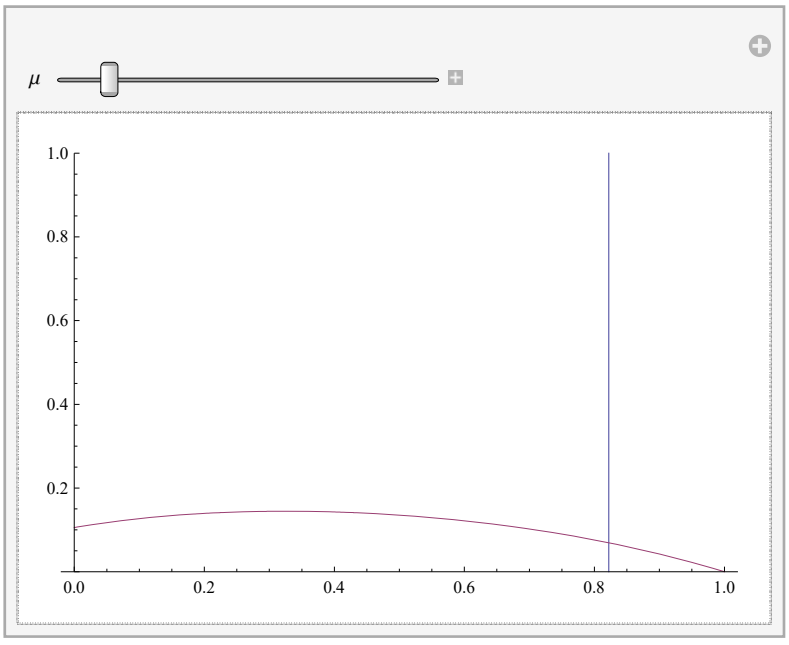
Out[280]=



In[281]:=
```
FPositive /. k → PROFITBEequ ;
FullSimplify[Solve[% == 0, c]]
Manipulate[
 Plot[{%[[1, 1, 2]], %[[2, 1, 2]]}, {γ, 0, 1}, PlotRange → {0, 1}], {{μ, 1}, 0.01, 10}]
cE1RegimeEMix = %%[[2, 1, 2]];
```

Out[282]=
$$\left\{\left\{c \to -\left(-\gamma^6 \mu^4+\gamma^3 \mu^2 (1+\mu)^2+\gamma (1+\mu)^2 (1+2\mu)+\mu (1+2\mu)^2 (2+3\mu)+\right.\right.\right.$$
$$\gamma^4 \mu^2 (2+\mu (8+9\mu))+\sqrt{\left(-1+\gamma^2\right)^2 \mu (1+\mu)^2 (2+3\mu)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2}-$$
$$\left.\left.\gamma^2 (1+\mu (6+7\mu) (1+\mu (2+3\mu)))\right)\Big/\left(\gamma^6 \mu^4-\mu (1+2\mu)^2 (2+3\mu)-\right.\right.$$
$$\left.\left.\gamma^4 \mu^2 (2+\mu (8+9\mu))+\gamma^2 (1+\mu (6+7\mu) (1+\mu (2+3\mu)))\right)\right\},$$
$$\left\{c \to \left(\gamma^6 \mu^4-\gamma^3 \mu^2 (1+\mu)^2-\gamma (1+\mu)^2 (1+2\mu)-\mu (1+2\mu)^2 (2+3\mu)-\right.\right.$$
$$\gamma^4 \mu^2 (2+\mu (8+9\mu))+\sqrt{\left(-1+\gamma^2\right)^2 \mu (1+\mu)^2 (2+3\mu)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2}+$$
$$\left.\gamma^2 (1+\mu (6+7\mu) (1+\mu (2+3\mu)))\right)\Big/$$
$$\left.\left.\left(\gamma^6 \mu^4-\mu (1+2\mu)^2 (2+3\mu)-\gamma^4 \mu^2 (2+\mu (8+9\mu))+\gamma^2 (1+\mu (6+7\mu) (1+\mu (2+3\mu)))\right)\right\}\right\}$$

Out[283]=



In[285]:=
```
(*Summarizing if c≤μ/(1+2 μ)=p1Eequ so that k1<PROFITBEequ,
then k2≤k≤k1 is admissible in the area below the green line of the
  graphs seen above and above the black area and below the black line.*)
```

In[286]:=
```
(*We conclude this Regime 2,
studying the case in which c>μ/(1+2 μ)=
 p1Eequ so that Regime 2 is defined by k1≤k≤PROFITBEequ. Nothing changes for E1.
    Hence, there are no constraint for c>μ/(1+2 μ) .*)
```

Regime 3) of exclusivity, “RegimeEConstr”, for k1<k≤PROFITBEequ both contracts are constrained with F1E=F2E=0. This exists only if $c \leq \frac{\mu}{1+2\mu}$

In[287]:=
```
RegimeECorn = {F1E → 0, p1E -> pE1kcorn, F2E → 0, p2E -> pE2kcorn};
```

In[288]:=
```
(*We first study constraint E1.

   This requires some algebraic manipulations to find closed
  form solutions for the threshold value of k that makes E1=0*)
```

In[289]:=
```
Btmp = FullSimplify[E1M /. RegimeECorn /. k → 0];
E1M /. RegimeECorn;
FullSimplify[% - (-√2 √k + (k + 2 k μ)/(1 + μ) + B /. B -> Btmp)]
```

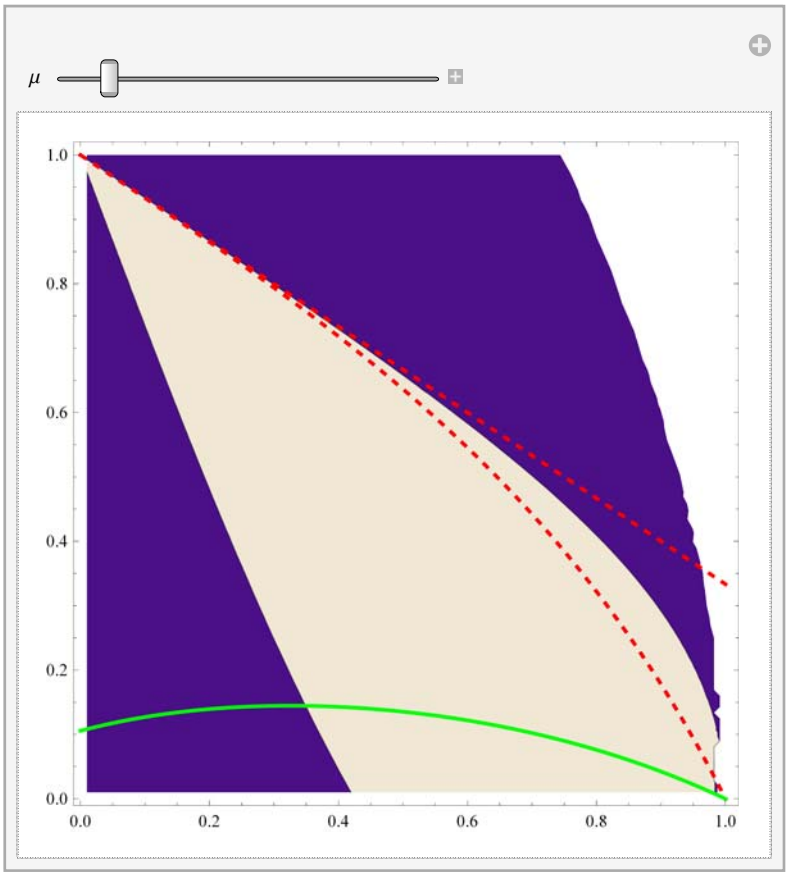
Out[291]=
```
0
```

In[292]:=
```
tmpCorn = FullSimplify[Solve[-√2 √k + (k + 2 k μ)/(1 + μ) + B == 0, k]]
```

Out[292]=
$$\left\{\left\{k \to \left(1+\mu\,(2+\mu)-B\,(1+\mu)\,(1+2\mu)-\sqrt{-(1+\mu)^3\,(-1+2B-\mu+4B\mu)}\right)\Big/(1+2\mu)^2\right\},\right.$$
$$\left.\left\{k \to \left(1+\mu\,(2+\mu)-B\,(1+\mu)\,(1+2\mu)+\sqrt{-(1+\mu)^3\,(-1-\mu+B\,(2+4\mu))}\right)\Big/(1+2\mu)^2\right\}\right\}$$

In[293]:=
```
(*The relevant solution is the largest one: tmpCorn[[2,1,2]]

   Moreover, when the discriminant in the square root is negative,
then E1M is always positive for any k in
 this case. When instead the discriminant is positive,
then we must have k larger than this new threshold tmpCorn[[2,1,2]].*)

Discr = -(1 + μ)^3 (-1 + 2 B - μ + 4 B μ) /. B → Btmp;
cE1RegimeEMix;
clim;
cdrast;
Manipulate[Show[ContourPlot[%%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}],
  Plot[{%, %%, %%%}, {γ, 0, 1},
   PlotStyle → {{Red, Thick, Dashed}, {Red, Thick, Dashed}, {Green, Thick}}]],
 {{μ, 1}, 0, 10}](*Notably, this analysis shows that the discriminant
 is negative when the previous  Regimes (with k<k1)*)
```

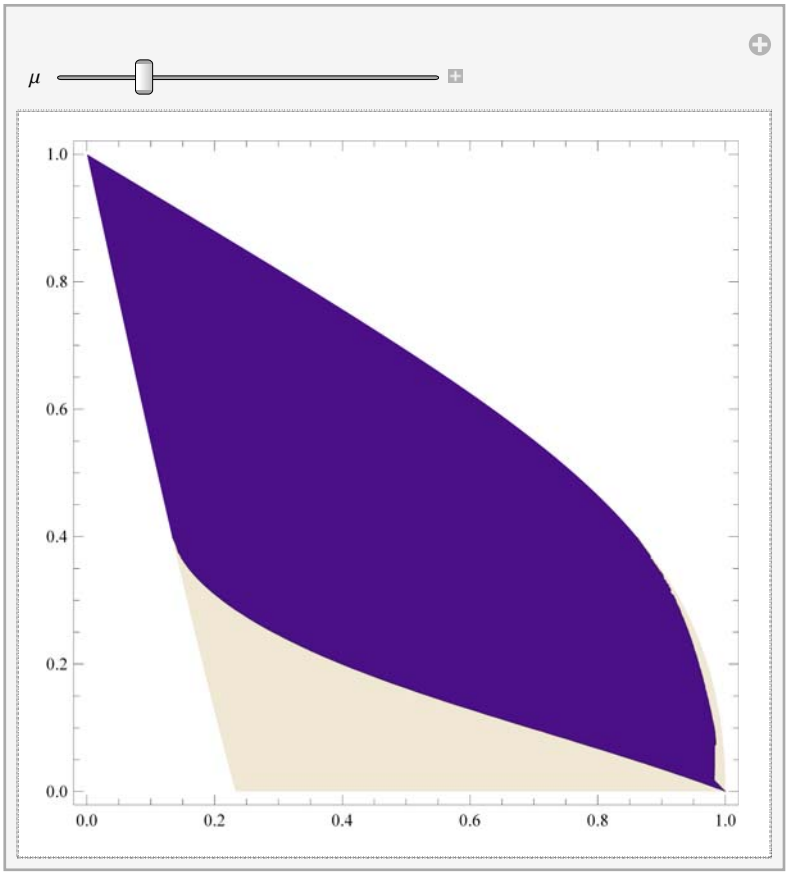
Out[297]=



In[298]:=
```
(*When the Discriminant is positive then we
 must be sure to be in an admissible condition as for k,
in particular it must be k larger than the following threshold kE1MConstr*)
kE1MConstr = tmpCorn[[2, 1, 2]] /. B → Btmp;
```

In[299]:=
```
(*Hence we have that if PROFITBEequ-kE1MConstr ≥ 0,
we can always find a k such that PROFITBEequ > k > kE1MConstr *)
PROFITBEequ - kE1MConstr;
Manipulate[ContourPlot[%, {γ, 0, 1}, {c, 0, 1}, Contours → {0}], {{μ, 2}, 0.01, 10}]
```

Out[300]=



In[301]:=
```
(*We explicitely determine the threshold that appears in the previous graph:*)
tmpCorndiff = Simplify[PROFITBEequ - kE1MConstr, {1 ≥ c ≥ 0, 1 ≥ γ ≥ 0, μ ≥ 0}]
```

Out[301]=

$$\frac{1}{2}(-1+c)^2-\frac{1}{(1+2\mu)^2}\Big(1+\mu(2+\mu)+\Big((1+\mu)^2(1+2\mu)\Big(-2c(-1+\gamma)(1+2\mu)(-1+\gamma\mu)(1+(2+\gamma)\mu)+2(-1+\gamma)(-1+(-1+\gamma)\mu)(1+(2+\gamma)\mu)^2+c^2(1+2\mu)\Big(1+\mu\Big(2+\gamma^2\mu\Big)\Big)\Big)\Big)\Big/\Big(2\Big(-1+\gamma^2\Big)(-1+(-2+\gamma)\mu)^2(1+(2+\gamma)\mu)^2\Big)+(1+\mu)^{3/2}\sqrt{\Big(1+\mu+\Big((1+\mu)(2+4\mu)\Big(-2c(-1+\gamma)(1+2\mu)(-1+\gamma\mu)(1+(2+\gamma)\mu)+2(-1+\gamma)(-1+(-1+\gamma)\mu)(1+(2+\gamma)\mu)^2+c^2(1+2\mu)\Big(1+\mu\Big(2+\gamma^2\mu\Big)\Big)\Big)\Big)\Big/\Big(2\Big(-1+\gamma^2\Big)(-1+(-2+\gamma)\mu)^2(1+(2+\gamma)\mu)^2\Big)\Big)}$$

In[302]:=
```
(*To solve this expression for the limit
 value of c we need some algebraic manipulations:*)
h = 2 (-1 + γ^2) (1 + μ (4 - (-4 + γ^2) μ))^2; (*note this does not depend on c*)
tmpCorndiffH = 1/2 (-1 + c)^2 - 1/(1 + 2 μ)^2 (1 + μ (2 + μ) -
      ((1 + μ)^2 (1 + 2 μ) (2 c (-1 + γ) (1 + 2 μ) (-1 + γ μ) (1 + (2 + γ) μ) - 2 (-1 + γ)
            (-1 + (-1 + γ) μ) (1 + (2 + γ) μ)^2 - c^2 (1 + 2 μ) (1 + μ (2 + γ^2 μ))))/H + (1 + μ)^(3/2)
     Sqrt(1 + μ - ((1 + μ) (2 + 4 μ) (2 c (-1 + γ) (1 + 2 μ) (-1 + γ μ) (1 + (2 + γ) μ) - 2 (-1 + γ)
            (-1 + (-1 + γ) μ) (1 + (2 + γ) μ)^2 - c^2 (1 + 2 μ) (1 + μ (2 + γ^2 μ))))/(H)));
```

In[304]:=
```
(*Check*)FullSimplify[tmpCorndiffH - (tmpCorndiff) /. H → h]
```

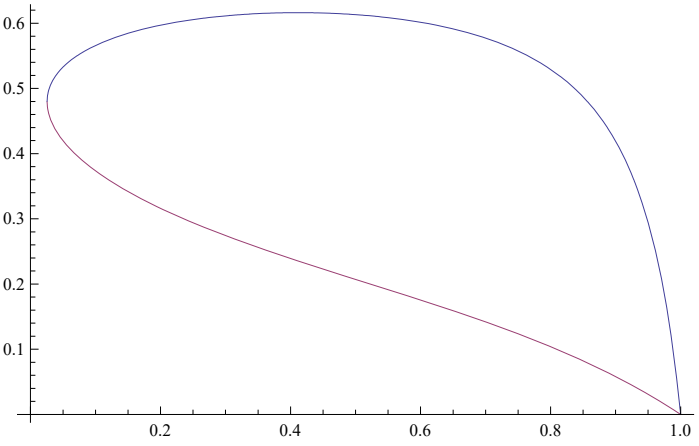
Out[304]=
```
0
```

In[305]:=
```
tmpCorndiffHsolve = Solve[tmpCorndiffH == 0, c];
```

In[306]:=
```
(*The relevant solutions are the first two:*)
tmpCorndiffHsolve /. H → h /. {γ → 0.5, μ → 10}
tmpCorndiffHsolve /. H → h /. {μ → 10};
Plot[{%[[1, 1, 2]], %[[2, 1, 2]]}, {γ, 0, 1}]
```

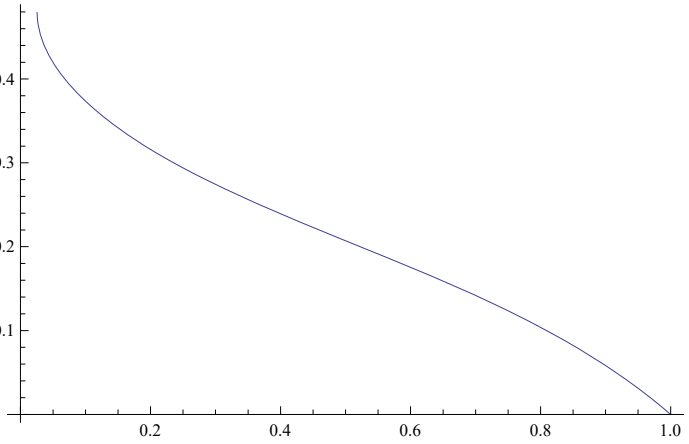
Out[306]=
```
{{c → 0.613123}, {c → 0.207127}, {c → 1.41467 + 0.367147 i}, {c → 1.41467 - 0.367147 i}}
```

Out[308]=



In[309]:=
```
(*Hence the threshold we are looking for is:*)
cRegime3E1 = FullSimplify[tmpCorndiffHsolve[[2, 1, 2]] /. H → h];
% /. {μ → 10};
Plot[%, {γ, 0, 1}]
```

Out[311]=



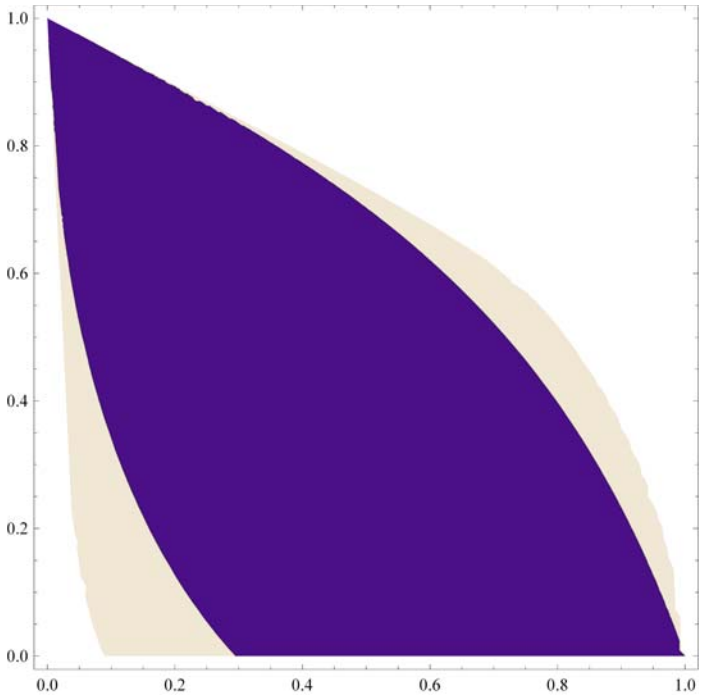
In[312]:=
```
(*Now we also have to check the non negativity of the fixed fees:*)
FPositive;
FullSimplify[D[FPositive, k]]
kFPositive = FullSimplify[Solve[%% == 0, k]][[1, 1, 2]]
(*Given the sign of the derivative, the constraint is verified for k<kFPositive*)
```

Out[313]=
$$-\frac{1}{1+\mu}$$

Out[314]=
$$-\left((1+\mu)^2\left(-2(-1+\gamma)(1+(2+\gamma)\mu)^2+2c(-1+\gamma)(1+(2+\gamma)\mu)^2+c^2\left(1+\mu\left(4(1+\mu)-\gamma^2(2+3\mu)\right)\right)\right)\right)\Big/\left(2\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)$$

In[315]:=
```
(*Hence we have the possibility kFPositive > k >
 kE1MConstr (so that F>0 and there are no deviations) when kFPositive-
  kE1MConstr is positive:*)
kFPositive - kE1MConstr /. μ → 10;
ContourPlot[%, {γ, 0, 1}, {c, 0, 1}, Contours → {0}]
```

Out[316]=



In[317]:=
```
(*We explicitely determine the previous
 threshold with some agelbraic manipulations:*)
```

In[318]:=
```
kFPositive - kE1MConstr
```

Out[318]=

$$-\Big((1+\mu)^2\left(-2(-1+\gamma)(1+(2+\gamma)\mu)^2+2c(-1+\gamma)(1+(2+\gamma)\mu)^2+c^2\left(1+\mu\left(4(1+\mu)-\gamma^2(2+3\mu)\right)\right)\right)\Big/\left(2\left(-1+\gamma^2\right)\left(1+\mu\left(4-\left(-4+\gamma^2\right)\mu\right)\right)^2\right)-\frac{1}{(1+2\mu)^2}\Big(1+\mu(2+\mu)+\Big((1+\mu)^2(1+2\mu)\left(-2c(-1+\gamma)(1+2\mu)(-1+\gamma\mu)(1+(2+\gamma)\mu)+2(-1+\gamma)(-1+(-1+\gamma)\mu)(1+(2+\gamma)\mu)^2+c^2(1+2\mu)\left(1+\mu\left(2+\gamma^2\mu\right)\right)\right)\Big)\Big/\left(2\left(-1+\gamma^2\right)(-1+(-2+\gamma)\mu)^2(1+(2+\gamma)\mu)^2\right)+\sqrt{\Big(-(1+\mu)^3\Big(-1-\mu-\Big((1+\mu)(2+4\mu)\left(-2c(-1+\gamma)(1+2\mu)(-1+\gamma\mu)(1+(2+\gamma)\mu)+2(-1+\gamma)(-1+(-1+\gamma)\mu)(1+(2+\gamma)\mu)^2+c^2(1+2\mu)\left(1+\mu\left(2+\gamma^2\mu\right)\right)\right)\Big)\Big/\left(2\left(-1+\gamma^2\right)(-1+(-2+\gamma)\mu)^2(1+(2+\gamma)\mu)^2\right)\Big)\Big)$$

In[319]:=
```
(*simplifying...*)
hb = 2 (-1 + γ^2) (1 + μ (4 - (-4 + γ^2) μ))^2;
hc = (2 (-1 + γ^2) (-1 + (-2 + γ) μ)^2 (1 + (2 + γ) μ)^2);
(*note hb and hc do not depend on c*)
manip2 = -1/Hb (1 + μ)^2 (-2 (-1 + γ) (1 + (2 + γ) μ)^2 +
        2 c (-1 + γ) (1 + (2 + γ) μ)^2 + c^2 (1 + μ (4 (1 + μ) - γ^2 (2 + 3 μ)))) - 1/(1 + 2 μ)^2
    (1 + μ (2 + μ) + ((1 + μ)^2 (1 + 2 μ) (-2 c (-1 + γ) (1 + 2 μ) (-1 + γ μ) (1 + (2 + γ) μ) +
          2 (-1 + γ) (-1 + (-1 + γ) μ) (1 + (2 + γ) μ)^2 + c^2 (1 + 2 μ) (1 + μ (2 + γ^2 μ)))) / Hc +
      √(-(1 + μ)^3 (-1 - μ - ((1 + μ) (2 + 4 μ) (-2 c (-1 + γ) (1 + 2 μ) (-1 + γ μ)
            (1 + (2 + γ) μ) + 2 (-1 + γ) (-1 + (-1 + γ) μ)
            (1 + (2 + γ) μ)^2 + c^2 (1 + 2 μ) (1 + μ (2 + γ^2 μ)))) / Hc)));

(*check*) FullSimplify[manip2 - (kFPositive - kE1MConstr) /. Hb → hb /. Hc -> hc]
```

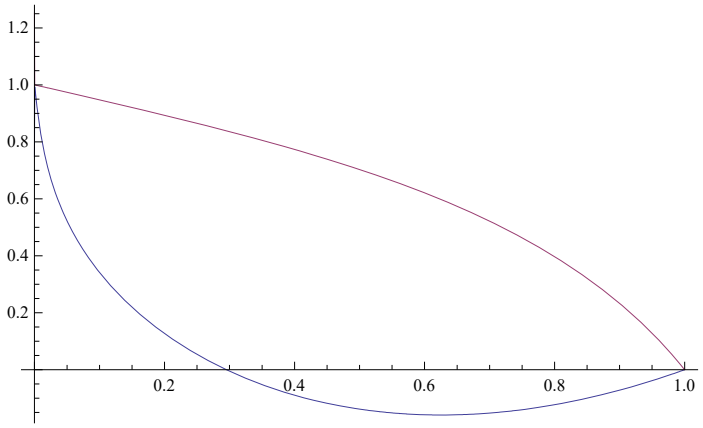
Out[322]=
```
0
```

In[323]:=
```
tmpmanip2 = Solve[manip2 == 0, c];
```

In[324]:=
```
tmpmanip2 /. {Hb → hb, Hc → hc} /. {γ → 0.5, μ → 10}
tmpmanip2 /. {Hb → hb, Hc → hc} /. {μ → 10};
Plot[{%[[1, 1, 2]], %[[2, 1, 2]]}, {γ, 0, 1}]
```

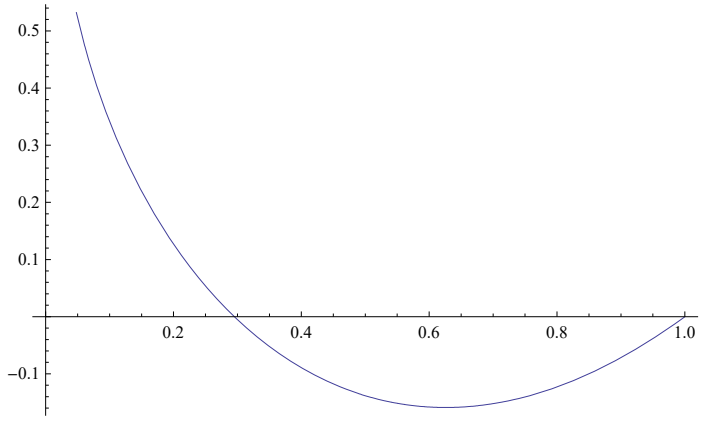
Out[324]=
```
{{c → -0.138325}, {c → 0.702703}, {c → 1.12322 - 1.11727 i}, {c → 1.12322 + 1.11727 i}}
```

Out[326]=



In[327]:=
```
(*Hence, the relevant solutions is the first one*)
cRegime3FPositive = tmpmanip2[[1, 1, 2]] /. {Hb → hb, Hc → hc};
cRegime3FPositive /. {μ → 10};
Plot[{%}, {γ, 0, 1}]
```

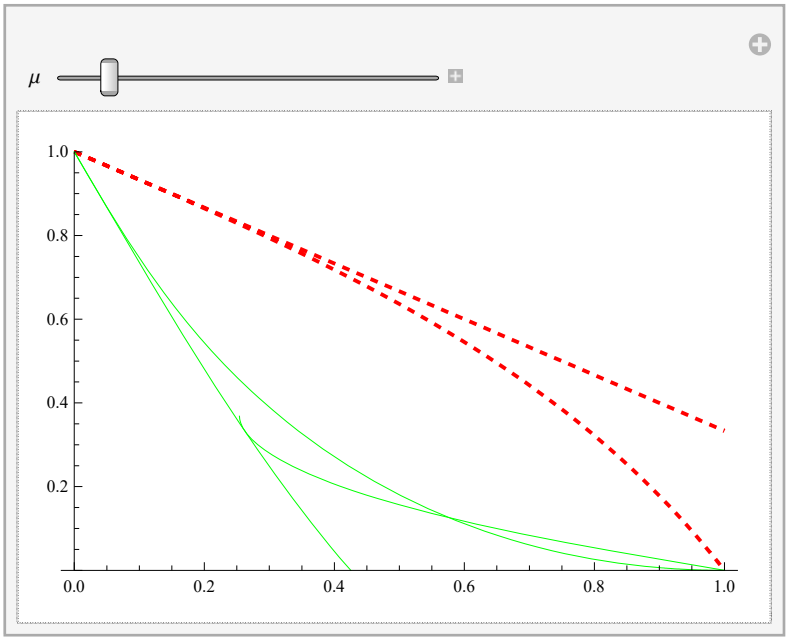
Out[329]=



In[330]:=
```
(*We can now summarize all the compatibility regions when c≤μ/(1+2 μ):
 there always exist a k that satisfies all the
  constraints for c that satisfies the following constraints:

 c<cE1   or  cE1 <c< Min[cRegime3E1, cRegime3FPositive]  *)
```

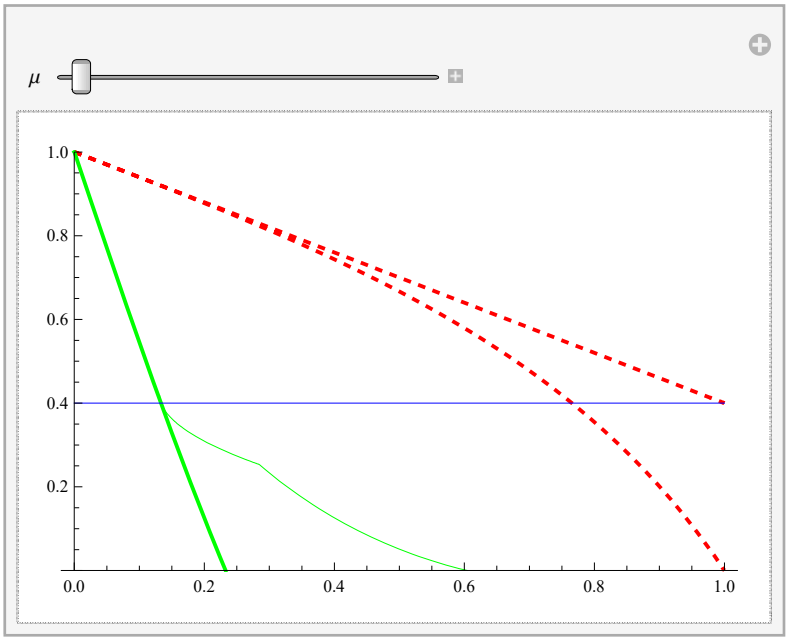
In[331]:=
```
cRegime3E1;
cRegime3FPositive;
cE1;
clim;
cdrast;
Manipulate[Plot[{%, %%, %%%, %%%%, %%%%%}, {γ, 0, 1},
  PlotStyle → {{Red, Thick, Dashed}, {Red, Thick, Dashed}, Green, Green, Green},
  PlotRange → {0, 1}], {{μ, 1}, 0.01, 10}]
 (*The admissibility region is below green lines*)
```

Out[336]=



In[337]:=
```
(*Now can now better graph these thresholds*)
p1m;
Min[cRegime3E1, cRegime3FPositive];
cE1;
clim;
cdrast;
Manipulate[
 Quiet[Plot[{%, %%, %%%, %%%%, %%%%%}, {γ, 0, 1}, PlotStyle → {{Red, Thick, Dashed},
     {Red, Thick, Dashed}, {Green, Thick}, Green, Blue}, PlotRange → {0, 1}]],
 {{μ, 2}, 0.01, 100}](*The admissibility region is below
 any of the green lines*)
```

Out[342]=



In[343]:=
```
(* SUMMARIZING: below the upper envelope of the
  green lines (which is the function γ(μ) in the paper) there
  is always an equilibrium that features Bertrand prices,
even if exclusive contracts are admissible.
  In particular,
- Above the Blue line (which is the Monopolist's price),
the admissibility region is just below the green line and the
  relevant regime for (out-of-equilibrium) exclusivity that grants
  the possibility to have Bertran-marginal prices is the Mixed Regime;
- Below the Blue line, the admissibility region is below the Green and thick
 line (which corresponds to the Mixed regime in exclusivity) and below the
 Green thin line (which corresponds to the Constrained Regime in exclusivity)
*)
```

Partial Coordination

In[344]:=
```
(*When Bertrand prices are not admissible, two cases can realize:
        1) There is no Common
     representation equilibrium and Exclusivity prevails;
        2) There is Common representation equilibrium with
   lower marginal prices than the Bertrand ones.
    We now study case 2) determining the boundary cED(μ) in the main
 text that delimits the region in which Common representation is
 still admissible although at marginal prices below the Bertrand ones
*)
```

In[345]:=
```
(*The general idea, to find this limit boundary, as explained in the text is:

   A) consider the largest possible exclusive
 profit that sellers can leave (out-of-equilibrium) to
 the buyer. Notice that independently of c≤ or >μ/(1+2 μ),
 the maximal k for exclusivity is always PROFITBEequ, which we now use as
  maximal profit that can be granted (out-of-equilibrium)in exclusivity.

   B) consider a marginal price for the seller
 with most incentives to deviate that is on his best-response,
 i.e. firm 1. So that being "out of" BR of firm 2 it must be F2=0.

   C) consider marginal prices such that the buyer is indifferent.

   D) consider a F1 as small as possible in checking no incentives to deviate,
in fact we will check F1=0 as limit case.
*)
```

In[346]:=
```
FullSimplify[ProfitBNE - PROFITBEequ /. p1 -> BR1 /. {F2 → 0, F1 → 0}];
FullSimplify[Solve[% == 0, p2]];
{%, p2NE} /. {c → 0.1, μ → 5, γ → 0.5}
(*clearly the relevant solution is the one that delivers a p2 lower than p2NE*)
p2dpartial = %%[[1, 1, 2]]
p1dpartial = FullSimplify[BR1 /. p2 -> p2dpartial]
```

Out[348]=
$$\{\{\{p2 \to 0.180348\}, \{p2 \to 1.45875\}\}, 0.351634\}$$

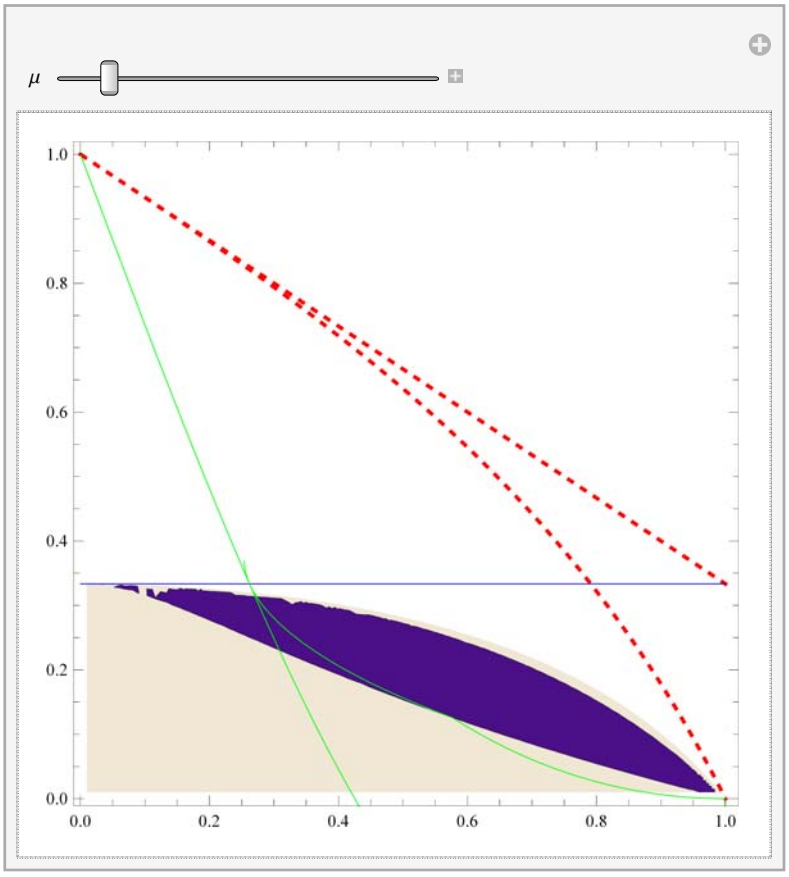
Out[349]=
$$\left(-1-4\mu\,(1+\mu)+\gamma\,(1+\mu)^2+\gamma^2\mu\,(2+3\mu)+\right.$$
$$\sqrt{\left(\left(-1+\gamma^2\right)\,(1+2\mu)^2\left(-(-2+c)\,c+2\left(-1+\gamma^2+(-2+c)\,c\left(-2+\gamma^2\right)\right)\mu+\right.\right.}$$
$$\left.\left.\left.\left(-3+8c-4c^2+3\,(-1+c)^2\gamma^2\right)\mu^2\right)\right)\Big/\left(-1+\mu\left(-4\,(1+\mu)+\gamma^2\,(2+3\mu)\right)\right)$$

Out[350]=
$$\frac{1}{1+2\mu}$$
$$\mu\left(1+\left(\gamma\left(\gamma\,(1+\mu)^2+\sqrt{\left(\left(-1+\gamma^2\right)\,(1+2\mu)^2\left(-(-2+c)\,c+2\left(-1+\gamma^2+(-2+c)\,c\left(-2+\gamma^2\right)\right)\mu+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left(-3+8c-4c^2+3\,(-1+c)^2\gamma^2\right)\mu^2\right)\right)\right)\Big/\left(-1+\mu\left(-4\,(1+\mu)+\gamma^2\,(2+3\mu)\right)\right)\right)$$

In[351]:=
```
(*Now we check the relevant constraints*)
```

In[352]:=
```
(*Check: No deviation to exclusivity for firm 1.*)
FullSimplify[
  E1 /. {p1 -> p1dpartial, p2 -> p2dpartial} /. F1 → 0 /. {p1E → p1Eequcorn, F1E → 0} ];
 (*Notice, the substitution for Exclusive guarantees that in E1
  for that profit we are in fact having Profit1Econstr/.k→kmax*)

clim;
cdrast;
Min[cRegime3E1, cRegime3FPositive];
cE1;
Manipulate[Quiet[Show[ContourPlot[%%%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}],
   Plot[{%%%, %%%%, μ/(1 + 2 μ), %, %%}, {γ, 0, 1}, PlotStyle → {{Red, Thick, Dashed},
     {Red, Thick, Dashed}, Blue, Green, Green}]]], {{μ, 1}, 0, 10}]
```

Out[357]=

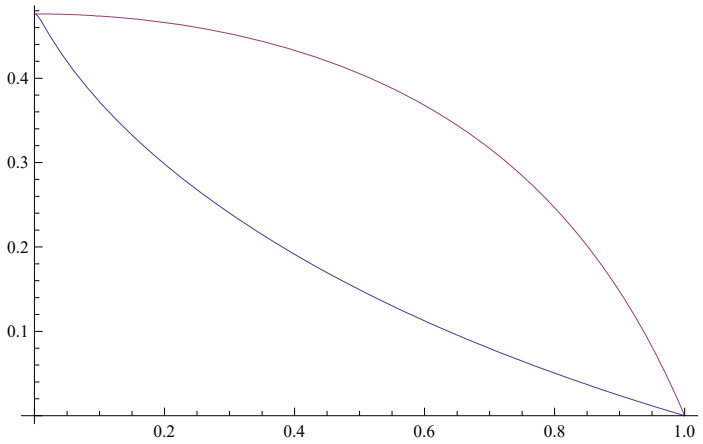


In[358]:=
```
(*To explicitely determine the new
 boundary we proceed with some simplifications.*)
FullSimplify[E1 /. {p1 -> p1dpartial, p2 -> p2dpartial} /. F1 → 0 /.
  {p1E → p1Eequcorn, F1E → 0} ]
```

Out[358]=
$$\left(-\mu\,(1+\mu)\,\left(\gamma^2\,(1+\mu)^2-(1+2\,\mu)^2+\gamma^4\,\mu\,(2+3\,\mu)+2\,\gamma\,\sqrt{\left(\left(-1+\gamma^2\right)\,(1+2\,\mu)^2\,\left(-(-2+c)\,c+2\,\left(-1+\gamma^2+(-2+c)\,c\,\left(-2+\gamma^2\right)\right)\,\mu+\left(-3+8\,c-4\,c^2+3\,(-1+c)^2\,\gamma^2\right)\,\mu^2\right)\right)\right)-c\,(-1+(-2+\gamma)\,\mu)\,(1+(2+\gamma)\,\mu)\,\left(-1+\mu\,\left(-4\,(1+\mu)+\gamma^2\,(2+3\,\mu)\right)\right)+c^2\,(1+2\,\mu)\,\left(-1+\left(-2+\gamma^2\right)\,\mu\right)\,\left(-1+\mu\,\left(-4\,(1+\mu)+\gamma^2\,(2+3\,\mu)\right)\right)\right)\Big/\left(-1+\mu\,\left(-4\,(1+\mu)+\gamma^2\,(2+3\,\mu)\right)\right)^2$$

In[359]:=
```
(*we simplify and then solve this expression for c, equal to zero:*)
tmppartial = Solve[
    (-μ (1 + μ) (γ^2 (1 + μ)^2 - (1 + 2 μ)^2 + γ^4 μ (2 + 3 μ) + 2 γ √((-1 + γ^2) (1 + 2 μ)^2 (-(-2 + c) c + 2 (-1 + γ^2 + (-2 + c) c (-2 + γ^2)) μ + (-3 + 8 c - 4 c^2 + 3 (-1 + c)^2 γ^2) μ^2))) -
      c (-1 + (-2 + γ) μ) (1 + (2 + γ) μ) z + c^2 (1 + 2 μ) (-1 + (-2 + γ^2) μ) z) / z^2 ==
     0, c] /. z -> (-1 + μ (-4 (1 + μ) + γ^2 (2 + 3 μ)));
{tmppartial[[1, 1, 2]], tmppartial[[2, 1, 2]]} /. μ → 10;
Plot[%, {γ, 0, 1}]
```

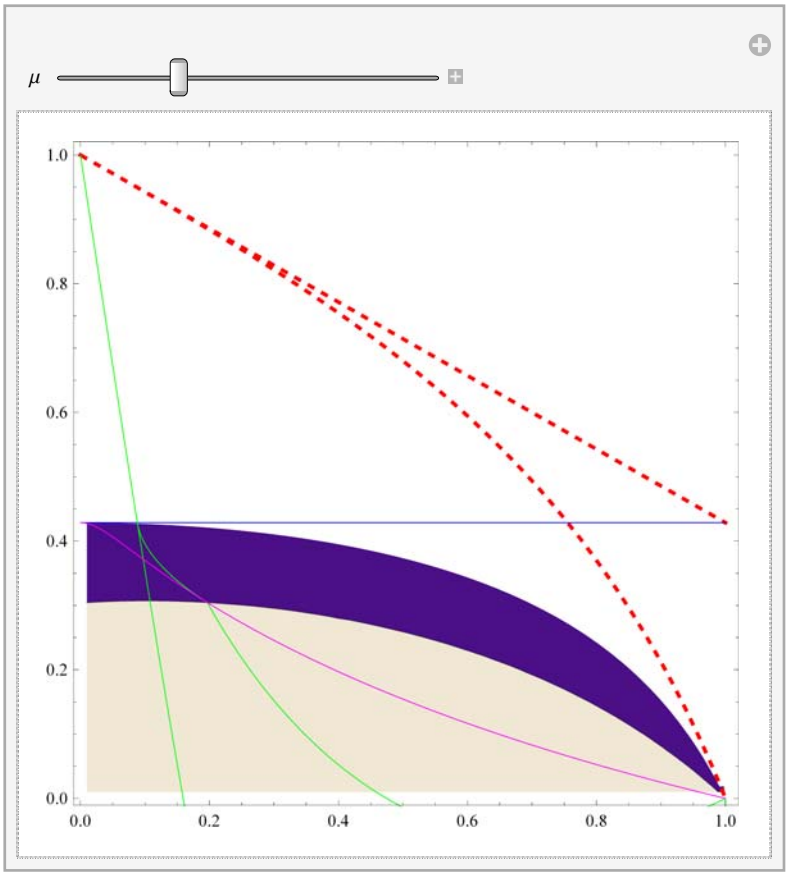
Out[361]=



In[362]:=
```
(*The relevant solution is the first:*)
cE1partial = tmppartial[[1, 1, 2]];
```

In[363]:=
```
(*Check: the solutions for marginal prices must satisfy BR2>p2.*)
BR2 - p2 /. {p1 -> p1dpartial, p2 -> p2dpartial} (*Notice,
 the substitution for Exclusive guarantees that in E1 for
   that pprofit we are in fact having Profit1Econstr/.k→kmax*);
cE1partial;
clim;
cdrast;
Min[cRegime3E1, cRegime3FPositive];
cE1;
Manipulate[
 Quiet[Show[ContourPlot[%%%%%%, {γ, 0.01, 1}, {c, 0.01, 1}, Contours → {0}], Plot[
    {%%%, %%%%, μ/(1 + 2 μ), %, %%, %%%%%}, {γ, 0, 1}, PlotStyle → {{Red, Thick, Dashed},
     {Red, Thick, Dashed}, Blue, Green, Green, Magenta}]]], {{μ, 3}, 0.01, 10}]
```

Out[369]=



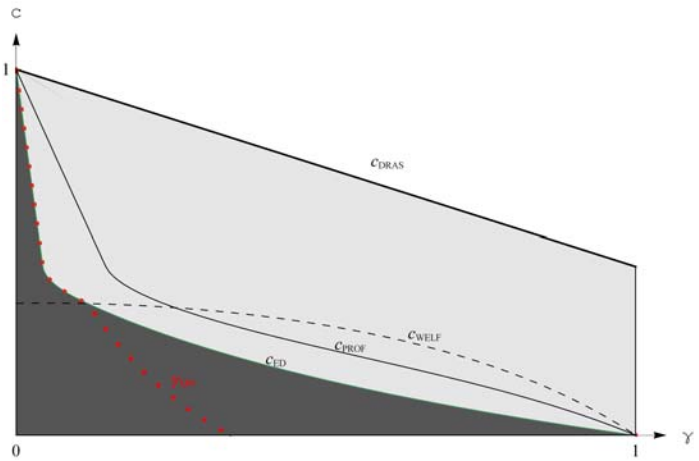
In[370]:=
```
(*This shows that the relevant boundary for
 partial coordination is just the one for E1 constraint,
i.e. the magenta line, which is the boundary c_ED(μ) in the main text.*)
```

Summary and figure 4

In[371]:=
```
(*Hence we can now draw Figure 4 in the main text.*)
```

In[372]:=
```
(*We plot for μ=6 to enhance visibility,
the plot is qualitatively unaffected with other values of μ as seen before.*)
μplot = 6;
Re[cE1partial];
Re[cE1];
Min[Re[cRegime3E1], Re[ cRegime3FPositive]];

vertline = {{1, 0}, {1, p1Em}} /. μ → μplot;
{cdrast, EffectonConsSurplMostCoop, Max[%%, %%%], Max[%%, %%%, %%%%]} /. μ → μplot;
{Profit1NEMostCoop - PROFIT1Eequ > 0 && c < cPPO ||
    Profit1NEMostCoop - PROFIT1Eequcorn > 0 && c < p1Em && c < cPPO} /. μ → μplot;
Figure4 = Show[


  Plot[%%, {γ, 0, 1}, PlotStyle → {{Black, Thickness[0.003]}, {Black, Dashed}},
   AxesLabel → {"γ", "c"},
   Ticks → {{0, 1}, {-1, 1}},
   PlotRange → {{0, 1.05}, {0, 1.1}},
   AxesStyle → Arrowheads[{0.0, 0.02}]
  ],


  Plot[%%, {γ, 0, 1}, PlotStyle → {{Black, Thickness[0.003]},
     {Black, Dashed}, {Red, Thick, AbsoluteDashing[{2, 8}]}, Null},
   Ticks → {{0, 1}, {0, 1}}, PlotRange → {0, 1},
   Filling → {1 → {4}},
   FillingStyle → {Lighter[Gray, 0.8]},
   PlotRange → {0, 1}
  ],

  Plot[%%, {γ, 0, 1}, PlotStyle → {{Black, Thickness[0.003]},
     {Black, Dashed}, {Red, Thick, AbsoluteDashing[{2, 8}]}, Null},
   Ticks → {{0, 1}, {0, 1}}, PlotRange → {0, 1},
   Filling → {4 → Bottom},
   FillingStyle → {Darker[Gray]}, PlotRange → {0, 1}
  ],

  RegionPlot[%, {γ, 0, 1}, {c, 0, 1},
   BoundaryStyle → {Black}, PlotStyle → Opacity[0], Frame → False],


  Graphics[{Text["c_DRAS", {0.6, 0.75}], Text["c_PROF", {0.54, 0.25}],
    Text["c_WELF", {0.66, 0.28}], Text["c_ED", {0.42, 0.21}],
    Text[Style["γ_QN", Red, Bold], {0.27, 0.15}]}],
```

```
  Graphics[{Line[vertline]}]
 ]
```

```
SetDirectory[NotebookDirectory[]];
SetDirectory[ParentDirectory[]];
SetDirectory["Figures"];
Export["Figure4.pdf", Figure4];
```

Out[379]=