Online Appendix 4 of Calzolari, Denicolò and Zanchettin “The demand-boost theory of exclusive dealing,” 2019

The model of Moral hazard model with Competitive Fringe

November 2019

Defining payoffs

In[1]:=
```
(*Buyer's utility*)
u = (q1 + q2) - 1 / (2 x) (q1^2 + q2^2) - (1 / x) γ q1 q2;
ue1 = u /. q2 → 0;
ue2 = u /. q1 → 0;
(*Uncertainty: x is uniform on [0,2],
so that E[x]=1. With the following density function:*)
f = 1 / 2 ;

(*Parameters: c in [0,1], γ in [0,1]*)

P1 = p1 q1 + F1; (*Two part tariff of dominant firm 1*)
P2 = c q2; (*The fringe is pricing competitively*)
```

In[7]:=
```
(*Duopoly*)
tmpq = FullSimplify[Solve[{D[u - P1 - P2, q1] == 0, D[u - P1 - P2, q2] == 0}, {q1, q2}]];
q1d = tmpq[[1, 1, 2]]
q2d = tmpq[[1, 2, 2]]
```

Out[8]=
$$\frac{x\,(-1+p1+\gamma-c\,\gamma)}{-1+\gamma^2}$$

Out[9]=
$$\frac{x\,(-1+c+\gamma-p1\,\gamma)}{-1+\gamma^2}$$

In[10]:=
```
(*Non exclusive payoffs*)
u - p1 q1 - p2 q2 /. {q1 → q1d, q2 → q2d} /. p2 → c;
ProfitBNEx = FullSimplify[%] - (F1 + F2) /. F2 → 0

P1 /. {q1 → q1d, q2 → q2d};
Profit1NE = FullSimplify[Integrate[% f, {x, 0, 2}]]
```

Out[11]=
$$-F1+\frac{-(2+(-2+c)\,c+(-2+p1)\,p1)\,x+2\,(-1+c)\,(-1+p1)\,x\,\gamma}{2\,(-1+\gamma^2)}$$

Out[13]=
$$F1+\frac{p1\,(-1+p1+\gamma-c\,\gamma)}{-1+\gamma^2}$$

In[14]:=
```
(*Monopoly/Exclusive firm 1*)
q1e = FullSimplify[Solve[D[ue1 - p1 q1, q1] == 0, q1]][[1, 1, 2]]

ue1 - p1 q1 - F1 /. q1 → q1e;
ProfitBE1x = FullSimplify[%]

P1 /. q1 → q1e;
Profit1Ex = FullSimplify[%]
```

Out[14]=
$$x-p1\,x$$

Out[16]=
$$-F1+\frac{1}{2}\,(-1+p1)^2\,x$$

Out[18]=
$$F1-(-1+p1)\,p1\,x$$

In[19]:=
```
(*Expected profits for the buyer in
 Exclusivity (E) and Common Representation (NE).*)
```

In[20]:=
```
xhatE1 = Solve[ProfitBE1x == 0, x][[1, 1, 2]]
(*Types x below this face losses, those above gains.*)

PROFITBE1 = FullSimplify[Integrate[ProfitBE1x * f, {x, xhatE1, 2}] +
    Integrate[ProfitBE1x * f, {x, 0, xhatE1}] (1 + λ)]
```

Out[20]=
$$\frac{2\,F1}{(-1+p1)^2}$$

Out[21]=
$$\frac{1}{2}\left((-1+p1)^2+F1\left(-2-\frac{F1\,\lambda}{(-1+p1)^2}\right)\right)$$

In[22]:=
```
(*Monopoly/Exclusive with the fringe*)
q2e = FullSimplify[Solve[D[ue2 - P2, q2] == 0, q2]][[1, 1, 2]]

FullSimplify[ue2 - P2 /. q2 → q2e];
ProfitBE2x = FullSimplify[%]
```

Out[22]= $x - c\,x$

Out[24]= $\frac{1}{2}(-1+c)^2 x$

In[25]:=
```
xhatE2 = Solve[ProfitBE2x == 0, x][[1, 1, 2]]
(*Types x below this face losses, those above gains.*)

PROFITBE2 = FullSimplify[Integrate[ProfitBE2x * f, {x, xhatE2, 2}] +
   Integrate[ProfitBE2x * f, {x, 0, xhatE2}] (1 + λ)]
```

Out[25]= 0

Out[26]= $\frac{1}{2}(-1+c)^2$

In[27]:=
```
(*Non-Exclusivity: *)
xhatNE = FullSimplify[Solve[ProfitBNEx == 0, x]][[1, 1, 2]]
(*Types x below this face losses, those above gains.*)
PROFITBNE = Simplify[Integrate[ProfitBNEx * f, {x, xhatNE, 2}] +
   Integrate[ProfitBNEx * f, {x, 0, xhatNE}] (1 + λ)]
```

Out[27]= $-\frac{2\,F1\,(-1+\gamma^2)}{2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma}$

Out[28]= $\frac{-(2-2\,c+c^2-F1-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma+F1\,\gamma^2)^2+F1^2\,(-1+\gamma^2)^2\,(1+\lambda)}{2\,(2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma)\,(-1+\gamma^2)}$

In[29]:=
```
Needs["PlotLegends`"]
```

Analysis: Firms compete under Exclusivity

In[30]:=
```
(*The dominant firm 1 wins the competition for exclusivity
 with a marginal price which is either the marginal cost c of
 the rival firms in the fringe or directly the monopoly price*)
```

In[31]:=
```
V0E2 = PROFITBE2 /. {F2 → 0, p2 → c};
tmp = FullSimplify[Solve[PROFITBE1 == V0E2, F1]]
tmp /. {c → 0.2, λ → 1, p1 → 0.1} (*a numerical example*)
```

Out[32]=
$$\left\{\left\{F1 \to -\frac{1+(-2+p1)\,p1+\sqrt{(-1+p1)^4+(-1+p1)^2\,(-c+p1)\,(-2+c+p1)\,\lambda}}{\lambda}\right\},\right.$$
$$\left.\left\{F1 \to \frac{-1-(-2+p1)\,p1+\sqrt{(-1+p1)^4+(-1+p1)^2\,(-c+p1)\,(-2+c+p1)\,\lambda}}{\lambda}\right\}\right\}$$

Out[33]=
```
{{F1 → -1.70095}, {F1 → 0.0809545}}
```

In[34]:=
```
(*The first solution is not relevant. Furthermore, as noted in text,
it must be F1>0, whenever expressions deliver an F1<
  0 we are in fact in a corner solution that requires F1=0.*)
F1E = FullSimplify[tmp[[2, 1, 2]] ]
% /. {c → 0.2, λ → 1, p1 → 0.1} (*a check*)
```

Out[34]=
$$\frac{-1-(-2+p1)\,p1+\sqrt{(-1+p1)^4+(-1+p1)^2\,(-c+p1)\,(-2+c+p1)\,\lambda}}{\lambda}$$

Out[35]= 0.0809545

In[36]:=
```
(*Now we show that the non negative fee is guaranteed by p1≤c.
  Furthermore, we have F1E→0 as λ→Infinity and as λ→
   0 F1E tends to the fixed fee that extracts the entire surplus from the buyer,
net of the reservation payoff V0=V0E2*)
Reduce[{F1E ≥ 0, λ > 0, c ≤ 1, p1 ≤ 1}, Reals]
Limit[F1E, λ → Infinity]
FullSimplify[Limit[D[Numerator[F1E], λ] / (D[Denominator[F1E], λ]), λ → 0], p1 ≤ 1]
FullSimplify[% + V0E2]
```

Out[36]= $(p1 < 1 \&\& p1 \le c \le 1 \&\& \lambda > 0) \,||\, (p1 == 1 \&\& c \le 1 \&\& \lambda > 0)$

Out[37]= 0

Out[38]=
$$\frac{1}{2}\,(-c+p1)\,(-2+c+p1)$$

Out[39]=
$$\frac{1}{2}\,(-1+p1)^2$$

In[40]:=
```
(*We verify that the critical type xhatE1 converges to 0 when λ→Infinity*)
FullSimplify[xhatE1 /. F1 → F1E]
Limit[%, λ → Infinity]
```

Out[40]=
$$\frac{2\left(-1+\frac{\sqrt{(-1+p1)^4+(-1+p1)^2\,(-c+p1)\,(-2+c+p1)\,\lambda}}{(-1+p1)^2}\right)}{\lambda}$$

Out[41]= 0

In[42]:=
```
(*Now we define profits*)
P1 /. q1 → q1e /. F1 → F1E;
PROFIT1E1 = FullSimplify[Integrate[ % f, {x, 0, 2}] ,  p1 ≤ 1]
```

Out[43]=
$$-\frac{(-1+p1)\left(-1+p1+p1\,\lambda+\sqrt{(-1+p1)^2+(-c+p1)\,(-2+c+p1)\,\lambda}\right)}{\lambda}$$

In[44]:=
```
(*Some checks*)
Limit[PROFIT1E1, λ → Infinity]
(*Profits tend to linear pricing profits when λ→Infinity*)
```

Out[44]= $-(-1+p1)\,p1$

In[45]:=
```
(*For λ→0, profits tend to the marginal contribution of the firm*)
FullSimplify[
 Limit[D[Numerator[PROFIT1E1], λ] / (D[Denominator[PROFIT1E1], λ]), λ → 0], p1 ≤ 1]
(*The definition of the marginal contribution:*)
FullSimplify[Integrate[FullSimplify[ue1 /. q1 -> q1e] f, {x, 0, 2}] - V0E2]
```

Out[45]= $c - \frac{c^2}{2} - \frac{p1^2}{2}$

Out[46]= $c - \frac{c^2}{2} - \frac{p1^2}{2}$

In[47]:=
```
(*Now we determine the optimal marginal price in exclusivity*)
tmp = Solve[D[PROFIT1E1, p1] == 0, p1];
% /. {c → 0.2, λ → 1}
Chop[%% /. {c → 0.2, λ → 1}]
(*this example shows that the correct solution is the first one. Note,
machine precision leaves an almost "zero" imaginary
 part which we eliminate by taking the real part only.*)
p1E = Re[tmp[[1, 1, 2]]];
p1E /. {c → 0.2, λ → 1}(*a counter-check*)
```

Out[48]= $\left\{\left\{p1 \to 0.0827441 + 4.30211 \times 10^{-16}\, i\right\}, \{p1 \to 0.665267 + 0.0992329\, i\}, \{p1 \to 0.665267 - 0.0992329\, i\}, \left\{p1 \to 1.58672 - 1.73472 \times 10^{-16}\, i\right\}\right\}$

Out[49]= $\{\{p1 \to 0.0827441\}, \{p1 \to 0.665267 + 0.0992329\, i\}, \{p1 \to 0.665267 - 0.0992329\, i\}, \{p1 \to 1.58672\}\}$

Out[51]= 0.0827441

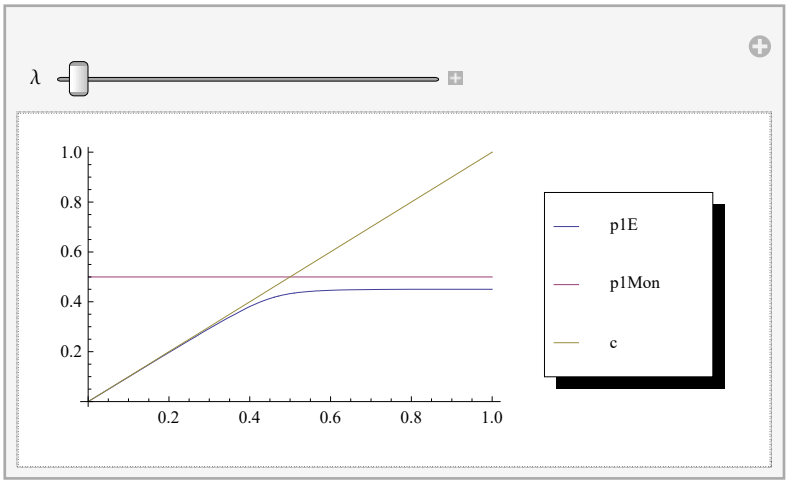
In[52]:=
```
(*The optimal marginal price converges
 to the monopoly price when λ is very large:*)

(*we determine the monopolist price first:*)
p1Mon = Solve[D[p1 Integrate[q1e f, {x, 0, 2}], p1] == 0, p1] [[1, 1, 2]]

(*Now comparison in an example*)
p1E;
Manipulate[Plot[{%, p1Mon, c}, {c, 0.001, 1}, PlotLegend → {"p1E", "p1Mon", "c"},
  LegendPosition → {1.1, -0.4}], {{λ, 100}, 0.001, 10 000}]
(*we need to exclude both c and λ too close to the extreme cases of zero*)
```

Out[52]=
$$\frac{1}{2}$$

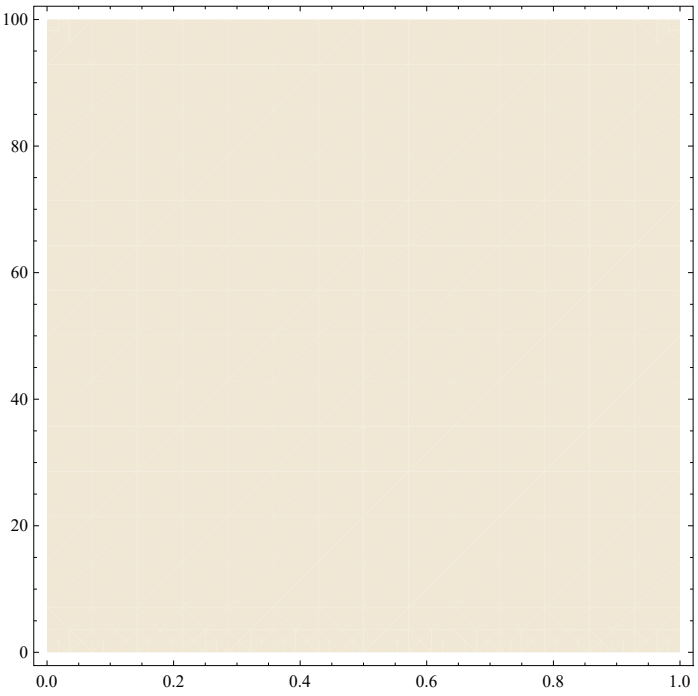
Out[54]=



In[55]:=
```
(*Note that the previous analysis shows that this is the optimal
 price for any c independently of the comparison with p1Mon*)
```

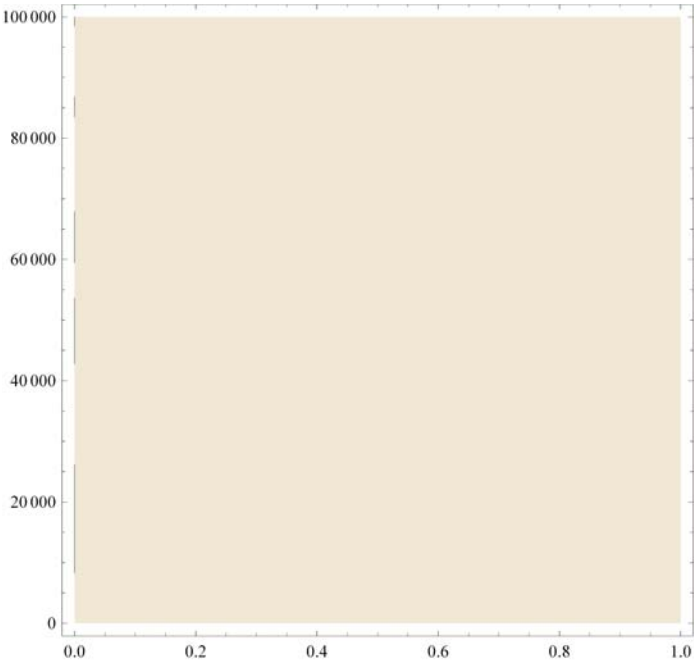
In[56]:=
```
(*and the price is always lower than the monopolist'
 s price for any λ: as shown in this simple check*)
p1Mon - p1E;
ContourPlot[%, {c, 0, 1}, {λ, 0, 100}, Contours → {0}]
```

Out[57]=



In[58]:=
```
(*We now show that the price is always lower than c*)
ContourPlot[c - p1E, {c, 0, 1}, {λ, 0, 100 000}, Contours → {0}]
```

Out[58]=



In[59]:=
```
(* Being the price lower than c,
it implies that the fixed fee is always postive and we
 do not have corner solutions in the case of exclusivity.*)
```

In[60]:=
```
(*We can now calculate the dominant firm's profit under exclusivity*)
PROFIT1Eopt1 = PROFIT1E1 /. p1 → p1E;
```

Analysis: Competition when exclusivity banned

In[61]:=
```
FullSimplify[Solve[{D[PROFITBNE, F1] == 0, D[PROFITBNE, p1] == 0}, {F1, p1}]]
```

Out[61]=
$$\left\{\left\{F1 \to -\frac{(-1+c)^2}{\lambda},\ p1 \to 1+(-1+c)\,\gamma\right\}\right\}$$

In[62]:=
```
{{F1 → (1 - c^2 - 2 p2 + 2 c p2 + F2 λ - γ^2 ((-1 + p2)^2 + F2 λ))/((-1 + γ^2) λ), p1 → 1 + γ (-1 + p2)}}
```

Out[62]=
$$\left\{\left\{F1 \to \frac{1-c^2-2\,p2+2\,c\,p2+F2\,\lambda-\gamma^2\,((-1+p2)^2+F2\,\lambda)}{(-1+\gamma^2)\,\lambda},\ p1 \to 1+(-1+p2)\,\gamma\right\}\right\}$$

In[63]:=
```
(*Indifference conditions of the buyer and the associated fixed fee:*)
FullSimplify[PROFITBE2 - PROFITBNE]
FullSimplify[Solve[% == 0, F1]]
% /. {λ → 1, γ → 0.5, c → 0.1, p1 → 0.1}
F1NEtmp = %%[[1, 1, 2]];
```

Out[63]=
$$\frac{1}{2}\left((-1+c)^2-\frac{-(2-2\,c+c^2-F1-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma+F1\,\gamma^2)^2+F1^2\,(-1+\gamma^2)^2\,(1+\lambda)}{(2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma)\,(-1+\gamma^2)}\right)$$

Out[64]=
$$\left\{\left\{F1 \to \frac{1}{(-1+\gamma^2)^2\,\lambda}\left(c^2\left(-1+\gamma^2\right)-2\,c\,(1+(-1+p1)\,\gamma)\left(-1+\gamma^2\right)+\left(-1+\gamma^2\right)(2-2\,\gamma+p1\,(-2+p1+2\,\gamma))+\sqrt{\left((2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma)\left(-1+\gamma^2\right)^2\left(2-2\,c+c^2-2\,p1+p1^2-2\,\gamma+2\,c\,\gamma+2\,p1\,\gamma-2\,c\,p1\,\gamma+(-1+p1+\gamma-c\,\gamma)^2\,\lambda\right)\right)}\right)\right\},\ \left\{F1 \to \frac{1}{(-1+\gamma^2)^2\,\lambda}\left(c^2\left(-1+\gamma^2\right)-2\,c\,(1+(-1+p1)\,\gamma)\left(-1+\gamma^2\right)+\left(-1+\gamma^2\right)(2-2\,\gamma+p1\,(-2+p1+2\,\gamma))-\sqrt{\left((2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma)\left(-1+\gamma^2\right)^2\left(2-2\,c+c^2-2\,p1+p1^2-2\,\gamma+2\,c\,\gamma+2\,p1\,\gamma-2\,c\,p1\,\gamma+(-1+p1+\gamma-c\,\gamma)^2\,\lambda\right)\right)}\right)\right\}\right\}$$

Out[65]=
$$\{\{F1 \to 0.127477\},\ \{F1 \to -2.28748\}\}$$

In[67]:=
```
Profit1NE /. F1 → F1NEtmp;
Simplify[D[%, p1]]
```

Out[68]=
$$\frac{1}{-1+\gamma^2}\Big(-1+2\,p1+\gamma-c\,\gamma+1/\lambda\Big(-2\,c\,\gamma+2\,(-1+p1+\gamma)+\Big((-1+p1+\gamma-c\,\gamma)\,\left(-1+\gamma^2\right)\,\left(2\,p1^2\,(1+\lambda)+4\,p1\,(-1+\gamma)\,(1+\lambda)+(-1+\gamma)\,(-4+(-3+\gamma)\,\lambda)+c^2\,\left(2+\lambda+\gamma^2\,\lambda\right)-2\,c\,\left(2+\lambda+\gamma^2\,\lambda+2\,(-1+p1)\,\gamma\,(1+\lambda)\right)\right)\Big/\left(\sqrt{\left((2-2\,c+c^2-2\,p1+p1^2-2\,(-1+c)\,(-1+p1)\,\gamma\right)\,\left(-1+\gamma^2\right)^2\,\left(2-2\,c+c^2-2\,p1+p1^2-2\,\gamma+2\,c\,\gamma+2\,p1\,\gamma-2\,c\,p1\,\gamma+(-1+p1+\gamma-c\,\gamma)^2\,\lambda\right)}\right)\Big)\Big)$$

In[69]:=
```
(*Taking the relevant part of the FOC:*)
Collect[2 p1 (1 + λ) + (-1 + γ) (2 + λ) - c γ (2 + λ) +
    ((-1 + p1 + γ - c γ) (-1 + γ^2) (2 p1^2 (1 + λ) + 4 p1 (-1 + γ) (1 + λ) + (-1 + γ)
          (-4 + (-3 + γ) λ) + c^2 (2 + λ + γ^2 λ) - 2 c (2 + λ + γ^2 λ + 2 (-1 + p1) γ (1 + λ))))/
     (Sqrt[((2 - 2 c + c^2 - 2 p1 + p1^2 - 2 (-1 + c) (-1 + p1) γ) (-1 + γ^2)^2
          (2 - 2 c + c^2 - 2 p1 + p1^2 - 2 γ + 2 c γ + 2 p1 γ - 2 c p1 γ + (-1 + p1 + γ - c γ)^2 λ))])), p1];
tmpp1 = Solve[% == 0, p1];
% /. {λ → 1, γ → 0.5, c → 0.1}
p1NE = %%[[3, 1, 2]];
% /. {λ → 1, γ → 0.5, c → 0.1}
F1NE = F1NEtmp /. p1 → p1NE;
% /. {λ → 1, γ → 0.5, c → 0.1}
(*this example shows that the correct solution is the first one. Note,
machine precision leaves an almost "zero" imaginary
 part which we eliminate by taking the real part only.*)
p1NE = Re[p1NE];
F1NE = Re[F1NE];
{p1NE, F1NE} /. {λ → 1, γ → 0.5, c → 0.1}
PROFIT1NE = Profit1NE /. {p1 -> p1NE, F1 -> F1NE};
PROFIT1NE /. {λ → 10, γ → 0.5, c → 0.5}
```

Out[71]=
$$\left\{\left\{p1 \to -0.25646 + 5.55112 \times 10^{-17}\,i\right\},\ \left\{p1 \to 1.57767 - 2.77556 \times 10^{-17}\,i\right\},\ \{p1 \to 0.32879 + 0.\,i\}\right\}$$

Out[73]=
$$0.32879 + 0.\,i$$

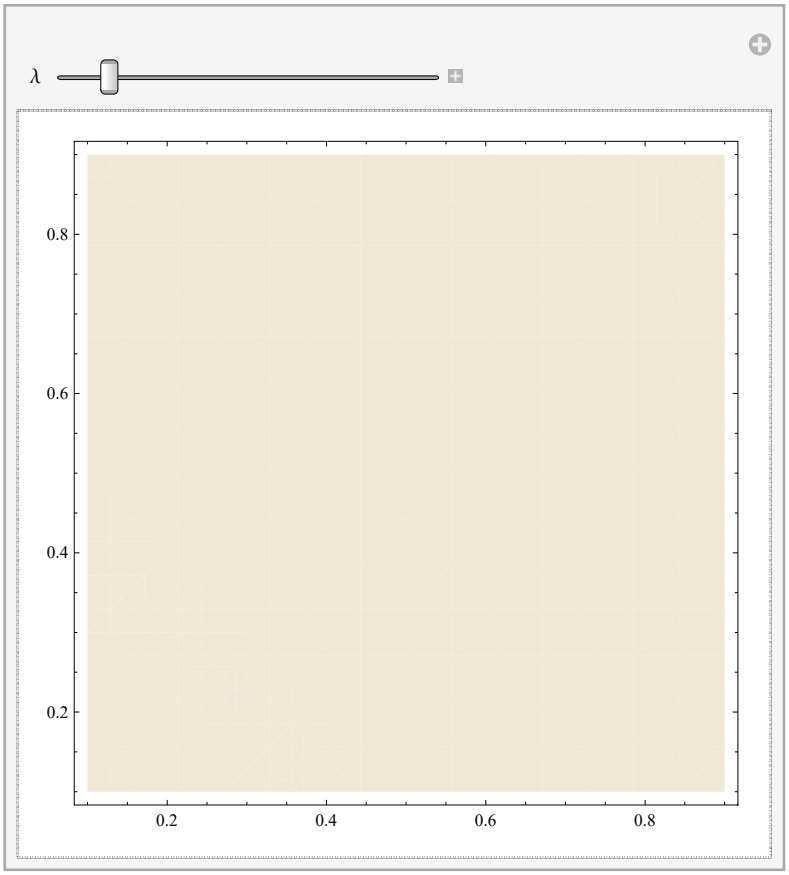
Out[75]=
$$0.0320363 + 0.\,i$$

Out[78]=
$$\{0.32879,\ 0.0320363\}$$

Out[80]=
$$0.255995$$

In[81]:=
```
(*Checking fixed fees are positive*)
F1NE;
Manipulate[
 ContourPlot[%, {c, 0.1, 0.9}, {γ, 0.1, 0.9}, Contours → {0}], {{λ, 1}, 0, 10}]
```

Out[82]=

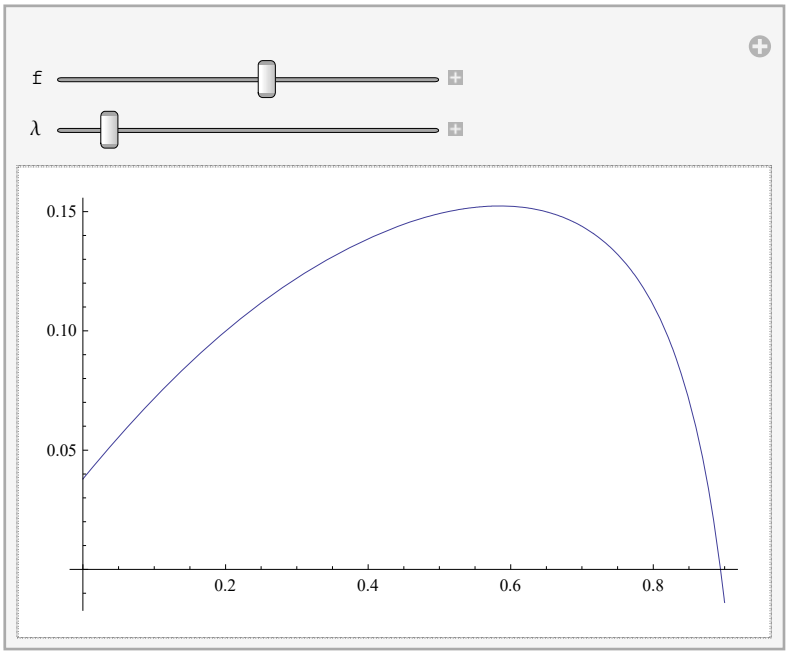


Comparisons: Exclusivity and Non-Exclusivity

In[83]:=
```
(*First we compare profits of Firm 1 seeing when exclusivity is profitable*)
```

In[84]:=
```
{PROFIT1Eopt1 - PROFIT1NE};
Manipulate[Plot[%, {γ, 0, 0.9}], {{c, 0.5}, 0, 0.9}, {{λ, 1}, 0.01, 10}]
```

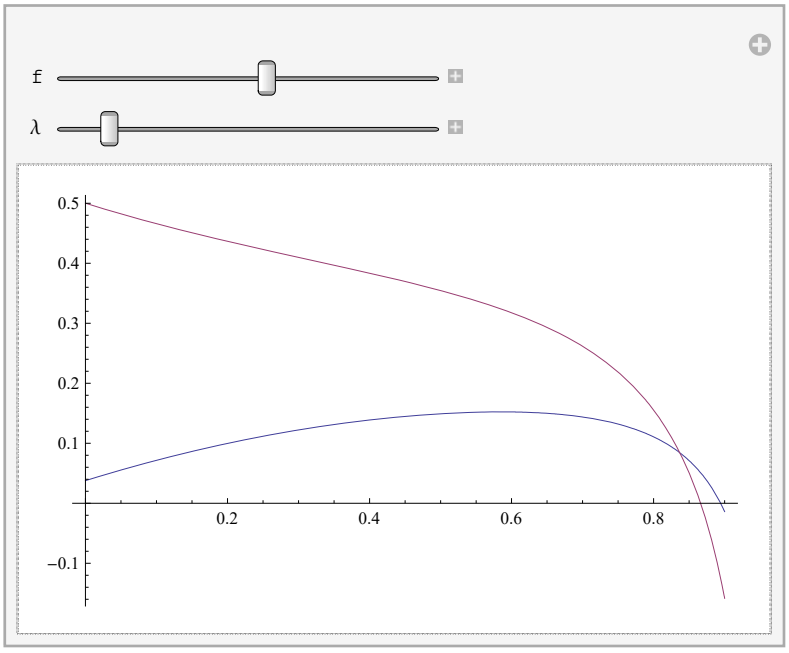
Out[85]=



In[86]:=
```
(*This shows that as expected, for given γ and λ,
exclusivity reduces profits when c small and then it increases it.
  Note that when the difference decreases in γ it is in
 fact associated with non positive q2 and it is irrelevant.*)
```

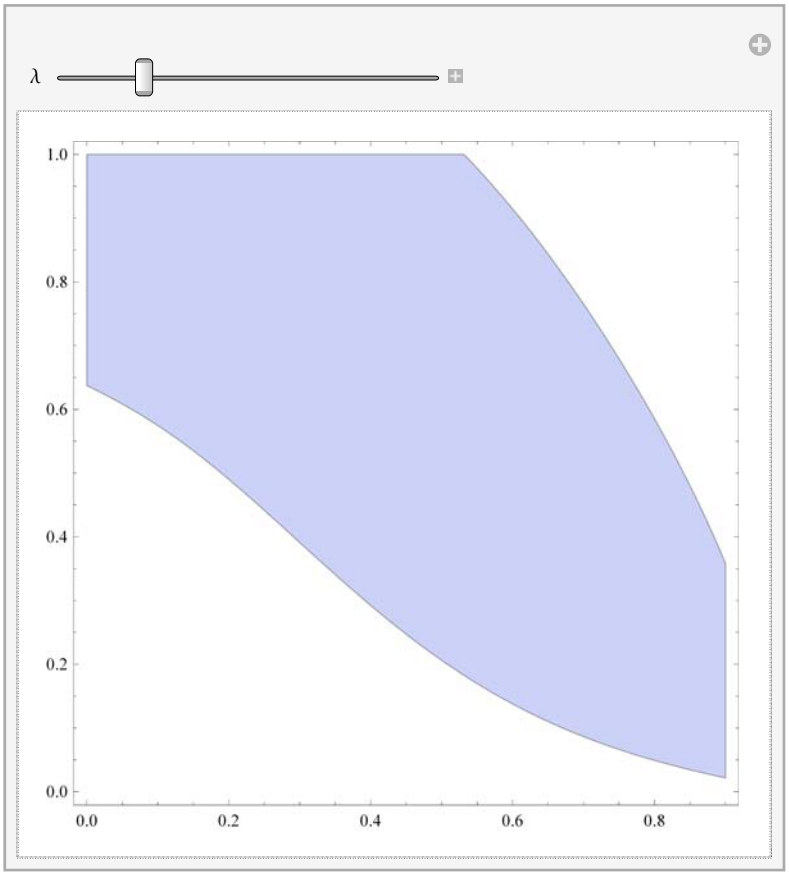
In[87]:=
```
{PROFIT1Eopt1 - PROFIT1NE, q2d /. {p2 → c, p1 → p1NE, x → 1}};
Manipulate[Plot[%, {γ, 0, 0.9}], {{c, 0.5}, 0, 0.9}, {{λ, 1}, 0.01, 10}]
```

Out[88]=



In[89]:=
```
PROFIT1Eopt1 - PROFIT1NE > 0;
Manipulate[RegionPlot[%, {γ, 0, 0.9}, {c, 0, 1}], {{λ, 2}, 0.01, 10}]
```

Out[90]=



In[91]:=
```
(*This shows that the relevant part of the comparison
 of the profits with and without exclusivity is when the
 difference between the two profits is increasing. In this case,
for low γ Non-Exclusive is better and then Excluivity is better for higher γ.*)
```