Data Science Training Webinars
Spatiotemporal Simulation

Application of Garin-Lowry Model in Simulating Urban Population and Employment Patterns

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Wuhan University

China Data Institute

Future Data Lab

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- Solving a system of linear equations

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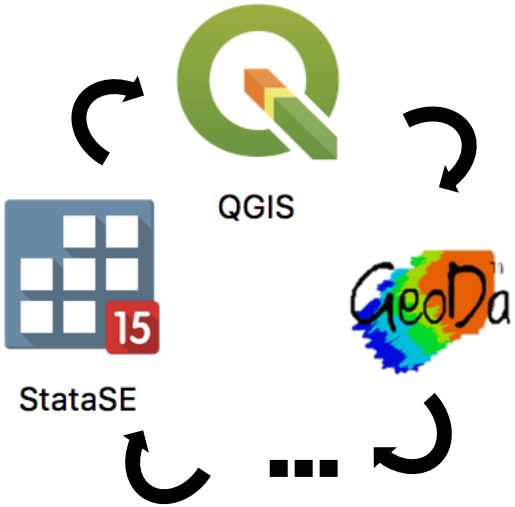
- Step by step workflow execution

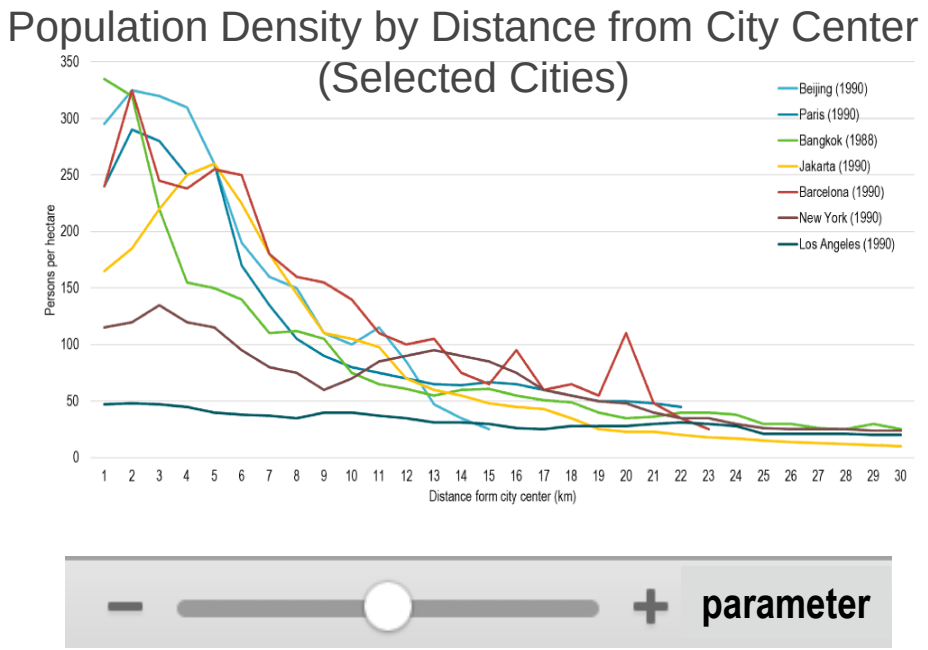
☐ Conclusion and discussion

Research Background

Data

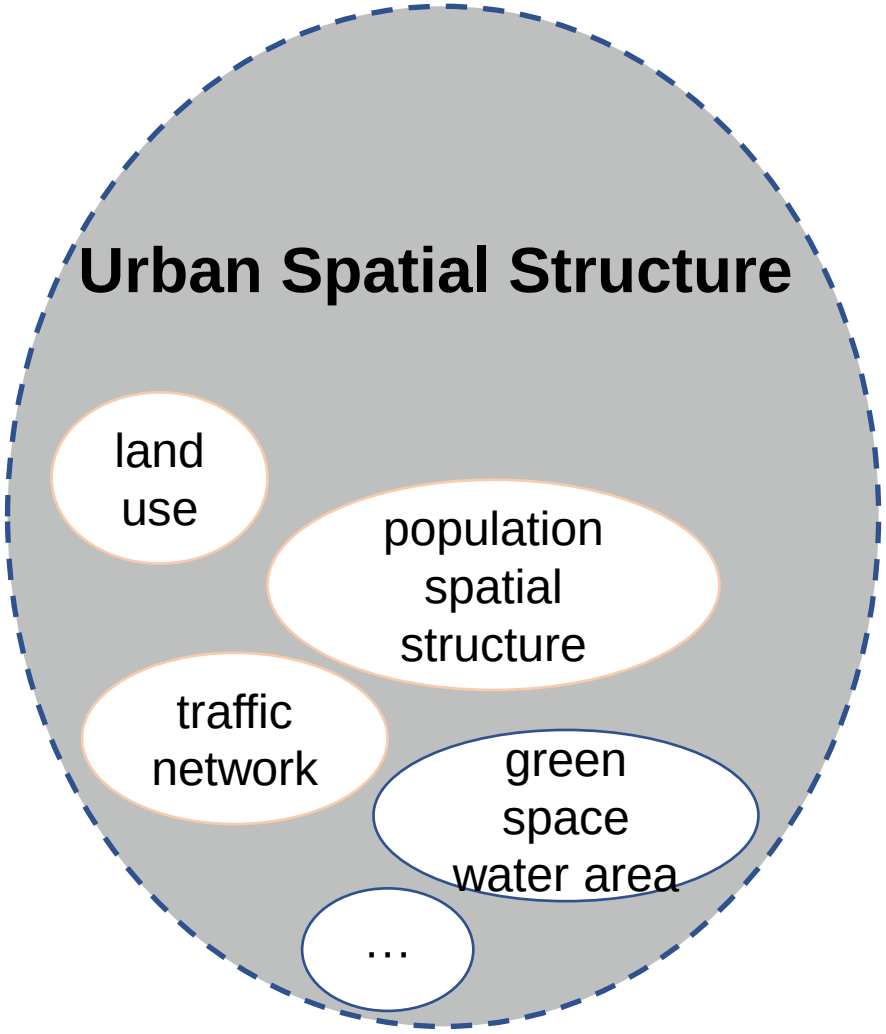


Technology



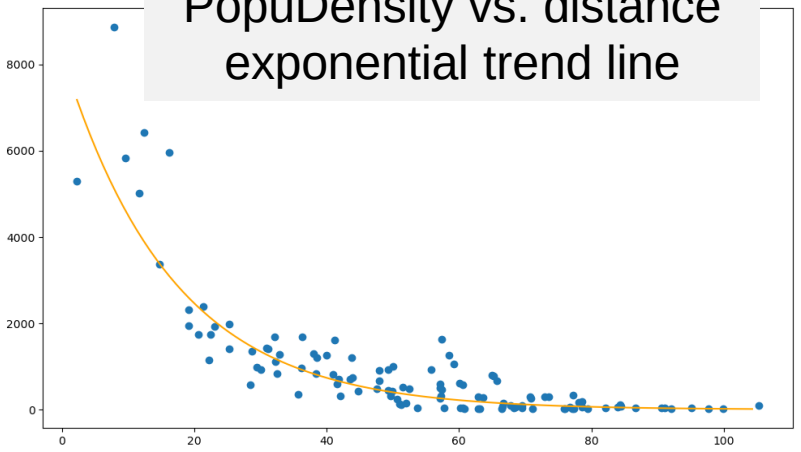
Simulation comparison of different scenarios

Literature Review

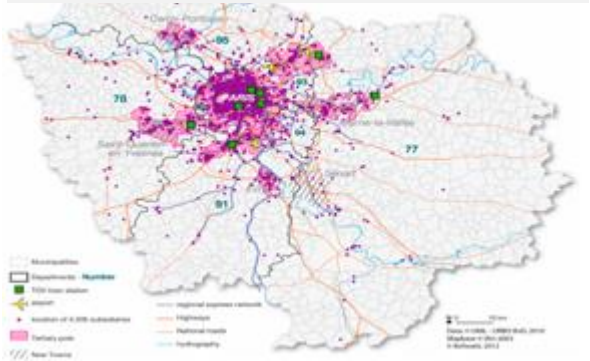


☐ Static urban spatial structure

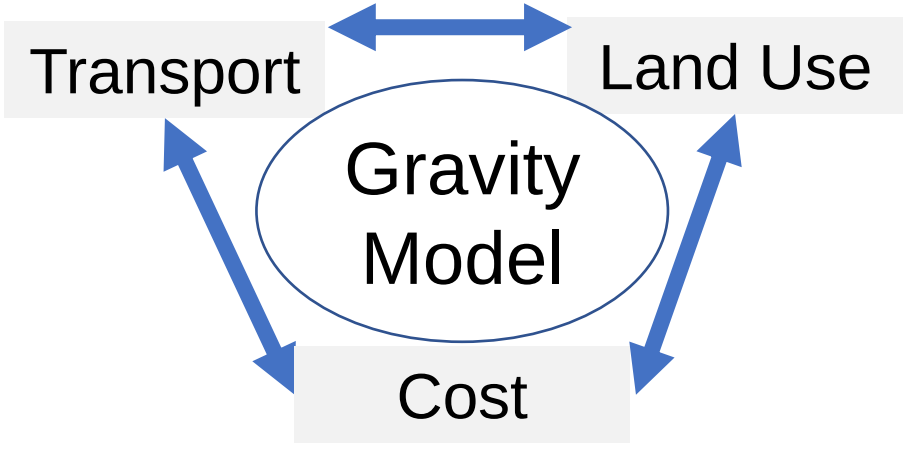
PopuDensity vs. distance exponential trend line



Location of multinational subsidiaries in the Paris



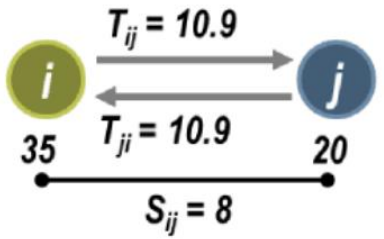
☐ Dynamic urban spatial structure



Literature Review

☐ Spatial Interaction Model

$$T_{ij} = \frac{V_i * W_j}{S_{ij}^2}$$



Gravity Model

☐ Lowry Model

- **A urban simulating model** of the spatial organization of human activities within a metropolitan area.
- Predict changes in **metropolitan form**:
  - the pattern of **employment**
  - the growth of **population**
  - the efficiency of transportation system

nonlinear optimization

☐ Garin-Lowry Model

$$y_2 = y_1 A_1 + x_2$$

From unique case studies

to multi-scenario simulations

Line of Thought

Data Collection → Data Curation → Model Simulation

- Study area：N tracts & Connected

- Basic employment distribution

- Travel time

- Manually set the parameter variation range

Visualization

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  - The Garin-Lowry model
  - Solving a system of linear equations
- **Conclusion and discussion**

The Garin-Lowry Model

☐ Basic Hypothesis

Population and employment distributions **interact** with each other and are **interdependent**.

I. Population follows employment

Residential
Population
population

Place of
Employment
employment for staff

II. Employment follows population

The Garin-Lowry Model

Basic Hypothesis

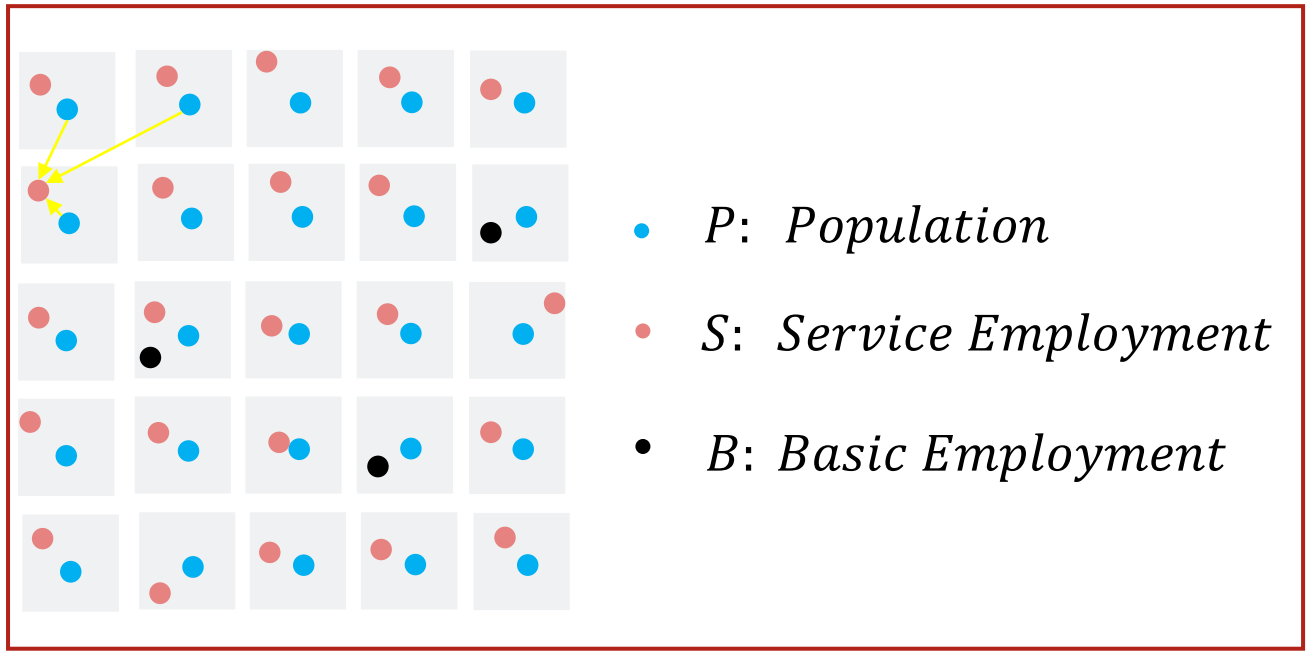
- Baisc Employment: independent
- Nonbasic/Service Employment: follows the distribution of residential population.

Interaction between population and employment distributions in a city

The Garin-Lowry Model

$$\boldsymbol{S = TP} \qquad S_i = e\sum_{j=1}^{n}(P_j t_{ij}) = e\sum_{j=1}^{n}[P_j\left(d_{ij}^{-\alpha} / \sum_{l=1}^{n} d_{lj}^{-\alpha}\right)]$$

$$\boldsymbol{P = GS + GB} \qquad P_i = h\sum_{i=1}^{n}(E_i g_{ij}) = h\sum_{i=1}^{n}[(B_i + S_i)\left(d_{ij}^{-\beta} / \sum_{k=1}^{n} d_{ik}^{-\beta}\right)]$$



$$e = \frac{Service\ Employment}{Population}$$

$$h = \frac{Population}{Employment}$$

d_{ij}: the distance between i and j, $(i \mid j = 1,2,\ldots,n)$

∂ : the distance friction coefficient $(resident - service)$

β: the distance friction coefficient $(resident - workplace)$

The Garin-Lowry Model

$$\boldsymbol{S = TP} \qquad S_i = e\sum_{j=1}^{n}(P_j t_{ij}) = e\sum_{j=1}^{n}[P_j\left(d_{ij}^{-\alpha} / \sum_{l=1}^{n} d_{lj}^{-\alpha}\right)]$$

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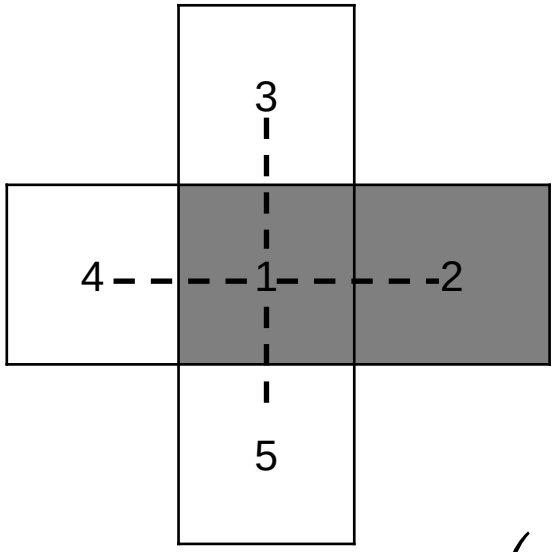

- *B1 = 1, B2 = B3 = B4 = B5 = 0*
- d_{1j}=1, d_{ii}=0.25
- *h=2.0 , e=0.3, α = 1.0, β = 1.0*

$$S_1 = 0.3\left(\frac{d_{11}^{-1}}{d_{11}^{-1} + d_{21}^{-1} + d_{31}^{-1} + d_{41}^{-1} + d_{51}^{-1}} P_1 + \frac{d_{12}^{-1}}{d_{12}^{-1} + d_{22}^{-1} + d_{32}^{-1} + d_{42}^{-1} + d_{52}^{-1}} P_2 * 4\right)$$

Solving A System of Linear Equations

LU-factorization

- the forward substitution algorithm

  is used to solve the system

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

- the back substitution algorithm

  is used to solve the system

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\begin{cases} S_1 = 0.1500P_1 + 0.1846P_2 \\ S_2 = 0.0375P_1 + 0.2538P_2 \\ P_1 = S_1 + 1.2308S_2 + 1 \\ P_2 = 0.25S_1 + 1.6823S_2 + 0.25 \end{cases}$$

$$\boldsymbol{Ax = b}$$

$$\begin{bmatrix} -1 & 0 & 0.15 & 0.1846 \\ 0 & -1 & 0.0375 & 0.2538 \\ 1 & 1.2308 & -1 & 0 \\ 0.25 & 1.6823 & 0 & -1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -0.25 \end{bmatrix}$$

$$\boldsymbol{A = LU}$$

$$\boldsymbol{Lz = b}$$

$$\boldsymbol{Ux = z}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.25 & 1 & 0 & 0 \\ -1 & 0.7316 & 1 & 0 \\ 0 & -0.5944 & -0.0681 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0.15 & 0.1846 \\ 0 & 1.6823 & 0.0375 & -0.9539 \\ 0 & 0 & -0.8774 & 0.8825 \\ 0 & 0 & 0 & -0.2531 \end{bmatrix}$$

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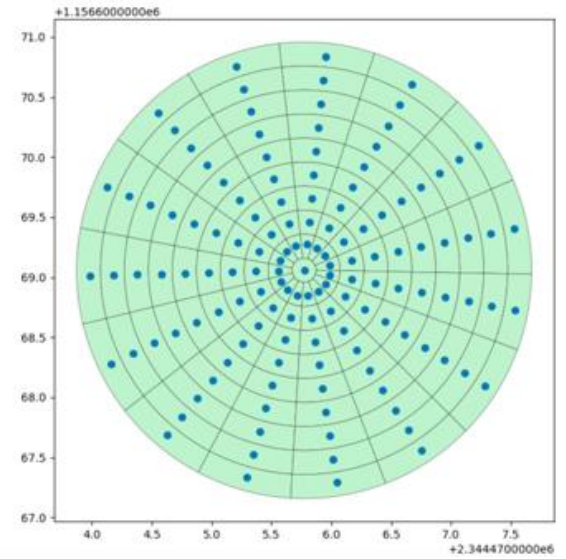
Case Study



- **Study area:** A Hypothetical City

- **Data**

Shapefile	Type	File Detail	Key Fields	Key Variable	Data Source
tract	Polygon	Base map for plotting			The case study is built upon the work reported in Wang (1998), but revised extensively for clarity of illustration.
trtpt	Point	Study area centroids	**BEMP_CBD** **BEMP_UNIF**	Basic employment per region：single center | uniform distribution	
road	line	The hypothetical city is here assumed to be partitioned by a transportation network made of 10 circular rings and 15 radial roads.	**LENGTH** **LENGTH1**	Length of each road segment	

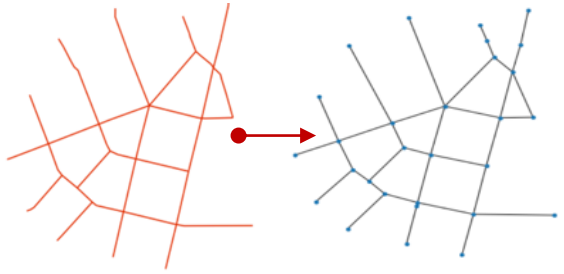
Technologic Flow

Data Input

- Road
- Tract Point
- Tract

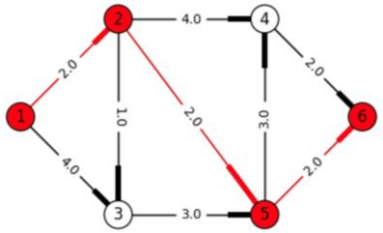
Data Preprocessing

Create a network dataset



Streets → Primal Graph

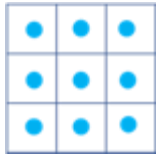
OD Cost Matrix



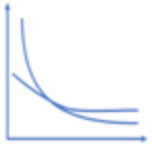
Distributions

Garin – Lowry Model

The basic case (Monocentric)



A uniform distribution



travel friction coefficients (α | ß)

transportation network

Compared

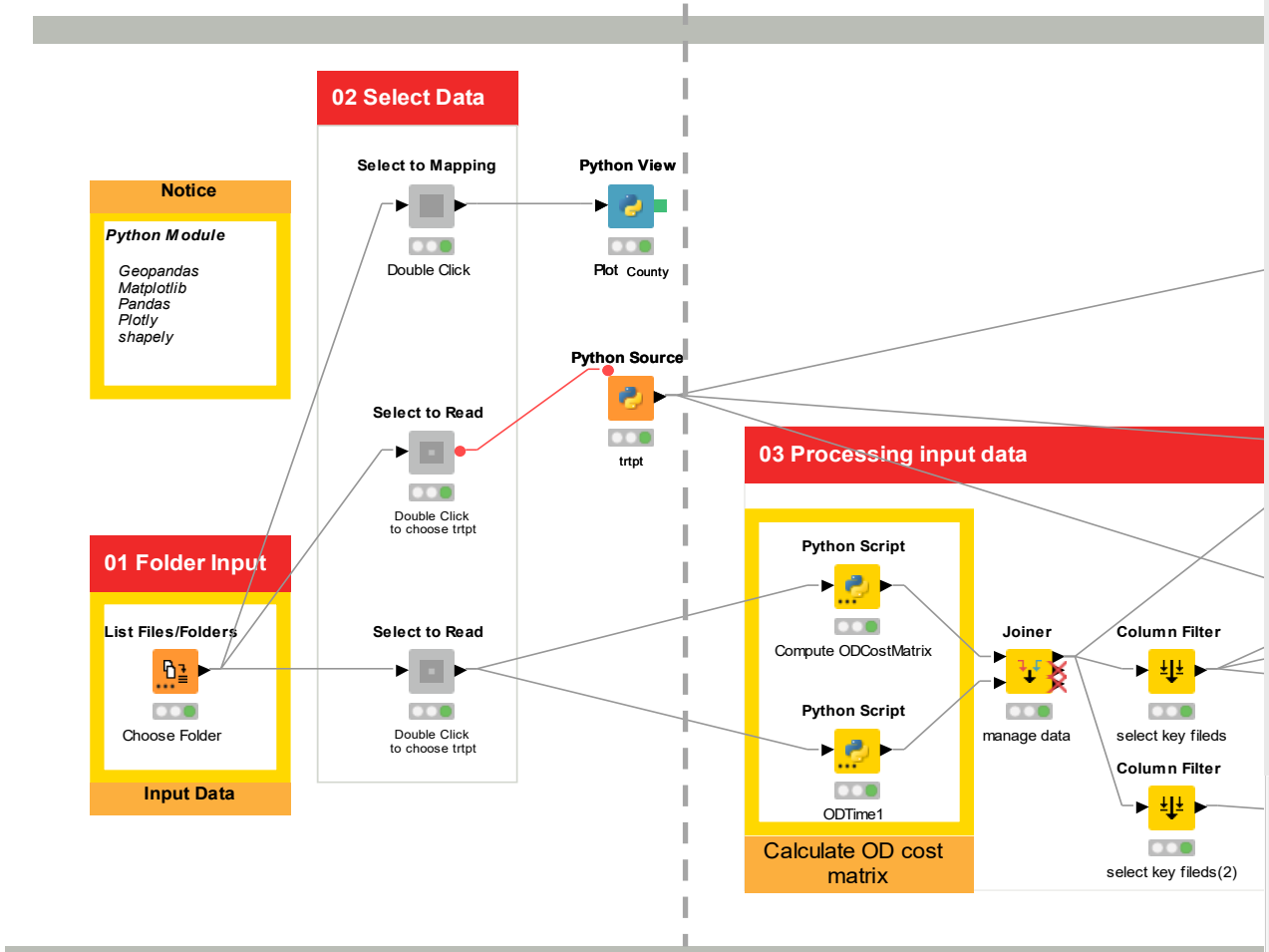
Population distributions in various scenarios

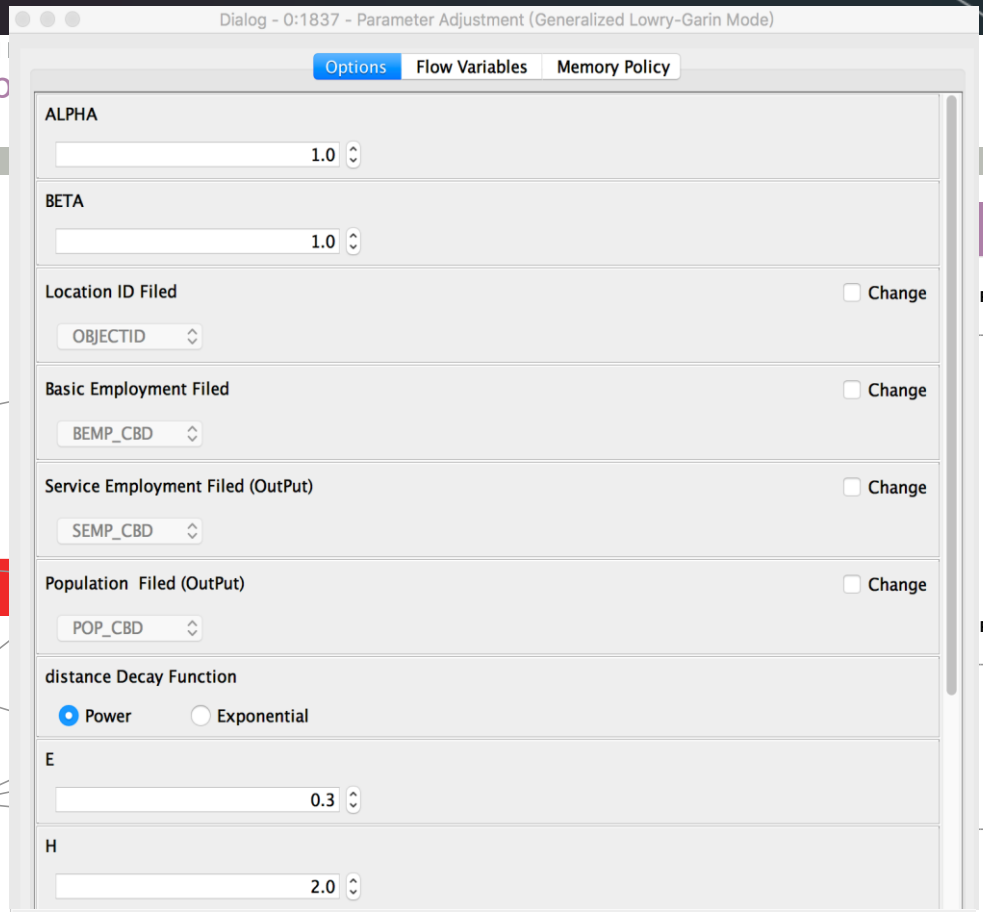
Service employment distributions in various scenarios

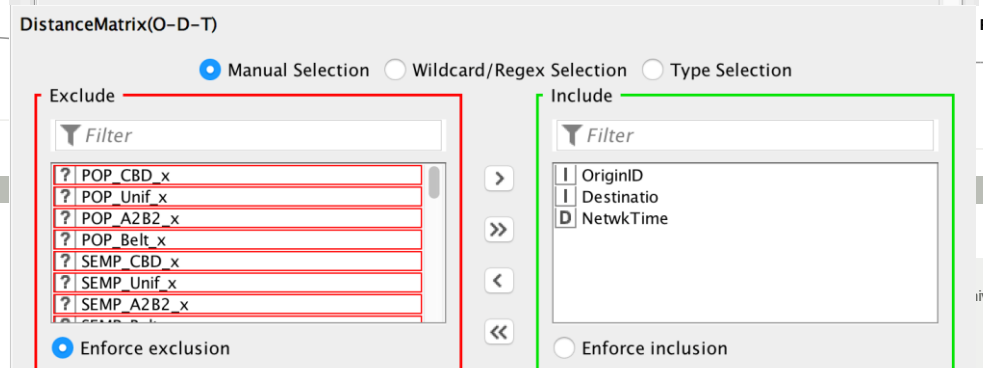
The Workflow Implementation

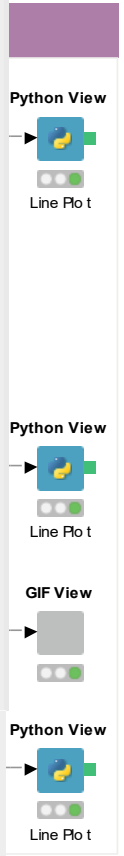
Chapter 10 System of Linear Equations and Application of Garin-Lowry Model in Simulating Urban Population and

Case Study 10 Simulating Population and Service Employment Distrib







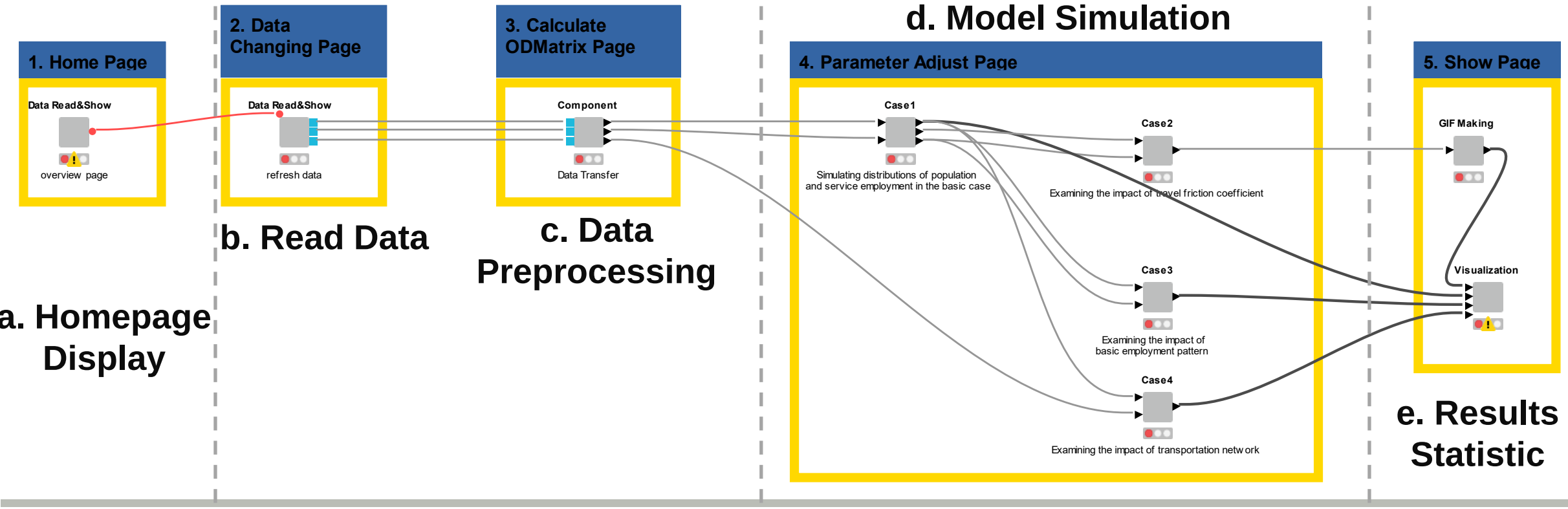


Computational Methods and Spatial Social Science in GIS

The Workflow Implementation

Chapter 10 System of Linear Equations and Application of Garin-Lowry Model in Simulating Urban Population and Employment Patterns

Case Study 10 Simulating Population and Service Employment Distributions in a Hypothetical City



Computational Methods and Spatial Social Science in GIS

Workflow Contributor

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Step by Step Workflow Execution I

a. Homepage Display

Home > lingbo > Chapter10_Final_web0521_postil > Chapter10_Final_web0521_postil 2022-05-30 02.09.08

Chapter 10:System of Linear Equations and Application of Garin-Lowry Model in Simulating Urban Population and Employment Patterns

This chapter introduces the method for solving a system of linear equations (SLE). The method is fundamental in numerical analysis (NA), and is often used as a building block in other NA tasks such as solving a system of nonlinear equations and the eigenvalue problem. Here, the SLE is illustrated in the Garin-Lowry model, a model widely used by urban planners and geographers for analyzing urban land use structure.
There has been an interesting debate on the relation between population and employment distributions in a city. Does population follows employment (i.e., workers find residences near their workplaces to save commuting)? Or vice versa (i.e., businesses locate near residents for recruiting workforce or providing services)? The Garin-Lowry model argues that population and employment distributions interact with each other and are interdependent. The Garin-Lowry model has the flexibility of simulating a population distribution pattern corresponding to any given basic employment pattern. It can be used to examine the impact of basic employment distribution on population as well as that of transportation network.

Title: Simulating Population and Service Employment Distributions in a Hypothetical City.

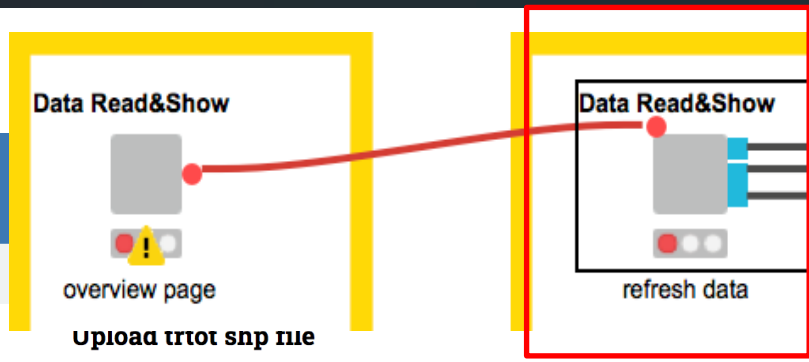
Objectives
Showing how the distributions of population and employment interact with each other and how the patterns can be affected by various transportation networks.

Methodology
LU-factorization & Garin - Lowry Model

Data
The project primarily uses the file geodatabase SimuCity.gdb under the folder SimuCity, which contains: (1)a polygon feature (136 tracts): tract.shp

Step by Step Workflow Execution II

b. Read Data



Monitoring

Home › Users › workbook02 › Chapter10_Final_web0327_2 › Chapter10_Final_web0327_2 2022-03-27 08.07.20

Refresh

Upload tract shp file

Select file tract.shp

Upload road shp file

Select file road.shp

Upload trtot shp file

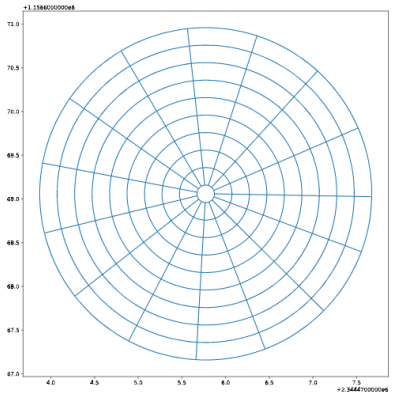
Select file trtpt.shp

Road data
Road data sample

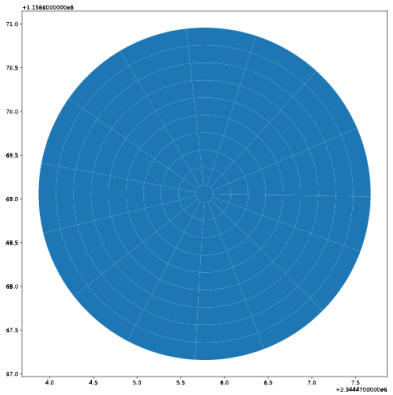
tract data
tract data sample

trtpt data
trtpt data sample

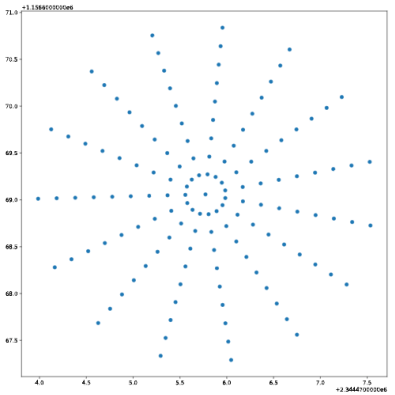
Road



tract



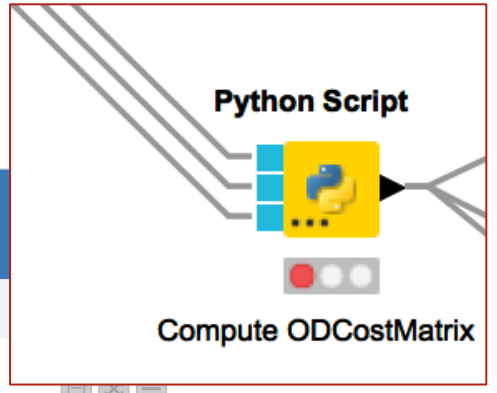
trtpt



Cancel Next

Step by Step Workflow Execution II

c. Data Preprocessing



Monitoring

Home › Users › workbook02 › Chapter10_Final_web0327_2 › Chapter10_Final_web0327_2 2022-03-27 08.07.20

Compute ODCostMatrix

O–D–T

Show 10 entries

Search:

RowID	OriginID	Destinatio	NetwkTime	NetwkTime1
0	1	1	0.9643491637846165	0.9643491637846165
1	1	2	1.157446947088816	1.1574469530684135
2	1	3	1.4931367129593789	1.4931367072868116
3	1	4	1.5278273244853613	1.5278273305005134
4	1	5	1.357446925294658	1.3574469292933056
5	1	6	1.8793322851331424	1.8793322842835474
6	1	7	1.6931367186960218	1.6931367070670296
7	1	8	1.7051099196591724	1.6165121132094844
8	1	9	1.557446904629664	1.5574469071380814

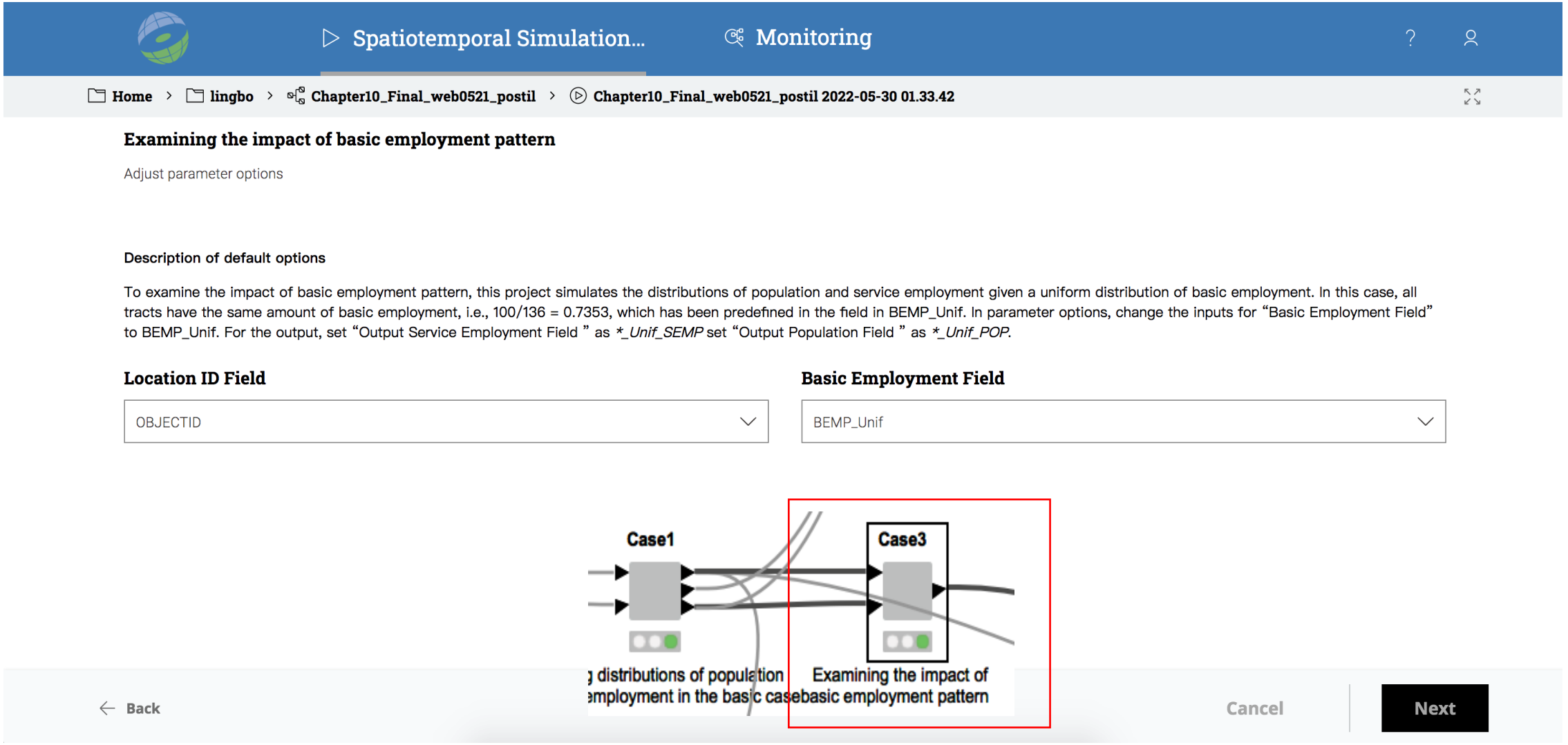
Back Cancel Next

Step by Step Workflow Execution III

d. Model Simulation

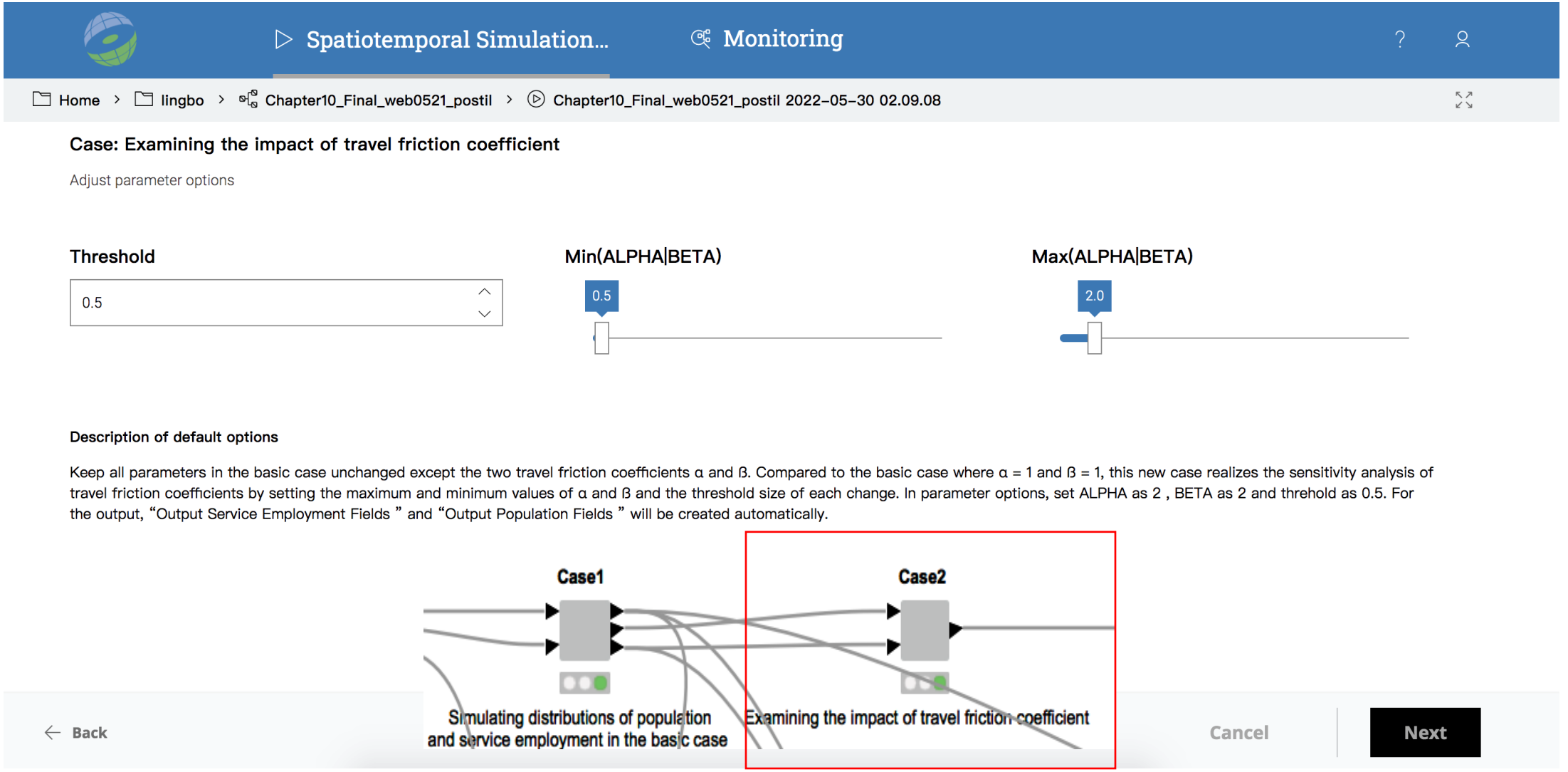
Step by Step Workflow Execution III

d. Model Simulation



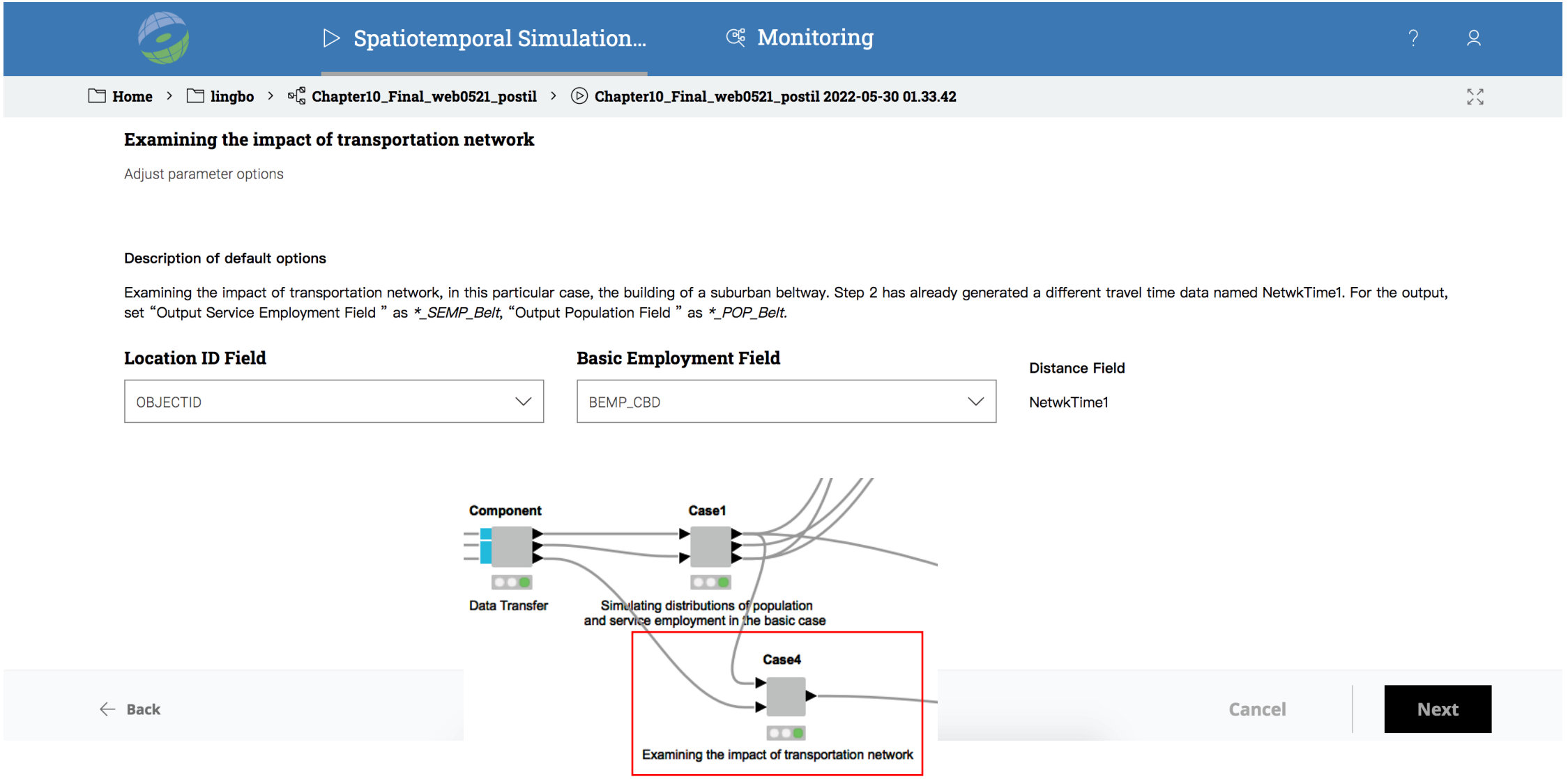
Step by Step Workflow Execution III

d. Model Simulation



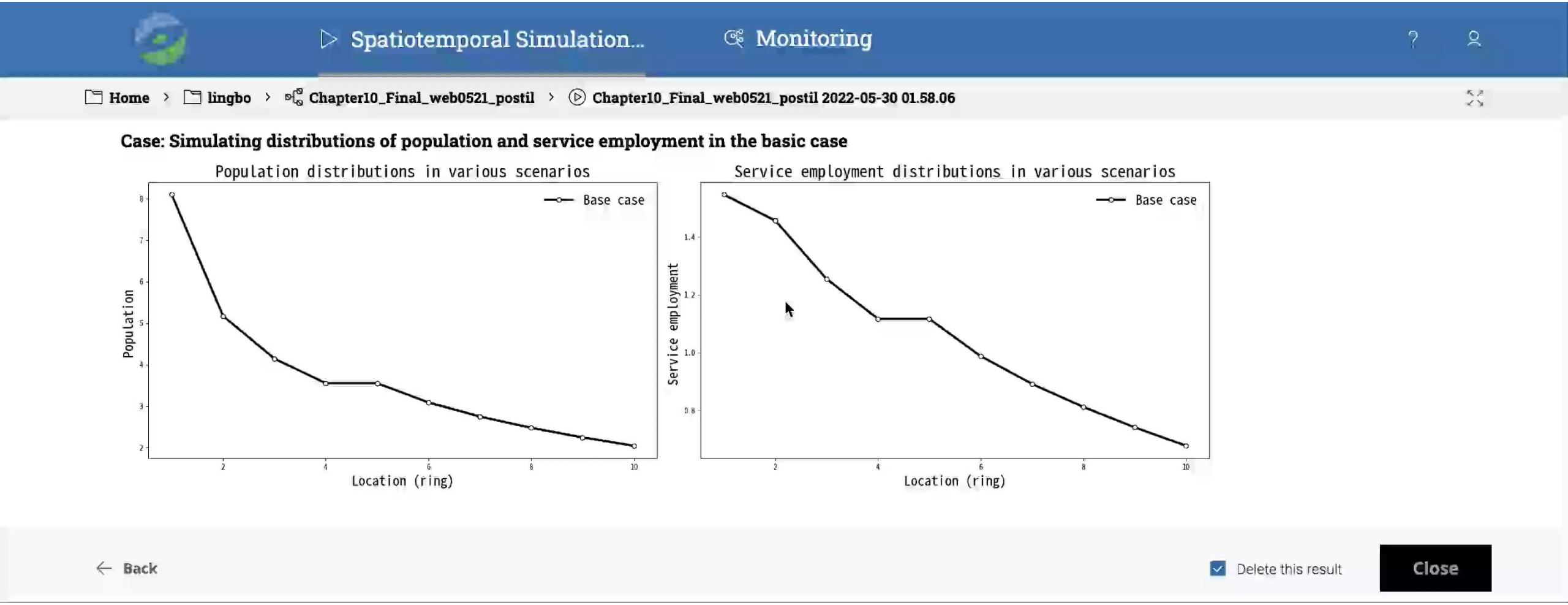
Step by Step Workflow Execution III

d. Model Simulation



Step by Step Workflow Execution IV

e. Results Statistic



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Conclusion

- The model is constructed as a system of linear equations, which is easy to calculate.
- The case study used a hypothetical city helps us understand the change of urban structure under various scenarios and explain many empirical observations in urban density studies.
- The distribution of service employment and population under **different distance friction coefficients** is **simulated**.

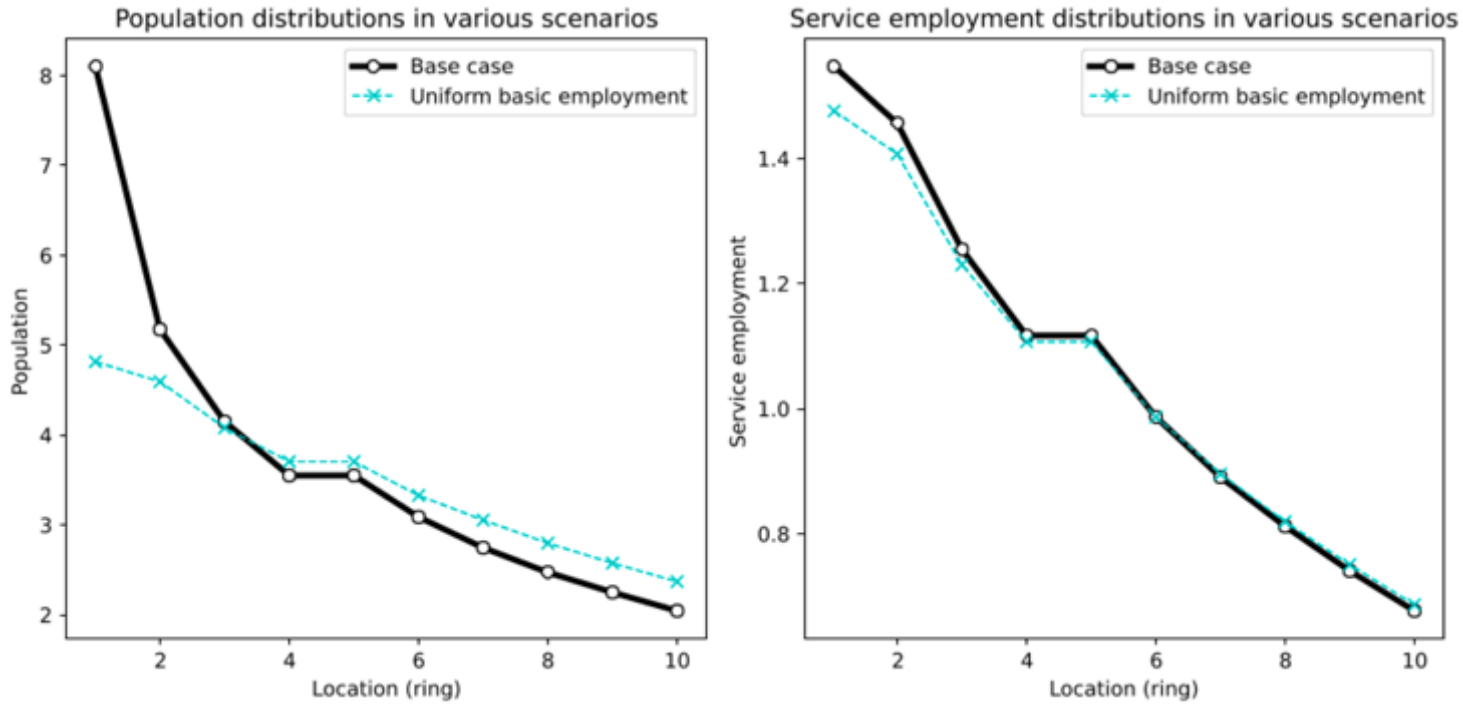
Discussion

☐ Comparison of different basic employment distributions

Case1: Simulating distributions of population and service employment in the basic case

vs

Case2: Examining the impact of basic employment pattern



- both the population and service employment remain **declining from the city**.
- both the population and service employment **exhibit steeper slopes** in the basic case than case2.

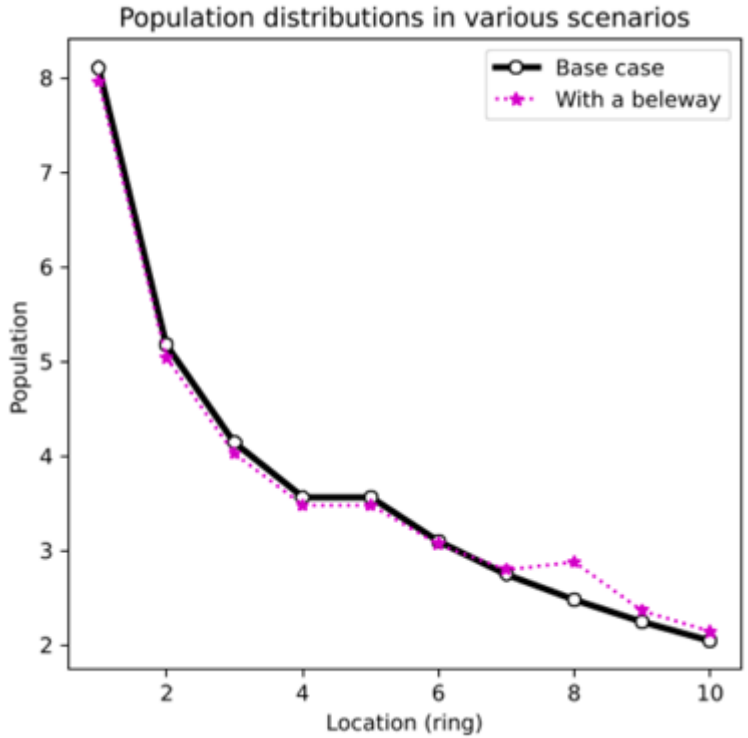
Discussion

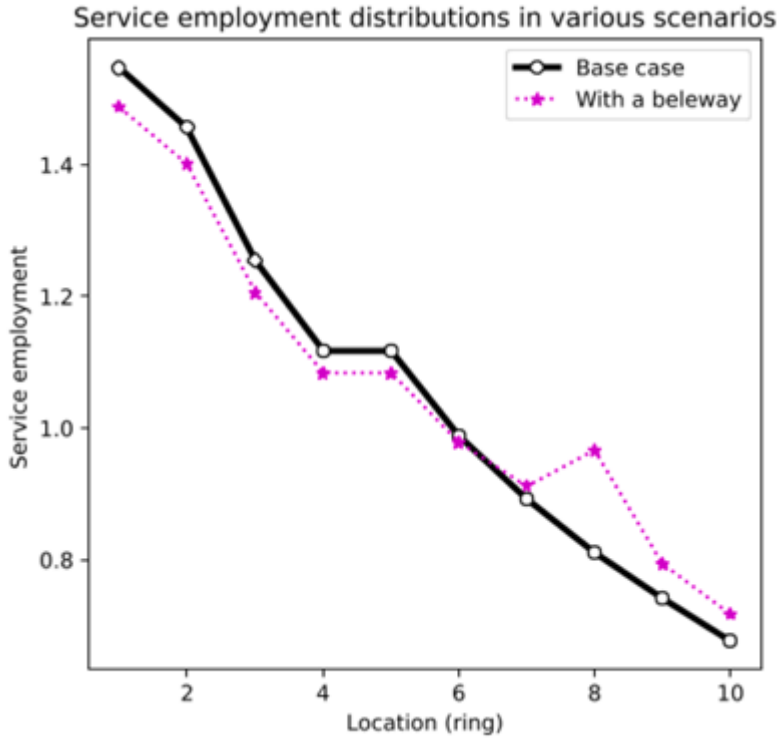
☐ Comparison of different transportation network

Case1: Simulating distributions of population and service employment in the basic case

vs

Case4: Examining the impact of transportation network





- The distribution patterns of population and service employment are **similar** to those in the basic case.
- The lower values before the 7th ring
- The higher values after the 7th ring

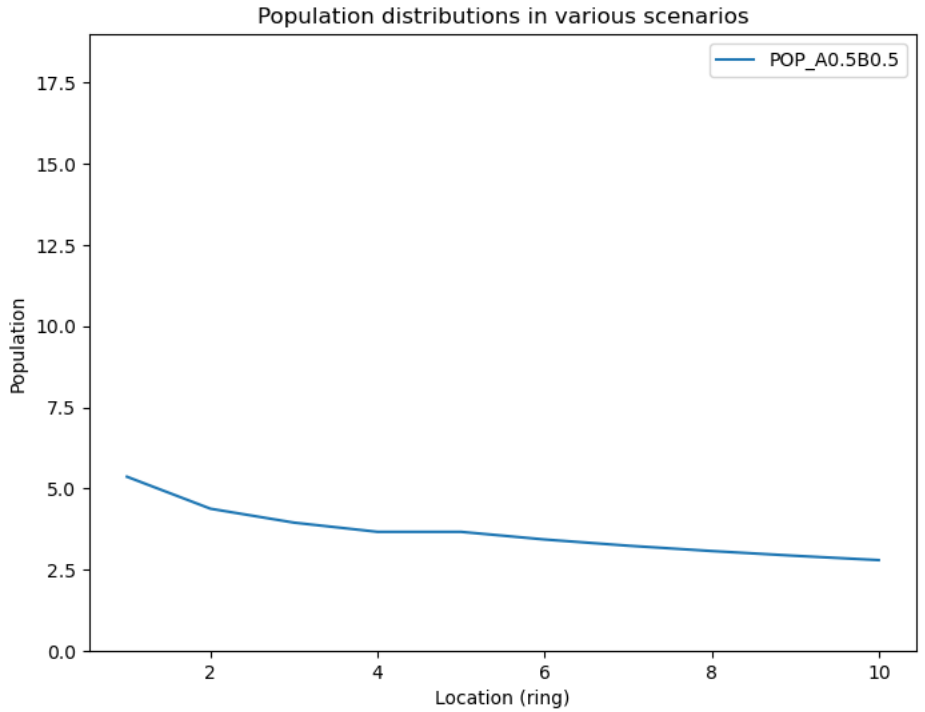
Discussion

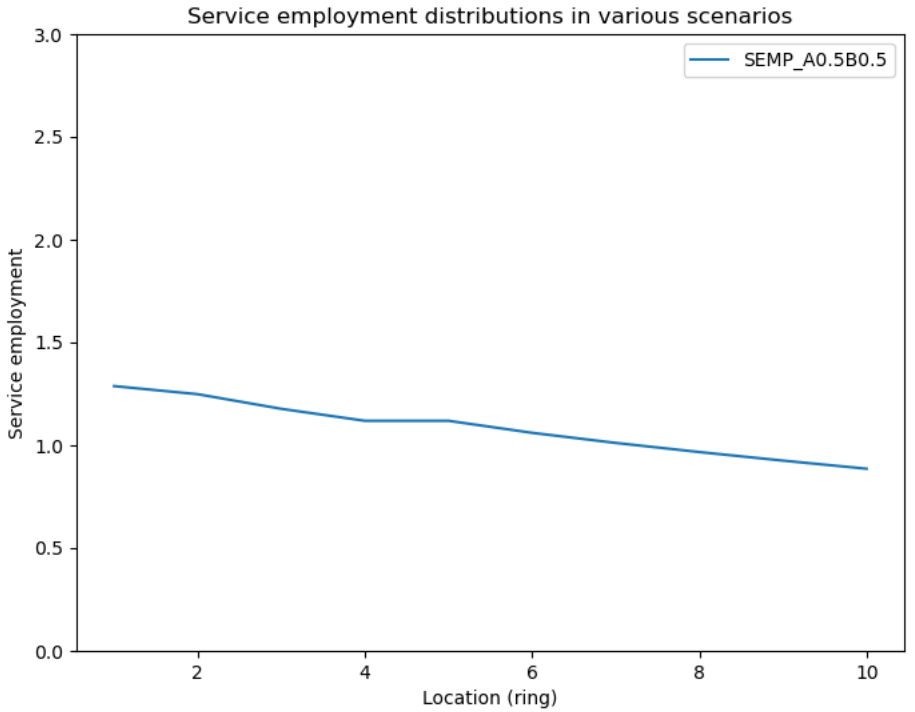
☐ Comparison of different distance friction coefficient

Case1: Simulating distributions of population and service employment in the basic case

vs

Case3: Examining the impact of travel friction coefficient

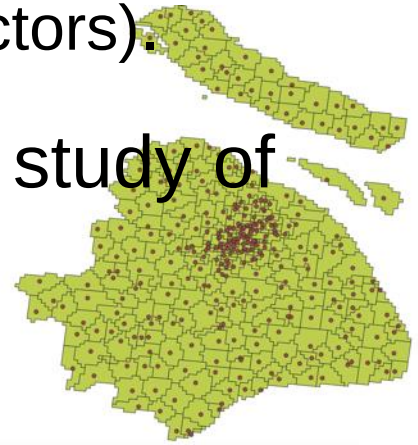




- The **steeper slopes** for both population and service employment in case3 with larger α and ß.

Future Work

- Applying the Garin-Lowry model in analyzing real-world cities：
  (required the division of employment into basic and nonbasic sectors).

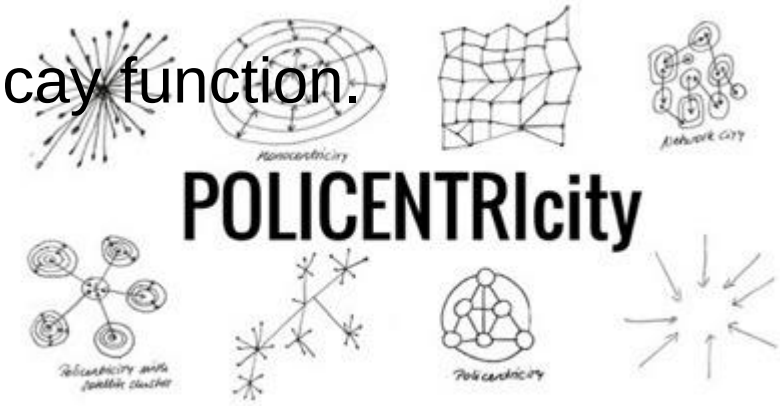


- The model may be used to examine more issues in the study of urban structure:

  the impact of road density;
  the impact of road network structure;
  exploring population and employment in polycentric cities.

- Model modification：

  The challenges of defining a realistic distance decay function.



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THANKS

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