armi.f90

Ari Anisfeld

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1 Define Variables (lines 1 - 166)

Conventions: I will match names from armi.f90, but use expressions and subscripts as implied by the code.

e.g. sigr = σ_r

Parameters that are inputs into the code will not be directly mentioned, while objects defined within the code will.

We will subscript matrices in row, column order.

Subscripts for matrices will sometime interfere with subscripts for objects. For example g_i is a 2x2 matrix. If we want to refer to the

top right corner, we'll use the following conventions. $g_{(1,1)i} = g_i[1,1]$. For vectors, $g_{2r} = g_r[2]$.

2 Quadrature (lines 168 - 386)

2.1 Quadrature

Set quadrature points $\mathbf{q}$ =(-1, 1)

2.2 One-period log inflation

Set 2x1 vector: $\mathbf{g_r} = \mathbf{q} \cdot \sigma_r + \mu_r = \begin{bmatrix} \mu_r - \sigma_r \\ \mu_r + \sigma_r \end{bmatrix}$

Get transition matrix. 2x2 $\mathbf{p}$

2.3 One-period log real yield

Set 2x1 vector: $\mathbf{g_{iinov}} = \mathbf{q} \cdot \sigma_i + \mu_i = \begin{bmatrix} \mu_i - \sigma_i \\ \mu_i + \sigma_i \end{bmatrix}$

Get transition matrix. 2x2 $\mathbf{tp}$.

Set 2x2 matrix: $\mathbf{g_i} = \begin{bmatrix} g_{1iinov} + \beta_i g_{1r} & g_{1iinov} + \beta_i g_{1r} \\ g_{2iinov} + \beta_i g_{2r} & g_{2iinov} + \beta_i g_{2r} \end{bmatrix}$

2.4 Innovations to log inflation

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Set 2x2 matrix: $\mathbf{in_r} = \begin{bmatrix} g_{1r} - (p_{(1,1)}g_{1r} + p_{(1,2)}g_{2r}) & g_{2r} - (p_{(1,1)}g_{1r} + p_{(1,2)}g_{2r}) \\ g_{1r} - (p_{(2,1)}g_{1r} + p_{(2,2)}g_{2r}) & g_{2r} - (p_{(2,1)}g_{1r} + p_{(2,2)}g_{2r}) \end{bmatrix}$

2.5 Innovations to real rate that are uncorrelated with inflation

Set 2x2 matrix: $\mathbf{in_i} = \begin{bmatrix} g_{1iinov} - (tp_{(1,1)}g_{1iinov} + tp_{(1,2)}g_{2iinov}) & g_{2iinov} - (tp_{(1,1)}g_{1iinov} + tp_{(1,2)}g_{2iinov}) \\ g_{1iinov} - (tp_{(2,1)}g_{1iinov} + tp_{(2,2)}g_{2iinov}) & g_{2iinov} - (tp_{(2,1)}g_{1iinov} + tp_{(2,2)}g_{2iinov}) \end{bmatrix}$

2.6 House prices

Let$\tilde{\mathbf{q}}$ = (1, -1). ie. q reversed.

Set 2x1 vector: $\mathbf{dlph} = \tilde{\mathbf{q}} \cdot \sigma_{\mathrm{dph}} + \mu_{dph} = \begin{bmatrix} \mu_{dph} + \sigma_{dph} \\ \mu_{dph} - \sigma_{dph} \end{bmatrix}$

(Exponentiate to get in meaningful units)

2.7 Income

Set 2x1 vector: $\mathbf{logy_p} = \mathbf{ly_p} = \tilde{q} \cdot \sigma_{py} = \begin{bmatrix} \sigma_{py} \\ -\sigma_{py} \end{bmatrix}$

Set 2x1 vector: $\mathbf{y_p p_{pyhouse}} = \mathrm{e}^{\mathbf{logy_p}}$

2.8 Correlation between innovations to permanent income and house prices

$n = (\rho_{pyhouse}\sigma_{py}\sigma_{dph} - 0.5 * (dlph_1 - \mu_{dph})lyp_2 - 0.5(dlph_2 - \mu_{dph})lyp_1) = \rho_{pyhouse}\sigma_{py}\sigma_{dpy} - \frac{1}{2}(\sigma_{dph}(lyp_2 - lyp_1)) = \rho_{pyhouse}\sigma_{py}\sigma_{dph} - \sigma_{dph}\sigma_{py} = \sigma_{dph}\sigma_{py}(1 - \rho_{pyhouse})$

$d = (0.5(dlph_1 - \mu_{dph})lyp_1 - 0.5(dlph_1 - \mu_{dph})lyp_2 - 0.5 * (dlph_2 - \mu_{dph})lyp_1 + 0.5(dlph_2 - \mu_{dph})lyp_2) = 2\sigma_{dph}\sigma_{py}$

$p_{pyhouse} = \frac{n}{d} = \frac{1-\rho_{pyhouse}}{2}$

Set 2x2 matrix: $\mathbf{pyh} = \begin{bmatrix} p_{pyhouse} & 1 - p_{pyhouse} \\ 1 - p_{pyhouse} & p_{pyhouse} \end{bmatrix} = \begin{bmatrix} \frac{1-\rho_{pyhouse}}{2} & \frac{1+\rho_{pyhouse}}{2} \\ \frac{1+\rho_{pyhouse}}{2} & \frac{1-\rho_{pyhouse}}{2} \end{bmatrix}$

2.9 Correlation between innovations to the real rate (that are uncorrelated with inflation) and real house prices

Set constant: $\sigma_{ini} = \sqrt{tp_{(1,1)}in^2_{(1,1)i} + tp_{(1,2)}in^2_{(1,2)i}}$

$n = \rho_{ratehouse}\sigma_{ini}\sigma_{dph} + (-tp_{(1,1)}(dlph_1 - \mu_{dph})in_{(1,1)i}) + (-tp_{(1,2)}(dlph_1 - \mu_{dph})in_{(1,2)i}) - (in_{(1,1)i}dlph_1 + \mu_{dph}\frac{1}{2tp_{(1,1)}})$

$n = \rho_{ratehouse}\sigma_{ini}\sigma_{dph} + (-tp_{(1,1)}(\sigma_{dph})in_{(1,1)i}) + (-tp_{(1,2)}(\sigma_{dph})in_{(1,2)i}) - (in_{(1,1)i}dlph_1 + \mu_{dph}\frac{1}{2tp_{(1,1)}})$

$d = (2*(dlph_1 - \mu_{dph})in_{(1,2)i}tp_{(1,2)} + (-\frac{tp_{(1,2)}}{tp_{(1,1)}})(-2tp_{(1,1)}in_{(1,1)i}(dlph_2 - \mu_{dph}))$
$p_{pratehouse2} = \frac{n}{d}$
$p_{pratehouse1} = \frac{1}{2tp_{(1,1)}} - \frac{tp_{(1,2)}}{tp_{(1,1)}}p_{pratehousese2}$
Set 2x2x2 tensor: prh (this is used once later)

2.10 Correlation between temporary income shocks and the level of real rates

Set constant: $\sigma_{realrate} = \sqrt{\sigma_i^2 + (\beta_i\sigma_r)^2}$
Set constant: $regcoef = correl\frac{\sigma_{ty}}{\sigma_{realrate}} = 0$ (correl = 0)
Set constant: $\sigma_{res} = \sqrt{\sigma_{ty}^2 - (regcoef\sigma_{realrate})^2} = \sigma_{ty}$
Set 2x2x2 tensor: $\mathbf{eyt}_{(.,.,\mathbf{1})} = \begin{bmatrix} -\sigma_{ty} & -\sigma_{ty} \\ -\sigma_{ty} & -\sigma_{ty} \end{bmatrix}$, because regcoef = 0
$\mathbf{eyt}_{(.,.,\mathbf{2})} = \begin{bmatrix} \sigma_{ty} & \sigma_{ty} \\ \sigma_{ty} & \sigma_{ty} \end{bmatrix}$
Set 21x1 vector with nth entry: $labinc_n = level_y(1 + growth_y)^{n-1}$
Set the 2x2x2x21x21: $\mathbf{fy}_{(:,:,:,\mathbf{k},\mathbf{n})} = eyt_{(:,:,:)} \cdot yp_2^{k-1} \cdot yp_1^{n-k}$
The interpretation of k is the number of times a low permanent income shock occured in the past ($k \leq n$). Because regcoef is 0, this is equivalent to a 2x21x21 tensor, where
the first dimension is temporary income shock, the second dimension measures the number of negative permanent shocks and the third tracks the number of years.

3 Construct Grids (lines 386 - 411)

Set 185x1 vector whose nth entry: $lgcash_n = (n-1) \cdot 0.035$
Set 185x1 vector whose nth entry : $gcash_n = e^{lgcash_n}$
Set 160x1 vector whose nth entry: $lgcons_n = (n-1) \cdot 0.036$
Set 160x1 vector whose nth entry : $gcons_n = e^{lgcons_n}$
Set gdebt = 0, this implies there is no additional credit usage.

4 Calculating annual payments (lines 411 - 629)

4.1 Construct term structure

Now we create 4 seperate paths, representing the term structure for the 4 tables in the paper. There is a variable called *indcase*
Case: 1 low inf low real; 2 low inf high real; 3 high inf low real; 4 high inf high real. (Subscript j will be reserved for these cases).
lyield is a 4x20 matrix.

$$lyield_{(1,:)} = \begin{bmatrix} g_{1r} + g_{1iinov} + \beta_i g_{1r} \\ g_{1r} + g_{2iinov} + \beta_i g_{1r} \\ g_{2r} + g_{1iinov} + \beta_i g_{2r} \\ g_{2r} + g_{2iinov} + \beta_i g_{2r} \end{bmatrix}$$

yield is a 4x20 matrix that is not used in the code except as a diagnostic to show $e^{lyield_{(1,:)}}$

4.1.1 Joint Probability

Now we use the transition probability matrices from above to create the 4x4x20 tensor *prob* which determines the potential term structures.

We think of this as 20 4x4 transition matrices

We use a 4x4 Matrix *probinfr* to help build *prob*.

$$probinfr = \begin{bmatrix} p_{(1,1)} \cdot tp_{(1,1)} & p_{(1,1)} \cdot tp_{(1,2)} & p_{(1,2)} \cdot tp_{(1,1)} & p_{(1,2)} \cdot tp_{(1,2)} \\ p_{(1,1)} \cdot tp_{(2,1)} & p_{(1,1)} \cdot tp_{(2,2)} & p_{(1,2)} \cdot tp_{(2,1)} & p_{(1,2)} \cdot tp_{(2,2)} \\ p_{(2,1)} \cdot tp_{(1,1)} & p_{(2,1)} \cdot tp_{(1,2)} & p_{(2,2)} \cdot tp_{(1,1)} & p_{(2,2)} \cdot tp_{(1,2)} \\ p_{(2,1)} \cdot tp_{(2,1)} & p_{(2,1)} \cdot tp_{(2,2)} & p_{(2,2)} \cdot tp_{(2,1)} & p_{(2,2)} \cdot tp_{(2,2)} \end{bmatrix}$$

We set $prob_{(:,:,1)} = \mathbf{0_{4x4}}$ and $prob_{(:,:,2)} = probinfr$. For $3 \leq n \leq 20$, and $a, b \in \{1, 2, 3, 4\}$:

$prob_{(a,b,n)} = prob_{(a,1,n-1)} probinfr_{(1,b)} + prob_{(a,2,n-1)} probinfr_{(2,b)} + prob_{(a,3,n-1)} probinfr_{(3,b)} + prob_{(a,4,n-1)} probinfr_{(4,b)}$

4.1.2 Getting log yield

Now we can use the transition probabilities to calculate the rest of log yield. For $2 \leq n \leq 20$, and $a, b \in \{1, 2, 3, 4\}$:

$lyield_{(a,n)} = lyield_{(a,n-1)} prob_{(1,b,n-1)} + lyield_{(a,2)} prob_{(2,b,n-1)} + lyield_{(a,3)} prob_{(3,b,n-1)} + lyield_{(a,4)} prob_{(4,b,n-1)}$

We will only use $lyield_{(a,20)}$for future calculations.

4.2 Anuity Factor and Anuity

For each case a we calculate: $anfactor_a = \sum_{k=1}^{20} (\frac{1}{e^{lyield_{(a,20)}+\theta}})^k$ and $anuity_a = \frac{loan}{anfactor_a}$

4.3 Remaining debt and annual payments

NOTE: the theta used here does not seem to correspond with preference for housing as in the paper.

With this in place, For $2 \leq n \leq 20$, and $a \in \{1, 2, 3, 4\}$ we can calculate

$remdebt_{(a,n)} = remdebt_{(a,n-1)} - (anuity_a - remdebt_{(a,n-1)} e^{lyield_{(a,20)}+\theta-1})$

with $remdebt_{(a,1)} = loan$

And so,

$loanrepaid_{(a,n-1)} = remdebt_{(a,n-1)} - remdebt_{(a,n)}$

with the remaining balance paid in year 20.

Then follows an if statement, where the first term of $loanrepaid, remdebt$,and *anuity* are replaced with the a^{th}term (as indicated with *indcase*).

5 Dynamic Programming (lines 629 - 959)

5.1 Terminal Period

Set t= 21, $1 \leq j, k \leq 21$,$1 \leq m \leq 185$.

vr is a 1x185x2x21x21x21x2 tensor that will captures utility from wealth including housing wealth. Note the code calls this vr2.

k and j capture the number of times the good/bad state of the world occurs.

m is a knot on the grid. Recall $gcash_1 = e^0 = 1$.

Note the loop has hardcoded 0.3, which seems to be the relative preference placed on housing. At one point, he also hard codes 21 where t would be.

$pricelevel = g_{2r}^{k-1} \cdot g_{1r}^{t-k}$

$reahousval = dlph_2^{j-1} \cdot dlph_1^{t-j}$

$tprice = (\sqrt{pricelevel} + \sqrt{0.3 \cdot reahousval \cdot pricelevel})^2$

$sv = \max\left(gcash_1, (gcash_m + house \cdot e^{dlph_2})^{j-1}(e^{dlph_1})^{t-j}\frac{pricelevel}{tprice})\right)$

$vr_{(:,m,:,k,:,j,:)} = bequest \cdot \frac{sv^{1-\gamma}}{1-\gamma}$

5.1.1 Calling irutil2

At this point, we call irutil2, a helper function that converts consumption to utility.

We get a 160 x 1 vector of utilities according to are grid values.

$util_n = \frac{gcons_n^{1-\gamma}}{1-\gamma}$

5.2 Lower bound on utility

Set 21 x 1 vector $utbd$

$utbd_{21} = bequest \cdot \frac{gcons_1^{1-\gamma}}{1-\gamma}$

$utbd_t = \frac{gcons_1^{1-\gamma}}{1-\gamma} + \beta utbd_{t+1}$

5.3 Other periods

This is a very large do-loop with many inner processes. For each year from year 20 to the start we do the following:

- We call spline() to get value function at t+1
- Backfill v_in_1
- Read in value function from default state (rent).
- Cycle through the state variables
- Write Cons choices and value function

5.3.1 Spline

Set 1 x 185 x 2 x 21 x 21 x 21 x 2 tensor: $secd2$

Use spline and cycle through

$secd2 = spline(gcash, vr)$

5.3.2 One-period utility

Note the loop has hardcoded 0.3, which seems to be the relative preference placed on housing.

For t = 20

$pricelevel = g_{2r}^{k-1} \cdot g_{1r}^{t-k+1}$

$reahousval = dlph_2^{j-1} \cdot dlph_1^{t-j+1}$

$tprice = (\sqrt{pricelevel} + \sqrt{0.3 \cdot reahousval \cdot pricelevel})^2$

$sv = \max\left(gcash_1, gcash_m \frac{pricelevel}{tprice})\right)$

$vin1_{(:,m,:,k,:,j,:)} = bequest \cdot \frac{sv^{1-\gamma}}{1-\gamma}$

For t $<$ 20:

They read in the files produced by the rent program and take in 185-length vectors and fill in $vin1$ and vin

5.4 Cycle state variables

Notes: the inflation in period 1 is not a state variable (but carried around in ind1).

Different consumption levels (ind6, 1:160)

Debt levels (ind7, 1:1).

Our main objects: cons, debt, vr are indexed (1,185,2,21,21,21,2)

(1, c, inflation, k, p, j, real interest)

5.4.1 What we cycle over

For each knot c of gcash (ind2, 1:185):

For f: current inflation rate (ind3, 1:2).

For k: Number of times of high inflation in the past (ind4, 1:21)

For p: Number of times of low permanent income in the past (ind5, 1:21)

For j: Number of low house prices in the past (ind8, 1:21)

For s: current real interest (ind9, 1:2)

5.4.2 Savings

Calculate savings at each level of consumption for the given level of cash.

Set 160 x 1 x 1 vector: $sav_n = gcash_c - gcons_n + gdebt$

Recall in armi.f90, there is no additional debt, so gdebt = 0

Then if $sav_n > 0$,

$u2_n = util_{(1,n)}$, else $u2_n = -\infty$ and set $sav_n = 0$

5.4.3 Nominal Mortgage Payment

Nominal mortgage payment = loan repayment + interest payment on remaining debt:

$anaux = loanrepaid_{(1,t)} + (e^{g_{(f)r}+g_{(s,f)i}} - 1 + \theta) \cdot remdebt_{1,t}$

$mortint = (e^{g_{(f)r}+g_{(s,f)i}} - 1 + \theta) \cdot remdebt_{1,t}$

5.4.4 Real Mortgage Payment

$pricelevel = e^{g_{(2)r}^{k-1}} e^{g_{(1)r}^{t+1-k}}$

$anaux2 = \frac{anaux}{pricelevel}$

$mortint2 = \frac{mortint}{pricelevel}$

5.4.5 Real and nominal house value

$reahousval = e^{dlph_2^{j-1}} e^{dlph_1^{t+1-j}}$

$nomhousval = house * pricelevel * reahousval$

5.4.6 Property tax and maintenance

$proptax = tax_p \cdot reahousval$

$propmaint = m_p \cdot reahousval$

5.4.7 Setting up and running irevi

We set pricelevel1 with $\tilde{k} = k + 1$ if current interest f is high. (ie. $f = 2$) and $\tilde{k} = k$ otherwise.

$pricelevel1 = e^{g_{(2)r}^{\tilde{k}-1}} e^{g_{(1)r}^{t+1-\tilde{k}}}$

We also set a forward looking 2 x 1 vector $ph1$where $\tilde{j} = j + 1$

With $ph1_1 = reahousval$ and $ph1_2 = e^{dlph_2^{\tilde{j}-1}} e^{dlph_1^{t+1-\tilde{j}}}$

And $remdebt1 = remdebt_{t+1}$recalling that remdebt holds the path of remaining debt given the state of the world set with *indcase*.

Now use $irevi()$ to produce a 160 x 1 vector which we assign to $v1$

5.4.8 Setting up choices (setting utility paths)

Set 160 x 2 vector:$vv_{(:,1)} = u2 + \beta v1$ and $vv_{(:,2)} = 0$

Set $vr_{state} = vr_{(1,c,f,k,p,j,s)} = max(vv_{(:,:)})$

5.4.9 consumption and debt

Strategy find the index into *gcons* and set

$cons_{state} = gcons_{index_{cons}}$

$debt_{state} = gdebt_{index_{debt}}$

5.5 Choices (still within cycle of state variables)

If $vr_{state} < vin_{state}$, then $def_{state} = 1$ and we set $vr_{state} = vin_{state}$

5.5.1 Default state

If $def_{state} = 1$ and $0.94 nomhousval > remdebt_t$, then

Set $cashrent = MAX\left(MIN\left(\left(gcash_c + \frac{0.94 nomhousval - remdebt_t}{pricelevel}\right), gcash_{185}\right), gcash_1\right)$; the purpose of the max/min is to ensure cash is kept within our grid.

Set $lcashrent = log(cashrent)$

Use $ntoil()$ which produces an index m and $m+1$ of where lcashrent falls in the lgcash grid, ensuring that the indices are within the grid.

Now take a convex combination of utility vin at m and $m+1$based on how close cashrent is to the knots.

$vrent = (1 - weight_{m+1}) vin_{current_m} + weight_{m+1} vin_{current_{m+1}}$

$vr_{state} = vrent$

5.5.2 Non-default state: Prepay?

Choose whether or not to prepay. The set up is identical to the previous section, but we get to choose:

If $vr_{current} < vrent$, prepay (set $def = 2$) and $vr_{current} = vrent$

5.5.3 Non-default state: Move?

Same set-up as previous sections. For ease of notation set $vstay = vr_{current}$

If $0.94 nomhousval > remdebt_t$ then

$vr_{current} = prob_{move} vrent + (1 - prob_{move}) vstay$

Else

$vr_{current} = ratiolock \cdot prob_{move} vrent + (1 - ratiolock \cdot prob_{move}) vstay$

5.5.4 Lower bound on utility

Do this if $vr_{current} < -10,000$.

6 Write output to file (lines 959 - end)

For each year t there is a file written called $year_t$.

First the level of consumption$cons_{index}$are written over various states of the world, then the default state def_{state}.

Finally the vr results are written to a seperate file (which is not used again).