1 Appendix

Proof that regressing legislative outcome measures on ideal point measures results in inconsistent estimates of unknown direction.[1]

For simplicity, consider the case where the only covariate is the function of (true) policy preferences x^* (e.g., gridlock interval). Let the true regression model be given by: $y = \beta x^* + \epsilon$ where y is some measure of output (e.g., roll rate, number of significant enactments). As we do not observe policy preferences, only measures of preferences based on roll calls, suppose $\hat{x} = x^* + u$ where $\hat{x}$ denotes the ideal point measure of policy preferences. This implies that the ideal point measure is a function of true policy preferences x^* plus error u. Since the same demands that shape policy outcomes also affect the translation of policy preferences into voting profiles on roll calls – for example, constituency demands – $cov(u, \epsilon) \neq 0$.

The proof is textbook and follows Greene *Econometric Analysis 5th Ed.* (2003) Section 5.6.1 with only trivial changes.

Claim: plim $\hat{\beta} \neq \beta$ and plim $\hat{\beta}$ is not necessarily attenuated (i.e., plim $\hat{\beta} < \beta$).

$$
\begin{aligned}
y &= \beta(\hat{x} - u) + \epsilon && \text{True Model} \\
&= \beta\hat{x} + \epsilon - \beta u \\
&= \beta\hat{x} + w && \text{where } w = \epsilon - \beta u \\
\\
\hat{\beta} &= \frac{(1/n)\sum_i \hat{x}_i y_i}{(1/n)\sum_i \hat{x}_i^2 y_i} && \text{OLS estimate} \\
\\
\text{plim } \hat{\beta} &= \frac{(1/n)\sum_i (x_i^* + u_i)(\beta x_i^* + \epsilon_i)}{(1/n)\sum_i (x_i^* + u_i)^2} && \text{by Slutsky} \\
&= \frac{(1/n)\sum_i (\beta x_i^{*2} + x_i^*\epsilon_i + u_i\beta x_i^* + u_i\epsilon_i)}{(1/n)\sum_i (x_i^{*2} + 2x_i^* u_i + u_i^2)^2} \\
&= \frac{(1/n)\sum_i (\beta x_i^{*2} + u_i\epsilon_i)}{(1/n)\sum_i (x_i^{*2} + u_i^2)^2} && \text{by indep. assumptions} \\
&= \frac{\beta Q^* + \sigma_{u\epsilon}}{Q^* + \sigma_u^2} && \text{where } Q^* = \text{ plim } (1/n)\textstyle\sum_i \beta x_i^{*2} \\
&= \frac{\beta + \sigma_{u\epsilon}/Q^*}{1 + \sigma_u^2/Q^*}
\end{aligned}
$$

If x^* was exogeneous, $\sigma_{u\epsilon} = 0$ and the last line would be: plim $\hat{\beta} = \frac{\beta}{1+\sigma_u^2/Q^*}$ which would imply that $\hat{\beta}$ is attenuated – $\beta >$ plim $\hat{\beta} = \frac{\beta}{1+\sigma_u^2/Q^*}$. However, since whether

[1] As claimed in footnote 14 of Clinton's (2006) "Testing Lawmaking Theories with (Endogenous) Roll Calls, 90th - 106th U.S. House."

plim $\hat{\beta}$ is too large or too small depends on the relationship between $\sigma_{u\epsilon}$ and σ_u^2, for the endogenous case this determination is impossible. Q.E.D.