Before the designation of the first successor, the relative position of these two candidates' powers will not change, and the power gap between them increases with time. Therefore, all the intuition and results in the main model can be preserved in this situation (Proposition 7 and Proposition 8). We simply repeat them as follows:

Proposition A.3. *If candidate 1 has lower initial power than candidate 2* ($\widetilde{S}^1 < \widetilde{S}^2$)*, then there exists a* $\hat{d'}_2 \geq 0$ *such that the ruler and his first successor will not immediately conflict only if* $\widetilde{S}^1 \geq \widetilde{S}^2 - \hat{d'}_2$.

Proposition A.4. (i) *If* $\widetilde{S}^1 > \widetilde{S}^2$*, then both the lower and the upper bounds of the first successor's designation interval are weakly greater than their counterparts in the single-candidate case.*

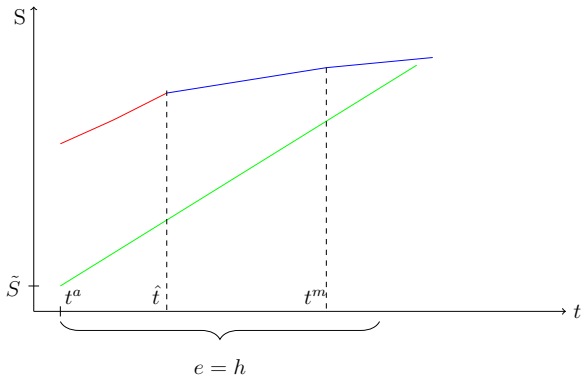
(ii) *If the optimal designation time for the first successor does not differ from that in the single-candidate case, then the duration of this successor–ruler's honeymoon phase will be weakly lower than that in the single-candidate case. Moreover, this honeymoon phase will be weakly less than that in Proposition 8 of the main model.*

Proposition A.3 is similar to Proposition 7. It indicates that the ruler has the incentive to strip the first successor as soon as possible when he is too weak to seize the power. Proposition A.4 is similar to Proposition 8. It indicates that the first successor is replaceable, and the ruler will prefer to choose any successor in a later period, thereby avoiding potential challenges. Moreover, the probability of conflict also increases in the presence of a backup candidate. The last part of this position shows that the probability of conflict in this situation is higher than in the main model with two candidates, because the backup candidate's power increases with time. Thus, the ruler is more likely to replace the successor with another candidate.

B Simulations

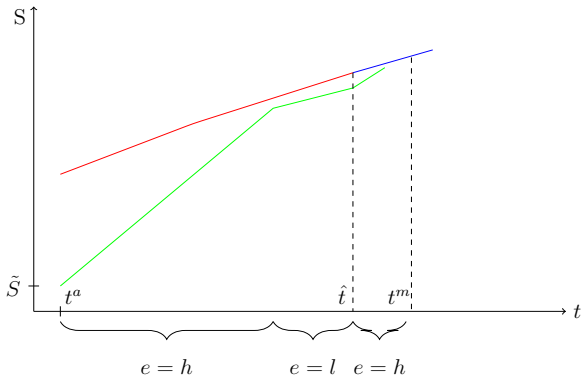
The conflict between the two parties is unlikely to occur under certain degenerated situations. Figure A.1 represents the scenario in which the ruler's health rapidly deteriorates, and the honeymoon phase is thus directly connected with the power transition phase. In this situation, the gap between the successor's power and the monitoring threshold is sufficient

Figure A.1



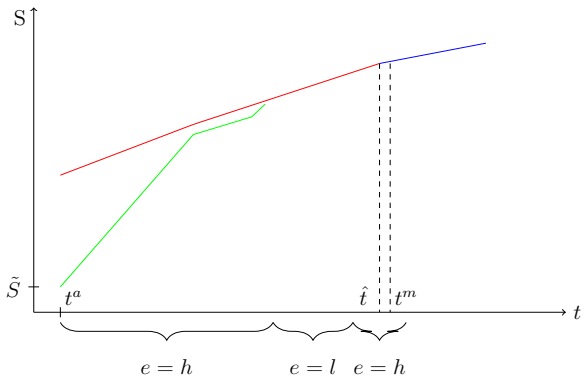
Notes: This figure represents the scenario that the ruler's health deteriorates quickly such that the honeymoon phase is connected with the power transition phase directly. This case indicate that the peaceful power transition is highly possible or the successor challenges the ruler with a high chance to win. The red curve represents the change of the monitoring thresholds $\bar{s}_t^m$ with time. The blue curve represents the change of the challenge thresholds $\bar{s}_t^c$ with time. When $t \leq \hat{t}$, $\bar{s}_t^m = \bar{s}_t^c$; when $\hat{t} < \bar{t}^m$, $\bar{s}_t^c < \bar{s}_t^m$; when $\bar{t}^m \leq t$, $\bar{s}_t^m$ does not exists. The green line represents the change of the expected power accrued by the successor with time. Since the successor always chooses high effort, the average power increase rate is $p_h(H - L) + L$. The parameters are chosen as follows: $b = 10$, $\delta = 0.7$, $R = 10$, $r = 0.1$, $L = 0.001$, $H = 0.01$, $p_h = 0.5$, $w = 0.05$, $\eta = 0.7$ $p_t = p_{t-1} + 0.01$, $p_0 = 0$, $\tilde{S} = 0.01$. Thresholds are calculated as: $\hat{t} = 6$, $t^m = 60$.

Figure A.2



Notes: This figure represents the scenario that the ruler's health deteriorates moderately such that after the honeymoon phase the successor has to keep a low profile to avoid the conflict and wait for the deterioration of the ruler's health. This situation indicates that the successor has a high chance to succeeds the throne. The red curve represents the change of the monitoring thresholds $\bar{s}_t^m$ with time. The blue curve represents the change of the challenge thresholds $\bar{s}_t^c$ with time. When $t \leq \hat{t}$, $\bar{s}_t^m = \bar{s}_t^c$; when $\hat{t} < \bar{t}^m$, $\bar{s}_t^c < \bar{s}_t^m$; when $\bar{t}^m \leq t$, $\bar{s}_t^m$ does not exists. The green line represents the change of the expected power accrued by the successor with time. When the successor chooses high effort, the average power increase rate is $p_h(H - L) + L$. When the successor chooses low effort, the power increase rate is L. The parameters are chosen as follows: $b = 10$, $\delta = 0.7$, $R = 10$, $r = 0.1$, $L = 0.001$, $H = 0.01$, $p_h = 0.5$, $w = 0.05$, $\eta = 0.7$ $p_t = p_{t-1} + 0.005$, $p_0 = 0$, $\tilde{S} = 0.29$. Thresholds are calculated as: $\hat{t} = 81$, $t^m = 119$.

Figure A.3



Notes: This figure depicts the scenario under which a ruler remains healthy for a relatively long time. This situation indicates that the successor has a high chance to conflict with the ruler and low chance to win. The red curve represents the change of the monitoring thresholds $\bar{s}_t^m$ with time. The blue curve represents the change of the challenge thresholds $\bar{s}_t^c$ with time. When $t \leq \hat{t}$, $\bar{s}_t^m = \bar{s}_t^c$; when $\hat{t} < \bar{t}^m$, $\bar{s}_t^c < \bar{s}_t^m$; when $\bar{t}^m \leq t$, $\bar{s}_t^m$ does not exists. The parameters are chosen as follows: $b = 10$, $\delta = 0.7$, $R = 10$, $r = 0.1$, $L = 0.001$, $H = 0.01$, $p_h = 0.5$, $w = 0.05$, $\eta = 0.7$ $p_t = p_{t-1} + 0.001$, $p_0 = 0$, $\tilde{S} = 0.01$. Thresholds are calculated as: $\hat{t} = 404$, $t^m = 592$.

to avoid the conflict during the period $\hat{t}$ so that the successor does not worry about being stripped of his title. Thereafter, the successor will challenge the throne only if his power reaches his challenge threshold (blue line in Figure A.1). This scenario can be observed in reality: if a weak (or young) successor is designated by an old ruler, then a conflict is unlikely to occur. Figure A.2 presents another scenario in which the ruler's health worsens at a moderate speed. Although the power increase of the successor is sufficiently large to raise the suspicion of the ruler, the successor can maintain his position until period $\hat{t}$ by keeping a low profile and lowering the power increase. Figure A.3 presents the scenario in which the ruler's health worsens slowly, then as the result in Proposition 5, the successor needs to maintain a low profile after the honeymoon phase, however if the ruler lives long enough, the conflict is unavoidable.