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Doctoral Dissertation

Capturing value from decentralized supply chain with third party reverse logistics

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“ Give value to every word in a context. Do not immediately reject those that do not conform to your ideas. Think that the idea might have been expressed from a different viewpoint, and be patient until the end… ”

M.F.Gülen

KENT STATE UNIVERSITY

Department of Management and Information Systems

Doctor of Philosophy in Business Administration (concentration in Operations Management)

Abstract

Capturing value from decentralized supply chain with third party reverse logistics

by Yertai Tanai

As competition in the global market continues to grow and customers are becoming more environmentally as well as cost conscious, recent trends in retail practices attest to the attention and resources devoted to the returns in supply chains. Topping the list is the economic incentive to reap as much value as possible from returns. With the growth of an internet consumer base, vigorous competition and the advance of online sales regardless of product type, size and locations, many firms provide generous return policies. This has caused a significant increase in the volume of reverse flows and therefore great potential for value recovery from returns. According to the National Retail Federation, the value of merchandise returned amounted to $260.5 billion in 2015. Hence, ways of improving the performance of a supply chain through effectively and efficiently closing the loop have received considerable attention both from academic researchers and industry practitioners over the past two decades. One way to recoup returns value as quickly as possible is to decentralize reverse logistics functions to third party reverse logistics providers (3PRLP). Outsourcing to a 3PRLP allows a firm to gain a state-of-the-art reverse logistics program immediately thereby avoiding the capital investment and start up delay required to implement an in-house RL program.

This dissertation proposes two models of a Closed-Loop Supply Chain (CLSC) with independent 3PRLP for returns processing. The first model presents a CLSC where demand is generated by a stochastic process. A fraction of the units that are initially sold are returned by the consumers for a full refund in every period. We model the forward flow interaction between the supplier, the retailer and 3PRLP by a widely accepted control policy that is lot size-reorder point inventory policy, which is detailed by the Markov process. We further propose a queuing network to capture reverse flow activities of the 3PRLP, which consists of customer decision delay and each of the 3PRLP activities. We characterize the expected profits for both firms and derive the effects of key parameters through set of numerical examples. The results of optimization based on numerical examples indicate that both firms' benefits from processing returns increase with an increasing returns rate. This is due to fact that the retailer captures more profits through selling processed returns at the price of new product. The 3PRLP unambiguously earns more profit from increasing product returns since the fee from processing returns is sole source of revenue. Furthermore, the directions of effects of changes in the holding cost are similar for both the retailer and 3PRLP. However, the magnitude of effects of the same parameter are quite opposite. Interestingly, the retailer's profit appears to be more sensitive to the holding cost than that of the 3PRLP's profit.

The second model analyzes coordination issues between a retailer and a 3PRLP to manage product returns. We formulate the returns processing capacity of a 3PRLP as a two-input production function where there is only one variable input. Crucially, this implies that the 3PRLP's short run marginal cost is strictly increasing. This key feature of the 3PRLP's short run cost function motivates two supply chain interaction scenarios. In an uncoordinated supply chain, the retailer acts as a market leader who makes a take-it-or-leave-it fee and quantity of returns offers to the 3PRLP. With increasing marginal cost of returns processing and retailer market power, the quantity of returns processed is inefficiently low due to a standard monopsony argument. In a coordinated supply chain, the retailer and the 3PRLP jointly decide on the returns quantity to be processed in order to maximize the total profit for the supply chain. An appealing approach to model how the benefit to coordination is shared between the two firms is Nash bargaining. Accordingly, we characterize the Nash bargaining solution with asymmetric bargaining powers, assuming that the disagreement payoffs are given by the uncoordinated supply chain profit levels. The underlying

model is one where the retailer and the 3PRLP negotiate the quantity of returns and the per unit fee, while both recognize that if they fail to reach an agreement, the retailer is poised to make a unilateral offer as in the uncoordinated case.

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Dedicated to my parents, Tanai and Aziman,
for their continuing unconditional support …

Chapter 1

Introduction

1.1 Background

As the competition in the global market continues to grow and customers becoming more environmental as well as cost conscious, several trends are adding to the attention and resources devoted to reverse flow of the products. Topping the list is the economic incentive. As the volume of returned products continues to grow rapidly, the big focus is on the product recovery to reap as much value as possible from returned products at which it is worthwhile to pour additional resources. The other one is the environmental incentive. As the consumers become more environmentally conscious as well as government regulations tighten, firms are under pressure to manage reverse flow of products in order either to recover or to dispose returns properly. Increasingly, many retailers and manufacturers are establishing economically and environmentally sustainable supply chains in order to maximize product recovery value.

Product returns are common and can occur as a result of commercial (consumer) returns by customers of the product within the return policy grace period, end-of-use returns when a functional product is upgraded technologically and end-of-life returns for products that are technologically obsolete or no longer contain any utility for the customers (Guide & Van Wassenhove, 2009). In terms of volumes of these returns, the volume of commercial product returns has significant economic value. According to National Retail Federation, the average customer return rate for North American retailers is 8.1 percent of sales (Greve, 2014). Varying among industries, Advanced Market Research estimates that the return rate is 8.5 percent for consumer electronics, 19.4 percent for apparel, 30 percent for catalog retailing, 3 percent for

durable goods (Christopher, 2003) and for fashion products it can be as high as 75 percent (Mostard & Teunter, 2006). For consumer electronics only, Accenture research estimated the annual cost of returns in the United States was $16.7 billion in 2011 and indicated that 95 percent of the returns may be classified as non-defective (Accenture, 2011). There are many reasons why customers return the products purchased. In many cases, customers return products because of failure to meet the expectations of consumers such as different quality, ending season, wrong model and wrong sizes. Such returns are especially apparent in online shopping where customers do not get to see physical product before making purchase decision. Consequently, the first rationale is the rapidly increasing online shopping. This rise of online sales relative to traditional brick-and-mortar sales means an overall increase in the probability of return (Ofek, Katona, & Sarvary, 2011). In particular, it is apparent in the items that are best experienced by in-person such as apparel and home goods. Generally it is difficult for a customer to judge artistic feature of an items design, texture and color without physically seeing or feeling it as opposed to merely viewing a digital image on a computer screen. As a result, the probability of returning an item purchased is higher for the online channel. Second factor that increases this probability is that some customers buy several similar items with the intention of keeping only the one they like the best.

Another reason for the increase in returns is the customer service policies of some of the largest retailers, which make the acceptance of returns effortless (e.g., no questions asked, no receipts necessary and no time limits). For instance, Zappos.com provides a full return within a year and pays for both original and return shipping. Amazon.com gives full refund within at least 30 days of purchase and also pays for both ways of shipping. The wholesaler giant Costco has no time limit returns policy on most of the non-perishable products, which allows customers to return any time. Furthermore, the fact that returns can be made via mail without traveling the physical distance makes the return process more anonymous and effortless. In fact, many firms realizing that effortless and seamless returns experience of the customer will offer a competitive advantage (Terry, 2014). As a result, this trend of increase in returns will likely to thrive further in future.

In the business literature the terms reverse logistics (RL) and closed-loop supply chain (CLSC) are often used interchangeably. According to American Reverse Logistics Executive Council, reverse logistics (RL)

can be viewed as the process of planning, implementing and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information, from the point of consumption back to the point of origin, for the purpose of recapturing their value or proper disposal (Rogers & Tibben-Lembke, 1998). A more recent definition of CLSC has been stated by Guide and Van Wassenhove (2009) "as the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with dynamic recovery of value from different types and volumes of returns over time".

Consequently, CLSCs focus on integrating the forward and reverse channel operations of the supply chain into one streamlined operation in order to maximize the value from product returns. Over the past two decades, CLSCs have received considerable attention both from academic researchers and industry practitioners, yet CLSCs are often characterized by a lack of formal systems and procedures to guide management decision-making (A. Atasu, Sarvary, & Van Wassenhove, 2008). The large number of products that are returned in any given year (even when focusing only on the United States) creates huge potential as well as challenges for value recovery. Guide and Van Wassenhove (2009) mention the example of a computer network manufacturer that destroyed more than $700 million worth of perfectly operational returned products. The computer manufacturer Hewlett-Packard estimated that they were able to recover less than half of the value of their returned products (Guide, Souza, Van Wassenhove, & Blackburn, 2006). Therefore, many firms and especially retailers are devoting more attention and resources to CLSC as they seek to extract as much value as possible from returned goods.

Decentralizing the reverse logistics

The main drivers for CLSC design are the volume of returns, the marginal value of time and the quality of returned product (Guide & Van Wassenhove, 2009). Given the fact that volume of returns is increasing, it should be clear that the most of the product returns, especially commercial returns, are not waste. However, if slow processes are used, the recoupable value from the returns is soon gone. In other words, commercial products that have been rarely used are best reintroduced market as quickly as possible in order to maximize the recovery value. In particular, speed of returns process is even more crucial in case of time-sensitive products such as PCs. It was estimated that the life cycle of a PC is 3 to 4 months, and its

value diminishes at 1% per week (Guide & Van Wassenhove, 2009).

Although the simplicity of returning a product for consumers is a competitive advantage at the front-end, the returns handling that CLSC should integrate is often complex to operate at the back-end. This complexity is compounded by the substantial uncertainties regarding the timing, volume and condition of the returned items (Serrato, Ryan, & Gaytán, 2007). The variability in characteristics and policies associated with each return indicates one of many possible outcomes when processing a returned item. For instance, the item may be returned to the OEM, transferred to another store, repackaged, repaired, liquidated, disassembled or reused. Returns management requires different processes and technologies, capabilities and expertise than forward operations (Terry, 2014; Greve & Davis, 2012). Thus firms with an effective forward logistics capability may not be able to operate a productive reverse operation for processing returns.

Consequently, given the time sensitivity of returns process and complexities of designing and managing the reverse logistics function in a supply chain, many firms have adopted a strategy to outsource the reverse logistics function to a third party reverse logistics provider (3PRLP). In fact, most Fortune 1000 retailers and consumer goods manufactures outsource part or all of their reverse logistics functions. Moreover, this trend is expected to grow globally over next 20 years (Greve, 2014). Especially most of the well-known retail chains and manufactures such as Walmart, Dell, Target, HP, Unilever, Pfizer and The Home Depot with well-established and developed forward logistics channels outsource their RL (Greve, 2014). The main reason for outsourcing RL is that RL is not considered to be a core competency of the firm (Terry, 2014; Serrato et al., 2007). By outsourcing its RL to a 3PRLP a firm can focus on doing what they do best i.e. producing and selling. Outsourcing to a 3PRLP also allows a firm to gain a state-of-the-art RL program immediately thereby avoiding the capital investment and start up delay required to implement an in-house RL program. Most 3PRLPs have existing facilities that can be leveraged depending on the situation, or will open facilities in the best locations to minimize processing costs. When 3PRLPs provide RL service, the costs of the entire infrastructure required, building facility, software and equipment can be consolidated to their price (Terry, 2014; Serrato et al., 2007). Additionally, most of the 3PRLP contracts include some form of price per item cap that makes budgeting and planning easy for the outsourcing firm. Finally, since the 3PRLPs are focused on the processing returns, they can provide cutting-edge analysis

and recommendations for an integrated and effective CLSC design.

1.2 Research gap and contributions to existing literature

This dissertation considers a CLSC where there is no distinction between a new product and a returned product once the returned product has undergone a series of RL processing activities to repair and repackage the product for reuse. Moreover, we consider that the returns processing activities are outsourced to a 3PRLP.

Our contributions to the current literature are multiple folds. Primarily, although there a is vast number of publications on the role of RL in supply chain management, most of the literature focuses on technical and operational issues of returns management. Furthermore these studies are in the context of manufacturing and remanufacturing. In this context, the manufacturer collects and remanufactures the returned items. Evidently, most of the approaches focus on cost minimization. Recently, studies have indicated that there is need to advance from technical perspective to business model view (Guide et al., 2006). Along with this need, in the research herein, we take more holistic approach that emphasizes on value recovery in addition operating costs. Our approach advances from technical perspective to integrated CLSC that maximizes the value for entire supply chain.

Moreover, in the mainstream literature returned items are end-of-use items, which the manufacturers disassemble, reuse, recycle or dispose of. But our modeling is generalizable to both commercial returns that are merely used and are resalable after processing, and end-of-use returns for which the user no longer has utility left for. In this dissertation, we model the case where the customer returns items to the retailer and not the manufacturer. The proposed model eliminates one leg of transportation back to the manufacturer and hence remanufacturing.

Additionally, the models in the current literature about product return, particularly remanufacturing, do not consider outsourcing explicitly. Since, product returns are managed by the manufacturer they involve in-house remanufacturing in parallel with manufacturing of new items. However, in this study, we model the CLSC where 3PRLP operates independently while coordinating activities with the retailer. We

propose a model that gives detailed analysis of 3PRLP activities, which can be specific to product or firm. Although there are studies that consider outsourcing of RL functions to 3PRLP, their modeling approaches are different. For example, models that evaluate when it is appropriate to outsource RL have been proposed but these models do not detail the processes of the RL (Atalay Atasu, Toktay, & Van Wassenhove, 2013; Serrato et al., 2007; Savaskan & Van Wassenhove, 2006; Savaskan, Bhattacharya, & Van Wassenhove, 2004). Our model assumes that the RL functions are already outsourced to a 3PRLP and optimizes the RL operations from the 3PRLP's point of view. We believe to the best of our knowledge, that the model herein represents the first quantitative examination of the detailed processes of RL by 3PRLP.

Another facet of this dissertation is the study of a coordination issues between a firm and a 3PRLP when a firm uses a 3PRLP to manage product returns. While a substantial amount of research focuses on modeling the decisions to control the returns flow from a manufacturing or a marketing perspective, few papers have addressed the pricing issues that arise between a 3PRLP and its clients. The research herein contributes to bridging this gap by examining a CLSC design with one retailer and an independent 3PRLP who processes returns that are under a full refund policy. Again, the focus is on the commercial returns for products that fail to meet consumer expectations, are barely used and are returned within the policy's return period. Hence the returns that are processed by the 3PRLP are considered to be as good as new and resold at the original retail price. The objective is to determine the optimal return quantity and returns processing fee in a CLSC where a retailer elects to use a 3PRLP to manage its product returns. The interaction between the retailer and 3PRLP with respect to the returns quantity and fee is investigated under coordinated and uncoordinated supply chain alignments.

1.3 Research objectives

The main objective is to investigate how customers return decisions and RL choices affect the forward channel decision and how the parties in the forward and reverse channels interact to process returns. For this, we consider a two-echelon CLSC, which consists of a supplier, a retailer and a 3PRLP who operates independently from the supplier and the retailer. In particular, we address the following research questions:

1. How are the inventory policy and total expected profits/costs of the retailer affected by the percentage of demand that is returned?

2. Given the stochastic returns, how does the 3PRLP allocate its labor into different types of processes that require different skills?

3. Can the retailer and the 3PRLP achieve overall profit gains by coordinating decisions regarding quantity and processing fees as compared to a take-it-or-leave-it, "arms length" contracts?

4. If gains exist, how are they affected by changes in exogenous demand and cost parameters?

5. What are the effects of demand and cost parameters on how the gains are shared between supply chain members?

In general this dissertation proposes two different models on the CLSC with independent 3PRLP for returns processing. The first model presents a CLSC where demand is generated by a stochastic process. A fraction of the units that are initially sold are returned by the consumers for a full refund in every period. We model the forward flow interaction between the supplier, the retailer and 3PRLP by a widely accepted control policy that is lot size-reorder point inventory policy, which is detailed by the Markov process. We further propose a queuing network to capture reverse flow activities of the 3PRLP, which consists of customer decision delay and each of the 3PRLP activities. We characterize the expected profits for both firms and derive the effects of key parameters through set of numerical examples. The results of optimization based on numerical examples indicate that both firms' profits increase with an increasing returns rate. This is due to fact that the retailer captures more profits through selling processed returns at the price of new product. The 3PRLP unambiguously earns more profit from increasing product returns since the fee from processing returns is sole source of revenue. Furthermore, the directions of effects of changes in the holding cost are similar for both the retailer and 3PRLP. However, the magnitude of effects of the same parameter are quite opposite. Interestingly, the retailer's profit appears to be more sensitive to the holding cost than that of the 3PRLP's profit.

The second model analyzes coordination issues between a retailer and a 3PRLP to manage product returns. We formulate the returns processing capacity of a 3PRLP as a two-input production function where there is only one variable input. Crucially, this implies that the 3PRLP's short run marginal cost is strictly increasing. This key feature of the 3PRLP's short run cost function motivates two supply chain interaction scenarios. In an uncoordinated supply chain, the retailer acts as a market leader who makes a take-it-or-leave-it fee and quantity of returns offers to the 3PRLP. With increasing marginal cost of returns processing and retailer market power, the quantity of returns processed is inefficiently low due to a standard monopsony argument. In a coordinated supply chain, the retailer and the 3PRLP jointly decide on the returns quantity to be processed in order to maximize the total profit for the supply chain. An appealing approach to model how the benefit to coordination is shared between the two firms is Nash bargaining. Accordingly, we characterize the Nash bargaining solution with asymmetric bargaining powers, assuming that the disagreement payoffs are given by the uncoordinated supply chain profit levels. The underlying model is one where the retailer and the 3PRLP negotiate the quantity of returns and the per unit fee, while both recognize that if they fail to reach an agreement, the retailer is poised to make a unilateral offer as in the uncoordinated case.

We find that regardless of bargaining powers, the coordinated supply chain processes greater quantities of returns than the uncoordinated supply chain, do so at a higher fee per unit, but yield mutual benefits to the parties. Our analysis also yields a rich set of analytical sensitivity results. For instance, increases in the manufacturer's wholesale price or the retailer's inventory cost lead to higher quantities of processed returns and in the uncoordinated case, to a higher fee. Interestingly an increase in demand has the opposite effect, leading to lower quantities and a lower uncoordinated fee. Moreover, for a wide range of relevant cases, an increase in the rate of product returns has no effect on the quantities or the uncoordinated fee. In the coordinated case, the effect on the fee of a change in a parameter cannot be determined analytically, thus we provide a set of numerical examples. In these examples, the coordinated fee responds in a similar manner as the uncoordinated fee. The numerical analysis also shows that the overall profit gain to coordination is increasing in the rate of product returns and the wholesale price, but decreasing in the price of the input used by the 3PRLP.

The remainder of the dissertation is organized as follows. In Chapter 2, we discuss the related literature and highlight this dissertation's contribution. In Chapter 3 we propose Stochastic CLSC with 3PRLP model that details the reprocessing activities of the 3PRLP based on the retailer's operational decisions. In Chapter 4, we propose Coordination in CLSC with 3PRLP. We derive the optimal quantities of processed returns and the fees in uncoordinated and coordinated supply chains. In Chapter 5 we discuss possible extensions and present concluding remarks.

Chapter 2

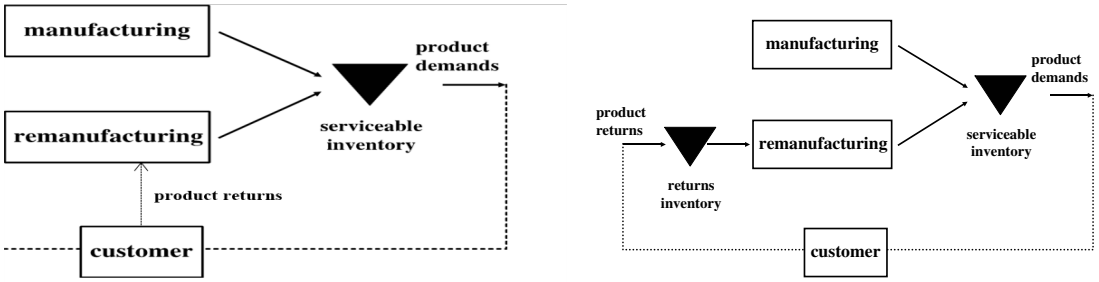
Literature Review

2.1 Stochastic models in CLSC

Most of the current literature on returns management is in the context of manufacturing and remanufacturing. These studies focus generally on technical aspect of RL that minimizes the average cost of inventory. From the retailer's inventory management point of view, our work is primarily related to the inventory and production planning streams of research in the CLSC area. We highlight that studies in inventory planning represent majority of research in CLSC and we acknowledge that many authors have provided excellent contributions to this theme. We shall discuss studies only related to our study. To provide more details about current literature, we ask readers to refer two recent literature review articles on CLSC. First, Akçalı and Çetinkaya (2011) review existing quantitative literature on inventory and production planning for CLSC systems up to year 2009. Next, Govindan, Soleimani, and Kannan (2014) review RL and CLSC models published between years 2007 and 2013.

Stochastic inventory control approaches that integrate returns are generally classified into to single-level versus two-level inventory structures. These are illustrated in Figure 2.1. We summarize studies of single-level inventory structures in Table 2.1 and two-level inventory structures in Table 2.2. As one can notice from these tables, most of the studies emphasize on the cost minimization. Conversely, we emphasize on value recovery by maximizing profit function. This approach puts more strategic lens on the overall supply chain in regards to changes in parameters. Furthermore, most of the studies assume

returns to be independent from the demand. However, we model returns as a fraction of demand and do not explicitly assume independence of returns.



(a) Single-level inventory

(b) Two-level inventory

Figure 2.1: Inventory structures

Article	Remarks	Demand & Return	Objective
Heyman (1977)	Continuous review model. No order lead-time.	Independent continuous RV for demand and return.	Cost minimization
Cohen et al. (1980)	Single period newsvendor model. No lead-time for ordering. Excess demand is lost.	Demand and returns are continuous iid.	Cost minimization
Muckstadt and Isaac (1981)	Continuous review model. Considers fixed order costs and non-zero procurement and repair lead-times. The values of the control parameters are determined via an approximation of the net inventory distribution.	Independent Poisson demand and return processes.	Cost minimization
Toktay et al. (2000)	CLSC model using queuing networks approach. Base-stock policy.	Independent Poisson demand and return processes.	Cost minimization

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Fleischmann et al. (2002)	Extension of Muckstadt and Isaac (1981) with closed form analytical solution.	Independent Poisson demand and return processes.	Cost minimization
Mostard and Teunter (2006)	Analyze a newsboy problem with resalable return.	Demand is continuous RV. Return is a fraction of demand.	Profit maximization
Guide et al. (2006)	Evaluate alternative reverse supply chain designs using network flow models capturing the effects of delays on costs and revenues.	Demand is Poisson. Return is fraction of demand.	Profit maximization
Karaer and Lee (2009)	CLSC model for single period newsboy problem. Quantify the value of information visibility on the reverse supply chain using RFID	Demand and returns are continuous iid.	Cost minimization
Alinovi et al. (2011)	Formulate stochastic EOQ model under discrete time domain	Demand is continuous RV. Return is a fraction of demand.	Cost minimization

Table 2.1: Single-level inventory articles

Our work is closely related to first line of research i.e. one-level inventory models. Specifically, we advance our CLSC model using inventory control approach studied by Muckstadt and Isaac (1981) and Fleischmann et al. (2002). Then, we detail the RL activities of 3PRLP using queuing network. This approach differs from the studies that use queuing network in two ways. First, while we use queuing network for RL activities only, others use queuing network for the entire supply chain (Guide et al., 2006; Toktay et al., 2000). Second, in our RL modeling we do not necessarily implement any specific inventory control policy

in order to analyze economic performance. The costs processing of returns depend on the number of labor and items in each queue node.

Article	Remarks	Demand & Return	Objective
Inderfurth (1997)	A fixed deterministic lead-time for remanufacturing as well as manufacturing. Simultaneous procurement, remanufacturing and disposal decisions.	Demand and return are continuous RVs. Dependent demand and return.	Cost minimization
van der Laan and Salomon (1997)	A PULL strategy had been investigated. Disposal policy when the system inventories become too high. Non-zero manufacturing and remanufacturing lead-times.	Demand and return are Poisson processes. Dependent demands and returns.	Cost minimization
van der Laan et al. (1999)	Examines both PUSH and PULL strategies	Demands and returns are continuous RVs. Dependent demand and return.	Cost minimization
Teunter and Vlachos (2002)	Average cost is discounted to the beginning of time. Extend van der Laan and Salomon (1997)	Independent Poisson demands and returns.	Cost minimization
van der Laan and Teunter (2006)	A heuristical approach for remanufacturing. Multiple types of policies are analyzed. Generalizes PUSH and PULL strategies.	Independent Poisson demands and returns.	Cost minimization

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Behret and Korugan (2009)	Simulation approach using queue nodes for each process of the remanufacturing	Demand is Poisson. Return is fraction of demand.	Cost minimization
Shi et al. (2011)	Study the production-planning problem for a multi-product closed loop system.	Independent continuous demands and returns.	Profit maximization

Table 2.2: Two-level inventory articles

Studies that explicitly model the 3PRLP or RL activities are summarized in Table 2.3. Most of these articles address the interaction between the manufacturer and the retailer. They analyze the problem of choosing appropriate reverse channel structures (centralized vs. decentralized) using various collection cost functions (linear vs. non-linear) under different types of economic environments (monopoly vs. competitive) (see for examples Atalay Atasu et al., 2013; Savaskan and Van Wassenhove, 2006; Savaskan et al., 2004). On the contrary, we focus on the decentralized CLSC where RL activities are outsourced to 3PRLP. Furthermore, our analysis is based on activities of the 3PRLP, which contains collection activity as well. Other studies evaluate the decision process of when it is appropriate to outsource the RL activities (for example see Serrato et al., 2007). Nevertheless, our modeling is different from this line of research that we assume RL activities are already outsourced to 3PRLP.

Article	Remarks	Objective
Savaskan et al. (2004)	Address the problem of choosing the appropriate reverse channel structure for the collection of used products from customers. Game theory approach.	Profit maximization
Savaskan and Van Wassenhove (2006)	Extends the findings of Savaskan et al. (2004) work to a competitive retailing environment.	Profit maximization

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Serrato et al. (2007)	A Markov decision model to evaluate outsourcing in reverse logistics.	Cost minimization
Efendigil et al. (2008)	A 3PRLP selection in the presence of vagueness. A two phase model based on artificial neural networks and fuzzy logic.	Performance maximization
Atalay Atasu et al. (2013)	Analysis of the impact of collection cost structure on the optimal reverse channel choice of manufacturers.	Profit maximization

Table 2.3: 3PRLP articles

2.2 Coordination models in CSLC

In terms of coordination in CLSCs, our paper draws on the recent research stream that models CLSC from a business economics perspective (see for examples Guide and Van Wassenhove, 2009; A. Atasu et al., 2008). In this regard, Savaskan et al. (2004) address the problem of choosing the appropriate reverse channel structure for the collection of used products from customers. They analyze three different reverse channel formats deployed by the manufacturer. These formats include returns collection from the customer by the manufacturer, the retailer or a third party. They compare the performance of the models with respect to the wholesale price, the retail price and the product returns rate. They find that from manufacturer's standpoint, the closest agent who is closest to the consumers (i.e. the retailer) is the most effective returns collector. Savaskan and Van Wassenhove (2006) extend the model of Savaskan et al. (2004) to a competitive retail environment with two retailers. They show that in order to price discriminate between the retailers, the manufacturer can use buy-back payments transferred to the retailers in exchange for the returns. Finally, Atalay Atasu et al. (2013) analyze the impact of different collection cost structures on the optimal reverse channel choice of manufacturers. In the above models the key decision variables are the wholesale price, the selling price and the amount of returns collected from customers. Furthermore they formulate

the rate of return as a function of collection effort measured by a firm's investment in collection activities. The authors note that in principle such investments should exhibit diminishing returns in the sense that an increase in the rate of returns is slower than the increase in investment.

Our modeling approach differs and complements this research stream in several respects. First, unlike remanufacturing environments where returns occur due to either end-of-life issues when a product becomes technologically obsolete, end-of-use issues when a functional product is upgraded technologically or issues of product failure and warranty, we focus on consumer (commercial) returns management where returns occur due to unmet expectations of consumers about the product. We analyze the CLSC from a different perspective, namely the interaction between the retailer and the 3PRLP given fixed wholesale and retail prices. Second, in contrast to existing work, we model a situation where a full refund policy applies and the retailer takes the returns rate as given. Indeed, in today's competitive retail environment retailers are forced to adopt generous refund policies. Moreover, unlike end-of-life returns, for consumer returns it is relatively difficult to control the rate of product returns since it depends on the likelihood that the product fails to meet consumers' expectations. Third, the 3PRLP's marginal cost to process returns is increasing. Thus we make a similar assumption regarding processing costs as the literature on collection and reverse logistics costs (for examples Ferguson, Guide, and Souza, 2006; Savaskan et al., 2004). Namely we assume some form of diminishing returns. However in our model, increasing marginal cost only applies to the short run, while in the long run, either economies or diseconomies of scale are possible. This is important because Atalay Atasu et al. (2013)'s examples show that both scale patterns can occur depending on the industry. Fourth, we complement the above studies by considering operational aspects of the firm such as inventory management by means of an EOQ policy for the retailer.

We note that in the related literature from marketing and consumer behavior, researchers have focused on the problem of setting optimum returns policy between a manufacturer and a retailer as well as the use of incentives to control the returns flow. Some examples of setting return policies include using markdown discount (Tsay, 2001), target rebate (Ferguson et al., 2006), restocking fee (Jeffrey D Shulman, Foster, Coughlan, & Savaskan, 2011; Jeffrey D. Shulman, Coughlan, & Savaskan, 2009; Su, 2008), and returns flow (Ketzenberg & Zuidwijk, 2009). Again, in contrast to these studies, we assume fixed returns flow with full

refund policy and simply model the interaction between the retailer and the 3PRLP.

Chapter 3

Stochastic CLSC with 3PRLP model

3.1 Description and notation

In this chapter we present a stochastic CLSC with 3PRLP model. The product flow starts at the top echelon, where the supplier directly supplies to the retailer who is at the bottom echelon. Further product flows continues from the retailer to the customer. However, the flow of products doest not stop upon their distribution to retailers as well as from retailer to the customers. Beside typical forward channel of the products from supplier to the customers, there is reverse channel of the products those being returned back to the market. Hence, the proposed CLSC model consists of a supplier, a retailer and a 3PRLP who operates independently from the supplier and the retailer. The generic product flow of proposed CLSC is illustrated in Figure 3.1. The 3PRLP is involved with RL activities, which are initiated by the customers' decision to return the item that he/she has purchased.

During each time period, after a delay customers decide whether to keep the product or return it to the retailer. We call this delay as customers decision delay and note that this delay is commonly dictated by the return policy of the product. By deciding to return the items, customers implicitly activate the return process. As a result, a certain proportion of demand is returned each period. The reverse channel activities can be described as follows. The returned items are collected by the 3PRLP or customers can bring the items to either retailer or 3PRLP location. Once collection, the 3PRLP internally processes the items in its facility and send them back to the retailer in same as new conditions. There are different types of activities ranging from collection to sorting to repair and to repackaging. We note that processing of returns by the

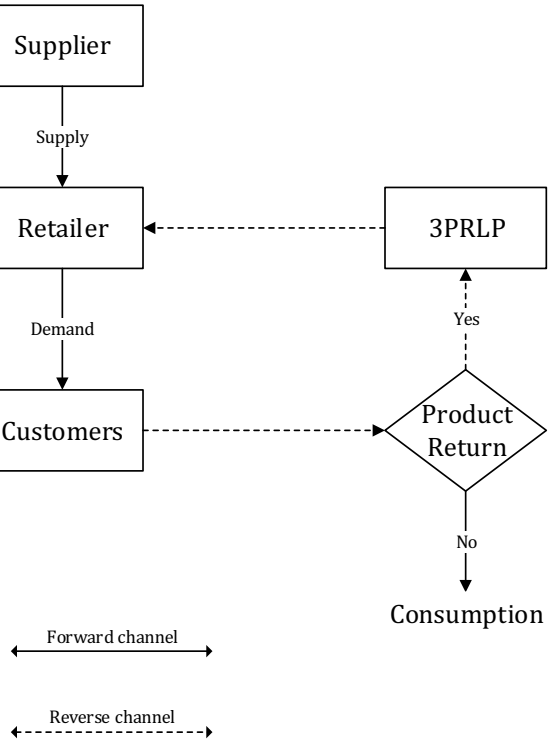


Figure 3.1: Product flow

3PRLP is specific to a particular item, and that the processing activities can be modified or generalized to more different kinds of RL functions.

For each type of activity, the 3PRLP employs different skill and number of labor and capital. For instance, the low skilled workers in sorting and repackaging activity are paid less than the highly skilled technicians in repair facility due to their skill and allowance through OEMs. Therefore, the fee charged and wage given by the 3PRLP is based on the activity that each return undergone. Furthermore, the processing times at each activity are also different from each other. Once the 3PRLP finishes processing the returns, it sends them back to the retailer or disposes.

The forward flow interaction between the supplier, the retailer and 3PRLP is governed by a widely accepted control policy that is lot size-reorder point inventory policy. We note that this policy is detailed by the Markov process. The backward flow or RL activities by 3PRLP is modeled as a queuing network,

which consists of customer decision delay and each of the 3PRLP activities. We summarize the notation for the M1 in Table 3.1. In section 3.2 we describe the lot size-reorder point model for the retailer and in section 3.3 we construct the queuing model for the 3PRLP to process the returns.

Retailer and forward flow	
Notation	**Description**
λ	Demand rate per unit time
τ	Fraction of demand that is returned per unit time
γ	3PRLP's processed returns supply rate per unit time
c	Unit cost to purchase an item ($)
p	Unit selling price ($)
a	Fixed cost to place an order ($)
b	Penalty cost for one unit backordered per unit time ($)
h	Cost to hold one unit in inventory per unit time ($)
r	Reorder point units
Q	Order size units
LT	Supplier lead time (constant)
3PRLP and reverse flow	
α_i	Percentage of returns that undergone through activity i per unit time
$1/\mu_d$	Customer mean decision delay to whether return the item
$1/\mu_i$	Mean processing time at activity i per unit time
k_i	Number of workers in activity i per unit time
f_i	Fee ($) charged from the retailer per unit return that undergone in activity i per unit time
w_i	Wage ($) per worker in activity i per unit time
h_i	Holding/storing cost of one unit at each activity i per unit time

Table 3.1: Notation for Stochastic CLSC with 3PRLP model

3.2 Retailer and forward flow

Each period the demand for the retailer is according to Poisson process with mean λ. Demand is satisfied from on-hand inventory based on availability of items whereas unsatisfied demand is backordered. Products returns represent τ percentage of total. Hence, the input in each period for the RL activity follows a Poisson process with mean $\tau\lambda$. We assume that returned products serviced by the 3PRLP emerge are

as good as new items and are indistinguishable from the new items being made available to customers by the supplier. Without loss of generality, let γ be the mean rate of items serviced by the 3PRLP that are sent back to retailer each period. Thus as illustrated in Figure 3.1, the retailer has two sources of product replenishment. The first source is products received from the supplier in the forward channel of the supply chain; the second source is products that have undergone processing by the 3PRLP in the reverse channel of the supply chain. The first one is the supplier with a constant lead-time LT. The second source is the 3PRLP, which supplies returns as good as new items according to Poisson process with a mean rate of γ units per period. This is, in fact, not an explicit assumption, it is because Poisson input process of a queue imposes that output process is also Poisson (see section 3.3 for derivation of γ). The retailer's inventory structure is illustrated in Figure 3.2.

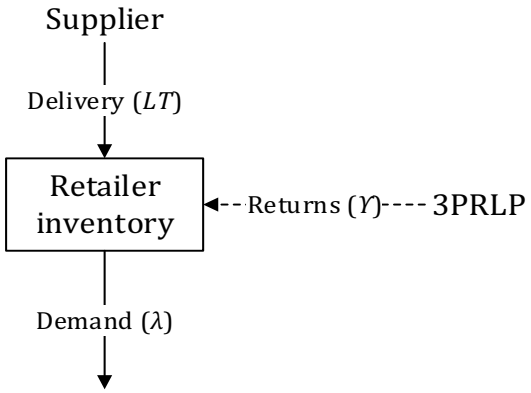


Figure 3.2: Retailer's inventory

The inventory cost factors are backorder cost b (per item per unit time), the holding cost h (per item per unit time) and the fixed order cost a (per order). We assume that the net inventory is continuously reviewed and that an (r, Q) inventory control policy is applied. On-hand inventory cannot be used rigorously to define the reorder point, since when a heavy demand occurs during some periods and a large number of backorders were incurred, then arrival of outstanding orders can never bring the on-hand inventory back to the reorder point. Therefore, the inventory position is a suitable level to apply control policy for defining reorder point. In other words, it is true that a heavy demand and a large number of backorders

during some period cause substantial number order placement. In this case, a reorder point in terms of inventory position will be crossed multiple times whereas a reorder point in terms of on-hand inventory may not be crossed at all (for more details see Hadley and Whitin, 1963).

Ultimately, to analyze the system, we are interested in the average number of on-hand inventory and average backorders. Since the lead times are constant and are not generated by Poisson process; we can not describe transitions between the on-hand inventory states. Instead we formulate a continuous-time Markov process for the inventory position. The advantage of this approach is that the supplier lead-time does not enter into the computation of steady-state probabilities. Hence a demand decreases the inventory position by one item; the arrival of a returned item from the 3PRLP increases the inventory position by one unit. Suppose when in a state $r+1$ and a demand occurs, the system moves from state $r+1$ to state $r+Q$ since the demand triggers the placement of the order. Note that unlike traditional inventory models, the state space is unbounded above. This is due to returns that are being supplied continuously according to Poisson process. The inventory position state transition diagram is depicted in Figure 3.3.

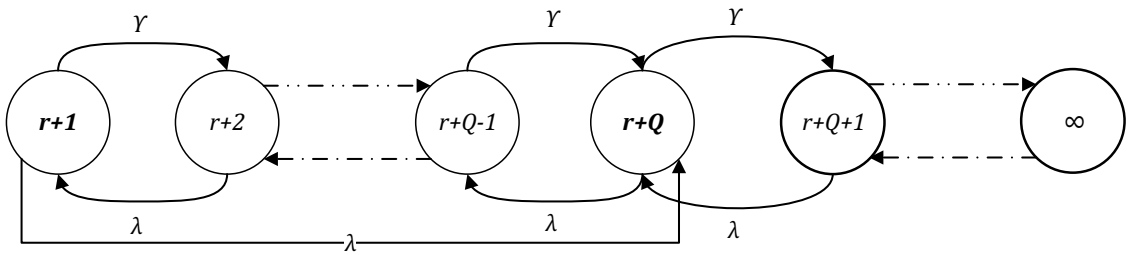


Figure 3.3: Inventory position transition diagram

Let $IP(t)$ be an inventory position at time t. Then we can write down the balance equations (transition rates) for this case as,

$$\lim_{\Delta_t \to 0} \Pr[IP(t+\Delta_t) = k | IP(t) = j] = \begin{cases} \lambda & \text{for } k = j-1, j \geq r+2, \\ \lambda & \text{for } k = r+Q, j = r+1, \\ \gamma & \text{for } k = j+1, j \geq r+1. \end{cases} \tag{3.1}$$

For $0 \leq \frac{\gamma}{\lambda} < 1$ the inventory position is ergodic and using balance equations, the steady-state distribution can be derived as (Fleischmann et al., 2002; Muckstadt & Isaac, 1981):

Next, following Fleischmann et al. (2002), we can write down the the distribution of net demand during the lead-time $D(t - LT, t)$ as $D(LT)$ since, it is independent of t. It follows that

$$\Pr[D(LT) = n] = \exp(-LT(\lambda + \gamma))(LT\lambda)^n \sum_{y=0}^{\infty} \frac{LT^{2y}(\lambda\gamma)^y}{y!(y+n)!}. \tag{3.2}$$

Furthermore $E[D(LT)] = LT(\lambda - \gamma)$ and $Var[D(LT)] = LT(\lambda + \gamma)$.

We now have all the preconditions for expected inventory cost function. Denote $IC_{RE}(r, Q)$ be the expected inventory cost per unit time in a steady-state. This function can be written as,

$$\begin{aligned} IC_{RE}(r,Q) &= a\frac{\lambda-\gamma}{Q} + \sum_{l=r+1}^{\infty} \eta(l)G(l) \\ &= \frac{1}{Q}\left[a(\lambda-\gamma) + \sum_{l=1}^{Q}(1-\rho^l)G(r+l) + (\gamma^{-Q}-1)\sum_{l=Q+1}^{\infty}\gamma^l(r+l)\right] \\ &= \frac{1}{Q}\left[a(\lambda-\gamma) + \sum_{k=s+1}^{s+Q} H(k)\right] \end{aligned} \tag{3.3}$$

where $H(k) = (1-\rho)\sum_{l=0}^{\infty}\rho^l G(k+l)$, which is convex in k and $G(l)$ is the sum of average holding and backorder costs at the inventory position l:

$$\begin{aligned} G(l) &= (h+b)\sum_{j=-\infty}^{l-1}(l-j)d_j + b(E[D(LT)] - l) \\ &= (h+b)\sum_{j=-\infty}^{l-1}\sum_{t=-\infty}^{j} d_j + b(E[D(LT)] - l). \end{aligned} \tag{3.4}$$

Furthermore, we now define the expected total cost for the retailer. Let $TC_{RE}(\cdot)$ be the expected total cost of retailer that is sum of expected inventory cost, purchase cost and the total fee paid to 3PRLP to process returns. Hence, assuming there are n number of activities in reverse flow (see section 3.3 for

details)

$$TC_{RE}(\cdot) = IC_{RE}(r, Q) + (\lambda - \gamma)c + \sum_{i=0}^{n} f_i \alpha_i \tau \lambda. \tag{3.5}$$

Finally, let $\Pi_{RE}(\cdot)$ be the expected profit per unit time for the retailer such that,

$$\Pi_{RE}(\cdot) = (1-\tau)\lambda p + \gamma p - \left[IC_{RE}(r, Q) + (\lambda - \gamma)c + \sum_{i=0}^{n} f_i \alpha_i \tau \lambda \right]. \tag{3.6}$$

The optimization problem for the retailer is to maximize the expected profit defined (3.6). Note that for a given set of cost parameters, the maximization problem above becomes minimization of expected inventory cost. Hence objective is to find non-negative integer pair (r^*, Q^*) that minimizes the expected inventory cost. The optimal solutions (r^*, Q^*) that minimize (3.3) can be found in a using complete enumeration (for details see Fleischmann et al., 2002; Muckstadt and Isaac, 1981). However, it is worth to mention that the implications of change in those parameters may not be reflected if only cost minimization model is adopted. The rationale here is that maximizing the profit or value for the entire supply chain.

Another cost component incurred by the retailer that we have not discussed this section is the fee $f_{3P}^{(i)}$ paid to 3PRLP to process returns in each activity i. At the same time, this fee turns out to be the only source of revenue for the 3PRLP. Hence we discuss this cost in next section 3.3 in detail since it does not affect the optimization problem for retailer in (3.6). In other words, fees paid to process returns do not influence reorder point or ordering quantities. This is intuitive such that the fee is exogenous and hence there is no influence of fee to the expected inventory cost.

3.3 3PRLP and reverse flow

In this section we use a queuing network to model the reverse flow of products in the CLSC. As discussed in model description 3.1, we assume that after a delay, τ fraction of the demand is returned during each period. Furthermore, we assume that the RL is independently managed by the 3PRLP who assumes all managerial responsibility for the return product flow. An example of a RL models with some sequential activities by the 3PRLP is illustrated in Figure 3.4.

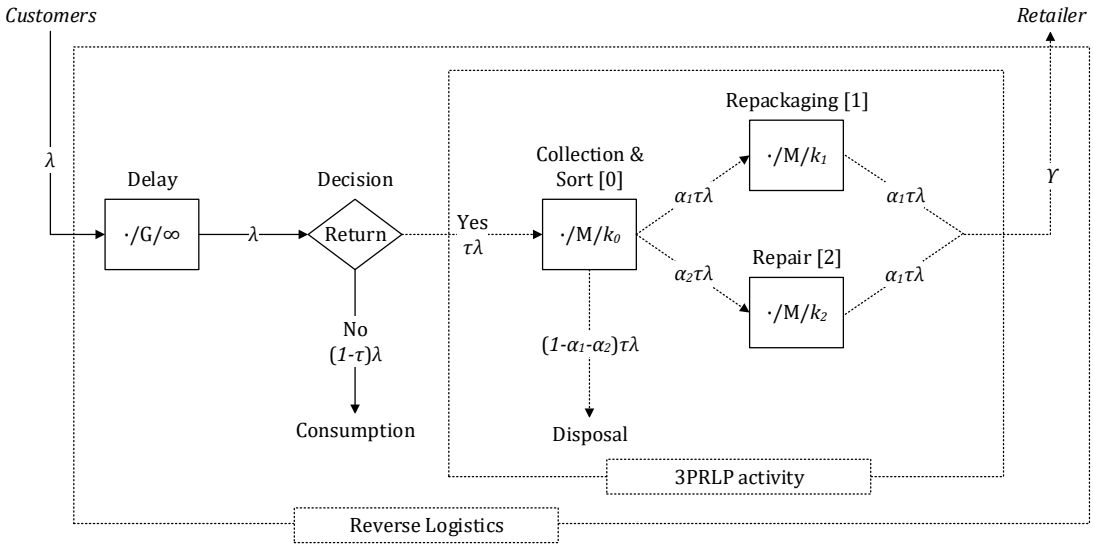


Figure 3.4: Reverse flow

Details of product flow in the RL are as follows. After the customer purchases an item, he or she may keep the item or return the item to the 3PRLP who is acting as the agent of the retailer. The time required by the customer to decide whether to keep or return the item is represented in the model as the customers' decision delay node and is modeled as an infinite-server queue with a general service time distribution with mean delay time equal to equal $1/\mu_d$. Toktay et al. (2000) also assumes the infinite-server queue with general service time distribution used to model customer return delays. Now if the customer decides not to return the item, then that item exists and never comes back to the system. Hence, $1-\tau$ portion of the demand exits the system each period which makes remaining $\tau\lambda$ as the input for the 3PRLP activities.

In order to keep track of input and output processes of 3PRLP activities we note following theorems. According to Mirasol (1963), the output process of infinite server queues with general service time is Poisson when the input is Poisson. In the same way, based on Burke (1956), the output process of multi-server queue is also Poisson and it is known as Burke's theorem. As indicated in above example 3.4, since the output of customers' decision delay is the input for the 3PRLP's first activity i.e. collection/sort node, it means 3PRLP receives returns according to Poisson process with mean $\tau\lambda$. Moreover, based on above theorems, we momentarily establish that all the input and output processes of 3PRLP activities are

according to Poisson process.

To exploit the tractability of the product-form queuing network theory (Baskett, Chandy, Muntz, & Palacios, 1975; Jackson, 1963), we assume that queues designated for the 3PRLP activities have exponential service times. Hence, the performance of the RL processing depends on the four service time distributions only through their means. Furthermore, we model the each activity of the 3PRLP as a multi-server queue. Also note that splitting a Poisson process using a Bernoulli switch is Poisson as well as merging multiple independent Poisson processes results in a Poisson process with a rate equal to the sum of individual rates (Ross, 1997). Hence, we have established that all the input and output processes of the 3PRLP activities are Poisson processes. Now let α_i be the fraction of return that is gone through activity i. This means $1-\sum_i \alpha_i$ fraction of returns is disposed. Consequently, after processing all the returns, the 3PRLP replenishes items back to the retailer according to Poisson process with mean $\sum_i \alpha_i \tau \lambda$. In other words, each period $\sum_i \alpha_i \tau$ fraction of the demand is processed and sent back to the retailer by the 3PRLP. Also we note that $\sum_i \alpha_i \tau \lambda$ corresponds to γ in previous section 3.2.

As mentioned above, we model each of 3PRLP activity as a multi-server queue in order to detail the activity (such that processing rate, number of workers in each department, average processing rates etc.) and derive economic performance based on these details. Therefore, each activity $i = 1..n$ has k_i workers and each worker has an independently and identically distributed exponential service-time distribution with mean $1/\mu_i$. Moreover, the 3PRLP charges a fee f_i per item processed from the retailer and pays wage w_i for every item processed, which both a fee and wages are specific to a particular activity. Furthermore, a fee charged, an wage paid as well as a cost incurred to hold an item are specific such that an activity i's costs can be different from j's. For instance, a repair activity might incur highest fee and wage due to the technical skills of an employee such that he/she is a certified technician in repair department and hence should be paid higher than a worker operating in collection or sorting department. Now, we derive the expected total profit for the 3PRLP. In order to derive this expression, we formulate the steady-state probability that there are no returns in the 3PRLP activity i such that,

$$\Pr[N^{(i)}(t)=0] = \left[\frac{z_i^{k_i}}{k_i!(1-\delta_i)} + \sum_{j=0}^{k_i-1}\frac{z_i^n}{j!}\right]^{-1} \quad \text{for } i = 0..2 \tag{3.7}$$

where $z_i = \begin{cases} \tau\lambda/\mu_i & \text{if } i=0 \\ \alpha_i\tau\lambda/\mu_i & \text{otherwise} \end{cases}$ and $\delta_i = \frac{z_i}{k_i}$.

Note that the condition for the existence of a steady-state solution for each activity is $\delta_i < 1$ (for details see Gross, Shortle, Thompson, and Harris, 2013). That is, the mean input rate for any 3PRLP activity must be less than the mean maximum potential processing rate of that activity. Next, we derive the expected number of returns in each activity. Denote $L_i(k_i)$ as the expected number of returns in activity i and,

$$L_i(k_i) = z_i^{k_i} + \frac{z_i^{k_i}\delta_i}{k_i!(1-\delta_i)^2}\Pr[N^{(i)}(t)=0]. \tag{3.8}$$

Note that the derivation of (3.8) is straightforward result of Little's formula for multi-server queues and can be found in most of queuing textbooks such as Gross et al. (2013).

Once, we have the expression for expected number of returns in each activity, we now construct expected total cost for the 3PRLP. As discussed above, we define a cost structure that takes into account two different operational cost factors, the wage provided to workers as well as the expected storage costs for returns incurred in each of the activities. Hence, the expected total cost for 3PRLP is sum of wages and storage costs incurred in every activity. That is for activities $i = 0..n$, the total expected cost is

$$TC_{3P}(\cdot) = \sum_{i=0}^{n}\left(h_i L_i(k_i) + w_i k_i\right). \tag{3.9}$$

Finally, the expected total profit for the 3PRLP is the difference of total fees collected from the retailer and the total expected cost expressed in (3.9):

$$\Pi_{3P}(k_i) = \sum_{i=0}^{n}\left[f_i\alpha_i\tau\lambda - (h_i L_i(k_i) + w_i k_i)\right] \tag{3.10}$$

.

We are interested in an optimization problem for the 3PRLP that is to obtain number of workers in each activity (integer values k_i^*), which maximizes the expected profit defined in (3.10). Because, the total revenue specified in the first part of (3.10) does not depend on the number of workers employed by the 3PRLP but it depends on the number of returns that are processed by the 3PRLP. This is intuitively appealing since, the retailer does not have any influence on the 3PRLP's internal decision making. Hence, the retailer should only incur costs according to number of returns processed by the 3PRLP. By the same token, the 3PRLP does not control the flow of returns $\tau\lambda$ to its processing facility, which implies that its total revenue on the returns is fixed. Therefore, given this fixed revenue, the 3PRLP will try to maximize its profit by determining the optimal number of workers to staff in its returns processing activities.

Due to the complexity of optimizing (3.10), closed form solutions for optimal number of labors for each activity of the 3PRLP are intractable. However, Proposition 3.1 ascertains that the optimal number of workers required to staff each process can be determined by complete enumeration.

Observation 3.1. *We can observe that from total cost function in* (3.9)*, cost of wage, $w_i k_i$, for any activity i is increasing in k_i, while $L_i(k_i)$ is decreasing in k_i. Hence, the expected profit in an activity i is a concave function in number of workers k_i since revenue for each activity is not a function of k_i. Furthermore, the maximum total expected profit can be achieved by summing all individual activity profits since each activity is independent.*

As noted earlier, both total profit functions for the retailer and the 3PRLP are highly non-linear and this complexity limits the ability to obtain closed form solutions of the decision variables. Therefore, in the next section, we conduct numerical example to derive solutions to the model and illustrate the impacts of the key parameters on the decision variables.

3.4 Numerical Example

In this section we conduct set of 18 numerical examples to demonstrate the model developed in Sections 3.2 and 3.3. Rather than analyzing the complete set all the parameters, for simplicity and tracking purposes, we limit the number of processing activities for the 3PRLP to $i = 1$. That is one could speculate that the

3PRLP firm is only responsible for repackaging the returned items and hence these items can be sold at the same price of new item. Furthermore, we vary the returns rate and the inventory holding costs for both the retailer and the 3PRLP, but fix other parameters. We consider values for the returns rate ranging from $\tau = 0.1$ to $\tau = 0.6$. Note that we do not consider values greater than 0.6 since returns rates higher than 60 percent are uncommon in practice. We vary unit holding cost ranging from \$2.5 to \$7.5 in order to illustrate the firm specific impacts. The parameters values used in the numerical example are summarized in Table 3.2.

Retailer and forward flow		
Notation	**Description**	**Values**
λ	Demand	20
τ	Fraction of demand that is returned	0.1, 0.2, 0.3, 0.4, 0.5, 0.6
c	Unit cost to purchase an item	\$25
p	Unit selling price	\$40
a	Fixed cost to place an order	\$400
b	Penalty cost for one unit backordered	\$100
h	Cost to hold one unit in inventory	\$2.5, \$5, \$7.5
LT	Supplier lead time (constant)	10
3PRLP and reverse flow		
α_1	Percentage of returns that undergone through sorting	1
$1/\mu_1$	Mean processing time at sorting	1
f_1	Fee (\$) charged to sort per unit return	\$20
w_1	Wage (\$) per worker at sorting	\$2.5
h_1	Holding cost of per unit at sorting	\$2.5, \$5, \$7.5

Table 3.2: Parameter values

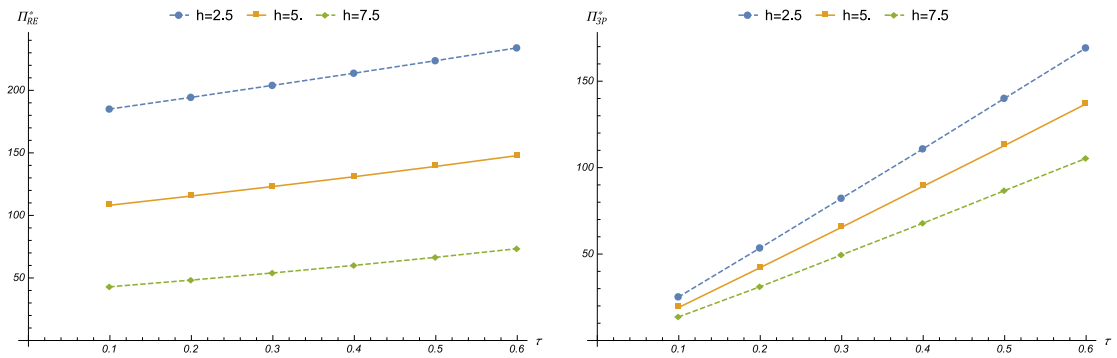
The results of optimization based above parameters are presented in Table 3.3. We discuss the results for each firm separately. For the retailer, as the amount of flow for processed returns increases, both reorder point units r^* and order size units Q^* decrease. This result is expected since the processed returns are considered as the second source for the retailer and are sold at the undifferentiated price. Therefore, the retailer's overall profit improves when returns rate increases as the more goods can be sold at a new product price. Furthermore, as expected the inventory holding cost has a negative impact on the overall

retailer profit. It is crucial to we note that the retailer's profit is affected more excessively with changes in the inventory holding cost from the case of 3PRLP's profit. This is due to the fact that retailer's inventory cost defined in (3.3) is sensitive to the changes in holding cost. These results are depicted in panel A of Figure 3.5.

The directions of effects of changes in both the returns rate and the holding cost are similar for the 3PRLP. However, the magnitude of effects are quite opposite from the case of retailer. In other words, even though the returns rate has positive effect on the 3PRLP's profit, it is quite excessive than the effect of decreasing holding cost. This is because the effect of changes in τ is direct to the 3PRLP's profit function in 3.10 whereas the effect of changes in h_1 is subdued by the average number of returns being processed. To put simply, the amount returned items is always higher or equal to amount of returns to be processed at the 3PRLP's facility.

$h = h_1$	τ	r^*	Q^*	Π^*_{RE}	k_1^*	Π^*_{3P}
	0.1	31	199	\$185.08	3	\$25.28
	0.2	30	180	\$194.42	6	\$53.58
	0.3	29	162	\$204.00	8	\$82.32
\$2.50	0.4	28	143	\$213.74	10	\$110.91
	0.5	26	124	\$223.73	13	\$140.12
	0.6	24	106	\$233.99	15	\$169.31
	0.1	24	196	\$108.15	4	\$19.13
	0.2	24	177	\$115.51	6	\$42.15
	0.3	23	158	\$123.10	9	\$65.54
\$5.00	0.4	22	139	\$130.95	11	\$89.23
	0.5	20	121	\$139.20	14	\$112.82
	0.6	19	102	\$147.81	16	\$136.93
	0.1	21	193	\$42.95	4	\$13.70
	0.2	21	174	\$48.30	7	\$31.15
	0.3	20	156	\$54.02	9	\$49.56
\$7.50	0.4	19	137	\$60.07	12	\$67.90
	0.5	19	118	\$66.51	14	\$86.74
	0.6	18	99	\$73.42	16	\$105.40

Table 3.3: Results for optimum values and profits



(a) Retailer's Profit (b) 3PRLP's Profit

Figure 3.5: Returns rate vs. holding cost

3.5 Summary

In this chapter we present a CLSC model where demand is generated by a stochastic process. A fraction of the units that are initially sold are returned by the consumers for a full refund in every period. The retailer may then contract out the services of a 3PRLP that has the capability of bringing the returned products back to their original condition (these processed returns are then "like new"). The processed returns are sold again at the full price.

We model the forward flow interaction between the supplier, the retailer and 3PRLP by a widely accepted control policy that is lot size-reorder point inventory policy, which is detailed by the Markov process. We further propose a queuing network to capture backward flow activities of the 3PRLP, which consists of customer decision delay and each of the 3PRLP activities. We characterize the expected profits for both firms and derive the effects of key parameters through set of numerical examples. We note that due to intractability of deriving closed form solutions to decision variables, we relied on running numerical examples.

The results of optimization based on numerical examples indicate that both firms' profits increase with an increasing returns rate. This is due to fact that the retailer captures more profits through selling processed returns at the price of new product. The 3PRLP unambiguously earns more profit from increasing product returns since the charging fee from processing returns is sole source of revenue. Consequently,

as indicated by numerical examples, the changes in the return rate has more impact to the 3PRLP's profit than the retailer's profit.

In addition to analyzing the effects of the returns rate we also examine the effects of holding cost per item. The directions of effects of changes in the holding cost are similar for both firms that is negative. However, the magnitude of effects are quite opposite. Interestingly, the retailer's profit appears to be more sensitive to the holding cost than that of the 3PRLP's profit. We believe this is due to structural setup of profit functions for both firms and articulate that 3PRLP's profit is subdued by the average number of returns being processed.

Chapter 4

Coordination in CLSC with 3PRLP

4.1 Description and notation

In this chapter, we take a different perspective to analyze CLSC with 3PRLP who manages the product returns. In particular, we investigate the interaction between the retailer and the 3PRLP with respect to the returns quantity and fee under coordinated and uncoordinated supply chain alignments.

As opposed to using queuing network in the previous model in chapter 3, here we approach the returns processing capacity of a 3PRLP as a two-input production function where there is only one variable input. Crucially, this implies that the 3PRLP's short run marginal cost is strictly increasing. This key feature of the 3PRLP's short run cost function motivates two supply chain interaction scenarios. In an *uncoordinated supply chain*, the retailer acts as a market leader who makes a take-it-or-leave-it fee and quantity of returns offers to the 3PRLP. With increasing marginal cost of returns processing and retailer market power, the quantity of returns processed is inefficiently low due to a standard monopsony argument. In a *coordinated supply chain*, the retailer and the 3PRLP jointly decide on the returns quantity to be processed in order to maximize the total profit for the supply chain. An appealing approach to model how the benefit to coordination is shared between the two firms is Nash bargaining. Accordingly, we characterize the Nash bargaining solution with asymmetric bargaining powers, assuming that the disagreement payoffs are given by the uncoordinated supply chain profit levels. The underlying model is one where the retailer and the 3PRLP negotiate the quantity of returns and the per unit fee, while both recognize that if they fail to reach an agreement, the retailer is poised to make a unilateral offer as in the uncoordinated case.

Similar to Stochastic CLSC model in chapter 3, the proposed supply chain in this model consists of three parties: a manufacturer, a retailer and a 3PRLP. We note that the manufacturer does not explicitly play any role at the returns management stage. Moreover, we conduct the analysis for a given wholesale and selling prices, but in section 4.6, we briefly discuss the implications of letting the wholesale price be a choice variable for the manufacturer. Unlike the stochastic demand in chapter 3, in this model we assume that the demand for new items is perfectly inelastic and equal to D as long as the price is less than some reservation price p. In what follows, p will also be the retail price. After purchasing the good from the retailer, consumers return a certain fraction τ of D to the retailer for a full refund. The supply chain structure is such that the retailer acquires new items from the manufacturer at wholesale price $c < p$, but the 3PRLP collects and processes the returns. Our model captures a scenario where the tasks undertaken by the 3PRLP are specific to the type of product being returned and may range from sorting to repairing and to repackaging. After completing these tasks, the 3PRLP ships R units of returns on behalf of the retailer in "as-good-as-new" condition to the consumers who bought items from the retailer. Therefore, the retailer receives the same price p both for new products and for processed returns. In contrast, for unprocessed returns, the retailer earns some given salvage price s where $s < p$. While reverse logistics operations generate substantial costs for OEM's, those costs can also be quite large at the retail level. For instance, Accenture research provides the example of a consumer electronics item generating $10 billion in revenue and for which the returns processing costs were $396 million for its retailers compared to $466 million for its OEM (Accenture, 2011). In this model we focus on the interactions between the retailer and the 3PRLP with no manufacturer involvement. Hence our results apply to supply chains in which reverse logistics operations are managed primarily by the retailer.

After collecting the returns, the 3PRLP processes a certain quantity and charges a fee f per processed unit. This implies that the only source of revenue for the 3PRLP is the total payment collected from processing the returns. The 3PRLP's returns processing capacity is represented by a standard two-input production function. We focus on the short run where there is only one variable input (e.g. labor) with per unit price w (the wage). Below we make additional assumptions on the production function.

The retailer procures additional new items from the manufacturer since number of returned items

cannot cover the entire demand. Hence, given the demand is deterministic, the retailer procures $D - R$ new items from the manufacturer in addition to R items processed by the 3PRLP to supply the entire demand D in the market. This total supply to cover the demand is illustrated in Figure 4.1. Consequently, in addition to the wholesale price c, an ordering cost of a per order and a holding cost of h per item are incurred for the retailer. We assume that new items must be held in the retailer's inventory, but processed returns are held by the 3PRLP. Hence, the retailer does not incur the holding cost on returned items. Given the quantity of returns processed by the 3PRLP, the retailer implements an EOQ inventory control policy to order new products from the manufacturer. Without loss of generality, we assume no backordering of inventory and no lead-time for new purchases from the manufacturer. In Section 4.6, we relax the assumption that the retailer does not incur holding costs on processed returns. The list of variables and parameters is summarized in Table 4.1.

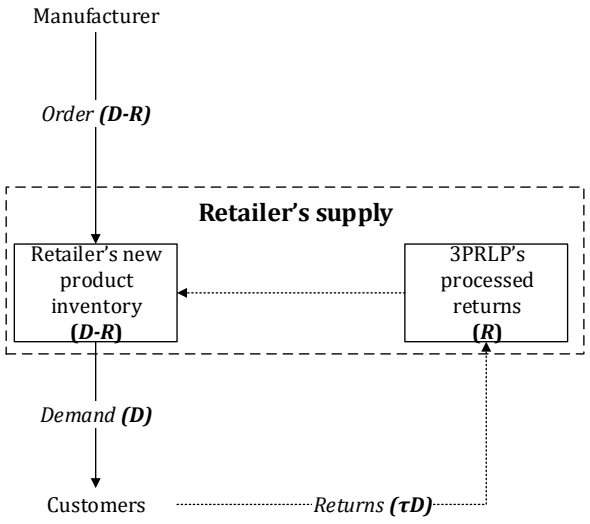


Figure 4.1: Retailer's supply

4.2 Retailer and 3PRLP

In this section, we characterize the profit functions for the retailer and the 3PRLP. Let R denote the quantity of returns processed by the 3PRLP. Then, considering an EOQ policy under deterministic demand without

Symbol	Description
D	Demand (units/period)
τ	Fraction of demand returned per period
R	Quantity of returns processed by the 3PRLP (units/period), $R \leq \tau D$
p	Unit selling price of an item (\$)
c	Unit cost to purchase an item (\$)
a	Fixed cost to place an order (\$)
h	Unit cost to hold an item (\$)
s	Unit salvage price of unprocessed returns (\$)
f	Unit processing fee (\$)
w	Price of variable input (\$)
β	Bargaining power of the retailer, $0 \leq \beta \leq 1$

Table 4.1: Notation for Coordination in CLSC with 3PRLP model

backordering, it is straightforward to show that the retailer's profit function is given by

$$\Pi_{RE}(R) = p(1-\tau)D + pR + s(\tau D - R) - c(D - R) - fR - \sqrt{2ah(D-R)}. \tag{4.1}$$

The first two terms represent the revenue earned from selling new and returned but processed items, the third term is the revenue from salvaged items, the fourth term is the purchasing cost of new items, the fifth term is the payment to the 3PRLP and the last term is the inventory cost.

Next we turn to how returns are processed by the 3PRLP. In the model, the 3PRLP's returns processing capacity is given by a function $F(L, K)$ where L is a variable input and K is another input whose quantity is fixed in the short run. We assume that this production function is strictly increasing and strictly concave in each input, though it is not necessarily strictly concave as a function of (L, K). Therefore, we make no specific assumption on returns to scale in the long run. For instance, a Cobb-Douglas production function $F(L, K) = L^{\theta}K^{\kappa}$ satisfies strict monotonicity and concavity with respect to L and K as long as $\theta \in (0, 1)$ and $\kappa \in (0, 1)$. Moreover, if $\theta + \kappa < 1 (> 1, = 1)$, the production function exhibits decreasing (increasing, constant) returns to scale leading to diseconomies of scale (economies of scale, constant average cost) in the long run. We do not analyze the long run in this paper, but in a different model, Atalay Atasu et al. (2013) examine the implications of economies vs. diseconomies of scale in the context of collection costs.

Letting L^* denote the minimum variable input requirement for a given R, that is, $R = F(L^*, K)$, the variable cost of returns processing is

$$V(R) = wL^*. \tag{4.2}$$

The cost of the fixed input is sunk and we ignore it in the analysis below. Furthermore, since the fee charged to the retailer is the 3PRLP's only source of revenue, its profit function is simply

$$\Pi_{3P}(R) = fR - V(R). \tag{4.3}$$

We denote the 3PRLP's marginal cost by $\Phi(R) = V'(R)$. Due to the strict concavity of the production function in the variable input L, marginal cost is strictly increasing, which is similar to the collection cost functions assumed in Savaskan et al. (2004) and Savaskan and Van Wassenhove (2006). To avoid corner solutions where the optimal quantity processed is zero, we assume that given the 3PRLP's production function, marginal cost satisfies $\Phi(0) = 0$.

In the next section, we examine the case of an uncoordinated supply chain, where the retailer first chooses f and the 3PRLP chooses the quantity R that maximizes its profit taking f as given. Before we proceed, we make the following assumptions, which ensure that all relevant objective functions are strictly concave.

Assumption 4.1. *Marginal cost satisfies the elasticity condition* $-\frac{\Phi''(R)}{\Phi(R)}R < 1$ *for every* $R \in [0, \tau D]$.

Assumption 4.1 is automatically satisfied if marginal cost is strictly convex, though convexity is not necessary. We make one additional assumption that also involves the first derivative of marginal cost.

Assumption 4.2. *Marginal cost is sufficiently steep and demand is sufficiently large so that* $\Phi'(R)\left((1-\tau)D\right)^{3/2} > \frac{\sqrt{ah}}{2\sqrt{2}}$ *holds for every* $R \in [0, \tau D]$.

4.3 Uncoordinated supply chain with retailer unilateral offer

In this section, we consider a scenario where the retailer has the market power to determine the processing fee received by the 3PRLP. Specifically, we analyze the supply chain model where the retailer makes a

unilateral take-it-or-leave-it offer to the 3PRLP firm for processing the returns. The 3PRLP firm takes the fee as given and supplies the quantity that maximizes its profit $\Pi_{3P}(R)$ subject to $R \leq \tau D$. Because the 3PRLP is a price taker, its optimal quantity supplied R^* is the quantity where the fee is equal to marginal cost, or $f = \Phi(R)$, as long as the solution is strictly less than τD and $R^* = \tau D$ otherwise. It will be useful to define f^{3P} as the lowest fee at which the 3PRLP firm would be willing to supply the entire quantity of returned products or $f^{3P} \equiv \Phi(\tau D)$.

The retailer chooses f to maximize $\Pi_{RE}(R^*)$ where (R^*) is a function of f as defined above. In the analysis below, it will be more convenient to focus on the equivalent problem of maximizing Π_{RE} with respect to R whereby f is given by the inverse of R^*.

Proposition 4.1. *In an uncoordinated supply chain where the retailer has monopsony power and makes a unilateral offer to the 3PRLP, the optimal quantity R_{un} is either interior and given by the unique solution to*

$$p + c - s + \frac{\sqrt{ah}}{\sqrt{2(D-R)}} = \Phi'(R)R + \Phi(R) \tag{4.4}$$

or it is a corner solution such that $R_{un} = \tau D$. In the former case $f_{un} = \Phi(R_{un})$ and in the latter case $f_{un} = f^{3P}$.

Proof: See the Appendix A.

In the next section, we show that the quantity of returns that is processed in an uncoordinated supply chain is inefficiently low. This under supply is due to monopsony pricing by the retailer who marks down the processing fee in order to maximize profit. Therefore, in general, there is a benefit to coordination since coordination eliminates the inefficiency.

4.4 Coordinated supply chain with bargaining

We now formulate a coordinated supply chain where the retailer and the 3PRLP maximize joint profit, which represents the total surplus from processing the returns. Following for instance Baron and Berman (2014), Nash bargaining is an appealing approach for modeling how the two parties share the benefit to

coordination. Accordingly, we calculate the Nash bargaining solution with asymmetric bargaining powers, assuming that the disagreement payoffs are given by the uncoordinated supply chain profit levels. We note that allowing asymmetric bargaining power is crucial since in practice the retailer and the 3PRLP will often have unequal strengths (e.g. a large retailer is likely to employ a highly experienced and effective procurement staff).

The underlying model is one where the two firms negotiate the quantity of returns and the per unit fee, while both recognize that if they fail to reach an agreement the retailer is poised to make a unilateral offer as in the previous section. Hence, as in Bernstein and Marx (2006), the disagreement payoffs are endogenous in the sense that they depend on the demand, cost and technology parameters. Under Nash bargaining, the firms will agree to the quantity and fee that solve the following maximization problem

$$\max_{R\leq\tau D,f}[\Pi_{RE}(R)-\Pi_{RE}(R_{un})]^{\beta}[\Pi_{3P}(R)-\Pi_{3P}(R_{un})]^{1-\beta} \tag{4.5}$$

where β is the retailer's bargaining power. In other words, the retailer and the 3PRLP maximize "the size of the pie" (i.e., joint profit) and each party's share is a function of its bargaining power and threat point. It is clear that the outcome of the bargaining problem is the quantity of processed returns that maximizes joint profit,

$$\max_{R\leq\tau D}\Pi_{co}(R)=\Pi_{RE}(R)+\Pi_{3P}(R). \tag{4.6}$$

Under Assumptions 4.1 and 4.2, joint profit is strictly concave and thus, the optimal quantity of returns R_{co} is either interior and given by the unique solution to

$$p+c-s+\frac{\sqrt{ah}}{\sqrt{2(D-R)}}=\Phi(R) \tag{4.7}$$

or it is given by $R_{co}=\tau D$.

Defining $\pi_{RE}(R_{co}) = \Pi_{RE}(R_{co}) + f_{co}R_{co}$ as the retailer's gross profit (excluding the processing fee payment) in the coordinated supply chain and $\Delta \equiv \Pi_{co}(R_{co}) - (\Pi_{RE}(R_{un}) - \Pi_{3P}(R_{un}))$ as the benefit

to coordination, the Nash bargaining fee is equal to

$$f_{co} = \frac{1}{R_{co}}[\pi_{RE}(R_{co}) - \Pi_{RE}(R_{un}) - \beta\Delta]. \tag{4.8}$$

Proposition 4.2. *In a coordinated supply chain where the quantity of returns to be processed by the 3PRLP firm and the per-unit processing fee are the outcome of Nash Bargaining, the quantity R_{co} is given by the solution to equation* (4.7) *if that solution is less than τD or by $R_{co} = \tau D$ otherwise. The fee f_{co} is given by equation* (4.8)*. Moreover, $R_{co} \geq R_{un}$ and $f_{co} \geq f_{un}$ with $f_{co} = f_{un}$ if and only if $R_{co} = R_{co} = \tau D$.*

Proof: See the Appendix B.

It is crucial to note that in general, the quantity of processed returns differs between on the one hand, the case where negotiations succeed and the Nash bargaining solution is implemented (quantity R_{co}) and on the other hand, the case where the parties fail to agree (quantity R_{un}). Indeed Nash bargaining results in the efficient quantity being processed, whereas under retailer's unilateral offer (which serves as the disagreement outcome), the quantity processed is inefficiently low due to standard monopsony pricing. There is a set of special cases for which supply chain coordination does not yield any profit gains over what an uncoordinated supply chain can achieve. Specifically, whenever both the coordinated and the uncoordinated quantity are corner solutions such that all returns are processed, then there are no gains to coordination.

To illustrate the properties of the Nash bargaining outcome, we find it is useful to define the set of (Pareto) efficient payoff vectors as the set of all (Π_{RE}, Π_{3P}) such that, $\Pi_{RE} + \Pi_{3P} = \Pi_{co}(R_{co})$ and the set of individually rational payoffs as (Π_{RE}, Π_{3P}) such that $\Pi_{RE} \geq \Pi_{RE}(R_{un})$ and $\Pi_{3P} \geq \Pi_{3P}(R_{un})$. Assume for a moment that the uncoordinated quantity is strictly less than the total quantity of returned products, $R_{un} < \tau D$. Since R_{co} is the unique maximizing quantity of joint-profit and $R_{un} < R_{co}$, it follows that $\Pi_{co}(R_{co}) > \Pi_{RE}(R_{un}) + \Pi_{3P}(R_{un})$. Indeed, a fully coordinated supply chain yields greater total profit than an uncoordinated supply chain. This case is illustrated by Figure 4.2, which also shows the benefit to coordination Δ. The only situation in which coordination provides no improvement over

an uncoordinated supply chain is when the optimal processed quantities are equal to the total quantity of returns: $R_{un} = R_{un} = \tau D$. This will occur either when demand or the returns rate are relatively low.

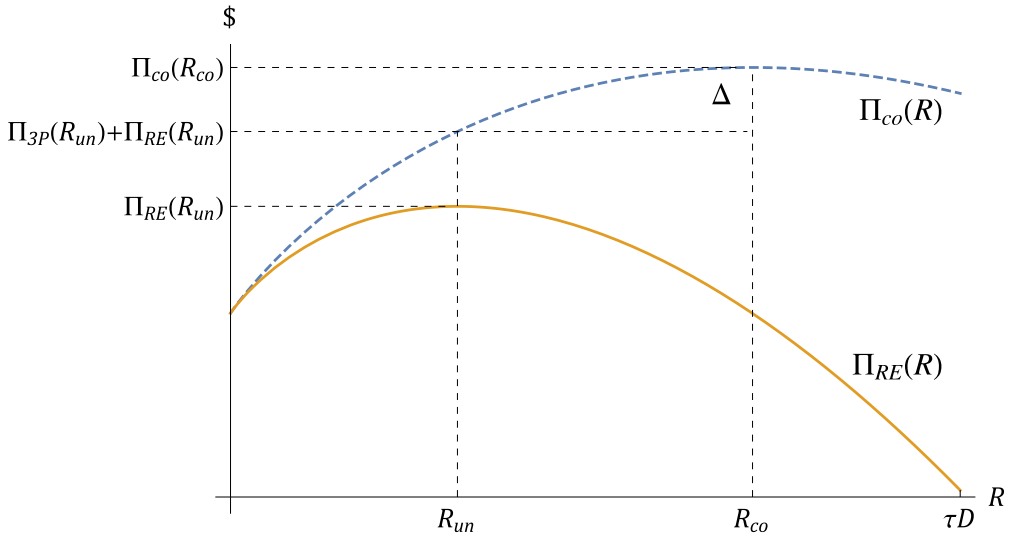


Figure 4.2: Profits comparison

Figure 4.3 illustrates the profit sharing role played by the fee. Clearly when the retailer has all the bargaining power ($\beta = 1$), then the fee is set to the minimum acceptable level. Specifically, in this case, the fee is such that the retailer's profit equals the supply chain's total profit minus the 3PRLP's uncoordinated profit (i.e., its threat point or disagreement payoff). Similarly, at the other extreme, if the 3PRLP has almost all the bargaining power ($\beta = 0$) then the fee is set to the highest acceptable level, where the retailer earns its reservation profit level given by its uncoordinated supply chain threat point. For intermediate values of β, the fee is such that each firm earns a payoff on the straight line (the Pareto frontier) and its profit is strictly more than its disagreement payoff.

In the next section we analyze the sensitivity of supply chain performance to changes in exogenous parameters such as demand, inventory costs and the prices (purchasing, retail, salvage and the input price).

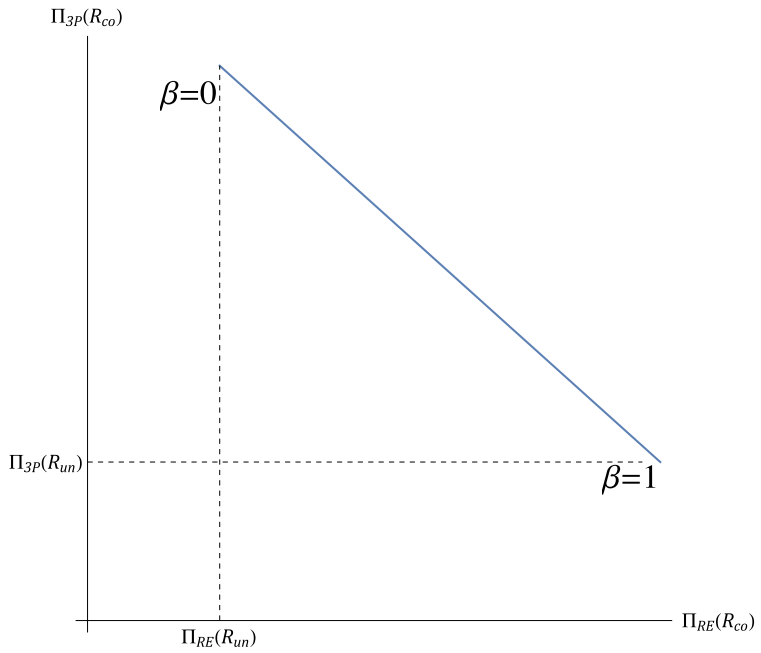


Figure 4.3: Benefits sharing under coordination

4.5 The effect of supply chain characteristics

4.5.1 Relationship between optimal values and parameters

Propositions 4.1 and 4.2 provide the full characterization of the optimal quantity and the fee of returns to be processed in which dictates interaction between the retailer and the 3PRLP. Turning to the effects of changes in the key parameters, Proposition 4.3 summarizes our results.

Proposition 4.3. *Assume that the optimal quantities are less than the total amount returned. The effects of a change in the value of a parameter on the optimal quantities of returns processed (R_{un} and R_{co}) and on the optimal fee (f_{un}) are summarized as follows.*

Positive	***Negative***
Ordering and Holding costs	*Demand*
Selling	*Salvage price*
Purchasing prices	*Price of variable input*

However, in general the effect of a parameter on the coordinated fee (f_{co}) is ambiguous.

Proof: See the Appendix C.

The results for R_{un}, R_{co}, and f_{un} are intuitive. First, when the inventory costs, both holding and ordering, are higher, the retailer is willing to pay more for the returns and thus it demands a larger quantity of processed returns from the 3PRLP. Accordingly, the retailer orders fewer units from the manufacturer since the inventory ordering costs associated with new items are higher. Second, when the demand for the good increases, both the fee and the optimal quantity of processed returns decreases. When the consumers return a sufficiently large quantity of the good, the demand affects the optimal fee and quantity of returns solely through the particular inventory control policy. In other words, a change in demand has no effect on the 3PRLP's supply of returns as long as $R_{co} < \tau D$ in a coordinated supply chain or $R_{un} < \tau D$ in an uncoordinated supply chain. Therefore, when the demand is sufficiently large, the retailer will not rely as much on returned items to satisfy current demand. Third, product prices for purchasing, selling and salvaging have unambiguous effects on the optimal quantity of processed returns. Under a fixed demand, when both the purchasing and selling prices of new products increase, the retailer relies more heavily on the returns because of increasing cost of new item due to former as well as increasing opportunity to earn additional profit due to latter. Similarly, when there is a higher salvage value from returns, the retailer avoids the processing of returns; hence there is less willingness to pay for processing the returns. Lastly, an increase in the price of variable input will decrease the supply of returns.

The effect of a parameter on f_{co} is ambiguous since the sign of $\partial f_{co}/\partial x$ cannot be determined without additional structure on the production function for processed returns. Therefore, in the next section, we conduct a numerical analysis where we formulate the returns processing capacity of the 3PRLP as a two-input Cobb Douglas production function. We assume constant returns to scale and focus on the short run where labor is the only variable input. In particular, we employ $F(L,K) = TL^{\theta}K^{1-\theta}$ where $T > 0$ is a technological parameter, L and K are labor and capital hours respectively and θ is between 0 and 1. Hence, for a given R the labor input requirement is $L^{*} = (\frac{R}{TK^{1-\theta}})^{1/\theta}$ and marginal cost is $\Phi(R) = \frac{w}{\theta}(\frac{R}{TK^{1-\theta}})^{1/\theta-1}$ where w is the wage per hour.

4.5.2 Numerical Analysis

In this section, we illustrate the impact of the parameters on the key variables, as described in the previous section. Rather than analyze the complete set of parameters, we focus on the individual firms' bargaining powers, the returns rate, the inventory costs and the prices. In the numerical analysis presented below, we vary these parameters over a wide range of values. The other parameters are fixed at the following values: $D = 40000; h = \$2.5; p = \$20; s = \$10; T = 5; K = 200; \theta = 0.6$

We consider values for the returns rate ranging from $\tau = 0.1$ to $\tau = 0.6$. Note that we do not consider values greater than 0.6 since returns rates higher than 60 percent are uncommon in practice. The bargaining power coefficient β takes on six different values between 0 and 1, whereby $\beta = 0$ indicates that the 3PRLP has all the bargaining power and $\beta = 1$ indicates that the retailer has all the bargaining power. We vary the remaining parameters between three and four levels. We combine ordering a and holding h costs since they appear as a product in all of the expressions. Moreover, we vary the unit purchase cost from $c = \$5$ to $c = \$15$. Note that in contrast to inventory costs, unit selling and purchasing prices do not have identical effects on the optimal results. Finally, we examine three different levels of wage from $w = \$10$ to $w = \$15$.

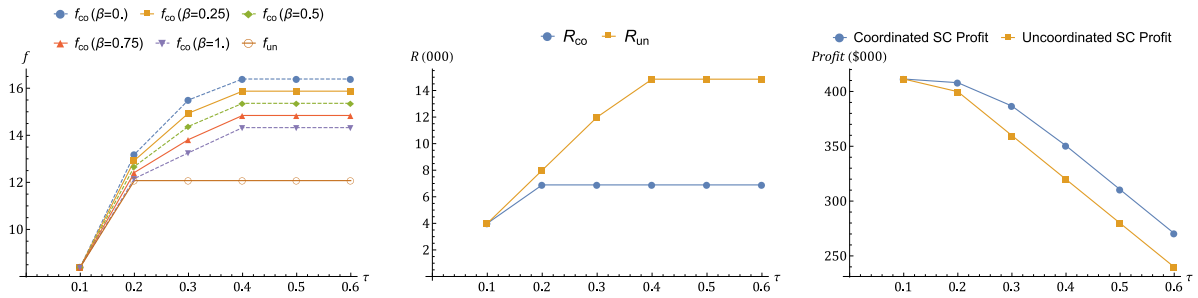
The parameter values used in the numerical analysis are summarized in Table 4.2. We compare coordinated and uncoordinated supply chains by systematically reporting optimal returns quantities, fees and profits. We report results for each parameter separately and for the cases of interesting interactions.

Parameter	**Symbol**	**Values**
Returns rate	τ	0.1, 0.2, 0.3, 0.4, 0.5, 0.6
Bargaining power of the retailer	β	0, 0.25, 0.5, 0.75, 1
Inventory costs	a	\$400, \$600, \$800, \$1000
Unit purchasing price	c	\$5, \$10, \$15
Wage	w	\$10, \$12.5, \$15

Table 4.2: Parameter values

Bargaining power vs. Returns rate

Figure 4.4 illustrates the effect of the returns rate τ on the optimal fees, quantities and profits for a given level of retailer bargaining power. As shown in Section 4.4, regardless of the bargaining power the optimal quantity and fee are always higher in a coordinated than in an uncoordinated supply chain. That is because in a coordinated supply chain, the firms jointly determine the returns quantity that maximizes supply chain profit. Moreover, in a coordinated supply chain, an increase in the retailer's bargaining power results in a lower fee, but has no effect on the quantity. Finally, it must be noted that the optimal fee is often higher than the purchasing price. Indeed, in our model, outsourcing reverse logistics operations allows the retailer to save not only on new product procurement, but also on inventory costs.



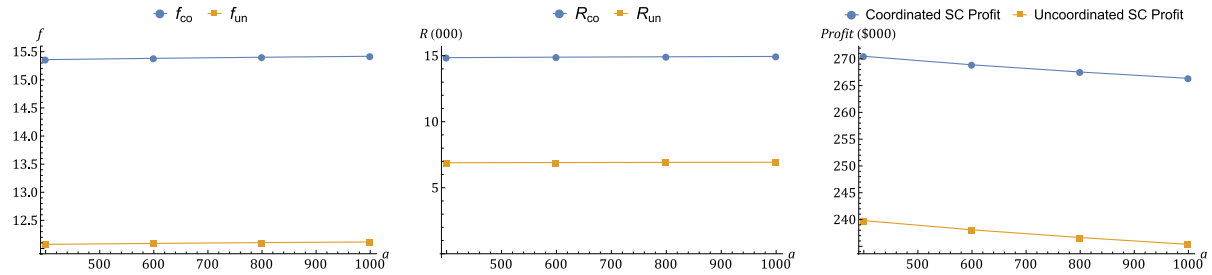
(a) Optimal fee (b) Optimal quantity (c) Profit

Figure 4.4: Bargaining power vs. returns rate

The relationship between the optimal quantities, fees, profits and return rate depends on whether the inequalities $R_{un} \leq \tau D$, $R_{co} \leq \tau D$ and $R_{un} \leq R_{co}$ are strict. If the returns rate is low $\tau = 0.1$ then $R_{un} = R_{co} = \tau D$ as shown in panel B of Figure 4.4. In this case there is no benefit to coordination and optimal fees are equal across supply chains (see panels A and C). For intermediate values of the returns rate ($\tau = 0.2$ and 0.3) then $R_{un} < R_{co} = \tau D$. In this case the benefit to coordination is positive and the fee is higher in the coordinated supply chain. Moreover, the quantity processed in an uncoordinated supply chain is independent of the returns rate. This follows from processing less than one hundred percent of returns under uncoordinated supply chain. Finally, the benefits to coordination increase with τ. When the returns rate is high ($\tau = 0.3$) then $R_{un} < R_{co} < \tau D$. In this case, the optimal fees and quantities do not depend on the returns rate and they are solely based on the processing capability of the 3PRLP and other cost and price parameters. Furthermore, the benefit to coordination is maximized and is independent of τ.

Inventory cost

We vary inventory costs across four different levels such that the increase in ordering cost a is between 50% and 150% of its initial value. The various effects of this change given equal bargaining powers ($\beta = 0.5$) are depicted in Figure 4.5. Overall, regardless of supply chain coordination, the optimal fees and quantities of returns are insensitive to changes in the ordering cost. This is due to fact that the optimal order quantity for new items is unresponsive to changes in inventory ordering and holding costs. In fact, the relative change in the optimal order quantity is the square root of the relative change in ordering cost; i.e. $Q'^*/Q^* = \sqrt{a'/a}$ (see Zipkin, 2000). Consequently, the total inventory cost is relatively unresponsive to changes in and as a result, the retailer's demand for processed returns is unresponsive as well, regardless of supply chain coordination.



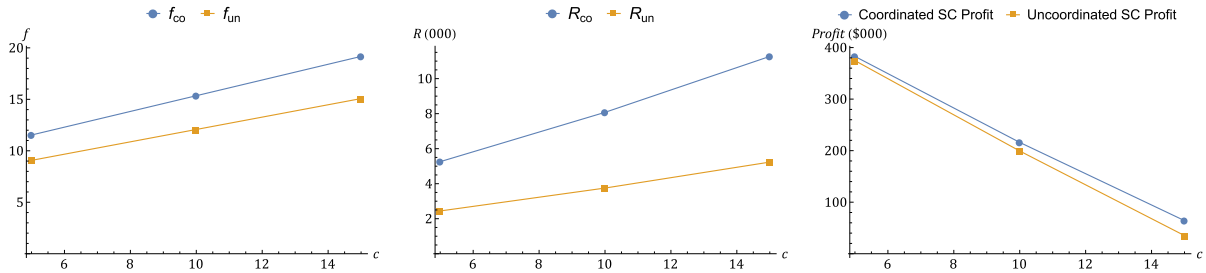
(a) Optimal fee (b) Optimal quantity (c) Profit

Figure 4.5: Ordering cost effects

Purchasing Price

As determined analytically in Proposition 4.3, the purchasing price has a positive effect on the optimal quantities regardless of supply chain coordination. However, this positive effect applies to the optimal fee only in the uncoordinated supply chain. We recall that when the purchasing price of a new item increases, the retailer's response is to order a greater quantity of returns to offset the rising cost of purchasing. Because the 3PRLP's marginal cost function is increasing, the retailer is forced to increase the fee to process the returns.

Figure 4.6 illustrates these effects. It also indicates that the purchasing price has a positive effect on the optimal fee in the coordinated supply chain. An interesting result emerges from panel C. While supply chain profit falls when the purchasing price rises, the benefit to coordination increases. This is intuitive because, as the purchasing price goes up the quantity of processed returns increases, and thus the efficiency loss from failing to coordinate is greater.



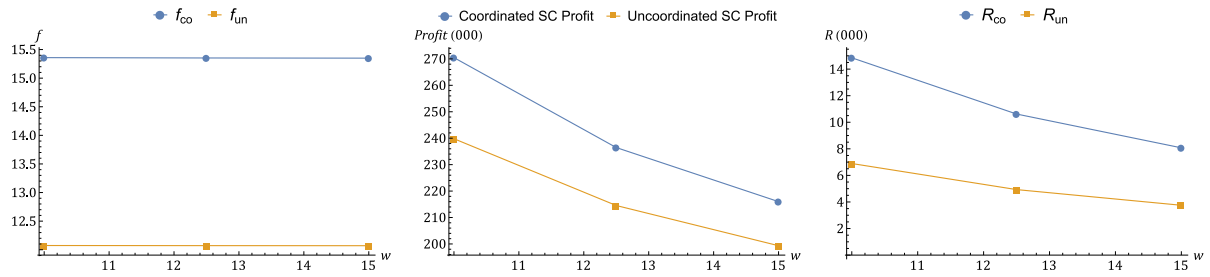
(a) Optimal fee (b) Optimal quantity (c) Profit

Figure 4.6: Purchasing price effects

Price of the variable input (wage)

A change in the wage directly affects the marginal cost of processing returns. As determined analytically in Proposition 4.3, the wage has a negative effect on the optimal quantities regardless of supply chain coordination. Similar to the purchasing price, this negative effect also applies to the optimal fee in the uncoordinated supply chain but not necessarily in the coordinated case. Figure 4.7 illustrates the negative impact of w on the quantities and the uncoordinated fee although it shows that f_{un} is rather unresponsive. The numerical analysis also indicates that in the coordinated supply chain, the effect of the wage on f_{co} is negative but quite unresponsive as well (see panel A).

Similar to the effect of a change in the ordering cost, the unresponsiveness of the optimal fee stems from the robustness of the EOQ model. Figure 4.8 helps to illustrate this in the case of f_{un}. Recall that f_{un} is determined by equation (4.4) and $f_{un} = \Phi(R_{un})$. The left-hand side of (4.4) is the retailer's *marginal willingness to pay for returns* (MW_{RE}). This term does not depend on the wage and is insensitive to changes in R due to the EOQ structure. Following the convention in monopsony pricing, we refer the



(a) Optimal fee (b) Optimal quantity (c) Profit

Figure 4.7: Wage effects

right-hand side as the *marginal buyer cost (MBC)*. This term depends on the wage and in general may be quite sensitive to R. Thus as illustrated in the figure, a change in w, which shifts the *MBC* curve will not impact f_{un} as much as it affects R_{un}. The unresponsiveness of f_{co} is due to a similar mechanism in the coordinated supply chain.

Lastly, turning to profit in panel C of Figure 4.7, the analysis indicates that profits in a coordinated supply chain decrease at a faster rate than in an uncoordinated supply chain, which results in lower coordination benefits. This is due to the shrinking gap between the quantities processed as the wage increases (see panel B).

4.6 Extensions

4.6.1 Optimal wholesale price

So far we have taken the manufacturer's behavior as given. However, it is important to ask how the wholesale price is affected by the retailer and the 3PRLP's choice of supply chain organization. Indeed much of the prior literature has focused on the manufacturer's choice of a wholesale price (for example see Savaskan and Van Wassenhove, 2006; Savaskan et al., 2004). To gain some insight into this question, recall that the demand from final consumers is perfectly inelastic and that the retail price p is equal to the consumers' reservation price. Therefore, in our model the retail price is unaffected by changes in the wholesale price. Consider the scenario in which the retailer and the 3PRLP move first and make an

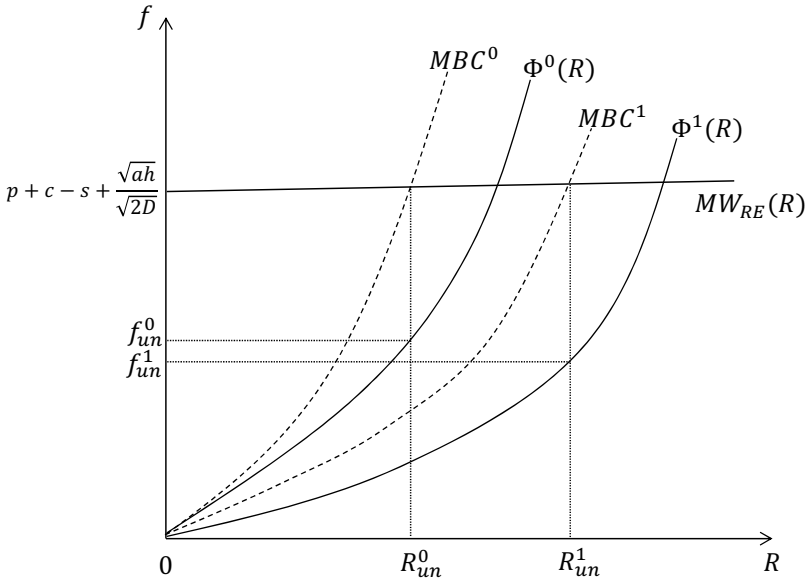


Figure 4.8: Marginal buyer cost vs. marginal willingness to pay for returns in an uncoordinated supply chain

irreversible commitment whether or not to coordinate. The manufacturer observes this decision and sets the wholesale price. With knowledge of the wholesale price, the retailer and the 3PRLP choose the fee and the quantity processed. Assume that manufacturing costs are given by $C_m(y) = c_m y + M$ where y is the quantity produced and sold by the manufacturer, c_m is the marginal cost of production (assumed to be constant for simplicity) and M is a fixed cost. Then, under supply chain structure $j, j \in \{un, co\}$, the manufacturer solves

$$\max_{c} \; cy - c_m y - M \tag{4.9}$$
$$\text{subject to} \quad y = D - R_j.$$

To the extent that R_j is not "too concave" in c (so that the second order condition is satisfied), this is a standard monopoly problem and the optimal price c_j satisfies

$$\frac{c_j - c_m}{c_j} = -\frac{1}{\epsilon_j} \tag{4.10}$$

where ϵ_j is the elasticity of the derived demand from the retailer, $D-R_j$, with respect to the wholesale price c. Thus, whether a change from an uncoordinated to a coordinated supply chain leads to an increase or a decrease in wholesale price is generally ambiguous. Indeed the wholesale price depends on the elasticity of the retailer's demand for new items under the different supply chain scenarios.

4.6.2 Returns and holding costs

In our model, the retailer does not incur holding or ordering costs on returned products. The underlying assumption is that the 3PRLP handles the collection, storage and distribution of returned products on behalf of the retailer. We now briefly discuss the implications of relaxing this assumption. To this effect, consider a scenario whereby the retailer and the 3PRLP first determine R, the quantity of returns to be processed. Then the 3PRLP collects all returned products and processes the amount R, which it ships to the retailer. The retailer must then hold R in its inventory before these "as good as new" items are sold to final consumers. Under the assumption that R is received at the beginning of the period and that the retailer orders new items from the manufacturer when and only when the inventory of processed returns has been depleted, in the modified EOQ the optimal order quantity Q^* is unchanged, but the inventory cost function is now given by

$$\sqrt{\frac{2ah}{D}}(D-R)+\frac{hR^2}{2D}+c(D-R) \tag{4.11}$$

so that the retailer's profit function is

$$p(1-\tau)D+pR+s(\tau D-R)-c(D-R)-\sqrt{\frac{2ah}{D}}(D-R)-\frac{hR^2}{2D}. \tag{4.12}$$

The key difference with the baseline model is the term corresponding to holding costs from processed returns. Hence, the holding cost and ordering cost no longer have identical effects and an increase in holding costs may have a negative effect on the optimal quantities processed.

4.7 Summary

In this model, the retailer sells a single good in a market where demand is deterministic. A fraction of the units that are initially sold are returned by the consumers for a full refund in every period. The retailer may then contract out the services of a 3PRLP that has the capability of bringing the returned products back to their original condition (these processed returns are then "like new"). The processed returns are sold again at the full price.

We examine a short run scenario in which the 3PRLP faces a strictly increasing marginal cost for returns processing, which acts as a flexible capacity constraint. We characterize the profit gains that can be achieved when the retailer and the 3PRLP coordinate supply chain decisions and share the profits according to the Nash bargaining solution. We derive the effects on the results of changes in key parameters analytically and we also examine a comprehensive set of numerical examples. We show analytically that under supply chain coordination the quantity of processed returns is always higher than in the case where the retailer makes a unilateral take-it-or-leave it offer to the 3PRLP. When the retailer and the 3PRLP have equal bargaining powers, the numerical analysis also shows that the unit processing fee paid by the retailer is higher with supply chain coordination.

The numerical analysis further highlights the effects of key parameters on the quantities processed and the fee charged by the 3PRLP and yields several managerial insights. When the consumers return the product at a relatively low rate, processing all returned items may be optimal regardless of supply chain coordination so that coordinating does not result in any benefits. In other words, supply chain coordination yields strictly positive benefits only when the consumers' propensity to return items is sufficiently high. This implies that the expected amount of returns plays an important role in the firms' decision to coordinate. In fact, devising a method to estimate the returns rate above which there exist benefits to coordination is an interesting empirical question to pursue in the future.

When there are gains to coordination, we find that a higher bargaining power allows retailer to obtain returns for a lower fee without affecting the optimal amount of returns processed by a 3PRLP. Second, regardless of supply chain coordination, the optimal fees and quantities of returns are insensitive to changes

in the ordering and holding costs due to the nature of the EOQ model. Third, the purchasing price has a positive effect on the optimal fees and quantities of returns regardless of coordination. More interestingly, a rise in the purchasing price will increase the benefits to coordination even if overall profits fall. In other words, a rising cost of purchasing new items increases the incentives for coordination.

Chapter 5

Conclusion

This dissertation proposes two models of a closed-loop supply chain and focuses on the interaction between a retailer and a 3PRLP who must determine a quantity of consumer returns to process and a per unit fee for this service. Every period, a fraction of the units that are initially sold are returned by the consumers for a full refund. The retailer may then contract out the services of a 3PRLP that has the capability of bringing the returned products back to their original condition (these processed returns are then "like new"). The processed returns are sold again at the full price.

In Chapter 3, we propose a stochastic CLSC model, where demand for the retailer is generated by Poisson process and the returns processing activities of the 3PRLP are designed by queuing network. The forward channel interaction between the supplier, the retailer and 3PRLP is governed by a widely accepted control policy that is lot size-reorder point inventory policy, which is detailed by the Markov process. The backward flow or RL activities by 3PRLP consists of customer decision delay and each of the 3PRLP internal activities. These activities range from collection to sorting to repair and to repackaging. For each type of activity, the 3PRLP employs different skill and number of labor and capital. For instance, the low skilled workers in sorting and repackaging activity are paid less than the highly skilled technicians in repair facility due to their skill and allowance through Original Equipment Manufacturers. Furthermore, the processing times at each activity are also different from each other and hence the 3PRLP incurs differentiated holding costs for the returns that are being allocated. We note that proposed optimizations for both firms are mathematically intractable and hence we analyze the results under a set of numerical examples.

The results numerical examples indicate that both firms' profits increase with an increasing returns rate. We explain this due to fact that the retailer will try to capture more revenue through selling processed returns at the price of new product given a stochastic demand. But this increasing profit for the 3PRLP is unambiguous and more direct since the total fee charged from processing returns is sole source of revenue for 3PRLP. Consequently, as indicated by numerical examples, the changes in the return rate has more impact to the 3PRLP's profit than retailer's profit. In addition to analyzing the effects of the returns rate we also examine the effects of holding cost per item. The direction of effects of changes in the holding cost are similar for both firms and is negative. However, the magnitude of effects are quite opposite. Interestingly, the retailer's profit appears to be more sensitive to the holding cost than that of the 3PRLP's profit. We extrapolate this is due to structural setup of profit functions for both firms and articulate that 3PRLP's profit is subdued by the average number of returns being processed.

In Chapter 4 we investigate the coordination between the retailer and the 3PRLP with respect to the returns quantity and fee where demand is deterministic. We formulate the returns processing capacity of a 3PRLP as a two-input production function where there is only one variable input, which implies that the 3PRLP's short run marginal cost is strictly increasing. This key feature of the 3PRLP's short run cost function motivates two supply chain interaction scenarios. In an *uncoordinated supply chain*, the retailer acts as a market leader who makes a take-it-or-leave-it fee and quantity of returns offers to the 3PRLP. In a *coordinated supply chain*, the retailer and the 3PRLP jointly decide on the returns quantity to be processed in order to maximize the total profit for the supply chain. Using Nash bargaining solution with asymmetric bargaining powers and assuming that the disagreement payoffs are given by the uncoordinated supply chain profit levels, we characterize the supply chain coordination where the retailer and the 3PRLP negotiate the quantity of returns and the per unit fee.

We derive the effects on the results of changes in key parameters analytically and we also examine a comprehensive set of numerical examples. We show analytically that under supply chain coordination the quantity of processed returns is always higher than in the case where the retailer makes a unilateral take-it-or-leave it offer to the 3PRLP. We numerically highlight that when the retailer and the 3PRLP have equal bargaining powers, the numerical analysis also shows that the unit processing fee paid by the retailer

is higher with supply chain coordination. Furthermore, supply chain coordination yields strictly positive benefits only when the consumers' propensity to return items is sufficiently high. This implies that the expected amount of returns plays an important role in the firms' decision to coordinate. Moreover, we find that a higher bargaining power allows retailer to obtain returns for a lower fee without affecting the optimal amount of returns processed by a 3PRLP.

We note it is not immediately clear how our results would change in a model where there is competition between 3PRLPs. The extension of our basic model to 3PRLP competition is not trivial. In the short run, the 3PRLPs have increasing marginal cost so that in the uncoordinated case, competition in fees would be as in the models of Maskin (1986), Dixon (1984) or similar to Bernstein and de Véricourt (2008). Moreover, supply chain coordination would now involve three parties. The analysis of 3PRLP competition promises to be fruitful and we leave it for future research.

Appendix A

Proof of Proposition 4.1

Setting $f = \Phi(R)$, the retailer's profit function is

$$\Pi_{RE}(R) = p(1-\tau)D + pR + s(\tau D - R) - c(D-R) - fR - \sqrt{2ah(D-R)}$$

We have

$$\Pi'_{RE}(R) = p + c - s + \frac{\sqrt{ah}}{\sqrt{2(D-R)}} - \Phi'(R)R - \Phi(R)$$

and

$$\Pi''_{RE}(R) = \frac{\sqrt{ah}}{2\sqrt{2}}(D-R)^{-\frac{3}{2}} - 2\Phi'(R) - \Phi''(R)R$$

We now show that under Assumptions 4.1 and 4.2, $2\Phi'(R) + \Phi''(R)R > \frac{\sqrt{ah}}{2\sqrt{2}}(\tau D)^{-\frac{3}{2}}$ holds for every $R \leq \tau D$, which will in turn establish that the retailer's profit function is strictly concave; $\Pi''_{RE}(R) < 0$. First, using Assumption 1, it follows that $2\Phi' - \Phi''R > \Phi'$ Second, using Assumption 4.2, $2\Phi' - \Phi''R > \Phi' > \frac{\sqrt{ah}}{2\sqrt{2}}(\tau D)^{-\frac{3}{2}}$. Finally, we note that $\frac{\sqrt{ah}}{2\sqrt{2}}(D-R)^{-\frac{3}{2}}$ is maximized at $R = \tau D$. Hence it follows from Assumptions 4.1 and 4.2 that $2\Phi' - \Phi''R > \Phi' > \frac{\sqrt{ah}}{2\sqrt{2}}((1-\tau)D)^{-\frac{3}{2}} > \frac{\sqrt{ah}}{2\sqrt{2}}(D-R)^{-\frac{3}{2}}$ which implies $\Pi''_{RE}(R) < 0$. Therefore, the optimal quantity is either interior and given by the unique solution to or it is a corner solution at $R = \tau D$. In the interior case, the fee is given by $f = \Phi(R)$ and in the corner case, to maximize profit, the retailer sets the lowest fee at which the 3PRLP is willing to supply $f = \tau D$ or $f = f^{3P}$.

Appendix B

Proof of Proposition 4.2

To prove the proposition, we first note that for a given quantity of processed returns, the fee simply acts as a transfer from the 3PRLP to the retailer. Second, because the Nash bargaining solution satisfies Pareto efficiency, the quantity processed under Nash bargaining must be a maximizer of total profit (or "total surplus"). Otherwise, for any given the firms could alter the quantity so as to increase one firm's profit without affecting the other firm's; thus contradicting Pareto efficiency. Therefore, consider the problem

$$max_{R \leq \tau D} \Pi_{co}(R) = \Pi_{RE}(R) + \Pi_{3P}(R)$$

and note that $\Pi_{co}(R)$ does not depend on f since again, f is simply a per unit transfer. The first and second derivatives are given by

$$\Pi'_{co}(R) = p + c - s + \frac{\sqrt{ah}}{\sqrt{2(D-R)}} - \Phi(R)$$

$$\Pi''_{co}(R) = \frac{\sqrt{ah}}{2\sqrt{2}}(D-R)^{-\frac{3}{2}} - \Phi'(R)$$

Under Assumption 2, $\Pi''_{co}(R), 0$ so that the problem has a unique solution R_{co} that is either interior and defined by $\Pi'_{co}(R) = 0$ or at a corner and given by $R_{co} = \tau D$. Therefore the Nash bargaining solution is characterized by the quantity $R = R_{co}$.

Now with Nash bargaining, given retailer bargaining power $\beta \in [0, 1]$, the firms' profits are given by

$$\Pi_{RE}(R_{co}) = \Pi_{RE}(R_{un}) + \beta\Delta$$

and

$$\Pi_{3P}(R_{co}) = \Pi_{3P}(R_{un}) + (1 - \beta)\Delta$$

where Δ is the gain from coordination, $\Delta = \Pi_{co}(R_{co}) - (\Pi_{RE}(R_{un}) + \Pi_{3P}(R_{un}))$. Using $\Pi_{RE}(R_{co}) = \pi_{RE}(R_{co}) - f_{co}R_{co}$ and solving for f_{co} yields the expression in equation (4.8).

The fact that $R_{co} \geq R_{un}$ follows from a comparison of equations (4.4) and (4.7). The left-hand sides are identical. However, because $\Phi'(R) > 0$, the right-hand side of (4.7) is strictly greater than the right-hand side of (4.4). It follows that if R_{co} satisfies (4.7) then $\Pi'_{RE}(R_{co}) < 0$. But since $\Pi_{RE}(R)$ is a strictly concave function, it must be the case that $\Pi'_{RE}(R_{un}) = 0$ occurs at $R_{un} < R_{co}$. Hence, if R_{co} and R_{un} are both given by interior solutions, then $R_{un} < R_{co}$. If R_{co} is given by a corner solution but R_{un} is not, then *a fortiori*, $R_{co} = \tau D > R_{un}$. Thus, we have shown that $R_{un} < R_{co}$, with an equality if and only if $R_{un} = \tau D = R_{co}$.

The fact that $f_{un} \leq f_{co}$ follows from revealed preference. From the discussion preceding Proposition 4.1, we have $f_{un} = \Phi(R_{un})$ (fee equals marginal cost). Thus, given $f = f_{un}$, the quantity $R = R_{un}$ maximizes the 3PRLP's profit. But then we know

$$f_{co}R_{co} - V(R_{co}) \geq f_{un}R_{un} - V(R_{un}) \geq f_{un}R_{co} - V(R_{co}).$$

The first inequality is simply $\Pi_{3P}(R_{co}) \geq \Pi_{3P}(R_{un})$, which follows from the fact that in a coordinated supply chain, the 3PRLP earns no less than its uncoordinated supply chain profit. The second inequality follows from the fact that R_{un} is optimal given $f = f_{un}$, as claimed above. But then the first and third expression imply $f_{co} \geq f_{un} + (V(R_{co}) - V(R_{un}))/R_{co}$. Now since $R_{un} \geq R_{co}$ and V is an increasing function, then $f_{un} \leq f_{co}$. Moreover, whenever $R_{un} < R_{co}$, $V(R_{un}) < V(R_{co})$ and thus $f_{un} < f_{co}$. Finally, if $R_{un} = R_{co}$, which only occurs if these quantities are equal to τD, then $\Delta = 0$, so that both

firms earn their uncoordinated supply chain profit for all β. This immediately implies $f_{co} = f_{un} = f^{3P}$.

Appendix C

Proof of Proposition 4.3

Suppose $R_{un} < R_{co} < \tau D$. In this case, the effect of any change in parameter, x, on optimal quantities R_{co} and R_{un} can be derived as

$$\frac{\partial R_{co}}{\partial x} = -\frac{\partial^2 \Pi_{co}/\partial R \partial x}{\partial^2 \Pi_{co}/\partial R^2}$$

$$\frac{\partial R_{un}}{\partial x} = -\frac{\partial^2 \Pi_{RE}/\partial R \partial x}{\partial^2 \Pi_{RE}/\partial R^2}$$

The denominators in the above equations are the second order conditions to the profit maximization problems. They are strictly negative under Assumptions 4.1 and 4.2 (and we denote them by SOC_j, where $-SOC_j > 0$ for $j \in \{co, un\}$). Hence, the numerators determine the direction of the effects. Using equation (4.4) for $j = un$ and equation (4.7) for $j = co$, we have

x	**Positive**	x	**Negative**
ah	$\frac{\partial R_j}{\partial ah} = -\frac{1}{2SOC_j\sqrt{2ah(D-R_j)}} > 0$	D	$\frac{\partial R_j}{\partial D} = \frac{\sqrt{ah}}{2\sqrt{2}SOC_j(D-R_j)^{3/2}} < 0$
p	$\frac{\partial R_j}{\partial p} = -\frac{1}{SOC_j} > 0$	s	$\frac{\partial R_j}{\partial s} = -\frac{1}{SOC_j} < 0$
c	$\frac{\partial R_j}{\partial c} = -\frac{1}{SOC_j} > 0$	w	$\frac{\partial R_{un}}{\partial w} = \frac{\frac{\partial \Phi'}{\partial w}R + \frac{\partial \Phi}{\partial w}}{SOC_{un}} < 0$, $\frac{\partial R_{co}}{\partial w} = \frac{\frac{\partial \Phi}{\partial w}}{SOC_{co}} < 0$

Lastly, the effect of a change in a parameter on the optimal coordinated fee f_{co} is given by

$$\frac{\partial f_{co}}{\partial x} = \frac{1}{R_{co}^2}\left[\left(\pi'_{RE}(R_{co})\frac{\partial R_{co}}{\partial x} - \Pi'_{RE}(R_{un})\frac{\partial R_{un}}{\partial x} - \beta\frac{\partial \Delta}{\partial x}\right)R_{co} - \frac{\partial R_{co}}{\partial x}(\pi_{RE}(R_{co}) - \Pi_{RE}(R_{un}) - \beta\Delta)\right].$$

Expanding terms and simplifying yields

$$\frac{\partial f_{co}}{\partial x} = \frac{1}{R_{co}^2}\Bigg[\left(\pi'_{RE}(R_{co})\frac{\partial R_{co}}{\partial x} - (1-\beta)\Pi'_{RE}(R_{un})\frac{\partial R_{un}}{\partial x} - \beta(\pi'_{RE}(R_{co}) - V'(R_{co}))\frac{\partial R_{co}}{\partial x} + \beta\Pi'_{3P}(R_{un})\frac{\partial R_{un}}{\partial x}\right)R_{co} - \frac{\partial R_{co}}{\partial x}(\pi_{RE}(R_{co}) - \Pi_{RE}(R_{un}) - \beta\Delta)\Bigg].$$

From the envelope theorem, we have

$$\Pi'_{RE}(R_{un}) = 0$$

and

$$-\beta(\pi'_{RE}(R_{co} - V'(R_{co})) = 0.$$

Therefore,

$$\frac{\partial f_{co}}{\partial x} = \frac{1}{R_{co}^2}\Bigg[\left(\pi'_{RE}(R_{co})\frac{\partial R_{co}}{\partial x} + \beta\Pi'_{3P}(R_{un})\frac{\partial R_{un}}{\partial x}\right)R_{co} - \frac{\partial R_{co}}{\partial x}(\pi_{RE}(R_{co}) - \Pi_{RE}(R_{un}) - \beta\Delta)\Bigg].$$

Now,

$$\Pi_{3P}(R_{un}) = \Phi(R_{un})R_{un} - V_{3P}(R_{un})$$

so

$$\Pi'_{3P}(R_{un}) = \Phi(R_{un}) + \Phi'(R_{un})R_{un} - \Phi(R_{un}) = \Phi'(R_{un})R_{un} > 0$$

Summarizing, we have,

$$\pi'_{RE}(R_{co}) > 0$$

$$\beta\Pi'_{3P}(R_{un}) > 0$$

$$\pi_{RE}(R_{co}) - \Pi_{RE}(R_{un}) - \beta\Delta \geq V_{3P}(R_{co}) + \Pi_{3P}(R_{un}) > 0$$

$$\text{sign}\left\{\frac{\partial R_{co}}{\partial x}\right\} = \text{sign}\left\{\frac{\partial R_{un}}{\partial x}\right\}$$

and again

$$\frac{\partial f_{co}}{\partial x} = \frac{1}{R_{co}^2}\Big[\underbrace{\left(\pi'_{RE}(R_{co})\frac{\partial R_{co}}{\partial x} + \beta\Pi'_{3P}(R_{un})\frac{\partial R_{un}}{\partial x}\right)R_{co}}_{A} - \underbrace{\frac{\partial R_{co}}{\partial x}\left(\pi_{RE}(R_{co}) - \Pi_{RE}(R_{un}) - \beta\Delta\right)}_{B}\Big].$$

But since $\text{sign}\{A\} = \text{sign}\{B\} = \text{sign}\left\{\frac{\partial R_{co}}{\partial x}\right\}$ it is not possible to determine the sign of $\frac{\partial f_{co}}{\partial x}$ without imposing additional structure on the production function (which determines the marginal cost function $\Phi(R)$) and restricting the parameter space.

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