Ecological Inference and Entropy-Maximizing: An Alternative Estimation Procedure for Split-Ticket Voting

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Publication of King's *A Solution to the Ecological Inference Problem* has rekindled interest in the estimation of unknown cell values in two- and three-dimensional matrices from knowledge of the marginal sums. This paper outlines an entropy-maximizing (EM) procedure which employs more constraints than King's EI method and produces mathematical rather than statistical procedures: the estimates are maximum-likelihood values. The mathematics are outlined, and the procedure's use illustrated with a study of ticket-splitting at New Zealand's first (1996) general election using the mixed-member proportional representation system, for which official figures provide a check against the EM estimate of the number voting a straight party ticket in each constituency.

1 Introduction

THE PUBLICATION IN 1997 of Gary King's *A Solution to the Ecological Inference Problem* promised major breakthroughs in the estimation not only of individual behavior from ecological data but also of spatial variations in that individual behavior. This is illustrated in a recent paper by Burden and Kimball (1998), which used King's procedure to estimate the amount and direction of ticket-splitting in the 1988 U.S. Presidential and Congressional elections. They demonstrated substantial variation in the proportions of Bush and Dukakis supporters who voted for other parties in the 264 House of Representatives and 33 Senate contested seats—which they related to the intensity of Republican and Democratic campaigning in those congressional contests.

The potential offered by King's method is very substantial across a wide range of fields within the social sciences. Its value has been contested, however [see, e.g., the review by Freedman et al. (1988), and King's (1999) response, and Tam Cho (1999)], and other

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methods are being explored. In this paper, we bring researchers' attention to a method that has been in use for almost two decades and that is based on well-established maximum-likelihood entropy-maximizing (EM) estimation procedures, as developed by Mosteller (1968; see also Bishop et al. 1975). The method has been used in a number of electoral studies since it was first set out by Johnston and Hay (1982, 1983) and full details are given by Johnston (1985). It has not been widely adopted, however, although the only substantial critique was positive in its evaluation (Cleave et al. 1995). The case for it is developed here by setting out the nature of the procedure and then exemplifying it using a study of ticket-splitting in the New Zealand 1996 general election, the first there using the mixed-member proportional system—for which official data provide a check of the validity of the estimates.

2 The Entropy-Maximizing Procedure

Based on the thermodynamic concept of entropy as maximum disorder, the EM procedure seeks the most likely configuration of elements within a constrained situation. It is widely used in a range of disciplines, but its potential for the social sciences has been but slightly recognized. [An early example of its promotion in the social sciences is Dempster et al. (1977).] It can be illustrated by a simple example.

Take a 2×2 ($N \times M$) matrix, in which the row and column totals are known but the internal cell values are unknown.

	M_1	M_2	
N_1	.	.	4
N_2	.	.	6
	3	7	

This is termed the system's *macrostate*.

That macrostate can be produced by a number of *mesostates*, distributions of individuals across the four internal matrix cells that are consistent with the system constraints—the row and column totals. There are four such feasible mesostates.

	M_1	M_2	M_1	M_2	M_1	M_2	M_1	M_2
N_1	3	1	2	2	1	3	0	4
N_2	0	6	1	5	2	4	3	3

The EM procedure identifies which of those mesostates is most likely to occur—given no other constraints or influences on which individual is allocated to which cell.

That procedure involves identifying the *microstate* that is most likely to occur because it is associated with the largest number of different allocations of the individuals in any one mesostate. For example, in the first mesostate there are four individuals in row N_1—*a*, *b*, *c* and *d*. There are four ways in which they can be allocated to the two cells in that row: one of them appears in column M_2 and the other three in column M_1. Whichever allocation occurs, all six of the individuals in row N_2 must be allocated to column M_2, hence there is a total of four microstates associated with the first mesostate.

For the second mesostate, there are six ways of allocating the four individuals in row N_1 to the two columns (the first pair refers to M_1 and the second to M_2: (1) *ab*, *cd*; (2) *ac*, *bd*; (3) *ad*, *bc*; (4) *cd*, *ab*; (5) *bd*, *ac*; and (6) *bc*, *ad*. In addition, there are six ways in which the six individuals in row N_2 can be divided between the two columns at the ratio of 1:5. This gives a total of 36 microstates associated with that mesostate (i.e., the product of

the two separate allocations). There are 60 microstates associated with the third mesostate (4 ways of dividing the four individuals in N_1 at the ratio 1:3 and 15 of dividing the six in N_2 at the ratio 2:4) and 18 associated with the fourth (there are 18 ways of dividing the six individuals in N_2 at the ratio 3:3 and just 1 way of allocating the four in N_1 at the ratio 0:4). The maximum-likelihood solution is thus the third of the four mesostates: this has the most microstates associated with it.

The method is applied here to a three-dimensional data cube for which the three sets of sums are known. Formally, this can be described in the following way. [This presentation is based on that of Wilson and Bennett (1985, pp. 237–243).] Assume a split-ticket voting matrix $\mathbf{V}$ with m rows and n columns that applies to the entire country (this is matrix ${}_m\mathbf{V}_n$). That country is divided into k states, each of which will have its own split-ticket matrix ${}_m\nu_n$. Thus

ν_{ijk} is the value for the ith row, in the jth column, and the kth state, and

$$\sum \nu_{ijk} = \mathbf{V}_{ij}$$

If the total number of voters in the country is T, then the number of combinations of voters in the first cell—ν_{111}—of the data cube, given a fixed value for ν_{111}, is

$$T!/[\nu_{111}!(T - \nu_{111})!]$$

For the next cell—ν_{112}, say—the number of combinations is

$$\{T - \nu_{111}\}!/[\nu_{111}!(T - \nu_{111} - \nu_{112})!]$$

The procedure continues to identify all the possible ways of allocating T voters across the ijk cells, until all of the cells are filled. The total number of possible combinations is W, defined as

$$W = T!/\prod \nu_{ijk}!$$

which is the factorial of the total number of voters divided by the product of the factorials of all of the individual ν_{ijk} values. Stirling's approximation can be used to derive this as

$$\log W = (T \log T - T) - \sum(\nu_{ijk} \log \nu_{ijk} - \nu_{ijk})$$

This is an unconstrained solution, however. We need the cell values to sum to the relevant row and column totals—$\nu_{i.k}$, which is the number of votes for candidate i in state k in the first contest; and $\nu_{.jk}$, which is the number of votes for candidate j in state k in the second contest. Thus

$\sum \nu_{ijk} = \nu_{i.k}$, over all m rows in each state; and

$\sum \nu_{ijk} = \nu_{.jk}$, over all n columns in each state.

In addition, the sum for any one cell across all states must equal the sum in the national split-ticket matrix ($\mathbf{V}$):

$\sum \nu_{ijk} = V_{ij}$, over all k states.

These three constraints are then used to limit the distribution of all T voters across the ν_{ijk} cells of the data cube, giving a statistic L, defined as

$$\begin{aligned} L = {} & (T \log T - T) - \sum(\nu_{ijk} \log \nu_{ijk} - \nu_{ijk}) + \sum \lambda_{ij}(V_{.j} - \nu_{ij.}) \\ & + \sum \gamma_{ik}(V_{i.} - \nu_{i.k}) + \sum \Phi_{jk}(V_{ij} - \nu_{ijk}) \end{aligned}$$

in which the three parameters λ, γ, and Φ constrain the distribution of T to the known totals:

$$\exp(-\lambda_{ij}) = A_{ij}V_{ij}$$
$$\exp(-\gamma_{ik}) = B_{ik}V_{i.}$$
$$\exp(-\Phi_{jk}) = C_{jk}V_{.j}$$

These are scalars for each row in each state (B_{ik}), for each column in each state (C_{jk}), and for each cell in the national split-ticket matrix (A_{ij}) that ensure that the constraints are met. The maximum-likelihood value for each allocation of T across the v_{ijk} cells is then calculated as

$$v_{ijk} = A_{ij}V_{ij}B_{ik}V_{i.}C_{jk}V_{.j}$$

Calculation of these v_{ijk} values is achieved using the iterative biproportionate matrix scaling procedure described by Bacharach (1970) [this procedure has been used for "smoothing" two-dimensional matrices in political science—Särlvik and Crewe (1983) term it "Mostellerisation"]. The three-dimensional data cube is assembled, and initial estimates of the internal cell values—i.e., of each v_{ijk} value—are added to the cube: the easiest way of doing this is to designate all cell values 1.0. These are then summed across one of the dimensions in the cube, and those sums compared with the relevant constraint values (the "actual" values). If the rows of the matrix for each state were chosen, this would give a matrix of row estimates (*ER*),

$ER_{i.k} = \sum v_{ijk}$ for all j columns in the kth state.

These are then compared with the relevant constraint for each row, and a row scalar (*SR*) is produced as a ratio between the sum of the estimates and the actual value—

$$SR_{i.k} = V_{i.k}/ER_{i.k}$$

The cell values are then inflated/deflated by the relevant scalar to give a new estimate of their values,

$$e_{ijk} = v_{ijk} * SR_{i.k}$$

These are then summed across the other two dimensions of the cube, to produce estimates of the column sums (matrix *EC*) and the slice sums (matrix *ES*). These are compared with the relevant constraints, to produce a new set of scalars (*SC* and *SS*):

$$SC_{.ik} = V_{.ik}/EC_{.jk}$$
$$SS_{ij.} = V_{ij.}/ES_{ij.}$$

The process is then repeated by producing a new set of cell value estimates—e_{ijk}—using one of these two scalars—either

$$e_{ijk} = e_{ijk} * SC_{.jk}$$

or

$$e_{ijk} = e_{ijk} * SS_{.jk}$$

The decision on which to use is determined by a goodness-of-fit statistic; we have used the average difference between the estimated and the constraint sum, expressed as a percentage of the constraint sum.

The procedure continues until a stopping rule is applied—either after a set number of iterations or when each of the two goodness-of-fit statistics is below a predefined maximum—an average of 1%, perhaps. (The third will always be zero, since the cell values have been scaled to fit that set of constraints.)

The EM method differs from King's in one major respect, therefore; it is based on three rather than two sets of constraints. [Constraints along further dimensions can be added (see Johnston et al. 1982).] This is a major advantage over ecological inference (EI) because, in addition to the row and column totals for each slice of the three-dimensional data cube employed in that method, EM also has the sums for each cell on the third face of the cube. This means that the EM estimates are based on additional information, with the likelihood that, as a consequence, they are closer to the "real" numbers than might be the case if EI were used. Other mathematical programming solutions using the row and column constraints only as in EI—as presented by Irwin and Meeter (1969) and McCarthy and Ryan (1976, 1977)—tend to underestimate the off-diagonal cell values—on which see Upton (1978). This could well be the case were they—or King's EI method—applied to that example. In the New Zealand case study presented below, for example, it is unlikely that the estimated amount of split-ticket voting would be as large were it not for the additional information provided by the national split-ticket matrix. Other situations in which a matrix acts as a third set of constraints can be applied, including flow-of-the-vote matrices, showing gross changes in voters' preferences over time, and vote by socioeconomic characteristic matrices. Examples of both are given by Johnston et al. (1988): flow-of-the-vote matrices have the number of votes cast for each candidate/party at each of the two elections for each area as the row and column constraints; vote by socioeconomic characteristic matrices use census or similar data for the relevant areas for the row constraints and vote data for the columns.

One major assumption of the EM method is that there is no three-way interaction involved—that there are no other factors influencing the distribution of elements through the data cube other than those identified by the constraints. It makes no other assumptions about individual behavior and how it may vary from place to place, however. Freedman et al. (1991), for example, have pointed out that studies which use ecological regression to estimate missing values imply a constancy assumption that there is no variation in behavior within each group of voters (apart from random variation) across states (using the terminology applied above) so that the pattern of split-ticket voting would be the same everywhere. They suggest an alternative, neighborhood, model in which "people who live near each other are like each other with respect to . . . voting behavior" (Freedman et al. 1991, p. 682). This allows for "state effects", which may be unique to individual states or which may vary systematically across states—as illustrated in the case study below. The EM method allows for these state effects, within the constraints discussed above; further, the additional constraints not included in King's EI method but incorporated within EM are based on survey data, which Freedman et al. (1991, p. 701) argue "offer a better approach to estimating voting than do models because surveys start from data on individuals". Thus EM, unlike EI (Freedman et al. 1998), is not subject to the criticism that it is based on the constancy assumption: it allows for variation across states that may be consistent with the neighborhood model and provides estimates that can be tested against neighborhood effect hypotheses.

Finally, in contrast to King's EI method, EM is a mathematical rather than a statistical procedure: it produces an exact solution to the problem. Evans (1973) has shown that the biproportional matrix scaling procedure is equivalent to that produced by the transportation problem in linear programming, which seeks to minimize the costs of moving goods/people between origins and destinations. Johnston (1985) discusses the equivalence between EM

and loglinear modeling. Thus there are no error terms associated with the estimates. In many of the applications to electoral data such as that discussed below, however, one of the sets of constraints (almost invariably the slice totals) is based on survey data. These have error terms associated with them, and it could be that a number of matrices consistent with the other constraints could be used in the EM procedure. Each may produce a different set of estimates for each of the internal cell values of the data cube, from which a sampling distribution could be derived. This would be of particular value if the purpose of using the procedure was to estimate particular cell values. In all of the applications to date, however, the goal has been to produce estimates for all cells to allow study of variations across space (of systemwide variations in state effects). For these, the important thing is whether the relative size of the values are the same with different starting constraints, and experiments have shown this to be the case (Johnston and Pattie 1991a). Additional tests could be undertaken by, for example, splitting the states into two or more groups; each would have its own survey-based slice constraints, so that tests for robustness could be conducted depending on the split.

3 The New Zealand Case Study

In 1992 and 1993 the New Zealand electorate voted in two referendums for their electoral system to be reformed; first-past-the-post in single-member constituencies was replaced by an MMP (mixed-member proportional) system akin to that in use in (West) Germany since democracy was reestablished in the 1950s and now used in a substantial number of ex-Communist states (Moser 1999). Just over half of the members of the House of Representatives (65) were to be elected from single-member constituencies by the first-past-the-post system, and the other 55 from national party lists. The electors had two votes, one for the constituency candidate and one for the party list: the final composition of the House of Representatives reflects the proportional distribution of party votes, with each party being allocated additional seats to those which it had won in the constituency contests to achieve the proportional outcome. [On the process involved in the introduction of MMP, see Jackson and McRobie (1998). To be allocated list seats a party must win either at least 5% of the national list votes or one constituency seat.]

This system was used for the first time in 1996 when the general election was contested by seven main political parties/groupings—Labour, New Zealand First, Alliance, National, Association of Consumers and Taxpayers (ACT), United, and the Christian Coalition. The first three of these contested all 65 constituencies in that section of the election, National contested 64, ACT 63, the Christian Coalition 37, and United 25: all seven were on the list for the party votes.

The voting patterns for the two sections of the election are very similar (Table 1: the index of dissimilarity between the two columns is 5.5), with the implication that the majority of electors voted for the same party in both contests. Some split-ticket voting was inevitable because only three of the seven major parties contested every constituency seat, but, in general, it appears from Table 1 as if the great majority of people voted for a constituency candidate from the party that they supported in the party vote contest. This was not the case, however. Before the votes were counted they were divided into two groups: those which were ‘straight ticket’ (i.e., the same party was supported in both contests) and those which were ‘split ticket’. The numbers in each group were tabulated by constituency and published in the Chief Electoral Officer’s report on the election. In the country as a whole, only 63% of the valid votes were ‘straight ticket’—with the corollary that more than one-third split their votes. Across the 65 constituencies, the percentage of ‘straight ticket’ votes ranged from a low of only 30.67 to a high of 76.77.

Table 1 The result of the 1996 New Zealand General Election: Percentage of the votes won by the parties in the two parts of the contest

	Party votes	*Constituency votes*
National	33.8	33.9
Labour	28.2	31.1
New Zealand First	13.4	13.5
Alliance	10.1	11.3
ACT	6.1	3.7
Christian Coalition	4.3	1.5
United	0.9	2.1
Other	3.2	2.9

3.1 *Estimating the Pattern of Split-Ticket Voting*

The net pattern of voting shown in Table 1 conceals a great deal of split-ticket voting. Its volume was made even more explicit by post-election surveys, of which two have been reported (in collections of papers on the election by Boston et al. 1998 and Vowles et al. 1998). One of those split-ticket matrices (see Levine and Roberts, 1998) is reported in Table 2, where the cell values are percentages of the row totals: 77.6% of those who voted for National on the party list also voted for a National constituency candidate, for example, whereas 10.2% of them voted for a Labour candidate, 2.4% for a New Zealand First candidate, etc. In general, the larger the party (in terms of its performance in the party vote section of the contest) and the more constituencies it contested, the greater the level of "straight ticket" voting (or party loyalty), but even for the three parties that contested all 65 constituencies, their percentages of loyalists were only 66.7, 58.5, and 56.8. Many New Zealand electors were clearly operating strategically within the context of their new electoral system, responding to the parties' general electioneering that the party vote was most important in determining the overall election outcome (see Banducci et al. 1998; Cousins and McLeay 1998) but in many cases determining the allocation of their constituency vote

Table 2 The national pattern of split-ticket voting: Percentage distribution of each party's constituency votes according to the party supported in the party vote (derived from a sample survey of the electorate)[a]

	Constituency vote								
Party vote	*NAT*	*LAB*	*NZF*	*ALL*	*ACT*	*UTD*	*CHR*	*OTH*	*N*
NAT	77.6	10.2	2.4	2.7	3.1	2.0	0.4	1.6	254
LAB	6.5	66.7	7.5	14.8	0.0	1.5	0.5	2.5	201
NZF	7.9	20.2	58.5	10.1	0.0	1.1	2.2	0.0	89
ALL	8.1	27.0	6.5	56.8	0.0	0.0	0.0	1.8	111
ACT	65.5	10.9	1.8	0.0	18.2	0.0	0.0	3.6	55
UTD	57.1	14.3	0.0	0.0	14.3	14.3	0.0	0.0	7
CHR	25.0	21.9	9.4	3.1	3.1	0.0	37.5	0.0	32
OTH	21.4	35.7	7.1	7.1	0.0	0.0	0.0	28.7	14

[a]NAT, National; LAB, Labour; NZF, New Zealand First; ALL, Alliance: ACT; Association of Consumers and Taxpayers; UTD; United New Zealand Party; CHR, Christian Coalition; OTH, other. *N*, number of voters in sample. Source: Levine and Roberts (1997, p. 185).

on different criteria. This is illustrated by the row for the ACT in Table 2; of those who voted for National in the party contest, almost two-thirds (65.5%) voted for a National party candidate in the constituency contests—their party votes would win seats allocated according to the list procedure, but they did not expect to get constituency seats. [Their 6.1% of the party votes entitled them to eight seats in the House of Representatives. Seven of these were allocated from the list and one resulted from victory in a constituency contest: their leader won in Wellington Central with 34.8% of the votes (see Fraser and Zangouropolous 1998). Only three other ACT candidates won more than 10% of the constituency votes, and none achieved more than 15%.]

The published election results give the number of votes cast in each constituency both for each candidate in the constituency ballot and for each party in the national list contest. Using those as the column and row total constraints, respectively, and the matrix in Table 2 as the basis for the internal cell constraints (i.e., as the slice of the three-dimensional data cube), we have used the EM method to estimate the split-ticket matrix for each constituency. (The percentages were converted into numbers of votes, consistent with the row and column totals for the whole of New Zealand.) Because not all of the parties contested all of the seats, the interconstituency variations in these estimates are best appreciated by looking separately at different groups of seats. Table 3 presents the summary statistics for the flow variations in the 35 constituencies contested by candidates from National, Labour, New Zealand First, the Alliance, ACT and the Christian Coalition. There are four rows for each matrix cell: the first two give the minimum and maximum percentages, respectively, and the other two give the mean and standard deviation. Thus, for example, the percentage of National loyalists (who voted for National in the party contest and for the National candidates in the constituency contests) ranged from 29.7 to 94.8, with a mean of 74.7 and a standard deviation of 15.2.

The clear impression gained from Table 3 is of very considerable interconstituency variation in the pattern of flows. Most of the standard deviations are large relative to the means, and there are very substantial ranges, even in the percentages who are loyal and who shift between the largest parties. National and Labour were by far the largest in the final outcomes (Table 1): they are also the longest-established parties and were the only contestants for power in the two-party system that dominated New Zealand from the 1930s until the end of the 1980s. Still, in one constituency, as many as 29.2% of National party voters in the list contest gave their support to the Labour candidate in their constituency contest, whereas in another, 23.2% of Labour's party voters in the list contest supported the National candidate for that constituency seat. In the former case (the Waimakariri constituency, on the northern fringe of Christchurch), National obtained 12,824 of the party votes to Labour's 10,631, with no other party getting more than 4100: in the constituency contest there, however, Labour's candidate got 19,875 votes to National's 9269. (The Labour candidate was a former party leader.) In the latter case (Nelson, in the north of the South Island), Labour and National got 11,012 and 10,652 votes, respectively, in the party contest, but National's candidate defeated Labour's by 20,481 to 8057 in the contest for the constituency seat. In both cases, the two parties' candidates were both incumbents: clearly in each there was a strong personal vote for one candidate.

Our major goal in this paper is not to analyze the spatial variations in split-ticket voting in detail but, rather, to illustrate the EM method. Many of the individual features—such as those in Wellington Central, Nelson, and Waimakariri referred to above—reflect particular aspects of the local campaigns, such as the popularity of incumbent candidates. But there may also be systemwide relationships. Burden and Kimball (1998) found very strong relationships between the pattern of split-ticket voting in the 1988 U.S. elections and the volume of

Table 3 Variations in split-ticket voting: Summary statistics for the percentage distribution of each party's constituency votes according to the party supported in the constituency vote, in the 35 constituencies contested by candidates from National, Labour, New Zealand First, Alliance, ACT and Christian Coalition[a]

	NAT	*LAB*	*NZF*	*ALL*	*ACT*	*CHR*	*OTH*
NAT							
Min	29.7	2.5	0.3	0.5	0.7	0.2	0.1
Max	94.8	29.2	12.9	8.9	54.1	1.2	1.1
Mean	74.7	10.9	3.1	2.2	7.1	0.6	0.2
SD	15.2	7.3	2.8	1.5	11.7	0.2	0.3
LAB							
Min	2.4	31.7	1.2	4.1	*[b]	0.3	0.1
Max	23.2	89.8	51.1	36.4	*	1.3	2.9
Mean	8.1	64.6	10.9	14.0	*	0.8	0.4
SD	5.2	17.3	9.7	9.2	*	0.3	0.6
NZF							
Min	1.6	2.2	19.6	1.2	*	1.4	*
Max	33.2	50.5	92.9	28.7	*	5.8	*
Mean	8.6	21.4	57.6	8.2	*	3.1	*
SD	5.9	14.2	17.8	5.4	*	1.0	*
ALL							
Min	2.8	8.9	1.9	28.3	*	*	0.1
Max	28.1	60.2	45.5	78.1	*	*	2.0
Mean	10.0	31.2	9.2	49.3	*	*	0.3
SD	5.2	16.4	8.2	15.1	*	*	0.5
ACT							
Min	7.0	2.6	0.1	*	5.0	*	0.1
Max	90.8	31.7	9.9	*	89.1	*	2.6
Mean	61.0	10.6	2.1	*	25.9	*	0.4
SD	19.7	7.3	2.0	*	20.3	*	0.6
CHR							
Min	6.1	4.9	1.1	0.6	1.2	19.0	*
Max	55.9	39.5	30.3	7.5	49.4	54.7	*
Mean	21.9	17.9	9.2	2.2	5.7	43.1	*
SD	10.0	9.1	6.1	1.5	9.6	7.7	*
OTH							
Min	12.4	16.7	1.7	*	2.9	*	0.3
Max	63.9	78.4	49.1	*	22.3	*	28.2
Mean	30.1	45.0	12.0	*	8.0	*	5.0
SD	13.0	18.7	9.6	*	5.1	*	6.4

[a]NAT, National; LAB, Labour; NZF, New Zealand First; ALL, Alliance; ACT, Association of Consumers and Taxpayers; CHR, Christian Coalition; OTH, other. Min, Minimum percentage; max, maximum percentage; mean, mean percentage; SD, standard deviation.
[b]No estimate.

campaign spending by candidates for the House of Representatives and the Senate. In New Zealand in 1996, candidates were allowed to spend up to NZ$20,000 on their personal campaigns in the 3 months before the election. We have used the spending data collected by the Electoral Commission to test the hypotheses that in each of the 35 constituencies considered in Table 3.

Table 4 Results of regressions of party loyalty (straight-ticket voting) versus campaign spending in the 35 constituencies considered in Table 3[a]

Party vote	*NAT*	*LAB*	*NZF*	*ALL*	*ACT*	*CHR*
Constituency vote	*NAT*	*LAB*	*NZF*	*ALL*	*ACT*	*CHR*
Spending by						
NAT	1.26	*	*	*	−1.42	*
LAB	−0.75	1.26	−1.03	*	*	*
NZF	*	−1.67	1.95	0.97	*	*
ALL	1.01	−0.85	*	1.77	−1.14	*
ACT	−1.33	*	−0.78	*	2.89	−0.68
CHR	*	*	*	*	*	1.04
R^2	0.58	0.63	0.62	0.51	0.58	0.30
(Adjusted)	(0.49)	(0.56)	(0.54)	(0.40)	(0.50)	(0.15)

[a]Significant coefficients at the 0.05 level are underlined; those not underlined are significant at the 0.10 level. An insignificant coefficient is indicated by *.

1. The more a party spent, the greater the level of loyalty to it across the two contests (i.e., the larger its "straight-ticket" percentage).
2. The more its opponents spent, the lower the level of loyalty to a party across the two contests (i.e., the smaller its "straight-ticket" percentage).

Table 4 presents the results of the multiple regressions. The first hypothesis is validated in each of the six equations. Five of the regression coefficients for the relevant party's expenditure on its own performance are significant at the 0.05 level and the other is significant at the 0.10 level: all six are positive. The more that a party spent on a local campaign, the greater the proportion of its party voters who also supported its constituency candidate. Regarding the second hypothesis, only 11 of the 30 coefficients are significant, 6 of them at the 0.05 level; in addition, 2 of those 11 are positive instead of negative (the more the Alliance spent, the larger the National loyalty, and the more the New Zealand First party spent, the larger the level of Alliance loyalty). Thus whereas there is strong evidence that intensive local campaigns (as indexed by expenditure) produced greater levels of voter loyalty across the two contests, there was less consistent evidence that the more one party spent on its constituency campaign, the lower the loyalty to its opponents. [A fuller discussion of the interparty shifts and their relationships with campaign expenditure is given by Johnston and Pattie (1999).]

3.2 *Validating the Straight-Ticket Estimates*

Publication of the number of "straight-ticket" and "split-ticket" votes in each constituency by the Chief Electoral Officer provides an opportunity to evaluate the output of the EM method, which has not been possible with most of its other electoral applications [Johnston et al. (1983, 1984) report on the only tests conducted; Cleave et al. (1995) report an evaluation using data on changes in housing tenure]. The actual number of straight- and split-ticket votes in each constituency can be compared with the estimated number. We have done this using the number of straight-ticket votes; Fig. 1 shows the relationship. The fit is clearly very close and there are no obvious outliers. (The largest positive residual was 2481 votes in a constituency where the actual number was 21,237, and the largest negative value was 1915, where the actual figure was 18,366.) The overall pattern shown by the scatter diagram and the r^2 value suggests considerable confidence in the EM estimates: where they can be

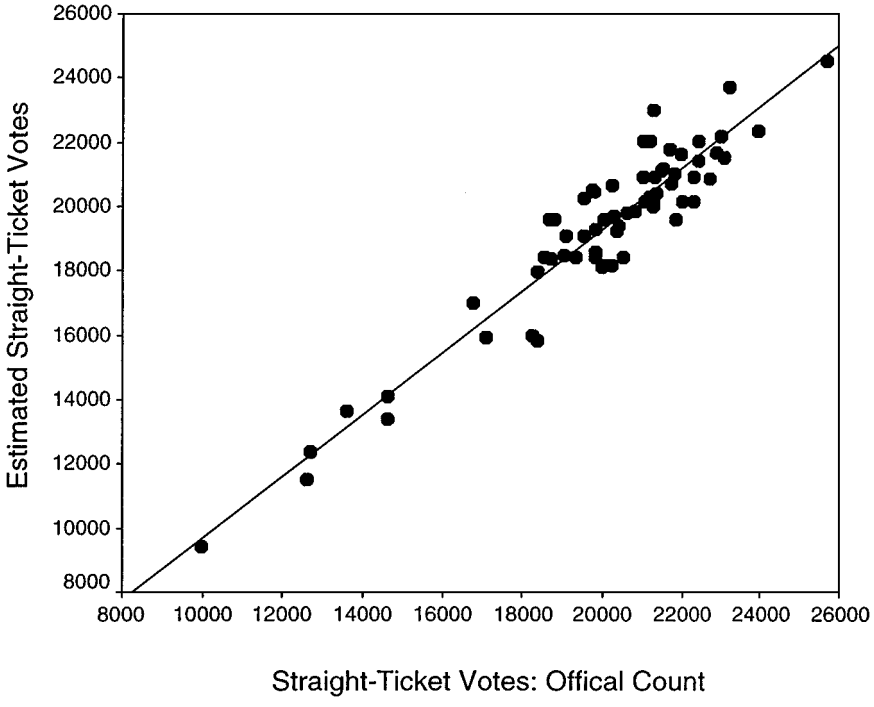

Fig. 1 Regression of estimated on actual number of straight-ticket votes, by constituency—New Zealand 1996 General Election. The regression equation is $Y = 98.70 + 0.96X$ ($SE = 0.04$), with an r^2 of 0.89.

checked against actual data, they are very close to them, allowing us to infer that the values that cannot be checked—the interparty flows—are also excellent estimates of the actual patterns.

4 Conclusions

The twin associated problems of EI and the estimation of known from unknown values have long concerned social scientists interested in, among other things, spatial variations in a wide range of behaviors. The EM method discussed here was introduced to social scientists as a means of estimating traffic flows and other movement patterns, providing maximum-likelihood estimates of flow patterns that could be used in transport planning. [The seminal work was by Wilson (1970); see also Wilson (1981).] It has been applied in a range of studies of electoral behavior, notably in Great Britain (as by Johnston et al. 1988), but also in work on split-ticket voting in the United States (Johnston and Hay 1984), but has not been widely appreciated and adopted. [Johnston and Pattie (1991b), for example, used it to estimate the volume of tactical—or strategic—voting at the 1987 British general election; their estimate has recently been confirmed by Alvarez and Nagler (2000).] This paper has therefore set out the basic procedures involved, illustrated their application with a case study of a novel electoral situation, and reported on a validation exercise that suggests considerable confidence in the estimates produced.

Different estimation procedures make different assumptions and have different data requirements. The EM method reported here requires more data than King's EI procedure,

in that it needs a systemwide matrix of the cell values that are to be estimated for the constituent subareas. It is therefore not a resolution of the "classic" ecological inference problem first identified by Robinson (1950) and Goodman (1953, 1959) involving estimates of that systemwide matrix—on which King (1997) focuses—but it does offer an alternative method of estimating the matrix values in the various subareas, which is a by-product of King's method. With these, it is possible to design a wide range of analyses of spatial variations in voting, as illustrated here.

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