Price equation

$p = a - q_i - q_j$

$$p = a - q_i - q_j \tag{1}$$

Firm profit function

$\Pi_i = (p - w_i) \cdot q_i$

$$\Pi_i = (p - w_i)\, q_i \tag{2}$$

$subs\left(p = a - q_i - q_j, \Pi_i = (p - w_i)\, q_i\right)$

$$\Pi_i = (a - q_i - q_j - w_i)\, q_i \tag{3}$$

Union utility

$V_i = (w_i)^{\theta} \cdot q_i$

$$V_i = w_i^{\theta}\, q_i \tag{4}$$

Efficient bargaining

$NP = \Pi_i^{(1-b)} \cdot (V_i)^b$

$$NP = \Pi_i^{(1-b)}\, V_i^b \tag{5}$$

$subs\left(\Pi_i = (a - q_i - q_j - w_i)\, q_i, V_i = w_i^{\theta}\, q_i, NP = \Pi_i^{(1-b)}\, V_i^b\right)$

$$NP = \left((a - q_i - q_j - w_i)\, q_i\right)^{1-b} \left(w_i^{\theta}\, q_i\right)^b \tag{6}$$

differentiate w.r.t. w[i] →

$$0 = -\frac{\left((a - q_i - q_j - w_i)\, q_i\right)^{1-b} (1-b) \left(w_i^{\theta}\, q_i\right)^b}{a - q_i - q_j - w_i} + \frac{\left((a - q_i - q_j - w_i)\, q_i\right)^{1-b} \left(w_i^{\theta}\, q_i\right)^b b\, \theta}{w_i} \tag{7}$$

solve for w[i] →

$$\left[\left[w_i = \frac{b\, \theta\, (a - q_i - q_j)}{b\, \theta - b + 1}\right]\right] \tag{8}$$

differentiate w.r.t. q[i] →

$$0 = \frac{\left((a - q_i - q_j - w_i)\, q_i\right)^{1-b} (1-b) \left(-2\, q_i + a - q_j - w_i\right) \left(w_i^{\theta}\, q_i\right)^b}{(a - q_i - q_j - w_i)\, q_i} + \frac{\left((a - q_i - q_j - w_i)\, q_i\right)^{1-b} \left(w_i^{\theta}\, q_i\right)^b b}{q_i} \tag{9}$$

solve for q[i] →

$$\left[\left[q_i = -\frac{a - w_i - q_j}{b - 2}\right]\right] \tag{10}$$

Reaction functions

$q_i = -\frac{a - w_i - q_j}{b - 2}$

$$q_i = -\frac{a - w_i - q_j}{b - 2} \tag{11}$$

$q_j = -\frac{a - w_j - q_i}{b - 2}$

$$q_j = -\frac{a - w_j - q_i}{b - 2} \tag{12}$$

$$subs\left(q_j = -\frac{a - w_j - q_i}{b - 2}, q_i = -\frac{a - w_i - q_j}{b - 2}\right)$$

$$q_i = -\frac{a - w_i + \frac{a - w_j - q_i}{b - 2}}{b - 2} \tag{13}$$

solve for q[i] →

$$\left[\left[q_i = -\frac{a\,b - b\,w_i - a + 2\,w_i - w_j}{b^2 - 4\,b + 3}\right]\right] \tag{14}$$

$$subs\left(q_i = -\frac{a - w_i - q_j}{b - 2}, q_j = -\frac{a - w_j - q_i}{b - 2}\right)$$

$$q_j = -\frac{a - w_j + \frac{a - w_i - q_j}{b - 2}}{b - 2} \tag{15}$$

solve for q[j] →

$$\left[\left[q_j = -\frac{a\,b - b\,w_j - a - w_i + 2\,w_j}{b^2 - 4\,b + 3}\right]\right] \tag{16}$$

$$subs\left(q_i = -\frac{a\,b - b\,w_i - a + 2\,w_i - w_j}{b^2 - 4\,b + 3}, q_j = -\frac{a\,b - b\,w_j - a - w_i + 2\,w_j}{b^2 - 4\,b + 3}, w_i = \frac{b\,\theta\,(a - q_i - q_j)}{b\,\theta - b + 1}\right)$$

$$w_i = \frac{b\,\theta\left(a + \frac{a\,b - b\,w_i - a + 2\,w_i - w_j}{b^2 - 4\,b + 3} + \frac{a\,b - b\,w_j - a - w_i + 2\,w_j}{b^2 - 4\,b + 3}\right)}{b\,\theta - b + 1}$$

solve for w[i] →

$$\left[\left[w_i = \frac{(a\,b - a - w_j)\,b\,\theta}{b^2\,\theta - b^2 - 2\,b\,\theta + 4\,b - 3} \right]\right]$$

$$subs\left(w_j = \frac{(a\,b - a - w_i)\,b\,\theta}{b^2\,\theta - b^2 - 2\,b\,\theta + 4\,b - 3} \,,\, w_i = \frac{(a\,b - a - w_j)\,b\,\theta}{b^2\,\theta - b^2 - 2\,b\,\theta + 4\,b - 3} \right)$$

$$w_i = \frac{\left(b\,a - a - \frac{(a\,b - a - w_i)\,b\,\theta}{b^2\,\theta - b^2 - 2\,b\,\theta + 4\,b - 3} \right) b\,\theta}{b^2\,\theta - b^2 - 2\,b\,\theta + 4\,b - 3}$$

$\xrightarrow{\text{solve for w[i]}}$ equilirium wage

$$\left[\left[w_i = \frac{a\,b\,\theta}{b\,\theta - b + 3} \right]\right] \tag{20}$$

Equilibrium quantity

$$subs\left(w_i = \frac{a\,b\,\theta}{b\,\theta - b + 3} \,,\, w_j = \frac{a\,b\,\theta}{b\,\theta - b + 3} \,,\, q_i = - \frac{a\,b - b\,w_i - a + 2\,w_i - w_j}{b^2 - 4\,b + 3} \right)$$

$$q_i = - \frac{b\,a - \frac{b^2\,a\,\theta}{b\,\theta - b + 3} - a + \frac{a\,b\,\theta}{b\,\theta - b + 3}}{b^2 - 4\,b + 3} \tag{21}$$

$\overset{\text{simplify}}{=}$

$$q_i = \frac{a}{3 + b\,(\theta - 1)} \tag{22}$$

Equilibrium profits

$$subs\left(w_i = \frac{a\,b\,\theta}{b\,\theta - b + 3} \,,\, q_i = \frac{a}{3 + b\,(\theta - 1)} \,,\, q_j = \frac{a}{3 + b\,(\theta - 1)} \,,\, \Pi_i = (a - q_i - q_j - w_i)\,q_i \right)$$

$$\Pi_i = \frac{\left(a - \frac{2\,a}{3 + b\,(\theta - 1)} - \frac{a\,b\,\theta}{b\,\theta - b + 3} \right) a}{3 + b\,(\theta - 1)} \tag{23}$$

$$\Pi_i = \frac{(1 - b)\,a^2}{(3 + b\,(\theta - 1))^2}$$

$$\Pi_i = \frac{(1 - b)\,a^2}{(3 + b\,(\theta - 1))^2} \tag{24}$$

Competitive firms

Firm profit function

$\Pi_i = p \cdot q_i$

$$\Pi_i = p\,q_i \tag{25}$$

$$subs\left(p = a - q_i - q_j \,,\, \Pi_i = p\,q_i \right)$$

$$\Pi_i = (a - q_i - q_j)\, q_i \tag{26}$$

differentiate w.r.t. q[i] →

$$0 = -2\, q_i + a - q_j \tag{27}$$

solve for q[i] →

$$\left[\left[q_i = \frac{1}{2}\, a - \frac{1}{2}\, q_j\right]\right] \tag{28}$$

$$subs\left(q_j = \frac{1}{2}\, a - \frac{1}{2}\, q_i\,,\, q_i = \frac{1}{2}\, a - \frac{1}{2}\, q_j\right)$$

$$q_i = \frac{1}{4}\, a + \frac{1}{4}\, q_i \tag{29}$$

solve for q[i] →

$$\left[\left[q_i = \frac{1}{3}\, a\right]\right] \tag{30}$$

$$subs\left(q_i = \frac{1}{3}\, a\,,\, q_j = \frac{1}{3}\, a\,,\, \Pi_i = (a - q_i - q_j)\, q_i\right)$$

$$\Pi_i = \frac{1}{9}\, a^2 \tag{31}$$

Mixed duopoly

$$subs\left(q_j = \frac{1}{2}\, a - \frac{1}{2}\, q_i\,,\, q_i = -\frac{a - w_i - q_j}{b - 2}\right)$$

$$q_i = -\frac{\frac{1}{2}\, a - w_i + \frac{1}{2}\, q_i}{b - 2}$$

solve for q[i] →

$$\left[\left[q_i = -\frac{a - 2\, w_i}{-3 + 2\, b}\right]\right] \tag{33}$$

$$subs\left(q_i = -\frac{a - w_i - q_j}{b - 2}\,,\, q_j = \frac{1}{2}\, a - \frac{1}{2}\, q_i\right)$$

$$q_j = \frac{1}{2}\, a + \frac{1}{2}\, \frac{a - w_i - q_j}{b - 2} \tag{34}$$

solve for q[j] →

$$\left[\left[q_j = \frac{a\, b - a - w_i}{-3 + 2\, b}\right]\right] \tag{35}$$

$$subs\left(q_i = -\frac{a - 2\, w_i}{-3 + 2\, b}\,,\, q_j = \frac{a\, b - a - w_i}{-3 + 2\, b}\,,\, w_i = \frac{b\, \theta\, (a - q_i - q_j)}{b\, \theta - b + 1}\right)$$

$$w_i = \frac{b\,\theta\left(a + \frac{a - 2\,w_i}{-3 + 2\,b} - \frac{a\,b - a - w_i}{-3 + 2\,b}\right)}{b\,\theta - b + 1} \tag{36}$$

solve for w[i] →

$$\left[\left[w_i = \frac{a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}\right]\right] \tag{37}$$

$$subs\left(w_i = \frac{a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}\,,\ q_i = -\frac{a - 2\,w_i}{-3 + 2\,b}\right)$$

$$q_i = -\frac{a - \frac{2\,a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}}{-3 + 2\,b} \tag{38}$$

simplify =

$$q_i = \frac{a}{(2\,\theta - 2)\,b + 3} \tag{39}$$

$$subs\left(w_i = \frac{a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}\,,\ q_j = \frac{a\,b - a - w_i}{-3 + 2\,b}\right)$$

$$q_j = \frac{b\,a - a - \frac{a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}}{-3 + 2\,b} \tag{40}$$

simplify symbolic →

$$q_j = \frac{(b\,\theta - b + 1)\,a}{2\,b\,\theta - 2\,b + 3} \tag{41}$$

$$subs\left(q_i = \frac{a}{(2\,\theta - 2)\,b + 3}\,,\ q_j = \frac{(b\,\theta - b + 1)\,a}{2\,b\,\theta - 2\,b + 3}\,,\ w_i = \frac{a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}\,,\ \Pi_i = (a - q_i - q_j - w_i)\,q_i\right)$$

$$\Pi_i = \frac{\left(a - \frac{a}{(2\,\theta - 2)\,b + 3} - \frac{(b\,\theta - b + 1)\,a}{2\,b\,\theta - 2\,b + 3} - \frac{a\,b\,\theta}{2\,b\,\theta - 2\,b + 3}\right)a}{(2\,\theta - 2)\,b + 3} \tag{42}$$

simplify =

$$\Pi_i = -\frac{(-1 + b)\,a^2}{(2\,b\,\theta - 2\,b + 3)^2} \tag{43}$$

$$\Pi_i = \frac{(1 - b)\,a^2}{(2\,b\,\theta - 2\,b + 3)^2}$$

$$\Pi_i = \frac{(1 - b)\,a^2}{(2\,b\,\theta - 2\,b + 3)^2} \tag{44}$$

$$subs\left(q_i = \frac{a}{(2\theta-2)\,b+3},\ q_j = \frac{(b\theta-b+1)\,a}{2b\theta-2b+3},\ \Pi_j = (a-q_i-q_j)\,q_j\right)$$

$$\Pi_j = \frac{\left(a - \frac{a}{(2\theta-2)\,b+3} - \frac{(b\theta-b+1)\,a}{2b\theta-2b+3}\right)(b\theta-b+1)\,a}{2b\theta-2b+3} \tag{45}$$

simplify

$$\Pi_j = \frac{(b(\theta-1)+1)^2 a^2}{(2b\theta-2b+3)^2} \tag{46}$$

$$\Delta\Pi_1 = \frac{a^2}{9} - \frac{(1-b)\,a^2}{(3+b(\theta-1))^2}$$

$$\Delta\Pi_1 = \frac{1}{9}a^2 - \frac{(1-b)\,a^2}{(3+b(\theta-1))^2} \tag{47}$$

simplify

$$\Delta\Pi_1 = -\frac{1}{9}\,\frac{a^2 b\left((\theta-1)^2 b+6\theta+3\right)}{(3+b(\theta-1))^2} \tag{48}$$

$$\Delta\Pi_2 = \frac{(b(\theta-1)+1)^2 a^2}{(2b\theta-2b+3)^2} - \frac{(1-b)\,a^2}{(3+b(\theta-1))^2}$$

$$\Delta\Pi_2 = \frac{(b(\theta-1)+1)^2 a^2}{(2b\theta-2b+3)^2} - \frac{(1-b)\,a^2}{(3+b(\theta-1))^2}$$

simplify symbolic

$$\Delta\Pi_2 = \frac{1}{(2b\theta-2b+3)^2(b\theta-b+3)^2}\left(a^2 b\left(b^3\theta^4 - 4b^3\theta^3 + 6b^3\theta^2 + 8b^2\theta^3 - 4b^3\theta - 20b^2\theta^2 + b^3 + 16b^2\theta + 18b\theta^2 - 4b^2 - 24b\theta + 6b + 12\theta - 3\right)\right)$$

$$b^3\theta^4 - 4b^3\theta^3 + 6b^3\theta^2 + 8b^2\theta^3 - 4b^3\theta - 20b^2\theta^2 + b^3 + 16b^2\theta + 18b\theta^2 - 4b^2 - 24b\theta + 6b + 12\theta - 3$$

$$b^3\theta^4 - 4b^3\theta^3 + 6b^3\theta^2 + 8b^2\theta^3 - 4b^3\theta - 20b^2\theta^2 + b^3 + 16b^2\theta + 18b\theta^2 - 4b^2 - 24b\theta + 6b + 12\theta - 3$$

simplify

$$(\theta-1)^4 b^3 + \left(8\theta^3 - 20\theta^2 + 16\theta - 4\right)b^2 + \left(18\theta^2 - 24\theta + 6\right)b + 12\theta - 3 \tag{52}$$

$$\Delta\Pi_2 = \frac{a^2 b\left((\theta-1)^4 b^3 + \left(8\theta^3 - 20\theta^2 + 16\theta - 4\right)b^2 + \left(18\theta^2 - 24\theta + 6\right)b + 12\theta - 3\right)}{(2b\theta-2b+3)^2(b\theta-b+3)^2}$$

$$\Delta\Pi_2 = \frac{a^2 b \left((\theta - 1)^4 b^3 + \left(8\theta^3 - 20\theta^2 + 16\theta - 4\right) b^2 + \left(18\theta^2 - 24\theta + 6\right) b + 12\theta - 3 \right)}{(2b\theta - 2b + 3)^2 (b\theta - b + 3)^2} \quad (53)$$

$$\Delta\Pi_3 = \frac{(1-b)\, a^2}{(2b\theta - 2b + 3)^2} - \frac{a^2}{9}$$

$$\Delta\Pi_3 = \frac{(1-b)\, a^2}{(2b\theta - 2b + 3)^2} - \frac{1}{9} a^2$$

simplify symbolic →

$$\Delta\Pi_3 = -\frac{1}{9} \frac{a^2 b \left(4b\theta^2 - 8b\theta + 4b + 12\theta - 3\right)}{(2b\theta - 2b + 3)^2} \quad (55)$$

$\Delta\Pi_3 = 0$

$$\frac{a^2 \cdot (1-b)}{(2b\theta - 2b + 3)^2} = \frac{a^2}{9}$$

$$\frac{a^2 (1-b)}{(2b\theta - 2b + 3)^2} = \frac{1}{9} a^2$$

solve for b →

$$\left[[b = 0], \left[b = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1} \right] \right]$$

$$b = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1}$$

$$b = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1} \quad (58)$$

evaluate at point →

$$0 = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1} \quad (59)$$

solve for theta →

$$\left[\left[\theta = \frac{1}{4} \right] \right] \quad (60)$$

$\Delta\Pi_2 = 0$

$$0 = \frac{a^2 b \left((\theta - 1)^4 b^3 + \left(8\theta^3 - 20\theta^2 + 16\theta - 4\right) b^2 + \left(18\theta^2 - 24\theta + 6\right) b + 12\theta - 3 \right)}{(2b\theta - 2b + 3)^2 (b\theta - b + 3)^2}$$

$$0 = \frac{a^2 b \left((\theta - 1)^4 b^3 + \left(8\theta^3 - 20\theta^2 + 16\theta - 4\right) b^2 + \left(18\theta^2 - 24\theta + 6\right) b + 12\theta - 3 \right)}{(2b\theta - 2b + 3)^2 (b\theta - b + 3)^2} \quad (61)$$

$$\frac{1-b}{(2b\theta - 2b + 3)^2} = \frac{1}{9}$$

$$\frac{1-b}{(2b\theta - 2b + 3)^2} = \frac{1}{9}$$

solve for b

$$\left[[b = 0], \left[b = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1} \right] \right] \quad (63)$$

$$0 = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1}$$

$$0 = -\frac{3}{4} \frac{-1 + 4\theta}{\theta^2 - 2\theta + 1}$$

solve for theta

$$\left[\left[\theta = \frac{1}{4} \right] \right] \quad (65)$$

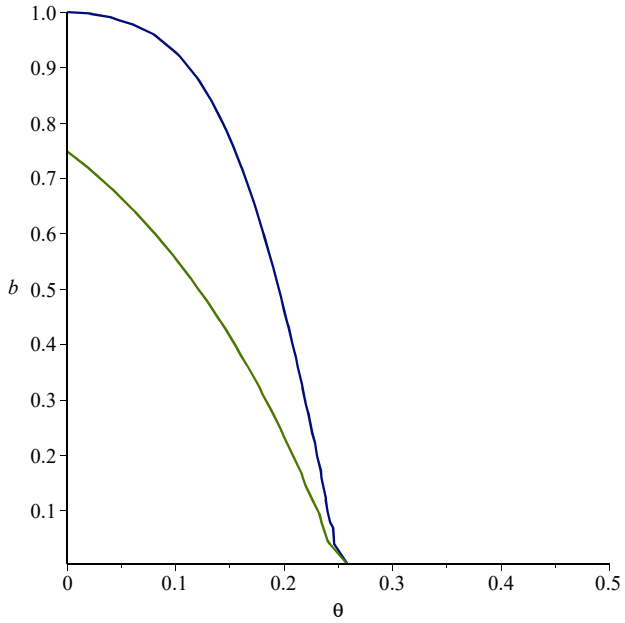
$$\frac{1}{9} a^2 = \frac{(1-b) a^2}{(3 + b(\theta - 1))^2}, \frac{(b(\theta - 1) + 1)^2 a^2}{(2b\theta - 2b + 3)^2} = \frac{(1-b) a^2}{(3 + b(\theta - 1))^2}, \frac{a^2 \cdot (1-b)}{(2b\theta - 2b + 3)^2} = \frac{a^2}{9}$$

$$\frac{1}{9} a^2 = \frac{(1-b) a^2}{(3 + b(\theta - 1))^2}, \frac{(b(\theta - 1) + 1)^2 a^2}{(2b\theta - 2b + 3)^2} = \frac{(1-b) a^2}{(3 + b(\theta - 1))^2}, \frac{(1-b) a^2}{(2b\theta - 2b + 3)^2} = \frac{1}{9} a^2 \quad (66)$$

evaluate at point

$$\frac{1}{9} = \frac{1-b}{(3 + b(\theta - 1))^2}, \frac{(b(\theta - 1) + 1)^2}{(2b\theta - 2b + 3)^2} = \frac{1-b}{(3 + b(\theta - 1))^2}, \frac{1-b}{(2b\theta - 2b + 3)^2} = \frac{1}{9} \quad (67)$$

→



$$\frac{1}{9} = \frac{1-b}{(3+b(\theta-1))^2}, \frac{1-b}{(2b\theta-2b+3)^2} = \frac{1}{9}$$

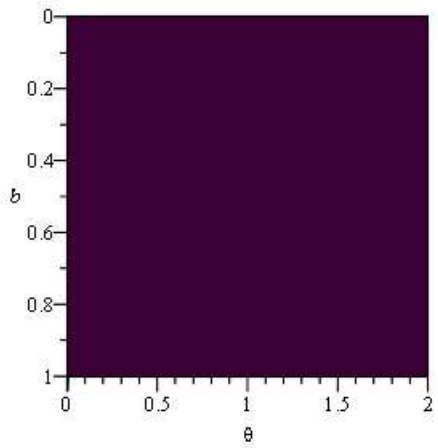
$$\frac{1}{9} = \frac{1-b}{(3+b(\theta-1))^2}, \frac{1-b}{(2b\theta-2b+3)^2} = \frac{1}{9} \quad \textbf{(68)}$$

Entry
Committed bargaining, blockaded entry
EB vs CF

$$\frac{(1-b)}{(3+b(\theta-1))^2}, \frac{1}{9}$$

$$\frac{1-b}{(3+b(\theta-1))^2}, \frac{1}{9} \tag{69}$$

→



Committed bargaining, threat of entry
Monopoly EB vs Duopoly CF
Monopoly EB profits

$$subs\left(\Pi_i = (a - q_i - w_i)\, q_i,\ V_i = w_i^{\theta} q_i,\ NP = \Pi_i^{(1-b)}\, V_i^{b}\right)$$

$$NP = \left((a - q_i - w_i)\, q_i\right)^{1-b} \left(w_i^{\theta} q_i\right)^{b}$$

differentiate w.r.t. w[i] →

$$0 = -\frac{\left((a - q_i - w_i)\, q_i\right)^{1-b} (1-b) \left(w_i^{\theta} q_i\right)^{b}}{a - q_i - w_i} + \frac{\left((a - q_i - w_i)\, q_i\right)^{1-b} \left(w_i^{\theta} q_i\right)^{b} b\,\theta}{w_i}$$

solve for w[i] →

$$\left[\left[w_i = \frac{b\,\theta\,(a - q_i)}{b\,\theta - b + 1}\right]\right]$$

differentiate w.r.t. q[i] →

$$0 = \frac{\left(\left(a - q_i - w_i\right) q_i\right)^{1-b} (1-b) \left(-2\,q_i + a - w_i\right) \left(w_i^{\theta} q_i\right)^{b}}{\left(a - q_i - w_i\right) q_i} + \frac{\left(\left(a - q_i - w_i\right) q_i\right)^{1-b} \left(w_i^{\theta} q_i\right)^{b} b}{q_i}$$

solve for q[i] →

$$\left[\left[q_i = -\frac{a - w_i}{b - 2}\right]\right] \tag{74}$$

$$q_i = -\frac{a - w_i}{b - 2}$$

$$q_i = -\frac{a - w_i}{b - 2}$$

solve for w[i] →

$$\left[\left[w_i = b\,q_i + a - 2\,q_i\right]\right] \tag{76}$$

$$\frac{b\,\theta\,(a - q_i)}{b\,\theta - b + 1} = b\,q_i + a - 2\,q_i$$

$$\frac{b\,\theta\,(a - q_i)}{b\,\theta - b + 1} = b\,q_i + a - 2\,q_i$$

solve for q[i] →

$$\left[\left[q_i = \frac{a}{b\,\theta - b + 2}\right]\right] \tag{78}$$

$$subs\left(q_i = \frac{a}{b\,\theta - b + 2},\ w_i = \frac{b\,\theta\,(a - q_i)}{b\,\theta - b + 1}\right)$$

$$w_i = \frac{b\,\theta\left(a - \frac{a}{b\,\theta - b + 2}\right)}{b\,\theta - b + 1} \tag{79}$$

simplify =

$$w_i = \frac{b\,\theta\,a}{b\,(\theta - 1) + 2} \tag{80}$$

$$subs\left(w_i = \frac{b\,\theta\,a}{b\,(\theta-1)+2}\,,\ q_i = \frac{a}{b\,\theta-b+2}\,,\ \Pi_i = (a-q_i-w_i)\,q_i\right)$$

$$\Pi_i = \frac{\left(a - \frac{a}{b\,\theta-b+2} - \frac{b\,\theta\,a}{b\,(\theta-1)+2}\right)a}{b\,\theta-b+2}$$

simplify

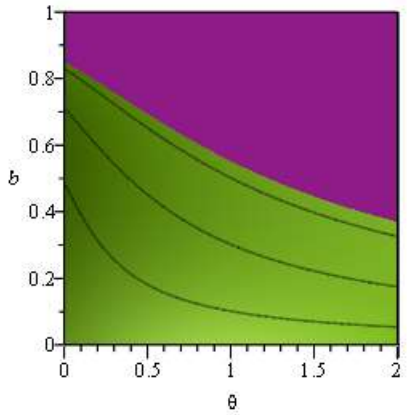
$$\Pi_i = -\frac{a^2\,(-1+b)}{(b\,(\theta-1)+2)^2} \tag{82}$$

Monopoly profit EB vs Duopoly profits CF

$$\frac{(1-b)}{(b\,(\theta-1)+2)^2}\,,\ \frac{1}{9}$$

$$\frac{1-b}{(b\,(\theta-1)+2)^2}\,,\ \frac{1}{9} \tag{83}$$

→



Green Monopoly profits under EB
Purple Duopoly profits under PM

→

→

$$\frac{1-b}{(b(\theta-1)+2)^2}=\frac{1}{9}$$

$$\frac{1-b}{(b(\theta-1)+2)^2}=\frac{1}{9} \tag{84}$$

$$\theta=\frac{b-2+3\sqrt{1-b}}{b}$$

$$\theta=\frac{b-2+3\sqrt{1-b}}{b}$$

evaluate at point →

$$\theta=-3+3\sqrt{2}$$

at 5 digits →

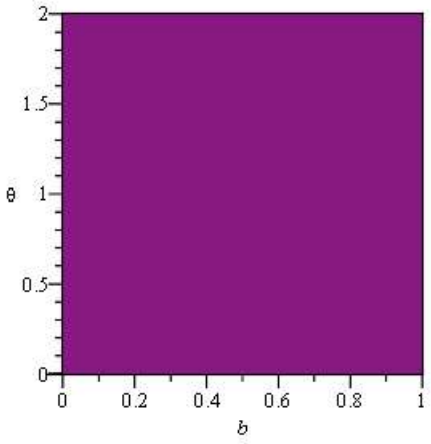
$$\theta=1.2426 \tag{87}$$

Monopoly profit CF vs duopoly profits EB

$$\frac{1}{4},\frac{(1-b)}{(3+b(\theta-1))^2}$$

$$\frac{1}{4},\frac{1-b}{(3+b(\theta-1))^2} \tag{88}$$

→

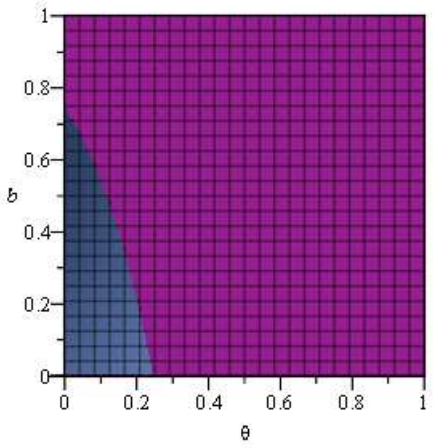


Duopoly EB/EB vs EB/PM vs PM/PM vs PM/EB

$$\frac{(1-b)}{(3+b(\theta-1))^2}, \frac{(1-b)}{(2b\theta-2b+3)^2}, \frac{1}{9}, \frac{(b(\theta-1)+1)^2}{(2b\theta-2b+3)^2}$$

$$\frac{1-b}{(3+b(\theta-1))^2}, \frac{1-b}{(2b\theta-2b+3)^2}, \frac{1}{9}, \frac{(b(\theta-1)+1)^2}{(2b\theta-2b+3)^2} \quad \textbf{(89)}$$

→



$$\theta^2 - 2\,\theta + 1$$

$$\theta^2 - 2\,\theta + 1 \tag{90}$$

$\overset{\text{factor}}{=}$

$$(\theta - 1)^2 \tag{91}$$

Threat of entry, flexible bargaining

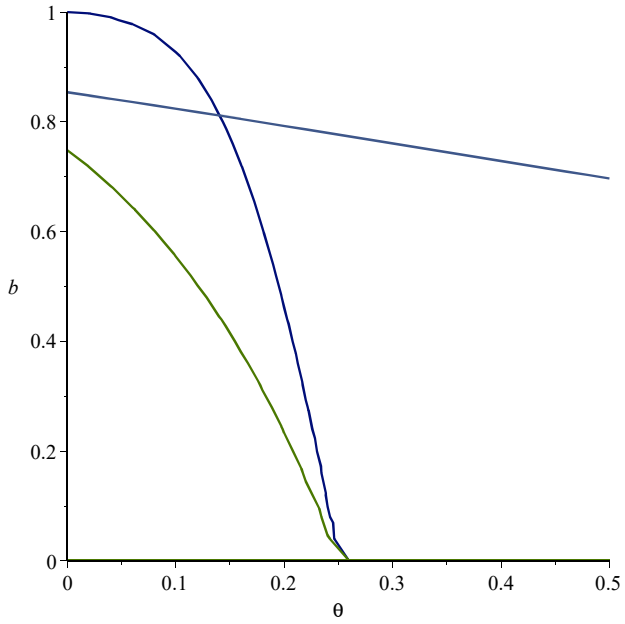
$$\frac{1}{9} = \frac{1-b}{(3+b\,(\theta-1))^2},\ \frac{(b\,(\theta-1)+1)^2}{(2\,b\,\theta-2\,b+3)^2} = \frac{1-b}{(3+b\,(\theta-1))^2},\ \frac{1-b}{(2\,b\,\theta-2\,b+3)^2} = \frac{1}{9}\ ,\ b$$

$$= \frac{1}{2}\ \frac{-4\,\theta-5+3\sqrt{4\,\theta^2+5}}{\theta^2-2\,\theta+1}$$

$$\frac{1}{9} = \frac{1-b}{(3+b\,(\theta-1))^2},\ \frac{(b\,(\theta-1)+1)^2}{(2\,b\,\theta-2\,b+3)^2} = \frac{1-b}{(3+b\,(\theta-1))^2},\ \frac{1-b}{(2\,b\,\theta-2\,b+3)^2} \tag{92}$$

$$= \frac{1}{9}, b = \frac{1}{2} \quad \frac{-4\,\theta - 5 + 3\sqrt{4\,\theta^2 + 5}}{\theta^2 - 2\,\theta + 1}$$

→



Network industry
Simultaneous EB with network effects

$$NP = \left(\left(n\left(y_1 + y_2\right) + a - q_1 - q_2 - w_1\right) q_1\right)^{1-b} \left(\left(w_1\right)^{\theta} \cdot q_1\right)^{b}$$

$$NP = \left(\left(n\left(y_1 + y_2\right) + a - q_1 - q_2 - w_1\right) q_1\right)^{1-b} \left(w_1^{\theta} q_1\right)^{b}$$

differentiate w.r.t. w[1] →

$$0 = -\frac{\left(\left(n\left(y_1 + y_2\right) + a - q_1 - q_2 - w_1\right) q_1\right)^{1-b} (1-b) \left(w_1^{\theta} q_1\right)^{b}}{n\left(y_1 + y_2\right) + a - q_1 - q_2 - w_1}$$

$$+ \frac{\left(\left(n\left(y_1 + y_2\right) + a - q_1 - q_2 - w_1\right) q_1\right)^{1-b} \left(w_1^{\theta} q_1\right)^{b} b\,\theta}{w_1}$$

$\xrightarrow{\text{solve for w[1]}}$ rent sharing curve

$$\left[\left[w_1 = \frac{b\,\theta\,(n\,y_1 + n\,y_2 + a - q_1 - q_2)}{b\,\theta - b + 1}\right]\right]$$

$\xrightarrow{\text{differentiate w.r.t. q[1]}}$ contract curve

$$0 = \frac{1}{(n\,(y_1 + y_2) + a - q_1 - q_2 - w_1)\,q_1}\left(\left((n\,(y_1 + y_2) + a - q_1 - q_2 - w_1)\,q_1\right)^{1-b}(1 - b)\left(-2\,q_1 + n\,(y_1 + y_2) + a - q_2 - w_1\right)\left(w_1^{\theta}\,q_1\right)^{b}\right) + \frac{\left((n\,(y_1 + y_2) + a - q_1 - q_2 - w_1)\,q_1\right)^{1-b}\left(w_1^{\theta}\,q_1\right)^{b} b}{q_1}$$

$\xrightarrow{\text{solve for q[1]}}$

$$\left[\left[q_1 = -\frac{n\,y_1 + n\,y_2 + a - q_2 - w_1}{b - 2}\right]\right] \tag{97}$$

$$q_1 = -\frac{n\,y_1 + n\,y_2 + a - q_2 - w_1}{b - 2}$$

$$q_1 = -\frac{n\,y_1 + n\,y_2 + a - q_2 - w_1}{b - 2} \tag{98}$$

$$q_2 = -\frac{n\,y_1 + n\,y_2 + a - q_1 - w_2}{b - 2}$$

$$q_2 = -\frac{n\,y_1 + n\,y_2 + a - q_1 - w_2}{b - 2} \tag{99}$$

Rational expectations realize

$$q_1 = -\frac{n\,q_1 + n\,q_2 + a - q_2 - w_1}{b - 2}$$

$$q_1 = -\frac{n\,q_1 + n\,q_2 + a - q_2 - w_1}{b - 2}$$

$\xrightarrow{\text{solve for q[1]}}$

$$\left[\left[q_1 = -\frac{n\,q_2 + a - q_2 - w_1}{n + b - 2}\right]\right] \tag{101}$$

$$q_2 = -\frac{n\,q_1 + n\,q_2 + a - q_1 - w_2}{b - 2}$$

$$q_2 = -\frac{n\,q_1 + n\,q_2 + a - q_1 - w_2}{b - 2}$$

$\xrightarrow{\text{solve for q[2]}}$

$$\left[\left[q_2 = -\frac{n\,q_1 + a - q_1 - w_2}{n + b - 2}\right]\right] \tag{103}$$

$$subs\left(q_1 = -\frac{n\,q_2 + a - q_2 - w_1}{n + b - 2},\ q_2 = -\frac{n\,q_1 + a - q_1 - w_2}{n + b - 2}\right)$$

$$q_2 = -\frac{-\frac{n\,(n\,q_2 + a - q_2 - w_1)}{n + b - 2} + a + \frac{n\,q_2 + a - q_2 - w_1}{n + b - 2} - w_2}{n + b - 2} \tag{104}$$

solve for q[2] →

$$\left[\left[q_2 = -\frac{a\,b - b\,w_2 + n\,w_1 - n\,w_2 - a - w_1 + 2\,w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3}\right]\right] \tag{105}$$

$$subs\left(q_2 = -\frac{n\,q_1 + a - q_1 - w_2}{n + b - 2},\ q_1 = -\frac{n\,q_2 + a - q_2 - w_1}{n + b - 2}\right)$$

$$q_1 = -\frac{-\frac{n\,(n\,q_1 + a - q_1 - w_2)}{n + b - 2} + a + \frac{n\,q_1 + a - q_1 - w_2}{n + b - 2} - w_1}{n + b - 2}$$

solve for q[1] →

$$\left[\left[q_1 = -\frac{a\,b - b\,w_1 - n\,w_1 + n\,w_2 - a + 2\,w_1 - w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3}\right]\right] \tag{107}$$

Rent-sharing curve

$$subs\left(q_1 = -\frac{a\,b - b\,w_1 - n\,w_1 + n\,w_2 - a + 2\,w_1 - w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3},\ q_2 = -\frac{a\,b - b\,w_2 + n\,w_1 - n\,w_2 - a - w_1 + 2\,w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3},\ w_1 = \frac{b\,\theta\,(n\,q_1 + n\,q_2 + a - q_1 - q_2)}{b\,\theta - b + 1}\right)$$

$$w_1 = \frac{1}{b\,\theta - b + 1}\left(b\,\theta\left(-\frac{n\,(a\,b - b\,w_1 - n\,w_1 + n\,w_2 - a + 2\,w_1 - w_2)}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3} - \frac{n\,(a\,b - b\,w_2 + n\,w_1 - n\,w_2 - a - w_1 + 2\,w_2)}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3} + a + \frac{a\,b - b\,w_1 - n\,w_1 + n\,w_2 - a + 2\,w_1 - w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3} + \frac{a\,b - b\,w_2 + n\,w_1 - n\,w_2 - a - w_1 + 2\,w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3}\right)\right) \tag{108}$$

solve for w[1] →

$$\left[\left[w_1 = \frac{(a\,b + n\,w_2 - a - w_2)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3}\right]\right] \tag{109}$$

$$w_1 = \frac{(a\,b + n\,w_2 - a - w_2)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3}$$

$$w_1 = \frac{(a\,b + n\,w_2 - a - w_2)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3} \tag{110}$$

$$w_2 = \frac{(a\,b + n\,w_1 - a - w_1)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3}$$

$$w_2 = \frac{(a\,b + n\,w_1 - a - w_1)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3} \tag{111}$$

Equilibrium wages

$$subs\left(w_2 = \frac{(a\,b + n\,w_1 - a - w_1)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3}, w_1 = \frac{(a\,b + n\,w_2 - a - w_2)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3}\right)$$

$$w_1 = \left(\left(a\,b + \frac{n\,(a\,b + n\,w_1 - a - w_1)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3} - a - \frac{(a\,b + n\,w_1 - a - w_1)\,b\,\theta}{b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3}\right) b\,\theta\right) \Big/ \left(b^2\,\theta + b\,n\,\theta - b^2 - 2\,b\,n - 2\,b\,\theta + 4\,b + 2\,n - 3\right)$$

$$\xrightarrow{\text{solve for w[1]}}$$

$$\left[\left[w_1 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3}\right]\right] \tag{113}$$

$$w_1 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3}$$

$$w_1 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3} \tag{114}$$

$$w_2 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3}$$

$$w_2 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3} \tag{115}$$

Equilibrium quantities

$$subs\left(w_1 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3}, w_2 = \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3}, q_1 = -\frac{a\,b - b\,w_1 - n\,w_1 + n\,w_2 - a + 2\,w_1 - w_2}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3}\right)$$

$$q_1 = -\frac{a\,b - \frac{b^2\,\theta\,a}{b\,\theta - b - 2\,n + 3} - a + \frac{\theta\,a\,b}{b\,\theta - b - 2\,n + 3}}{b^2 + 2\,b\,n - 4\,b - 2\,n + 3} \tag{116}$$

simplify

$$q_1 = \frac{a}{(\theta - 1)\, b - 2\, n + 3} \tag{117}$$

$$subs\left(q_1 = \frac{a}{(\theta - 1)\, b - 2\, n + 3},\ q_2 = \frac{a}{(\theta - 1)\, b - 2\, n + 3},\ w_1 = \frac{\theta\, a\, b}{b\, \theta - b - 2\, n + 3},\ \Pi = \left(n\, \left(q_1 + q_2\right) + a - q_1 - q_2 - w_1\right)\, q_1\right)$$

$$\Pi = \frac{\left(\frac{2\, n\, a}{(\theta - 1)\, b - 2\, n + 3} + a - \frac{2\, a}{(\theta - 1)\, b - 2\, n + 3} - \frac{\theta\, a\, b}{b\, \theta - b - 2\, n + 3}\right) a}{(\theta - 1)\, b - 2\, n + 3} \tag{118}$$

simplify

$$\Pi = -\frac{a^2\, (b - 1)}{\left((\theta - 1)\, b - 2\, n + 3\right)^2} \tag{119}$$

Competitive labor market

$\Pi_1 = \left(n\, \left(y_1 + y_2\right) + a - q_1 - q_2\right)\, q_1$

$$\Pi_1 = \left(n\, \left(y_1 + y_2\right) + a - q_1 - q_2\right)\, q_1$$

differentiate w.r.t. q[1] →

$$0 = -2\, q_1 + n\, \left(y_1 + y_2\right) + a - q_2$$

solve for q[1] →

$$\left[\left[q_1 = \frac{1}{2}\, n\, y_2 + \frac{1}{2}\, n\, y_1 + \frac{1}{2}\, a - \frac{1}{2}\, q_2\right]\right] \tag{122}$$

$\Pi_2 = \left(n\, \left(y_1 + y_2\right) + a - q_1 - q_2\right)\, q_2$

$$\Pi_2 = \left(n\, \left(y_1 + y_2\right) + a - q_1 - q_2\right)\, q_2 \tag{123}$$

differentiate w.r.t. q[2] →

$$0 = -2\, q_2 + n\, \left(y_1 + y_2\right) + a - q_1 \tag{124}$$

solve for q[2] →

$$\left[\left[q_2 = \frac{1}{2}\, n\, y_2 + \frac{1}{2}\, n\, y_1 + \frac{1}{2}\, a - \frac{1}{2}\, q_1\right]\right] \tag{125}$$

$$subs\left(q_1 = \frac{1}{2}\, n\, y_2 + \frac{1}{2}\, n\, y_1 + \frac{1}{2}\, a - \frac{1}{2}\, q_2,\ q_2 = \frac{1}{2}\, n\, y_2 + \frac{1}{2}\, n\, y_1 + \frac{1}{2}\, a - \frac{1}{2}\, q_1\right)$$

$$q_2 = \frac{1}{4}\, n\, y_2 + \frac{1}{4}\, n\, y_1 + \frac{1}{4}\, a + \frac{1}{4}\, q_2 \tag{126}$$

solve for q[2] →

$$\left[\left[q_2 = \frac{1}{3}\, n\, y_2 + \frac{1}{3}\, n\, y_1 + \frac{1}{3}\, a\right]\right] \tag{127}$$

$$subs\left(q_2 = \frac{1}{2}\, n\, y_2 + \frac{1}{2}\, n\, y_1 + \frac{1}{2}\, a - \frac{1}{2}\, q_1\,,\, q_1 = \frac{1}{2}\, n\, y_2 + \frac{1}{2}\, n\, y_1 + \frac{1}{2}\, a - \frac{1}{2}\, q_2\right)$$

$$q_1 = \frac{1}{4}\, n\, y_2 + \frac{1}{4}\, n\, y_1 + \frac{1}{4}\, a + \frac{1}{4}\, q_1 \tag{128}$$

solve for q[1] →

$$\left[\left[q_1 = \frac{1}{3}\, n\, y_2 + \frac{1}{3}\, n\, y_1 + \frac{1}{3}\, a\right]\right] \tag{129}$$

Rational expectations realize, equilibrium quantities

$$q_1 = \frac{1}{3}\, n\, q_2 + \frac{1}{3}\, n\, q_1 + \frac{1}{3}\, a$$

$$q_1 = \frac{1}{3}\, n\, q_2 + \frac{1}{3}\, n\, q_1 + \frac{1}{3}\, a$$

solve for q[1] →

$$\left[\left[q_1 = -\frac{n\, q_2 + a}{n - 3}\right]\right] \tag{131}$$

$$subs\left(q_2 = -\frac{n\, q_1 + a}{n - 3}\,,\, q_1 = -\frac{n\, q_2 + a}{n - 3}\right)$$

$$q_1 = -\frac{-\frac{n\,(n\, q_1 + a)}{n - 3} + a}{n - 3} \tag{132}$$

solve for q[1] →

$$\left[\left[q_1 = -\frac{a}{2\, n - 3}\right]\right] \tag{133}$$

$$q_2 = \frac{1}{3}\, n\, q_2 + \frac{1}{3}\, n\, q_1 + \frac{1}{3}\, a$$

$$q_2 = \frac{1}{3}\, n\, q_2 + \frac{1}{3}\, n\, q_1 + \frac{1}{3}\, a$$

solve for q[2] →

$$\left[\left[q_2 = -\frac{n\, q_1 + a}{n - 3}\right]\right]$$

$$subs\left(q_1 = -\frac{n\, q_2 + a}{n - 3}\,,\, q_2 = -\frac{n\, q_1 + a}{n - 3}\right)$$

$$q_2 = -\frac{-\frac{n\,(n\, q_2 + a)}{n - 3} + a}{n - 3} \tag{136}$$

solve for q[2]

$$\left[\left[q_2 = -\frac{a}{2\,n-3}\right]\right] \tag{137}$$

Equilibrium profits

$$subs\left(q_1 = -\frac{a}{2\,n-3},\ q_2 = -\frac{a}{2\,n-3},\ \Pi_1 = \left(n\left(q_1+q_2\right)+a-q_1-q_2\right)q_1\right)$$

$$\Pi_1 = -\frac{\left(-\frac{2\,n\,a}{2\,n-3}+a+\frac{2\,a}{2\,n-3}\right)a}{2\,n-3} \tag{138}$$

simplify

$$\Pi_1 = \frac{a^2}{(2\,n-3)^2} \tag{139}$$

Mixed duopoly

Reaction functions

$$q_1 = -\frac{n\,y_1+n\,y_2+a-q_2-w_1}{b-2}$$

$$q_1 = -\frac{n\,y_1+n\,y_2+a-q_2-w_1}{b-2} \tag{140}$$

$$q_2 = \frac{1}{2}\,n\,y_2+\frac{1}{2}\,n\,y_1+\frac{1}{2}\,a-\frac{1}{2}\,q_1$$

$$q_2 = \frac{1}{2}\,n\,y_2+\frac{1}{2}\,n\,y_1+\frac{1}{2}\,a-\frac{1}{2}\,q_1 \tag{141}$$

$$subs\left(q_2 = \frac{1}{2}\,n\,y_2+\frac{1}{2}\,n\,y_1+\frac{1}{2}\,a-\frac{1}{2}\,q_1,\ q_1 = -\frac{n\,y_1+n\,y_2+a-q_2-w_1}{b-2}\right)$$

$$q_1 = -\frac{\frac{1}{2}\,n\,y_1+\frac{1}{2}\,n\,y_2+\frac{1}{2}\,a+\frac{1}{2}\,q_1-w_1}{b-2} \tag{142}$$

solve for q[1]

$$\left[\left[q_1 = -\frac{n\,y_1+n\,y_2+a-2\,w_1}{-3+2\,b}\right]\right] \tag{143}$$

$$subs\left(q_1 = -\frac{n\,y_1+n\,y_2+a-q_2-w_1}{b-2},\ q_2 = \frac{1}{2}\,n\,y_2+\frac{1}{2}\,n\,y_1+\frac{1}{2}\,a-\frac{1}{2}\,q_1\right)$$

$$q_2 = \frac{1}{2}\,n\,y_2+\frac{1}{2}\,n\,y_1+\frac{1}{2}\,a+\frac{1}{2}\,\frac{n\,y_1+n\,y_2+a-q_2-w_1}{b-2}$$

solve for q[2]

$$\left[\left[q_2 = \frac{b\,n\,y_1+b\,n\,y_2+a\,b-n\,y_1-n\,y_2-a-w_1}{-3+2\,b}\right]\right] \tag{145}$$

$$q_1 = -\frac{n\,q_1 + n\,q_2 + a - 2\,w_1}{-3 + 2\,b}$$

$$q_1 = -\frac{n\,q_1 + n\,q_2 + a - 2\,w_1}{-3 + 2\,b}$$

solve for q[1] →

$$\left[\left[q_1 = -\frac{n\,q_2 + a - 2\,w_1}{n - 3 + 2\,b}\right]\right] \tag{147}$$

$$q_2 = \frac{b\,n\,q_1 + b\,n\,q_2 + a\,b - n\,q_1 - n\,q_2 - a - w_1}{-3 + 2\,b}$$

$$q_2 = \frac{b\,n\,q_1 + b\,n\,q_2 + a\,b - n\,q_1 - n\,q_2 - a - w_1}{-3 + 2\,b}$$

solve for q[2] →

$$\left[\left[q_2 = -\frac{b\,n\,q_1 + a\,b - n\,q_1 - a - w_1}{b\,n - 2\,b - n + 3}\right]\right] \tag{149}$$

$$q_2 = -\frac{b\,n\,q_1 + a\,b - n\,q_1 - a - w_1}{b\,n - 2\,b - n + 3}$$

$$q_2 = -\frac{b\,n\,q_1 + a\,b - n\,q_1 - a - w_1}{b\,n - 2\,b - n + 3} \tag{150}$$

simplify =

$$q_2 = \frac{(-n\,q_1 - a)\,b + n\,q_1 + a + w_1}{(n - 2)\,b - n + 3} \tag{151}$$

$$subs\left(q_2 = -\frac{b\,n\,q_1 + a\,b - n\,q_1 - a - w_1}{b\,n - 2\,b - n + 3},\ q_1 = -\frac{n\,q_2 + a - 2\,w_1}{n - 3 + 2\,b}\right)$$

$$q_1 = -\frac{-\frac{n\,(b\,n\,q_1 + a\,b - n\,q_1 - a - w_1)}{b\,n - 2\,b - n + 3} + a - 2\,w_1}{n - 3 + 2\,b} \tag{152}$$

solve for q[1] →

$$\left[\left[q_1 = \frac{n\,w_1 + a - 2\,w_1}{b\,n - 2\,b - 2\,n + 3}\right]\right] \tag{153}$$

$$subs\left(q_1 = -\frac{n\,q_2 + a - 2\,w_1}{n - 3 + 2\,b},\ q_2 = -\frac{b\,n\,q_1 + a\,b - n\,q_1 - a - w_1}{b\,n - 2\,b - n + 3}\right)$$

$$q_2 = -\frac{-\frac{b\,n\,(n\,q_2 + a - 2\,w_1)}{n - 3 + 2\,b} + a\,b + \frac{n\,(n\,q_2 + a - 2\,w_1)}{n - 3 + 2\,b} - a - w_1}{b\,n - 2\,b - n + 3}$$

solve for q[2]

$$\left[\left[q_2 = -\frac{a\,b + n\,w_1 - a - w_1}{b\,n - 2\,b - 2\,n + 3}\right]\right] \tag{155}$$

Rent-sharing curve, rational expectations realized

$$subs\left(q_1 = \frac{n\,w_1 + a - 2\,w_1}{b\,n - 2\,b - 2\,n + 3},\ q_2 = -\frac{a\,b + n\,w_1 - a - w_1}{b\,n - 2\,b - 2\,n + 3},\ w_1 = \frac{b\,\theta\,(n\,q_1 + n\,q_2 + a - q_1 - q_2)}{b\,\theta - b + 1}\right)$$

$$w_1 = \frac{1}{b\,\theta - b + 1}\left(b\,\theta\left(\frac{n\,(n\,w_1 + a - 2\,w_1)}{b\,n - 2\,b - 2\,n + 3} - \frac{n\,(a\,b + n\,w_1 - a - w_1)}{b\,n - 2\,b - 2\,n + 3} + a - \frac{n\,w_1 + a - 2\,w_1}{b\,n - 2\,b - 2\,n + 3} + \frac{a\,b + n\,w_1 - a - w_1}{b\,n - 2\,b - 2\,n + 3}\right)\right)$$

solve for w[1]

$$\left[\left[w_1 = -\frac{a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3}\right]\right] \tag{157}$$

Equilibrium quantities

$$subs\left(w_1 = -\frac{a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3},\ q_1 = \frac{n\,w_1 + a - 2\,w_1}{b\,n - 2\,b - 2\,n + 3}\right)$$

$$q_1 = \frac{-\frac{n\,a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3} + a + \frac{2\,a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3}}{b\,n - 2\,b - 2\,n + 3} \tag{158}$$

simplify

$$q_1 = -\frac{a}{(\theta - 1)\,(n - 2)\,b + 2\,n - 3} \tag{159}$$

$$subs\left(w_1 = -\frac{a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3},\ q_2 = -\frac{a\,b + n\,w_1 - a - w_1}{b\,n - 2\,b - 2\,n + 3}\right)$$

$$q_2 = -\frac{a\,b - \frac{n\,a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3} - a + \frac{a\,b\,\theta}{b\,n\,\theta - b\,n - 2\,b\,\theta + 2\,b + 2\,n - 3}}{b\,n - 2\,b - 2\,n + 3}$$

simplify

$$q_2 = -\frac{a\,(1 + (\theta - 1)\,b)}{(\theta - 1)\,(n - 2)\,b + 2\,n - 3} \tag{161}$$

Equilibrium profits

$$subs\left(q_1 = -\frac{a}{(\theta - 1)\,(n - 2)\,b + 2\,n - 3},\ q_2 = -\frac{a\,(1 + (\theta - 1)\,b)}{(\theta - 1)\,(n - 2)\,b + 2\,n - 3},\ w_1 =\right)$$

$$-\frac{a\,b\,\theta}{b\,n\,\theta-b\,n-2\,b\,\theta+2\,b+2\,n-3},\ \Pi_1=\left(n\left(q_1+q_2\right)+a-q_1-q_2-w_1\right)q_1\Bigg)$$

$$\Pi_1=-\frac{1}{(\theta-1)\,(n-2)\,b+2\,n-3}\left(\left(n\left(-\frac{a}{(\theta-1)\,(n-2)\,b+2\,n-3}-\frac{a\left(1+(\theta-1)\,b\right)}{(\theta-1)\,(n-2)\,b+2\,n-3}\right)+a+\frac{a}{(\theta-1)\,(n-2)\,b+2\,n-3}+\frac{a\left(1+(\theta-1)\,b\right)}{(\theta-1)\,(n-2)\,b+2\,n-3}+\frac{a\,b\,\theta}{b\,n\,\theta-b\,n-2\,b\,\theta+2\,b+2\,n-3}\right)a\right) \tag{162}$$

simplify

$$\Pi_1=-\frac{a^2\,(b-1)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} \tag{163}$$

$$subs\left(q_1=-\frac{a}{(\theta-1)\,(n-2)\,b+2\,n-3},\ q_2=-\frac{a\left(1+(\theta-1)\,b\right)}{(\theta-1)\,(n-2)\,b+2\,n-3},\ \Pi_2=\left(n\left(q_1+q_2\right)+a-q_1-q_2\right)q_2\right)$$

$$\Pi_2=-\frac{1}{(\theta-1)\,(n-2)\,b+2\,n-3}\left(\left(n\left(-\frac{a}{(\theta-1)\,(n-2)\,b+2\,n-3}-\frac{a\left(1+(\theta-1)\,b\right)}{(\theta-1)\,(n-2)\,b+2\,n-3}\right)+a+\frac{a}{(\theta-1)\,(n-2)\,b+2\,n-3}+\frac{a\left(1+(\theta-1)\,b\right)}{(\theta-1)\,(n-2)\,b+2\,n-3}\right)a\left(1+(\theta-1)\,b\right)\right)$$

simplify

$$\Pi_2=\frac{a^2\left(1+(\theta-1)\,b\right)^2}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} \tag{165}$$

Entry game with network externalities
Monopoly profits with EB
Simultaneous EB

$$N=\left((n\,y+a-q-w)\,q\right)^{1-b}\left(\left(w^{\theta}q\right)\right)^{b}$$

$$N=\left((n\,y+a-q-w)\,q\right)^{1-b}\left(w^{\theta}\,q\right)^{b} \tag{166}$$

differentiate w.r.t. w

$$0=-\frac{\left((n\,y+a-q-w)\,q\right)^{1-b}(1-b)\left(w^{\theta}q\right)^{b}}{n\,y+a-q-w}+\frac{\left((n\,y+a-q-w)\,q\right)^{1-b}\left(w^{\theta}\,q\right)^{b}b\,\theta}{w} \tag{167}$$

solve for w

$$\left[\left[w=\frac{b\,\theta\,(n\,y+a-q)}{b\,\theta-b+1}\right]\right] \tag{168}$$

differentiate w.r.t. q →

$$0 = \frac{((n\,y + a - q - w)\,q)^{1-b}\,(1-b)\,(n\,y + a - 2\,q - w)\,\left(w^{\theta} q\right)^{b}}{(n\,y + a - q - w)\,q} + \frac{((n\,y + a - q - w)\,q)^{1-b}\,\left(w^{\theta} q\right)^{b} b}{q} \tag{169}$$

solve for q →

$$\left[\left[q = -\frac{n\,y + a - w}{b-2}\right]\right] \tag{170}$$

$$w = \frac{b\,\theta\,(n\,q + a - q)}{b\,\theta - b + 1}$$

$$w = \frac{b\,\theta\,(n\,q + a - q)}{b\,\theta - b + 1}$$

simplify =

$$w = \frac{b\,((n-1)\,q + a)\,\theta}{1 + (\theta - 1)\,b} \tag{172}$$

$$subs\left(w = \frac{b\,((n-1)\,q + a)\,\theta}{1 + (\theta - 1)\,b},\ q = -\frac{n\,q + a - w}{b-2}\right)$$

$$q = -\frac{n\,q + a - \frac{b\,((n-1)\,q + a)\,\theta}{1 + (\theta - 1)\,b}}{b-2} \tag{173}$$

simplify =

$$q = \frac{((n - \theta)\,q + a)\,b - n\,q - a}{(b-2)\,(1 + (\theta - 1)\,b)} \tag{174}$$

solve for q →

$$\left[\left[q = \frac{a}{b\,\theta - b - n + 2}\right]\right] \tag{175}$$

$$subs\left(q = \frac{a}{b\,\theta - b - n + 2},\ w = \frac{b\,((n-1)\,q + a)\,\theta}{1 + (\theta - 1)\,b}\right)$$

$$w = \frac{b\left(\frac{(n-1)\,a}{b\,\theta - b - n + 2} + a\right)\theta}{1 + (\theta - 1)\,b} \tag{176}$$

simplify =

$$w = \frac{b\,a\,\theta}{(\theta - 1)\,b - n + 2} \tag{177}$$

$$subs\left(w=\frac{b\,a\,\theta}{(\theta-1)\,b-n+2},\ q=\frac{a}{b\,\theta-b-n+2},\ \pi=(n\,q+a-q-w)\,q\right)$$

$$\pi=\frac{\left(\frac{n\,a}{b\,\theta-b-n+2}+a-\frac{a}{b\,\theta-b-n+2}-\frac{b\,a\,\theta}{(\theta-1)\,b-n+2}\right)a}{b\,\theta-b-n+2} \tag{178}$$

$\overset{\text{simplify}}{=}$

$$\pi=-\frac{a^2\,(b-1)}{((\theta-1)\,b-n+2)^2} \tag{179}$$

$$\pi=\frac{a^2\,(1-b)}{((\theta-1)\,b-n+2)^2}$$

$$\pi=\frac{a^2\,(1-b)}{((\theta-1)\,b-n+2)^2} \tag{180}$$

Threat of entry, committed bargaining
Monopoly profits EB vs duopoly CF

$$\frac{a^2\,(1-b)}{((\theta-1)\,b-n+2)^2},\ \frac{a^2}{(2\,n-3)^2}$$

$$\frac{a^2\,(1-b)}{((\theta-1)\,b-n+2)^2},\ \frac{a^2}{(2\,n-3)^2} \tag{181}$$

$$\frac{a^2\,(1-b)}{((\theta-1)\,b-n+2)^2}=\frac{a^2}{(2\,n-3)^2}$$

$$\frac{a^2\,(1-b)}{((\theta-1)\,b-n+2)^2}=\frac{a^2}{(2\,n-3)^2} \tag{182}$$

$\xrightarrow{\text{solve for n}}$

$$\left[\left[n \tag{183}\right.\right.$$

$$=\frac{1}{4\,b-3}\left(b\,\theta\right.$$

$$\left.+\sqrt{-4\,b^3\,\theta^2+8\,b^3\,\theta+4\,b^2\,\theta^2-4\,b^3-12\,b^2\,\theta+8\,b^2+4\,b\,\theta-5\,b+1}+5\,b-4\right)\Big],\left[n\right.$$

$$=$$

$$-\frac{1}{4\,b-3}\left(-b\,\theta\right.$$

$$+\sqrt{-4\,b^3\,\theta^2+8\,b^3\,\theta+4\,b^2\,\theta^2-4\,b^3-12\,b^2\,\theta+8\,b^2+4\,b\,\theta-5\,b+1}\;-5\,b+4\Big)\Big]\Big]$$

$$-\frac{-b\,\theta+\sqrt{-4\,b^3\,\theta^2+8\,b^3\,\theta+4\,b^2\,\theta^2-4\,b^3-12\,b^2\,\theta+8\,b^2+4\,b\,\theta-5\,b+1}\;-5\,b+4}{4\,b-3}\ \overset{\text{simplify}}{=}$$

$$\frac{-\sqrt{-4\left(\frac{1}{2}+(\theta-1)\,b\right)^2(-1+b)}\;-4+(\theta+5)\,b}{4\,b-3}\ \xrightarrow{\text{simplify symbolic}}$$

$$-\frac{2\sqrt{1-b}\;b\,\theta-2\sqrt{1-b}\;b-b\,\theta+\sqrt{1-b}\;-5\,b+4}{4\,b-3}$$

$$\overset{\text{simplify}}{=}\ \frac{(-1+(-2\,\theta+2)\,b)\sqrt{1-b}\;-4+(\theta+5)\,b}{4\,b-3}$$

$$n^{\mathrm{O}}=\frac{(-1+(-2\,\theta+2)\,b)\sqrt{1-b}\;-4+(\theta+5)\,b}{4\,b-3}$$

$$\theta=\frac{b+n-2+\sqrt{-4\,b\,n^2+12\,b\,n+4\,n^2-9\,b-12\,n+9}}{b}$$

$$\theta=\frac{b+n-2+\sqrt{-4\,b\,n^2+12\,b\,n+4\,n^2-9\,b-12\,n+9}}{b}$$

$$\xrightarrow{\text{solve for b}}$$

$$\Big[\Big[b=\frac{1}{2}\;\frac{1}{\theta^2-2\,\theta+1}\Big(-4\,n^2+2\,n\,\theta+10\,n-4\,\theta-5 \qquad \textbf{(185)}$$

$$+\left(16\,n^4-16\,n^3\,\theta+16\,n^2\,\theta^2-80\,n^3+48\,n^2\,\theta-48\,n\,\theta^2+152\,n^2-36\,n\,\theta\right.$$

$$\left.+36\,\theta^2-132\,n+45\right)^{1/2}\Big)\Big],\Big[b=-\frac{1}{2}\;\frac{1}{\theta^2-2\,\theta+1}\Big(4\,n^2-2\,n\,\theta$$

$$+\left(16\,n^4-16\,n^3\,\theta+16\,n^2\,\theta^2-80\,n^3+48\,n^2\,\theta-48\,n\,\theta^2+152\,n^2-36\,n\,\theta\right.$$

$$\left.+36\,\theta^2-132\,n+45\right)^{1/2}-10\,n+4\,\theta+5\Big)\Big]\Big]$$

$$b=\frac{1}{2}\;\frac{1}{\theta^2-2\,\theta+1}\Big(-4\,n^2+2\,n\,\theta+10\,n-4\,\theta-5$$

$$+$$

$$\left(16\,n^4-16\,n^3\,\theta+16\,n^2\,\theta^2-80\,n^3+48\,n^2\,\theta-48\,n\,\theta^2+152\,n^2-36\,n\,\theta+36\,\theta^2\right.$$

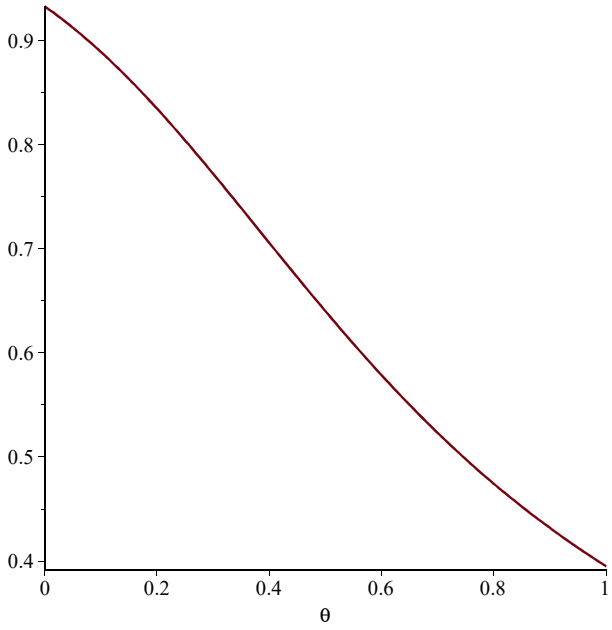
$$\left.-132\,n+45\right)^{1/2}\Big)$$

$$b = \frac{1}{2}\ \frac{1}{\theta^2 - 2\,\theta + 1}\left(-4\,n^2 + 2\,n\,\theta + 10\,n - 4\,\theta - 5 + \left(16\,n^4 - 16\,n^3\,\theta + 16\,n^2\,\theta^2 - 80\,n^3 + 48\,n^2\,\theta - 48\,n\,\theta^2 + 152\,n^2 - 36\,n\,\theta + 36\,\theta^2 - 132\,n + 45\right)^{1/2}\right)$$

evaluate at point

$$b = \frac{1}{2}\ \frac{-0.44 - 2.8\,\theta + \sqrt{12.96\,\theta^2 - 7.776\,\theta + 5.3136}}{\theta^2 - 2\,\theta + 1} \qquad \textbf{(187)}$$

→



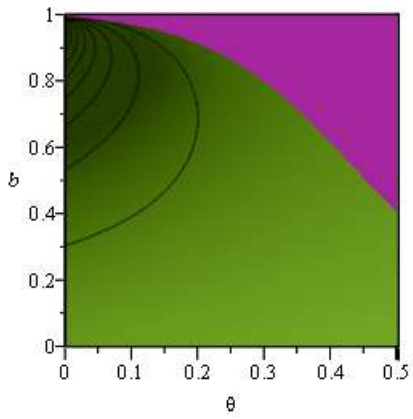
$$\frac{1-b}{\left((\theta-1)\,b - n + 2\right)^2},\ \frac{1}{(2\,n-3)^2}$$

$$\frac{1-b}{((\theta-1)\,b-n+2)^2},\ \frac{1}{(2\,n-3)^2}$$

evaluate at point → n=.95

$$\frac{1-b}{((\theta-1)\,b+1.05)^2},\ 0.8264462810$$

→

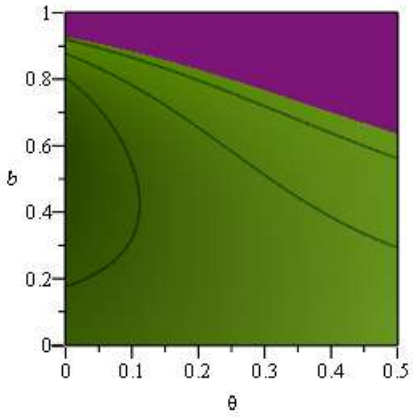


Green Monopoly profits under EB
Purple Duopoly profits under PM

evaluate at point → n=.6

$$\frac{1-b}{((\theta-1)\,b+1.4)^2},\ 0.3086419753 \qquad \textbf{(190)}$$

→



Green Monopoly profits under EB
Purple Duopoly profits under PM

Profit differentials and entry

PM/PM-EB/EB

$$\Delta\Pi_1 = \frac{a^2}{(2n-3)^2} - \left(\frac{a^2(1-b)}{((\theta-1)b-2n+3)^2} \right)$$

$$\Delta\Pi_1 = \frac{a^2}{(2n-3)^2} - \frac{a^2(1-b)}{((\theta-1)b-2n+3)^2}$$

simplify symbolic →

$$\Delta\Pi_1 = \frac{a^2 b\left(b\theta^2 - 2b\theta + 4n^2 - 4n\theta + b - 8n + 6\theta + 3\right)}{(2n-3)^2\left(b\theta - b - 2n + 3\right)^2} \quad \textbf{(192)}$$

PM/EB-EB/EB

$$\Delta\Pi_2 = \frac{a^2(1+(\theta-1)b)^2}{((\theta-1)(n-2)b+2n-3)^2} - \left(\frac{a^2(1-b)}{((\theta-1)b-2n+3)^2} \right)$$

$$\Delta\Pi_2 = \frac{a^2(1+(\theta-1)b)^2}{((\theta-1)(n-2)b+2n-3)^2} - \frac{a^2(1-b)}{((\theta-1)b-2n+3)^2} \quad \textbf{(193)}$$

$$\Delta\Pi_3 = \frac{a^2\,(1-b)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} - \frac{a^2}{(2\,n-3)^2}$$

$$\Delta\Pi_3 = \frac{a^2\,(1-b)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} - \frac{a^2}{(2\,n-3)^2} \qquad \textbf{(194)}$$

$$b = \frac{1}{2}\;\frac{1}{\theta^2-2\,\theta+1}\left(-4\,n^2+2\,n\,\theta+10\,n-4\,\theta-5 + \left(16\,n^4-16\,n^3\,\theta+16\,n^2\,\theta^2-80\,n^3+48\,n^2\,\theta-48\,n\,\theta^2+152\,n^2-36\,n\,\theta+36\,\theta^2-132\,n+45\right)^{1/2}\right)$$

Relevant areas for the entry game
delta pi1, delta pi2, delta pi3, threshold entry deterrence

$$\frac{a^2\,b\left(b\,\theta^2-2\,b\,\theta+4\,n^2-4\,n\,\theta+b-8\,n+6\,\theta+3\right)}{(2\,n-3)^2\left(b\,\theta-b-2\,n+3\right)^2},\ \frac{a^2\,(1-b)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} - \frac{a^2}{(2\,n-3)^2},\ \frac{a^2\left(1+(\theta-1)\,b\right)^2}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} - \frac{a^2\,(1-b)}{\left((\theta-1)\,b-2\,n+3\right)^2},\ b$$

$$= \frac{1}{2}\;\frac{1}{\theta^2-2\,\theta+1}\left(-4\,n^2+2\,n\,\theta+10\,n-4\,\theta-5 + \left(16\,n^4-16\,n^3\,\theta+16\,n^2\,\theta^2-80\,n^3+48\,n^2\,\theta-48\,n\,\theta^2+152\,n^2-36\,n\,\theta+36\,\theta^2-132\,n+45\right)^{1/2}\right)$$

$$\frac{a^2\,b\left(b\,\theta^2-2\,b\,\theta+4\,n^2-4\,n\,\theta+b-8\,n+6\,\theta+3\right)}{(2\,n-3)^2\left(b\,\theta-b-2\,n+3\right)^2},\ \frac{a^2\,(1-b)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} - \frac{a^2}{(2\,n-3)^2},\ \frac{a^2\left(1+(\theta-1)\,b\right)^2}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2} - \frac{a^2\,(1-b)}{\left((\theta-1)\,b-2\,n+3\right)^2},\ b$$

$$= \frac{1}{2}\;\frac{1}{\theta^2-2\,\theta+1}\left(-4\,n^2+2\,n\,\theta+10\,n-4\,\theta-5 + \left(16\,n^4-16\,n^3\,\theta+16\,n^2\,\theta^2-80\,n^3+48\,n^2\,\theta-48\,n\,\theta^2+152\,n^2-36\,n\,\theta+36\,\theta^2-132\,n+45\right)^{1/2}\right)$$

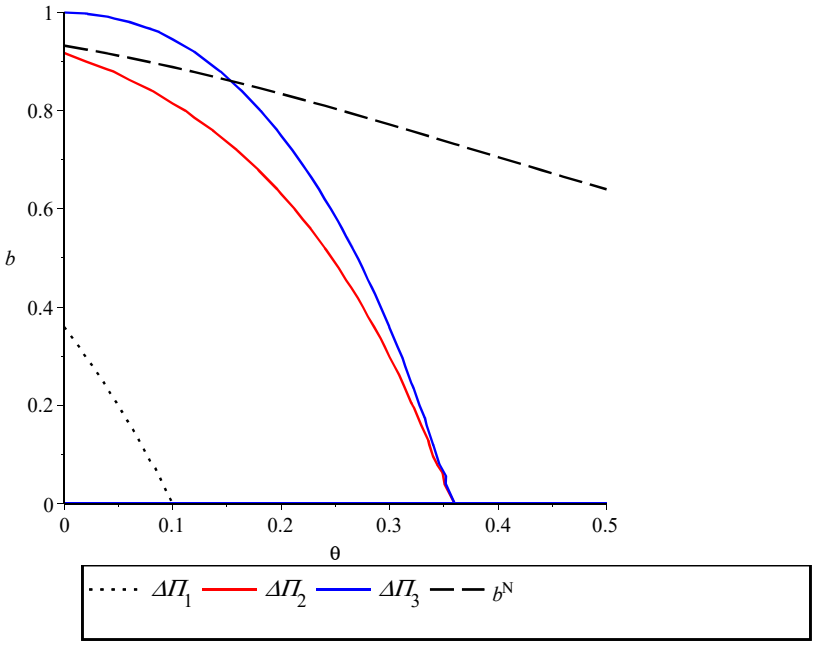
evaluate at point $\longrightarrow$

$$\frac{0.3086419753\, b\left(b\,\theta^2-2\,b\,\theta+b+3.6\,\theta-0.36\right)}{\left(b\,\theta-b+1.8\right)^2},\ \frac{1-b}{\left(-1.4\,(\theta-1)\,b-1.8\right)^2}$$

$$-0.3086419753,\ \frac{\left(1+(\theta-1)\,b\right)^2}{\left(-1.4\,(\theta-1)\,b-1.8\right)^2}-\frac{1-b}{\left((\theta-1)\,b+1.8\right)^2},\ b$$

$$=\frac{1}{2}\ \frac{-0.44-2.8\,\theta+\sqrt{12.96\,\theta^2-7.776\,\theta+5.3136}}{\theta^2-2\,\theta+1}$$

→



evaluate at point ⟶ n=0.95

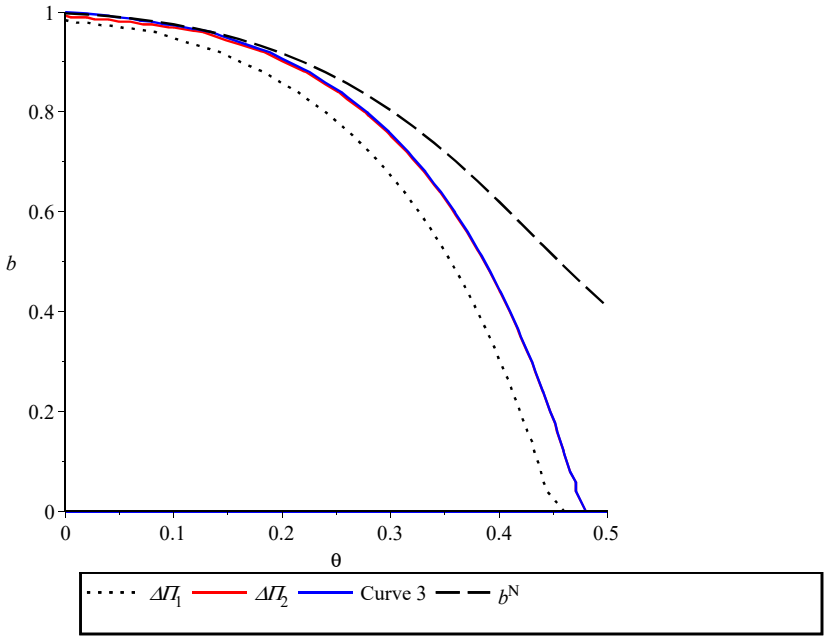
$$\frac{0.8264462810\, b\left(b\,\theta^2-2\,b\,\theta+b+2.20\,\theta-0.9900\right)}{\left(b\,\theta-b+1.10\right)^2},\ \frac{1-b}{\left(-1.05\,(\theta-1)\,b-1.10\right)^2}$$

$$-0.8264462810,\ \frac{\left(1+(\theta-1)\,b\right)^2}{\left(-1.05\,(\theta-1)\,b-1.10\right)^2}-\frac{1-b}{\left((\theta-1)\,b+1.10\right)^2},\ b$$

(197)

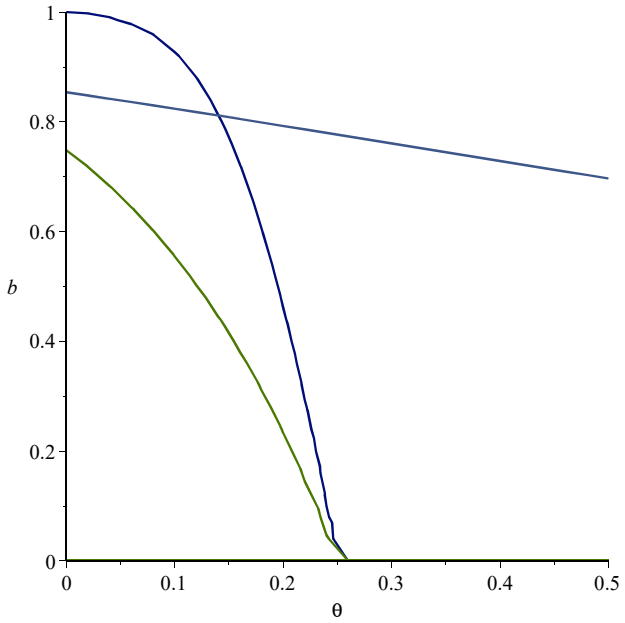
$$= \frac{1}{2}\ \frac{0.8900 - 2.10\,\theta + \sqrt{4.8400\,\theta^2 - 4.598000\,\theta + 1.22210000}}{\theta^2 - 2\,\theta + 1}$$

$\rightarrow$



$$\frac{1}{9}\ \frac{b\left(b\,\theta^2 - 2\,b\,\theta + b + 6\,\theta + 3\right)}{\left(b\,\theta - b + 3\right)^2},\ \frac{\left(1 + (\theta - 1)\,b\right)^2}{\left(-2\,(\theta - 1)\,b - 3\right)^2} - \frac{1 - b}{\left((\theta - 1)\,b + 3\right)^2},$$

$$\frac{1 - b}{\left(-2\,(\theta - 1)\,b - 3\right)^2} - \frac{1}{9},\ b = \frac{1}{2}\ \frac{-5 - 4\,\theta + \sqrt{36\,\theta^2 + 45}}{\theta^2 - 2\,\theta + 1}$$



Result 4

$$\frac{a^2\,(1-b)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2}-\frac{a^2}{(2\,n-3)^2}=0$$

$$\frac{a^2\,(1-b)}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2}-\frac{a^2}{(2\,n-3)^2}=0 \qquad \textbf{(198)}$$

evaluate at point

$$\frac{1-b}{\left((\theta-1)\,(n-2)\,b+2\,n-3\right)^2}-\frac{1}{(2\,n-3)^2}=0 \qquad \textbf{(199)}$$

solve for b

$$\left[[b=0],\left[b=-\frac{4\,n^2\theta-14\,n\theta+2\,n+12\,\theta-3}{n^2\theta^2-2\,n^2\theta-4\,n\theta^2+n^2+8\,n\theta+4\,\theta^2-4\,n-8\,\theta+4}\right]\right] \qquad \textbf{(200)}$$

$$b=-\frac{4\,n^2\theta-14\,n\theta+2\,n+12\,\theta-3}{n^2\theta^2-2\,n^2\theta-4\,n\theta^2+n^2+8\,n\theta+4\,\theta^2-4\,n-8\,\theta+4}$$

$$b = -\frac{4\,n^2\,\theta - 14\,n\,\theta + 2\,n + 12\,\theta - 3}{n^2\,\theta^2 - 2\,n^2\,\theta - 4\,n\,\theta^2 + n^2 + 8\,n\,\theta + 4\,\theta^2 - 4\,n - 8\,\theta + 4} \tag{201}$$

simplify

$$b = \frac{\left(-4\,n^2 + 14\,n - 12\right)\theta - 2\,n + 3}{(\theta - 1)^2\,(n - 2)^2} \tag{202}$$

$-4\,n^2 + 14\,n - 12$

$$-4\,n^2 + 14\,n - 12 \tag{203}$$

$$\frac{\left(-4\,n^2 + 14\,n - 12\right)\theta - 2\,n + 3}{(\theta - 1)^2\,(n - 2)^2}$$

$$\frac{\left(-4\,n^2 + 14\,n - 12\right)\theta - 2\,n + 3}{(\theta - 1)^2\,(n - 2)^2} \tag{204}$$

limit

$$-\frac{2\,\theta - 1}{(\theta - 1)^2} \tag{205}$$

Result 5

$$\frac{a^2\,b\left(b\,\theta^2 - 2\,b\,\theta + 4\,n^2 - 4\,n\,\theta + b - 8\,n + 6\,\theta + 3\right)}{(2\,n - 3)^2\left(b\,\theta - b - 2\,n + 3\right)^2} = 0$$

$$\frac{a^2\,b\left(b\,\theta^2 - 2\,b\,\theta + 4\,n^2 - 4\,n\,\theta + b - 8\,n + 6\,\theta + 3\right)}{(2\,n - 3)^2\left(b\,\theta - b - 2\,n + 3\right)^2} = 0$$

evaluate at point

$$\frac{b\left(b\,\theta^2 - 2\,b\,\theta + 4\,n^2 - 4\,n\,\theta + b - 8\,n + 6\,\theta + 3\right)}{(2\,n - 3)^2\left(b\,\theta - b - 2\,n + 3\right)^2} = 0$$

solve for n

$$\left[\left[n = \frac{1}{2}\,\theta + 1 + \frac{1}{2}\sqrt{-b\,\theta^2 + 2\,b\,\theta + \theta^2 - b - 2\,\theta + 1}\right], \left[n = \frac{1}{2}\,\theta + 1 - \frac{1}{2}\sqrt{-b\,\theta^2 + 2\,b\,\theta + \theta^2 - b - 2\,\theta + 1}\right]\right] \tag{208}$$

$$n = \frac{1}{2}\,\theta + 1 - \frac{1}{2}\sqrt{-b\,\theta^2 + 2\,b\,\theta + \theta^2 - b - 2\,\theta + 1}$$

$$n = \frac{1}{2}\,\theta + 1 - \frac{1}{2}\sqrt{-b\,\theta^2 + 2\,b\,\theta + \theta^2 - b - 2\,\theta + 1}$$

evaluate at point

$$n = 1 - \frac{1}{2}\sqrt{1 - b}$$

simplify

$$n = \frac{1}{2}\,\theta + 1 - \frac{1}{2}\,\sqrt{-(\theta-1)^2\,(b-1)}$$

evaluate at point

$$n = \frac{1}{2} \tag{212}$$

Double check
PM/PM, EB/EB, EB/PM, PM/EB

$$\frac{a^2}{(2n-3)^2},\ \frac{a^2\,(1-b)}{((\theta-1)\,b-2n+3)^2},\ \frac{a^2\,(1-b)}{((\theta-1)\,(n-2)\,b+2n-3)^2},\ \frac{a^2\,(1+(\theta-1)\,b)^2}{((\theta-1)\,(n-2)\,b+2n-3)^2}$$

evaluate at point

$$\frac{1}{(2n-3)^2},\ \frac{1-b}{((\theta-1)\,b-2n+3)^2},\ \frac{1-b}{((\theta-1)\,(n-2)\,b+2n-3)^2},\ \frac{(1+(\theta-1)\,b)^2}{((\theta-1)\,(n-2)\,b+2n-3)^2}$$

Flexible bargaining
incumbnt EB

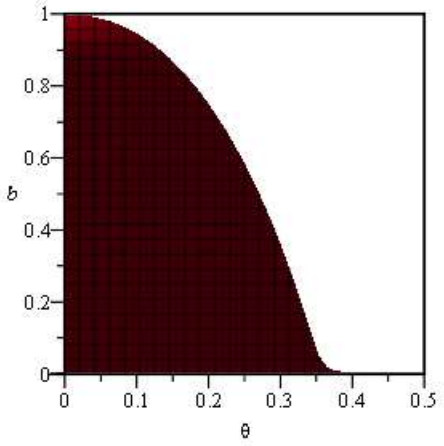
$$\frac{1-b}{((\theta-1)\,b-2n+3)^2} - \frac{(1+(\theta-1)\,b)^2}{((\theta-1)\,(n-2)\,b+2n-3)^2}$$

$$\frac{1-b}{((\theta-1)\,b-2n+3)^2} - \frac{(1+(\theta-1)\,b)^2}{((\theta-1)\,(n-2)\,b+2n-3)^2} \tag{213}$$

evaluate at point

$$\frac{1-b}{((\theta-1)\,b+1.8)^2} - \frac{(1+(\theta-1)\,b)^2}{(-1.4\,(\theta-1)\,b-1.8)^2} \tag{214}$$

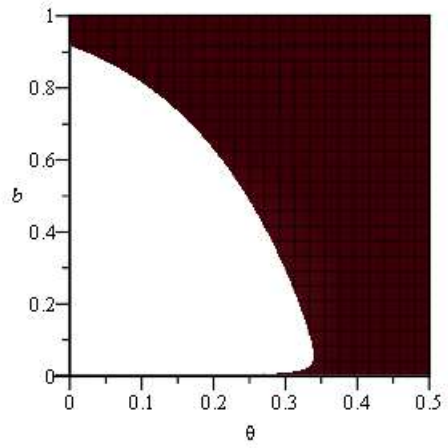
→



incumbent PM

$$\frac{1}{(2n-3)^2} - \frac{1-b}{\left((\theta-1)(n-2)b+2n-3\right)^2} \xrightarrow{\text{evaluate at point}}$$

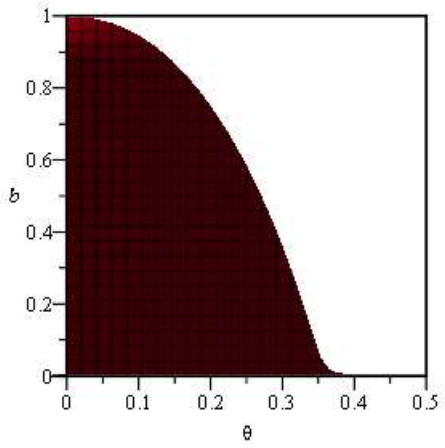
$$0.3086419753 - \frac{1-b}{\left(-1.4(\theta-1)b-1.8\right)^2} \rightarrow$$



incumbent choice

$$\frac{1-b}{((\theta-1)\,b-2\,n+3)^2}-\frac{(1+(\theta-1)\,b)^2}{((\theta-1)\,(n-2)\,b+2\,n-3)^2}\xrightarrow{\text{evaluate at point}}$$

$$\frac{1-b}{((\theta-1)\,b+1.8)^2}-\frac{(1+(\theta-1)\,b)^2}{(-1.4\,(\theta-1)\,b-1.8)^2}\rightarrow$$



$$\frac{1}{(2-n)^2} = \frac{1-b}{((\theta-1)\,b-2\,n+3)^2} - F$$

$$\frac{1}{(2-n)^2} = \frac{1-b}{((\theta-1)\,b-2\,n+3)^2} - F$$

$\xrightarrow{\text{solve for F}}$

$$\left[\left[F = -\frac{b^2\,\theta^2 - 2\,b^2\,\theta + b\,n^2 - 4\,b\,n\,\theta + b^2 + 6\,b\,\theta + 3\,n^2 - 2\,b - 8\,n + 5}{(-2+n)^2\,(b\,\theta - b - 2\,n + 3)^2}\right]\right] \qquad \textbf{(216)}$$

$$\frac{1}{(2-n)^2} = \frac{1-b}{((\theta-1)\,b-2\,n+3)^2} \xrightarrow{\text{solve for n}}$$

$$\left[\left[n = \frac{2\,b\,\theta + \sqrt{-b^3\,\theta^2 + 2\,b^3\,\theta + b^2\,\theta^2 - b^3 - b^2 - 2\,b\,\theta + b + 1} + 4}{b+3}\right], \left[n = -\frac{-2\,b\,\theta + \sqrt{-b^3\,\theta^2 + 2\,b^3\,\theta + b^2\,\theta^2 - b^3 - b^2 - 2\,b\,\theta + b + 1} - 4}{b+3}\right]\right]$$

$$-\frac{-2\,b\,\theta+\sqrt{-b^3\,\theta^2+2\,b^3\,\theta+b^2\,\theta^2-b^3-b^2-2\,b\,\theta+b+1}-4}{b+3}$$

$$-\frac{-2\,b\,\theta+\sqrt{-b^3\,\theta^2+2\,b^3\,\theta+b^2\,\theta^2-b^3-b^2-2\,b\,\theta+b+1}-4}{b+3}$$

simplify

$$=\frac{2\,b\,\theta-\sqrt{-(-1+b)\,(-1+(\theta-1)\,b)^2}+4}{b+3}$$

simplify symbolic

$$-\frac{\sqrt{1-b}\;b\,\theta-\sqrt{1-b}\;b-2\,b\,\theta-\sqrt{1-b}-4}{b+3}$$

simplify

$$=\frac{(1+(-\theta+1)\,b)\sqrt{1-b}+2\,b\,\theta+4}{b+3}$$

evaluate at point

$$-\frac{-4+\sqrt{-b^3-b^2+b+1}}{b+3}$$

differentiate w.r.t. b

$$-\frac{1}{2}\,\frac{-3\,b^2-2\,b+1}{\sqrt{-b^3-b^2+b+1}\;(b+3)}+\frac{-4+\sqrt{-b^3-b^2+b+1}}{(b+3)^2}$$

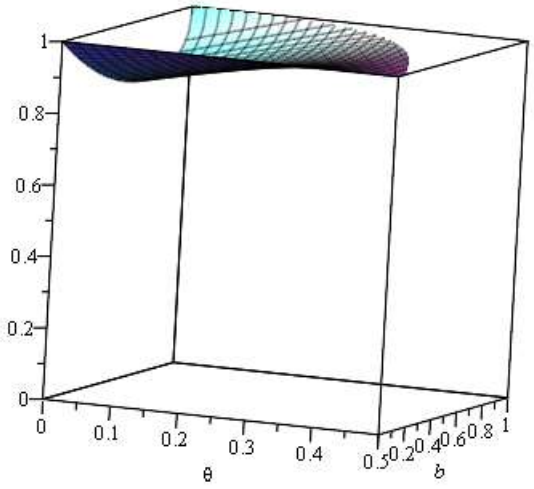
equate to 0

$$-\frac{1}{2}\,\frac{-3\,b^2-2\,b+1}{\sqrt{-b^3-b^2+b+1}\;(b+3)}+\frac{-4+\sqrt{-b^3-b^2+b+1}}{(b+3)^2}=0$$

solve

$$0.6568542495$$

→



$$\frac{2\,b\,\theta+\sqrt{-b^3\,\theta^2+2\,b^3\,\theta+b^2\,\theta^2-b^3-b^2-2\,b\,\theta+b+1}+4}{b+3}$$

$$\frac{2\,b\,\theta+\sqrt{-b^3\,\theta^2+2\,b^3\,\theta+b^2\,\theta^2-b^3-b^2-2\,b\,\theta+b+1}+4}{b+3} \tag{225}$$

Threat of market entry
Monopoly EB, PM, duopoly EB, PM

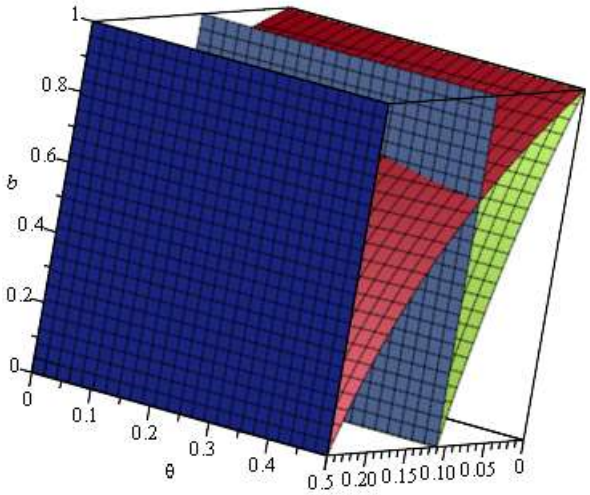
$$\frac{1-b}{\left((\theta-1)\,b-n+2\right)^2},\ \frac{1}{(2-n)^2},\ \frac{1-b}{\left((\theta-1)\,b-2\,n+3\right)^2},\ \frac{1}{(2\,n-3)^2}$$

$$\frac{1-b}{\left((\theta-1)\,b-n+2\right)^2},\ \frac{1}{(2-n)^2},\ \frac{1-b}{\left((\theta-1)\,b-2\,n+3\right)^2},\ \frac{1}{(2\,n-3)^2} \tag{226}$$

$\xrightarrow{\text{evaluate at point}}$ n=0

$$\frac{1-b}{((\theta-1)\,b+2)^2}, \frac{1}{4}, \frac{1-b}{((\theta-1)\,b+3)^2}, \frac{1}{9} \tag{227}$$

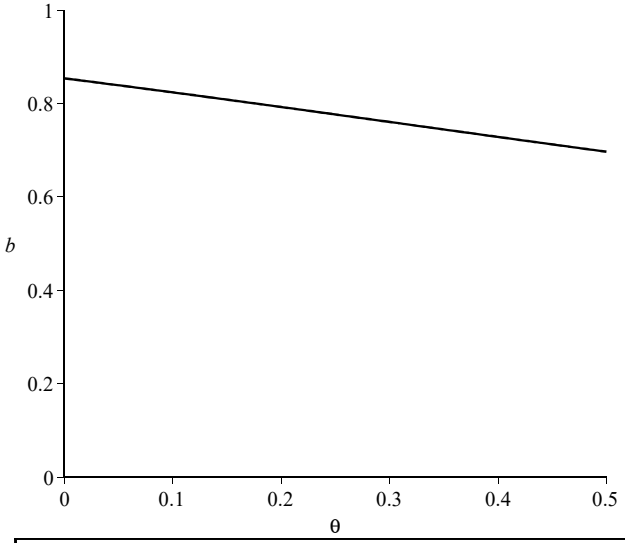
→



$$\frac{1-b}{((\theta-1)\,b+2)^2} = \frac{1}{9}$$

$$\frac{1-b}{((\theta-1)\,b+2)^2} = \frac{1}{9} \tag{228}$$

→

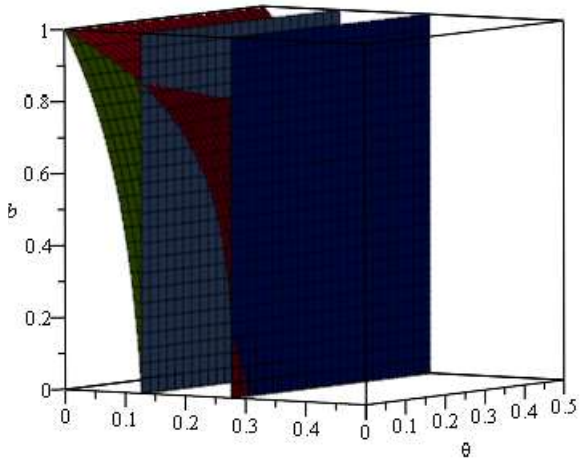


$$n^{O} = \frac{\left(-1+\left(-2\,\theta+2\right)b\right)\sqrt{1-b}-4+\left(\theta+5\right)b}{4\,b-3}$$

evaluate at point

$$\rightarrow \quad \frac{1-b}{\left(\left(\theta-1\right)b+1.9\right)^{2}}, 0.2770083102, \frac{1-b}{\left(\left(\theta-1\right)b+2.8\right)^{2}}, 0.1275510204 \qquad \textbf{(229)}$$

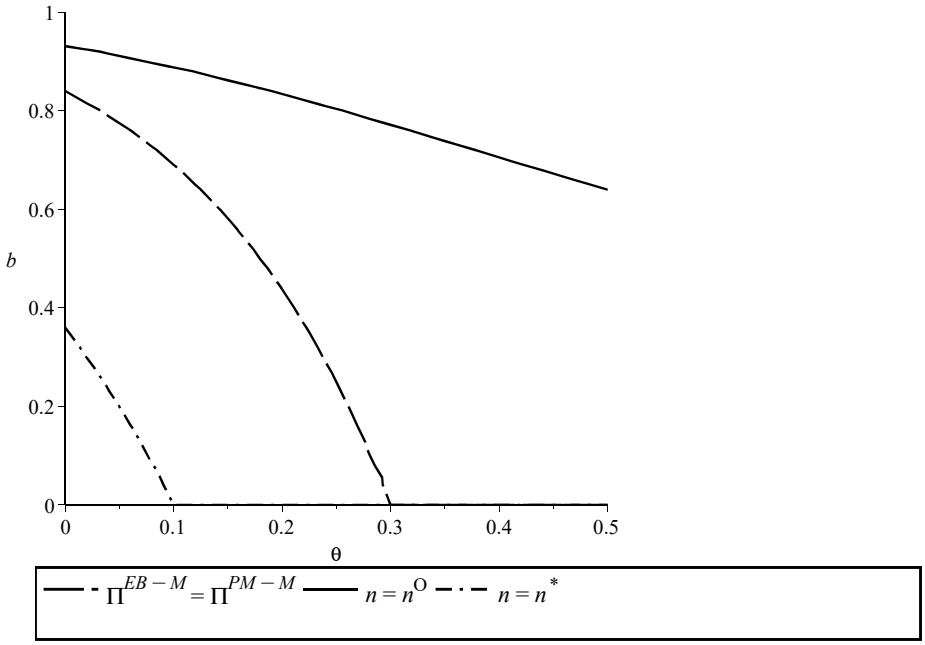
$\rightarrow$



$$\frac{1-b}{((\theta-1)\,b+1.4)^2}=0.5102040816\,,\ \frac{1-b}{((\theta-1)\,b+1.4)^2}=0.3086419753\,,\ \frac{1-b}{((\theta-1)\,b+1.8)^2}=0.3086419753$$

$$\frac{1-b}{((\theta-1)\,b+1.4)^2}=0.5102040816,\ \frac{1-b}{((\theta-1)\,b+1.4)^2}=0.3086419753,\ \frac{1-b}{((\theta-1)\,b+1.8)^2}=0.3086419753 \quad \textbf{(230)}$$

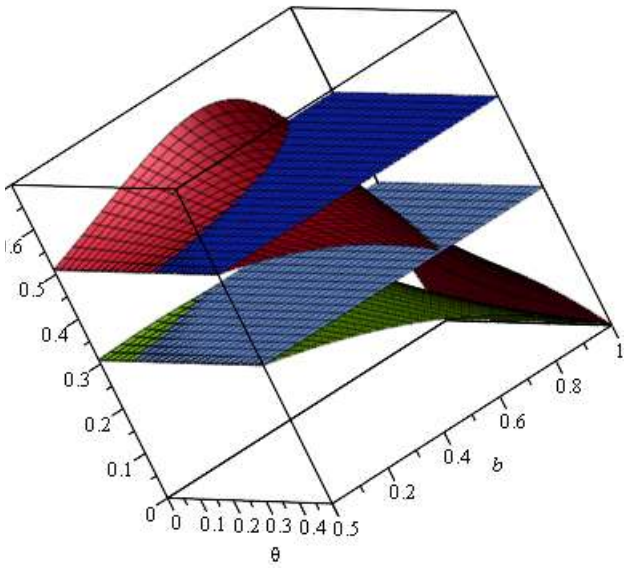
→



evaluate at point → n=.6

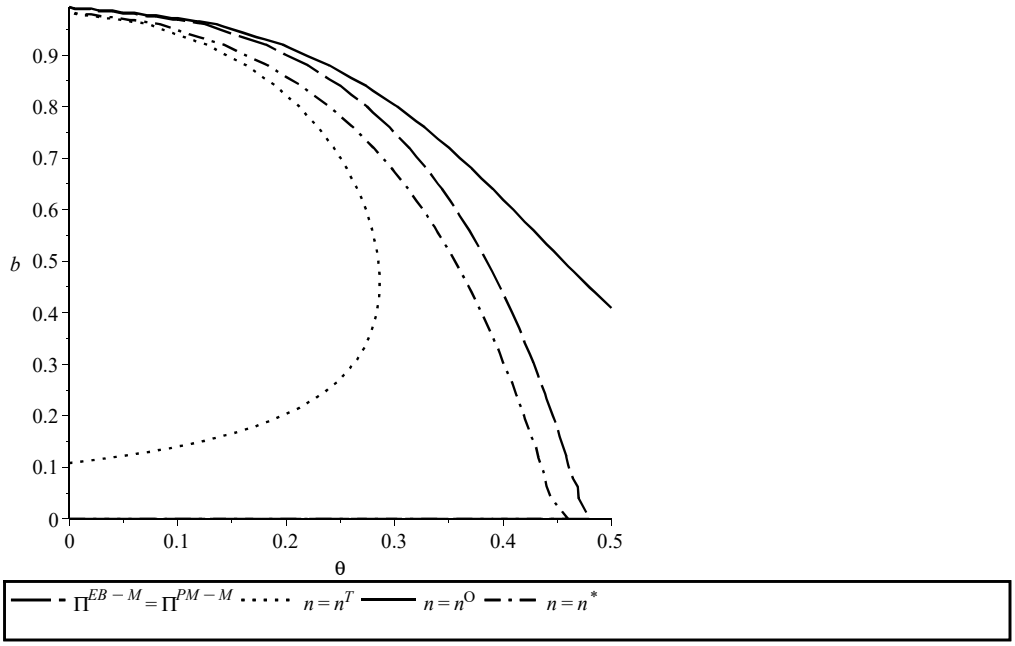
$$\frac{1-b}{((\theta-1)\,b+1.4)^2}, 0.5102040816, \frac{1-b}{((\theta-1)\,b+1.8)^2}, 0.3086419753 \quad \textbf{(231)}$$

→



$$\frac{1-b}{((\theta-1)\,b+1.05)^2}=0.9070294785\,,\ 0.9070294785=\frac{1-b}{((\theta-1)\,b+1.10)^2}\,,$$
$$\frac{1-b}{((\theta-1)\,b+1.05)^2}=0.8264462810\,,\ \frac{1-b}{((\theta-1)\,b+1.10)^2}=0.8264462810$$
$$\frac{1-b}{((\theta-1)\,b+1.05)^2}=0.9070294785,\ 0.9070294785=\frac{1-b}{((\theta-1)\,b+1.10)^2}, \qquad \textbf{(232)}$$
$$\frac{1-b}{((\theta-1)\,b+1.05)^2}=0.8264462810,\ \frac{1-b}{((\theta-1)\,b+1.10)^2}=0.8264462810$$

→

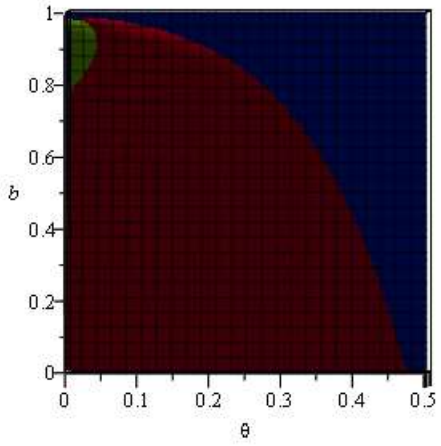


evaluate at point

$\longrightarrow$ n.95

$$\frac{1-b}{\left(\left(\theta-1\right)b+1.05\right)^2}, 0.9070294785, \frac{1-b}{\left(\left(\theta-1\right)b+1.10\right)^2}, 0.8264462810 \quad \textbf{(233)}$$

$\rightarrow$



→

$$\frac{1-b}{\left((\theta-1)\,b-2\,n+3\right)^2}=\frac{1}{(2\,n-3)^2}$$

$$\frac{1-b}{\left((\theta-1)\,b-2\,n+3\right)^2}=\frac{1}{(2\,n-3)^2}$$

solve for n →

$$\left[\left[n=\frac{1}{2}\,\theta+1+\frac{1}{2}\sqrt{-b\,\theta^2+2\,b\,\theta+\theta^2-b-2\,\theta+1}\right],\left[n=\frac{1}{2}\,\theta+1-\frac{1}{2}\sqrt{-b\,\theta^2+2\,b\,\theta+\theta^2-b-2\,\theta+1}\right]\right] \quad \textbf{(235)}$$

$$\frac{1}{2}\,\theta + 1 - \frac{1}{2}\,\sqrt{-b\,\theta^2 + 2\,b\,\theta + \theta^2 - b - 2\,\theta + 1}$$

$$\frac{1}{2}\,\theta + 1 - \frac{1}{2}\,\sqrt{-b\,\theta^2 + 2\,b\,\theta + \theta^2 - b - 2\,\theta + 1} \tag{236}$$

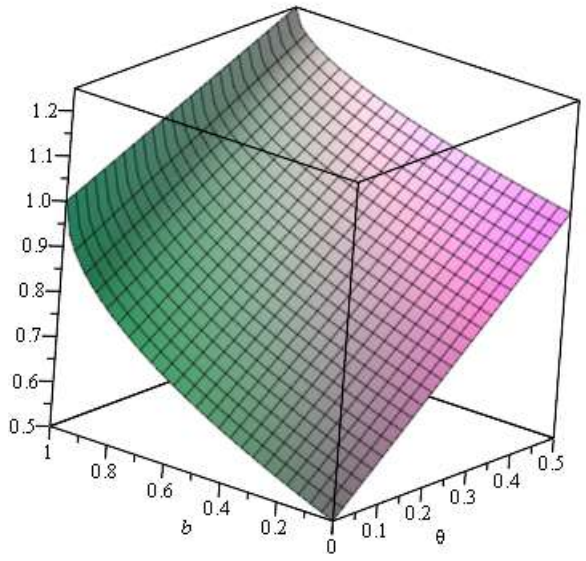
simplify

$$= \frac{1}{2}\,\theta + 1 - \frac{1}{2}\,\sqrt{-(\theta - 1)^2\,(-1 + b)} \tag{237}$$

simplify symbolic

$$\longrightarrow \frac{1}{2}\,\theta + 1 - \frac{1}{2}\,\sqrt{1 - b}\,\theta + \frac{1}{2}\,\sqrt{1 - b} \tag{238}$$

simplify

$$= \frac{1}{2}\,(-\theta + 1)\,\sqrt{1 - b} + \frac{1}{2}\,\theta + 1 \tag{239}$$

$\rightarrow$