*Political Analysis*, 9:3

# Ideal Point Estimation with a Small Number of Votes: A Random-Effects Approach

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Many conventional ideal point estimation techniques are inappropriate when only a limited number of votes are available. This paper presents a covariate-based random-effects Bayesian approach that allows scholars to estimate ideal points based on fewer votes than required for fixed-effects models. Using covariates brings more information to bear on the estimation; using a Bayesian random-effects approach avoids incidental parameter problems. Among other things, the method allows us to estimate directly the effect of covariates such as party on preferences and to estimate standard errors for ideal points. Monte Carlo results, an empirical application, and a discussion of further applications demonstrate the usefulness of the method.

## 1 Introduction

SCHOLARS WORKING WITH spatial theories of politics place great importance on preference estimation. As Krehbiel (1993, p. 21) argues,

> Most or all theories of legislative politics are fundamentally driven by a notion of preferences—whether electoral preferences, party-induced preferences, ideology, or some combination of the above. Thus, empirical research that tests theoretically derived hypotheses must inevitably, if only implicitly, address the question of how legislative preferences should be measured.

Unfortunately, addressing this question is not always easy. One specific problem facing researchers is that they may observe only a small number of votes. For example, researchers interested in legislative preferences on trade or agriculture may have only a handful of votes to use to estimate ideal points.

In such situations, several conventional approaches to preference estimation are problematic. Consider first standard probit/logit analyses that estimate ideal points with fitted

*Author's note*: An early version of this paper was presented as a poster at the 1996 Summer Methods Meeting, Ann Arbor, Michigan. I am grateful to Doug Rivers for his guidance on the project. In addition, Ed Haertel, Trevor Hastie, Tony Lancaster, and Eric Lawrence have provided helpful comments. I am responsible for any errors or omissions that may remain. The model is estimated using S-Plus. The data and program are given at the *Political Analysis* website.

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values. The conventional options are to pool the votes into a single estimation or to use ordered probit analysis. Simple pooling comes at the cost of disregarding the correlation across votes by the same individuals, causing estimation to be inefficient and standard errors to be incorrect (Maddala 1987). In addition, the fitted values can be misleading preference estimates for individuals who do not vote as their covariates would predict. Ordered probit models come at the cost of requiring data to be Guttman scalable—something that is seldom true.

Another option is to use fixed-effect models that directly estimate each individual ideal point. These models face the "incidental parameters" problem that arises because the number of parameters increases as the sample size increases. In the case of voting analysis, adding roll calls to the sample increases the number of roll-call parameters; adding legislators to the sample increases the number of legislator parameters. Not only do standard maximum-likelihood results not apply (Neyman and Scott 1948; Anderson 1972; Londregan 2000), but it is intuitively clear that directly estimating separate ideal points for many individuals will be difficult with only a small number of votes per person.

This paper presents a Bayesian random effects approach (Bock and Aitkin 1981; Mislevy 1987) to the problem. The key to the approach is that it models ideal points as stochastic functions of district and personal characteristics. This means that adding legislators no longer adds legislator parameters to estimate; instead doing so adds more data to be used in estimating the effects of covariates on ideal points. Among other things, this allows the method to handle autocorrelation across individuals and heteroscedasticity across votes. In addition, standard errors of ideal points estimates can be calculated via the delta method (Greene 1997, p. 124).

The method fills a gap in the literature by providing covariate-based ideal point estimates when there is a relatively small number of votes on a particular issue dimension. Poole and Rosenthal (1997), Heckman and Snyder (1997), and others provide estimation strategies for data sets with many votes and many legislators. Londregan (2000) estimates ideal points when there is a small number of voters but, as discussed below, does so in a way that is unattractive for the case with a small number of votes. Clinton et al. (2000) and Jackman (2000a) use a Bayesian approach but do not include covariates, something that is important when one is faced with the limited information contained in a small number of votes. Lewis (1998) applies a similar approach to estimating distributions of voter preferences.

This paper proceeds as follows. Section 2 discusses conventional approaches to ideal point estimation. Section 3 presents a spatial model of voting that underlies the statistical model. Section 4 develops the statistical model and discusses its estimation via an EM algorithm. Section 5 presents a Monte Carlo comparison of techniques. Section 6 presents an application that demonstrates the practical usefulness of the method. Section 7 concludes.

## 2 Conventional Approaches to Ideal Point Estimation

Spatial models have come to provide the language of legislative scholarship. Translating empirical data into this language, however, has proven to be an important challenge. The next two subsections use Krehbiel's (1993) distinction between "constituency-characteristic measures" and "vote-based measures" to discuss conventional strategies for turning observed behavior into preference estimates.

### 2.1 *Constituency-Characteristic Measures*

Most constituency-characteristic measures have been based on standard logit/probit estimation methods. These methods model ideal points as unobserved latent variables that are

functions of observed covariates. Krehbiel and Rivers (1988) formulated ideal points as

$$\theta_i = \mathbf{X_i'}\beta + \epsilon_i \tag{1}$$

where $\theta_i$ is legislator $i$'s ideal point, $\mathbf{X_i}$ is a vector of district and personal characteristics associated with each legislator, $\epsilon_i$ is a random variable, and $\beta$ is a vector of parameters to be estimated. Only the vote, a dichotomous variable $y_i$, is observed. If $\theta_i$ is above $C$—the vote cutpoint—then $y_i = 1$; otherwise $y_i = 0$. Assuming that the $\epsilon_i$ are distributed normally with mean zero, the likelihood of the $i$th vote is

$$P[Y_i = y_i] = \Phi(C - \mathbf{X_i'}\beta)^{y_i}[1 - \Phi(C - \mathbf{X_i'}\beta)]^{1-y_i} \tag{2}$$

where $\Phi$ is the cumulative density function (CDF) for the normal distribution. The estimation procedure maximizes the log likelihood function with respect to $\beta$ and $C$. The fitted values, $\mathbf{X_i'}\hat{\beta}$, constitute ideal point estimates.

One problem with such an approach is that it constitutes a limited use of available information. Each vote tells us only whether a legislator's position is above or below some cutpoint. Consider a 60–40 vote on a measure to increase the minimum wage. The observed voting behavior of all 60 yea voters is the same, with the most ardent supporters of the bill indistinguishable from more moderate supporters. Better estimates would be based on multiple votes; knowing, for example, that a legislator voted yes on both a 60–40 and a 20–80 vote provides a clearer picture of his or her preferences.

However, standard probit/logit models are not well suited to the analysis of multiple votes. One option is to pool the observations across all votes into a single logit or probit equation. This approach produces consistent estimates of effect parameters, but the correlation of errors across votes by the same individual means that estimation will be inefficient and standard errors will be incorrect (Maddala 1987, p. 321). More subtly, this approach fails to tap information in votes fully. Consider a legislator whose district and personal characteristics are generally associated with free trade. The fitted trade ideal point for the legislator will be very pro-free trade, *even if he or she voted against free trade every time.*

A better way to apply probit/logit techniques to multiple votes is to use Krehbiel and Rivers' (1988) ordered probit model. In it legislator ideal points are the same as above, but $Y_i$ is now a categorical variable indicating the number of times a legislator voted in a certain direction. For example, suppose we observe members' votes on two bills. On the first vote members with ideal points less than $C_1$ voted no and on the second vote members with ideal points less than $C_2$, a point higher than $C_1$, voted no. The categorical variable, $S_i$, is the sum of yes votes. If $\theta_i < C_1$, then $S_i = 0$. If $C_1 < \theta_i < C_2$, then $S_i = 1$ and if $\theta_i > C_2$, then $S_i = 2$. When errors are assumed to be standard normal, the probability that $S_i$ will be in the $j$th category is

$$P[Y_i = j] = \Phi(C_j - \mathbf{X_i'}\beta) - \Phi(C_{j-1} - \mathbf{X_i'}\beta) \tag{3}$$

This method has three limitations. The most serious is the requirement that votes are Guttman scalable; that is, the cutpoints must *always* distinguish between members with ideal points above and members with ideal points below them. To see this, consider a 2-vote index of voting for which the first vote was 80–20 and the second vote was 50–50. Clearly the first cutpoint, $C_1$, must be lower than the second cutpoint, $C_2$. For ordered probit to work, all 20 members who voted no on the first vote must also vote no on the second vote (which had a higher cutpoint). If a member votes no on the first vote and yes on the second,

that would mean that his or her ideal point is below $C_1$ and above $C_2$, a contradiction. The second limitation is that the estimated ideal points, $\mathbf{X}_i'\hat{\beta}$, condition only on covariates. Hence, the ideal points of legislators who voted inconsistently with their covariates can, in some cases, be far from the true values. The third limitation is that the ordered probit model requires deletion of all legislators who missed one or more votes in the sample. This can become a serious problem as votes are added to the sample.

## **2.2** *Vote-Based Methods*

### **2.2.1** Voting Indices

The simplest and most popular vote-based estimators are interest group ratings. While convenient, they suffer from serious shortcomings. First, they do not provide the interval-level data necessary for statistical analysis (Snyder 1992). Suppose that legislator ideal points are uniformly distributed from 0 to 100. Suppose also that interest groups focus on controversial votes that tend to divide the chamber nearly in half. Members with ideal points around 60 will get very high scores since they will vote for the higher alternative on all votes in the index; legislators with ideal points around 40 will get very low scores. The differences among these moderate legislators will be, in Snyder's terminology, "artificially extreme."

Missing votes are a second problem for index scores. Voting with an interest group 100% of the time has a different meaning for a member voting "correctly" on 10 votes than for a member voting "correctly" on 8 votes. If the two votes missed by the latter member were votes on which he or she would have voted incorrectly, a score of 100 is misleading; if the legislator would have voted correctly on the two votes, a score of 80 is also inappropriate. There is no way to be certain of what would have happened, so researchers using indices must choose one of two imperfect alternatives.

A third issue is that some votes are more strongly affected by random elements than are others. For example, a vote on affirmative action may divide members according to civil rights preferences more cleanly than a vote on a welfare bill. Not taking this into account could even mean that indices do not provide ordinal-level data. For example, it is possible that issues not related to civil rights introduce enough noise in the index that some members with anti-civil rights ideal points vote more frequently in a "pro-civil rights" manner than members with pro-civil rights ideal points. One possible way to avoid this is to place more weight on the votes more clearly associated with civil rights. Generally, indices do not do this, however, and when they do, they rely on ad hoc double or triple counting of specific votes. A better alternative is to model and estimate differences in the ability of votes to discriminate preferences on a given issue.

Groseclose et al. (1999) provide a clever method to generate scale and stretch parameters to make index scores comparable over time. However, the inputs into the process are interest group ratings, meaning that we cannot be sure to what extent the output has been affected by the above imperfections in the original indices. Herron (2000) provides another approach to recalculating index scores, arguing that one needs to take into account differences in cutpoint distributions of votes in the indices.

### **2.2.2** Fixed-Effects Models

A fundamental problem with indices is that they are not based on a model that characterizes how preferences are manifested on each vote. In response to this shortcoming, several scholars have developed measurement techniques based on random utility choice models. In these

models, voting is based on legislator ideal points, vote characteristics, and a parametrically defined random variable.

Poole and Rosenthal (1997) have conducted pathbreaking work in this area. They define the probability of a yea vote to be

$$P[y_{it} = \text{yea}] = \frac{e^{ze^{-0.125(\theta_i - \gamma_{ty})^2}}}{e^{ze^{-0.125(\theta_i - \gamma_{ty})^2}} + e^{ze^{-0.125(\theta_i - \gamma_{tn})^2}}} \tag{4}$$

where $z$ is a parameter to be estimated, $\theta_i$ is the ideal point of legislator $i$, and $\gamma_{ty}$ and $\gamma_{tn}$ are the spatial locations of the yea and nay alternatives.[1] This structure is used because the roll-call locations are not identified for more conventional quadratic utility. Poole and Rosenthal (1991, p. 237) acknowledge that identification based solely on functional form is tenuous but justify it based on Monte Carlo results.

To estimate the model, Poole and Rosenthal use an alternating algorithm. First, they hold ideal points and $z$ fixed and estimate vote parameters. Then they hold vote parameters and $z$ fixed and estimate ideal points. Finally, they hold ideal points and vote parameters fixed and estimate $z$. They repeat this process until the parameters correlate at 0.99 with the previous iteration (Poole and Rosenthal 1997, p. 237). Although this may appear similar to an EM algorithm, it is not; instead it has much more in common with Birnbaum's (1968) joint maximum-likelihood approach.

This approach is remarkably effective at predicting votes based on one dimension, or sometimes two. Based on almost all votes by virtually every member of Congress in the history of the United States, Poole and Rosenthal found that a single dimension explained about 80% of the variation in voting. A second dimension explained about 3% more of the variation.

The findings about dimensionality are strong but not uncontested. First, the interpretation of these dimensions is more of an art than a science. Poole and Rosenthal (1991, p. 235) prefer to interpret the first dimension in terms of a standard left–right continuum (although it can also be interpreted as a party dimension) and the second dimension in terms of region. Second, other methods find that additional dimensions are more important (Heckman and Snyder 1997).

Behind the whole effort (and efforts like it) looms the "incidental parameters problem" (Neyman and Scott 1948). This problem arises because there is at least one parameter for each legislator and each roll call, meaning that the number of parameters increases as the sample size increases. Therefore, standard maximum-likelihood estimation results based on models with a fixed number of parameters are not valid (Anderson 1972; Londregan 2000).[2]

When there are many votes and many legislators, Monte Carlo experiments indicate that the ideal point estimates are quite accurate (Poole and Rosenthal 1991, p. 272). However, when either the number of votes or the number of legislators is small, joint estimation of

[1] Their notation has been changed to make it consistent with the rest of this paper. Their later work includes a time covariate such that $\theta_{itp} = \theta_{it}^0 + \theta_{it}^1 p + \theta_{it}^2 p^2$, where $p$ is measured in terms of congresses served by a member at time $t$.

[2] A similar dynamic is at work in the findings of consistency for related models. Haberman (1977) has shown that MLE estimates of a Rasch model are consistent if $N \to \infty$, $T \to \infty$, and $N/T \to \infty$. Anderson (1972) proves that the MLE estimator will be biased by a factor of two when $T = 2$; he states (without proof) that the bias of the MLE estimator is $T/(T-1)$, meaning that, with enough votes, the bias of the estimator should go to zero. These models differ from the model below by assuming $\alpha_t = 1$ for all $t$.

ideal points and vote parameters is problematic. Dealing with this situation is important, as scholars studying how district interests relate to legislator positions on specific policies or how committee members' preferences on committee-related issues compare to floor preferences often will not have enough votes to make using fixed effect methods reasonable.

To overcome this within the NOMINATE framework one could assume that the one or two dimensions of NOMINATE scores based on all votes accurately capture preferences for the issue of concern. This approach is often untenable, however. Consider trade politics. Both sides of the issue completely span the conventional left–right spectrum: protectionists include Paul Wellstone, Fritz Hollings, and Strom Thurmond, while free traders include Joseph Lieberman, Mark Hatfield, and Phil Gramm. Poole and Rosenthal's general ideology measures do not capture these positions. Researchers interested in other areas, ranging from milk supports to superconductors to abortion, will find, to varying degrees, similar differences from general ideological orientations.

## 3 A Spatial Model of Legislative Voting

This paper presents a method that is useful for scholars interested in estimating one-dimensional preferences based on a small number of votes. The theoretical foundation for the approach is a unidimensional spatial choice model. Legislator utility is

$$u_i(\gamma_{tk}) = -(\theta_i - \gamma_{tk})^2 + \epsilon_{itk} \tag{5}$$

where $\theta_i$ is the ideal point of legislator $i$, $\gamma_{tk}$ is the spatial location of alternative $k$ on vote $t$, and $\epsilon_{itk}$ is a random shock to $i$'s utility on vote $t$ from alternative $k$. Legislator $i$ votes for the alternative that yields higher utility. Let $y_{it} = 1$ if $i$ votes for $\gamma_{t1}$ over $\gamma_{t0}$. Then

$$P[y_{it} = 1] = P[u_i(\gamma_{t1}) > u_i(\gamma_{t0})] \tag{6}$$

$$= P\left[ -\left(\theta_i^2 - 2\theta_i\gamma_{t1} + \gamma_{t1}^2\right) + \epsilon_{it}^1 > -\left(\theta_i^2 - 2\theta_i\gamma_{t0} + \gamma_{t0}^2\right) + \epsilon_{it}^0 \right] \tag{7}$$

$$= P\left[ \frac{\epsilon_{it0} - \epsilon_{it1}}{\sigma_t} < \frac{2(\gamma_{t1} - \gamma_{t0})}{\sigma_t}\left(\theta_i - \frac{(\gamma_{t1} + \gamma_{t0})}{2}\right)\right] \tag{8}$$

The $\epsilon_{itk}$ are independent, identically distributed Type I Extreme Value random variables. Because the difference of two such random variables is a logistic random variable, we replace $(\epsilon_{it0} - \epsilon_{it1})/\sigma_t$ with a logistic random variable, $\epsilon_{it}$.[3]

To make Eq. (8) more intuitive, let $\kappa_t = (\gamma_{t1} + \gamma_{t0})/2$ and $\alpha_t = 2(\gamma_{t1} - \gamma_{t0})/\sigma_t$. The first term, $\kappa_t$, is the cutpoint for vote $t$. Legislators with ideal points greater than $\kappa_t$ will tend to vote for the bill; legislators with ideal points less than $\kappa_t$ will tend to vote against the bill. If $\theta_i = \kappa_t$, then legislator $i$ will be equally likely to vote in favor of or against bill $t$.

The second term, $\alpha_t$, magnifies the difference between ideal point and cutpoint. Votes with high values of $\alpha_t$ discriminate relatively well between individuals with ideal points above and individuals with ideal points below $\kappa_t$; votes with low values discriminate poorly. For example, if $\alpha_t$ were very large, an individual with an ideal point just below $\kappa_t$ would almost certainly vote against the bill and an individual with an ideal point just above it would almost certainly vote for the bill. For values of $\alpha_t$ close to zero (meaning either that

[3] If $\epsilon_{itk}$ were independent normal random variables, $(\epsilon_{it0} - \epsilon_{it1})\sigma_t$ would also be normal. Such an assumption is often made in the literature (Poole 2000; Bailey and Chang 2000). It is unlikely that results will differ substantially under the two assumptions because the logistic and normal distributions are virtually indistinguishable under a simple transformation (Baker 1992, 15).

the bill positions are very close or that the bill-specific variance is very high), all individuals, regardless of their ideal points, would have a roughly 50% chance of voting for the bill. Estimating this value addresses heteroscedasticity across votes as $\sigma_t$ is implicitly allowed to vary.

Substituting $\epsilon_{it}$, $\alpha_t$, and $\kappa_t$ into Eq. (8), we get

$$P[y_{it} = 1 \mid \alpha_t, \kappa_t, \theta_i] = P[\epsilon_{it} < \alpha_t(\theta_i - \kappa_t)] \tag{9}$$

$$= \frac{1}{1 + e^{-\alpha_t(\theta_i - \kappa_t)}} \tag{10}$$

## 4 A Bayesian Random-Effects Framework

Directly estimating the parameters in Eq. (10) will not work if one has only a small number of votes. A handful of votes simply does not give enough information about the ideal point of legislators. Therefore, this paper turns to random-effects methods that avoid direct estimation of parameters for each individual.

Londregan (2000) uses random-effects modeling to deal with the case in which there is a small number of legislators. The approach presented here is distinctive in several ways. First, the data structure implies different estimation strategies. When there are a small number of legislators and many votes, as there are in Londregan's data, we have many data for each legislator, but only a few data for each vote. Hence it makes sense to treat vote parameters as nuisance parameters and to concentrate on directly estimating only the ideal points. When there are a small number of votes and many legislators, on the other hand, we have a lot of information about each vote but only a little about each legislator. Hence, it makes sense to treat the legislator parameters as nuisance parameters and to estimate the vote parameters directly. At first glance, this seems counterproductive, as the goal is to generate ideal points. However, as discussed below, we therefore have to add additional steps to the process, a process known as *expected a posteriori* (EAP) estimation. In this process, adding covariates and using Bayes' rule makes effective use of data.

Second, the approach here does not rely on a major simplification used by Londregan. His strategy is to treat vote parameters as functions of covariates. Londregan (2000, p. 50) assumes that one of the random vote parameters is not random in order to make the model tractable. In the case in which the only covariate is the identity of the bill proposer, this means that the estimation will treat all bills proposed by a certain entity as having the same ideological distance from the status quo, even if the observed proposals from an individual induce markedly different behavior. This contrasts with a truly random-effects approach that would allow for differing proposals even as it estimated a central tendency for proposals from each individual. However, such an assumption is not necessary for conventional two-parameter item response theory (IRT) approaches; Bailey and Chang (2000) estimate a random-effects model in which both vote parameters are random for the case with a small number of voters.

Third, the approach here works within a Bayesian context. As discussed below, it is well known in the item response literature that certain parameters can become unbounded. In the frequentist approach, one must deal with such problems either by excluding individuals or by imposing constraints on parameters. In the Bayesian context, reasonable priors can be explicitly explained and modeled.

The most direct way to implement a Bayesian random-effects approach would be to assume that all legislators' ideal points are drawn randomly from the same distribution. However, we have much more information, including information about district, party, and

other factors. Hence a more reasonable—and inclusive—assumption is to assume that the ideal points are random functions of district and personal characteristics and a random shock (Mislevy 1987). In fact, this assumption is identical to that underlying Eq. (1), the fundamental assumption in a probit model. Specifically, let

$$\theta_i = \mathbf{X}_\mathbf{i}'\beta + \eta_i \tag{11}$$

where $\eta_i$ is an i.i.d. normal random variable with mean zero and variance $\sigma_\theta^2$. This means that, conditional on covariates, the ideal points of individuals are independent.

With this setup, we can tap into an extensive literature on marginal maximum likelihood (MMLE) (Bock and Aitkin 1983; Mislevy 1987).[4] The key is *marginalization*. That is, the stochastic ideal points are integrated out of the data likelihood to produce the marginal likelihood. This means that one no longer needs to estimate a separate ideal point for every legislator. Instead, each legislator simply adds information about the relationship between covariates and ideal points. This allows us to address the autocorrelation across individual's votes without moving to fixed-effect methods that are inappropriate with few votes.

A second—and separate—element of the approach presented here is the use of a Bayesian framework. An important issue in the random effects panel logit literature is that some $\alpha_t$ increase indefinitely and others go to zero (Baker 1992, pp. 97–98; Mislevy and Bock 1990, p. 8).[5] In other words, using conventional MLE methods, the model estimates that some votes perfectly distinguish between high and low types while others add on information. Typical approaches to dealing with this are to impose assumptions about the distribution of $\alpha_t$ (e.g., $\alpha_t = 1$ for all $t$) or to posit a "penalty function" in the likelihood that keeps the estimates bounded. Since frequentist theory provides no guidance, however, we move to a Bayesian perspective that explicitly enables us to incorporate our belief that no $\alpha$'s are either extremely big or extremely small.

Bayesian analysis is multifaceted and increasingly common in the social sciences. Jackman (2000a, b) provides an overview and applications of Bayesian analysis, including a Markov chain Monte Carlo (MCMC) estimation of ideal points. The form of Bayesian analysis used here proceeds in three steps. First, one posits prior distributional assumptions about parameters. These priors may be diffuse (e.g., "any value between $-\infty$ and $+\infty$ is equally likely") or more specific (e.g., "the value is greater than 0 but almost certainly less than 10"). Second, one writes down a likelihood-like equation for the posterior probability of the parameters conditional on the data. (An example is provided below.) Third, one generates point estimates by finding the modal value of the posterior parameter probability density.

In this case, the prior information about parameters is an assumption that the $\alpha_t$ are positive but tend not to be extremely large; Mislevy (1986, p. 193) motivates this assumption

[4]There is an alphabet soup of other procedures to deal with the incidental parameters problem including Birnbaum's (1968) joint maximum likelihood (JMLE) and Anderson's (1972) and Chamberlain's (1980) conditional maximum likelihood (CMLE). For a survey of these methods applied to estimation of ideal points, see Bailey and Rivers (1997). Of these, MMLE merits the most attention because it has a broader range of applicability than CMLE (Zwinderman 1997) and performs better than JMLE when there is a limited number of votes (Drasgow 1989). Specific issues with CMLE are that it drops all observations for which legislators vote all-yea or all-nay (Greene 1993, p. 656); it cannot handle missing data; it cannot estimate covariates that vary between persons but not within persons (Zwinderman 1997, p. 248) and becomes computationally very burdensome as the number of votes increases (Baker 1992, p. 143).

[5]These kind of problems are not unique to random-effects models. Fixed-effect methods produce infinite ideal point estimates for all legislators who vote consistently in one or the other directions. The standard—and *ad hoc*—way to deal with this is to delete such legislators from the sample or to impose arbitrary constraints.

more concretely: "If most of the items have $\alpha$'s between $\frac{1}{3}$ and 3, then the $\alpha$ for this particular item is not 957." We implement this by assuming that $\alpha$ is distributed lognormally, with mean = variance = 0.25. This characterizes a prior distribution with a mode at 1. The other parameters are well behaved, so we opt for diffuse priors.[6]

### 4.1 *The Posterior Density Function*

To present the posterior density function that serves as the foundation for Bayesian analysis, we follow the development given by Tsutukawa and Lin (1986). The probability of observing the data given ideal points and parameters is

$$P[\mathbf{Y} \mid \theta, \alpha, \kappa] = \prod_{n=1}^{N} \prod_{t=1}^{T} P_{it}^{y_{it}} (1 - P_{it})^{(1-y_{it})} \tag{12}$$

where $P_{it} = P[y_{it} = 1 \mid \alpha_t, \kappa_t, \theta_i]$.

Bayesian methods focus on the probability density of the parameters conditional on the data. By Bayes' theorem and independence of vote parameters, the joint probability of all unobservable parameters is

$$P[\theta, \alpha, \kappa, \beta \mid \mathbf{Y}] = \frac{P[\mathbf{Y} \mid \theta, \alpha, \kappa] P[\theta \mid \mathbf{X}'\beta] P[\beta] P[\alpha] P[\kappa]}{P[\mathbf{Y}]} \tag{13}$$

where $\mathbf{Y}$ is the $N \times T$ matrix of responses and $\mathbf{X}$ is the $N \times P$ matrix of covariates. The unconditional probability of $\mathbf{Y}$ is fixed (and unknowable) and, together with the flat priors on $\kappa$ and $\beta$, is incorporated into a proportionality constant, yielding

$$P[\theta, \alpha, \kappa, \beta \mid \mathbf{Y}] \propto P[\mathbf{Y} \mid \theta, \alpha, \kappa] P[\theta \mid \mathbf{X}'\beta] P[\alpha] \tag{14}$$

To formulate the posterior probability in terms of $\alpha, \kappa$, and $\beta$ only, Eq. (14) is integrated with respect to $\theta$, yielding the marginal posterior:

$$P[\alpha, \kappa, \beta \mid \mathbf{Y}] \propto P[\alpha] \int P[\mathbf{Y} \mid \theta, \alpha, \kappa] P[\theta \mid \mathbf{X}'\beta] \, d\theta \tag{15}$$

### 4.2 *EM Estimation of the Model*

An EM algorithm simplifies estimation of the complex posterior density. The most influential work on this method is that by Dempster et al. (1977). Bock and Aitkin (1981) apply it to marginal models such as this one. McLachlan and Krishnan (1997) and Wu (1983) provide more extensive discussions of the EM approach.

An important part of the method is the "complete data posterior," the posterior probability we would have if we observed $\theta$. If we were able to observe $\theta$, maximizing with respect to vote parameters would be relatively simple. Or if we knew the vote parameters, we could infer the $\theta$. We are not so fortunate, however. Therefore, the strategy is to use provisional estimates of $\alpha, \kappa$, and $\beta$ to allow us to calculate the expected value of the complete data

[6] Other parameters may not always be well behaved. For example, if one has a dummy variable for race of legislator and all African-American representatives vote liberally on all votes, the coefficient on the variable may increase indefinitely.

posterior. We then choose new estimates of $\alpha$, $\kappa$, and $\beta$ to maximize the expected value of the complete data posterior probability.

A complete review of the process is available at the *Political Analysis* website. Here we present an overview of the process. It proceeds as follows.

1. Choose provisional parameter estimates of $\alpha$, $\kappa$, and $\beta$.
2. Calculate the expected value of the log of the complete data posterior over all values of $\theta$ given provisional vote and covariate parameter estimates (this is the "E step").
3. Maximize this function with respect to the vote and effect parameters to yield new estimates of $\alpha$, $\kappa$, and $\beta$ (the "M step"). This is simplified due to the relatively simple complete posterior formulation; for each vote, vote parameters can be estimated separately using standard maximization routines.
4. Use these new estimates as provisional parameter estimates and repeat from Step 2 until convergence.

The approach presented here is estimated with a program written for S-Plus; it is available at the *Political Analysis* website. The EM algorithm is not the only way to estimate a Bayesian IRT model. Sampling Bayesian methods generate a specific characterization of the posterior distribution from which standard errors can be calculated directly (Jackman 2000a).

### 4.3 *Characteristics of the Estimates*

The random-effects estimation method produces three classes of estimates: ideal points, effect parameters, and vote parameters. First, and most important for our purposes, the method estimates ideal points. These estimates are indirect, as they are expected ideal points given the estimated parameters. They are sometimes referred to as expected *a posteriori* estimators, or EAP estimators, for this reason.[7] They are calculated at the final iteration as

$$E(\theta_i \mid \mathbf{Y_i}, \mathbf{X_i}, \hat{\alpha}, \hat{\kappa}, \hat{\beta}) = \int \theta P[\theta \mid \mathbf{Y_i}, \mathbf{X_i}, \hat{\alpha}, \hat{\kappa}, \hat{\beta}]\, d\theta \tag{16}$$

where $P[\theta \mid \mathbf{Y_i}, \mathbf{X_i}, \hat{\alpha}, \hat{\kappa}, \hat{\beta}]$ is derived from Bayes' law,

$$P[\theta \mid \mathbf{Y_i}, \mathbf{X_i}, \hat{\alpha}, \hat{\kappa}, \hat{\beta}] = \frac{P[\mathbf{Y_i} \mid \theta, \hat{\alpha}, \hat{\kappa}] P[\theta \mid \mathbf{X_i}'\hat{\beta}]}{f P[\mathbf{Y_i} \mid \theta, \hat{\alpha}, \hat{\kappa}] P[\theta \mid \mathbf{X_i}'\hat{\beta}]\, d\theta} \tag{17}$$

All estimates come with estimated standard errors. As Clinton et al. (2000) make clear, the ideal point estimation literature has paid relatively little attention to assessing uncertainty. This is clearly a shortcoming and a hindrance in hypothesis testing. With the model discussed here, we can produce such assessments. The standard errors of vote parameters are based on an outer-product-of-the-gradient estimator of the posterior covariance matrix evaluated at the estimated parameter values.[8] The standard errors of ideal points are calculated via the delta method (Greene 1993, p. 297). The website appendix gives an extended discussion of this process.

This estimator conditions on *both* covariates and votes. That is, a legislator with district and personal characteristics generally associated with free trade who voted "against" his or

[7] Another EAP estimator is $E(\theta_i \mid \mathbf{X_i}, \hat{\alpha}, \hat{\kappa}, \hat{\beta})$; it is calculated simply as the estimated effect parameters multiplied by the covariate parameters. It is similar to the covariate-based methods discussed earlier. It is useful for estimating ideal points of out-of-sample legislators. It is also possible to calculate ideal points based only on votes, $E(\theta_i \mid \mathbf{Y_i}, \hat{\alpha}, \hat{\kappa}, \hat{\beta})$.

[8] Tsutakawa and Lin (1986) use the inverse Hessian; we rely here on the asymptotic equivalence of the two.

her covariates will have an estimated ideal point that is not strongly free trade due to the inclusion of the individual's votes in the expectation calculation. In contrast, probit/logit and ordered probit estimates condition only on covariates. For example, legislators whose districts and personal characteristics are generally associated with free trade will have very pro-free trade estimates *even if they voted against free trade every time*.

The method directly estimates effect parameters that link legislator and district characteristics to estimated ideal points. Because these are based on multiple votes, these estimates use more information than single-vote probit/logit models. They are also superior to models that use indices or fixed-effect estimates as dependent variables because the ideal point estimates based on those methods may be unreliable (Zwinderman 1991, p. 597).

Finally, the random-effects method estimates vote cutpoints and vote discrimination parameters. As is clear from the spatial choice model, these estimates are a necessary byproduct of the estimation process. These estimates are also useful in their own right. For example, simulation of future votes (or past votes in different circumstances) requires estimates of vote parameters (see Bailey 2001). In addition, the vote parameters can be used to study agenda formation, providing directly comparable data on the cutpoints of votes over periods of time or across committee jurisdictions.

Several things should be kept in mind when using these estimates. First, the estimates are based on a one-dimensional model. Because of this, any application of this method should pick votes and covariates carefully. If another dimension systematically affects legislators, then multidimensional models may be more appropriate. The costs of using multidimensional models are that more data are required and the ideal points are more difficult to interpret.

Second, the estimation process presented here is not based on a full-fledged model of the legislative process in which vote trading, sophisticated voting, and other complicated interactions occur. While such matters do not necessarily cause problems for the estimation [see the excellent discussion by Poole and Rosenthal (1997, Chap. 7)], they can complicate matters. For example, the approach discussed here assumes that unmeasurable factors such as log-rolling among legislators do not induce correlations across errors. If such factors are a primary concern, one can consider approaches such as that of Zorn (2001), which uses a General Estimating Equation model to estimate directly the correlation of errors in voting by Supreme Court members. The method requires many votes for each individual in order to estimate a full correlation matrix across all individuals. Another option is to follow Smith and McGillivray (1996) and use MCMC methods to estimate models that allow for correlation (conditional on covariates) across votes by senators from the same states.

Third, analysts must be aware that the random-effects approach does not definitively solve all problems. Consistency requires large numbers of observations per individual and there is, by definition, no way around this when one has a small number of observations per individual. The point here is to make optimal use of the information we do have, information contained in multiple votes and covariates. In addition, a misspecification of the randomness may contaminate the whole process. In this case, the specification of the randomness is relatively benign; it is of the same form of specifications we typically make in one-vote probit models. If it is wrong, however, the estimates will be wrong as well.

## 5 Monte Carlo Comparison of Estimation Methods

It is clear that naive pooled logit and index scores are wrong, but some may wonder if they are so wrong as to justify more involved methods. Similarly, it is also clear that fixed-effect estimators are inappropriate with a small number of votes, but some may wonder if their

**Table 1** Monte Carlo results—Mean squared error

| *Number of votes* | *Pooled logit* | *Index* | *Fixed effects* | *Random effects* |
|---|---|---|---|---|
| 2 | 0.57 | 0.93 | 0.89 | 0.51 |
| 4 | 0.54 | 0.61 | 0.72 | 0.43 |
| 6 | 0.54 | 0.53 | 0.62 | 0.41 |
| 8 | 0.52 | 0.49 | 0.58 | 0.37 |
| 10 | 0.54 | 0.43 | 0.49 | 0.37 |
| 50 | 0.53 | 0.11 | 0.12 | 0.31 |

small sample properties are "close enough." This section uses Monte Carlo simulations to evaulate such possibilities; the setup follows Zwinderman (1991).

Each iteration of the simulation process consisted of two steps. First, vote data were simulated based on ideal points that were generated according to

$$\theta_i = 0.5X_{1i} - 0.5X_{2i} + \eta_i \tag{18}$$

where $X_{1i}$ and $X_{2i}$ are simulated covariates and $\eta$ is an $N(0, 0.5)$ random variable. The covariates were drawn from $N(0, 1)$ and fixed throughout. They could be drawn from any distribution. Discrimination and cutpoint parameters for the votes were then chosen and votes were simulated as binomial draws with probability from Eq. (10). Details on vote parameters and other aspects of the simulation are available at the *Political Analysis* website. Second, ideal points were estimated using four methods:

(1) a naive pooled logit model that disregards correlation of votes by each individual,
(2) index scores calculated as the percentage of yes votes,
(3) a fixed-effects model that directly estimates the $\theta_i$, and
(4) the random-effects method discussed above.

Table 1 presents the mean of the mean squared errors (MSE) over all iterations for each method.[9] The reported results are based on 200 iterations with 100 legislators; the results are essentially the same with different settings.[10] When there is a small number of votes, the random-effects approach is substantially better, sometimes decreasing the MSE by more than 30%. Correlations of the estimates with the true values exhibit similar patterns. For example, for eight votes, the pooled logit estimates correlate at 0.67 with the true values, the index correlates at 0.75, the fixed-effect estimates correlate at 0.70, and the random-effects estimates correlate at 0.81.

Table 1 also indicates the relative strengths of the various methods. The superiority of random effects over the pooled logit increases as the number of votes increases, which is not surprising, as the pooled logit does not acknowledge dependence across votes. The

[9]The index scores and the simulated (true) ideal points were both standardized before the MSE for the index method was calculated. The MSE for the fixed-effect model was based only on legislators who did not vote always yes or always no. Fixed-effect methods cannot estimate ideal points for all-nay or all-yea individuals because the maximum-likelihood estimate is $\pm\infty$.

[10]For $N = 435$, the mean squared errors are lower across the board, but improvement of the random-effects model over the others is generally the same as reported in Table 1. For example, with four votes the MSE for pooled logit was 0.60, the MSE for index was 0.51, and the MSE for random effects was 0.34; the fixed-effect model was computationally infeasible.

random-effects estimates are better than the fixed-effects estimates when the number of votes is small; when the number of votes is large, however, the fixed-effects estimator performs better. Again, this is not surprising, as the fixed-effect estimator has the most flexibility and, given enough information, is ideal. The relatively strong performance of the index is interesting, although it may not generalize, as the vote parameters used in the analysis were relatively well behaved; indices have the most trouble when this is not the case. Information on the vote parameters is available in the website appendix.

As with many Monte Carlo simulations, these results must be taken with a grain of salt, as they are based on the assumptions of the underlying statistical model. However, note that the simulation equation is consistent with the models underlying the other methods. Those who run naive pooled logit models would accept ideal points as a function of covariates; they disregard the individual specific effects for convenience. Those who run fixed-effects models assume that they can directly estimate ideal points with or without covariates, a claim that encompasses the above specification. Studies of more complicated situations, involving various degrees of dependence across individuals, for example, are an area for future studies.

## 6 Estimating Trade Preferences

This section presents a random-effects estimation of international trade ideal points for the 1993–1994 Senate that highlights how the approach works in practice. For a substantive discussion of the effect of district characteristics on trade preferences, see Bailey (2001). The method offers several practical benefits: it does not require votes to be Guttman scalable, it allows for missing votes, and, unlike fixed-effect and CML methods, it does not have to exclude information from the 55 senators who voted all yea or all nay.

During this congress, the Senate had five substantive votes on trade:

(1) an amendment to delete environmental and labor side agreements from the North American Free Trade Agreement (NAFTA) implementing legislation,
(2) final passage of the NAFTA,
(3) an extension of fast-track authority for the Uruguay Round negotiations,
(4) a budget waiver for the implementation of the Uruguay Round agreement, and
(5) final passage of the Uruguay Round agreement.

The covariates are state and legislator characteristics that would be expected to affect trade preferences. The state characteristics are export share of production, district skill levels, changes in import penetration, Perot vote in 1992, and unionization. The legislator characteristics are campaign contributions from multinationals and labor, party, and an interaction between Democrat and unionization.

Table 2 reports votes and expected ideal points. The vote numbers in the table correspond to the vote numbers in the list above. The ideal point estimates, $E(\theta \mid X, Y)$, are expected ideal points conditional on covariates and votes. Not surprisingly, those most opposed to trade liberalization such as Senators Feingold, Helms, Byrd, and Wellstone have the low expected ideal points. Those associated with trade liberalism such as Senators Lugar and Packwood have high ideal points. Note that the method produces estimates for those such as Senators Boren, Dorgan, and Inhofe who voted on only a subset of the votes.[11]

[11] This assumes abstentions are random. Those uncomfortable with such an assumption may exclude legislators who missed votes or deal directly with missing data (King et al. 2001).

**Table 2** Votes and expected trade ideal points, 1993–1994 Senate

| *Senator* | *1* | *2* | *3* | *4* | *5* | $\hat{\theta}$ | *SE* |
|---|---|---|---|---|---|---|---|
| Akaka | 0 | 1 | 0 | 1 | 1 | −0.24 | 0.45 |
| Baucus | 1 | 1 | 1 | 0 | 0 | −0.04 | 0.50 |
| Bennett | 1 | 1 | 1 | 1 | 1 | 1.35 | 0.75 |
| Biden | 1 | 0 | 1 | 1 | 1 | 1.18 | 0.33 |
| Bingaman | 1 | 1 | 1 | 1 | 1 | 1.63 | 0.74 |
| Bond | 1 | 1 | 1 | 1 | 1 | 2.22 | 0.78 |
| Boren | 1 | 1 | 1 | NA | NA | 1.91 | 0.72 |
| Boxer | 0 | 1 | 1 | 1 | 1 | 0.47 | 0.42 |
| Bradley | 1 | 1 | 1 | 1 | 1 | 1.71 | 0.77 |
| Breaux | 1 | 1 | 1 | 1 | 1 | 1.36 | 0.78 |
| Brown | 1 | 1 | 1 | 0 | 0 | −0.01 | 0.52 |
| Bryan | 0 | 1 | 1 | 0 | 0 | −1.07 | 0.61 |
| Bumpers | 1 | 1 | 1 | 1 | 0 | 0.26 | 0.48 |
| Burns | 0 | 0 | 1 | 0 | 0 | −1.13 | 0.83 |
| Byrd | 0 | 1 | 0 | 0 | 0 | −1.53 | 1.01 |
| Campbell | 0 | 1 | 0 | 0 | 0 | −1.03 | 1.07 |
| Chafee | 1 | 1 | 1 | 1 | 1 | 1.64 | 0.84 |
| Coats | 1 | 1 | 1 | 1 | 1 | 3.50 | 0.36 |
| Cochran | 1 | 1 | 1 | 1 | 1 | 1.31 | 0.87 |
| Cohen | 0 | 0 | 1 | 1 | 1 | 0.34 | 0.33 |
| Conrad | 0 | 1 | 0 | 1 | 1 | −0.06 | 0.42 |
| Coverdell | 1 | 1 | 1 | 1 | 1 | 1.39 | 0.66 |
| Craig | 0 | 0 | 1 | 0 | 0 | −1.21 | 0.81 |
| Damato | 0 | 0 | 1 | 1 | 1 | 1.56 | 0.96 |
| Danforth | 1 | 1 | 1 | 1 | 1 | 1.98 | 0.79 |
| Daschle | 1 | 1 | NA | 1 | 1 | 1.20 | 0.85 |
| DeConcini | 1 | 1 | 1 | 1 | 1 | 1.93 | 0.79 |
| Dodd | 1 | 1 | 1 | 1 | 1 | 2.19 | 0.87 |
| Dole | 1 | 1 | 1 | 1 | 1 | 1.47 | 0.69 |
| Domenici | 1 | 1 | 1 | 1 | 1 | 0.98 | 0.78 |
| Dorgan | NA | NA | 0 | 0 | 0 | −1.74 | 1.44 |
| Durenberger | 1 | 1 | 1 | 1 | 1 | 1.18 | 0.62 |
| Exon | 0 | 0 | 1 | 0 | 0 | −1.03 | 0.89 |
| Faircloth | 0 | 0 | 0 | 1 | 0 | −0.81 | 0.80 |
| Feingold | 0 | 0 | 0 | 0 | 0 | −2.50 | 1.63 |
| Feinstein | 0 | 1 | 0 | 1 | 1 | 0.16 | 0.37 |
| Ford | 0 | 1 | 1 | 1 | 0 | −0.13 | 0.44 |
| Glenn | 0 | 0 | 1 | 1 | 1 | 0.07 | 0.41 |
| Gorton | 1 | 1 | 1 | 1 | 1 | 2.42 | 0.68 |
| Graham | 1 | 1 | 1 | 1 | 1 | 1.42 | 0.77 |
| Gramm | 1 | 1 | 1 | 1 | 1 | 1.25 | 0.62 |
| Grassley | 1 | 1 | 1 | 1 | 1 | 1.29 | 0.73 |
| Gregg | 1 | 1 | 1 | 1 | 1 | 2.15 | 0.78 |
| Harkin | 1 | 1 | 1 | 1 | 0 | 0.04 | 0.46 |
| Hatch | 1 | 1 | 1 | 1 | 1 | 1.33 | 0.75 |
| Hatfield | 1 | 1 | 1 | 1 | 1 | 1.49 | 0.66 |
| Heflin | 0 | 0 | 0 | 0 | 0 | −1.42 | 1.41 |
| Helms | 0 | 0 | 0 | 0 | 0 | −1.96 | 1.68 |
| Hollings | 0 | 0 | 0 | 0 | 0 | −1.26 | 1.43 |
| Hutchison | 1 | 1 | 1 | 1 | 1 | 1.65 | 0.60 |
| Inhofe | NA | NA | NA | 0 | 0 | −1.50 | 1.04 |

**Table 2** Votes and expected trade ideal points, 1993–94 Senate *(Continued)*

| *Senator* | *1* | *2* | *3* | *4* | *5* | $\hat{\theta}$ | *SE* |
|---|---|---|---|---|---|---|---|
| Inouye | 0 | 0 | 1 | 1 | 0 | −0.22 | 0.53 |
| Jeffords | 1 | 1 | 0 | 0 | 0 | −0.97 | 0.59 |
| Johnston | 1 | 1 | 1 | 1 | 1 | 1.30 | 0.79 |
| Kassebaum | 1 | 1 | 1 | 1 | 1 | 1.29 | 0.69 |
| Kempthorne | 0 | 0 | 1 | 0 | 0 | −0.90 | 0.95 |
| Kennedy | 1 | 1 | 1 | 1 | 1 | 1.57 | 0.81 |
| Kerrey | 1 | 1 | 1 | 1 | 1 | 1.15 | 0.74 |
| Kerry | 1 | 1 | 1 | 1 | 1 | 1.58 | 1.01 |
| Kohl | 0 | 1 | 1 | 1 | 1 | 0.45 | 0.51 |
| Lautenberg | 0 | 1 | 1 | 1 | 1 | 1.12 | 0.27 |
| Leahy | 1 | 1 | 1 | 0 | 0 | −0.34 | 0.47 |
| Levin | 0 | 0 | 1 | 1 | 1 | 0.19 | 0.46 |
| Lieberman | 1 | 1 | 1 | 1 | 1 | 2.64 | 0.87 |
| Lott | 1 | 0 | 1 | 1 | 1 | 0.66 | 0.47 |
| Lugar | 1 | 1 | 1 | 1 | 1 | 3.51 | 0.35 |
| Mach | 1 | 1 | 1 | 1 | 1 | 0.70 | 0.76 |
| Mathews | 1 | 1 | 1 | 1 | 1 | 1.05 | 0.76 |
| McCain | 1 | 1 | 1 | 1 | 1 | 0.99 | 0.67 |
| McConnell | 1 | 1 | 1 | 1 | 1 | 1.22 | 0.86 |
| Metzenbaum | 0 | 0 | 0 | 0 | 0 | −1.61 | 1.99 |
| Mikulski | 0 | 1 | 1 | 1 | 1 | 0.92 | 0.38 |
| Mitchell | 1 | 1 | 1 | 1 | 1 | 0.92 | 1.09 |
| Moseley | 1 | 1 | 1 | 1 | 1 | 1.09 | 0.75 |
| Moynihan | 0 | 1 | 1 | 1 | 1 | 1.45 | 0.13 |
| Murkowski | 1 | 0 | NA | 1 | 0 | 0.29 | 0.44 |
| Murray | 1 | 1 | 1 | 1 | 1 | 0.48 | 0.73 |
| Nickles | 1 | 1 | 1 | 1 | 1 | 1.73 | 0.70 |
| Nunn | 1 | 1 | 1 | 1 | 1 | 2.20 | 0.69 |
| Packwood | 1 | 1 | 1 | 1 | 1 | 1.69 | 0.65 |
| Pell | 1 | 1 | 1 | 1 | 1 | 0.89 | 0.90 |
| Pressler | 1 | 0 | 1 | 1 | 0 | −0.44 | 0.40 |
| Pryor | 1 | 1 | NA | 1 | 1 | 0.91 | 0.73 |
| Reid | 0 | 1 | 1 | 0 | 0 | −1.31 | 0.32 |
| Riegle | 0 | 0 | 1 | 1 | 1 | 0.37 | 0.40 |
| Robb | 1 | 1 | 1 | 1 | 1 | 2.33 | 0.86 |
| Rockefeller | 0 | 1 | 1 | 1 | 1 | 0.23 | 0.46 |
| Roth | 1 | 1 | 1 | 1 | 1 | 1.56 | 0.86 |
| Sarbanes | 0 | 1 | 1 | 1 | 1 | 0.77 | 0.43 |
| Sasser | 0 | 1 | 1 | 1 | 1 | 0.44 | 0.43 |
| Shelby | 0 | 0 | 0 | 0 | 0 | −1.36 | 1.41 |
| Simon | 1 | 1 | 1 | 1 | 1 | 1.10 | 0.71 |
| Simpson | 1 | 1 | 1 | 1 | 1 | 1.16 | 0.76 |
| Smith | 0 | 0 | 1 | 0 | 0 | −0.82 | 0.86 |
| Specter | 1 | 0 | NA | 1 | 1 | 1.79 | 0.18 |
| Stevens | 0 | 0 | 1 | 0 | 0 | −1.04 | 1.02 |
| Thurmond | 0 | 1 | 0 | 0 | 0 | −1.43 | 0.98 |
| Wallop | 1 | 1 | 1 | 1 | 0 | 0.78 | 0.54 |
| Warner | 1 | 0 | 1 | 1 | 1 | 1.22 | 0.27 |
| Wellstone | 0 | 0 | 0 | 0 | 0 | −2.25 | 1.56 |
| Wofford | 0 | 1 | 0 | 1 | 1 | −0.34 | 0.43 |

**Table 3** Senate trade voting, 1993–1994: Effect parameters[a]

| | *Coefficient* | t *statistic* |
|---|---|---|
| Free trade interests | | |
| Skill levels | 0.56 | 2.85 |
| Exports | 0.15 | 0.58 |
| Multinational contributions | 0.59 | 3.01 |
| Protectionist interests | | |
| Change in import penetration | −0.10 | −0.38 |
| Unionization | 1.12 | 3.53 |
| Democrat ∗ Unionization | −0.98 | −2.56 |
| Labor contributions | −0.43 | −2.39 |
| Perot vote | −0.14 | −0.68 |
| Control variables | | |
| Democrat | 0.36 | 0.92 |
| Constant | 0.46 | 1.26 |
| Number of senators | 101 | |
| Percentage correctly predicted | 87% | |
| Proportional reduction in error | 55% | |

[a]Continuous variables are standardized.

In addition, the senators from Alaska provide an (almost) perfect example of how conditioning on both votes and ideal points is important. Both have identical district and personal covariates, yet their estimated ideal points differ due to their different voting patterns. Murkowski voted for free trade two of four times (and skipped the easiest free trade vote, GATT Fast Track), for an estimate of 0.29; Stevens voted for free trade only on GATT Fast Track, for an estimate of −1.04.

The estimated ideal points are also quite different from the typical one-dimensional summary ideal points. The trade ideal points correlate at only 0.10 with Poole and Rosenthal's first dimension, −0.36 with Poole and Rosenthal's second dimension, and 0.20 with Chamber of Commerce ratings. Poole and Rosenthal's first dimension and Chamber of Commerce scores, in contrast, correlate at 0.93.

Tables 3 and 4 report the effect and vote parameters. All continuous variables are standardized; their coefficients indicate the standard deviation change in trade ideal points

**Table 4** Senate trade voting, 1993–1994: Vote parameters

| | *Coefficient* | t *statistic* |
|---|---|---|
| Discrimination parameters | | |
| NAFTA | 1.00 | — |
| NAFTA side agreements | 0.94 | 6.03 |
| GATT Fast Track | 1.26 | 2.97 |
| GATT 1994 | 1.66 | 1.33 |
| GATT budget waiver | 1.58 | 1.53 |
| Cutpoint parameters | | |
| NAFTA | 0.00 | — |
| NAFTA side agreements | −0.85 | −1.66 |
| GATT Fast Track | −1.30 | −2.45 |
| GATT 1994 | −0.72 | −1.28 |
| GATT budget waiver | −0.28 | −0.59 |

associated with a one-standard deviation increase in the independent variable. Several covariates have a significant and substantial effect on ideal points. For example, a 1-standard deviation increase in skill levels is associated with a 0.56-standard deviation increase in trade ideal points.

The vote parameters can be of interest as well. The discrimination parameters (the $\alpha$'s) indicate that the NAFTA side agreement vote had more noise in the voting than the other votes, a fairly sensible result, as this vote had a lot to do with side agreements associated with factors not always lumped with trade. Based on this result, some may choose to focus the analysis on the other votes. In addition, the cutpoint parameters indicate the dividing lines between the alternatives on each vote, a useful value when one wants to simulate what votes would have been in various counterfactual situations.

## 7 Conclusion

Political scientists have become increasingly aware of the importance—and difficulty—of measuring ideology and its determinants. For broad measures of ideology, Poole and Rosenthal (1997), Heckman and Snyder (1997), and Groseclose et al. (1999) offer a healthy menu of options. For the common case in which a scholar has only a limited number of votes, however, this paper fills a hole in the literature. This conclusion briefly summarizes the random-effects approach, discusses its robustness, and explores some of the areas where it could be fruitfully applied.

A main advantage of the random-effects method is that it avoids the incidental parameter problem. By making the reasonable assumption that ideal points are random functions of district and legislator characteristics—the *exact* assumption underlying probit models—scholars can avoid the parameter proliferation that haunts conventional multivote analysis.

With the incidental parameter problem out of the way, scholars can use much more of the available information. First, concerns about correlations across individual votes need no longer trap researchers into focusing on single votes. Instead, the random-effects model lets them use multiple votes, even when there are few of them. Second, they can use actual voting patterns in generating ideal points, avoiding situations in which conditioning only on covariates yields poor predictions of observed votes. In addition, unlike some options, the method can handle problems such as missing data and large numbers of legislators who always vote for the same "side" (e.g., always vote for free trade). The method also produces estimates of uncertainty associated with ideal points and other parameters, something often missing in the ideal point estimation literature.

As with all statistical methods, there are assumptions that may not hold in reality. First, the method assumes that policy is unidimensional. Therefore, it is best suited for cases in which there are a small number of votes selected for their concern with a specific issue. Second, there may be a correlation across ideal points and voting beyond that which is captured with covariates. No method will be totally immune from such critiques, especially one based on a relatively small number of votes. If this is a major concern, one will probably need more votes and a different method. Finally, specification error when modeling the random effects may introduce biases into the estimated parameters.

There are many endeavors where this specific approach, or the general random-effects approach, can be extremely useful. First, random-effects models can improve existing research. Countless articles are based on 5- or 10-vote index scores or vote-by-vote analysis. For theoretical and practical reasons, we may find that such efforts could be made more accurate and efficient by applying a random-effects ideal point estimation model. This is particularly true when one wants to estimate the effect of covariates on preferences; instead

of using ideal point estimates as dependent variables, the method here allows us to estimate directly the effect of constituency, party and personal variables on preferences.

Second, the model can be useful beyond the traditional confines of legislative politics. Consider two classes of research designs involving dichotomous data. In one case, there are data on many individuals, but only a few observations per individual. Here, direct application of the method in this paper generates ideal points, choice and covariate parameters. This is true whether the concern is with legislators voting on roll calls, voters voting on candidates and referenda, survey respondents answering questions, or states choosing institutional reform. Lewis (1998) has applied a similar method to estimate the distribution of voter ideal points based on almost 1 million ballots in Los Angeles County in 1992. In the other case, there are few individuals and many observations per individual. Here, further assumptions and modifications are required (see Londregan 2000; Bailey and Chang 2000), but the random-effects approach is again very useful. It allows researchers to integrate out nuisance parameters and generate parameters of interest, be they ideal points of political parties, Supreme Court justices, Federal Open Market Committee members, or nations in the European Union. By improving estimation in these cases, the random-effects method broadens the political science toolkit and enables scholars to test more effectively questions of substantive importance.

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