Optimal Travel Restrictions in Epidemics

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Introduction

- Population flows have been shown to strongly predict the spread of COVID-19 across space (e.g., Fang et al. (2020), Jia et al. (2020), Qiu et al. (2020), Wu et al. (2020)).
- With the advent of mobile technology, travel histories of individuals have become easily accessible, aggressive contact tracing, targeted screening and timely quarantine can be applied to people with certain travel histories.
- While studies have shown that city lockdowns have contributed to significant reductions in the number of infections in China (Tian et al. (2020)), travel restrictions that are imposed on routes with specific origins or destinations could be less restrictive and potentially be more cost-effective than lockdowns of the entire cities.

Introduction

- In this paper, we explore the possibility that travel restrictions are imposed on specific routes and examine the optimal designs of such policies.
- Besides incurring lower social and economic costs, route-specific travel restrictions can still be feasible when lockdowns are not.

Outline

We embed a parsimonious model of the spatial spread of the disease in an optimization problem that takes into account the costs from disease infections and the travel restriction measures.

- The optimization problem is solved from the perspective of a social planner that can dictate the levels of restrictions on the population flows between each pair of cities.
- We also solve the optimization problem under the constraint that each city acts independently to maximize its own payoff while taking the actions of other cities as given, i.e., a simultaneous move Nash equilibrium outcome.
- Due to the externality of policy effect, uncoordinated travel restrictions can be suboptimal and we illustrate this in an empirical application.

Related literature

Optimal policies in response to COVID-19:

- Holtz er al. (2020): coordinated and uncoordinated shelter-in-place orders
- Fajgelbaum et al. (2020): endogenize the costs of lockdowns by modeling individuals' commuting decisions, which provide labor supply to firms, generate incomes, and also spread diseases.
- Optimal control with epidemiology models (SIR models): lockdowns of certain sections of the population (Acemoglu et al. (2020), Alfaro et al. (2020), Alvarez et al. (2020)), testing and quarantine (Berger et al. (2020))

Policies on networks:

- Ballester et al. (2006) characterize the node in a network whose removal has the largest impact on the aggregate outcome. We consider the marginal effect of varying the strength of a directed link between two nodes on the individual and aggregate outcomes, and show that this can be characterized as a product between the Bonacich centralities of the origin and the destination.
- Our results can be extended to spatial interaction models where agents take the network as given (e.g, Lee (2007)) and a planner can alter the strength of network links.

Spatial spread of the disease

Our model is built upon a parsimonious description of how infectious diseases spread through space.
Let y_i denote the cumulative number of COVID-19 cases in city i in logs. We assume the following model of disease transmissions within and across cities,

$$y_i = \left(\sum_{j=1}^{n} w_{ij} y_j \right) \lambda + t_i \gamma + x_i' \beta + u_i.$$

$w_{ij} \geq 0$, $i \neq j$ is a measure of the intensity of population flows from city j to i.
$w_{ii} = 0$. $t_i \geq 0$ is a measure of the population movement within city i. x_i is a vector of control variables. u_i is the error term.

Assuming that the matrix $I - \lambda W$ is invertible, we have an equilibrium system of $\{y_i\}_{i=1}^n$ for given $\eta = t\gamma + x\beta + u$. The reduced form is

$$y_i = \ell_i'(I - \lambda W)^{-1}\eta,$$

where ℓ_i an $n \times 1$ vector with i-th entry 1 and all other entries 0. The effect of varying $w_{j_0 j_1}$ on the equilibrium y_i, while keeping other network links fixed, is given by

$$\frac{\partial y_i}{\partial w_{j_0 j_1}} = \lambda \ell_i' (I - \lambda W)^{-1} \ell_{j_0} \ell_{j_1}' (I - \lambda W)^{-1} \eta.$$

The marginal effect of varying the intensity of the network link $w_{j_0 j_1}$ on the aggregate equilibrium outcome, $\sum_i y_i$, is

$$\frac{\partial \sum_i y_i}{\partial w_{j_0 j_1}} = \lambda \mathbf{1}' (I - \lambda W)^{-1} \ell_{j_0} \ell_{j_1}' (I - \lambda W)^{-1} \eta,$$

where $\mathbf{1}$ is a vector of ones.

Connections with Bonacich centralities

In a model of network interactions, Ballester et al. (2006) show that individual outcomes are proportional to their Bonacich centralities and that the marginal contribution of a unit to the aggregate outcome is given by its own Bonacich centrality and its contribution to the Bonacich centralities of other units.

- Lockdown of an entire city can be viewed as removing all network links that originate or point to the city.
- In our case, the policy targets the values of directed network links, for example, through ex ante restriction of route-specific transportation or ex post contact tracing and quarantine measures contingent on specific travel histories.

$$\frac{\partial \sum_i y_i}{\partial w_{j_0 j_1}} = \lambda \underbrace{\ell'_{j_0} (I - \lambda W)^{-1} \mathbf{1}}_{\text{Bonacich centrality of the destination}} \underbrace{\ell'_{j_1} (I - \lambda W)^{-1} \eta}_{\approx \text{Bonacich centrality of the origin}} .$$

- Intuitively, the intensity of population flows between two cities has a stronger effect on the aggregate outcome if the origin city has a higher infection risk or the destination city can affect many other cities.

Estimates of parameters

We estimate the model parameters using data on confirmed COVID-19 cases and the intensities of between and within city population flows in China.

$$y_i = \left(\sum_{j=1}^{n} w_{ij} y_j \right) \lambda + t_i \gamma + x_i' \beta + u_i.$$

We consider two empirical specifications:

	Model A	Model B
y_i	$log(1 + \#$ of confirmed COVID-19 cases by February 29)	
z_i	local weather variables that affect infection rates	
w_{ij}, t_i	average pop. flows, Jan 1-Feb 29	average pop. flows, Jan 1-Jan 22
$\tilde{w}_{ij}$, $\tilde{t}_i$	average pop. flows, same lunar calendar days in 2019 as Jan 1-Feb 29, 2020	
Endogenous	w_{ij}, t_i, $\sum_{j=1}^{n} w_{ij} y_j$	$\sum_{j=1}^{n} w_{ij} y_j$
IV	$\tilde{t}_i$, $\sum_{j=1}^{n} \tilde{w}_{ij} z_j$	$\sum_{j=1}^{n} w_{ij} z_j$

Estimates of parameters: Model A

	(1) OLS	(2) IV	(3) OLS	(4) IV
λ	0.252***	0.258***	0.240***	0.245***
	(0.0406)	(0.0269)	(0.0373)	(0.0329)
	[0.277]	[0.284]	[0.264]	[0.270]
γ	-0.365	0.294	-0.387**	-0.144
	(0.365)	(0.209)	(0.143)	(0.136)
Precipitation	-0.987***	-0.614***	-0.249***	-0.237***
	(0.253)	(0.207)	(0.0731)	(0.0660)
Precipitation $\times$ wind speed	0.329***	0.245***	0.0594*	0.0547*
	(0.0709)	(0.0702)	(0.0346)	(0.0322)
Bad weather	3.434***	2.959**	0.562	0.619
	(1.131)	(1.175)	(0.592)	(0.605)
Observations	360	360	360	360
Province FE	NO	NO	YES	YES

Elasticity of infection spillovers per 100000 daily population movements are reported in brackets. Standard errors in parentheses are clustered by provinces. *** p<0.01,** p<0.05, * p<0.1.

Estimates of parameters: Model B

	(1) OLS	(2) IV	(3) OLS	(4) IV
λ	0.179***	0.163***	0.170***	0.170***
	(0.0165)	(0.0126)	(0.0136)	(0.0125)
	[0.197]	[0.179]	[0.187]	[0.187]
γ	0.288*	0.294**	0.131	0.131
	(0.148)	(0.141)	(0.164)	(0.153)
Precipitation	-0.606***	-0.638***	-0.229***	-0.228***
	(0.140)	(0.152)	(0.0731)	(0.0685)
Precipitation $\times$ wind speed	0.234***	0.242***	0.0513	0.0513
	(0.0568)	(0.0610)	(0.0360)	(0.0337)
Bad weather	2.387**	2.531**	0.439	0.436
	(0.973)	(1.052)	(0.752)	(0.704)
Observations	360	360	360	360
Province FE	NO	NO	YES	YES

Elasticity of infection spillovers per 100000 daily population movements are reported in brackets. Standard errors in parentheses are clustered by provinces. *** p<0.01,** p<0.05, * p<0.1.

Social planner's problem

The planner's optimization problem is to minimize the cost from the COVID-19 infections and the travel restrictions imposed,

$$\min_{0\leq\phi_{ij}\leq 1, 1\leq i\leq n, 1\leq j\leq n, i\neq j} \sum_i g_i y_i + \sum_{i,j,i\neq j} (\phi_{ij}\varphi_1 + \frac{1}{2}\phi_{ij}^2\varphi_2) w_{ij},$$

subject to

$$y_i = \left(\sum_{j=1}^{n} \phi_{ij} w_{ij} y_j\right) \lambda + \eta_i, \qquad \text{for all } i.$$

Restrictions on between city travels can be imposed so that w_{ij} is reduced to $\phi_{ij} w_{ij}$ with $0 \leq \phi_{ij} \leq 1$. The marginal cost of such restrictions is assumed to be increasing with the amount of travel that could have happened without the restrictions but did not happen with the restrictions imposed. The cost of the restriction on route w_{ij} is modeled by $\left(\phi_{ij}\varphi_1 + \frac{1}{2}\phi_{ij}^2\varphi_2\right) w_{ij}$, with $\varphi_1 < 0$ and $\varphi_2 > 0$. Summarize travel restrictions by matrix Φ, $(\Phi)_{ij} = \phi_{ij}$ and the diagonal elements of Φ are restricted to zeros. The social planner applies weight g_i to city i and $g = \begin{pmatrix} g_1 & \cdots & g_n \end{pmatrix}'$.

Social planner's problem

- The planner's problem is a static one-period model. We have assumed that the planner's information set is $\sigma(W, \{\eta_i\}_{i=1}^n)$. In this parsimonious model, spatial distribution of infections is determined by the initial viral spread (η) and the population flows between cities which are the targets of travel restrictions.
- In reality, the planner can adjust its decisions as new information become available with the passage of time, and the future evolution of the variables may depend on the decisions made in the past.
  - Jeong and Lee (2020): forward looking agents interact with others over time on a fixed network.

Our model can accommodate uncertainties in the initial viral spread. Suppose that W and the distribution of η are known to the decision maker. The planner's problem is

$$\min_{0\leq\phi_{ij}\leq 1, 1\leq i\leq n, 1\leq j\leq n, i\neq j} \mathbb{E}\sum_i g_i y_i + \sum_{i,j,i\neq j}(\phi_{ij}\varphi_1 + \frac{1}{2}\phi_{ij}^2\varphi_2)w_{ij}.$$

Notice that $\sum_i g_i y_i$ is linear in η, $\sum_i g_i y_i = g'(I - \lambda\Phi \odot W)^{-1}\eta$. Let $\bar{\eta} = \mathbb{E}\eta$. We can rewrite the above equation as

$$\min_{0\leq\phi_{ij}\leq 1, 1\leq i\leq n, 1\leq j\leq n, i\neq j} \sum_i g_i \bar{y}_i + \sum_{i,j,i\neq j}(\phi_{ij}\varphi_1 + \frac{1}{2}\phi_{ij}^2\varphi_2)w_{ij},$$

subject to

$$\bar{y}_i = \left(\sum_{j=1}^n \phi_{ij} w_{ij} \bar{y}_j\right)\lambda + \bar{\eta}_i, \qquad \text{for all } i.$$

Optimal solutions

By the extreme value theorem, a continuous function attains a minimum on a compact set, and therefore the optimization problem always has a solution. The first order condition with respect to ϕ_{ij} for $w_{ij} \neq 0$ and $i \neq j$ is

$$\lambda w_{ij} g' \left(I - \lambda \Phi \odot W\right)^{-1} \ell_i \ell_j' \left(I - \lambda \Phi \odot W\right)^{-1} \eta + (\varphi_1 + \phi_{ij}\varphi_2) w_{ij} = 0$$

for interior $0 < \phi_{ij} < 1$. For the boundary solutions,

$$\lambda w_{ij} g' \left(I - \lambda \Phi \odot W\right)^{-1} \ell_i \ell_j' \left(I - \lambda \Phi \odot W\right)^{-1} \eta + (\varphi_1 + \varphi_2) w_{ij} \leq 0, \qquad \text{if } \phi_{ij} = 1,$$
$$\lambda w_{ij} g' \left(I - \lambda \Phi \odot W\right)^{-1} \ell_i \ell_j' \left(I - \lambda \Phi \odot W\right)^{-1} \eta + \varphi_1 w_{ij} \geq 0, \qquad \text{if } \phi_{ij} = 0.$$

Optimal solutions

Let Γ be a function from an $n \times n$ matrix to an $n \times n$ matrix. Define the following functions,

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases},$$

$$(\Gamma(\Phi))_{ij} = \begin{cases} f\left(-\varphi_2^{-1}\left(\lambda g'\left(I - \lambda\Phi \odot W\right)^{-1} \ell_i \ell_j' \left(I - \lambda\Phi \odot W\right)^{-1} \eta + \varphi_1\right)\right) \\ \qquad \text{if } i \neq j,\ w_{ij} \neq 0 \\ 0 \qquad \text{otherwise} \end{cases}.$$

- The first order necessary conditions above can be summarized by $\Gamma(\Phi) = \Phi$ for $\Phi \in [0,1]^{n \times n}$.
- Assuming $\lambda > 0$, the travel from city j to i tends to be more severely restricted (i.e., smaller ϕ_{ij}) if the infection risk is high for travelers from city j, or city i is influencing many other cities.

Φ contains $n^2 - n$ unknown parameters and directly solving for them can be infeasible. However, there are restrictions on Φ which can greatly reduce the number of free parameters, as we can write

$$\Phi = \Gamma(\Phi) = \Lambda\left(-\varphi_2^{-1}\lambda\left(I - \lambda(\Phi \odot W)'\right)^{-1} g\eta'\left(I - \lambda(\Phi \odot W)'\right)^{-1} - \frac{\varphi_1}{\varphi_2}\mathbf{1}_{n\times n}\right),$$

where $\Lambda()$ is a function from $\mathbb{R}^{n\times n} \to [0,1]^{n\times n}$ with $(\Lambda(M))_{ij} = f(M_{ij})$ if $i \neq j, w_{ij} \neq 0$ and $(\Lambda(M))_{ij} = 0$ otherwise. Because the rank of matrix $\left(I - \lambda(\Phi \odot W)'\right)^{-1} g\eta'\left(I - \lambda(\Phi \odot W)'\right)^{-1}$ is only 1, we can parameterize Φ as

$$\Phi = \Lambda\left(ab' - \frac{\varphi_1}{\varphi_2}\mathbf{1}_{n\times n}\right),$$

with unknown parameters $a = \begin{pmatrix}1 & a_2 & \cdots & a_n\end{pmatrix}'$ and $b = \begin{pmatrix}b_1 & b_2 & \cdots & b_n\end{pmatrix}'$. The number of unknown parameters is now $2n-1$, which can be much smaller than $n^2 - n$.

Coordination between cities

The presence of policy effect externality calls for coordination between local jurisdictions in the adoption of travel restrictions, and aggregate social welfare could be lower if local decisions do not account for their spillover effects.

- Holtz et al. (2020), on shelter-in-place orders

Travel restrictions without coordination

Suppose that each city can choose the level of population inflows to the city. The optimization problem of city i is to choose $\{\phi_{ij}\}_{j=1,j\neq i}^{n}$ while taking the choices of other cities as given,

$$\min_{0\leq\phi_{ij}\leq 1, 1\leq j\leq n, j\neq i} g_i y_i + \sum_{j,i\neq j}(\phi_{ij}\varphi_1 + \frac{1}{2}\phi_{ij}^2\varphi_2)w_{ij},$$

subject to

$$y_i = \left(\sum_{j=1}^{n}\phi_{ij}w_{ij}y_j\right)\lambda + \eta_i.$$

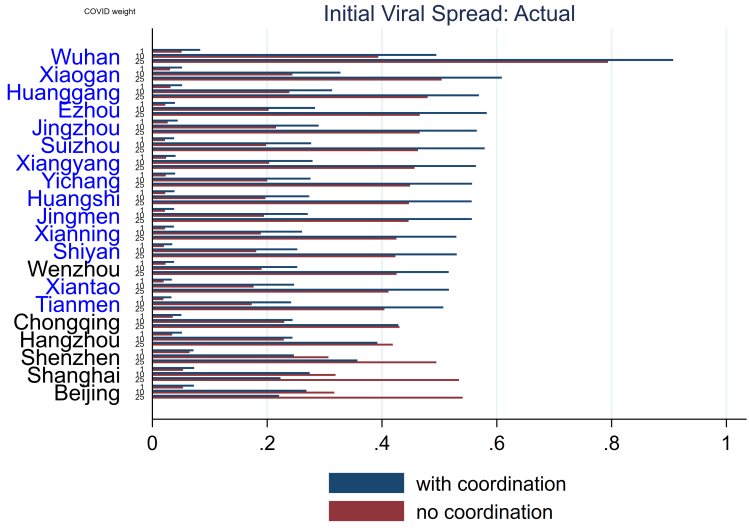
- The resulting set of policies is a complete information Nash equilibrium.
- In theory, the set of travel restrictions with coordination may not be more stringent than the restrictions without coordination on all routes. With coordination, it may be optimal that with certain routes strictly controlled, travels on other routes can be restricted less.

Empirical illustration

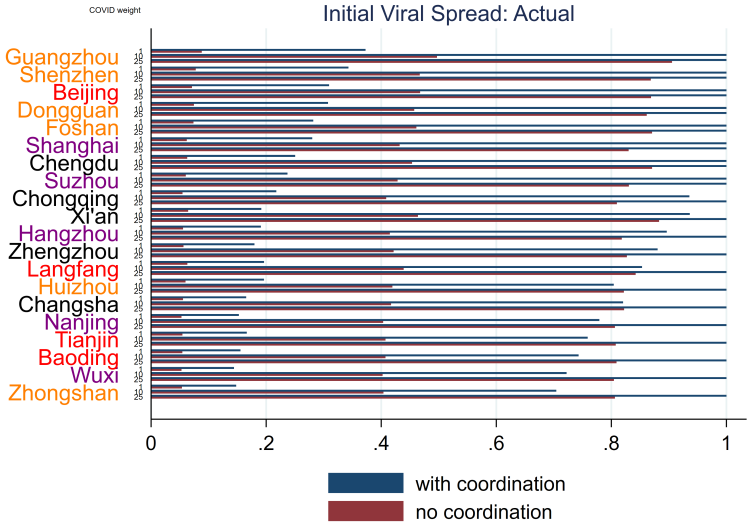
To illustrate how the set of optimal travel restrictions relate to the most influential travel routes and the potential loss from the lack of coordination between cities, we numerically solve the optimization problems. The following are assumptions:

- Counterfactual population flow patterns without any government intervention: the averages of intercity population flows in 2019 corresponding to the same lunar calendar days between January 1, 2020 and February 29, 2020.
- The costs of travel restrictions are parameterized by $\varphi_1 = -1$ and $\varphi_2 = 1$.
  - For the $j \to i$ route, the marginal cost of new travel restrictions starting at no restrictions is zero, and the marginal cost is increasing with the levels of restrictions imposed, until it reaches w_{ij} at the maximum level of restrictions.
- The coefficient of the spatial spillover effect (λ) is 0.258.
- The relative costs of COVID-19 infections and travel restrictions are measured by the vector g, and we consider $g = \tilde{g}\mathbf{1}$ with $\tilde{g} \in \{1, 10, 25\}$.

Average optimal levels of outbound restrictions



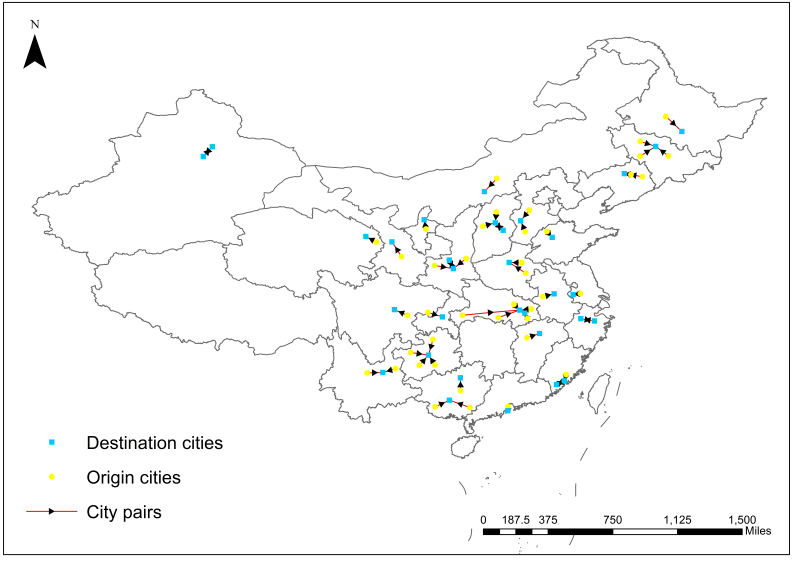
Optimal intensities of outbound restrictions from Wuhan



With coordination between cities that takes into account the external effects of local policies, the optimal set of travel restrictions tends to place greater restrictions on routes whose destinations have large population outflows, and less restrictions on routes that have a lower likelihood of generating large spillovers, which is confirmed in the following figure that compares the actual population flow patterns in 2020 after government interventions with those that are predicted to be optimal according to the model.

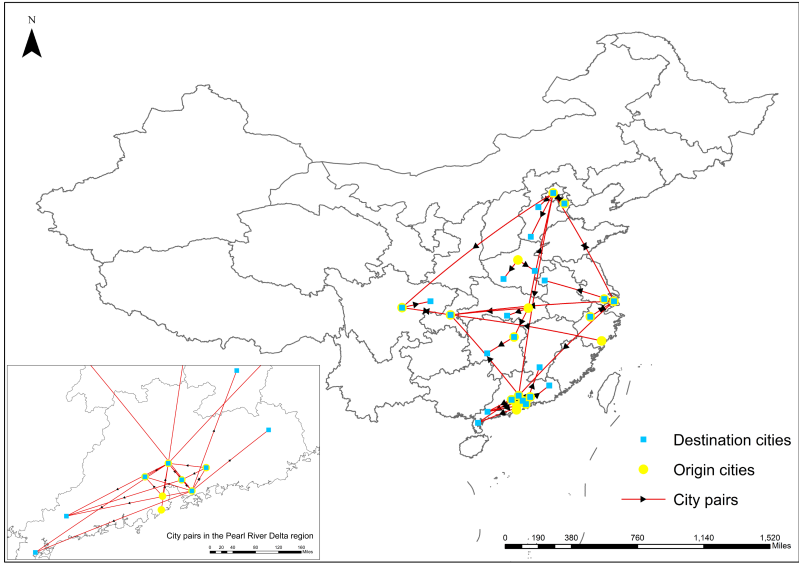
Actual and optimal levels of intercity population flows

50 routes with most negative differences



Actual and optimal levels of intercity population flows

50 routes with most positive differences



Conclusions and discussion

- The marginal effect of decreasing intercity population flows in reducing total infections is not homogeneous and depends on the positions of the origin and destination cities in the network of population flows.
  - The marginal benefit of population flow restrictions is larger if the origin city is closely connected to areas with severe infections, or the infections in the destination city can spill over to many other cities.
  - Population flow restrictions that target these links could be more cost effective.
- We apply the theoretical model to the data on the spread of COVID-19 in China.
  - When cities can coordinate their policies, faced with an outbreak of infectious diseases in one city, travels out of the city, and those whose destinations are cities with large population outflows, will be restricted the most according to the optimal solution of travel restriction policies.
  - Without coordination, the set of travel restriction policies are suboptimal.
- The modeling framework of this paper could also be applied to optimize the public health measures in response to other infectious diseases where population flows significantly contribute to the spread.