Appendix

Lecture on Conditional Probabilities and expected value (Study 2 only)

In Study 2, in order to enhance participants' understanding of probabilities and expected value, we showed them a lecture video (about 20 minutes) on probability and expected value calculations before the task began. The following are the lecture content and the slides used in the lecture.

Lecture on Conditional probabilities

1. Problem Setup and Components

This analysis investigates the probabilistic structure of a lottery draw defined by the following components. The lottery bag contains one white box (W) and one black box (B).

- White Box Composition (W): 2 red balls and 1 blue ball (Total 3 balls).
- Black Box Composition (B): 1 red ball and 1 blue ball (Total 2 balls).

The player first randomly selects a box from the bag, and subsequently, draws one ball randomly from the chosen box. The boxes and balls are identical in shape and size, making selection rely solely on touch, and the choice of box cannot be changed mid-draw. Drawing a red ball constitutes a "Win" (A), and drawing a blue ball constitutes a "Loss".

2. Probability Calculation

2.1. Basic Selection Probability and Conditional Probability

Since the box selection is random, the probability of choosing the white box, $P(W)$, is 1/2, and the probability of choosing the black box, $P(B)$, is also 1/2.

The conditional probability of winning given that the white box was selected, $P(A \mid W)$, is 2/3, based on the ball composition inside the white box. Similarly, the conditional probability of winning given the black box was selected, $P(A \mid B)$, is 1/2.

2.2. Joint Probability

The joint probability of selecting the white box and winning, $P(W \cap A)$, is calculated as the product of the basic selection probability and the conditional probability.

$P(W \cap A) = P(W) \times P(A|W) = 1/2 \times 2/3 = 1/3$

Similarly, the joint probability of selecting the black box and winning, $P(B \cap A)$, is:

$P(B \cap A) = P(B) \times P(A|B) = 1/2 \times 1/2 = 1/4$

2.3. Total Probability

The overall probability of winning, $P(A)$, is the sum of the mutually exclusive events of winning via the white box and winning via the black box.

$P(A) = P(W \cap A) + P(B \cap A) = 1/3 + 1/4 = 7/12$

2.4. Derivation of Conditional Probability using Bayes' Theorem

Given the information that a player has drawn a winning ball (Event A), the probability that this winning ball originated from the white box, $P(W \mid A)$, is calculated as the ratio where the total probability of winning $P(A)$ is the denominator, and the joint probability of winning from the white box $P(W \cap A)$ is the numerator.

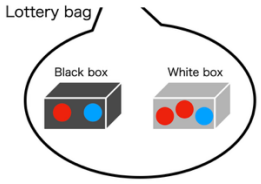
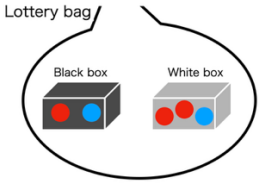
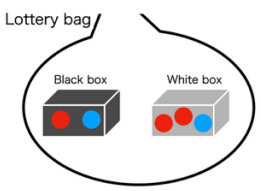
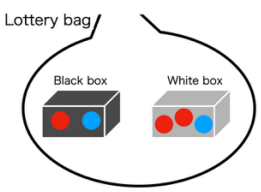
$P(W|A) = P(W \cap A) / P(A) = (1/3) / (7/12)$

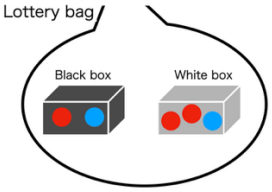
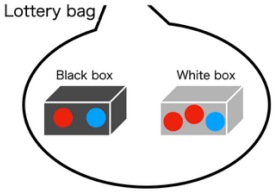
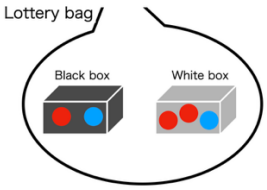
The calculation yields:

$P(W|A) = 1/3 \times 12/7 = 4/7$

Therefore, the probability that the winning ticket originated from the white box, given a win has occurred, is 4/7.

Slides

1	2
We will play a lottery game. Inside the lottery bag, there is one white box and one black box. Furthermore, inside the white box are 2 red balls and 1 blue ball, and inside the black box are 1 red ball and 1 blue ball. Both the boxes and the balls are identical in shape and size, and cannot be distinguished by touch alone. After thoroughly mixing the boxes within the bag and the balls within each box, reach into the bag without looking and first select one box. Then, from the selected box, reach in without looking and choose one ball (lottery). Note: You cannot change the box you initially selected midway through. If the ball you draw is red, you win; if it's blue, you lose. Answer the following questions about this lottery game. 	Question 1: In this lottery game, what is the probability that the person drawing a lot will choose the white box? 
3	**4**
Question 2: In this lottery game, suppose the person drawing a lot has chosen the white box. In this case, what is the probability that this person will win? (Red is a win) 	Question 3: In this lottery game, what is the probability of drawing a winning lot from the white box? (Red is a win) 

5	6
Question 4: In this lottery game, what is the probability that the drawn lot (ball) was taken from the black box? (Red is a win) 	Question 5: In this lottery game, what is the probability of winning? (Red is a win) 
7	
Question 6: In this lottery game, suppose the person drawing a lot has drawn a winning lot. In this case, what is the probability that this winning lot was a ball taken from the white box? (Red is a win) 	

Lecture on expected value

1. Definition of Expected Value

This explanation uses a concrete roulette example to describe the Expected Value (E). Expected Value is defined as the average value of the amount of money a player would acquire if the trial (the roulette spin) were repeated many times (e.g., 10,000 times).

Mathematically, the Expected Value is calculated by multiplying the probability of obtaining a certain value by the amount of money obtained for that value, and then summing these results across all possible outcomes.

In the following examples, we assume the roulette wheel is unbiased, meaning the needle has an equal likelihood of stopping at any point on the circumference.

2. Case Study 1: Equal Probabilities

In the initial case, we assume the roulette wheel is equally divided into four sections: 100, 200, 300, and 400. The probability of winning each amount is thus 1/4.

Calculation
The Expected Value E_1 is calculated according to the formula:

$$\begin{aligned} E_1 &= (1/4 \times 100) + (1/4 \times 200) + (1/4 \times 300) + (1/4 \times 400) \\ &= 1/4 \times (100 + 200 + 300 + 400) \\ &= 1000/4 = 250 \end{aligned}$$

Result: 250.

Analysis: When all outcomes have equal probability, the Expected Value is equivalent to the simple arithmetic average of the possible amounts. This serves as the most straightforward case.

3. Case Study 2: Increased Probability for a Small Amount

Next, consider a scenario where the area corresponding to 100 is doubled, the 200 section is eliminated, and the 300 and 400 sections remain unchanged. In this case, the probability of obtaining 100 increases to 1/2 (2/4). The probabilities for 300 and 400 remain 1/4.

Calculation
The Expected Value E_2 is calculated as follows:

$$E_2 = (1/2 \times 100) + (1/4 \times 300) + (1/4 \times 400)$$

For easier calculation, 1/2 × 100 is rewritten as 1/4 × 200 to factor out 1/4.

$$\begin{aligned} E_2 &= 1/4 \times (200 + 300 + 400) \\ &= 900/4 = 225 \end{aligned}$$

Result: 225.

Analysis: Compared to Case 1 (250), the Expected Value decreased. This reduction is interpreted as a consequence of the probability of obtaining the smaller amount (100) increasing to 1/2, thereby pulling down the overall average value.

4. Case Study 3: Increased Probability for a Large Amount

Finally, consider a scenario where the area for 400 occupies 3/4 of the wheel, and the area for 100 occupies the remaining 1/4.

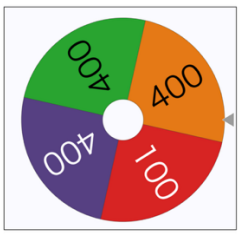
Calculation
The Expected Value E_3 is calculated as follows:

$$\begin{aligned} E_3 &= (1/4 \times 100) + (3/4 \times 400) \\ &= 1/4 \times (100 + 1200) \\ &= 1300/4 = 325 \end{aligned}$$

Result: 325.

Analysis: Compared to Case 1 (250), the Expected Value increased. This demonstrates that when the probability associated with a larger monetary amount (400) increases, the weight of that amount in the calculation grows, leading to a higher overall Expected Value of the winnings.

Slides

1	2
Question • Consider a lottery where you spin the roulette wheel shown in the figure below and receive the amount of money where the roulette stops. Assume that the roulette wheel is not biased. 	• What is the expected value of the amount you can win with this roulette wheel? Expected value is the average amount you would expect to win if you could spin the roulette wheel many times. Mathematically, the expected value is known to be calculated by summing [probability of winning × amount won].

3	4
• Now, let's calculate it. (It is known that the expected value can be calculated by summing [probability of winning × amount won].) 	• What is the expected value of the amount for the following roulette wheel?
5	
• What is the expected value of the amount for the following roulette wheel?