Measuring Collateral Posting: A Model and Empirical Examination

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ABSTRACT

This paper studies the economic impact and implications of collateral posting in financial markets. A model is presented for valuation with collateralization. The model characterizes a collateral process directly based on the foundations of collateral margin and bankruptcy laws, that allows us to decompose asset prices into economic factors or drivers. We extract information about credit risk as well as collateral risk and find empirical evidence that empirical evidence of collateral agreements in reducing economic capital, freeing up line of credit, and expanding range of counterparties.

Key words: collateral margin, collateral posting, collateralization, asset pricing, plumbing of the financial system, swap premium spread.

JEL Classification: E44, G21, G12, G24, G32, G33, G18, G28

1. Introduction

Collateral posting is one of the primary credit risk mitigation techniques. The effect of collateral margin is to substitute the credit risk of a transaction. In a collateral arrangement, the secured obligation is periodically marked to the market, and the collateral is adjusted to reflect the changes in value. The collateral pledged to secure a transaction can be claimed if a counterparty defaults on its payment obligations during the life of the transaction.

The counterparty exposures are significantly affected by the existence of collateral agreements, the type of derivatives and collateral assets, and the definition of collateral agreements.

Some collateralized products are instruments designed to allow investors to participate in the hedge fund market. After posting collateral, clients are entitled to the leveraged returns on a reference basket of hedge funds. At the same time, the investors agree to pay a fee that is based on a floating rate of interest applied to a notional amount and possibly a spread on the basket level.

There is a vast literature on collateralization. Luck and Santos (2023) study the valuation of collateral by comparing spreads on loans issued by the same bank at the same issuing date but backed by different types of collateral. They find that pledging collateral reduces borrowing costs.

Calomiris et al. (2017) find the creditors' ability for borrowing improves quite a bit by using movable assets as collateral rather than using immovable real estate. They also find that loan-to-values of loans collateralized with movable assets are lower in countries with weak collateral laws.

Donaldson et al. (2020) develop a model to collateral to protect creditors from the claims of other creditors. They find that paradoxically borrowers rely most on collateral when pledgeability is high. Creditors thus require collateral for protection against possible dilution by collateralized debt.

Demarzo (2019) studies how collateral mitigates conflicts of interest and enforcement frictions in lending. He finds that collateral is valuable as a low-cost commitment device and observed capital structure dynamics will exhibit hysteresis and depart significantly from standard predictions.

Cerqueiro et al. (2016) demonstrate that collateral plays an important role in debt, credit, and landing. They also study the bank's response to a legal reform that exogenously reduced collateral value by tightening credit limits and reducing the intensity of its monitoring of collateral.

Traditional derivatives valuation using no arbitrage pricing theory may not be suitable for these collateralized products for at least two reasons. First, they are implicitly assumed to be sufficiently deep in-the-money that a thorough, but unnecessarily complex analysis is not warranted. Second, the collateral and diligent oversight of the products ensure that it is highly improbable for a user to suffer large losses from these transactions.

This article presents a model based on the principle and legal structure of the Credit Support Annex (CSA). Given this model, we can explain credit-related spreads and provide an important tool for credit value adjustment (CVA).

First, we assess the impact of collateralization on valuation and check whether collateralization is sufficient to explain market spreads. We use a regression model where market spreads are used as the dependent variable and CDS premia as the explanatory variable. The empirical evidence shows that credit risk is main but not all driving forces on market spreads. But the joint effect of counterparty risk and collateralization have overwhelming explanatory power regarding the market spreads.

Second, we find empirically that collateral pledging can improve credit risk and reduce credit valuation adjustment (CVA) by comparing collateralized and non-collateralized portfolios. We also find collateralization can make some contracts, such as interest rate swaps risk-free, but may not be able to get rid of counterparty risk entirely for other contracts, such as credit defaults swaps.

Finally, we gauge the effect of collateralization on market risk and liquidity risk. Empirically we find that both market risk and liquidity risk decrease when a portfolio is collateralized.

The remainder of this paper is organized as follows: Section 2 presents a new model for pricing collateralized financial contracts. Section 3 discusses empirical evidence. The conclusions and discussion are provided in Section 4.

2. The Model

The posting of collateral is regulated by the CSA that specifies a variety of terms including the threshold, the independent amount, and the minimum transfer amount (MTA), etc. Margin Period of Risk (MPR) is defined as the time period from the last exchange of collateral covering a netting set of transactions with a defaulting counterpart until that counterpart is closed out and the resulting market risk is re-hedged.

The margin period of risk can be divided into two sub-periods, i.e., the period after the last collateral exchange and the liquidation period. The main risk in the first sub-period is under-collateral (when the Bank receiving) or over-collateral (when the Bank pledging). The main risk in the second sub-period is price divergence between portfolio and collateral during the liquidation.

The timeline of potential collateral calls around default is shown below. Here, s_c represents the settlement time between portfolio valuation and delivery of collateral required, and s_g the settlement grace period under the agreement.

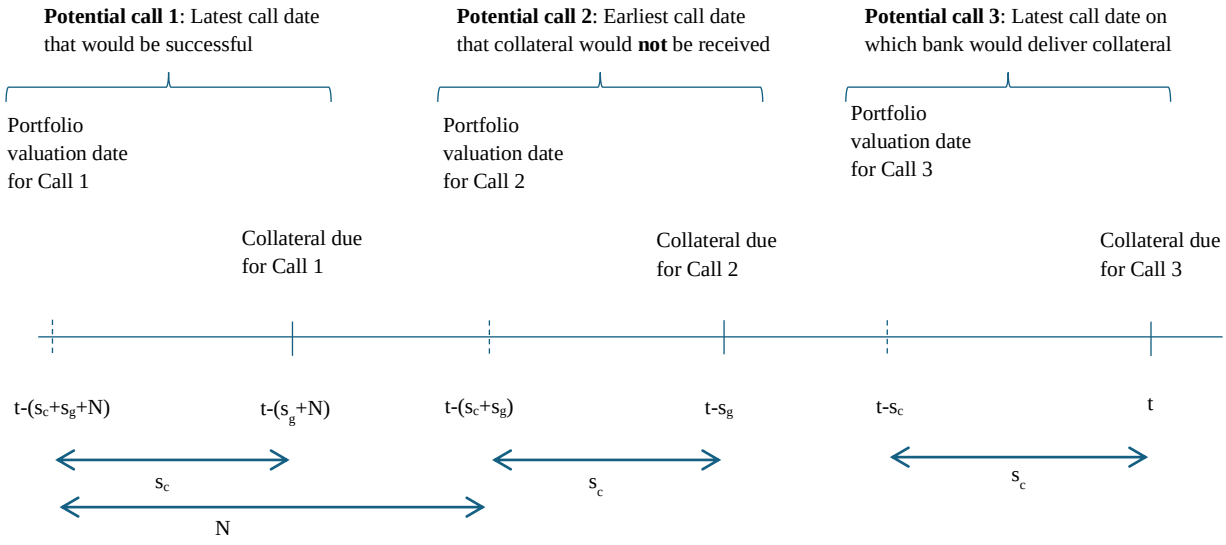


Figure 1. The timeline of potential collateral calls around default.

To be conservative, we need to derive the earliest possible date of the last successful exchange of collateral (call 1 in the diagram) as this is the date which results in the longest margin period of risk. To do this we first consider the earliest possible failed exchange of collateral (i.e. call 2 in the diagram), without causing a default prior to *t*.

Since collateral may be delivered up to s_g days late without triggering default, the collateral must have been due to be received no earlier than $t - s_g$ (otherwise the counterparty would have been in default before *t*). The call must therefore have been called based on the portfolio value no earlier than $t - (s_c + s_g)$.

Given this result, we can conclude that the last successful exchange (call 1 in the diagram) must have been at the previous call date, which was *N* days prior. Therefore, we can determine the portfolio valuation date for the last successful exchange of collateral is $t - (s_c + s_g + N)$, and so p_1 is:

$$p_1 = s_c + s_g + N \qquad (1)$$

The latest possible margin call where the bank would post additional collateral (call 3 in the diagram) is where the collateral delivery date is at *t*, i.e. just before the counterparty is placed in default. By the same logic above, this would be in response to a margin call based on the portfolio valuation as at $t - s_c$.

Collateralized valuation determines the collateralized exposure. The portfolio value at time *t* equals its liquidation value plus the cash flow adjustments. Although the MTM (market-to-market) value at *t* is observable, the liquidation value observable at the end of liquidation period reflects the true replacement cost. These two values may become very different when the market is very volatile, or the liquidity becomes very low (e.g. during the 2008 financial crisis).

The timeline of trade valuation is shown in Figure 2. The exposure reporting time *t* is at the end of the settlement period and the beginning of the liquidation period.

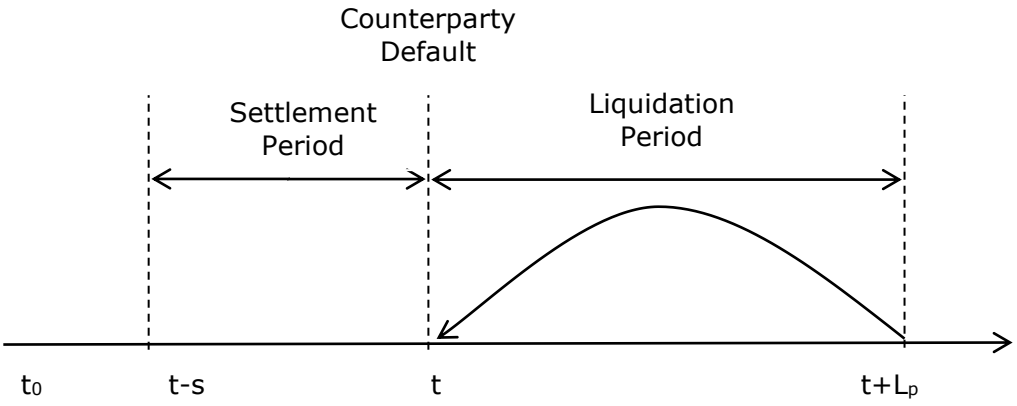


Figure 2 Timeline of Portfolio Valuation

Collateral assets value is used to calculate the collateralized exposure. The collateral valuation is very similar to portfolio valuation. The counterparty is assumed to be in default at time *t*. The collateral value equals its liquidation value plus cash flow adjustments.

Although the market price of collateral is observable at *t*, the Bank need some time to liquidate the collateral assets. The realized selling price more realistically reflects the collateral value at *t*. The cash flows during the liquidation period (e.g. coupons and yields) are counted as part of the collateral value, since all cash flows belongs to the party whoever takes hold of the asset.

Assuming that there are *I* collateral assets posted, the collateral value is calculated as the aggregation of individual collateral asset value plus an initial margin (*IM*):

$$C(t) = IM(t) + \sum_{i=1}^{I} C_i(t), \quad \text{and} \tag{2}$$

$$C_i(t) = n_{c,i}(t) * \left(p_{c,i}(t + L_{c,i}) * DF(t, t + L_{c,i}) + \sum_{t < t_j < t + L_{c,i}} CF_{c,i}(t_j) * DF(t, t_j) \right) + \left(n_{c,i}(t)\right)^{-} * \sum_{t - s \le t_k \le t} CF_{c,i}(t_k) / DF(t_k, t)$$

where $CF_{c,i}$ stands for cash flow yields from one unit of the collateral asset, $n_{c,i}$ is the collateral quantity, $p_{c,i}$ is unit price. *IM* stands for the initial margin, whose valuation is assumed to be a fixed amount in this method.

Unlike derivative trades, cash yields from the collateral assets are always positive. The collateral quantity is a signed amount: $n_{c,i} \geq 0$ when the Bank receives collateral, $n_{c,i} \leq 0$ when the Bank posts collateral. To be conservative, it is assumed that only cash yields from collateral posted by the Bank are subject to settlement risk. The function $(\ )^-$ represents the case when the Bank posts collateral.

3. **Empirical Results**

We use interest rate swaps for our empirical study, as they account for around two-thirds of OTC outstanding derivatives. We examine if counterparty credit risk and collateralization are sufficient to explain swap premium spreads.

Swap rate is the fixed rate that sets the market value of a given swap at initiation to zero. The mid-market swap rates are based on a rigorously organized daily poll from a panel of dealers for their par swap rate quotes.

In practice, the mid-market swap rates are generally not the actual swap rates transacted with counterparties as a swap dealer needs to add a spread to the mid-market swap rate to compensate trading expenses.

We obtained data set from FinPricing (https://finpricing.com/lib/IrCurve.html). The trading dates are from May 6, 2015, to May 11, 2020. We find some swap pairs in the data set, where the two contracts in each pair have the same terms and conditions but are traded with different counterparties under different collateral arrangements. The two contracts in each pair is the difference in unsecured credit risk between two counterparties, as all other risks and costs are identical.

By taking credit risk and collateralization into account only, we calculate the model-implied swap rates. We find that the model-implied swap premia fluctuate randomly around the market swap premia. The summary statistics of the market quoted premium spreads, the model-implied premium spreads, and the model-market premium spread differentials are presented in Table 7, where we refer to the differences between the model-implied premium spreads and the market quoted premium spreads as the *model-market premium spread differentials*.

As can be seen from Table 1, the average of the model-market spread differentials is only -0.03 bps, which can be partly attributed to noises. The results indicate prima facie that the model performs quite well. The empirical tests corroborate the theoretical prediction on the impact of collateralization on swap rates.

Table 7: Summary statistics of implied spreads and market spreads

	Max	Min	Mean	Median	Std
Market quoted swap spreads	3.09	-5.23	-0.46	-0.16	1.81
Model-implied swap spreads	2.09	-5.31	-0.43	0.03	1.73
Spread differentials	0.99	-1.17	-0.03	0.11	0.43

Each collateral asset has its own liquidity period. The length of liquidation period depends on collateral asset type and characteristics (e.g. notional amount, term, market turnover, etc.). It is also impacted by economic environments. The length of the liquidation horizon should be defined at the asset level. The liquidation period can be determined using some mapping rules. For illustration purposes, a few different collateral liquidation period categories can be defined in Table 2.

Table 2: Collateral asset liquidation horizons

Collateral Liquidation Period	Length
Short	3 Days
Median	8 Days
Long	18 Days

Trade Type	Length
U.S. T-Bill	Short
Corporate Bond: (issuer rating = AA or plus)	Median
Corporate Bond: (issuer rating = BBB or lower)	Long
Cash	0
…	…

At the exposure calculation time *t*, the collateral has been posted on a past day, and its quantity and component are observable. On the base date, the detailed information of these collateral assets should be obtained from some source system, and the initial posting date should be observable.

If the initial posting date is missing, we need to set a default value for this date. For example, it can be set equal to t_0- *(margining frequency + settlement period)* for conservative purposes. On a future date $t > t_0$, we need a method to decide how much collateral has been posted by the counterparty (or the Bank). The future posing dates can be derived based on the initial posting date and the posting schedule (frequency).

Consistent with portfolio valuation, the counterparty is deemed to be in default at the exposure calculation time *t*. The Bank needs some time to liquidate the collateral assets or replace (re-purchase) the lost collaterals posted by the Bank. This period is called the collateral liquidation period. The length of this liquidation period will depend on collateral asset type and traits (e.g. notional, term, etc.). It also

depends on the economic environments. The length of the liquidation horizon should be defined for each collateral asset according to some rules.

The gross and collateralized exposures are calculated using some dummy netting sets including interest rate swaps and FX forwards. Bilateral collateralization is assumed. The settlement period is 2 days, liquidation period for trades and collaterals are both 15 days, margin call frequency is weekly. The collateral asset is an AA bond. The threshold, MTA and haircut are all assumed to be zero. The results are listed in the following figures.

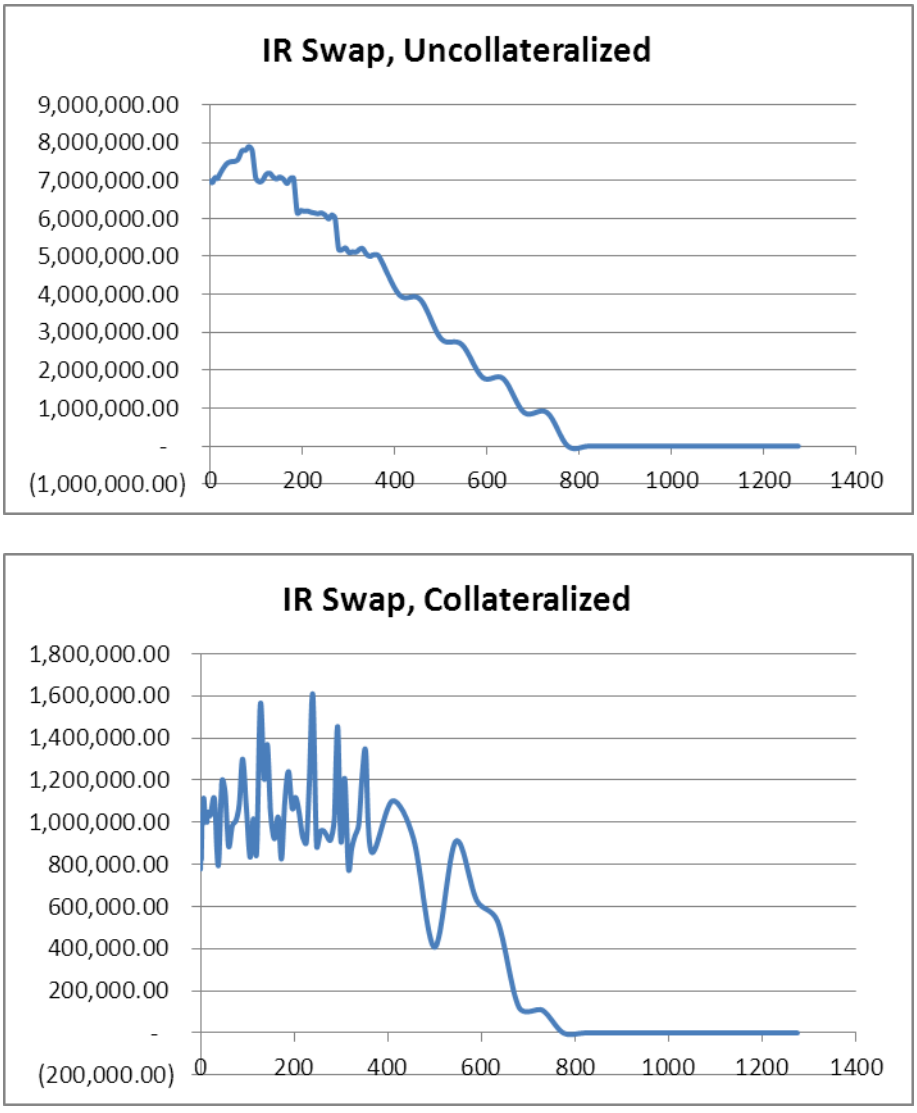
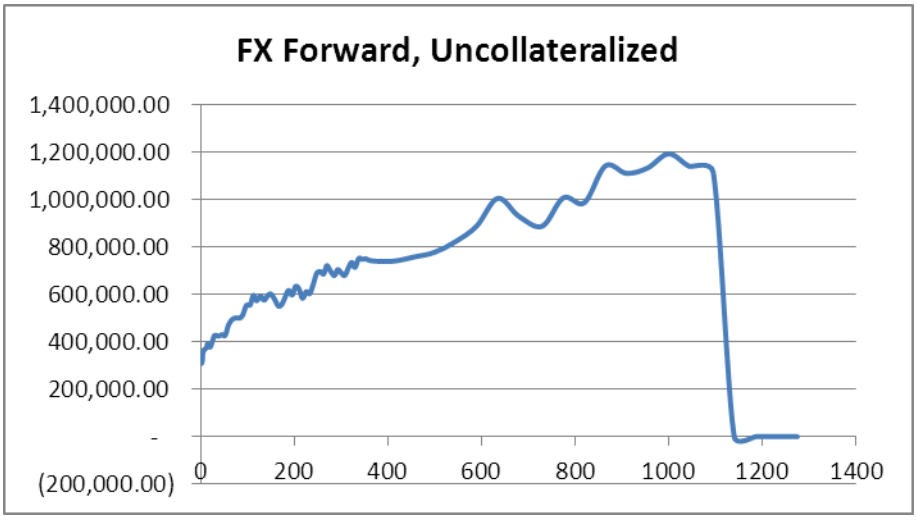


Figure 3: Credit risk exposures for collateralized and uncollateralized swaps



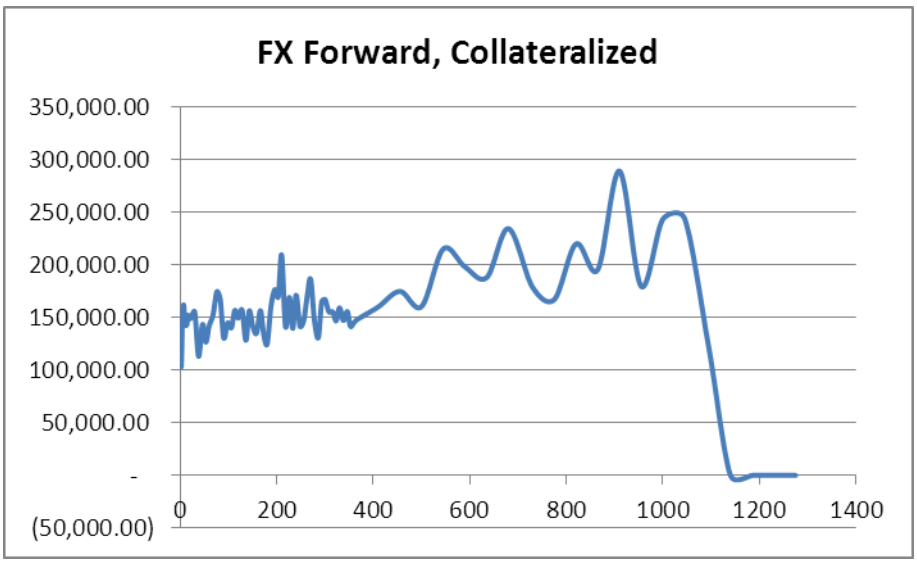
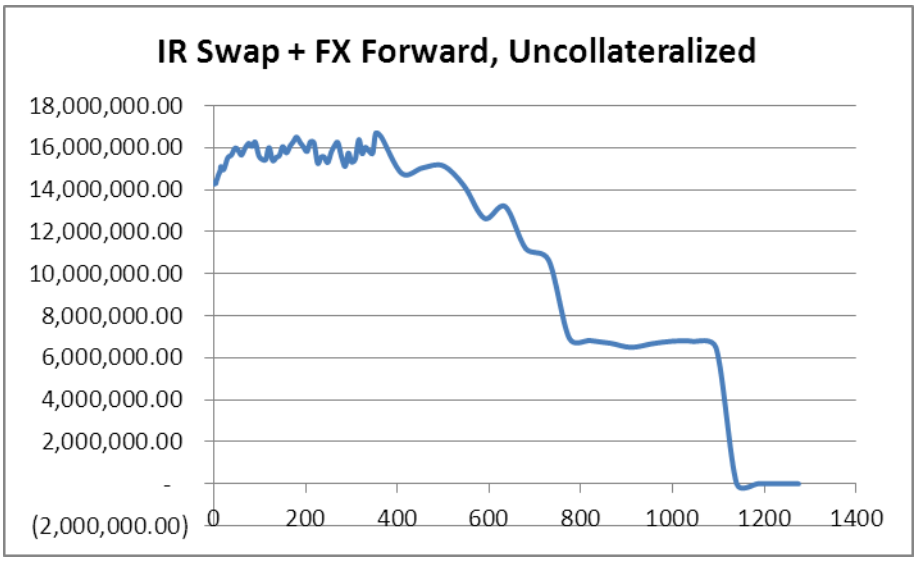


Figure 4: Credit risk exposures for collateralized and uncollateralized FX forwards



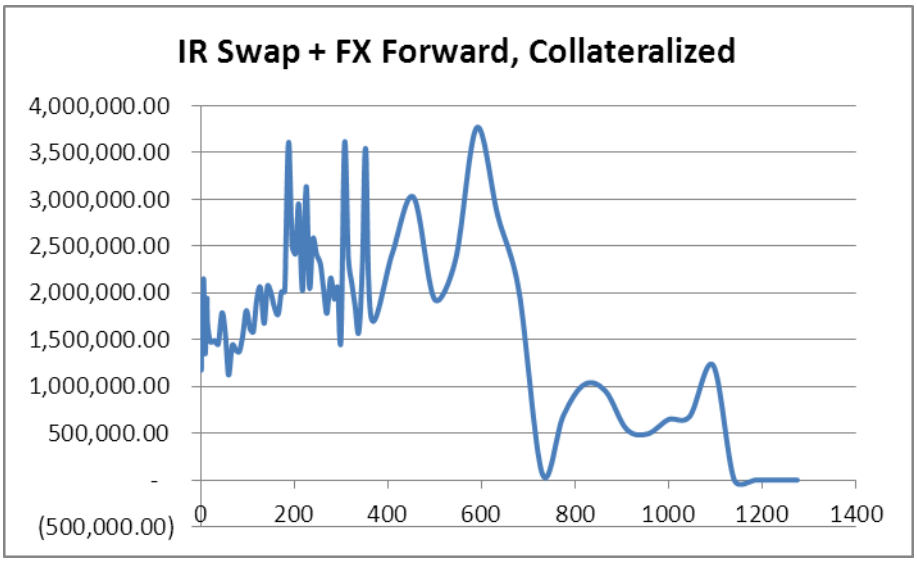


Figure 5: Credit risk exposures for collateralized and uncollateralized swaps and FX forwards

Making these calculations for all scenarios leads to a distribution of exposures for a given future time (time bucket) t. The 95th percentile or peak exposure for a given time bucket is calculated by taking the specified quintile of this distribution, while expected exposure is calculated as the mean value of the distribution. Therefore, the 95th percentile and expected exposures calculated in such a way represent term profiles across all maturity time buckets that are relevant to the counterparty portfolio.

For the counterparty with collateral agreement in place, the collateralized exposure (1.1) is the most appropriate measure of the exposure and hence the exposure to which limits can be most meaningfully applied.

Typical term profiles for Uncollateralized (Gross) and Collateralized OTC derivatives are presented below.

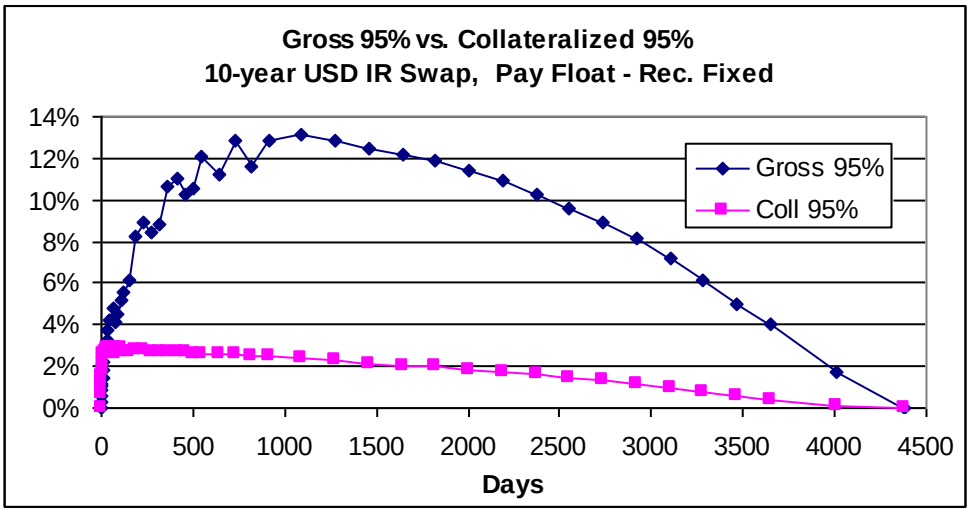


Figure 6. Exposure for collateralized and uncollateralized swap

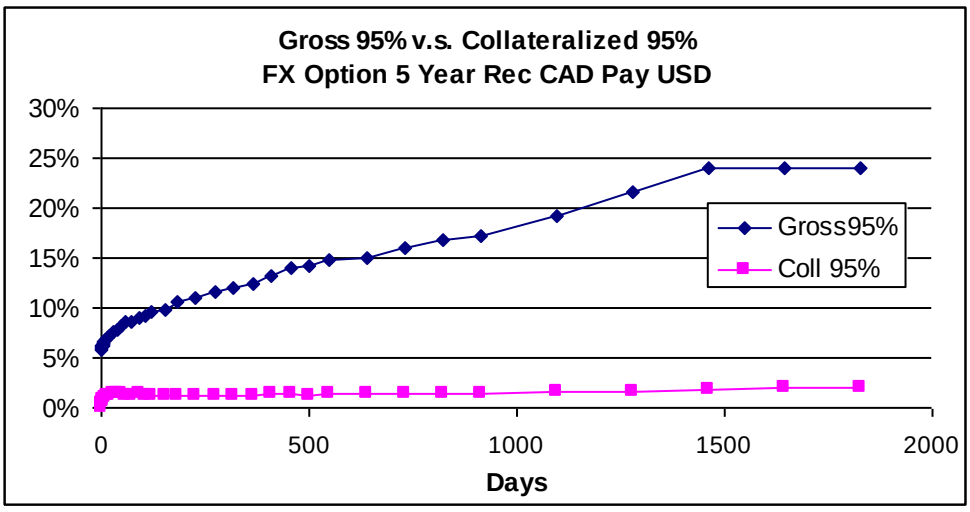


Figure 6. Exposure for collateralized and uncollateralized FX option

Next, we examine the effects of credit risk and collateralization on swap spreads. For each swap pair, the differences in CDS spreads should mainly represent differences in credit risk. We present a regression model to determine the strength of the statistical relationship between market swap spreads and CDS spreads:

$$Y = a + bX + \varepsilon \tag{3}$$

where Y is the market swap spread, X is the CDS spreads, a is the intercept, b is the slope, and ε is the regression residual.

Table 3 shows the estimation results where the adjusted R^2 is 0.758, implying that approximately 76% of market swap spreads can be explained by credit risk alone. Empirically we find that counterparty risk is the main but not all driving factor for determining swap spreads when contracts are under collateral agreements. Furthermore, the slope is 0.0126, implying that a CDS spread of about 100 basis points translates into a swap spread of about 1.26 basis points.

Table 3: Marginal credit risk regression results for the OTC data

Slope	Intercept	Adjusted R^2	Significance F
0.0126	-2.7E-04	0.7472	1.1E-05

We model where the market swap spreads are regressed on the implied swap spreads. Since the model considers both credit risk and collateralization, the implied spreads should refer to the joint effect of counterparty risk and collateralization. The regression results are shown in Table 4.

Table 4 tells us that the adjusted R^2 value is 0.988, implying that approximately 98.8% of the market spreads can be explained by the implied spreads. The empirical evidence demonstrates that the

economic significance of collateralization on determining unsecured credit spread. Only considering collateralization can provide a sufficient explanation for these credit spreads.

Table 4: Credit risk and collateralization combined regression results.

Slope	Intercept	Adjusted R^2	Significance F
0.986	3.98E-05	0.988	2.86E-08

Furthermore, we study the impact of collateralization on credit valuation adjustment (CVA). CVA is a direct measurement of counterparty credit risk and allows institutions to quantify counterparty risk as a single measurable P&L number. CVA banks dynamically manage, price and hedge counterparty risk.

The settlement risk also needed to be considered in the portfolio valuation. When the counterparty becomes default at *t*, there might be some outstanding trade cash flows payments still not settled, and the other party may never be able to collect them. These cash flows are vulnerable to the settlement risk and should be included as part of the counterparty exposure. The length of settlement period is defined at trade level when each OTC derivative is negotiated. Under normal business conventions, the settlement period may last for only a few days.

Upon a margin call, the collateral quantity is calculated using the portfolio value and collateral asset price. Besides, threshold, valuation percentage and minimum transfer amount (*MTA*) also affect the quantity called. Denoting t_p as any margin call day, the quantity of collateral can be calculated as:

$$\sum_i n_{c,i}(t_p) * p_{c,i}(t_p) * v_i = (V(t_p) - D_{cpty}(t_p) - MTA_{cpty})^+ + (V(t_p) - D_{bank}(t_p) - MTA_{bank})^-$$

The *i* denotes the i^{th} asset inside the collateral pool, *v* is valuation percentage (reflecting haircut), *MTA* is the minimum-transfer-amount and *D* is the threshold. The values of *MTA* and *D* might be different

depending on the Bank receiving or pledging collateral. The valuation percentage *v* is usually between 0 and 1. It is defined at the asset level to reflect its volatility and liquidity characteristics.

We select a portfolio with collateral agreement. First, we compute the risk-free value V^F that is relatively straightforward as the risk-free portfolio value is what brokers quote or what trading systems or models normally report.

First we compute risky value V^N of the portfolio. The CVA without collateralization is equal to $V^F - V^N$. Them, we calculate the risk value V^C of the portfolio with a margin agreement. The CVA with collateralization is given by $V^F - V^C$. The results are displayed in Table 5.

Table 5. The Impact of Collateral Threshold on CVA

Collateral Threshold	0	10.3 Mil	15.2 Mil	20.6 Mil	Infinite (∞)
CVA	0	23,685.3	33,503.8	42,253.9	49,686.3

Table 5 tells us that collateral posting can reduce CVA. The collateral threshold reflects the unsecured credit exposure that a party is willing to bear. As collateral threshold increases, unsecured credit exposure rises. Consequently, CVA increases. Full collateralization makes a portfolio appear to be risk-free.

Finally we study the impact of collateralization on market risk. Value-at-risk (VaR) is the standard measurement for assessing market risk. It is defined as the maximum loss likely to be suffered on a portfolio for a given probability defined as a confidence level over a given period time. In its most general form, VaR measures 10-day 99th percentile of potential loss that can be incurred, stemming from fluctuations in market prices.

desk. A market risk portfolio may contain many derivative products belonging to different counterparties. We select a large trading portfolio with positions across multiple counterparties. We

extract all positions traded with one counterparty and construct a sub-portfolio. The sub-portfolio contains interest rate derivatives only.

The impact of collateral pledging on VaR is shown in Table 6. Table 6 tells us that counterparty credit risk increases market risk, e.g., VaR from $386,632 to $419.236. Collateralization can reduce the impact of counterparty risk on market risk. Market risk increases with collateral threshold. In particular, VaR reaches maximum when the threshold is infinite representing no collateral arrangement.

Table 6. The Impact of Collateral Threshold on VaR

Collateral Threshold	0	10.3 Mil	15.2 Mil	20.6 Mil	Infinite (∞)
CVA	-386,578	-398,233	-404,421	-410,768	-418,962

4. Conclusion

In general, collateral is very effective at mitigating portfolio credit risk, but it leaves two residual risks. Both of these risks arise because a period of uncertainty, called here a close-out period, during which a counterparty may have failed to meet a collateral call but cannot legally be said to be in default. The first risk is the basis in market risk between the portfolio and the collateral; a change in market conditions over the closeout period may cause an increase in exposure. The second risk is that a failure to meet a collateral call results in uncollateralized exposure. Events such as payments or deal exposures that result in collateral transfers can cause temporary but significant jumps in exposure.

This article presents a model for pricing collateralized financial contracts. The model can back out market prices for a position subject to collateralization. This is very useful for pricing outstanding financial contracts with collateral agreements.

The empirical results show that model-implied prices are quite close to market-quoted prices, which suggests the model is fairly accurate in pricing collateralized contracts.

We find strong evidence that counterparty risk alone is the main but not all factors in determining credit spreads for collateralized contracts. Only the joint effect of collateralization and credit risk can provide a sufficient explanation for unsecured credit costs.

We also find evidence that cleared contracts may not be economically equivalent to their OTC counterparts because the clearing mechanics of a clearinghouse change the risk structure and thereby the asset prices. In fact, we find that cleared derivatives are not economically equivalent to their OTC counterparts.

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