The New Era of Counterforce: Technical Appendix

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Counterforce in the Age of Accuracy

The following sections explain the calculations we used to model the consequences of accuracy improvements for counterforce targeting.

Missile Lethality

The calculations behind Figure 1 and throughout Table 1 are based on four standard formulas.[1] The lethal radius (LR) of a given warhead against a given target can be estimated as:

$$(1)\ LR = 2.62 * Y^{1/3} / H^{1/3},$$

where Y is the warhead's yield in megatons, H is the target's hardness in psi, and LR is expressed in nautical miles (nm).

The odds that a given delivery system (for example, a missile) will deliver the warhead within the LR, the so-called "single shot probability of kill" or SSPK, is:

$$(2)\ SSPK = 1 - 0.5^{(LR/CEP)^2},$$

where CEP, "circular error probable," is the delivery system's accuracy.[2]

The probability of destroying the target must also take into account the reliability (R) of the weapon system. The variable R is an estimate of the probability that the delivery system

[1] The seminal unclassified work on the effects of nuclear weapons is Samuel Glasstone and Philip J. Dolan, *The Effects of Nuclear Weapons* (Washington, D.C.: U.S. Government Printing Office, 1977). See also Lynn E. Davis and Warner R. Schilling, "All You Ever Wanted to Know about MIRV and ICBM Calculations but Were Not Cleared to Ask," *Journal of Conflict Resolution*, Vol. 17, No. 2 (June 1973), pp. 207–242; and Keir A. Lieber and Daryl G. Press, "The End of Mad? The Nuclear Dimension of U.S. Primacy," *International Security*, Vol. 30, No. 4 (Spring 2006), pp. 7-44, Appendix 1.

[2] CEP is the median miss-distance: half the warheads land closer to the target than the CEP and half land farther away.

and warhead function correctly. The variable "terminal kill probability" (TKP) incorporates SSPK and R, where:

$$(3)\ TKP = R * SSPK.$$

If multiple warheads are sent to destroy a single target, then the target only survives if all the warheads fail. Therefore the odds of destroying the target with n-shots is 1 minus the probability that every warhead misses, or:

$$(4)\ p(kill)_n = 1 - (1- TKP)^n.$$

These four formulas can be used, along with the weapons characteristics given in Table A1, to estimate the capabilities of U.S. missiles against a variety of targets.

Table A1. U.S. Strategic Ballistic Missile Systems and Key Characteristics[3]

Type	Year	Delivery Vehicle	Warhead	Yield (kt)	CEP (m)
ICBM	1985	MMIII/Mk-12A	W78	335	183
	2017	MMIII/Mk-12A	W78	335	120
		MMIII/Mk-21	W87	300	90
SLBM	1985	Trident I C-4/Mk-4	W76	100	380
	2017	Trident II D-5/Mk-4A	W76	100	90
		Trident II D-5/Mk-5	W88	455	90

Reprogramming

To estimate the effects of reprogramming on counterforce strikes (for Table 1, line 5), we assume that the attacker detects "boost phase" malfunctions and then reassigns the targets that would have been struck by the faulty missiles to new weapons. The probability of a

[3] See Matthew G. McKinzie, Thomas B. Cochran, Robert S. Norris, and William M. Arkin, *The U.S. Nuclear War Plan: A Time for Change* (Washington, DC: Natural Resources Defense Council, 2001), p. 19; Miller, *The Cold War*, Appendix 7; Lennox, *IHS Jane's Weapons;* Hans M. Kristensen, "Nuclear Forces Guide," Federation of American Scientists, <http://www.fas.org/nuke/guide>; and "LGM-30G Minuteman III," Missilethreat.com.

malfunction is (1-R), and following Steinbrunner and Garwin, we assume that 75% of those failures occur during boost phase.[4] We allow the reserve missile to be replaced (up to three times) if it too experiences a boost phase malfunction.

Fratricide

To assess the effects of fratricide during a many-on-1 attack (for Table 1, line 6), we assess the outcomes for individual warheads in the sequence they arrive at the target. We assume the attacker adopts standard targeting strategies to minimize fratricide. For example, incoming warheads are separated by 3-5 seconds: far enough apart to prevent one warhead's detonation from interfering with nearby reentry vehicles, but close enough together to allow each weapon to arrive before the dust cloud from the immediately preceding weapon can form.[5] Despite these strategies, if warhead n arrives near the target but "misses" (that is, falls outside of the LR), the dust cloud it creates may interfere with subsequent warheads.

Warhead n will create a dust cloud that shields the target if two conditions are met: (i) warhead n misses the target – calculated as 1-(TKP+(1-R)) – *and* (ii) the location of the miss generates a dust cloud along the trajectory of subsequent incoming reentry vehicles. If these two conditions are met, then warhead n will block subsequent warheads beginning with n+2. (Warhead n+1 will not be affected, because, as described above, it arrives within 3-5 seconds, before the dust cloud from warhead n solidifies.)

To create a conservative estimate of fratricide risks (in other words, one that errs on the side of overstating the fratricide danger), we assume that all "misses" that fall short of the target, and half of those that miss "long," create a dust cloud along the trajectory of the other incoming reentry vehicles. Because misses are equally likely to be "short" and "long," 75% of the time that warhead n misses, subsequent weapons are blocked, beginning with n+2.

Compensated Fuses

[4] Steinbrunner and Garwin, ""Strategic Vulnerability," p. 150. Steinbrunner and Garwin calculate failure rates differently. Their method leads them to estimate that roughly 76% of all malfunctions occur during boost phase. We simplify that and use 75%.

[5] The dust cloud does not form until the temperature in the fireball cools sufficiently to allow debris to solidify, which takes roughly 6-8 seconds. These targeting strategies, and details about target debris and dust clouds, are described in Bennett, "How To Assess the Survivability of U.S. ICBMs: Appendixes," pp. 34-39. There are additional steps that attackers would take to minimize fratricide. For example, they would likely attack groups of nearby targets (for example, silos in a missile field) from back to front to prevent the dust clouds generated in an attack on one target from interfering with reentry vehicles aimed at a nearby target.

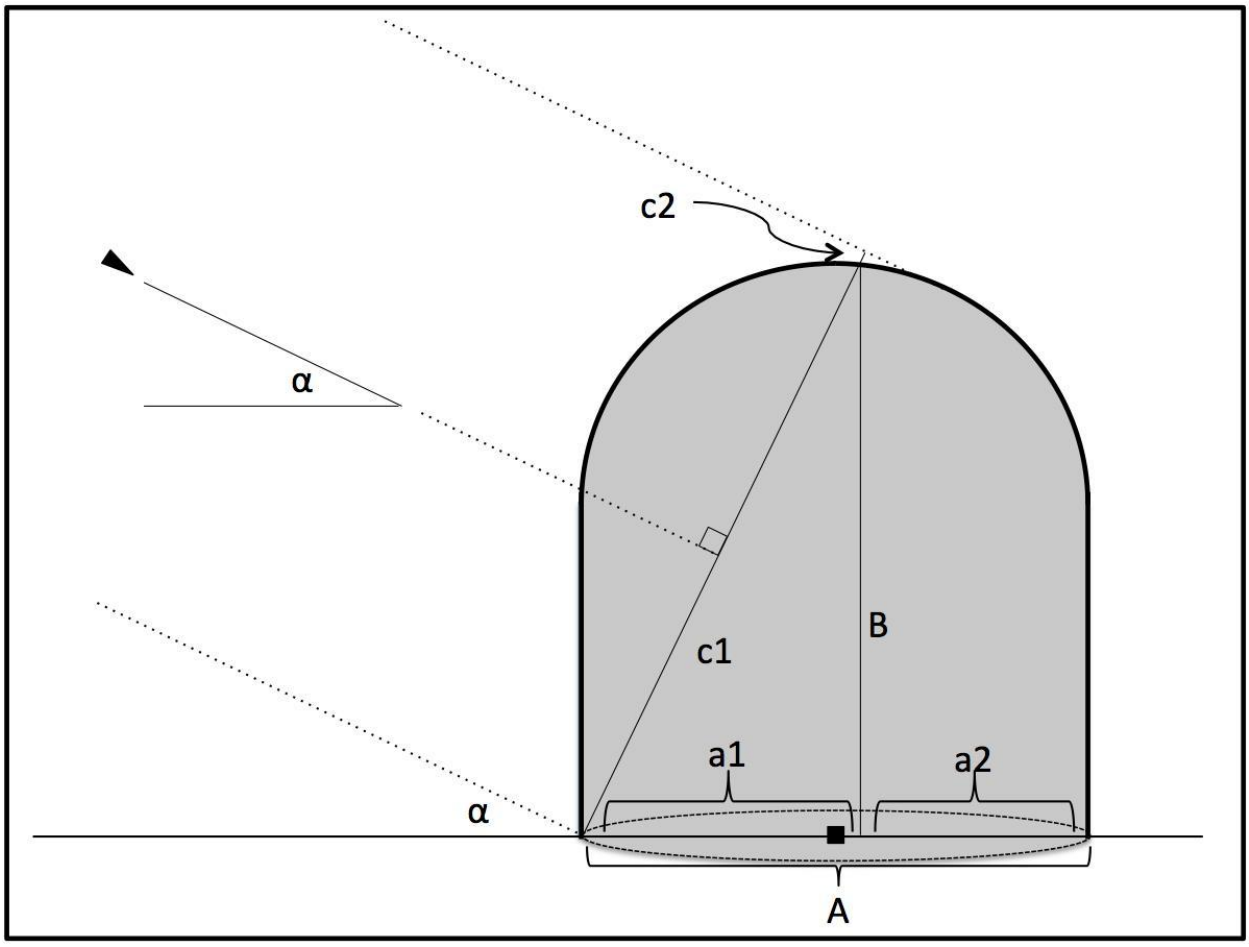
Reentry vehicles equipped with a compensated fuse use an altimeter to measure the difference between the actual and expected trajectory of the reentry vehicle as it approaches the target. The fuse then compensates for inaccuracies by adjusting the warhead's height of burst (HoB).[6] If the altimeter reveals that the warhead will detonate "short" of the target, the fusing system lowers the HoB, allowing the weapon to travel farther (hence, closer to the aim point) before detonation. Alternatively, if the reentry vehicle is going to detonate beyond the target, the HoB is adjusted upward to allow the weapon to detonate before it travels too far.

As a result of its ability to adjust its height of burst, a reentry vehicle equipped with a compensated fuse will destroy a target as long as the RV passes through the lethal volume around the target at some point during its flight path (and hence can detonate – if the HoB is set appropriately – within the lethal volume). To estimate the probability of destroying the target, therefore, one needs to calculate the probability that the reentry vehicle will pass through the lethal volume.

The lethal volume around a hardened target is roughly cylindrical with a curved top (because it is the lethal area on the ground—roughly a circle—projected upward into the air). The dimensions of the lethal cylinder for hardened targets can be estimated using the curves in Glasstone and Dolan (henceforth G/D) figure 3.73a, which indicate the maximum distance at which a weapon can destroy a target as a function of the target's hardness and the weapon's height of burst.[7] To estimate the probability of a reentry vehicle passing through the cylindrical lethal volume, we calculate the dimensions of the two-dimensional rectangular projection of the cylinder, perpendicular to the reentry angle of the warhead. (See Figure A1, below).[8]

[6] See Postol, "Monte Carlo;" Donald A. Price and Charles A Louis, "Burst Height Compensation," United States Patent US4456202 A, June 26, 1984; and John Ainsle, "Sharpening Trident," unpublished document, 2009, available at swordofdamocles.org, and published sources cited therein.
[7] The figures in Glasstone and Dolan are for one kiloton weapons, and hence must be scaled up for any given weapon's yield. The scaling rules are in Glasstone and Dolan, *Effects of Nuclear Weapons*, pp. 111-115.
[8] To be clear, as long as the RV passes through the lethal volume, there is a height of burst that the fuse can select that will destroy the target. The RV's target, therefore, is defined by the projection of the cylindrical lethal area perpendicular to the RV's flight path, which is a rectangle.

Figure A1: Method for estimating the effectiveness of a compensated fuse



Note: In Figure A1, a warhead with a compensated fuse (the triangle in the upper left corner) is aimed at a target, indicated by the black square. The warhead must detonate within the shaded lethal volume to destroy the target.

The rectangular projection of the lethal volume is defined by its height, C (which is c1+c2), and its width, which is A. A is known (it is twice the distance along the x-axis in G/D, 3.73a), so we simply need to calculate C. The lower portion of C (c1) can be calculated using trigonometry. B, c1, and a1 make a right triangle, so c1 = B / sin(90-α). Calculating c2 requires more manipulations, but c2 = a2 * cos(90-α).

To estimate the probability that a warhead with a given CEP will strike a rectangular area, one can use the Polnya Calculation method given in Formula (5) below

$$(5)\ SSPK_{rectangle} = (1 - 0.5^{(1/\pi)(W/CEP)^2})^{1/2} * (1 - 0.5^{(1/\pi)(L/CEP)^2})^{1/2},$$

where W is the rectangle's width and L is its length. We can use this formula to estimate the probability of striking the rectangular projection of the shaded area by substituting C for L and A for W.

Line 7 of Table 1 reports the outcome of an attack using 100-kiloton W76s against 3,000 psi targets. Assuming the W76s, approached the ground at approximately 30 degrees, a warhead would have to hit a rectangular target 357 meters high by 297 meters wide to destroy a silo. With a CEP of 90 meters, each warhead would do so 94% of the time. Assuming 90% reliability and a 2-on-1 attack, 1 silo out of 200 would be expected to survive.

Low Casualty Counterforce

The solid black line in Figure 2 shows the maximum weapon yield that can destroy a given target from above the fallout threshold (*FT*). Generating that line requires two formulas. First, if one knows the maximum height of burst (*MHoB*) at which a 1-kiloton weapon can destroy a target, then *MHoB* for *W* kilotons can be calculated using Formula (6)[9]:

$$(6)\ MHoB_{W\text{-}kilotons} = MHoB_{1\text{-}kiloton} * W^{(1/3)}.$$

Figure 3.73a in G/D indicates *MHoB* for a few, discrete levels of target hardness (in the G/D figure, *MHoB* is the y-value where x = 0), but to generalize to a broader set of possible targets, we used the software "Blast Effects" to calculate *MHoB* for a 1-kiloton weapon against targets whose hardness ranges from 500 to 3,000 psi.[10]

Second, the fallout threshold (*FT)* for a weapon depends on its yield, and can be estimated using Formula (7)[11]:

$$(7)\ FT = 180 * W^{(0.4)}.$$

To generate the line in Figure 2, therefore, one needs to find, for every level of hardness, the yield at which "*MHoB*" = "*FT*", meaning solve for *W* in the following equation:

$$180 * W^{(0.4)} = MHoB * W^{(1/3)}$$

[9] Glasstone and Dolan, *Effects of Nuclear Weapons*, pp. 110.
[10] "Blast Effects" was created by Horizons Technology for the Defense Nuclear Agency in 1984, and is available on the web.
[11] Glasstone and Dolan, *Effects of Nuclear Weapons*, p. 71.

To estimate the accuracy (CEP) required for low-yield weapons to conduct low-casualty counterforce strikes against hardened targets, one must (i) choose a height of burst at or just below *FT*, (ii) estimate the lethal radius for the weapon at that height of burst (e.g., using G/D Figures 3.73a or "Blast Effects"), and (iii) manipulate Formulas 1-4 *backwards*: that is, instead of starting with hardness and yield (in Formulas 1 and 2) and solving for $p(kill)_n$ in Formula (4), start with the desired $p(kill)_n$ (in this case, 95% likely that each target is destroyed), plug that into Formula 4, and solve backwards to CEP. To estimate the fatalities in the two nuclear strikes, we use Hazard Prediction and Assessment Capability (HPAC) 5.1.

Counterforce in the Age of Transparency

The following sections explain the analysis that we conducted to illustrate some of the consequences of the revolution in remote sensing for counterforce missions.

Roads

There is no comprehensive, unclassified digital map of North Korean roads, so we created a map by merging the road data from OpenStreetMap (OSM) and DIVA-GIS. We included eight classes of road from the OSM database: motorways, primary roads, secondary roads, and trunk roads – plus the "linkage" roads associated with each of those classes. We merged the resulting set of roads with the road data in DIVA-GIS data, thereby adding roads that were missed by OSM. Merging the data, of course, creates duplicates, which we reduced by having ArcGIS remove any road segment if it remained within 1 km distance of an existing segment from start to finish.

Satellite Analysis

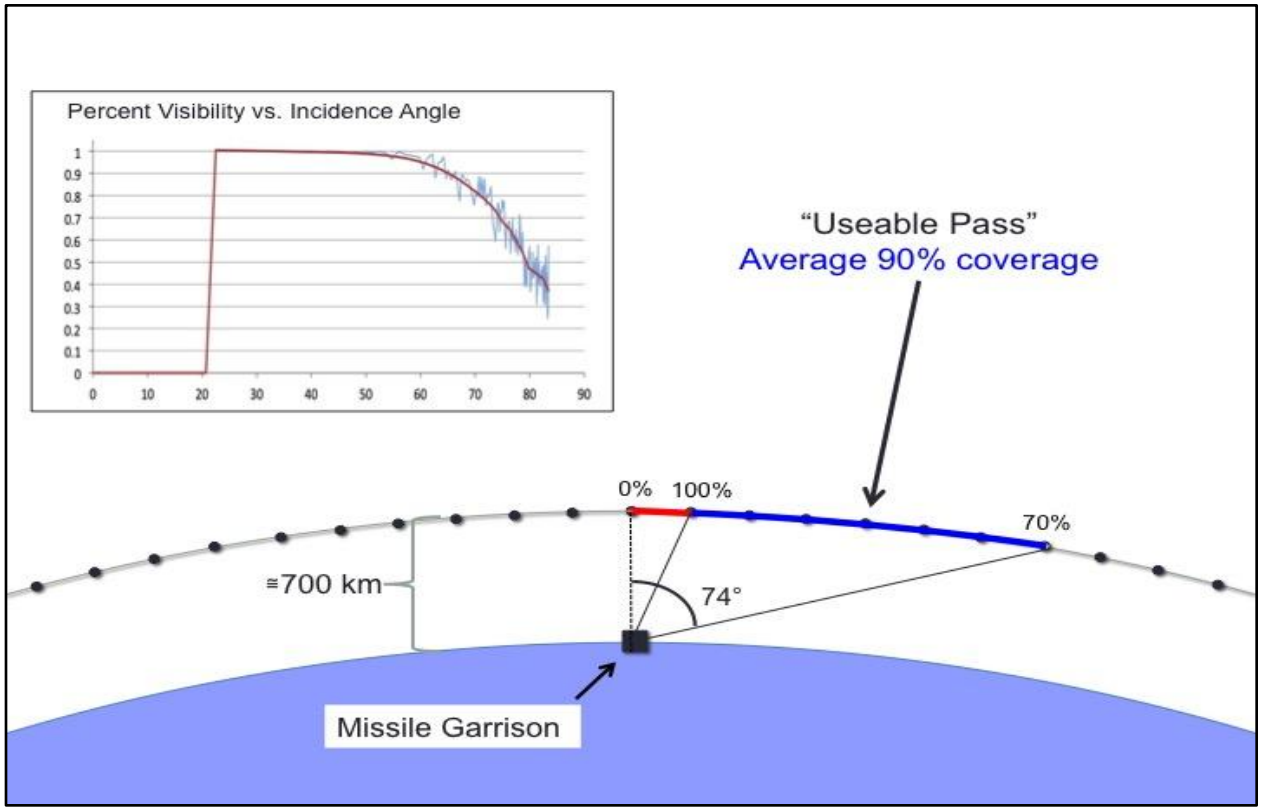
Satellites in low earth orbit do not pass over the same spot on earth with each orbit. Instead, relative to a fixed location on the ground (such as a missile garrison), subsequent passes by a given satellite may be several hundred kilometers to one side or the other. We used ArcGIS to explore the relationship between the lateral distance of a given satellite pass and the percentage of North Korean roads that are visible.

The graph inset in Figure A2 shows the results. Radar satellites that are directly overhead a given target cannot view the target or the nearby roads, because the target would be within the satellite's "nadir hole."[12] As the graph in the inset reveals, orbits that are nearly

[12] "Alternatives for Military Space Radar," Congressional Budget Office, Publication 1609, January 2007.

overhead – but far enough laterally such that the target is not in the nadir hole – have near-perfect visibility of the road network, even in a place as mountainous as North Korea. As the incidence angle grows beyond 50 degrees, the coverage drops.[13]

Figure A2: Defining a "Useable Pass" Based on Road Network Visibility



Note: In the figure, the satellite orbits are moving through the paper or computer screen toward (or away from) the reader. The dots represent possible satellite positions as the satellite passes perpendicular to a hypothetical target (e.g., a missile garrison). The orbital band directly above a target (extending out to 22 degrees incidence angle) are not useable because from those satellite positions the target would lie within the nadir hole. The orbital band from 22 degrees out to 74 degrees is the band we define as "useable passes" in that they provide 90% visibility of N. Korean roads.

A "useable pass" occurs when a satellite crosses through an orbital band that is defined such that the set of passes through the band provides 90% coverage of the road network below. According to the ArcGIS results, the orbital band in which useable passes occur

[13] Radar satellites have a blind spot – called the "nadir hole" – directly below them. We assume that radar satellites cannot see any targets that lie within a zone defined by the "look angle" of 0-20 degrees. See Li Bin, "Tracking Chinese Strategic Mobile Missiles," pp. 13. The ArcGIS calculations are conducted using the "incidence angle" rather than the "look angle" – the former being the angle from the ground to the satellite, which is slightly larger than the angle from the "look angle" because of the curvature of the earth (Lillesand et al, *Remote Sensing*, p. 394-95). For a satellite orbiting at 700 km altitude, a 20-degree look angle corresponds roughly to a 22-degree incidence angle, which is why the graph in Figure A1 shows 0% visibility out to 22 degrees incidence angle.

stretches from 22 to 74 degrees incidence angle.[14] For a satellite in a typical radar satellite orbit, roughly 8.5% of its downward passes (north-to-south) and the same percentage of upward passes will be within the "useable pass" zone. At 700 km altitude, a satellite completes an orbit (with one downward and one upward pass) every 99 minutes, leading to 2.5 useable passes per day, on average.

The inner border of the useable pass zone (defined by a 22 degrees incidence angle) puts the satellite 250 km lateral planar distance away from a given point in North Korea; the outer border is 1,500 km lateral distance. For the average useable pass, therefore, the satellite is 900 km lateral planar distance from a given target ((1,500 km + 250 km) / 2).

Given that on an average "useable pass", a satellite is roughly 900 km lateral distance from a given target in North Korea, and assuming that a satellite can aim its radar 45 degrees forward or back, it will be able to image sites in North Korea approximately 900 km before it becomes parallel to North Korea (by aiming the radar forward by 45 degrees), for the 350 km ground track distance as it passes along the length of North Korea, and for 900 km as it passes beyond the Peninsula (by aiming its radar backwards), for a total of 2,150 km per pass. A satellite's ground track speed can be calculated by taking the earth's circumference (40,075 km) and dividing it by the satellite's orbital period (99 minutes), which results in 400 km / minute. At that speed, a satellite will cover the 2,150 km along which it can image North Korean targets in approximately 4.8 minutes. In that time, it will theoretically be able to image 14 150km x 150km "boxes" of North Korea (2,150km / 150km), but because there will be inefficiencies, we assume 12 images per pass.

UAV Analysis

In order for a UAV to detect a moving target, it needs to observe the target for some period of time. We assume that a SAR/GMTI radar on a RQ-4 can detect a TEL if it can observe it for at least 5 minutes, with minimal obstructions. Therefore, we define a "visible" road segment as one with at least 3.3 km of continuous visibility (which is the distance that a TEL can move in 5 minutes at 40 km/hour). We allowed visible segments to have small areas of blocked sight (up to 500 meters) because dropping out of sight momentarily would not necessarily break the UAVs track of the TEL.

[14] At 22 degrees, the North Korean road network is 100% visible. At 74 degrees, coverage is approximately 70%. The average visibility within that band is 90%, meaning that on a typical pass through that band a satellite will observe 90% of the North Korean road network.