Appendix: Proof of Reported Results

This appendix contains proofs of the equilibria and propositions presented in the main text. The equilibria are derived using the perfect Bayesian equilibrium solution concept: each actor's strategies are best responses to each other at all decision nodes given their beliefs, which must be derived using Bayes' rule.

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1 Baseline Model: Bargaining with a Third-Party Rival

This section presents the equilibria and comparative statics in the baseline model specified and analyzed in Section 2 of the main text.

Proposition 1 *The equilibria of the baseline model are as characterized in Section 2 of the main text.*

We first establish the threshold of α that delineates supporters and abandoners (lemma 1) and the conditions under which P grants an alliance in the separating and pooling equilibria, respectively (lemma 2). Subsection 1.2 then characterizes the three separating equilibria, two with alliance at stake (lemmata 3 and 4) and one without (lemma 5). Subsection 1.3 characterizes the four equilibria in which supporters and abandoners pool on acceptance, one with alliance at stake (lemma 6) and three without (lemmata 7-9), as well as a fifth equilibrium that does not survive the divinity refinement (lemma 10). Subsection 1.4 characterizes the pooling equilibrium in which supporters and abandoners pool on rejection (lemma 11). Finally, Subsection 1.5 presents proofs of the formal propositions underpinning the results in the main text. With alliance at stake: 1) A's threshold for accepting R's offer increases (proposition 2); no offers are acceptable to A when alliance is sufficiently valuable (proposition 3); and the separating equilibria expand when alliance is sufficiently valuable (proposition 4).

1.1 General Thresholds

Lemma 1 *The threshold of α below which A would abandon its alliance commitment to P against R and above which it would sustain it is given by $\alpha^* \equiv \frac{c}{w}$.*

Proof. At the terminal node of the game given that P has granted an alliance, A decides whether to support P against R or not. If A abandons, P and R bargain bilaterally and reach a settlement at π. On the other hand, if A supports then its capabilities are added to P's, yielding a settlement with R that falls at $\pi + w$. These settlements are multiplied by α to determine A's settlement payoff. In addition, the expenditure of resources involved in supporting the alliance imposes cost c on A. Setting $\alpha\pi$ equal to $\alpha(\pi + w) - c$ and solving for α yields the threshold, α^*, above which A is a supporter and below which it is an abandoner:

$$\alpha^* \equiv \frac{c}{w} \tag{1}$$

Thus, by definition, $\overline{\alpha} > \frac{c}{w}$ and $\underline{\alpha} < \frac{c}{w}$. ■

Lemma 2 *In a pooling equilibrium A's posterior belief $q' = q$, and P allies iff $q' \geq \frac{u}{w} \equiv q^*$. In a separating equilibrium, P never allies in response to A's acceptance of R's offer, and allies in response to A's rejection of R's offer iff $w \geq u$.*

Proof. In a PE, P retains its prior beliefs about A's type, such that $q' = q$. Thus, its payoff for allying is $q(\pi + w) + (1 - q)(\pi) - u$, whereas its payoff for not allying is π. Setting these payoffs equal and solving for q yields q^*, the prior belief that A will abandon the alliance below which P allies in a PE:

$$q^* \equiv \frac{u}{w} \tag{2}$$

In an SE, P is completely informed of A's type. If A is an abandoner that has accepted R's offer, P's payoff for allying is $\pi - u$, which is always outweighed by its payoff for not allying of π. On the other hand, if A is a supporter that has rejected R's offer, P's payoff for allying is $\pi + w - u$, whereas its payoff for not allying is π. From these payoffs it is obvious that P allies iff $w \geq u$. ■

1.2 Separating Equilibria

1.2.1 SE — no alliance off path

Lemma 3 *The following strategies and beliefs form a PBE.*

***Strategies**:*

R: Offers $x^*_{\underline{A}} = w - \frac{k_A - u}{\phi\underline{\alpha}}$

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to rejection but not in response to acceptance. P does not ally off path in response to pooling on either acceptance or rejection following a sufficiently high or low offer from R, respectively.

***Beliefs**:*

R: $\rho < \min\left\{\frac{k_A - u + \underline{\alpha}k_R}{\underline{\alpha}(k_R+w)+k_A-u}, \frac{(\overline{\alpha}-\underline{\alpha})(k_A-u)+\underline{\alpha}(\overline{\alpha}w-c)}{\overline{\alpha}[\underline{\alpha}(k_R+w)+k_A-u]}\right\}$

P: Forms posterior beliefs $q' = 1$ *in response to rejection and* $q' = 0$ *in response to acceptance.*

***Conditions**:*

1. $q < \frac{u}{w}$
2. $\max\left\{u, \frac{(\overline{\alpha}-\underline{\alpha})(u-k_A)+\rho\overline{\alpha}(k_A-u+\underline{\alpha}k_R)+\underline{\alpha}c}{(1-\rho)\underline{\alpha}\overline{\alpha}}\right\} < w < \frac{(1-\rho)(k_A-u+\underline{\alpha}k_R)}{\underline{\alpha}\rho}$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 2, P does not ally in response to A's acceptance of R's offer in an SE, and allies in response to rejection iff $w > u$ (as specified in condition 2). Given that P would ally in response, rejection yields a payoff for the supporter of $\phi w\overline{\alpha} - k_A + \overline{\alpha}(\pi + w) + u - c$, whereas its payoff for acceptance given that P would not ally in response is $\phi\overline{\alpha}x + \overline{\alpha}\pi$. Setting these payoffs equal and solving for x reveals that the supporter accepts iff

$$x \geq w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}} = x^*_{\overline{A}} \tag{3}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer high enough to be accepted by the supporter will also be accepted by the less-resolved abandoner. On the other hand, the abandoner's payoff for accepting a lower offer that the supporter would reject is $\phi\underline{\alpha}x + \underline{\alpha}\pi$, while its payoff for rejecting — given that P would ally in response — would be $\phi w\underline{\alpha} - k_A + \underline{\alpha}\pi + u$. Setting these payoffs equal and solving for x reveals that the abandoner rejects iff

$$x \leq w - \frac{k_A - u}{\phi\underline{\alpha}} = x^*_{\underline{A}} \tag{4}$$

R clearly prefers $x^*_{\overline{A}}$ to any greater offer, and prefers $x^*_{\underline{A}}$ to any offer $x \in \left(x^*_{\underline{A}}, x^*_{\overline{A}}\right)$. At its decision node, R's plausible equilibrium offers are therefore $x^*_{\overline{A}}$, $x^*_{\underline{A}}$, and a range of lower offers $x < x^*_{\underline{A}}$ that will be rejected by both types (among which R is indifferent). R's expected payoff from offer $x^*_{\overline{A}}$ given that P holds prior beliefs that preclude alliance in response to pooling on acceptance is

$$U_R\left(x^*_{\overline{A}}\right) = \phi(1 - x^*_{\overline{A}}) + 1 - \pi \tag{5}$$

and its expected payoff from offer $x^*_{\underline{A}}$ given that P will conditionally ally in response is

$$U_R\left(x^*_{\underline{A}}\right) = \rho\left[\phi\left(1-w\right) - k_R + 1 - \pi - w\right] + \left(1-\rho\right)\left[\phi\left(1 - x^*_{\underline{A}}\right) + 1 - \pi\right] \tag{6}$$

Setting these equal and solving for ρ reveals that R prefers offer $x^*_{\underline{A}}$ to offer $x^*_{\overline{A}}$ iff

$$\rho < \frac{\phi\left(x^*_{\overline{A}} - x^*_{\underline{A}}\right)}{k_R + w - \phi\left(x^*_{\underline{A}} - w\right)}$$

Substituting line 3 for $x^*_{\overline{A}}$ and line 4 for $x^*_{\underline{A}}$ and reducing yields

$$\rho < \frac{\left(\overline{\alpha} - \underline{\alpha}\right)\left(k_A - u\right) + \underline{\alpha}\left(\overline{\alpha}w - c\right)}{\overline{\alpha}\left[\underline{\alpha}\left(k_R + w\right) + k_A - u\right]} \tag{7}$$

which is equivalent to

$$w > \frac{\left(\overline{\alpha} - \underline{\alpha}\right)\left(u - k_A\right) + \rho\overline{\alpha}\left(k_A - u + \underline{\alpha}k_R\right) + \underline{\alpha}c}{\left(1-\rho\right)\underline{\alpha}\overline{\alpha}}$$

Finally, we check to see if R prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$. Given that P holds priors that will prompt it not to ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1-w) - k_R + 1 - \pi$$

Setting this equal to R's payoff from $x^*_{\underline{A}}$ in line 6 reveals that R prefers $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$ iff

$$\rho < \frac{k_R + \phi\left(w - x^*_{\underline{A}}\right)}{k_R + w + \phi\left(w - x^*_{\underline{A}}\right)}$$

Substituting line 4 for $x^*_{\underline{A}}$ and reducing yields

$$\rho < \frac{k_A - u + \underline{\alpha}k_R}{\underline{\alpha}\left(k_R + w\right) + k_A - u}$$

which is equivalent to

$$w < \frac{\left(1-\rho\right)\left(k_A - u + \underline{\alpha}k_R\right)}{\underline{\alpha}\rho}$$

■

1.2.2 SE — alliance off path

Lemma 4 *The following strategies and beliefs form a PBE.*

***Strategies**:*

R: Offers $x^*_{\underline{A}} = w - \frac{k_A - u}{\phi\underline{\alpha}}$

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha} + u - c}{\phi\overline{\alpha}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to rejection but not in response to acceptance. P allies off path in response to pooling on either acceptance or rejection following a sufficiently high or low offer from R, respectively.

***Beliefs**:*

R: $\rho \leq \frac{(\overline{\alpha}-\underline{\alpha})(k_A-u)+\underline{\alpha}(\overline{\alpha}w-c)}{\overline{\alpha}(\underline{\alpha}k_R+k_A-u)}$

P: Forms posterior beliefs $q'=1$ *in response to rejection and* $q'=0$ *in response to acceptance.*

***Conditions**:*

1. $q \geq \frac{u}{w}$
2. $w \geq \max\left\{u, \frac{u-k_A}{\underline{\alpha}} - k_R, \frac{(\overline{\alpha}-\underline{\alpha})(u-k_A)+\rho\overline{\alpha}(k_A-u+\underline{\alpha}k_R)+\underline{\alpha}c}{\overline{\alpha}\underline{\alpha}}\right\}$

Proof. We solve by backward induction. A's payoffs are identical to lemma 3, yielding the same threshold of $x^*_{\overline{A}}$ as in line 3 and the same threshold of $x^*_{\underline{A}}$ as in line 4:

$$x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}}$$

$$x^*_{\underline{A}} = w - \frac{k_A-u}{\phi\underline{\alpha}}$$

As above, R's plausible equilibrium offers are $x^*_{\overline{A}}$, $x^*_{\underline{A}}$, and a lower offer $x < x^*_{\underline{A}}$ that will be rejected by both types. R's expected payoff from offer $x^*_{\overline{A}}$ given that P holds prior beliefs that support alliance in response to pooling on acceptance is

$$U_R\left(x^*_{\overline{A}}\right) = \phi(1-x^*_{\overline{A}}) + 1 - \pi - \rho w \tag{8}$$

and its expected payoff from offer $x^*_{\underline{A}}$ given that P will conditionally ally in response is the same as in line 6:

$$U_R\left(x^*_{\underline{A}}\right) = \rho\left[\phi\left(1-w\right) - k_R + 1 - \pi - w\right] + (1-\rho)\left[\phi\left(1-x^*_{\underline{A}}\right) + 1 - \pi\right]$$

Setting these equal and solving for ρ reveals that R prefers offer $x^*_{\underline{A}}$ to offer $x^*_{\overline{A}}$ iff

$$\rho < \frac{\phi\left(x^*_{\overline{A}} - x^*_{\underline{A}}\right)}{k_R - \phi\left(x^*_{\underline{A}} - w\right)}$$

Substituting line 3 for $x^*_{\overline{A}}$ and line 4 for $x^*_{\underline{A}}$ and reducing yields

$$\rho < \frac{\left(\overline{\alpha}-\underline{\alpha}\right)\left(k_A-u\right) + \underline{\alpha}\left(\overline{\alpha}w-c\right)}{\overline{\alpha}\left(\underline{\alpha}k_R + k_A - u\right)} \tag{9}$$

which is equivalent to

$$w > \frac{\left(\overline{\alpha}-\underline{\alpha}\right)\left(u-k_A\right) + \rho\overline{\alpha}\left(k_A - u + \underline{\alpha}k_R\right) + \underline{\alpha}c}{\underline{\alpha}\overline{\alpha}}$$

Finally, we check to see if R prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$. Given that P holds prior beliefs that support alliance in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1-w) - k_R + 1 - \pi - w$$

Setting this equal to R's payoff from $x^*_{\underline{A}}$ in line 6 reveals that R prefers $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$ iff

$$k_R > \phi\left(x^*_{\underline{A}} - w\right) - w$$

Substituting line 4 for $x^*_{\underline{A}}$ and reducing yields

$$w > \frac{u-k_A}{\underline{\alpha}} - k_R$$

■

1.2.3 SE — P never allies

Lemma 5 *The following strategies and beliefs form a PBE.*

***Strategies**:*

R: Offers $x^*_{\underline{A}} = w - \frac{k_A}{\phi\underline{\alpha}}$

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Never allies.

***Beliefs**:*

R: $\rho < \frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$

P: Forms posterior beliefs $q' = 1$ *in response to rejection and* $q' = 0$ *in response to acceptance.*

***Conditions**:*

1. $w < u$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 2, P never allies when $w < u$ (as given in condition 1). Thus, rejection yields a payoff for the supporter of $\phi w\overline{\alpha} - k_A + \overline{\alpha}\pi$, whereas its payoff for acceptance is $\phi\overline{\alpha}x + \overline{\alpha}\pi$. Setting these payoffs equal and solving for x reveals that the supporter accepts iff

$$x \geq w - \frac{k_A}{\phi\overline{\alpha}} = x^*_{\overline{A}} \tag{10}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer high enough to be accepted by the supporter will also be accepted by the less-resolved abandoner. On the other hand, the abandoner's payoff for accepting a lower offer that the supporter would reject, given that P would not ally in response, is $\phi\underline{\alpha}x + \underline{\alpha}\pi$, while its payoff for rejecting would be $\phi w\underline{\alpha} - k_A + \underline{\alpha}\pi$. Setting these payoffs equal and solving for x reveals that the abandoner rejects iff

$$x \leq w - \frac{k_A}{\phi\underline{\alpha}} = x^*_{\underline{A}} \tag{11}$$

As above, R's plausible equilibrium offers are $x^*_{\overline{A}}$, $x^*_{\underline{A}}$, and a lower offer $x < x^*_{\underline{A}}$ that will be rejected by both types. R's expected payoff from offer $x^*_{\overline{A}}$ given that P holds prior beliefs that preclude alliance in response to pooling on acceptance is the same as in line 5:

$$U_R\left(x^*_{\overline{A}}\right) = \phi(1 - x^*_{\overline{A}}) + 1 - \pi$$

and its expected payoff from offer $x^*_{\underline{A}}$ given that P will not ally in response to either action by A is

$$U_R\left(x^*_{\underline{A}}\right) = \rho\left[\phi\left(1 - w\right) - k_R\right] + \left(1 - \rho\right)\phi\left(1 - x^*_{\underline{A}}\right) + 1 - \pi \tag{12}$$

Setting these equal and solving for ρ reveals that R prefers offer $x^*_{\underline{A}}$ to offer $x^*_{\overline{A}}$ iff

$$\rho < \frac{\phi\left(x^*_{\overline{A}} - x^*_{\underline{A}}\right)}{k_R - \phi\left(x^*_{\underline{A}} - w\right)}$$

Substituting line 10 for $x^*_{\overline{A}}$ and line 11 for $x^*_{\underline{A}}$ and reducing yields

$$\rho < \frac{\overline{\alpha} - \underline{\alpha}}{\underline{\alpha} k_R + k_A} \frac{k_A}{\overline{\alpha}} \tag{13}$$

Finally, we check to see if R prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$. Given that P will not ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1 - w) - k_R + 1 - \pi$$

Setting this equal to R's payoff from $x^*_{\underline{A}}$ in line 12 reveals that R prefers $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$ iff

$$k_R > \phi\left(x^*_{\underline{A}} - w\right)$$

Substituting line 11 for $x^*_{\underline{A}}$ and reducing reveals that R always prefers $x^*_{\underline{A}}$ to $x < x^*_{\underline{A}}$ in this equilibrium. ■

1.3 Pooling Equilibria on Acceptance

1.3.1 Acceptance PE — alliance off path, not on path

Lemma 6 *The following strategies and beliefs form a PBE.*

***Strategies**:*

R: Offers $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}}$

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}} = w - \frac{k_A - u}{\phi\underline{\alpha}}$. *Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to rejection off path, but not in response to acceptance on path.

***Beliefs**:*

R: $\rho \geq \frac{(\overline{\alpha}-\underline{\alpha})(k_A-u)+\underline{\alpha}(\overline{\alpha}w-c)}{\overline{\alpha}[\underline{\alpha}(k_R+w)+k_A-u]}$

P: On path P retains its prior beliefs $q < \frac{u}{w}$. *In response to rejection off path it forms posterior beliefs* $q' \geq \frac{u}{w}$.

***Conditions**:*

1. $q < \frac{u}{w}$
2. $u \leq w \leq \min\left\{k_R + \frac{k_A-u+c}{\overline{\alpha}}, \frac{\phi\overline{\alpha}+k_A-u+c}{(1+\phi)\overline{\alpha}}, \frac{(\overline{\alpha}-\underline{\alpha})(u-k_A)+\rho\overline{\alpha}(k_A-u+\underline{\alpha}k_R)+\underline{\alpha}c}{(1-\rho)\underline{\alpha}\overline{\alpha}}\right\}$

Proof. This equilibrium is the complement of the one in lemma 3. A's conditional payoffs are identical in each, yielding thresholds of x each type will accept that are the same as lines 3 and 4: $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}}$ and $x^*_{\underline{A}} = w - \frac{k_A-u}{\phi\underline{\alpha}}$, respectively. R's expected payoffs are also identical to lemma 3, but its beliefs fall on the opposite side of the threshold in line 7, such that it prefers offer $x^*_{\overline{A}}$ to offer $x^*_{\underline{A}}$:

$$\rho \geq \frac{(\overline{\alpha} - \underline{\alpha})(k_A - u) + \underline{\alpha}(\overline{\alpha}w - c)}{\overline{\alpha}[\underline{\alpha}(k_R + w) + k_A - u]}$$

which is equivalent to

$$w \leq \frac{(\overline{\alpha} - \underline{\alpha})(u - k_A) + \rho\overline{\alpha}(k_A - u + \underline{\alpha}k_R) + \underline{\alpha}c}{(1-\rho)\underline{\alpha}\overline{\alpha}}$$

Finally, we check to see if R prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$. Given that P holds priors that will prompt it not to ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1-w) - k_R + 1 - \pi$$

Setting this equal to R's payoff from $x^*_{\overline{A}}$ in line 5 reveals that R prefers $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$ iff

$$w \geq x^*_{\overline{A}} - \frac{k_R}{\phi}$$

Substituting line 3 for $x^*_{\overline{A}}$ and reducing yields

$$w \leq k_R + \frac{k_A - u + c}{\overline{\alpha}}$$

An additional condition for this equilibrium to hold is that settlements must exist that the supporter prefers to war. Setting line 3 equal to 1 reveals that there are settlements supporters prefer to war iff

$$w \leq \frac{\phi\overline{\alpha} + k_A - u + c}{(1+\phi)\overline{\alpha}}$$

Finally, w must of course be greater than u for P to ally in response to A's rejection of the separating offer off path, as specified in condition 2. ■

1.3.2 Acceptance PE — alliance on and off path

Lemma 7 *The following strategies and beliefs form a PBE.*

***Strategies**:*

R: Offers $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}}$.

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}} = w - \frac{k_A - u}{\phi\underline{\alpha}}$. *Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to both acceptance on path and rejection off path.

***Beliefs**:*

R: $\rho \geq \frac{(\overline{\alpha}-\underline{\alpha})k_A - \overline{\alpha}u}{\overline{\alpha}[\underline{\alpha}k_R + k_A - u]}$

P: On path P retains its prior beliefs $\left(q' \geq \frac{u}{w}\right)$. *In response to rejection off path it forms posterior beliefs* $q' \geq \frac{u}{w}$.

***Conditions**:*

1. $q \geq \frac{u}{w}$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From equation 2, P allies in response to pooling iff $q \geq \frac{u}{w}$ (which is given in

condition 1) and allies off path if it forms off-path beliefs $q' \geq \frac{u}{w}$. Given that P holds priors that would lead it to ally in response, pooling on acceptance yields a payoff for the supporter of $\phi\overline{\alpha}x + \overline{\alpha}(\pi + w) + u - c$, whereas its payoff for rejection given that P would also ally off path in response is $\phi w\overline{\alpha} - k_A + \overline{\alpha}(\pi + w) + u - c$. Setting these payoffs equal and solving for x reveals that the minimum offer the supporter accepts is the same as in line 10:

$$x \geq w - \frac{k_A}{\phi\overline{\alpha}} = x^*_{\overline{A}}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer high enough to be accepted by the supporter will also be accepted by the less-resolved abandoner. At its decision node, R's plausible equilibrium offers are $x^*_{\overline{A}}$, separating offer $x^*_{\underline{A}}$ (the minimum offer that will be accepted by only the abandoner), and a lower offer $x < x^*_{\underline{A}}$ that will be rejected by both types.[1] Off path following the separating offer P would form off-path beliefs of $q' = 1$ (certainty that A is a supporter) in response to rejection and of $q' = 0$ (certainty that A is an abandoner) in response to acceptance. Given $w > u$ (as necessitated by condition 1), P therefore allies iff A rejects. This yields payoffs for the abandoner of $\phi\underline{\alpha}x + \underline{\alpha}\pi$ for accepting offer $x^*_{\underline{A}}$, while its payoff for rejecting it is $\phi w\underline{\alpha} - k_A + \underline{\alpha}\pi + u$. Setting these payoffs equal and solving for x reveals that the abandoner's threshold for rejection off path is the same as in line 4:

$$x \leq w - \frac{k_A - u}{\phi\underline{\alpha}} = x^*_{\underline{A}}$$

R's expected payoff from offer $x^*_{\overline{A}}$ given that P will ally in response to A's pooling behavior is the same as in line 8:

$$U_R\left(x^*_{\overline{A}}\right) = \phi(1 - x^*_{\overline{A}}) + 1 - \pi - \rho w$$

and its expected payoff from offer $x^*_{\underline{A}}$ given that P will conditionally ally in response when $w > u$ is the same as in line 6:

$$U_R\left(x^*_{\underline{A}}\right) = \rho\left[\phi\left(1 - w\right) - k_R + 1 - \pi - w\right] + (1 - \rho)\left[\phi\left(1 - x^*_{\underline{A}}\right) + 1 - \pi\right]$$

Setting these equal and solving for ρ reveals that R prefers offer $x^*_{\overline{A}}$ to offer $x^*_{\underline{A}}$ iff

$$\rho \geq \frac{\phi\left(x^*_{\overline{A}} - x^*_{\underline{A}}\right)}{k_R - \phi\left(x^*_{\underline{A}} - w\right)}$$

Substituting lines 10 and 4 for $x^*_{\overline{A}}$ and $x^*_{\underline{A}}$, respectively, and reducing yields

$$\rho \geq \frac{\left(\overline{\alpha} - \underline{\alpha}\right)k_A - \overline{\alpha}u}{\overline{\alpha}\left[\underline{\alpha}k_R + k_A - u\right]} \tag{14}$$

Finally, we check to see if R prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$. Given that P holds priors that will prompt it to ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1 - w) - k_R + 1 - \pi - \rho w$$

Setting this equal to R's payoff from $x^*_{\overline{A}}$ in line 8 reveals that R strictly prefers $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$. ■

1.3.3 Acceptance PE — no alliance on or off path

Lemma 8 *The following strategies and beliefs form a PBE.*

Strategies:

R: Offers $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}}$

[1] This follows the same argument as the proof of lemma 3

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}} = w - \frac{k_A - u}{\phi \underline{\alpha}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Does not ally in response to either acceptance on path or rejection off path.

***Beliefs**:*

R: $\rho \geq \frac{(\overline{\alpha} - \underline{\alpha}) k_A - \overline{\alpha} u}{\overline{\alpha}[\underline{\alpha}(k_R + w) + k_A - u]}$

P: On path P retains its prior beliefs $\left(q' < \frac{u}{w}\right)$*. In response to rejection off path it forms posterior beliefs* $q' < \frac{u}{w}$*.*

***Conditions**:*

1. $q < \frac{u}{w}$

2. $w \geq u$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From equation 2, P allies in response to pooling iff $q \geq \frac{u}{w}$ and allies off path iff it forms off-path beliefs $q' \geq \frac{u}{w}$. Given that P would not ally either on or off path, the supporter's payoff are the same as in lemma 5. The minimum offer the supporter accepts is therefore the same as in line 10:

$$x \geq w - \frac{k_A}{\phi \overline{\alpha}} = x^*_{\overline{A}}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer high enough to be accepted by the supporter will also be accepted by the less-resolved abandoner. As above, R's plausible equilibrium offers are $x^*_{\overline{A}}$, separating offer $x^*_{\underline{A}}$ (the minimum offer that will be accepted by only the abandoner), and a lower offer $x < x^*_{\underline{A}}$ that will be rejected by both types. The abandoner's payoffs given that P will not ally off path are identical to those in lemma 7, yielding the same threshold for rejection off path (from line 4) of

$$x \leq w - \frac{k_A - u}{\phi \underline{\alpha}} = x^*_{\underline{A}}$$

R's expected payoff from offer $x^*_{\overline{A}}$ given that P will not ally in response to A's pooling behavior is the same as in line 5:

$$U_R\left(x^*_{\overline{A}}\right) = \phi(1 - x^*_{\overline{A}}) + 1 - \pi$$

and its expected payoff from offer $x^*_{\underline{A}}$ given that P will conditionally ally in response is the same as in line 6:

$$U_R\left(x^*_{\underline{A}}\right) = \rho\left[\phi\left(1 - w\right) - k_R + 1 - \pi - w\right] + (1 - \rho)\left[\phi\left(1 - x^*_{\underline{A}}\right) + 1 - \pi\right]$$

Setting these equal and solving for ρ reveals that R prefers offer $x^*_{\overline{A}}$ to offer $x^*_{\underline{A}}$ iff

$$\rho \geq \frac{\phi\left(x^*_{\overline{A}} - x^*_{\underline{A}}\right)}{k_R + w - \phi\left(x^*_{\underline{A}} - w\right)}$$

Substituting line 10 for $x^*_{\overline{A}}$ and line 4 for $x^*_{\underline{A}}$ and reducing yields

$$\rho \geq \frac{\left(\overline{\alpha} - \underline{\alpha}\right) k_A - \overline{\alpha} u}{\overline{\alpha}\left[\underline{\alpha}\left(k_R + w\right) + k_A - u\right]} \tag{15}$$

Finally, we check to see if R prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$. Given that P holds priors that will prompt it not to ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1-w) - k_R + 1 - \pi$$

Setting this equal to R's payoff from $x^*_{\overline{A}}$ in line 5 reveals that R strictly prefers $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$. ■

1.3.4 Acceptance PE — P never allies

Lemma 9 *The following strategies and beliefs form a PBE.*

***Strategies**:*

R: Offers $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}}$

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}} = w - \frac{k_A}{\phi\underline{\alpha}}$. *Conditional on P granting an alliance,* $\overline{A}$ *would support and* $\underline{A}$ *would abandon at the terminal node.*

P: Never allies.

***Beliefs**:*

R: $\rho \geq \frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$

P: Retains prior beliefs in response to pooling on acceptance; off-path beliefs unrestricted.

***Conditions**:*

1. $w < u$

Proof. Because P will never ally when $w < u$, A's payoffs are identical to lemma 5. Thus, from lines 10 and 11, respectively, $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}}$ and $x^*_{\underline{A}} = w - \frac{k_A}{\phi\underline{\alpha}}$. R's expected payoffs are also identical to lemma 5, but its beliefs fall on the opposite side of the threshold in line 13, such that it prefers offer $x^*_{\overline{A}}$ to offer $x^*_{\underline{A}}$:

$$\rho \geq \frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$$

Finally, we check to see if R prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$. Given that P will not ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1-w) - k_R + 1 - \pi$$

Setting this equal to R's payoff from $x^*_{\overline{A}}$ in line 5 reveals that R prefers $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$ iff

$$x^*_{\overline{A}} < w + \frac{k_R}{\phi}$$

Substituting line 10 for $x^*_{\overline{A}}$ and reducing reveals that R always prefers $x^*_{\overline{A}}$ to $x < x^*_{\underline{A}}$ in this equilibrium. ■

1.3.5 Acceptance PE — alliance on path, not off path (not supported)

Lemma 10 *Any PBE with the following characteristics is eliminated by the divine equilibrium refinement.*

***Strategies**:*

R: Offers $x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}}$.

A: On A's initial move, both types accept R's equilibrium offer. Conditional on P granting an alliance, $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to acceptance on path and does not ally in response to rejection off path.

***Beliefs**:*

R: $\rho \geq \frac{(\overline{\alpha}-\underline{\alpha})k_A - \overline{\alpha}u}{\overline{\alpha}(\underline{\alpha}k_R + k_A - u)}$

P: On path P retains its prior beliefs $q \geq \frac{u}{w}$. *In response to rejection off path it forms posterior beliefs* $q' < \frac{u}{w}$.

***Conditions**:*

1. $q \geq \frac{u}{w}$

Proof. This equilibrium requires P to update its beliefs in response to A's rejection of R's offer that A is more likely to be an abandoner than P originally believed was the case. However, such beliefs violate the D_1-criterion, because the supporter benefits more from rejection than the abandoner would and should thus be seen as more likely to reject. We can therefore eliminate this equilibrium as requiring implausible beliefs. ■

1.4 Pooling Equilibrium on Rejection

Lemma 11 *The following strategies and beliefs form a PBE*

***Strategies**:*

R: Offers $x < w - \frac{k_A}{\phi\underline{\alpha}}$.

A: On A's initial move, the supporter accepts R's offer iff $x \geq x^*_{\overline{A}} = w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}}$ *and the abandoner accepts R's offer iff* $x \geq x^*_{\underline{A}} = w - \frac{k_A}{\phi\underline{\alpha}}$. *Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Does not ally in response to rejection or acceptance following on-path offer $x < w - \frac{k_A}{\phi\underline{\alpha}}$. *Does ally in response to rejection of an off-path offer* $x \geq w - \frac{k_A}{\phi\underline{\alpha}}$, *but not in response to acceptance of such an offer.*

***Beliefs**:*

R: $\rho \geq \frac{k_A + \underline{\alpha}k_R}{k_A + \underline{\alpha}k_R + w\underline{\alpha}}$

P: On path P retains its prior beliefs ($q' < \frac{u}{w}$). *In response to acceptance off path it forms posterior beliefs* $q' < \frac{u}{w}$.

***Conditions**:*

1. $q < \frac{u}{w}$
2. $w > \min\left\{\frac{\phi\overline{\alpha}+k_A-u+c}{(1+\phi)\overline{\alpha}}, k_R + \frac{k_A+u-c}{\overline{\alpha}}\right\}$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From line 2, P allies in response to pooling iff $q \geq \frac{u}{w}$ and allies off path iff it forms off-path beliefs $q' \geq \frac{u}{w}$. To find the maximum offer the abandoner would reject, given that the supporter would necessarily reject that offer, we compare the abandoner's payoff for pooling on rejection to its payoff for accepting. Given that P would not ally in either case, the abandoner's payoff for rejection is $\phi\underline{\alpha}x + \underline{\alpha}\pi$, while its payoff for accepting would be $\phi w\underline{\alpha} - k_A + \underline{\alpha}\pi$. This yields the same threshold as in line 11:

$$x \leq w - \frac{k_A}{\phi\underline{\alpha}} = x^*_{\underline{A}}$$

To find the minimum offer the supporter would accept, given that the abandoner would necessarily accept the same offer, we compare the supporter's payoff for pooling on acceptance to its payoff for separating by rejecting the offer. Given that P holds priors that would lead it not to ally in response, pooling on acceptance yields payoffs for the supporter of $\phi\overline{\alpha}x + \overline{\alpha}\pi$. On the other hand, since $w \geq u$ (as implied by condition 1), rejecting an off-path offer that would separate from the abandoner would result in alliance, yielding payoffs to the supporter of $\phi w\overline{\alpha} - k_A + \overline{\alpha}(\pi + w) + u - c$. Setting these payoffs equal reveals that the minimum offer the supporter accepts is the same as in line 3:

$$x \geq w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha} + u - c}{\phi\overline{\alpha}} = x^*_{\overline{A}}$$

As above, R's plausible equilibrium offers are $x^*_{\overline{A}}$, separating offer $x^*_{\underline{A}}$, and a lower offer $x < x^*_{\underline{A}}$ that will be rejected by both types. R's expected payoff from separating offer $x^*_{\underline{A}}$ given that P will conditionally ally in response is the same as in line 6:

$$U_R\left(x^*_{\underline{A}}\right) = \rho\left[\phi\left(1-w\right) - k_R + 1 - \pi - w\right] + \left(1-\rho\right)\left[\phi\left(1 - x^*_{\underline{A}}\right) + 1 - \pi\right]$$

Given that P holds priors that will prompt it not to ally in response to pooling on reject, R's expected payoff from an offer $x < x^*_{\underline{A}}$ is

$$U_R\left(x < x^*_{\underline{A}}\right) = \phi(1-w) - k_R + 1 - \pi$$

Setting these equal and solving for ρ reveals that R prefers offer $x < x^*_{\underline{A}}$ to offer $x^*_{\underline{A}}$ iff

$$\rho \geq \frac{k_R + \phi\left(w - x^*_{\underline{A}}\right)}{w + k_R + \phi\left(w - x^*_{\underline{A}}\right)}$$

Substituting line 11 for $x^*_{\underline{A}}$ and reducing yields

$$\rho \geq \frac{k_A + \underline{\alpha}k_R}{k_A + \underline{\alpha}k_R + w\underline{\alpha}} \tag{16}$$

Finally, we examine the conditions under which R prefers $x < x^*_{\underline{A}}$ to $x^*_{\overline{A}}$. R's expected payoff from offer $x^*_{\overline{A}}$ given that P will not ally in response is

$$U_R\left(x^*_{\overline{A}}\right) = \phi\left(1 - x^*_{\overline{A}}\right) + 1 - \pi$$

Setting this equal to R's payoff for $x < x^*_{\underline{A}}$ and substituting line 3 for $x^*_{\overline{A}}$ reveals that R prefers $x < x^*_{\underline{A}}$ to $x^*_{\overline{A}}$ iff

$$w > k_R + \frac{k_A + u - c}{\overline{\alpha}}$$

In addition, when $w > \frac{\phi\overline{\alpha}+k_A-u+c}{(1+\phi)\overline{\alpha}}$ (as stipulated in condition 2), $x^*_{\overline{A}} > 1$, such that this offer cannot be in equilibrium. ■

1.5 Comparative Statics

1.5.1 A's threshold for acceptance

Proposition 2 *When an alliance with P is at stake,*[2] *both supporters and abandoners have higher thresholds for accepting R's offer.*

Proof. From the main text, alliance is defined to be at stake when P will ally in response to A's rejection of R's offer but not in response to its acceptance. By this definition, alliance is at stake on path in the equilibria characterized in lemmata 3, 4, and 6. Conversely, alliance is not at stake on path in the equilibria characterized in lemmata 5, 7, 8, and 9.

It is straightforward to show that proposition 2 holds for the abandoner. With alliance at stake, the abandoner's payoff for accepting R's offer (which would result in no alliance) is $\phi\underline{\alpha}x + \underline{\alpha}\pi$, and its payoff for rejecting (which would elicit an alliance) is $\phi\underline{\alpha}w - k_A + \underline{\alpha}\pi + u$. Setting these equal and solving for x yields $\overline{x^*_{\underline{A}}}$, the minimum offer the abandoner will accept with alliance at stake:

$$\overline{x^*_{\underline{A}}} \equiv w - \frac{k_A - u}{\phi\underline{\alpha}} \tag{17}$$

This is identical to the threshold of $x^*_{\underline{A}}$ given in line 4. Conversely, alliance is not at stake if P would either 1) ally in response to both acceptance and rejection; 2) not ally in response to both acceptance and rejection. In the former case, the abandoner's payoff for accepting R's offer is $\phi\underline{\alpha}x + \underline{\alpha}\pi + u$, and its payoff for rejecting is $\phi\underline{\alpha}w - k_A + \underline{\alpha}\pi + u$. In the latter case, the abandoner's payoff for accepting R's offer is $\phi\underline{\alpha}x + \underline{\alpha}\pi$, and its payoff for rejecting is $\phi\underline{\alpha}w - k_A + \underline{\alpha}\pi$. In either case, setting these payoffs equal and solving for x yields $\underline{x^*_{\underline{A}}}$, the minimum offer the abandoner will accept with no alliance at stake:

$$\underline{x^*_{\underline{A}}} \equiv w - \frac{k_A}{\phi\underline{\alpha}} \tag{18}$$

This is identical to the threshold of $x^*_{\underline{A}}$ given in line 11. Comparing the two, $\overline{x^*_{\underline{A}}}$ is clearly greater than $\underline{x^*_{\underline{A}}}$, such that abandoners have a greater incentive to reject R's offer with alliance at stake.

The same directional effect obtains for the supporter. With alliance at stake, the supporter's payoff for accepting R's offer (which would result in no alliance) is $\phi\overline{\alpha}x + \overline{\alpha}\pi$, and its payoff for rejecting (which would elicit an alliance) is $\phi\overline{\alpha}w - k_A + \overline{\alpha}(\pi + w) + u - c$. Setting these equal and solving for x yields $\overline{x^*_{\overline{A}}}$, the minimum offer the supporter will accept with alliance at stake:

$$\overline{x^*_{\overline{A}}} \equiv w - \frac{k_A}{\phi\overline{\alpha}} + \frac{w\overline{\alpha} + u - c}{\phi\overline{\alpha}} \tag{19}$$

This is identical to the threshold of $x^*_{\overline{A}}$ given in line 3. Now consider the supporter's payoffs with no alliance at stake. If P would ally in response to both acceptance and rejection, the supporter's payoff for accepting R's offer is $\phi\overline{\alpha}x + \overline{\alpha}(\pi + w) + u - c$, and its payoff for rejecting is $\phi\overline{\alpha}w - k_A + \overline{\alpha}(\pi + w) + u - c$. If P would not ally in response to both acceptance and rejection, the supporter's payoff for accepting R's offer is $\phi\overline{\alpha}x + \overline{\alpha}\pi$, and its payoff for rejecting is $\phi\overline{\alpha}w - k_A + \overline{\alpha}\pi$. In either case, setting these payoffs equal and solving for x yields $\underline{x^*_{\overline{A}}}$, the minimum offer the supporter will accept with no alliance at stake:

$$\underline{x^*_{\overline{A}}} \equiv w - \frac{k_A}{\phi\overline{\alpha}} \tag{20}$$

This is identical to the threshold of $x^*_{\overline{A}}$ given in line 10. Again, $\overline{x^*_{\overline{A}}}$ is strictly greater than $\underline{x^*_{\overline{A}}}$, since by definition $\overline{\alpha} > \frac{c}{w}$ (see lemma 1). Thus, supporters also have greater incentive to reject with alliance at stake than without. ■

[2] Recall that alliance is defined to be at stake when P will ally in response to A's rejection of R's offer but not in response to its acceptance.

1.5.2 Elimination of bargaining range

Proposition 3 *The prospect of an alliance with P completely eliminates the range of first-round offers acceptable to A iff* $w \geq \frac{\phi\overline{\alpha}-u+k_A+c}{(1+\phi)\overline{\alpha}}$ *for supporters and* $w \geq 1-\frac{u-k_A}{\phi\underline{\alpha}}$ *for abandoners.*

Proof. No bargain is acceptable to A when its minimum offer is ≥ 1. Lines 18 and 20 show that with no alliance at stake, A's minimum acceptable offer is $w-\frac{k_A}{\phi\overline{\alpha}} \equiv \underline{x_A^*}$ for supporters and $w-\frac{k_A}{\phi\underline{\alpha}} \equiv \underline{\underline{x_A^*}}$ for abandoners, both of which are clearly less than 1. Conversely, lines 17 and 19 show that with alliance at stake, the supporter's minimum acceptable offer increases to $\overline{\underline{x_A^*}} \equiv \underline{x_A^*}+\frac{w\overline{\alpha}+u-c}{\phi\overline{\alpha}}$, and the abandoner's minimum acceptable offer increases to $\overline{\underline{\underline{x_A^*}}} \equiv \underline{\underline{x_A^*}}+\frac{u}{\phi\underline{\alpha}}$. Setting $\overline{\underline{x_A^*}}$ and $\overline{\underline{\underline{x_A^*}}}$ equal to 1 reveals that supporters reject all offers iff $w \geq \frac{\phi\overline{\alpha}-u+k_A+c}{(1+\phi)\overline{\alpha}}$, while abandoners reject all offers iff $w \geq 1-\frac{u-k_A}{\phi\underline{\alpha}}$. ■

1.5.3 Expansion of SE

Proposition 4 *The parameter space supporting the SE with alliance at stake is broader than the parameter space supporting the equilibria without alliance at stake under the following conditions:*

1. *For the SE in lemma 3,* $w \in \left[\frac{k_A c+k_R[(\overline{\alpha}-\underline{\alpha})u+\underline{\alpha}c]}{\underline{\alpha}(k_A+\overline{\alpha}k_R)}, \frac{k_A-u+\underline{\alpha}k_R}{k_A}\frac{k_A+\overline{\alpha}k_R}{\overline{\alpha}-\underline{\alpha}}\right]$
2. *For the SE in lemma 4,* $w \geq \frac{k_A c+k_R[(\overline{\alpha}-\underline{\alpha})u+\underline{\alpha}c]}{\overline{\alpha}(k_A+\underline{\alpha}k_R)}$

Proof. We analyze the effect of having alliance at stake on the range of the SE by comparing the thresholds of ρ below which R makes the separating offer, x_A^*, across the SE with and without alliance at stake. The greater this threshold, the greater R's incentive to make the separating offer. This corresponds to greater credibility of A's signals to P, since A's signals are only informative in the SE.

The SE in which alliance is not at stake is the one characterized in lemma 5, where $w < u$. There, the threshold for R to make the separating offer is $\rho < \frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$. We compare this threshold to the thresholds of ρ in each of the other two separating equilibria where alliance is at stake to identify the conditions under which having alliance at stake increases the credibility of A's signals. In the equilibrium in lemma 3, R makes the separating offer iff $\rho \leq \min\left\{\frac{k_A-u+\underline{\alpha}k_R}{\underline{\alpha}(k_R+w)+k_A-u}, \frac{(\overline{\alpha}-\underline{\alpha})(k_A-u)+\underline{\alpha}(\overline{\alpha}w-c)}{\overline{\alpha}[\underline{\alpha}(k_R+w)+k_A-u]}\right\}$. $\frac{k_A-u+\underline{\alpha}k_R}{\underline{\alpha}(k_R+w)+k_A-u} \geq \frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$ iff

$$w \leq \frac{k_A-u+\underline{\alpha}k_R}{k_A}\frac{k_A+\overline{\alpha}k_R}{\overline{\alpha}-\underline{\alpha}}$$

and $\frac{(\overline{\alpha}-\underline{\alpha})(k_A-u)+\underline{\alpha}(\overline{\alpha}w-c)}{\overline{\alpha}[\underline{\alpha}(k_R+w)+k_A-u]} \geq \frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$ iff

$$w \geq \frac{k_A c+k_R\left[(\overline{\alpha}-\underline{\alpha})u+\underline{\alpha}c\right]}{\underline{\alpha}\left(k_A+\overline{\alpha}k_R\right)}$$

Combining these statements, the parameter space supporting the SE in lemma 3 expands iff

$$\frac{k_A c+k_R\left[(\overline{\alpha}-\underline{\alpha})u+\underline{\alpha}c\right]}{\underline{\alpha}\left(k_A+\overline{\alpha}k_R\right)} \leq w \leq \frac{k_A-u+\underline{\alpha}k_R}{k_A}\frac{k_A+\overline{\alpha}k_R}{\overline{\alpha}-\underline{\alpha}}$$

In the SE in lemma 4, R makes the separating offer iff $\rho < \frac{(\overline{\alpha}-\underline{\alpha})(k_A-u)+\underline{\alpha}(\overline{\alpha}w-c)}{\overline{\alpha}(\underline{\alpha}k_R+k_A-u)}$. This is greater than $\frac{\overline{\alpha}-\underline{\alpha}}{\underline{\alpha}k_R+k_A}\frac{k_A}{\overline{\alpha}}$ iff

$$w \geq \frac{k_A c+k_R\left[(\overline{\alpha}-\underline{\alpha})u+\underline{\alpha}c\right]}{\overline{\alpha}\left(k_A+\underline{\alpha}k_R\right)}$$

■

2 Extension: Bargaining with Patron's Existing Client

This section presents the equilibria and comparative statics in the extension of the baseline model in Section 3 of the main text, in which A initially bargains with P's existing client, C. In addition to the specification in the main text, note that C holds prior beliefs that A is a supporter with probability ψ and an abandoner with probability $1-\psi$.

Proposition 5 *The equilibria of the model extension are as characterized in Section 3 of the main text.*

We first establish the conditions under which P grants an alliance in the separating and pooling equilibria, respectively (lemma 13). Subsection 2.1 then characterizes the three separating equilibria, two with alliance at stake (lemmata 13 and 14) and one without (lemma 15). Subsection 2.2 characterizes the four equilibria in which supporters and abandoners pool on acceptance, one with alliance at stake (lemma 16) and three without (lemmata 17-19), as well as a fifth equilibrium that does not survive the divinity refinement (lemma 21). Subsection 2.3 shows that pooling on rejection is *not* supported (lemma 21). Finally, Subsection 2.4 presents a proof of the formal proposition underpinning Result 4 in the main text. With alliance at stake, A's threshold for accepting C's offer decreases, such that all offers are acceptable to A when alliance is sufficiently valuable (proposition 6).

Lemma 12 *In a pooling equilibrium, P allies iff $q' = q \geq \frac{u}{w} \equiv q^*$. In a separating equilibrium, P never allies in response to A's rejection of C's offer, and allies in response to A's acceptance of C's offer iff $w \geq u$.*

Proof. This follows from the same logic as lemma 2, except that in the SE supporters accept C's offer and abandoners reject it. The reverse was the case in lemma 2 under pre-alliance bargaining with R. ■

2.1 Separating Equilibria

2.1.1 SE — no alliance off path

Lemma 13 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\overline{A}} = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta} - \frac{w\overline{\alpha}+u-c}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}}$ and the abandoner accepts C's offer iff $x \geq x^*_{\underline{A}} = zw - \frac{f\underline{\alpha}+k_A+u}{\phi\beta}$. Conditional on P granting an alliance, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node.

P: Allies in response to acceptance but not in response to rejection. P does not ally off path in response to pooling on either acceptance or rejection following a sufficiently high or low offer from C, respectively.

***Beliefs**:*

C: $\psi \geq \frac{(\underline{\alpha}+\beta)f+\beta k_C+k_A+u}{(\overline{\alpha}+\beta)(f+w)+\beta k_C+k_A+u-c}$

P: Forms posterior beliefs $q' = 1$ in response to acceptance and $q' = 0$ in response to rejection.

***Conditions**:*

1. $q < \frac{u}{w}$
2. $w \geq u$
3. $k_C \geq \frac{(w+f)(\beta-\overline{\alpha})-k_A-u+c}{\beta}$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 12, P does not ally in response to A's rejection of C's offer in an SE, and allies in response to acceptance iff $w \geq u$ (as specified in condition 2). Given that P would ally in response, acceptance yields a payoff for the abandoner of $\phi\beta x + \underline{\alpha}\pi + u$, whereas its payoff for rejection given that P would not ally in response is $\phi z w \beta - k_A + \underline{\alpha}(\pi - f)$. Setting these payoffs equal and solving for x reveals that the abandoner rejects iff

$$x \leq zw - \frac{f\underline{\alpha} + k_A + u}{\phi\beta} = x^*_{\underline{A}} \tag{21}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer large enough to be accepted by the abandoner will also be accepted by the more-committed supporter. On the other hand, the supporter's payoff for accepting a lower offer that the abandoner would reject is $\phi\beta x + \overline{\alpha}(\pi + w) + u - c$, while its payoff for rejecting — given that P would not ally in response — would be $\phi z w \beta - k_A + \overline{\alpha}(\pi - f)$. Setting these payoffs equal and solving for x reveals that the supporter accepts iff

$$x \geq zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} - \frac{w\overline{\alpha} + u - c}{\phi\beta} = x^*_{\overline{A}} \tag{22}$$

C clearly prefers $x^*_{\underline{A}}$ to any greater offer, and prefers $x^*_{\overline{A}}$ to any offer $x \in \left(x^*_{\overline{A}}, x^*_{\underline{A}}\right)$. At its decision node, C's plausible equilibrium offers are therefore $x^*_{\underline{A}}$, $x^*_{\overline{A}}$, and a range of lower offers $x < x^*_{\overline{A}}$ that will be rejected by both types (among which C is indifferent). C's expected payoff from offer $x^*_{\underline{A}}$ given that P holds prior beliefs that preclude alliance in response to pooling on acceptance is

$$U_C\left(x^*_{\underline{A}}\right) = \phi\left(1 - x^*_{\underline{A}}\right) + \pi \tag{23}$$

and its expected payoff from offer $x^*_{\overline{A}}$ given that P will conditionally ally in response is

$$U_C\left(x^*_{\overline{A}}\right) = \psi\left[\phi\left(1 - x^*_{\overline{A}}\right) + \pi + w\right] + (1 - \psi)\left[\phi(1 - zw) - k_C + \pi - f\right] \tag{24}$$

Setting these equal and solving for ψ reveals that C prefers separating offer $x^*_{\overline{A}}$ to pooling offer $x^*_{\underline{A}}$ iff

$$\psi \geq \frac{k_C + f + \phi\left(zw - x^*_{\underline{A}}\right)}{k_C + f + w + \phi\left(zw - x^*_{\overline{A}}\right)}$$

Substituting line 21 for $x^*_{\underline{A}}$ and line 22 for $x^*_{\overline{A}}$ and reducing yields

$$\psi \geq \frac{(\underline{\alpha} + \beta) f + \beta k_C + k_A + u}{(\overline{\alpha} + \beta)(f + w) + \beta k_C + k_A + u - c}$$

Finally, we check to see if C prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$. Given that P holds priors that will prompt it not to ally in response to pooling on reject, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi(1 - zw) - k_C + \pi - f$$

Setting this equal to C's payoff from $x^*_{\overline{A}}$ in line 24 reveals that C prefers $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$k_C \geq f + w - \phi\left(zw - x^*_{\overline{A}}\right)$$

Substituting line 22 for $x^*_{\overline{A}}$ and reducing reveals that C prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$k_C \geq \frac{(w + f)(\beta - \overline{\alpha}) - k_A - u + c}{\beta}$$

■

2.1.2 SE — alliance off path

Lemma 14 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\overline{A}} = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta} - \frac{w\overline{\alpha}+u-c}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}}$ *and the abandoner accepts C's offer iff* $x \geq x^*_{\underline{A}} = zw - \frac{f\underline{\alpha}+k_A+u}{\phi\beta}$. *Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to acceptance but not in response to rejection. P allies off path in response to pooling on either acceptance or rejection following a sufficiently high or low offer from C, respectively.

***Beliefs**:*

C: $\psi \geq \frac{f\underline{\alpha}+\beta(k_C+f)+k_A+u}{\overline{\alpha}(w+f)+\beta(k_C+f)+k_A+u-c}$

P: Forms posterior beliefs $q' = 1$ *in response to acceptance and* $q' = 0$ *in response to rejection.*

***Conditions**:*

1. $q \geq \frac{u}{w}$

Proof. We solve by backward induction. A's payoffs are identical to lemma 13, yielding the same threshold of $x^*_{\underline{A}}$ as in line 21 and the same threshold of $x^*_{\overline{A}}$ as in line 22:

$$x \leq zw - \frac{f\underline{\alpha} + k_A + u}{\phi\beta} = x^*_{\underline{A}}$$

$$x \geq zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} - \frac{w\overline{\alpha} + u - c}{\phi\beta} = x^*_{\overline{A}}$$

As above, C's plausible equilibrium offers are $x^*_{\underline{A}}$, $x^*_{\overline{A}}$, and a range of lower offers $x < x^*_{\overline{A}}$ that will be rejected by both types. C's expected payoff from offer $x^*_{\underline{A}}$ given that P holds prior beliefs that support alliance in response to pooling on acceptance is

$$U_C\left(x^*_{\underline{A}}\right) = \phi\left(1 - x^*_{\underline{A}}\right) + \pi + \psi w \tag{25}$$

and its expected payoff from offer $x^*_{\overline{A}}$ given that P will conditionally ally in response is the same as in line 24

$$U_C\left(x^*_{\overline{A}}\right) = \psi\left[\phi\left(1 - x^*_{\overline{A}}\right) + \pi + w\right] + (1-\psi)\left[\phi\left(1 - zw\right) - k_C + \pi - f\right]$$

Setting these equal and solving for ψ reveals that C prefers separating offer $x^*_{\overline{A}}$ to pooling offer $x^*_{\underline{A}}$ iff

$$\psi \geq \frac{k_C + f + \phi\left(zw - x^*_{\underline{A}}\right)}{k_C + f + \phi\left(zw - x^*_{\overline{A}}\right)}$$

Substituting line 21 for $x^*_{\underline{A}}$ and line 22 for $x^*_{\overline{A}}$ and reducing yields

$$\psi \geq \frac{f\underline{\alpha} + \beta\left(k_C + f\right) + k_A + u}{\overline{\alpha}\left(w + f\right) + \beta\left(k_C + f\right) + k_A + u - c}$$

Finally, we check to see if C prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$. Given that P holds priors that will prompt it to ally in response to pooling on reject, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi\left(1 - zw\right) - k_C + \pi + \psi w - f$$

Setting this equal to C's payoff from $x^*_{\overline{A}}$ in line 24 reveals that C prefers $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$k_C \geq \phi\left(x^*_{\overline{A}} - zw\right) - f$$

Substituting line 22 for $x^*_{\overline{A}}$ and reducing reveals that the RHS of this inequality is negative, such that C strictly prefers $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$. ■

2.1.3 SE — P never allies

Lemma 15 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\overline{A}} = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}}$ *and the abandoner accepts C's offer iff* $x \geq x^*_{\underline{A}} = zw - \frac{f\underline{\alpha}+k_A}{\phi\beta}$. *Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Never allies.

***Beliefs**:*

C: $\psi \geq \frac{(\underline{\alpha}+\beta)f+\beta k_C+k_A}{(\overline{\alpha}+\beta)f+\beta k_C+k_A}$

P: Forms posterior beliefs $q' = 1$ *in response to acceptance and* $q' = 0$ *in response to rejection.*

***Conditions**:*

1. $w < u$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 12, P never allies when $w < u$ (as specified in condition 1). Acceptance therefore yields a payoff for the abandoner of $\phi\beta x + \underline{\alpha}\pi$, whereas its payoff for rejection given that P would not ally in response is $\phi zw\beta - k_A + \underline{\alpha}\left(\pi - f\right)$. Setting these payoffs equal and solving for x reveals that the abandoner rejects iff

$$x \leq zw - \frac{f\underline{\alpha} + k_A}{\phi\beta} = x^*_{\underline{A}} \tag{26}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer large enough to be accepted by the abandoner will also be accepted by the more-committed supporter. On the other hand, given that P would never ally, the supporter's payoff for accepting a lower offer that the abandoner would reject is $\phi\beta x + \overline{\alpha}\pi$, while its payoff for rejecting would be $\phi zw\beta - k_A + \overline{\alpha}\left(\pi - f\right)$. Setting these payoffs equal and solving for x reveals that the supporter accepts iff

$$x \geq zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} = x^*_{\overline{A}} \tag{27}$$

As above, C's plausible equilibrium offers are $x^*_{\underline{A}}$, $x^*_{\overline{A}}$, and a range of lower offers $x < x^*_{\overline{A}}$ that will be rejected by both types. C's expected payoff from offer $x^*_{\underline{A}}$ given that P will never ally is

$$U_C\left(x^*_{\underline{A}}\right) = \phi\left(1 - x^*_{\underline{A}}\right) + \pi \tag{28}$$

and its expected payoff from offer $x^*_{\overline{A}}$ given that P will never ally is

$$U_C\left(x^*_{\overline{A}}\right) = \psi\left[\phi\left(1 - x^*_{\overline{A}}\right) + \pi\right] + (1-\psi)\left[\phi\left(1 - zw\right) - k_C + \pi - f\right] \tag{29}$$

Setting these equal and solving for ψ reveals that C prefers separating offer $x^*_{\overline{A}}$ to pooling offer $x^*_{\underline{A}}$ iff

$$\psi \geq \frac{k_C + f + \phi\left(zw - x^*_{\underline{A}}\right)}{k_C + f + \phi\left(zw - x^*_{\overline{A}}\right)}$$

Substituting line 26 for $x^*_{\underline{A}}$ and line 27 for $x^*_{\overline{A}}$ and reducing yields

$$\psi \geq \frac{\left(\underline{\alpha} + \beta\right) f + \beta k_C + k_A}{\left(\overline{\alpha} + \beta\right) f + \beta k_C + k_A} \tag{30}$$

Finally, we check to see if C prefers offer $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$. Given that P never allies, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi\left(1 - zw\right) - k_C + \pi - f$$

Setting this equal to C's payoff from $x^*_{\overline{A}}$ in line 29 reveals that C prefers $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$k_C \geq \phi\left(x^*_{\overline{A}} - zw\right) - f$$

Substituting line 27 for $x^*_{\overline{A}}$ and reducing reveals that the RHS of this inequality is negative, such that C strictly prefers $x^*_{\overline{A}}$ to $x < x^*_{\overline{A}}$. ■

2.2 Pooling Equilibria on Acceptance

2.2.1 PE — alliance on path, not off path

Lemma 16 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\underline{A}} = zw - \frac{f\underline{\alpha} + k_A + u}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}} = zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} - \frac{w\overline{\alpha} + u - c}{\phi\beta}$ *and the abandoner accepts C's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to acceptance but does not ally off path in response to rejection.

***Beliefs**:*

C: $\psi < \min\left\{\frac{(\underline{\alpha}+\beta)f + \beta k_C + k_A + u}{\overline{\alpha}w + (\overline{\alpha}+\beta)f + \beta k_C + k_A + u - c}, 1 - \frac{f\underline{\alpha} + \beta(f + k_C) + k_A + u}{w\beta}\right\}$

P: Retains prior beliefs in response to acceptance; forms off-path beliefs in response to rejection $q' < \frac{u}{w}$.

***Conditions**:*

1. $q \geq \frac{u}{w}$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 12, P allies in response to A's acceptance of C's offer in a PE iff $q \geq \frac{u}{w}$ (as specified in condition 1). Given that P will ally in response, acceptance yields a payoff for the abandoner of $\phi\beta x + \underline{\alpha}\pi + u$, whereas its payoff for rejection given that P would not ally in response is $\phi zw\beta - k_A + \underline{\alpha}(\pi - f)$. Setting these payoffs equal and solving for x reveals that the threshold offer the abandoner will accept — the minimum pooling offer — is the same as in line 21:

$$x \geq zw - \frac{f\underline{\alpha} + k_A + u}{\phi\beta} = x^*_{\underline{A}}$$

This is the minimum offer necessary to induce both types of A to accept, as any offer large enough to be accepted by the abandoner will also be accepted by the more-committed supporter. On the other hand, given the P would ally in response (since condition 1 implies $w \geq u$), the supporter's payoff for accepting a lower offer that the abandoner would reject is $\phi\beta x + \overline{\alpha}(\pi + w) + u - c$, while its payoff for rejecting such a separating offer — given that P would not ally in response — would be $\phi zw\beta - k_A + \overline{\alpha}(\pi - f)$. Setting these payoffs equal and solving for x yields the same threshold for the supporter to accept as in line 22:

$$x \geq zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} - \frac{w\overline{\alpha} + u - c}{\phi\beta} = x^*_{\overline{A}}$$

C clearly prefers $x^*_{\underline{A}}$ to any greater offer, and prefers $x^*_{\overline{A}}$ to any offer $x \in \left(x^*_{\overline{A}}, x^*_{\underline{A}}\right)$. At its decision node, C's plausible equilibrium offers are therefore $x^*_{\underline{A}}$, $x^*_{\overline{A}}$, and a range of lower offers $x < x^*_{\overline{A}}$ that will be rejected by both types (among which C is indifferent). C's expected payoff from offer $x^*_{\underline{A}}$ given that P's prior beliefs dictate alliance in response to pooling on acceptance is the same as in line 25:

$$U_C\left(x^*_{\underline{A}}\right) = \phi\left(1 - x^*_{\underline{A}}\right) + \pi + \psi w$$

and its expected payoff from offer $x^*_{\overline{A}}$ given that P will conditionally ally in response when $w \geq u$ (as is implied by condition 1) is the same as in line 24:

$$U_C\left(x^*_{\overline{A}}\right) = \psi\left[\phi\left(1 - x^*_{\overline{A}}\right) + \pi + w\right] + (1 - \psi)\left[\phi(1 - zw) - k_C + \pi - f\right]$$

Setting these equal and solving for ψ reveals that C prefers pooling offer $x^*_{\underline{A}}$ to separating offer $x^*_{\overline{A}}$ iff

$$\psi < \frac{k_C + f + \phi\left(zw - x^*_{\underline{A}}\right)}{k_C + f + \phi\left(zw - x^*_{\overline{A}}\right)}$$

Substituting line 26 for $x^*_{\underline{A}}$ and line 22 for $x^*_{\overline{A}}$ and reducing yields

$$\psi < \frac{(\underline{\alpha} + \beta)f + \beta k_C + k_A + u}{\overline{\alpha}w + (\overline{\alpha} + \beta)f + \beta k_C + k_A + u - c}$$

Finally, we check to see if C prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$. Given that P holds priors that will prompt it to ally in response to pooling on reject, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi(1 - zw) - k_C + \pi + w - f$$

Setting this equal to C's payoff from $x^*_{\underline{A}}$ in line 25 reveals that C prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$\psi < 1 - \frac{f + k_C + \phi\left(zw - x^*_{\underline{A}}\right)}{w}$$

Substituting line 21 for $x^*_{\underline{A}}$ and reducing reveals that C prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$\psi < 1 - \frac{f\underline{\alpha} + \beta(f + k_C) + k_A + u}{w\beta}$$

■

2.2.2 PE — alliance on and off path

Lemma 17 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\underline{A}} = zw - \frac{f\underline{\alpha}+k_A}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}} = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta} - \frac{w\overline{\alpha}+u-c}{\phi\beta}$ *and the abandoner accepts C's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Allies in response to acceptance and off path in response to rejection.

***Beliefs**:*

C: $\psi < \frac{(\underline{\alpha}+\beta)f+\beta k_C+k_A}{\overline{\alpha}(w+f)+\beta(k_C+f)+k_A+u-c}$

P: Retains prior beliefs in response to acceptance; forms off-path beliefs in response to rejection $q' \geq \frac{u}{w}$*.*

***Conditions**:*

1. $q \geq \frac{u}{w}$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 12, P allies in response to A's acceptance of C's offer in a PE iff $q \geq \frac{u}{w}$ (as specified in condition 1). Given that P would ally in response, acceptance yields a payoff for the abandoner of $\phi\beta x + \underline{\alpha}\pi + u$, whereas its payoff for rejection given that P would also ally in response is $\phi zw\beta - k_A + \underline{\alpha}(\pi - f) + u$. Setting these payoffs equal and solving for x reveals that the abandoner's threshold for accepting is the same as in line 26:

$$x \geq zw - \frac{f\underline{\alpha}+k_A}{\phi\beta} = x^*_{\underline{A}}$$

On the other hand, the supporter's payoffs for accepting and rejecting (respectively) a lower offer that the abandoner would reject are the same as in lemma 16, yielding the same threshold for the supporter to accept as in line 22:

$$x \geq zw - \frac{f\overline{\alpha}+k_A}{\phi\beta} - \frac{w\overline{\alpha}+u-c}{\phi\beta} = x^*_{\overline{A}}$$

As above, C's plausible equilibrium offers are $x^*_{\underline{A}}$, $x^*_{\overline{A}}$, and a range of lower offers $x < x^*_{\overline{A}}$ that will be rejected by both types. C's expected payoff from offer $x^*_{\underline{A}}$ given that P's prior beliefs dictate alliance in response to pooling on acceptance is the same as in line 25:

$$U_C\left(x^*_{\underline{A}}\right) = \phi\left(1 - x^*_{\underline{A}}\right) + \pi + \psi w$$

and its expected payoff from offer $x^*_{\overline{A}}$ given that P will conditionally ally in response when $w > u$ (as is implied by condition 1) is the same as in line 24:

$$U_C\left(x^*_{\overline{A}}\right) = \psi\left[\phi\left(1 - x^*_{\overline{A}}\right) + \pi + w\right] + (1-\psi)\left[\phi(1-zw) - k_C + \pi - f\right]$$

Setting these equal and solving for ψ reveals that C prefers pooling offer $x^*_{\underline{A}}$ to separating offer $x^*_{\overline{A}}$ iff

$$\psi < \frac{k_C + f + \phi\left(zw - x^*_{\underline{A}}\right)}{k_C + f + \phi\left(zw - x^*_{\overline{A}}\right)}$$

Substituting line 26 for $x^*_{\underline{A}}$ and line 22 for $x^*_{\overline{A}}$ and reducing yields

$$\psi < \frac{(\underline{\alpha}+\beta)\,f+\beta k_C+k_A}{\overline{\alpha}\,(w+f)+\beta\,(k_C+f)+k_A+u-c}$$

Finally, we check to see if C prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$. Given that P holds priors that will prompt it to ally in response to pooling on reject, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi\,(1-zw) - k_C + \pi + \psi w - f$$

Setting this equal to C's payoff from $x^*_{\underline{A}}$ in line 25 reveals that C prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$k_C \geq \phi\left(x^*_{\underline{A}} - zw\right) - f$$

Substituting line 26 for $x^*_{\underline{A}}$ and reducing reveals that the RHS of this inequality is negative, such that C strictly prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$. ■

2.2.3 PE — no alliance on or off path

Lemma 18 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\underline{A}} = zw - \frac{f\underline{\alpha}+k_A}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}} = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta} - \frac{w\overline{\alpha}+u-c}{\phi\beta}$ *and the abandoner accepts C's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Does not ally in response to acceptance or off path in response to rejection.

***Beliefs**:*

C: $\psi < \frac{(\underline{\alpha}+\beta)f+\beta k_C+k_A}{(\overline{\alpha}+\beta)w+(\overline{\alpha}+\beta)f+\beta k_C+k_A+u-c}$

P: Retains prior beliefs in response to acceptance; forms off-path beliefs in response to rejection $q' < \frac{u}{w}$*.*

***Conditions**:*

1. $q < \frac{u}{w}$
2. $w > u$

Proof. We solve by backward induction. By definition, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node following alliance from P. From lemma 12, P does not ally in response to A's acceptance of C's offer in a PE iff $q < \frac{u}{w}$ (as specified in condition 1). Given that P would not ally in response, acceptance yields a payoff for the abandoner of $\phi\beta x + \underline{\alpha}\pi$, whereas its payoff for rejection given that P would also not ally in response is $\phi zw\beta - k_A + \underline{\alpha}\,(\pi - f)$. Setting these payoffs equal and solving for x reveals that the threshold offer the abandoner will accept — the minimum pooling offer — is the same as in line 26:

$$x \geq zw - \frac{f\underline{\alpha}+k_A}{\phi\beta} = x^*_{\underline{A}}$$

On the other hand, the supporter's payoffs for accepting and rejecting (respectively) a lower offer that the abandoner would reject are the same as in lemmata 16 and 17, yielding the same threshold for the supporter to accept as in line 22:

$$x \geq zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} - \frac{w\overline{\alpha} + u - c}{\phi\beta} = x^*_{\overline{A}}$$

As above, C's plausible equilibrium offers are $x^*_{\underline{A}}$, $x^*_{\overline{A}}$, and a range of lower offers $x < x^*_{\overline{A}}$ that will be rejected by both types. C's expected payoff from offer $x^*_{\underline{A}}$ given that P's prior beliefs dictate no alliance in response to pooling on acceptance is the same as in line 28:

$$U_C\left(x^*_{\underline{A}}\right) = \phi\left(1 - x^*_{\underline{A}}\right) + \pi$$

and its expected payoff from offer $x^*_{\overline{A}}$ given that P will conditionally ally in response when $w > u$ (as is implied by condition 1) is the same as in line 24:

$$U_C\left(x^*_{\overline{A}}\right) = \psi\left[\phi\left(1 - x^*_{\overline{A}}\right) + \pi + w\right] + (1 - \psi)\left[\phi\left(1 - zw\right) - k_C + \pi - f\right]$$

Setting these equal and solving for ψ reveals that C prefers pooling offer $x^*_{\underline{A}}$ to separating offer $x^*_{\overline{A}}$ iff

$$\psi < \frac{k_C + f + \phi\left(zw - x^*_{\underline{A}}\right)}{k_C + f + w + \phi\left(zw - x^*_{\overline{A}}\right)}$$

Substituting line 26 for $x^*_{\underline{A}}$ and line 22 for $x^*_{\overline{A}}$ and reducing yields

$$\psi < \frac{\left(\underline{\alpha} + \beta\right) f + \beta k_C + k_A}{\left(\overline{\alpha} + \beta\right) w + \left(\overline{\alpha} + \beta\right) f + \beta k_C + k_A + u - c}$$

Finally, we check to see if C prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$. Given that P holds priors that will prompt it not to ally in response to pooling on reject, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi\left(1 - zw\right) - k_C + \pi - f$$

Setting this equal to C's payoff from $x^*_{\underline{A}}$ in line 28 reveals that C prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$k_C \geq \phi\left(x^*_{\underline{A}} - zw\right) - f$$

Substituting line 26 for $x^*_{\underline{A}}$ and reducing reveals that the RHS of this inequality is negative, such that C strictly prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$. ■

2.2.4 PE — P never allies

Lemma 19 *The following strategies and beliefs form a PBE.*

***Strategies**:*

C: Offers $x^*_{\underline{A}} = zw - \frac{f\underline{\alpha} + k_A}{\phi\beta}$

A: On A's initial move, the supporter accepts C's offer iff $x \geq x^*_{\overline{A}} = zw - \frac{f\overline{\alpha} + k_A}{\phi\beta}$ *and the abandoner accepts C's offer iff* $x \geq x^*_{\underline{A}}$*. Conditional on P granting an alliance,* $\overline{A}$ *supports and* $\underline{A}$ *abandons at the terminal node.*

P: Never allies.

***Beliefs**:*

C: $\psi < \frac{(\underline{\alpha}+\beta)f+\beta k_C+k_A}{(\overline{\alpha}+\beta)f+\beta k_C+k_A}$

P: Retains prior beliefs; off-path beliefs unrestricted.

***Conditions**:*

1. $w < u$

Proof. Because P will never ally when $w < u$, A's payoffs are identical to lemma 15. Thus, from lines 26 and 27, respectively, $x^*_{\underline{A}} = zw - \frac{f\underline{\alpha}+k_A}{\phi\beta}$ and $x^*_{\overline{A}} = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta}$. C's expected payoffs are also identical to lemma 15, but its beliefs fall on the opposite side of the threshold in line 30, such that it prefers offer $x^*_{\underline{A}}$ to offer $x^*_{\overline{A}}$:

$$\psi < \frac{(\underline{\alpha}+\beta)\,f+\beta k_C+k_A}{(\overline{\alpha}+\beta)\,f+\beta k_C+k_A}$$

Finally, we check to see if C prefers offer $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$. Given that P will never ally, C's expected payoff from an offer $x < x^*_{\overline{A}}$ is

$$U_C\left(x < x^*_{\overline{A}}\right) = \phi(1-zw) - k_C + \pi$$

Setting this equal to C's payoff from $x^*_{\underline{A}}$ in line 28 reveals that C prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$ iff

$$x^*_{\underline{A}} < zw + \frac{k_C}{\phi}$$

Substituting line 26 for $x^*_{\underline{A}}$ reveals that C always prefers $x^*_{\underline{A}}$ to $x < x^*_{\overline{A}}$ in this equilibrium. ■

2.2.5 PE — alliance off path, not on path (not supported)

Lemma 20 *Any PBE with the following characteristics is eliminated by the divine equilibrium refinement.*

***Strategies**:*

C: Offers $x^*_{\overline{A}} = zw - \frac{f\underline{\alpha}+k_A-u}{\phi\beta}$.

$\overline{A}$: On A's initial move, both types accept C's equilibrium offer. Conditional on P granting an alliance, $\overline{A}$ supports and $\underline{A}$ abandons at the terminal node.

P: Does not ally in response to acceptance on path but allies in response to rejection off path.

***Beliefs**:*

On path P retains its prior beliefs $q < \frac{u}{w}$. *In response to rejection off path it forms posterior beliefs* $q' \geq \frac{u}{w}$.

***Conditions**:*

1. $q < \frac{u}{w}$

2. $w > u$

Proof. This equilibrium requires P to update its beliefs in response to A's rejection of C's offer that A is more likely to be a supporter than P originally believed was the case. However, such beliefs violate the D_1-criterion, because the supporter benefits less from rejection than the abandoner would and should thus be seen as less likely to reject. We can therefore eliminate this equilibrium as requiring implausible beliefs.

■

2.3 Pooling Equilibria on Rejection (Not Supported)

Lemma 21 *Pooling on rejection is never in equilibrium.*

Proof. We prove lemma 21 by showing that a low offer $x < x_A^*$ that would induce pooling on rejection is dominated by separating offer x_A^*. Consider the following three sets of conditions, which are exhaustive: 1) $w < u$; 2) $w \geq u$ and $q < \frac{u}{w}$; 3) $w \geq u$ and $q \geq \frac{u}{w}$.

Case 1 $w < u$

Here, P never allies. Thus, alliance is not at stake for any offer such that $x_A^* = zw - \frac{f\overline{\alpha}+k_A}{\phi\beta}$ (from line 27). C's payoff for offer x_A^* is $\psi\left[\phi\left(1-x_A^*\right)+\pi\right]+(1-\psi)\left[\phi\left(1-zw\right)-k_C+\pi-f\right]$, whereas its offer for $x < x_A^*$ is war payoff $\phi(1-zw)-k_C+\pi-f$. Setting these equal and substituting $zw - \frac{f\overline{\alpha}+k_A}{\phi\beta}$ for x_A^* reveals that C always prefers offer x_A^* to offer $x < x_A^*$. This is intuitive, since without alliance at stake the possibility that supporters accept x_A^* is obviously preferable for C to fighting with certainty following offer $x < x_A^*$.

Case 2 $w \geq u$ *and* $q < \frac{u}{w}$

Here, P allies if it observes A accept offer x_A^* but not following $x < x_A^*$ or if A rejects offer x_A^*. Thus, offer x_A^* puts alliance at stake, such that $x_A^* = zw - \frac{f\overline{\alpha}+k_A+u}{\phi\beta}$ (from line 22). This means that C's payoff for offer x_A^* is $\psi\left[\phi\left(1-x_A^*\right)+\pi+w\right]+(1-\psi)\left[\phi\left(1-zw\right)-k_C+\pi-f\right]$, whereas its offer for $x < x_A^*$ is $\phi(1-zw)-k_C+\pi-f$. Setting these equal and substituting $zw - \frac{f\overline{\alpha}+k_A+u}{\phi\beta}$ for x_A^* reveals that C always prefers offer x_A^* to offer $x < x_A^*$. This is again intuitive: C's payoff for $x < x_A^*$ is identical to Case 1, but its payoff for offer x_A^* is better, since x_A^* is smaller with alliance at stake and supporters will only join the stage-2 coalition against R following offer x_A^*, which is beneficial to C.

Case 3 $w \geq u$ *and* $q \geq \frac{u}{w}$

Here, P allies if it observes A accept offer x_A^* and following $x < x_A^*$, but not if A rejects offer x_A^*. Thus, offer x_A^* again puts alliance is at stake, such that $x_A^* = zw - \frac{f\overline{\alpha}+k_A+u}{\phi\beta}$. This means that C's payoff for offer x_A^* is $\psi\left[\phi\left(1-x_A^*\right)+\pi+w\right]+(1-\psi)\left[\phi\left(1-zw\right)-k_C+\pi-f\right]$, whereas its offer for $x < x_A^*$ is $\phi(1-zw)-k_C+\pi+\psi w-f$. Setting these equal and substituting $zw - \frac{f\overline{\alpha}+k_A+u}{\phi\beta}$ for x_A^* reveals that C always prefers offer x_A^* to offer $x < x_A^*$. This is again intuitive: for either offer C's payoff increases by ψw relative to Case 1, since supporters but not abandoners will join the stage-2 coalition against R in either case. But C's payoff for offer x_A^* increases more than its payoff for $x < x_A^*$ relative to Case 1, since x_A^* is smaller with alliance at stake. ■

2.4 Comparative Statics

Proposition 6 *When an alliance with P is at stake,*[3] *both supporters and abandoners are more likely to accept C's offer. If the value of the alliance is high enough* $(u \geq \phi\beta zw + c - (f+w)\,\overline{\alpha} - k)$*, supporters accept all offers, which can avert conflict that would have counterfactually occurred for exogenous reasons.*

Proof. From the main text, alliance is defined to be at stake when P will ally in response to A's acceptance of C's offer but not in response to its rejection. By this definition, alliance is at stake on path in the equilibria characterized in lemmata 13, 14, and 16. Conversely, alliance is not at stake on path in the equilibria characterized in lemmata 15, 17, 18, and 19.

It is straightforward to show that proposition 6 holds for the abandoner. With alliance at stake, the abandoner's payoff for accepting C's offer (which would result in an alliance) is $\phi\beta x + \underline{\alpha}\pi + u$, and its payoff

[3] Alliance is defined to be at stake when P will ally in response to A's rejection of C's offer but not in response to its acceptance.

for rejecting (which would elicit no alliance) is $\phi\beta zw - k_A + \underline{\alpha}(\pi - f)$. Setting these equal and solving for x yields $\underline{\underline{x^*_A}}$, the minimum offer the abandoner will accept with alliance at stake:

$$\underline{\underline{x^*_A}} \equiv zw - \frac{f\underline{\alpha} + k_A + u}{\phi\beta} \tag{31}$$

This is identical to the threshold of $\underline{x^*_A}$ given in line 21. Conversely, alliance is not at stake if P would either 1) ally in response to both acceptance and rejection; 2) not ally in response to both acceptance and rejection. In the former case, the abandoner's payoff for accepting C's offer is $\phi\beta x + \underline{\alpha}\pi + u$, and its payoff for rejecting is $\phi\beta zw - k_A + \underline{\alpha}(\pi - f) + u$. In the latter case, the abandoner's payoff for accepting C's offer is $\phi\beta x + \underline{\alpha}\pi$, and its payoff for rejecting is $\phi\beta zw - k_A + \underline{\alpha}(\pi - f)$. In either case, setting these payoffs equal and solving for x yields $\overline{x^*_{\underline{A}}}$, the minimum offer the abandoner will accept with no alliance at stake:

$$\overline{x^*_{\underline{A}}} \equiv zw - \frac{f\underline{\alpha} + k_A}{\phi\beta} \tag{32}$$

This is identical to the threshold of $x^*_{\underline{A}}$ given in line 26. Comparing the two, $x^*_{\underline{A}}$ is clearly less than $\overline{x^*_{\underline{A}}}$, such that abandoners have a greater incentive to accept C's offer with alliance at stake.

The same directional effect obtains for the supporter. With alliance at stake, the supporter's payoff for accepting C's offer (which would result in an alliance) is $\phi\beta x + \overline{\alpha}(\pi + w) + u - c$, and its payoff for rejecting (which would elicit no alliance) is $\phi\beta w - k_A + \overline{\alpha}(\pi - f)$. Setting these equal and solving for x yields $\underline{x^*_{\overline{A}}}$, the minimum offer the supporter will accept with alliance at stake:

$$\underline{x^*_{\overline{A}}} \equiv zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} - \frac{w\overline{\alpha} + u - c}{\phi\beta} \tag{33}$$

This is identical to the threshold of $\underline{x^*_{\overline{A}}}$ given in line 22. Now consider the supporter's payoffs with no alliance at stake. If P would ally in response to both acceptance and rejection, the supporter's payoff for accepting C's offer is $\phi\beta x + \overline{\alpha}(\pi + w) + u - c$, and its payoff for rejecting is $\phi\beta w - k_A + \overline{\alpha}(\pi + w - f) + u - c$. If P would not ally in response to both acceptance and rejection, the supporter's payoff for accepting C's offer is $\phi\beta x + \overline{\alpha}\pi$, and its payoff for rejecting is $\phi\beta w - k_A + \overline{\alpha}(\pi - f)$. In either case, setting these payoffs equal and solving for x yields $\overline{x^*_{\overline{A}}}$, the minimum offer the supporter will accept with no alliance at stake:

$$\overline{x^*_{\overline{A}}} \equiv zw - \frac{f\overline{\alpha} + k_A}{\phi\beta} \tag{34}$$

This is identical to the threshold of $x^*_{\overline{A}}$ given in line 27. Again, $\underline{x^*_{\overline{A}}}$ is strictly less than $\overline{x^*_{\overline{A}}}$, since by definition $\overline{\alpha} > \frac{c}{w}$ (see lemma 1). Thus, supporters also have greater incentive to accept with alliance at stake than without.

Indeed, when the value of an alliance to A (proxied here by u) is large enough, the supporter accepts all offers from C. This occurs when $zw - \frac{(f+w)\overline{\alpha} + k_A + u - c}{\phi\beta} \leq 0$, which is satisfied iff

$$u \geq \phi\beta zw + c - (f + w)\overline{\alpha} - k_A$$

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