Supplemental Files

Contents **Pages**

Part 1: Trading Goods and Answer Sheets ……...2
Part 2: Estimating U_{LES} in Exp. 1 of C-Sample…….....6
Part 3: Estimating U_{Est} in Exp. 1 of C-Sample……12
Part 4: Estimating U_{LES} and U_{Est} in Exp. 1 of S-Sample……..... 24
Part 5: Estimating U_{LES} and U_{Est} in Exp. 2 of S-Sample…….....41

Part 1　Trading Goods and Answer Sheets

1　Mini-bagged pistachio, almond, and cashew nut for Exp. 1

A mini bag of pistachio

A mini bag of almond

A mini bag of cashew nut

2 Apple, pen, and facial tissue for Exp. 2

3 Answer sheets for Exp. 1

3.1 Answer sheet in Session A of Exp. 1

Name_____ Sex_____ Birth：month___ year_____

Item	*Price*	*Quantity*	*Money*	
Choice I：				
Pistachio	¥0.50 /bag	_____	_____	
Almond	¥0.50 /bag	_____	_____	
Cashew nut	¥0.50 /bag	_____	_____	
		Total money amount：	_____	（No more than ¥6.60）
Choice II：				
Pistachio	¥0.70 /bag	_____	_____	
Almond	¥0.30 /bag	_____	_____	
Cashew nut	¥0.50 /bag	_____	_____	
		Total money amount：	_____	（No more than ¥6.60）
Choice III：				
Pistachio	¥0.50 /bag	_____	_____	
Almond	¥0.30 /bag	_____	_____	
Cashew nut	¥0.70 /bag	_____	_____	
		Total money amount：	_____	（No more than ¥6.60）

3.2 Inquiry card in Session B of Exp. 1

The following is an example of the inquiry card used in Session B of Exp. 1.

Cashew nut: 8 bags Unit price: ¥0.50/bag Your bid unit price: (¥)

4 Answer sheets for Exp. 2

4.1 Answer sheet in Session A of Exp. 2

Name_____ Sex_____ Birth：month___ year_____

Item	*Price*	*Quantity*	*Money*	
Choice I：				
Apple	¥0.50 /piece	_____	_____	
Pen	¥0.50 /piece	_____	_____	
Facial tissue	¥0.50 /set	_____	_____	
		Total money amount：	_____	（No more than ¥8.00）
Choice II：				
Apple	¥0.70 /piece	_____	_____	
Pen	¥0.90 /piece	_____	_____	
Facial tissue	¥0.50 /set	_____	_____	
		Total money amount：	_____	（No more than ¥8.00）
Choice III：				
Apple	¥0.50 /piece	_____	_____	
Pen	¥0.90 /piece	_____	_____	
Facial tissue	¥0.70 /set	_____	_____	
		Total money amount：	_____	（No more than ¥8.00）

4.2 Inquiry card in Session B of Exp. 2

The following is an example of the inquiry card used in Session B of Exp. 2.

Apple: 8 pieces

Unit price: ¥0.50/piece

Your bid unit price: (¥)

Part 2 Estimating U_{LES} in Exp. 1 of C-Sample

1 Original data

Tables 2-1, 2-2, and 2-3 present the original data obtained from Session A in Exp. 1 of C-Sample for solving LES.

Table 2-1 Data of Choice I of Exp. 1 of C-Sample

N=38, Male 9, Female 29

Subject	Sex	Pistachio (q_{I1}) (p_{I1}=0.50)	Almond (q_{I2}) (p_{I2}=0.50)	Cashew nut (q_{I3}) (p_{I3}=0.50)	$p_{I1}q_{I1}$	$p_{I2}q_{I2}$	$p_{I3}q_{I3}$	$\Sigma p_{Ij}q_{Ij}$
S_1	F	4	4	4	2.00	2.00	2.00	6.00
S_2	M	4	5	4	2.00	2.50	2.00	6.50
S_3	M	4	4	4	2.00	2.00	2.00	6.00
S_4	F	5	4	4	2.50	2.00	2.00	6.50
S_5	F	4	4	5	2.00	2.00	2.50	6.50
S_6	F	5	4	3	2.50	2.00	1.50	6.00
S_7	F	3	3	6	1.50	1.50	3.00	6.00
S_8	M	2	5	6	1.00	2.50	3.00	6.50
S_9	F	4	4	5	2.00	2.00	2.50	6.50
S_{10}	F	3	5	5	1.50	2.50	2.50	6.50
S_{11}	M	7	2	4	3.50	1.00	2.00	6.50
S_{12}	F	4	5	4	2.00	2.50	2.00	6.50
S_{13}	F	5	4	4	2.50	2.00	2.00	6.50
S_{14}	F	4	5	4	2.00	2.50	2.00	6.50
S_{15}	F	6	4	3	3.00	2.00	1.50	6.50
S_{16}	F	4	4	5	2.00	2.00	2.50	6.50
S_{17}	F	5	4	4	2.50	2.00	2.00	6.50
S_{18}	F	5	4	4	2.50	2.00	2.00	6.50
S_{19}	F	5	5	3	2.50	2.50	1.50	6.50
S_{20}	F	4	4	5	2.00	2.00	2.50	6.50
S_{21}	F	4	5	4	2.00	2.50	2.00	6.50
S_{22}	F	4	3	6	2.00	1.50	3.00	6.50
S_{23}	F	5	4	4	2.50	2.00	2.00	6.50
S_{24}	F	3	5	4	1.50	2.50	2.00	6.50
S_{25}	M	4	4	5	2.00	2.00	2.50	6.50
S_{26}	F	4	4	5	2.00	2.00	2.50	6.50
S_{27}	F	4	4	5	2.00	2.00	2.50	6.50
S_{28}	M	3	5	5	1.50	2.50	2.50	6.50
S_{29}	M	4	4	5	2.00	2.00	2.50	6.50
S_{30}	F	4	4	5	2.00	2.00	2.50	6.50
S_{31}	F	4	4	5	2.00	2.00	2.50	6.50
S_{32}	F	6	5	2	3.00	2.50	1.00	6.50
S_{33}	F	3	6	3	1.50	3.00	1.50	6.00

S_{34}	F	3	2	5	1.50	1.00	2.50	5.00
S_{35}	M	2	2	4	1.00	1.00	2.00	4.00
S_{36}	M	2	2	2	1.00	1.00	1.00	3.00
S_{37}	F	4	1	2	2.00	0.50	1.00	3.50
S_{38}	F	2	3	2	1.00	1.50	1.00	3.50
Mean ($\overline{q}_{Ij}$)		**4.00**	**3.95**	**4.18**				

Table 2-2 Data of Choice II in Exp. 1 of C-Sample

N=37, Male 9, Female 28

Subject	Sex	Pistachio (q_{II1}) (p_{II1}=0.70)	Almond (q_{II2}) (p_{II2}=0.30)	Cashew nut (q_{II3}) (p_{II3}=0.50)	$p_{II1}q_{II1}$	$p_{II2}q_{II2}$	$p_{II3}q_{II3}$	$\Sigma p_{IIj}q_{IIj}$
S_1	F	3	4	4	2.10	1.20	2.00	5.30
S_2	M	2	14	2	1.40	4.20	1.00	6.60
S_3	M	4	5	4	2.80	1.50	2.00	6.30
S_4	F	3	4	4	2.10	1.20	2.00	5.30
S_5	F	4	4	5	2.80	1.20	2.50	6.50
S_6	F	5	7	2	3.50	2.10	1.00	6.60
S_7	F	2	8	5	1.40	2.40	2.50	6.30
S_8	M	1	11	4	0.70	3.30	2.00	6.00
S_9	F	4	6	4	2.80	1.80	2.00	6.60
S_{11}	M	1	6	8	0.70	1.80	4.00	6.50
S_{12}	F	1	10	5	0.70	3.00	2.50	6.20
S_{13}	F	4	5	4	2.80	1.50	2.00	6.30
S_{14}	F	3	8	4	2.10	2.40	2.00	6.50
S_{15}	F	1	8	7	0.70	2.40	3.50	6.60
S_{16}	F	4	6	4	2.80	1.80	2.00	6.60
S_{17}	F	3	5	6	2.10	1.50	3.00	6.60
S_{18}	F	4	6	4	2.80	1.80	2.00	6.60
S_{19}	F	2	7	6	1.40	2.10	3.00	6.50
S_{20}	F	3	8	4	2.10	2.40	2.00	6.50
S_{21}	F	1	11	5	0.70	3.30	2.50	6.50
S_{22}	F	2	7	6	1.40	2.10	3.00	6.50
S_{23}	F	4	4	5	2.80	1.20	2.00	6.00
S_{24}	F	2	8	4	1.40	2.40	2.00	5.80
S_{25}	M	1	18	1	0.70	5.40	0.50	6.60
S_{26}	F	3	6	5	2.10	1.80	2.50	6.40
S_{27}	F	3	6	5	2.10	1.80	2.50	6.40
S_{28}	M	3	10	3	2.10	3.00	1.50	6.60
S_{29}	M	4	6	4	2.80	1.80	2.00	6.60
S_{30}	F	2	10	4	1.40	3.00	2.00	6.40
S_{31}	F	2	9	5	1.40	2.70	2.50	6.60
S_{32}	F	2	14	2	1.40	4.20	1.00	6.60
S_{33}	F	1	5	2	0.70	1.50	1.00	3.20

S_{34}	F	2	2	8	1.40	0.60	4.00	6.00
S_{35}	M	1	5	4	0.70	1.50	2.00	4.20
S_{36}	M	1	3	4	0.70	0.90	2.00	3.60
S_{37}	F	2	4	3	1.40	1.20	1.50	4.10
S_{38}	F	1	5	2	0.70	1.50	1.00	3.20
Mean ($\overline{q}_{IIj}$)		**2.54**	**7.05**	**4.24**				

Table 2-3 Data of Choice III in Exp. 1 of C-Sample

N=36, Male 8, Female 28

Subject	Sex	Pistachio (q_{III1}) (p_{III1}=0.50)	Almond (q_{III2}) (p_{III2}=0.30)	Cashew nut (q_{III3}) (p_{III3}=0.70)	$p_{III1}q_{I1}$	$p_{III2}q_{III2}$	$p_{III3}q_{III3}$	$\Sigma p_{IIIj}q_{IIIj}$
S_1	F	4	4	3	2.00	1.20	2.10	5.30
S_2	M	3	14	1	1.50	4.20	0.70	6.40
S_3	M	5	6	3	2.50	1.80	2.10	6.40
S_4	F	5	4	3	2.50	1.20	2.10	5.80
S_5	F	5	4	4	2.50	1.20	2.80	6.50
S_7	F	5	8	2	2.50	2.40	1.40	6.30
S_8	M	3	10	3	1.50	3.00	2.10	6.60
S_9	F	4	6	4	2.00	1.80	2.80	6.60
S_{10}	F	5	11	1	2.50	3.30	0.70	6.50
S_{12}	F	3	10	3	1.50	3.00	2.10	6.60
S_{13}	F	4	5	4	2.00	1.50	2.80	6.30
S_{14}	F	4	8	3	2.00	2.40	2.10	6.50
S_{15}	F	7	8	1	3.50	2.40	0.70	6.60
S_{16}	F	5	9	2	2.00	2.70	1.40	6.10
S_{17}	F	6	5	3	3.00	1.50	2.10	6.60
S_{18}	F	4	6	4	2.00	1.80	2.80	6.60
S_{19}	F	6	7	2	3.00	2.10	1.40	6.50
S_{20}	F	4	8	3	2.00	2.40	2.10	6.50
S_{21}	F	5	9	2	2.50	2.70	1.40	6.60
S_{22}	F	6	4	3	3.00	1.20	2.10	6.30
S_{23}	F	5	4	4	2.00	1.20	2.80	6.00
S_{24}	F	3	8	2	1.50	2.40	1.40	5.30
S_{25}	M	1	18	1	0.50	5.40	0.70	6.60
S_{26}	F	5	6	3	2.50	1.80	2.10	6.40
S_{27}	F	7	8	1	3.50	2.40	0.70	6.60
S_{28}	M	3	10	3	1.50	3.00	2.10	6.60
S_{29}	M	4	7	3	2.00	2.10	2.10	6.20
S_{30}	F	4	10	2	2.00	3.00	1.40	6.40
S_{31}	F	4	7	3	2.00	2.10	2.10	6.20
S_{32}	F	3	14	1	1.50	4.20	0.70	6.40
S_{33}	F	2	5	1	1.00	1.50	0.70	3.20
S_{34}	F	8	2	2	4.00	0.60	1.40	6.00

S_{35}	M	2	5	2	1.00	1.50	1.40	3.90
S_{36}	M	4	3	1	2.00	0.90	0.70	3.60
S_{37}	F	3	4	1	1.50	1.20	0.70	3.40
S_{38}	F	2	5	1	1.00	1.50	0.70	3.20
Mean ($\overline{q}_{IIIj}$)		**4.25**	**7.28**	**2.36**				

To balance the contributions from different budget constraint groups (subjects with the same value of $\Sigma p_{ij}q_{ij}$ belong to the same group), Table 2-1-I presents the average values of $p_{I1}q_{I1}$, $p_{I2}q_{I2}$, and $p_{I3}q_{I3}$ of each budget constraint group in Table 2-1; Table 2-2-II the average values of $p_{II1}q_{II1}$, $p_{II2}q_{II2}$, and $p_{II3}q_{II3}$ of each budget constraint group in Table 2-2; and Table 2-3-III the average values of $p_{III1}q_{III1}$, $p_{III2}q_{III2}$, and $p_{III3}q_{III3}$ of each budget constraint group in Table 2-3. U_{LES} will be estimated from Tables 2-1-I, 2-2-II, and 2-3-III.

Table 2-1-I Category data of Choice I in Exp. 1 of C-Sample

$\overline{p_{I1}q_{I1}}$	$\overline{p_{I2}q_{I2}}$	$\overline{p_{I3}q_{I3}}$	$\sum p_{Ij}q_{Ij}$
2.14	2.13	2.21	6.50
2.00	1.88	2.13	6.00
1.50	1.00	2.50	5.00
1.00	1.00	2.00	4.00
1.50	1.00	1.00	3.50
1.00	1.00	1.00	3.00

Table 2-2-II Category data of Choice II in Exp. 1 of C-Sample

$\overline{p_{II1}q_{II1}}$	$\overline{p_{II2}q_{II2}}$	$\overline{p_{II3}q_{II3}}$	$\sum p_{IIj}q_{IIj}$
2.04	2.73	1.83	6.60
1.60	2.18	2.71	6.50
1.87	2.20	2.33	6.40
2.33	1.80	2.17	6.30
0.70	3.00	2.50	6.20
1.40	2.10	2.50	6.00
1.40	2.40	2.00	5.80
2.10	1.20	2.00	5.30
0.70	1.50	2.00	4.20
1.40	1.20	1.50	4.10
0.70	0.90	2.00	3.60
0.70	1.50	1.00	3.20

Table 2-3-III Category data of Choice III in Exp. 1 of C-Sample

$\overline{p_{III1}q_{III1}}$	$\overline{p_{III2}q_{III2}}$	$\overline{p_{III3}q_{III3}}$	$\sum p_{IIIj}q_{IIIj}$
2.15	2.70	1.75	6.60

2.40	2.27	1.82	6.50
2.00	3.00	1.40	6.40
2.50	1.70	2.10	6.30
2.00	2.10	2.10	6.20
2.00	2.70	1.40	6.10
3.00	0.90	2.10	6.00
2.50	1.20	2.10	5.80
1.75	1.80	1.75	5.30
1.00	1.50	1.40	3.90
2.00	0.90	0.70	3.60
1.50	1.20	0.70	3.40
1.00	1.50	0.70	3.20

2 Derivation for U_{LES}

Denoting b_{ij} and r_{ij} the estimated values in experimental data, and e_{ij} the residue, the estimated LES in Exp.1 is

$$p_{ij}q_{ij} = p_{ij}r_{ij} + b_{ij}(\sum_{j=1}^{3} p_{ij}q_{ij} - \sum_{j=1}^{3} p_{ij}r_{ij}) + e_{ij}, \qquad i=\text{I,II,III}, \quad j=1,2,3.$$

Approximately,

$$p_{ij}q_{ij} \approx p_{ij}r_{ij} + b_{ij}(\sum_{j=1}^{3} p_{ij}q_{ij} - \sum_{j=1}^{3} p_{ij}r_{ij}), \qquad i=\text{I,II,III}, \quad j=1,2,3.$$

Denoting

$$a_{ij} = p_{ij}r_{ij} - b_{ij}\sum_{j=1}^{3} p_{ij}r_{ij}, \tag{2-1}$$

a_{ij} is a constant. The LES becomes

$$p_{ij}q_{ij} \approx a_{ij} + b_{ij}\sum_{j=1}^{3} p_{ij}q_{ij}, \qquad i=\text{I,II,III}, \quad j=1,2,3. \tag{2-2}$$

Sum on the both sides in (2-1)

$$\sum_{j=1}^{3} a_{ij} \approx \sum_{j=1}^{3} p_{ij}r_{ij} - \sum_{j=1}^{3} b_{ij}\sum_{j=1}^{3} p_{ij}r_{ij} = \sum_{j=1}^{3} p_{ij}r_{ij}(1 - \sum_{i=1}^{3} p_{ij}q_{ij})$$

Transpose the terms

$$\sum_{j=1}^{3} p_{ij}r_{ij} \approx \frac{\sum_{j=1}^{3} a_{ij}}{1 - \sum_{j=1}^{3} b_{ij}}. \tag{2-3}$$

Substitute (2-3) in (2-1)

$$p_{ij}r_{ij} \approx a_{ij} + b_{ij}\frac{\sum_{j=1}^{3} a_{ij}}{1-\sum_{j=1}^{3} b_{ij}}, \qquad .i=\text{I,II,III}, \quad j=1,2,3. \tag{2-4}$$

Utilizing the ordinary least square method (OLS) to solve (2-2) in the data of Tables 2-1-I, 2-2-II, and 2-3-III, the estimated values of a_{ij} and b_{ij} will be obtained. Then, substituting these estimated values in (2-4) and solving for r_{ij}, the estimated values of r_{ij} will be derived out. And using b_{ij} and r_{ij} U_{LES} will be determined. This is one of standard estimation approaches to solve LES in observed data.

The above approximate approach relies on the residue $e_{ij}\neq 0$. In case of $e_{ij}=0$, from the definition of b_{ij}, $1-\sum_{j=1}^{3} b_{ij} = 0$ will exactly hold, and it will lead the fraction on the right-hand side of (2-4) diverge, namely, the estimation will become impossible. Such a case indeed occurs in Choice I of Exp. 2 (see Part 5).

Solving (2-2) in the data of Table 2-1-I for Choice I,

$\begin{cases} a_{I1}=0.096, \\ b_{I1}=0.306; \end{cases}$ $\begin{cases} a_{I2}=-0.205, \\ b_{I2}=0.330; \end{cases}$ $\begin{cases} a_{I3}=0.118, \\ b_{I3}=0.362. \end{cases}$

Solving (2-2) in the data of Table 2-2-II for Choice II,

$\begin{cases} a_{II1}=-0.279, \\ b_{II1}=0.316; \end{cases}$ $\begin{cases} a_{II2}=-0.287, \\ b_{II2}=0.407; \end{cases}$ $\begin{cases} a_{II3}=0.569, \\ b_{II3}=0.276. \end{cases}$

Solving (2-2) in the data of Table 2-3-III for Choice III,

$\begin{cases} a_{III1}=0.264, \\ b_{III1}=0.323; \end{cases}$ $\begin{cases} a_{III2}=0.045, \\ b_{III2}=0.330; \end{cases}$ $\begin{cases} a_{III3}=-0.307, \\ b_{III3}=0.346. \end{cases}$

Substituting the above results of a_{ij} and b_{ij} in (2-4) and solving for r_{ij}, obtain

$r_{I1}=2.95$, $r_{I2}=2.56$, $r_{I3}=3.49$;

$r_{II1}=0.96$, $r_{II2}=3.11$, $r_{II3}=2.79$.

$r_{III1}=1.82$, $r_{III2}=2.35$, $r_{III3}=0.55$.

Finally obtain each U_{LES} in Session A of Exp. 1 for C-Sample:

in Choice I,

$$U_{LES} = 0.31\ln(q_{I1}-2.95)+0.33\ln(q_{I2}-2.56)+0.36\ln(q_{I3}-3.49); \tag{2-1}$$

in Choice II,

$$U_{LES} = 0.32\ln(q_{II1}-0.96)+0.41\ln(q_{II2}-3.11)+0.28\ln(q_{II3}-2.79); \tag{2-2}$$

in Choice III,

$$U_{LES} = 0.32\ln(q_{II1}-1.82)+0.33\ln(q_{II2}-2.35)+0.35\ln(q_{II3}-0.55); \tag{2-3}$$

Part 3 Estimating U_{Est} in Exp. 1 of C-Sample

1 Original data

Tables 3-1, 3-2, 3-3, 3-4, 3-5, and 3-6 present the experimental data obtained from Session B in Exp. 1 of S-Sample. The data created by multiplying subjects' bid unit prices with corresponding assigned quantities. For example, in Table 3-1, the up-left datum "1.60" is created by multiplying subject S_1's bid unit price 0.40 with corresponding assigned quantity "4", that is, 0.40×4=1.60. Or reversely, this datum "1.60" means S_1's bid unit price for assigned quantity "4" is 0.40; similarly, on the first line of the data block in Table 3-1, the second datum "1.80" means S_1's bid unit price for assigned quantity "6" is 0.30, and so on.

Table 3-1 Data for the utility scales of pistachio in Choices I and III (u_{I1}, u_{III1})

N=38, Male 9, Female 29; Assigned unit price 0.50/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	F	1.60	1.80	2.40	2.00	2.40
S_2	M	2.00	2.40	4.00	2.00	3.60
S_3	M	2.00	3.00	2.40	3.10	3.60
S_4	F	1.60	2.10	3.20	3.00	4.80
S_5	F	2.00	2.70	3.60	3.50	4.80
S_6	F	1.60	2.10	3.60	4.00	3.60
S_7	F	1.40	1.50	2.40	3.00	3.60
S_8	M	1.60	2.10	2.80	3.00	4.20
S_9	F	1.20	2.40	3.20	3.50	3.00
S_{10}	F	1.20	2.10	2.40	3.00	3.60
S_{11}	M	1.80	2.70	3.60	4.00	4.80
S_{12}	F	1.00	1.20	1.60	2.00	1.80
S_{13}	F	1.60	1.80	2.40	3.00	3.60
S_{14}	F	1.40	1.80	2.40	3.50	2.40
S_{15}	F	1.00	1.50	2.00	2.50	2.40
S_{16}	F	1.84	2.52	2.40	2.00	3.60
S_{17}	F	1.96	2.70	3.20	3.60	4.80
S_{18}	F	1.60	3.00	3.20	4.00	4.80
S_{19}	F	1.40	1.50	2.80	2.50	3.00
S_{20}	F	2.00	2.70	3.20	4.00	4.20
S_{21}	F	1.60	1.50	2.00	4.00	3.00
S_{22}	F	1.20	1.80	2.40	4.00	3.60
S_{23}	F	1.00	2.40	2.00	2.50	3.00
S_{24}	F	1.20	1.20	3.36	2.00	3.60
S_{25}	M	0.40	1.50	3.36	2.50	2.40
S_{26}	F	1.80	2.40	2.80	2.50	3.60
S_{27}	F	1.60	2.10	2.80	2.50	3.60
S_{28}	M	1.20	2.40	2.40	3.00	3.60

S_{29}	M	1.60	2.70	2.40	3.50	2.76
S_{30}	F	1.60	2.40	3.20	2.00	2.40
S_{31}	F	1.00	2.70	2.40	3.00	3.00
S_{32}	F	1.40	2.10	2.80	2.00	4.20
S_{33}	F	1.60	2.40	2.80	3.00	3.00
S_{34}	F	1.40	3.00	3.20	3.50	3.60
S_{35}	M	0.80	1.20	1.60	1.50	1.80
S_{36}	M	1.00	1.80	2.00	2.50	2.40
S_{37}	F	1.60	2.40	3.20	3.50	3.60
S_{38}	F	1.40	2.10	2.80	3.00	3.00
Mean (u_{I1}, u_{III1})		**1.45**	**2.15**	**2.75**	**2.94**	**3.38**

Table 3-2 Data for the utility scale of almond in Choice I (u_{I2})

N=38, Male 9, Female 29; Assigned unit price 0.50/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	F	1.60	1.80	2.40	2.00	2.40
S_2	M	1.20	3.00	2.40	5.00	4.80
S_3	M	2.00	3.00	4.00	4.80	4.80
S_4	F	1.60	2.40	3.60	3.50	4.80
S_5	F	1.80	2.40	3.20	4.00	4.80
S_6	F	1.40	2.40	2.40	3.00	3.60
S_7	F	1.60	1.50	2.80	3.00	3.60
S_8	M	1.60	2.10	2.80	3.00	4.20
S_9	F	2.00	1.80	3.20	3.50	4.20
S_{10}	F	1.20	1.80	2.80	3.00	4.20
S_{11}	M	1.00	1.50	1.60	2.50	1.80
S_{12}	F	0.80	1.20	1.60	2.00	2.64
S_{13}	F	1.60	1.80	3.04	2.50	4.20
S_{14}	F	1.40	1.50	2.64	3.00	2.40
S_{15}	F	1.20	1.50	2.00	2.50	2.40
S_{16}	F	1.12	2.52	3.76	2.60	2.40
S_{17}	F	1.96	2.88	3.20	4.50	4.80
S_{18}	F	1.60	3.00	3.60	4.00	4.80
S_{19}	F	1.80	1.50	2.00	2.50	2.40
S_{20}	F	2.00	2.70	3.60	4.00	4.20
S_{21}	F	1.60	1.50	2.00	4.50	3.00
S_{22}	F	1.20	1.80	2.40	3.00	3.00
S_{23}	F	1.00	1.50	2.00	mistake	3.00
S_{24}	F	1.40	2.10	2.40	2.00	3.00
S_{25}	M	0.40	1.20	1.50	3.00	3.00
S_{26}	F	1.92	2.40	3.20	2.50	3.60
S_{27}	F	1.60	1.80	3.20	3.00	3.60
S_{28}	M	1.20	1.80	2.40	3.00	2.40

S_{29}	M	1.80	1.80	2.52	3.40	2.64
S_{30}	F	1.60	2.40	1.60	1.60	1.44
S_{31}	F	1.00	1.80	1.60	2.00	2.40
S_{32}	F	1.00	1.50	2.00	2.10	3.00
S_{33}	F	1.80	2.70	3.60	4.20	4.80
S_{34}	F	1.60	2.10	2.80	3.00	3.00
S_{35}	M	0.80	1.20	1.60	1.80	1.80
S_{36}	M	1.00	1.50	2.00	2.50	2.40
S_{37}	F	1.20	2.10	1.60	1.50	1.80
S_{38}	F	1.60	2.40	3.20	3.80	3.60
Mean (u_{I2})		**1.42**	**1.99**	**2.57**	**3.00**	**3.38**

Table 3-3 Data for the utility scales of cashew nut in Choices I and II (u_{I3}, u_{II3})

N=38, Male 9, Female 29; Assigned unit price 0.50/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	F	1.20	1.80	2.40	2.00	2.40
S_2	M	1.60	2.40	4.00	4.00	4.80
S_3	M	2.00	2.40	3.04	4.00	6.00
S_4	F	1.60	2.40	3.20	3.50	3.60
S_5	F	1.80	2.40	3.60	3.50	4.80
S_7	F	1.40	1.50	2.40	3.50	3.60
S_8	M	1.60	2.10	2.80	3.50	4.20
S_9	F	2.00	1.20	2.00	3.50	3.00
S_{10}	F	1.40	2.10	2.80	3.00	3.60
S_{11}	M	mistake	mistake	3.60	2.50	3.60
S_{12}	F	1.20	1.20	1.60	2.50	1.80
S_{13}	F	1.60	1.80	2.00	3.50	3.00
S_{14}	F	1.20	1.80	2.80	2.50	3.00
S_{15}	F	1.20	1.50	2.40	2.50	2.40
S_{16}	F	1.84	2.28	2.00	4.20	3.60
S_{17}	F	1.96	2.40	3.60	5.80	4.80
S_{18}	F	1.60	2.40	3.20	4.00	4.80
S_{19}	F	1.40	1.50	2.40	2.00	1.80
S_{20}	F	2.00	2.70	3.20	4.00	4.20
S_{21}	F	1.00	2.10	2.40	3.00	3.60
S_{22}	F	1.20	1.80	2.40	2.50	2.40
S_{23}	F	1.00	1.50	2.40	2.50	3.00
S_{24}	F	1.00	0.90	2.00	2.50	3.00
S_{25}	M	0.80	1.50	3.20	2.50	2.40
S_{26}	F	1.80	2.40	2.40	3.00	3.00
S_{27}	F	1.60	2.10	2.40	3.00	3.60
S_{28}	M	1.20	1.80	2.52	3.00	3.60
S_{29}	M	1.60	2.10	2.56	2.00	2.40

S_{30}	F	1.60	2.40	3.20	2.00	4.80
S_{31}	F	1.40	1.50	3.60	3.00	3.60
S_{32}	F	1.00	1.50	2.00	2.50	2.40
S_{33}	F	1.60	2.40	2.80	3.10	3.00
S_{34}	F	1.40	2.70	3.20	3.50	3.60
S_{35}	M	1.00	1.80	2.00	2.50	2.40
S_{36}	M	1.60	2.40	3.20	3.50	4.20
S_{37}	F	1.20	2.40	2.80	3.00	3.00
S_{38}	F	1.60	2.40	3.20	3.50	3.60
Mean (u_{I3}, u_{II3})		**1.46**	**1.98**	**2.73**	**3.08**	**3.41**

Table 3-4 Data for the utility scale of pistachio in Choices II (u_{II1})

N=37, Male 9, Female 28; Assigned unit price 0.70/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	F	2.00	3.00	3.20	3.00	3.60
S_2	M	2.00	3.00	4.00	4.00	2.40
S_3	M	2.00	2.40	4.00	5.00	3.60
S_4	F	2.00	3.00	4.80	5.00	4.80
S_5	F	2.40	3.00	4.80	4.50	4.80
S_6	F	2.00	3.00	4.00	3.50	6.00
S_7	F	2.20	3.00	3.20	5.50	2.40
S_8	M	2.20	3.30	4.40	5.50	6.00
S_9	F	1.40	2.40	3.20	3.50	3.60
S_{11}	M	mistake	mistake	3.60	4.00	6.00
S_{12}	F	1.40	1.80	3.20	3.00	3.60
S_{13}	F	mistake	2.40	3.04	3.50	4.20
S_{14}	F	1.40	2.10	3.04	4.00	3.60
S_{15}	F	2.00	1.50	4.00	3.50	2.40
S_{16}	F	2.64	3.36	5.20	6.20	3.60
S_{17}	F	2.60	3.60	5.20	6.50	7.20
S_{18}	F	2.00	3.00	4.00	4.00	6.00
S_{19}	F	1.80	2.70	2.80	3.50	4.20
S_{20}	F	2.40	3.00	4.80	5.00	6.00
S_{21}	F	1.60	2.40	3.20	3.50	4.80
S_{22}	F	1.60	2.40	2.40	3.00	3.60
S_{23}	F	1.40	2.70	2.80	3.50	4.20
S_{24}	F	2.00	2.10	3.50	4.00	3.60
S_{25}	M	1.20	1.50	2.00	2.00	3.60
S_{26}	F	2.60	3.00	4.80	4.50	4.80
S_{27}	F	2.00	2.40	3.60	3.00	4.80
S_{28}	M	2.00	3.00	2.40	3.00	3.60
S_{29}	M	2.00	3.00	3.60	3.00	4.80
S_{30}	F	0.80	2.40	1.60	2.00	2.40

S_{31}	F	mistake	2.40	2.00	3.00	5.40
S_{32}	F	1.40	2.10	2.80	3.50	4.20
S_{33}	F	2.00	3.00	3.60	4.00	4.20
S_{34}	F	2.40	3.60	4.40	5.50	6.00
S_{35}	M	2.00	3.00	3.20	3.50	4.20
S_{36}	M	2.00	3.00	3.60	4.00	4.20
S_{37}	F	2.00	3.00	3.84	4.50	5.40
S_{38}	F	2.00	2.70	3.20	3.50	4.20
Mean (u_{II1})		**1.92**	**2.70**	**3.54**	**3.94**	**4.38**

Table 3-5 Data for the utility scale of almond in Choices II (u_{II2}, u_{III2})

N=37, Male 9, Female 28; Assigned unit price 0.30/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	F	0.80	1.20	1.60	1.00	2.40
S_2	M	1.20	1.80	2.40	3.00	3.60
S_3	M	1.20	1.50	2.40	3.00	3.60
S_4	F	1.00	1.50	2.40	2.50	3.00
S_5	F	1.00	1.50	2.40	2.50	3.00
S_6	F	1.00	1.50	2.40	3.00	3.60
S_7	F	1.20	1.20	1.60	1.50	1.20
S_8	M	0.80	1.20	1.60	2.00	2.40
S_9	F	1.00	1.20	1.60	2.00	2.40
S_{10}	F	1.00	1.50	2.00	2.50	3.00
S_{11}	M	1.00	1.20	1.60	1.50	2.40
S_{12}	F	0.80	0.90	1.20	1.50	1.20
S_{13}	F	0.80	1.50	2.00	2.50	2.40
S_{14}	F	0.80	1.08	1.60	1.50	1.80
S_{15}	F	1.00	1.50	1.20	2.00	1.68
S_{16}	F	1.16	1.60	2.00	2.50	2.40
S_{17}	F	1.20	1.74	2.00	2.50	2.40
S_{18}	F	1.00	1.80	2.40	3.00	3.60
S_{19}	F	0.60	1.20	1.60	1.50	1.80
S_{20}	F	1.20	1.68	2.24	2.80	3.00
S_{21}	F	0.80	1.20	1.60	2.00	2.40
S_{22}	F	0.80	1.20	1.60	2.00	2.40
S_{23}	F	0.60	0.90	1.20	1.50	1.80
S_{24}	F	1.00	1.20	1.60	2.00	2.16
S_{25}	M	0.20	1.20	1.60	2.00	1.80
S_{26}	F	0.80	1.20	1.60	2.00	1.80
S_{27}	F	1.20	1.50	2.00	2.00	3.00
S_{28}	M	1.20	1.20	1.60	2.00	2.40
S_{29}	M	0.80	1.50	1.60	1.50	1.80
S_{30}	F	0.80	1.80	1.60	2.00	2.40

S_{31}	F	0.80	0.90	1.60	1.50	2.40
S_{32}	F	0.60	1.50	2.00	2.50	3.00
S_{33}	F	1.00	1.50	2.00	2.00	2.40
S_{34}	F	0.80	1.20	1.20	1.50	1.80
S_{35}	M	0.60	0.90	1.20	1.50	1.20
S_{36}	M	0.80	1.50	1.60	2.00	1.80
S_{37}	F	0.40	0.90	1.20	1.00	1.20
S_{38}	F	0.80	1.20	1.60	2.00	1.80
Mean (u_{II2}, u_{III2})		**0.89**	**1.34**	**1.75**	**2.03**	**2.33**

Table 3-6 Data for the utility scale of cashew nut in Choices II (u_{III3})

N=36, Male 8, Female 28; Assigned unit price 0.70/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	F	2.00	3.00	3.20	4.00	4.80
S_2	M	2.00	2.40	3.20	5.00	4.80
S_3	M	2.00	3.00	4.00	5.00	6.00
S_4	F	2.00	2.10	4.00	4.00	4.20
S_5	F	2.40	3.00	4.80	4.50	6.00
S_7	F	2.40	3.00	4.00	4.50	6.00
S_8	M	2.40	3.00	4.40	5.50	6.60
S_9	F	2.00	1.80	4.00	3.50	4.80
S_{10}	F	1.40	2.10	3.20	3.50	4.80
S_{12}	F	1.40	1.80	2.40	3.00	3.60
S_{13}	F	2.00	2.40	3.20	3.50	4.20
S_{14}	F	1.60	2.10	3.04	3.50	3.60
S_{15}	F	2.00	3.00	4.00	3.50	3.60
S_{16}	F	2.72	3.60	5.44	6.30	7.20
S_{17}	F	2.76	4.14	5.20	6.00	7.20
S_{18}	F	2.00	3.00	4.00	5.00	6.00
S_{19}	F	1.80	2.10	2.80	3.50	4.20
S_{20}	F	2.40	3.30	4.80	5.00	6.00
S_{21}	F	1.60	2.28	3.20	3.00	4.20
S_{22}	F	1.20	1.80	2.40	2.50	2.40
S_{23}	F	1.40	2.10	2.80	3.50	4.20
S_{24}	F	1.60	2.10	2.80	3.50	4.20
S_{25}	M	0.40	1.50	1.66	3.00	2.40
S_{26}	F	2.72	3.00	5.20	4.50	6.00
S_{27}	F	1.60	2.10	4.00	3.50	4.20
S_{28}	M	2.00	2.40	3.20	4.00	4.80
S_{29}	M	2.20	3.00	2.80	3.50	3.72
S_{30}	F	0.80	1.80	1.60	4.00	2.88
S_{31}	F	1.80	2.40	2.40	3.00	3.60
S_{32}	F	1.40	2.10	2.00	2.50	3.00

S_{33}	F	2.00	3.30	4.00	3.50	4.20
S_{34}	F	2.40	3.60	4.80	5.00	6.00
S_{35}	M	2.00	2.70	3.60	4.00	4.20
S_{36}	M	2.00	3.00	4.00	4.50	4.80
S_{37}	F	1.60	2.40	2.80	3.50	3.60
S_{38}	F	2.00	3.00	3.20	3.00	3.00
Mean (u_{III3})		**1.89**	**2.60**	**3.50**	**3.97**	**4.58**

2 Derivation for U_{Est}

2.1 Estimating r_{ij}

The estimated values of r_{ij} are contained in the utility curve estimations in the data of Session B. The key problem in derivation for U_{Est} is how to estimate r_{ij} by determining the utility curves.

To estimate r_{ij}, fit the logarithmic law $u_{ij}=c\ln(q_{ij}-r_{ij})+C$, i=I,II,III and j=1,2,3, in the average data of Session B (they are listed on the bottom lines of Tables 3-1, 3-2, 3-3, 3-4, 3-5, and 3-6). It is realized by the curve regression in SPSS, in which the optimal values of c and C in the logarithmic law are created automatically, but r_{ij} must be selected by hand. To isolate from the derivation of U_{LES}, the procedure for selecting r_{ij} is that select the values of r_{ij} to improve the regression results in SPSS until $R^2\geq 0.97$.

1) Estimating r_{I1}, r_{I2}, and r_{I3} in Choice I

For Choice I, by fitting the logarithmic law in the average data presented on the bottom lines in Tables 3-1, 3-2, and 3-3, the curve regression results in SPSS are as the following:

For the pistachio, the logarithmic law is

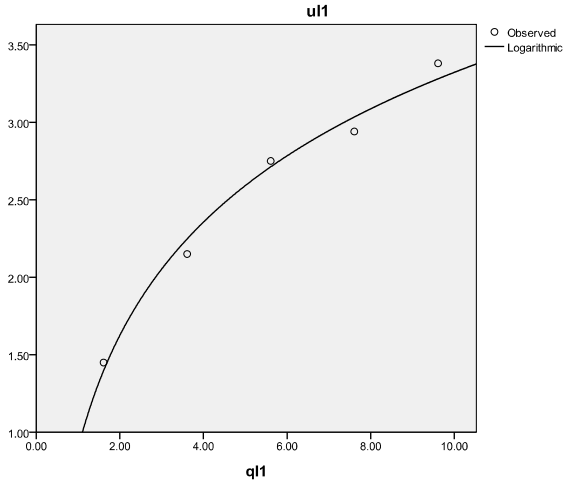
$u_{I1}=1.06\ln(q_{I1}-2.39)+0.89$, $r_{I1}=2.39$ ($R^2=0.985$).

Model Summary and Parameter Estimates

Dependent Variable:uI1

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.985	203.640	1	3	.001	.893	1.055

The independent variable is qI1.



For the almond, the logarithmic law is

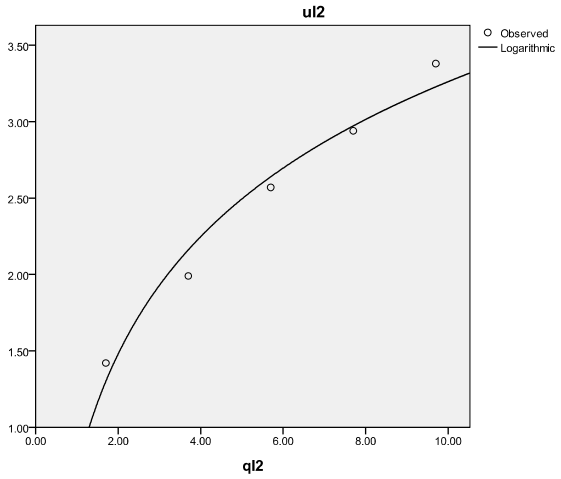
u_{I2}=1.11ln(q_{I2}-2.30)+0.71, r_{I2}=2.30 (R^2=0.970).

Model Summary and Parameter Estimates

Dependent Variable:VAR00003

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.970	95.810	1	3	.002	.713	1.107

The independent variable is VAR00004.



For the cashew nut, the logarithmic law is

u_{I3}=1.07ln(q_{I3}–2.50)+0.90, r_{I3}=2.50 (R^2≈0.970).

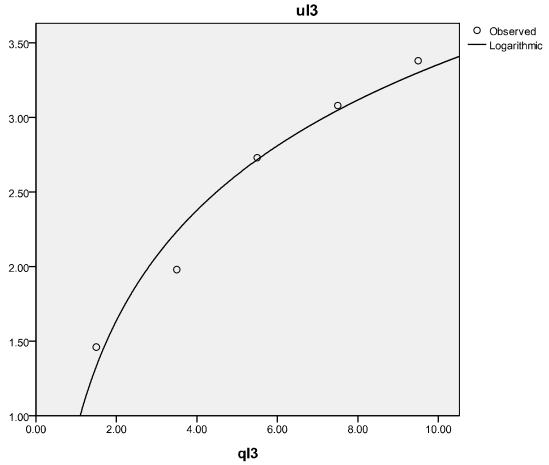
In this curve estimation for cashew nut, the value of R^2 is rounded from 0.965 to 0.97, because the value of R^2 is very insensitive to the value of r_{I3} in this point. It is caused by such a situation: among five fitted data, four of them have fitted to the logarithmic curve very well, but the second datum still deviates from the curve by a relatively big distance. The overall effect of the regression is very well with R^2=0.965.

Model Summary and Parameter Estimates

Dependent Variable:uI3

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.965	81.508	1	3	.003	.898	1.067

The independent variable is qI3.



Summing up, in Choice I of C-Sample, the estimated values of r_{Ij} for U_{Est} are r_{I1}=2.39, r_{I2}=2.30, r_{I3}=2.50.

2) Estimating r_{II1}, r_{II2}, and r_{II3} in Choice II

For Choice II, r_{II3}=r_{I3}=2.50 because their assigned quantities and prices are the same. By fitting the logarithmic law in the average data presented on the bottom lines in Tables 3-4 and 3-5, the curve regression results in SPSS for r_{II1} and r_{II2} are derived as the following:

For the pistachio, the logarithmic law is

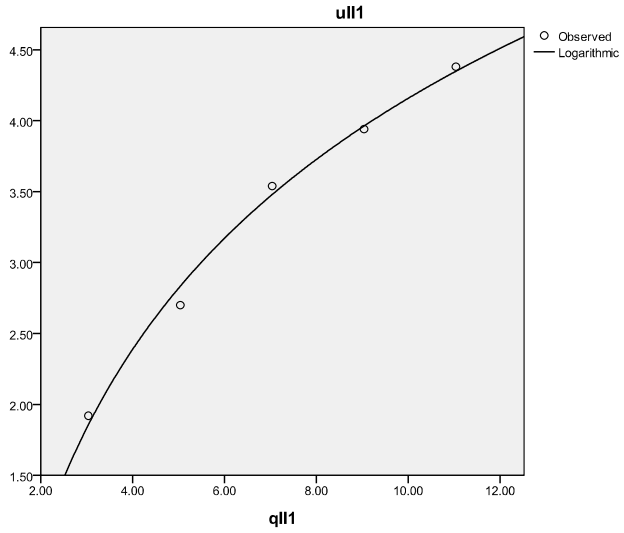
u_{II1}=1.93ln(q_{II1}–0.96)–0.29, r_{II1}=0.96 (R^2=0.993).

Model Summary and Parameter Estimates

Dependent Variable:uII1

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.993	429.911	1	3	.000	-.292	1.932

The independent variable is qII1.



For the almond, the logarithmic law is

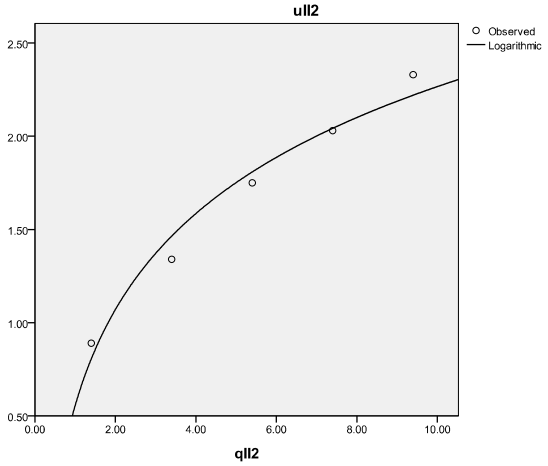
u_{II2}=0.74ln(q_{II2}–2.40)–0.56, r_{II2}=2.40 (R^2=0.970).

Model Summary and Parameter Estimates

Dependent Variable:uII2

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.970	97.930	1	3	.002	.555	.743

The independent variable is qII2.



Summing up, in Choice II of C-Sample, the estimated values of r_{IIj} for U_{Est} are r_{II1}=0.96, r_{II2}=2.40, r_{II3}= r_{I3}=2.50.

3) Estimating r_{III1}, r_{III2}, and r_{III3} in Choice III

For Choice III, r_{III1}=r_{I1}=2.39 and r_{III2}=r_{II2}=2.40 because their assigned quantities and prices are the same respectively. By fitting the logarithmic law in the average data presented on the bottom line in Table 3-6, the curve regression result in SPSS for r_{III3} is derived as the following:

For the cashew nut, the logarithmic law is

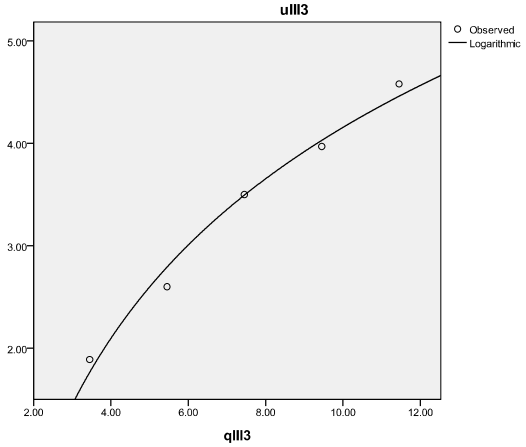
u_{III3}=2.25ln(q_{II1}–0.55)–1.02, r_{III3}=0.55 (R^2=0.985).

Model Summary and Parameter Estimates

Dependent Variable:uIII3

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.985	193.105	1	3	.001	-1.020	2.248

The independent variable is qIII3.



Summing up, in Choice III of C-Sample, the estimated values of r_{IIIj} for U_{Est} are r_{III1}=r_{I1}=2.39, r_{III2}=r_{II2}=2.40, r_{II3}=0.55.

Now the estimated values of r_{ij} for each U_{Est} in Choices I, II, and III of C-Sample all have been obtained. They are collected below:

in Choice I: r_{I1}=2.39, r_{I2}=2.30, r_{I3}=2.50;

in Choice II: r_{II1}=0.96, r_{II2}=2.40, r_{II3}= r_{I3}=2.50;

in Choice III: r_{III1}=r_{I1}=2.39, r_{III2}=r_{II2}=2.40, r_{II3}=0.55.

2.2 Estimating b_{ij}

b_{ij} follows the formula $b_{ij}=\dfrac{p_{ij}(q_{ij}-r_{ij})}{\sum p_{ij}(q_{ij}-r_{ij})}$, i=I,II,III and j=1,2,3. In the experiments, for U_{Est}, the formula is rewritten

$$b_{ij}=\frac{p_{ij}(\bar{q}_{ij}-r_{ij})}{\sum p_{ij}(\bar{q}_{ij}-r_{ij})}, \qquad i\text{=I,II,III and } j\text{=1,2,3.}$$

Where, $\bar{q}_{ij}$ is estimated by the data presented on the bottom lines in Tables 2-1, 2-2, and 2-3 in Part 2, and r_{ij} had been obtained above.

Using the data on the bottom line of Table 2-1 and the estimated values of r_{Ij}, b_{Ij} for U_{Est} in Exp. 1 of C-Sample is estimated for Choice I:

$$b_{I1}=\frac{p_{I1}(\bar{q}_{I1}-r_{I1})}{\sum p_{Ij}(\bar{q}_{Ij}-r_{Ij})}=\frac{0.5(4.0-2.39)}{0.5(4.0-2.39)+0.5(3.95-2.30)+0.5(4.18-2.50)}=0.32;$$

$$b_{I2}=\frac{p_{I2}(\bar{q}_{I2}-r_{I2})}{\sum p_{Ij}(\bar{q}_{Ij}-r_{Ij})}=\frac{0.5(3.95-2.30)}{0.5(4.0-2.39)+0.5(3.95-2.30)+0.5(4.18-2.50)}=0.33;$$

$$b_{I3}=\frac{p_{I3}(\bar{q}_{I3}-r_{I3})}{\sum p_{Ij}(\bar{q}_{Ij}-r_{Ij})}=\frac{0.5(4.18-2.50)}{0.5(4.0-2.39)+0.5(3.95-2.30)+0.5(4.18-2.50)}=0.34.$$

Using the data on the bottom line of Table 2-2 and the estimated values of r_{IIj}, b_{IIj} for U_{Est} in Exp. 1 of C-Sample is estimated for Choice II:

$$b_{II1} = \frac{p_{II1}(\bar{q}_{II1} - r_{II1})}{\sum p_{IIj}(\bar{q}_{IIj} - r_{IIj})} = \frac{0.7(2.54 - 0.96)}{0.7(2.54 - 0.96) + 0.3(7.05 - 2.40) + 0.5(4.24 - 2.50)} = 0.33 ;$$

$$b_{II2} = \frac{p_{II2}(\bar{q}_{II2} - r_{II2})}{\sum p_{IIj}(\bar{q}_{IIj} - r_{IIj})} = \frac{0.3(7.05 - 2.40)}{0.7(2.54 - 0.96) + 0.3(7.05 - 2.40) + 0.5(4.24 - 2.50)} = 0.41 ;$$

$$b_{II3} = \frac{p_{II3}(\bar{q}_{II3} - r_{II3})}{\sum p_{IIj}(\bar{q}_{IIj} - r_{IIj})} = \frac{0.5(4.24 - 2.50)}{0.7(2.54 - 0.96) + 0.3(7.05 - 2.40) + 0.5(4.24 - 2.50)} = 0.26 .$$

Using the data on the bottom line of Table 2-3 and the estimated values of r_{IIIj}, b_{IIIj} for U_{Est} in Exp. 1 of C-Sample is estimated for Choice III

$$b_{III1} = \frac{p_{III1}(\bar{q}_{III1} - r_{III1})}{\sum p_{IIIj}(\bar{q}_{IIIj} - r_{IIIj})} = \frac{0.5(4.35 - 2.39)}{0.5(4.35 - 2.39) + 0.3(7.28 - 2.40) + 0.7(2.36 - 0.55)} = 0.26$$

$$b_{III2} = \frac{p_{III2}(\bar{q}_{III2} - r_{III2})}{\sum p_{IIIj}(\bar{q}_{IIIj} - r_{IIIj})} = \frac{0.3(7.28 - 2.40)}{0.5(4.35 - 2.39) + 0.3(7.28 - 2.40) + 0.7(2.36 - 0.55)} = 0.39$$

$$b_{III3} = \frac{p_{III3}(\bar{q}_{III3} - r_{III3})}{\sum p_{IIIj}(\bar{q}_{IIIj} - r_{IIIj})} = \frac{0.7(2.36 - 0.55)}{0.5(4.35 - 2.39) + 0.3(7.28 - 2.40) + 0.7(2.36 - 0.55)} = 0.34$$

2.3 Results of U_{Est}

Using the above estimated values of r_{Ij} and b_{ij}, each U_{Est} for C-Sample is finally derived:

in Choice I,

$$U_{Est} = 0.32\ln(q_{I1} - 2.39) + 0.33\ln(q_{I2} - 2.30) + 0.34\ln(q_{I3} - 2.50) ;$$

in Choice II,

$$U_{Est} = 0.33\ln(q_{II1} - 0.96) + 0.41\ln(q_{II2} - 2.40) + 0.26\ln(q_{II3} - 2.50) ;$$

in Choice III,

$$U_{Est} = 0.26\ln(q_{III1} - 2.39) + 0.39\ln(q_{III2} - 2.40) + 0.34\ln(q_{III3} - 0.55) .$$

Part 4 Estimating U_{LES} and U_{Est} in Exp. 1 of S-Sample

1 Original data for U_{LES}

Tables 4-1, 4-2, and 4-3 present the original data obtained from Session A in Exp. 1 of S-Sample for solving LES.

Table 4-1 Data of Choice I in Exp. 1 of S-Sample

N=41, Male 22, Female 19

Subject	Sex	Pistachio (q_{I1}) (p_{I1}=0.50)	Almond (q_{I2}) (p_{I2}=0.50)	Cashew nut (q_{I3}) (p_{I3}=0.50)	$p_{I1}q_{I1}$	$p_{I2}q_{I2}$	$p_{I3}q_{I3}$	$\Sigma p_{Ij}q_{Ij}$
S_1	M	5	4	4	2.50	2.00	2.00	6.50
S_2	M	4	4	5	2.00	2.00	2.50	6.50
S_3	M	4	4	5	2.00	2.00	2.50	6.50
S_4	M	2	1	10	1.00	0.50	5.00	6.50
S_5	F	4	5	4	2.00	2.50	2.00	6.50
S_6	M	5	3	5	2.50	1.50	2.50	6.50
S_7	F	6	3	4	3.00	1.50	2.00	6.50
S_8	M	6	2	5	3.00	1.00	2.50	6.50
S_9	F	4	3	6	2.00	1.50	3.00	6.50
S_{10}	M	4	4	5	2.00	2.00	2.50	6.50
S_{11}	M	6	6	1	3.00	3.00	0.50	6.50
S_{12}	M	4	4	5	2.00	2.00	2.50	6.50
S_{13}	F	5	2	5	2.50	1.00	2.50	6.00
S_{14}	M	4	4	5	2.00	2.00	2.50	6.50
S_{15}	M	6	4	3	3.00	2.00	1.50	6.50
S_{16}	M	4	4	5	2.00	2.00	2.50	6.50
S_{17}	M	4	4	4	2.00	2.00	2.00	6.00
S_{18}	F	5	3	5	2.50	1.50	2.00	6.00
S_{19}	F	4	4	5	2.00	2.00	2.50	6.50
S_{20}	M	4	4	4	2.00	2.00	2.00	6.00
S_{21}	M	10	2	1	5.00	1.00	0.50	6.50
S_{22}	M	5	4	4	2.50	2.00	2.00	6.50
S_{23}	F	5	4	4	2.50	2.00	2.00	6.50
S_{24}	F	5	4	4	2.50	2.00	2.00	6.50
S_{25}	M	5	4	4	2.50	2.00	2.50	6.50
S_{26}	F	5	2	6	2.50	1.00	3.00	6.50
S_{27}	F	2	10	1	1.00	5.00	0.50	6.50
S_{28}	M	4	4	5	2.00	2.00	2.50	6.50
S_{29}	F	4	6	3	2.00	3.00	1.50	6.50
S_{30}	M	5	3	5	2.50	1.50	2.50	6.50
S_{31}	F	5	4	4	2.50	2.00	2.00	6.50
S_{32}	F	1	6	6	0.50	3.00	3.00	6.50
S_{33}	M	8	4	1	4.00	2.00	0.50	6.50

S_{34}	M	4	4	5	2.00	2.00	2.50	6.50
S_{35}	F	4	4	4	2.00	2.00	2.00	6.00
S_{36}	F	4	4	4	1.00	2.00	2.00	6.00
S_{37}	M	4	3	3	2.00	1.50	1.50	5.00
S_{38}	F	4	2	2	2.00	1.00	1.00	4.00
S_{39}	F	2	1	6	1.00	0.50	3.00	4.50
S_{40}	F	4	3	2	2.00	1.50	1.00	4.50
S_{41}	F	2	4	2	1.00	2.00	1.00	4.00
Mean ($\overline{q}_{Ij}$)		**4.44**	**3.76**	**4.17**				

Table 4-2 Data of Choice II in Exp. 1 of S-Sample

N=40, Male 22, Female 18

Subject	Sex	Pistachio (q_{II1}) (p_{II1}=0.70)	Almond (q_{II2}) (p_{II2}=0.30)	Cashew nut (q_{II3}) (p_{II3}=0.50)	$p_{II1}q_{II1}$	$p_{II2}q_{II2}$	$p_{II3}q_{II3}$	$\Sigma p_{IIj}q_{IIj}$
S_1	M	3	5	6	2.10	1.50	3.00	6.50
S_2	M	2	4	8	1.40	1.20	4.00	6.60
S_3	M	4	6	4	2.80	1.80	2.00	6.60
S_4	M	1	3	10	0.70	0.90	5.00	6.60
S_5	F	2	12	3	1.40	3.60	1.50	6.50
S_6	M	4	4	5	2.10	1.20	2.50	5.80
S_7	F	4	6	4	2.80	1.80	2.00	6.60
S_8	M	4	4	5	2.80	1.20	2.50	6.50
S_9	F	3	5	6	2.10	1.50	3.00	6.60
S_{10}	M	4	5	4	2.80	1.50	2.00	6.30
S_{11}	M	3	12	1	2.10	3.60	0.50	6.20
S_{12}	M	3	11	2	2.10	3.30	1.00	6.40
S_{13}	F	3	2	6	2.10	0.60	3.00	5.70
S_{14}	M	3	5	5	2.10	1.50	2.50	6.10
S_{15}	M	6	5	1	4.20	1.50	0.50	6.20
S_{16}	M	3	10	3	2.10	3.00	1.50	6.60
S_{17}	M	3	7	4	2.10	2.10	2.00	6.20
S_{18}	F	3	4	6	2.10	1.20	3.00	6.30
S_{19}	F	4	6	4	2.10	2.40	2.00	6.50
S_{20}	M	4	5	4	2.80	1.50	2.00	6.30
S_{21}	M	3	10	3	2.10	3.00	1.50	6.60
S_{22}	M	2	14	2	1.40	4.20	1.00	6.60
S_{23}	F	4	4	5	2.80	1.20	2.00	6.00
S_{24}	F	3	6	5	2.10	1.80	2.00	5.90
S_{25}	M	4	4	5	2.80	1.20	2.50	6.50
S_{26}	F	4	4	5	2.80	1.20	2.50	6.50
S_{27}	F	1	16	2	0.70	4.80	1.00	6.50
S_{28}	M	4	4	5	2.80	1.20	2.50	6.50
S_{29}	F	2	14	2	1.40	4.20	1.00	6.60

S_{30}	M	3	3	6	2.10	0.90	3.00	6.00
S_{31}	F	5	5	3	3.50	1.50	1.50	6.50
S_{33}	M	5	5	1	3.50	1.50	0.50	5.50
S_{34}	M	1	11	1	0.70	3.30	0.50	4.50
S_{35}	F	2	2	3	1.40	0.60	1.50	3.50
S_{36}	F	3	4	4	2.10	1.20	2.00	5.20
S_{37}	M	4	4	3	2.80	1.20	1.50	5.50
S_{38}	F	4	3	3	2.80	1.20	1.50	4.50
S_{39}	F	1	1	10	0.70	0.30	5.00	6.00
S_{40}	F	3	3	2	2.10	0.90	1.00	4.00
S_{41}	F	1	5	3	0.70	1.50	1.50	3.70
Mean ($\overline{q}_{IIj}$)		**3.13**	**6.08**	**4.10**				

Table 4-3 Data of Choice III in Exp. 1 of S-Sample

N=39, Male 20, Female 19

Subject	Sex	Pistachio (q_{III1}) (p_{III1}=0.50)	Almond (q_{III2}) (p_{III2}=0.30)	Cashew nut (q_{III3}) (p_{III3}=0.70)	$p_{III1}q_{I1}$	$p_{III2}q_{III2}$	$p_{III3}q_{III3}$	$\Sigma p_{IIIj}q_{IIIj}$
S_1	M	6	5	3	3.00	1.50	2.10	6.60
S_2	M	8	4	2	4.00	1.20	1.40	6.60
S_3	M	4	6	4	2.00	1.80	2.80	6.60
S_4	M	10	3	1	5.00	0.90	0.70	6.60
S_5	F	5	10	1	2.50	3.00	0.70	6.20
S_6	M	5	4	4	2.50	1.20	2.80	6.50
S_7	F	6	4	3	3.00	1.20	2.10	6.30
S_8	M	6	5	3	3.00	1.50	2.10	6.60
S_9	F	6	5	3	3.00	1.50	2.10	6.60
S_{11}	M	4	12	1	2.00	3.60	0.70	6.30
S_{12}	M	2	11	3	1.00	3.30	2.10	6.40
S_{13}	F	5	2	5	2.50	0.60	3.50	6.60
S_{14}	M	5	5	3	2.50	1.50	2.10	6.10
S_{15}	M	6	9	1	3.00	2.70	0.70	6.40
S_{16}	M	4	8	3	2.00	2.40	2.10	6.50
S_{17}	M	4	7	3	2.00	2.10	2.10	6.20
S_{18}	F	4	4	4	2.00	1.20	2.80	6.00
S_{19}	F	4	6	4	2.00	1.80	2.80	6.60
S_{20}	M	5	5	3	2.50	1.50	2.10	6.10
S_{21}	M	3	10	3	1.50	3.00	2.10	6.60
S_{22}	M	3	10	1	1.50	3.00	0.70	5.20
S_{23}	F	5	5	3	2.50	1.50	2.10	6.10
S_{24}	F	6	4	2	3.00	1.20	1.40	5.60
S_{25}	M	5	5	3	2.50	1.50	2.10	6.10
S_{26}	F	5	4	4	2.50	1.20	2.80	6.50
S_{27}	F	2	16	1	1.00	4.80	0.70	6.50

S_{28}	M	5	4	4	2.50	1.20	2.80	6.50
S_{29}	F	3	14	1	1.50	4.20	0.70	6.40
S_{30}	M	4	3	3	2.00	0.90	2.10	5.00
S_{31}	F	6	5	2	3.00	1.50	1.40	5.90
S_{32}	F	3	9	1	1.50	2.70	0.70	5.90
S_{34}	M	1	11	1	0.50	3.30	0.70	4.50
S_{35}	F	3	2	2	1.50	0.60	1.40	3.50
S_{36}	F	4	4	3	2.00	1.20	2.10	5.40
S_{37}	M	4	3	3	2.00	0.90	2.10	5.00
S_{38}	F	4	3	1	2.00	0.90	0.70	3.60
S_{39}	F	2	1	5	1.00	0.30	3.50	4.80
S_{40}	F	4	3	1	2.00	0.90	0.70	3.60
S_{41}	F	2	5	2	1.00	1.50	1.40	3.90
Mean ($\overline{q}_{IIIj}$)		**4.44**	**6.05**	**2.56**				

Table 4-1-I presents the average values of $p_{I1}q_{I1}$, $p_{I2}q_{I2}$, and $p_{I3}q_{I3}$ of each budget constraint group in Table 4-1; Table 4-2-II the average values of $p_{II1}q_{II1}$, $p_{II2}q_{II2}$, and $p_{II3}q_{II3}$ of each budget constraint group in Table 4-2; and Table 4-3-III the average values of $p_{III1}q_{III1}$, $p_{III2}q_{III2}$, and $p_{III3}q_{III3}$ of each budget constraint group in Table 4-3. U_{LES} will be estimated from Tables 4-1-I, 4-2-II, and 4-3-III.

Table 4-1-I Category data of Choice I in Exp. 1 of S-Sample

$\overline{p_{I1}q_{I1}}$	$\overline{p_{I2}q_{I2}}$	$\overline{p_{I2}q_{I2}}$	$\sum p_{Ij}q_{Ij}$
2.33	2.00	2.18	6.50
2.00	1.70	2.10	6.00
2.00	1.50	1.50	5.00
1.50	1.00	2.00	4.50
1.00	2.00	1.00	4.00

Table 4-2-II Category data of Choice II in Exp. 1 of S-Sample

$\overline{p_{II1}q_{II1}}$	$\overline{p_{II2}q_{II2}}$	$\overline{p_{II2}q_{II2}}$	$\sum p_{IIj}q_{IIj}$
1.87	2.40	2.33	6.60
2.33	2.07	2.11	6.50
2.10	3.30	1.00	6.40
2.57	1.40	2.33	6.30
2.80	2.40	1.00	6.20
2.10	1.50	2.50	6.10
1.87	0.80	3.33	6.00
2.10	1.80	2.00	5.90
2.10	1.20	2.50	5.80
2.10	0.60	3.00	5.70
3.15	1.35	1.00	5.50

2.10	1.20	2.00	5.20
1.75	2.25	1.00	5.00
0.70	3.30	0.50	4.50
2.10	0.90	1.00	4.00
0.70	1.50	1.50	3.70
1.40	0.60	1.50	3.50

Table 4-3-III Category data of Choice III in Exp. 1 of S-Sample

$\overline{p_{III1}q_{III1}}$	$\overline{p_{III2}q_{III2}}$	$\overline{p_{III2}q_{III2}}$	$\sum p_{IIIj}q_{IIIj}$
2.89	1.54	2.18	6.60
2.10	2.16	2.24	6.50
1.83	3.40	1.17	6.40
2.50	2.40	1.40	6.30
2.25	2.55	1.40	6.20
2.50	1.50	2.10	6.10
2.00	1.20	2.80	6.00
2.25	2.1	1.05	5.90
3.00	1.20	1.40	5.60
2.00	1.20	2.10	5.40
1.50	3.00	0.70	5.20
2.00	0.90	2.10	5.00
1.00	0.30	3.50	4.80
0.50	3.30	0.70	4.50
1.00	1.50	1.40	3.90
2.00	0.90	0.70	3.60
1.50	0.60	1.40	3.50

2 Derivation for U_{LES}

Solving (2-2) in the data of Table 4-1-I for Choice I,

$\begin{cases} a_{I1}=-0.627, \\ b_{I1}=0.460; \end{cases}$ $\begin{cases} a_{I2}=0.963, \\ b_{I2}=0.130; \end{cases}$ $\begin{cases} a_{I3}=-0.196, \\ b_{I3}=0.375. \end{cases}$

Solving (2-2) in the data of Table 4-2-II for Choice II,

$\begin{cases} a_{II1}=-0.131, \\ b_{II1}=0.388; \end{cases}$ $\begin{cases} a_{II2}=0.318, \\ b_{II2}=0.249; \end{cases}$ $\begin{cases} a_{II3}=-0.175, \\ b_{II3}=0.361. \end{cases}$

Solving (2-2) in the data of Table 4-3-III for Choice III,

$\begin{cases} a_{III1}=-0.264, \\ b_{III1}=0.408; \end{cases}$ $\begin{cases} a_{III2}=-0.276, \\ b_{III2}=0.376; \end{cases}$ $\begin{cases} a_{III3}=0.586, \\ b_{III3}=0.201. \end{cases}$

Substitute the above results in (2-4) and solve for r_{ij}

$r_{I1}=2.43$, $r_{I2}=2.97$, $r_{I3}=2.61$;

$r_{II1}=3.14$, $r_{II2}=6.04$, $r_{II3}=3.98$;

$r_{III1}=1.98$, $r_{III2}=2.93$, $r_{III3}=1.72$.

Finally obtain each U_{LES} in Session A of Exp. 1 for S-Sample:

in Choice I,

$$U_{LES} = 0.46\ln(q_{I1} - 2.43) + 0.13\ln(q_{I2} - 2.97) + 0.38\ln(q_{I3} - 2.61);$$

in Choice II,

$$U_{LES} = 0.39\ln(q_{II1} - 3.14) + 0.25\ln(q_{II2} - 6.04) + 0.6\ln(q_{II3} - 3.98);$$

in Choice III,

$$U_{LES} = 0.41\ln(q_{III1} - 1.98) + 0.381\ln(q_{III2} - 2.93) + 0.20\ln(q_{III3} - 1.72).$$

3 Derivation for U_{Est}

3.1 Original data for U_{Est}

Tables 4-4, 4-5, 4-6, 4-7, 4-8, and 4-9 present the experimental data obtained from Session B in Exp. 1 of S-Sample.

Table 4-4 Data for the utility scales of pistachio in Choices I and III (u_{I1}, u_{III1})

N=41, Male 22, Female 19; Assigned unit price 0.50/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	1.40	3.00	3.20	3.00	4.20
S_2	M	1.80	2.70	3.20	4.00	4.80
S_3	M	1.60	2.40	3.20	3.00	3.60
S_4	M	0.80	1.80	2.80	3.00	2.40
S_5	F	2.00	3.00	2.40	4.00	4.80
S_6	M	1.80	2.70	3.20	4.00	4.80
S_7	F	2.00	2.40	4.00	4.00	4.80
S_8	M	2.00	3.00	3.60	4.00	4.80
S_9	F	1.80	2.40	3.20	3.50	4.20
S_{10}	M	1.20	1.80	2.40	2.00	2.40
S_{11}	M	1.20	2.10	2.40	3.00	3.60
S_{12}	M	1.60	2.40	3.20	3.00	3.60
S_{13}	F	0.80	2.40	3.20	3.00	3.60
S_{14}	M	1.80	2.70	3.60	4.00	6.00
S_{15}	M	0.80	1.20	1.60	1.00	1.20
S_{16}	M	2.00	2.40	2.40	3.00	3.00
S_{17}	M	1.60	2.40	3.20	2.00	2.40
S_{18}	F	2.00	2.40	2.40	3.00	3.60
S_{19}	F	2.00	2.70	2.40	4.50	3.60
S_{20}	M	2.00	3.00	4.00	4.50	5.40
S_{21}	M	1.80	2.88	3.60	4.50	4.80
S_{22}	M	1.60	2.40	3.20	4.00	4.50
S_{23}	F	1.60	2.40	3.20	4.00	4.80
S_{24}	F	1.60	2.40	3.20	3.50	4.20
S_{25}	M	1.80	2.40	3.20	4.00	4.20
S_{26}	F	2.00	3.00	4.00	4.00	5.40
S_{27}	F	1.60	3.00	3.20	4.50	4.80
S_{28}	M	1.80	2.70	3.20	4.00	4.80

S_{29}	F	0.40	1.80	1.60	1.00	1.20
S_{30}	M	2.00	3.00	4.00	5.00	4.80
S_{31}	F	0.80	1.50	1.60	2.00	2.40
S_{32}	F	1.20	1.20	2.40	2.00	2.40
S_{33}	M	2.00	3.00	4.00	5.00	6.00
S_{34}	M	1.20	1.80	2.40	4.00	3.60
S_{35}	F	0.40	1.20	2.00	2.50	2.40
S_{36}	F	1.20	1.80	2.40	2.00	2.40
S_{37}	M	1.80	2.70	3.20	4.00	4.80
S_{38}	F	1.20	2.40	3.20	4.00	3.60
S_{39}	F	1.20	1.80	2.40	2.50	3.00
S_{40}	F	1.84	2.76	3.36	4.00	4.56
S_{41}	F	1.20	1.80	2.40	2.00	2.40
Mean (u_{I1}, u_{III1})		**1.52**	**2.36**	**2.96**	**3.37**	**3.85**

Table 4-5 Data for the utility scale of almond in Choice I (u_{I2})

N=41, Male 22, Female 19; Assigned unit price 0.50/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	1.40	2.10	2.80	3.00	4.20
S_2	M	1.80	2.70	3.20	4.00	4.80
S_3	M	1.60	2.40	3.20	3.00	3.60
S_4	M	1.00	1.80	2.00	2.00	2.40
S_5	F	1.60	2.40	2.40	4.00	3.60
S_6	M	1.80	2.70	3.20	4.00	4.80
S_7	F	1.60	2.40	2.40	2.00	2.40
S_8	M	1.60	2.40	3.20	4.00	4.80
S_9	F	1.80	2.40	3.20	3.50	4.20
S_{10}	M	1.20	1.80	2.40	2.00	2.40
S_{11}	M	1.20	1.80	2.40	2.50	3.00
S_{12}	M	1.60	2.40	3.20	3.00	3.60
S_{13}	F	0.80	1.80	2.40	3.00	2.40
S_{14}	M	1.80	2.70	3.20	4.00	4.80
S_{15}	M	1.60	2.40	2.40	3.00	1.20
S_{16}	M	1.60	2.40	4.00	4.00	4.80
S_{17}	M	2.00	3.00	3.20	2.00	2.40
S_{18}	F	2.00	2.40	2.40	4.00	2.40
S_{19}	F	2.00	2.70	2.40	4.50	3.60
S_{20}	M	2.00	3.00	4.00	4.50	5.40
S_{21}	M	1.92	2.70	3.60	4.50	4.80
S_{22}	M	1.80	2.40	3.20	4.00	4.20
S_{23}	F	1.60	2.40	3.20	4.00	4.80
S_{24}	F	1.60	2.40	3.20	3.50	3.60
S_{25}	M	1.80	2.52	3.20	4.00	4.20

S_{26}	F	1.60	2.10	3.20	3.50	4.80
S_{27}	F	2.00	2.70	4.00	4.80	5.76
S_{28}	M	1.20	2.40	3.20	2.00	3.00
S_{29}	F	0.80	1.80	1.60	3.00	1.20
S_{30}	M	1.60	2.40	3.20	4.00	3.60
S_{31}	F	0.40	1.20	0.80	1.00	1.20
S_{32}	F	1.20	1.80	2.40	2.50	3.00
S_{33}	M	2.00	3.00	3.20	4.00	3.60
S_{34}	M	0.80	1.80	2.40	4.00	3.60
S_{35}	F	0.40	1.20	1.60	2.00	1.20
S_{36}	F	0.80	1.80	1.60	2.00	1.20
S_{37}	M	1.60	2.40	3.20	3.50	3.60
S_{38}	F	1.20	1.80	2.40	2.00	2.40
S_{39}	F	1.00	1.50	1.60	2.00	2.40
S_{40}	F	1.76	2.82	3.36	4.20	4.20
S_{41}	F	1.20	1.80	2.40	2.00	2.40
Mean (u_{I2})		**1.47**	**2.26**	**2.78**	**3.23**	**3.40**

Table 4-6 Data for the utility scales of cashew nut in Choices I and II (u_{I3}, u_{II3})

N=41, Male 22, Female 19; Assigned unit price 0.50/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	1.40	2.40	2.80	3.50	4.20
S_2	M	1.80	2.70	3.20	4.00	4.80
S_3	M	1.60	2.40	2.40	3.00	3.60
S_4	M	1.00	1.80	2.80	3.00	3.20
S_5	F	1.60	2.40	4.00	3.00	3.60
S_6	M	1.60	2.88	3.60	4.50	5.40
S_7	F	1.60	2.40	2.40	3.00	3.60
S_8	M	2.00	2.70	3.20	4.00	4.80
S_9	F	1.60	2.40	2.80	3.50	4.20
S_{10}	M	1.20	1.80	2.40	2.00	2.40
S_{11}	M	1.00	1.50	2.00	2.00	2.40
S_{12}	M	1.00	2.40	3.20	4.00	4.80
S_{13}	F	1.60	2.40	4.00	3.00	3.60
S_{14}	M	1.80	2.70	3.60	2.00	4.80
S_{15}	M	0.80	1.80	1.60	2.00	1.20
S_{16}	M	1.60	1.80	3.20	3.00	3.60
S_{17}	M	1.40	2.10	2.00	2.00	2.40
S_{18}	F	2.00	3.00	5.60	4.00	3.60
S_{19}	F	2.00	3.00	3.60	3.00	4.80
S_{20}	M	2.00	3.00	3.360	4.50	5.52
S_{21}	M	1.80	2.88	3.60	4.00	4.80
S_{22}	M	1.60	2.70	3.20	4.00	4.20

S_{23}	F	1.60	2.40	3.20	4.00	4.80
S_{24}	F	1.60	2.40	3.20	3.50	3.60
S_{25}	M	2.00	2.70	3.20	3.50	4.20
S_{26}	F	2.00	3.00	4.00	4.50	4.80
S_{27}	F	2.00	3.00	3.36	4.10	4.80
S_{28}	M	2.00	2.70	3.20	4.00	4.80
S_{29}	F	0.80	2.40	1.60	2.00	1.20
S_{30}	M	2.00	3.00	4.00	5.00	4.80
S_{31}	F	0.80	1.20	0.80	1.00	1.20
S_{32}	F	1.60	2.40	2.80	3.00	3.60
S_{33}	M	1.60	1.80	2.40	4.00	3.60
S_{34}	M	1.20	1.20	2.40	3.00	1.20
S_{35}	F	0.80	1.80	2.00	2.50	2.40
S_{36}	F	1.20	1.80	2.40	2.00	2.40
S_{37}	M	1.80	2.70	3.20	4.00	3.60
S_{38}	F	1.60	2.40	3.20	3.00	3.60
S_{39}	F	1.40	2.10	2.80	3.00	3.60
S_{40}	F	1.84	2.82	3.60	4.20	4.20
S_{41}	F	1.20	1.80	2.40	2.00	2.40
Mean (u_{I3}, u_{II3})		**1.54**	**2.36**	**2.98**	**3.25**	**3.67**

Table 4-7 Data for the utility scale of pistachio in Choice II (u_{II1})

N=40, Male 22, Female 18; Assigned unit price 0.70/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	2.00	3.00	4.00	5.00	6.00
S_2	M	2.40	3.60	4.80	5.00	6.00
S_3	M	2.20	3.30	4.40	5.00	6.00
S_4	M	2.00	3.00	4.00	4.00	4.80
S_5	F	2.60	3.90	4.80	6.00	7.20
S_6	M	2.00	2.40	4.00	4.00	6.00
S_7	F	2.40	3.60	4.80	5.00	6.00
S_8	M	2.40	3.60	4.80	5.00	6.00
S_9	F	2.40	3.60	4.80	5.80	6.96
S_{10}	M	`1.20	1.80	2.40	3.00	2.40
S_{11}	M	2.00	3.00	2.40	3.00	3.60
S_{12}	M	2.40	3.60	4.00	5.00	6.00
S_{13}	F	1.60	2.40	1.60	2.00	2.40
S_{14}	M	2.70	3.90	4.80	6.00	7.20
S_{15}	M	0.80	1.20	1.60	2.00	1.80
S_{16}	M	2.00	2.40	4.00	3.00	3.60
S_{17}	M	2.00	3.60	4.80	3.00	3.00
S_{18}	F	2.80	3.60	5.60	6.00	6.00
S_{19}	F	2.00	3.00	4.00	3.00	4.80

S_{20}	M	2.80	4.20	5.44	6.50	7.56
S_{21}	M	2.72	3.90	5.20	6.00	7.20
S_{22}	M	2.40	3.60	4.00	5.00	6.00
S_{23}	F	2.00	3.00	4.00	5.00	6.00
S_{24}	F	2.00	3.00	3.60	4.50	4.80
S_{25}	M	2.60	3.60	4.48	5.50	6.60
S_{26}	F	2.00	3.00	4.80	6.00	6.00
S_{27}	F	2.36	3.78	4.64	6.00	7.32
S_{28}	M	2.00	3.60	4.00	5.00	6.00
S_{29}	F	0.40	2.40	3.20	2.00	1.20
S_{30}	M	2.80	4.20	5.60	6.00	7.20
S_{31}	F	0.80	1.20	1.60	2.00	2.40
S_{33}	M	2.00	3.00	4.00	5.00	6.00
S_{34}	M	0.40	1.80	4.00	3.00	4.80
S_{35}	F	0.80	1.80	2.40	3.00	3.00
S_{36}	F	1.20	2.40	2.40	2.50	3.00
S_{37}	M	2.70	3.90	4.80	6.00	6.60
S_{38}	F	1.60	2.40	3.20	4.00	3.60
S_{39}	F	1.40	2.10	2.80	3.00	3.60
S_{40}	F	2.68	3.96	5.28	6.20	6.60
S_{41}	F	1.20	1.80	2.40	3.00	2.40
Mean (u_{II1})		**1.99**	**3.03**	**3.94**	**4.40**	**5.09**

Table 4-8 Data for the utility scales of almond in Choices II and III (u_{II2}, u_{III2})

N=41, Male 22, Female 19; Assigned unit price 0.30/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	0.80	1.20	1.60	2.00	2.40
S_2	M	1.20	1.80	2.00	2.50	3.00
S_3	M	0.80	1.80	1.60	2.00	2.40
S_4	M	1.20	1.80	1.60	2.00	2.40
S_5	F	0.80	1.80	2.40	2.00	2.40
S_6	M	0.40	1.68	2.24	2.00	2.40
S_7	F	0.80	1.80	1.80	2.00	2.40
S_8	M	1.00	1.50	2.00	2.50	3.00
S_9	F	1.00	1.50	2.00	2.00	2.40
S_{10}	M	1.20	1.80	1.60	3.00	2.40
S_{11}	M	1.20	1.80	2.00	2.50	3.00
S_{12}	M	1.20	1.80	2.40	3.00	3.60
S_{13}	F	0.80	1.80	1.60	2.00	1.20
S_{14}	M	1.00	1.50	2.00	2.00	2.40
S_{15}	M	0.40	1.20	0.80	1.00	1.20
S_{16}	M	0.40	1.20	1.60	1.00	1.20
S_{17}	M	1.20	1.80	2.40	2.00	2.40

S_{18}	F	1.20	1.20	2.40	1.00	1.20
S_{19}	F	1.20	1.80	2.40	2.50	3.00
S_{20}	M	1.20	1.80	2.00	2.80	3.36
S_{21}	M	1.12	1.50	2.00	2.00	3.00
S_{22}	M	1.00	1.50	1.60	2.00	2.76
S_{23}	F	0.80	1.20	1.60	2.00	2.40
S_{24}	F	0.80	1.20	1.60	1.80	1.80
S_{25}	M	1.12	1.20	1.60	2.00	2.40
S_{26}	F	1.20	1.80	2.40	3.00	3.60
S_{27}	F	1.12	1.62	2.24	2.70	3.36
S_{28}	M	1.20	1.80	2.00	2.00	2.40
S_{29}	F	1.20	1.80	1.60	2.00	2.40
S_{30}	M	1.20	1.80	1.60	2.00	2.40
S_{31}	F	0.40	1.50	0.80	1.00	1.20
S_{32}	F	0.80	1.20	1.60	1.50	2.40
S_{33}	M	1.00	1.80	2.00	3.00	2.40
S_{34}	M	0.40	1.20	0.80	1.00	1.20
S_{35}	F	0.60	1.20	1.60	1.00	2.40
S_{36}	F	0.80	1.20	0.80	1.00	1.20
S_{37}	M	0.80	1.50	2.00	2.50	2.40
S_{38}	F	0.80	1.80	2.40	2.00	2.40
S_{39}	F	1.00	1.50	1.60	2.00	2.40
S_{40}	F	1.12	1.50	2.00	2.00	2.40
S_{41}	F	0.80	1.20	1.60	2.00	2.40
Mean (u_{II2}, u_{III2})		**0.93**	**1.55**	**1.80**	**2.01**	**2.37**

Table 4-9 Data for the utility scale of cashew nut in Choice II (u_{III3})

N=39, Male 20, Female 19; Assigned unit price 0.70/bag

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	2.00	3.00	4.00	5.00	6.00
S_2	M	2.40	3.60	4.40	5.50	6.60
S_3	M	2.40	3.60	4.00	5.00	6.00
S_4	M	1.20	2.40	2.80	3.50	4.20
S_5	F	1.20	2.40	3.20	3.00	3.60
S_6	M	2.40	3.90	4.80	5.50	6.60
S_7	F	1.60	3.00	4.00	5.00	4.80
S_8	M	2.40	3.00	4.80	5.00	6.00
S_9	F	2.40	3.60	4.80	5.80	6.96
S_{11}	M	1.00	1.80	1.60	2.50	2.40
S_{12}	M	2.40	3.60	4.00	4.00	4.80
S_{13}	F	1.60	2.40	2.40	3.00	3.60
S_{14}	M	2.70	3.90	4.80	6.00	7.20
S_{15}	M	1.20	1.20	4.00	2.00	1.20

S_{16}	M	2.00	3.00	4.80	6.00	6.00
S_{17}	M	2.40	3.60	4.00	3.00	2.40
S_{18}	F	2.80	3.60	5.60	6.00	6.00
S_{19}	F	2.00	3.00	4.00	3.00	3.60
S_{20}	M	2.80	3.96	5.44	6.50	7.92
S_{21}	M	2.40	3.90	5.20	6.00	7.20
S_{22}	M	2.40	3.00	4.00	5.00	5.20
S_{23}	F	2.00	3.00	4.00	5.00	6.00
S_{24}	F	2.00	3.00	3.60	4.00	4.80
S_{25}	M	2.70	3.60	4.80	5.50	6.00
S_{26}	F	2.00	3.00	4.00	4.50	4.80
S_{27}	F	2.00	3.00	3.36	4.10	4.80
S_{28}	M	2.40	3.60	4.00	5.00	6.00
S_{29}	F	0.80	1.80	2.40	3.00	1.20
S_{30}	M	2.80	4.20	5.60	7.00	7.20
S_{31}	F	0.80	1.20	1.60	1.00	1.20
S_{32}	F	1.60	2.40	3.20	3.50	3.60
S_{34}	M	1.40	2.40	2.40	3.00	2.40
S_{35}	F	0.80	1.80	2.40	3.00	3.00
S_{36}	F	1.20	2.40	2.40	2.50	3.00
S_{37}	M	2.70	3.90	4.80	6.00	6.60
S_{38}	F	1.60	2.40	2.40	3.00	3.60
S_{39}	F	1.60	2.40	3.20	3.50	4.20
S_{40}	F	2.64	3.96	4.96	5.70	6.60
S_{41}	F	1.20	1.80	2.40	2.00	2.40
Mean (u_{III3})		**1.95**	**2.96**	**3.80**	**4.30**	**4.76**

3.2 Estimating r_{ij}

To estimate r_{ij}, fit the logarithmic law $u_{ij}=c\ln(q_{ij}-r_{ij})+C$, i=I,II and j=1,2,3, in the average data of Session B (they are listed on the bottom lines of Tables 4-4, 4-5, 4-6, 4-7, and 4-8. It is realized by the curve regression in SPSS, in which the optimal values of c and C in the logarithmic law are created automatically, but r_{ij} must be selected by hand. To isolate from the derivation of U_{LES}, the procedure for selecting r_{ij} is that select the values of r_{ij} to improve the regression results in SPSS until $R^2\geq 0.97$.

1) Estimating r_{I1}, r_{I2}, and r_{I3} in Choice I

For Choice I, by fitting the logarithmic law in the average data presented on the bottom lines in Tables 4-4, 4-5, and 4-6, the curve regression results in SPSS are derived as the following:

For the pistachio in Choice I, the logarithmic law is

$u_{I1}=1.34\ln(q_{I1}-2.21)+0.66$, $r_{I1}=2.21$ ($R^2=0.989$).

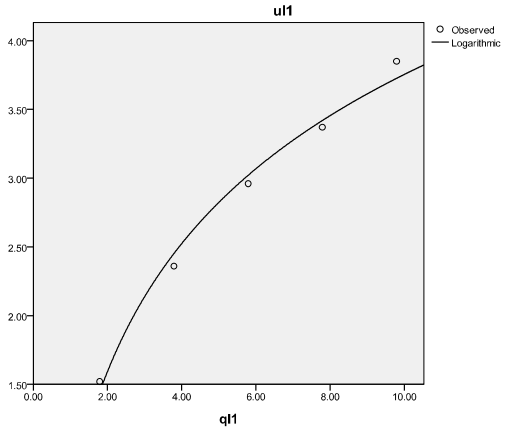
Model Summary and Parameter Estimates

Dependent Variable:uI1

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1

Logarithmic	.989	273.753	1	3	.000	.662	1.343

The independent variable is qI1.



For the almond in Choice I, the logarithmic law is

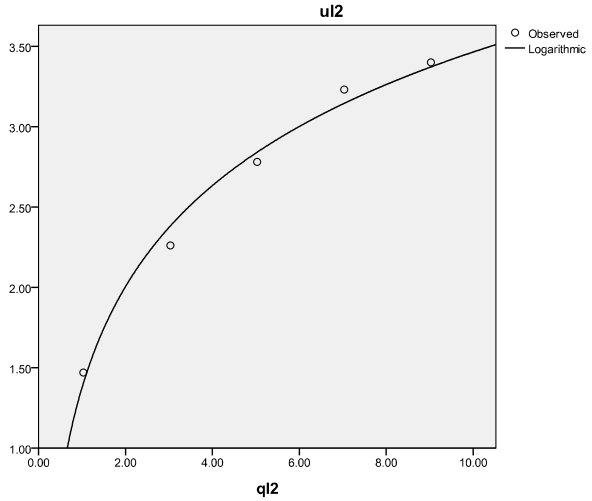
u_{I2}=0.91ln(q_{I2}–2.97)+1.38, r_{I2}=2.97 (R^2=0.987).

Model Summary and Parameter Estimates

Dependent Variable:uI2

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.987	234.426	1	3	.001	1.376	.907

The independent variable is qI2.



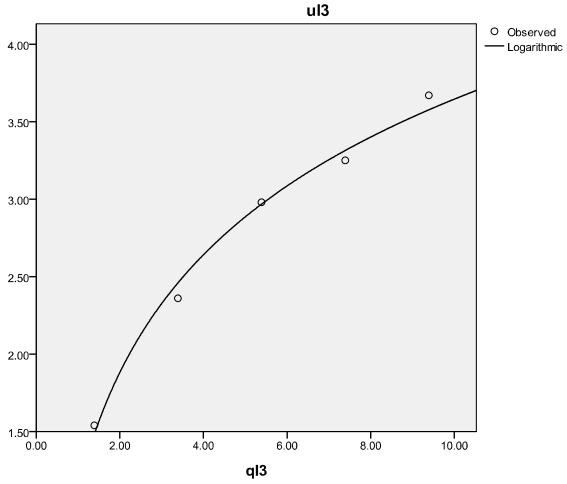
For the almond in Choice I, the logarithmic law is

u_{I3}=1.10 ln(q_{I3}–2.61)+1.21, r_{I3}=2.61 (R^2=0.991).

Model Summary and Parameter Estimates

Dependent Variable:uI3

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.991	313.249	1	3	.000	1.121	1.096



Summing up, in Choice I, Exp. 1 of S-Sample, the estimated values of r_{Ij} for U_{Est} are r_{I1}=2.21, r_{II2}=2.97, r_{I3}=2.61.

2) Estimating r_{II1}, r_{II2}, and r_{II3} in Choice II

For Choice II, r_{II3}=r_{I3}=2.61 because their assigned quantities and prices are the same. By fitting the logarithmic law in the average data presented on the bottom lines in Tables 4-7 and 4-8, the curve regression results in SPSS for r_{II1} and r_{II2} are derived as the following:

For the pistachio in Choice II, the logarithmic law is

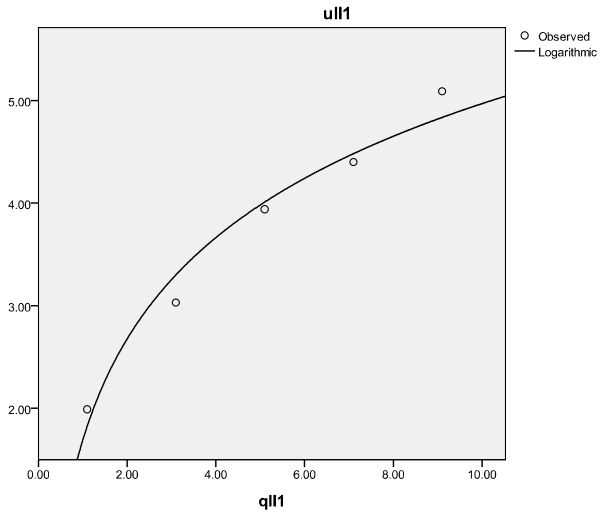
u_{II1}=1.43ln(q_{II1}–2.90)+1.69, r_{II1}=2.90 (R^2=0.970).

Model Summary and Parameter Estimates

Dependent Variable:uII1

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.970	96.067	1	3	.002	1.688	1.425

The independent variable is qII1.



For the almond in Choice II, the logarithmic law is

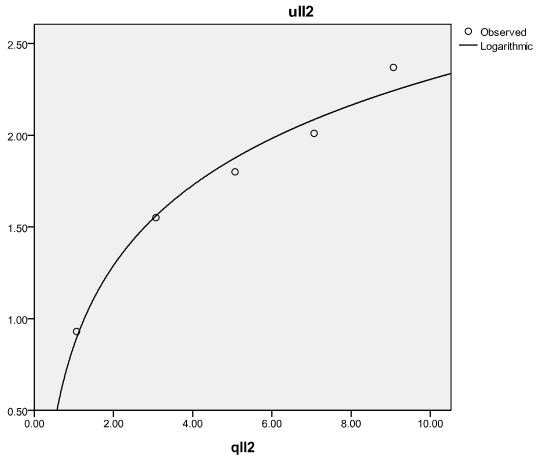
u_{II2}=0.63ln(q_{II2}–2.93)+0.85, r_{II2}=2.93 (R^2=0.975).

Model Summary and Parameter Estimates

Dependent Variable:uII2

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.975	117.509	1	3	.002	.853	.630

The independent variable is qII2.



Summing up, in Choice II, Exp. 1 of S-Sample, the estimated values of r_{Ij} for U_{Est} are r_{I1}=2.90, r_{II2}=2.93, r_{I3}=2.61.

3) Estimating r_{III1}, r_{III2}, and r_{III3} in Choice III

For Choice III, r_{III1}=r_{I1}=2.21 and r_{III2}=r_{II2}=2.93 because their assigned quantities and prices are the same respectively. By fitting the logarithmic law in the average data presented on the bottom line in Table 4-9, the curve regression results in SPSS for r_{III3} is derived as the following:

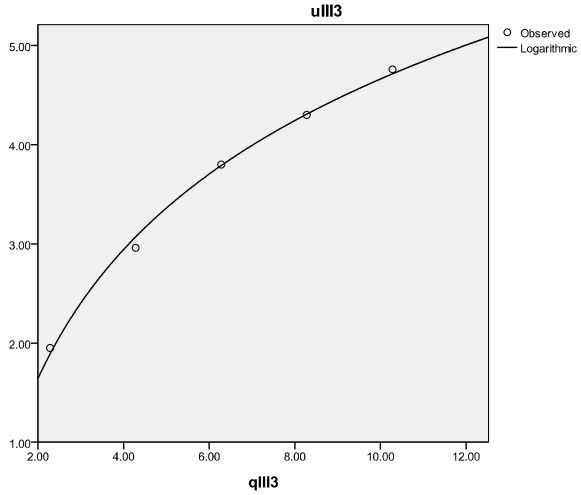
u_{III3}=1.88ln(q_{III1}–1.72)+0.34, r_{III3}=1.72 (R^2=0.996).

Model Summary and Parameter Estimates

Dependent Variable:uIII3

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.996	821.327	1	3	.000	.342	1.876

The independent variable is qIII3.



Summing up, in Choice III of S-Sample, the estimated values of r_{IIIj} for U_{Est} are r_{III1}=r_{I1}=2.21, r_{III2}=r_{II2}=2.93, r_{III3}=1.72.

In summary, the estimated values of r_{ij} for each U_{Est} in Choices I, II, and III of S-Sample all have been obtained. They are collected below:

in Choice I: r_{I1}=2.21, r_{I2}=2.97, r_{I3}=2.61;

in Choice II: r_{II1}=2.90, r_{II2}=2.93, r_{II3}= r_{I3}=2.61;

in Choice III: r_{III1}=r_{I1}=2.21, r_{III2}=r_{III2}=2.93, r_{III3}=1.72.

3.3 Estimating b_{ij}

Using the data on the bottom line of Table 4-1 and the estimated values of r_{Ij}, b_{Ij} for U_{Est} in Choice I, Exp. 1 of S-Sample is estimated

$$b_{I1} = \frac{0.5(4.44-2.21)}{0.5(4.44-2.21)+0.5(3.76-2.97)+0.5(4.17-2.61)} = 0.49\,;$$

$$b_{I2} = \frac{0.5(3.76-2.97)}{0.5(4.44-3.07)+0.5(3.76-2.97)+0.5(4.17-2.61)} = 0.17\,;$$

$$b_{I3} = \frac{0.5(4.17-2.61)}{0.5(4.44-2.21)+0.5(3.76-2.97)+0.5(4.17-2.61)} = 0.34\,.$$

Using the data on the bottom line of Table 4-2 and the estimated values of r_{IIj}, b_{IIj} for U_{Est} in Choice II, Exp. 1 of S-Sample is estimated

$$b_{II1} = \frac{0.7(3.13-2.90)}{0.7(3.13-2.90)+0.3(6.08-2.93)+0.5(4.10-2.61)} = 0.09\,;$$

$$b_{II2} = \frac{0.5(6.08-2.93)}{0.7(3.13-2.90)+0.3(6.08-2.93)+0.5(4.10-2.61)} = 0.51\,;$$

$$b_{II3} = \frac{0.5(4.10-2.61)}{0.7(3.13-2.90)+0.3(6.08-2.93)+0.5(4.10-2.61)} = 0.40\,.$$

Using the data on the bottom line of Table 4-3 and the estimated values of r_{IIIj}, b_{IIIj} for U_{Est} in Exp. 1 of S-Sample is estimated for Choice III

$$b_{III1} = \frac{0.5(4.44-2.21)}{0.5(4.44-2.21)+0.3(6.05-2.93)+0.7(2.56-1.72)} = 0.42$$

$$b_{III2} = \frac{0.3(6.05 - 2.93)}{0.5(4.44 - 2.21) + 0.3(6.05 - 2.93) + 0.7(2.56 - 1.72)} = 0.35$$

$$b_{III3} = \frac{0.7(2.56 - 1.72)}{0.5(4.44 - 2.21) + 0.3(6.05 - 2.93) + 0.7(2.56 - 1.72)} = 0.22$$

3.4 Results of U_{Est}

Using the above estimated values of r_{Ij} and b_{ij}, each U_{Est} in Exp. 1 for S-Sample is finally derived:

for Choice I,

$$U_{Est} = 0.49\ln(q_{I1} - 2.21) + 0.17\ln(q_{I2} - 2.97) + 0.34\ln(q_{I3} - 2.61);$$

for Choice II,

$$U_{Est} = 0.09\ln(q_{II1} - 2.90) + 0.51\ln(q_{II2} - 2.93) + 0.40\ln(q_{II3} - 2.61);$$

for Choice III,

$$U_{Est} = 0.42\ln(q_{III1} - 2.21) + 0.35\ln(q_{III2} - 2.93) + 0.22\ln(q_{III3} - 1.72).$$

Part 5 Estimating U_{LES} and U_{Est} in Exp. 2 of S-Sample

1 Original data for U_{LES}

Tables 5-1, 5-2, and 5-3 present the original data obtained from Session A in Exp. 2 of S-Sample, and the calculation data $p_{ij}q_{ij}$ and $\Sigma p_{Ij}q_{Ij}$ for solving LES.

Table 5-1 Data of Choice I inExp. 2 of S-Sample

N=28, Male 16, Female 12

Subject	Sex	Apple (q_{I1}) (p_{I1}=0.50)	Pen (q_{I2}) (p_{I2}=0.50)	Facial tissue (q_{I3}) (p_{I3}=0.50)	$p_{I1}q_{I1}$	$p_{I2}q_{I2}$	$p_{I3}q_{I3}$	$\Sigma p_{Ij}q_{Ij}$
S_1	M	8	2	6	4.00	1.00	3.00	8.00
S_2	F	4	4	8	2.00	2.00	4.00	8.00
S_3	F	7	2	7	3.50	1.00	3.50	8.00
S_4	M	10	5	1	5.00	2.50	0.50	8.00
S_5	M	13	2	1	6.50	1.00	0.50	8.00
S_6	M	10	2	4	5.00	1.00	2.00	8.00
S_7	M	13	1	2	6.50	0.50	1.00	8.00
S_8	M	5	5	5	2.50	2.50	2.50	7.50
S_9	M	5	5	5	2.50	2.50	2.50	7.50
S_{10}	M	8	4	4	4.00	2.00	2.00	8.00
S_{11}	M	6	6	4	3.00	3.00	2.00	8.00
S_{12}	M	10	3	3	5.00	1.50	1.50	8.00
S_{13}	F	5	5	6	2.50	2.50	3.00	8.00
S_{14}	M	8	4	4	4.00	2.00	2.00	8.00
S_{15}	M	8	4	4	4.00	2.00	2.00	8.00
S_{16}	F	8	2	6	4.00	1.00	3.00	8.00
S_{17}	F	7	3	6	3.50	1.50	3.00	8.00
S_{18}	F	10	5	1	5.00	2.50	0.50	8.00
S_{19}	F	10	2	2	5.00	1.00	1.00	7.00
S_{20}	F	6	2	4	3.00	1.00	2.00	6.00
S_{21}	M	10	4	2	5.00	2.00	1.00	8.00
S_{22}	F	4	6	4	2.00	3.00	2.00	7.00
S_{23}	F	6	6	2	3.00	3.00	1.00	7.00
S_{24}	F	5	4	4	2.50	2.00	2.00	6.50
S_{25}	M	10	1	1	5.00	0.50	0.50	6.00
S_{26}	M	6	2	2	3.00	1.00	1.00	5.00
S_{27}	F	3	4	1	1.50	2.00	0.50	4.00
S_{28}	M	10	1	1	5.00	0.50	0.50	6.00
Mean ($\overline{q}_{Ij}$)		**7.68**	**3.43**	**3.57**				

Table 5-2 Data of Choice II in Exp. 2 of S-Sample

N=28, Male 16, Female 12

Subject	Sex	Apple (q_{II1}) (p_{II1}=0.50)	Pen (q_{II2}) (p_{II2}=0.90)	Facial tissue (q_{II3}) (p_{II3}=0.70)	$p_{II1}q_{II1}$	$p_{II2}q_{II2}$	$p_{II3}q_{II3}$	$\Sigma p_{IIj}q_{IIj}$
S_1	M	8	1	4	4.00	0.90	2.80	7.70
S_2	F	6	2	3	3.00	1.80	2.10	6.90
S_3	F	8	2	3	4.00	1.80	2.00	7.80
S_4	M	11	2	1	5.50	1.80	0.70	8.00
S_5	M	12	1	1	6.00	0.90	0.70	7.60
S_6	M	11	1	2	5.50	0.90	1.40	7.80
S_7	M	12	1	1	6.00	0.90	0.70	7.60
S_8	M	8	2	3	4.00	1.80	2.10	7.90
S_9	M	5	3	4	2.50	2.70	2.80	8.00
S_{10}	M	8	2	3	4.00	1.80	2.10	7.90
S_{11}	M	8	1	4	4.00	0.90	2.80	7.70
S_{12}	M	12	1	1	6.00	0.90	0.70	7.60
S_{13}	F	5	2	5	2.00	1.80	3.50	7.20
S_{14}	M	10	1	3	5.00	0.90	2.10	8.00
S_{15}	M	10	1	3	5.00	0.90	2.10	8.00
S_{16}	F	8	1	4	4.00	0.90	2.80	7.70
S_{17}	F	8	1	4	4.00	0.90	2.80	7.70
S_{18}	F	9	3	1	4.50	2.70	0.70	7.90
S_{19}	F	10	2	1	5.00	1.80	0.70	7.50
S_{20}	F	6	1	2	3.00	0.90	1.40	5.30
S_{21}	M	10	1	1	5.00	0.90	0.70	6.60
S_{22}	F	6	3	2	3.00	2.70	1.40	7.10
S_{23}	F	6	3	2	3.00	2.70	1.40	7.10
S_{24}	F	6	2	3	3.00	1.80	2.10	6.90
S_{25}	M	10-	1	1	5.00	0.90	0.70	6.60
S_{26}	M	6	1	2	3.00	0.90	1.40	5.30
S_{27}	F	4	1	1	2.00	0.90	0.70	3.60
S_{28}	M	10	1	1	5.00	0.90	0.70	6.60
Mean ($\overline{q}_{IIj}$)		**8.32**	**1.57**	**2.36**				

Table 5-3 Data of Choice III in Exp. 2 of S-Sample

N=28, Male 16, Female 12

Subject	Sex	Apple (q_{III1}) (p_{III1}=0.70)	Pen (q_{III2}) (p_{III2}=0.90)	Facial tissue (q_{III3}) (p_{III3}=0.50)	$p_{III1}q_{I1}$	$p_{III2}q_{III2}$	$p_{III3}q_{III3}$	$\Sigma p_{IIIj}q_{IIIj}$
S_1	M	6	1	5	4.20	0.90	2.50	7.60
S_2	F	3	2	8	2.10	1.80	4.00	7.90
S_3	F	3	2	8	2.10	1.80	4.00	7.90
S_4	M	5	4	1	3.50	3.60	0.50	7.60
S_5	M	9	1	1	6.30	0.90	0.50	7.70

S_6	M	8	1	2	5.60	0.90	1.00	7.50
S_7	M	8	1	2	5.60	0.90	1.00	7.50
S_8	M	5	2	5	3.50	1.80	2.50	7.80
S_9	M	5	2	5	3.50	1.80	2.50	7.80
S_{10}	M	5	2	5	3.50	1.80	2.50	7.80
S_{11}	M	7	1	4	4.90	0.90	2.00	7.80
S_{12}	M	8	1	3	5.60	0.90	1.50	8.00
S_{13}	F	3	2	5	2.10	1.80	2.50	7.40
S_{14}	M	2	5	1	1.40	4.50	0.50	7.40
S_{15}	M	6	2	4	4.20	1.80	2.00	8.00
S_{16}	F	8	1	2	5.60	0.90	1.00	7.50
S_{17}	F	5	1	6	3.50	0.90	3.00	7.40
S_{18}	F	6	3	1	4.20	2.70	0.50	7.40
S_{19}	F	5	2	2	3.50	1.80	1.00	6.30
S_{20}	F	6	1	4	4.20	0.90	2.00	7.10
S_{21}	M	5	1	2	3.50	0.90	1.00	5.40
S_{22}	F	5	3	3	3.50	2.70	1.50	7.70
S_{23}	F	3	3	2	2.10	2.70	1.00	5.80
S_{24}	F	4	2	2	2.80	1.80	1.00	5.60
S_{25}	M	8	1	1	5.60	0.90	0.50	7.00
S_{26}	M	5	1	3	3.50	0.90	1.50	5.90
S_{27}	F	3	1	2	2.10	0.90	1.00	4.00
S_{28}	M	8	1	1	5.60	0.90	0.50	7.00
Mean ($\overline{q}_{IIIj}$)		**5.50**	**1.79**	**3.21**				

Table 5-1-I presents the average values of $p_{I1}q_{I1}$, $p_{I2}q_{I2}$, and $p_{I3}q_{I3}$ from each budget constraint group in Table 5-1; Table 5-2-II the average values of $p_{II1}q_{II1}$, $p_{II2}q_{II2}$, and $p_{II3}q_{II3}$ from each budget constraint group in Table 5-2; and Table 5-3-III the average values of $p_{III1}q_{III1}$, $p_{III2}q_{III2}$, and $p_{III3}q_{III3}$ from each budget constraint group in Table 5-3. U_{LES} will be estimated from Tables 5-1-I, 5-2-II, and 5-3-III.

Table 5-1-I Category data of Choice I in Exp. 2 of S-Sample

$\overline{p_{I1}q_{I1}}$	$\overline{p_{I2}q_{I2}}$	$\overline{p_{I3}q_{I3}}$	$\sum p_{Ij}q_{Ij}$
4.26	1.71	2.03	8.00
2.50	2.50	2.50	7.50
3.33	2.33	1.33	7.00
2.50	2.00	2.00	6.50
4.33	0.67	1.00	6.00
3.00	1.00	1.00	5.00
1.50	2.00	0.50	4.00

Table 5-2-II Category data of Choice II in Exp. 2 of S-Sample

$\overline{p_{II1}q_{II1}}$	$\overline{p_{II2}q_{II2}}$	$\overline{p_{II3}q_{II3}}$	$\sum p_{IIj}q_{IIj}$
4.50	1.58	1.93	8.00
4.17	2.10	1.63	7.90
4.75	1.35	1.70	7.80
4.00	0.90	2.80	7.70
6.00	0.90	0.70	7.60
5.00	1.80	0.70	7.50
2.00	1.80	3.50	7.20
3.00	2.70	1.40	7.10
3.00	1.80	2.10	6.90
5.00	0.90	0.70	6.60
3.00	0.90	1.40	5.30
2.00	0.90	0.70	3.60

Table 5-3-III Category data of Choice III in Exp. 2 of S-Sample

$\overline{p_{III1}q_{III1}}$	$\overline{p_{III2}q_{III2}}$	$\overline{p_{III3}q_{III3}}$	$\sum p_{IIIj}q_{IIIj}$
4.90	1.35	1.75	8.00
2.10	1.80	4.00	7.90
3.85	1.58	2.38	7.80
4.90	1.80	1.00	7.70
3.85	2.25	1.50	7.60
5.60	0.90	1.00	7.50
2.8	2.48	1.63	7.40
4.20	0.90	2.00	7.10
5.60	0.90	0.50	7.00
3.50	1.80	1.00	6.30
3.50	0.90	1.50	5.90
2.10	2.70	1.00	5.80
2.80	1.80	1.00	5.60
3.50	0.90	1.00	5.40
2.10	0.90	1.00	4.00

2 Derivation for U_{LES}

Solving (2-2) in the data of Table 5-1-I for Choice I,

$\begin{cases} a_{I1}=0.568, \\ b_{I1}=0.397; \end{cases}$ $\begin{cases} a_{I2}=-0.684, \\ b_{I2}=0.169; \end{cases}$ $\begin{cases} a_{I3}=-1.250, \\ b_{I3}=0.434. \end{cases}$

Solving (2-2) in the data of Table 5-2-II for Choice II,

$\begin{cases} a_{II1}=-0.157, \\ b_{II1}=0.581; \end{cases}$ $\begin{cases} a_{II2}=0.211, \\ b_{II2}=0.181; \end{cases}$ $\begin{cases} a_{II3}=-0.059, \\ b_{II3}=0.240. \end{cases}$

Solving (2-2) in the data of Table 5-3-III for Choice III,

$\begin{cases} a_{III1}=0.181, \\ b_{III1}=0.521; \end{cases}$ $\begin{cases} a_{III2}=0.712, \\ b_{III2}=0.122; \end{cases}$ $\begin{cases} a_{III3}=-0.814, \\ b_{III3}=0.341. \end{cases}$

In Choice I, $1-\sum_{j=1}^{3} b_{Ij}$ =1–(0.397+0.169+0.434)=0, namely, the fraction on the right-hand side of (2-4) diverges. Choice I in Exp. 2 cannot be tested in this way. Thus, for Exp. 2, only Choices II and III will be tested.

Substitute the above results of Choices II and III in (2-4) and solve for r_{ij}

r_{II1}=2.59, r_{II2}=0.74, r_{II3}=0.77;

r_{III1}=3.91, r_{III2}=1.46, r_{III3}=1.72.

Finally obtain each U_{LES} in Choices II and III of Session A, Exp. 2 for S-Sample:

in Choice II,

$$U_{LES} = 0.58\ln(q_{II1} - 2.59) + 0.18\ln(q_{II2} - 0.74) + 0.24\ln(q_{II3} - 0.77);$$

in Choice III,

$$U_{LES} = 0.52\ln(q_{III1} - 3.91) + 0.12\ln(q_{III2} - 1.46) + 0.34\ln(q_{III3} - 1.72).$$

3 Derivation for U_{Est}

3.1 Original data

Tables 5-4, 5-5, 2-6, 5-7, 5-8, and 5-9 present the experimental data obtained from Session B in Exp. 2 of S-Sample.

Table 5-4 Data for the utility scales of apple in Choices I and II (u_{I1}, u_{II1})

N=28, Male 16, Female 12; Assigned unit price 0.50/piece

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	1.60	1.80	3.20	4.00	4.80
S_2	F	2.00	3.00	3.20	4.00	3.60
S_3	F	1.60	2.40	3.20	3.00	3.60
S_4	M	2.00	3.00	4.00	4.00	4.80
S_5	M	2.00	2.40	3.20	4.00	2.40
S_6	M	2.00	3.00	4.00	4.00	3.60
S_7	M	1.40	2.10	2.80	3.50	4.20
S_8	M	1.60	3.00	2.80	3.50	3.60
S_9	M	1.60	2.40	3.20	3.00	3.60
S_{10}	M	2.00	3.00	4.00	4.50	5.40
S_{11}	M	2.00	3.00	3.20	4.00	4.80
S_{12}	M	1.60	1.80	3.20	3.00	3.60
S_{13}	F	1.60	3.00	4.00	5.00	4.80
S_{14}	M	1.80	2.70	3.60	4.00	4.80
S_{15}	M	2.00	3.00	2.40	3.00	3.60
S_{16}	F	2.00	2.40	3.20	3.00	3.60
S_{17}	F	2.04	3.12	3.92	5.10	6.12
S_{18}	F	1.20	2.40	2.40	4.00	3.60
S_{19}	F	1.20	1.80	2.40	3.00	2.40

S_{20}	F	2.00	3.00	3.20	3.50	4.20
S_{21}	M	1.60	3.00	4.00	4.00	4.80
S_{22}	F	1.60	2.40	3.20	4.00	4.80
S_{23}	F	1.20	1.80	2.40	2.00	2.40
S_{24}	F	1.84	2.64	3.36	4.20	4.56
S_{25}	M	2.00	2.40	2.80	4.00	4.20
S_{26}	M	1.80	2.70	3.20	4.00	4.80
S_{27}	F	1.60	2.40	2.40	3.50	4.20
S_{28}	M	1.60	2.40	3.20	4.00	4.20
Mean (u_{I1}, u_{II1})		**1.73**	**2.57**	**3.21**	**3.74**	**4.11**

Table 5-5 Data for the utility scale of pen in Choice I (u_{I2})

N=28, Male 16, Female 12; Assigned unit price 0.50/piece

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	1.60	2.40	3.20	4.00	4.80
S_2	F	2.00	3.00	2.40	4.00	2.40
S_3	F	1.20	2.40	2.40	3.50	4.20
S_4	M	1.60	2.40	3.20	4.00	4.80
S_5	M	2.00	2.40	3.20	2.00	2.40
S_6	M	1.60	2.40	3.20	3.00	3.60
S_7	M	1.40	2.10	2.40	3.00	3.60
S_8	M	1.60	2.70	3.60	4.50	4.80
S_9	M	1.60	2.40	3.20	3.00	3.60
S_{10}	M	2.00	3.00	4.00	4.50	5.40
S_{11}	M	2.00	3.00	3.20	4.00	4.20
S_{12}	M	1.20	1.20	2.40	2.00	2.40
S_{13}	F	1.60	3.00	4.00	5.00	4.80
S_{14}	M	1.80	2.70	3.20	4.00	4.80
S_{15}	M	2.00	3.00	4.00	3.00	3.60
S_{16}	F	1.20	1.80	3.20	2.00	3.60
S_{17}	F	2.00	2.94	3.84	4.50	5.40
S_{18}	F	1.60	2.10	2.40	3.50	3.60
S_{19}	F	1.20	1.80	2.40	3.00	3.60
S_{20}	F	2.00	2.40	2.40	3.00	3.00
S_{21}	M	2.00	3.00	3.20	4.00	4.80
S_{22}	F	1.60	2.40	3.20	4.00	4.80
S_{23}	F	0.80	1.20	1.60	1.00	1.20
S_{24}	F	1.76	2.58	3.36	4.00	4.56
S_{25}	M	2.00	2.70	3.20	3.50	4.20
S_{26}	M	1.80	2.70	3.20	3.50	4.20
S_{27}	F	1.60	2.10	3.36	3.00	3.60
S_{28}	M	1.20	1.80	2.00	2.50	3.00
Mean (u_{I2})		**1.64**	**2.42**	**3.03**	**3.39**	**3.89**

Table 5-6 Data for the utility scales of facial tissue in Choices I and II (u_{I3}, u_{III3})

N=28, Male 16, Female 12; Assigned unit price 0.50/set

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	1.60	2.40	2.40	4.00	3.60
S_2	F	2.00	3.00	3.20	4.00	3.60
S_3	F	1.60	1.50	3.20	4.00	4.80
S_4	M	1.60	1.80	2.40	5.00	3.60
S_5	M	2.00	2.40	1.60	2.00	2.40
S_6	M	1.60	2.40	2.80	3.00	3.60
S_7	M	1.40	2.10	2.80	3.00	3.60
S_8	M	1.60	2.40	3.60	4.00	4.80
S_9	M	1.60	2.40	3.20	3.00	3.60
S_{10}	M	2.00	3.00	3.68	4.30	5.16
S_{11}	M	2.00	2.40	3.20	3.00	3.60
S_{12}	M	1.20	1.80	1.60	3.00	1.20
S_{13}	F	1.60	3.00	4.00	5.00	6.00
S_{14}	M	1.80	2.70	3.20	4.00	4.80
S_{15}	M	2.00	3.00	2.40	3.00	3.60
S_{16}	F	1.60	1.80	3.20	3.00	3.60
S_{17}	F	2.00	2.82	3.84	4.50	5.40
S_{18}	F	1.20	1.80	3.20	3.00	2.40
S_{19}	F	1.20	1.80	1.60	2.50	3.00
S_{20}	F	1.40	2.40	4.20	1.60	3.60
S_{21}	M	2.00	2.40	4.00	3.00	3.60
S_{22}	F	1.60	2.40	3.20	4.00	4.80
S_{23}	F	0.80	1.20	2.40	1.00	1.20
S_{24}	F	1.84	2.76	3.68	4.00	4.80
S_{25}	M	2.00	3.00	4.00	4.00	4.80
S_{26}	M	1.80	2.40	3.20	3.50	4.20
S_{27}	F	1.40	2.40	3.20	3.00	4.20
S_{28}	M	1.20	1.80	2.00	2.50	3.00
Mean (u_{I3}, u_{III3})		**1.63**	**2.33**	**3.04**	**3.35**	**3.80**

Table 5-7 Data for the utility scales of pen in Choices II and III (u_{II2}, u_{III2})

N=28, Male 16, Female 12; Assigned unit price 0.90/piece

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	2.80	4.20	5.60	7.00	8.40
S_2	F	3.20	4.20	5.60	6.00	7.20
S_3	F	2.40	3.00	4.00	6.00	7.20
S_4	M	3.20	3.60	4.80	6.00	6.00
S_5	M	2.40	2.40	3.20	2.00	1.20

S_6	M	2.00	3.00	4.00	4.00	4.20
S_7	M	2.80	4.20	4.80	5.50	6.00
S_8	M	2.80	4.20	5.60	6.00	6.00
S_9	M	3.20	4.80	5.60	7.00	7.20
S_{10}	M	3.48	4.98	6.96	8.50	9.96
S_{11}	M	2.40	3.60	4.00	4.00	4.20
S_{12}	M	1.20	1.80	2.40	2.00	2.40
S_{13}	F	2.00	3.60	4.00	5.00	5.40
S_{14}	M	3.40	5.10	6.40	7.00	8.40
S_{15}	M	2.00	3.00	4.00	3.00	3.60
S_{16}	F	1.20	1.80	2.40	2.50	3.00
S_{17}	F	3.24	5.28	6.24	7.50	8.40
S_{18}	F	2.80	3.60	5.60	6.00	6.00
S_{19}	F	2.00	3.00	3.20	3.00	2.40
S_{20}	F	2.00	4.20	4.80	5.00	6.00
S_{21}	M	3.20	5.40	6.40	7.00	8.40
S_{22}	F	2.80	4.20	5.60	7.00	8.40
S_{23}	F	1.20	1.80	1.60	3.00	3.60
S_{24}	F	3.24	4.92	6.56	7.80	9.36
S_{25}	M	2.00	3.00	3.20	4.00	4.20
S_{26}	M	3.20	4.80	5.60	6.00	7.20
S_{27}	F	2.00	3.00	3.20	3.50	4.20
S_{28}	M	1.60	2.10	2.40	3.00	3.00
Mean (u_{II2}, u_{III2})		**2.49**	**3.67**	**4.56**	**5.15**	**5.77**

Table 5-8 Data for the utility scale of facial tissue in Choice II (u_{II3})

N=28, Male 16, Female 12; Assigned unit price 0.70/set

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	2.00	3.00	4.00	5.00	6.00
S_2	F	2.40	3.60	4.00	5.00	6.00
S_3	F	2.00	3.00	4.00	4.00	4.80
S_4	M	2.40	3.60	4.00	5.00	6.00
S_5	M	2.40	2.40	3.20	3.00	2.40
S_6	M	2.00	3.00	4.00	5.00	4.80
S_7	M	2.00	3.00	4.00	5.00	6.00
S_8	M	2.40	3.60	4.00	6.00	6.00
S_9	M	2.40	3.60	4.00	5.00	6.00
S_{10}	M	`2.80	3.96	5.44	6.50	7.80
S_{11}	M	2.40	3.60	4.80	5.00	6.00
S_{12}	M	1.60	2.40	3.20	3.00	3.60
S_{13}	F	2.00	3.00	4.00	5.00	6.00
S_{14}	M	2.70	3.96	5.20	6.00	7.20
S_{15}	M	2.00	3.00	4.00	3.00	3.60

S_{16}	F	1.60	1.80	3.20	3.00	3.60
S_{17}	F	2.64	4.20	5.760	6.70	7.56
S_{18}	F	2.40	3.60	3.60	5.00	6.00
S_{19}	F	1.20	1.80	2.40	2.50	3.00
S_{20}	F	2.00	3.00	4.00	4.00	4.80
S_{21}	M	2.80	4.20	4.80	5.00	6.00
S_{22}	F	2.00	3.00	4.00	5.00	6.00
S_{23}	F	1.60	2.40	3.20	3.00	3.60
S_{24}	F	2.64	3.78	4.88	6.00	6.96
S_{25}	M	2.00	3.00	3.60	4.00	4.80
S_{26}	M	2.40	3.60	4.80	5.00	6.00
S_{27}	F	1.60	2.40	3.20	3.50	4.20
S_{28}	M	2.00	3.00	4.00	4.50	6.00
Mean (u_{II3})		**2.13**	**3.16**	**4.05**	**4.60**	**5.38**

Table 5-9 Data for the utility scale of apple in Choices III (u_{III1})

N=28, Male 16, Female 12; Assigned unit price 0.70/piece

Subject	Sex	Quantity				
		4	6	8	10	12
S_1	M	2.00	3.60	4.00	5.00	6.00
S_2	F	2.80	3.60	5.60	5.00	4.80
S_3	F	2.00	3.00	4.00	5.00	4.20
S_4	M	1.60	3.00	3.20	4.00	4.80
S_5	M	1.20	3.00	3.20	2.50	2.40
S_6	M	1.60	2.40	3.20	4.00	4.80
S_7	M	2.00	3.00	4.00	4.50	5.40
S_8	M	2.00	3.00	4.00	4.50	5.40
S_9	M	2.00	3.00	3.20	4.00	4.80
S_{10}	M	2.80	3.96	5.44	6.00	7.20
S_{11}	M	2.40	3.60	4.80	5.00	6.00
S_{12}	M	0.80	1.80	2.40	3.00	2.00
S_{13}	F	2.40	3.00	4.80	5.00	6.00
S_{14}	M	2.70	3.96	4.80	6.00	7.20
S_{15}	M	2.00	3.00	4.00	3.00	3.60
S_{16}	F	1.60	2.40	3.20	3.00	3.60
S_{17}	F	2.72	3.90	5.44	6.50	7.20
S_{18}	F	2.40	3.00	3.20	3.00	3.60
S_{19}	F	2.00	3.00	4.00	3.00	3.60
S_{20}	F	2.00	3.00	4.00	4.00	4.20
S_{21}	M	2.80	3.00	4.00	3.00	3.60
S_{22}	F	2.00	3.00	4.00	5.00	6.00
S_{23}	F	1.20	1.80	2.40	2.00	2.40
S_{24}	F	2.00	3.72	4.88	6.00	6.96
S_{25}	M	2.00	3.00	2.80	3.50	4.20

S_{26}	M	2.40	3.30	4.00	4.50	5.40
S_{27}	F	2.00	2.40	2.40	3.50	3.60
S_{28}	M	1.40	2.10	2.40	3.00	3.00
Mean (u_{III1})		**2.03**	**3.02**	**3.83**	**4.16**	**4.71**

3.2 Estimating r_{ij}

To estimate r_{ij}, fit the logarithmic law $u_{ij}=c\ln(q_{ij}-r_{ij})+C$, i=II,III and j=1,2,3, in the average data of Session B (they are listed on the bottom lines of Tables 5-4, 5-6, 5-7, 5-8, and 5-9. It is realized by the curve regression in SPSS, in which the optimal values of c and C in the logarithmic law are created automatically, but r_{ij} must be selected by hand. To isolate from the derivation of U_{LES}, the procedure for selecting r_{ij} is that select the values of r_{ij} to improve the regression results in SPSS until $R^2\geq 0.97$.

1) Estimating r_{II1}, r_{II2}, and r_{II3} in Choice II

By fitting the logarithmic law in the average data presented on the bottom lines in Tables 5-4, 5-7, and 5-8, the curve regression results in SPSS for r_{II1}, r_{II2} and r_{II3} are derived as the following:

For the apple in Choice II, the logarithmic law is

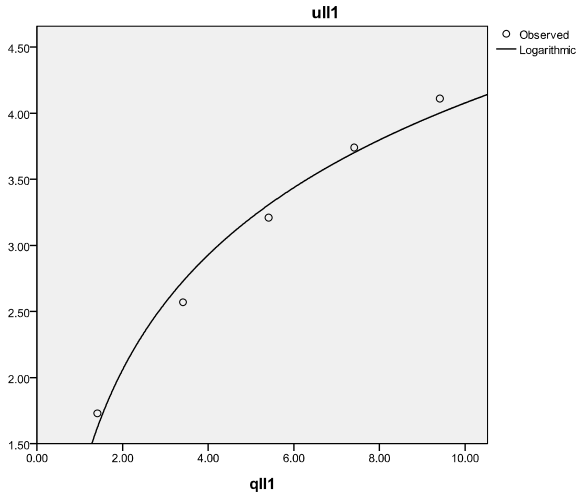
$u_{II1}=1.25\ln(q_{II1}-2.59)+1.19$, $r_{II1}=2.59$ ($R^2=0.983$).

Model Summary and Parameter Estimates

Dependent Variable:uII1

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.983	177.659	1	3	.001	1.191	1.253

The independent variable is qII1.



For the pen in Choice II, the logarithmic law is

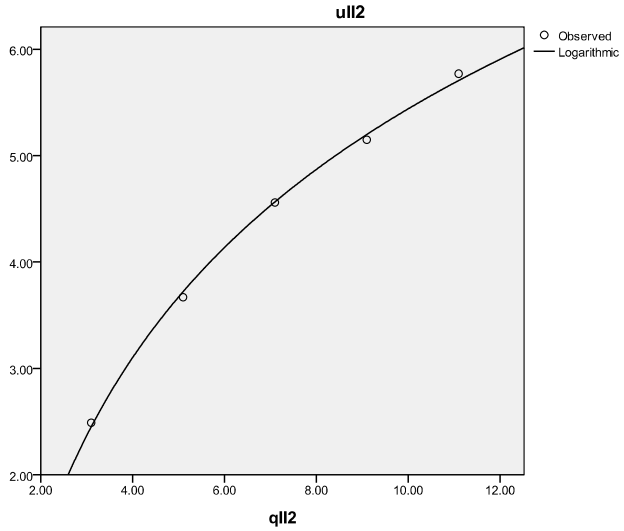
$u_{II2}=2.55\ln(q_{II2}-0.90)-0.44$, $r_{II2}=0.90$ ($R^2=0.998$).

Model Summary and Parameter Estimates

Dependent Variable:uII2

Equation	Model Summary	Parameter Estimates

	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.998	1854.299	1	3	.000	-.438	2.553



For the facial tissue in Choice II, the logarithmic law is

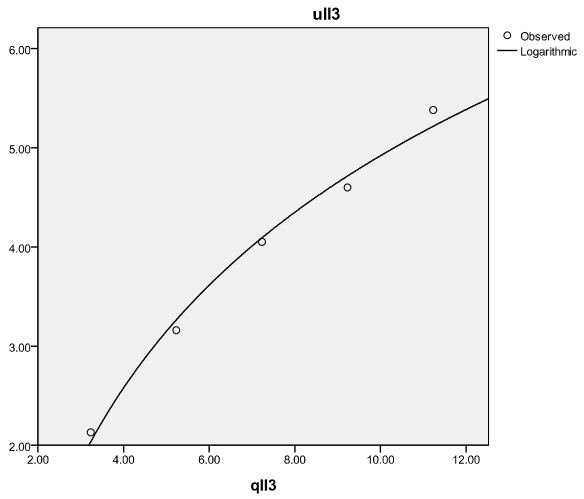
u_{II3}=2.55ln(q_{II3}–0.77)–0.96, r_{II3}=0.77 (R^2=0.990).

Model Summary and Parameter Estimates

Dependent Variable:uII3

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.990	305.313	1	3	.000	-.961	2.554

The independent variable is qII3.



Summing up, in Choice II, Exp. 2 of S-Sample, the estimated values of r_{IIj} for U_{Est} are r_{II1}=2.59, r_{II2}=0.90, r_{II3}=0.77.

2) Estimating r_{III1}, r_{III2}, and r_{III3} in Choice III

For Choice III, r_{III2}=r_{II2}=0.90 because their assigned quantities and prices are the same. By fitting the logarithmic law in the average data presented on the bottom lines in Tables 5-9 and 5-6, the curve regression results in SPSS for r_{III1} and r_{III3} are derived as the following:

For the apple in Choice III, the logarithmic law is

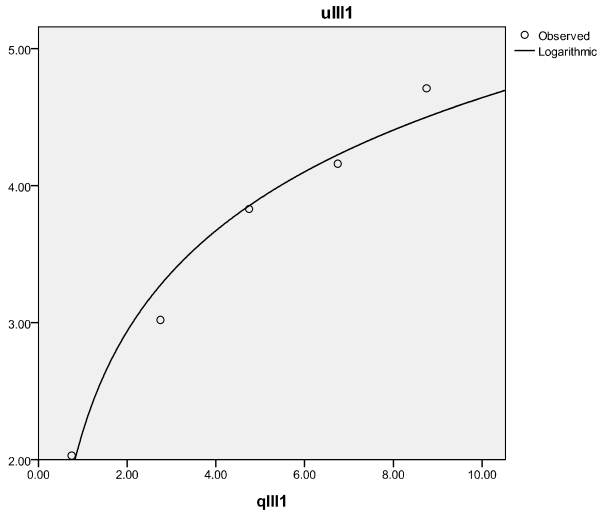
u_{III1}=1.06ln(q_{III1}–3.25)+2.20, r_{III1}=3.25 (R^2=0.970).

Model Summary and Parameter Estimates

Dependent Variable:uIII1

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.970	97.578	1	3	.002	2.202	1.059

The independent variable is qIII1.



For the facial tissue in Choice III, the logarithmic law is

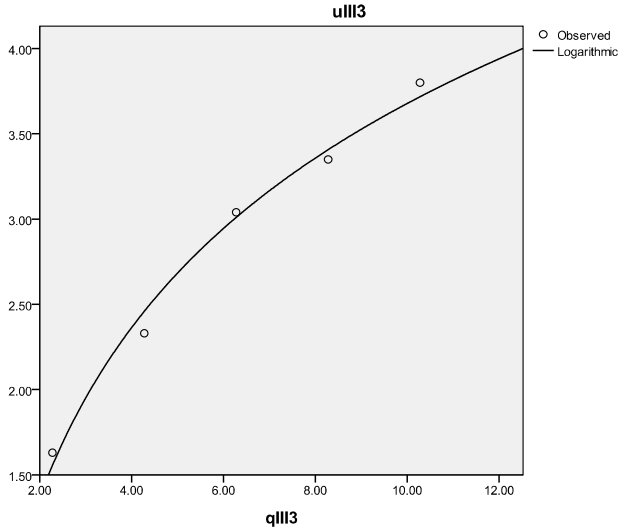
u_{III3}=1.44ln(q_{III3}–1.72)+0.37, r_{III3}=1.72 (R^2=0.989).

Model Summary and Parameter Estimates

Dependent Variable:uIII3

Equation	Model Summary					Parameter Estimates	
	R Square	F	df1	df2	Sig.	Constant	b1
Logarithmic	.989	262.458	1	3	.001	.373	1.435

The independent variable is qIII3.



Summing up, in Choice III, Exp. 2 of S-Sample, the estimated values of r_{IIIj} for U_{Est} are r_{III1}= 3.25, r_{III2}=r_{II2}=0.90, r_{III3}=1.72.

Summing up, the estimated values of r_{ij} for each U_{Est} in Choices I, II, and III of S-Sample all have been obtained. They are collected below:

in Choice II: r_{II1}=2.59, r_{II2}=0.90, r_{II3}= 0.77;

in Choice III: r_{III1}= 3.25, r_{III2}=r_{II2}=0.90, r_{II3}=1.72.

3.3 Estimating b_{ij}

Using the data on the bottom line of Table 5-2 and the estimated values of r_{IIj}, b_{IIj} for U_{Est} in Choice II, Exp. 2 of S-Sample is estimated

$$b_{II1} = \frac{0.5(8.32-2.59)}{0.5(8.32-2.59)+0.9(1.57-0.90)+0.7(2.36-0.77)} = 0.63;$$

$$b_{II2} = \frac{0.9(1.57-0.90)}{0.5(8.32-2.59)+0.9(1.57-0.90)+0.7(2.36-0.77)} = 0.13;$$

$$b_{II3} = \frac{0.7(2.36-0.77)}{0.5(8.32-2.59)+0.9(1.57-0.90)+0.7(2.36-0.77)} = 0.24.$$

Using the data on the bottom line of Table 5-3 and the estimated values of r_{IIIj}, b_{IIIj} for U_{Est} in Exp. 1 of S-Sample is estimated for Choice III

$$b_{III1} = \frac{0.7(5.50-3.25)}{0.7(5.50-3.25)+0.9(1.79-0.90)+0.5(3.21-1.72)} = 0.50$$

$$b_{III2} = \frac{0.9(1.79-0.90)}{0.7(5.50-3.25)+0.9(1.79-0.90)+0.5(3.21-1.72)} = 0.26$$

$$b_{III3} = \frac{0.5(3.21-1.72)}{0.7(5.50-3.25)+0.9(1.79-0.90)+0.5(3.21-1.72)} = 0.24$$

3.4 Results of U_{Est}

Using the above estimated values of r_{Ij} and b_{ij}, each U_{Est} in Exp. 1 for S-Sample is finally derived: in Choice II,

$$U_{Est} = 0.63\ln(q_{I1} - 2.59) + 0.13\ln(q_{I2} - 0.90) + 0.24\ln(q_{I3} - 0.77);$$

in Choice III,

$$U_{Est} = 0.50\ln(q_{II1} - 3.25) + 0.26\ln(q_{II2} - 0.90) + 0.24\ln(q_{II3} - 1.72).$$