Online Appendix to

Securities Auctions with Pre-project Information Management

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January 31, 2023

OA.1 The Welfare-Maximizing Mechanism

In this section, we characterize the securities auction that maximizes social welfare, which is defined as the sum of the seller's revenue and the agents' ex-ante payoffs.

$$\mathbb{E}_{c,z}\left[\sum_{i=1}^{n} P_i(c)\max_{a_i}\left(\alpha_i\left[\mathbb{E}\left[(v_i-c_i)q_i|z_i\right]-k\right]+(1-\alpha_i)(z_i-c_i)\bar{q}_i\right)\right]. \tag{OA.1}$$

The welfare maximization problem is to choose a mechanism to maximize (OA.1), subject to Bayesian incentive compatibility (3), individual rationality (4), feasibility, and the fact that the winning bidder will choose a_i^* to maximize the ex-post payoff (1). Proposition OA.1 describes this welfare-maximizing mechanism, and its proof follows the proof of Proposition 3.

Proposition OA.1. *The welfare-maximizing mechanism is the following:*

(a) The contingent payment is $S_i(v_i;c)=0$.

(b) The project is allocated to the bidder with the highest expected ex-post payoff:

$$P_i(c)=\begin{cases}1 & \text{if } i=\arg\max_j \mathbb{E}_{z_j}\left[u_j(c_j,z_j,0,a_j)\right],\\ 0 & \text{if otherwise.}\end{cases}$$

(c) The upfront payment $X_i(c)$ *is set such that* $V_i(\bar{c})=0$.

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The welfare-maximizing mechanism sets the contingent payment to zero everywhere. The reason is that any non-zero contingent payment distorts the winning bidder's post-auction optimal decision as shown in Section 3.2 and 5.1. The mechanism then allocates the project with probability 1 to the bidder with the highest expected ex-post payoff in order to realize the highest social surplus. The allocation rule compares the *expected* ex-post payoffs because the signal z_i is not observed yet at the auction stage.

In comparison, the revenue-maximizing limited-liability securities auction introduces two distortions compared to the welfare-maximizing benchmark. First, the investment threshold is $v_0^{Debt}(c_i) = c_i + \frac{F(c_i)}{f(c_i)} > v_0^{FB}(c_i) = c_i$. Thus, there is under-investment.[1] Second, it induces excessive information acquisition that prevents Type I errors: $z_I^{Debt} > z_I^{FB}$; and insufficient information acquisition that prevents Type II errors: $z_{II}^{Debt} > z_{II}^{FB}$, as stated in Proposition 1.

OA.2 The Impact of an Ex-Ante Private Signal

Consider the private signal z regarding the project value v is received *before* the auction. For example, z could present private information on the common value v of an oil field. To simplify the analysis, suppose there is only one bidder and the project cost c is public information in that private information is single dimensional. With these assumptions, we use the original notation F for c to denote the distribution of the private signal z; and as before, $G(v|z)$ is the conditional distribution of the project value given the ex-ante signal. To highlight the point in the main text, we further assume away the option to acquire information ex-post. The insights we develop in this section can be easily generalized to the setting with information acquisition.

In this setting, the agent's ex-post payoff with a true type z and a reported type $\hat{z}$ is

$$u(z, S(\hat{z}), \bar{q}) = (z - \mathbb{E}\left[S(v;\hat{z})|z\right] - c)\,\bar{q}, \tag{OA.2}$$

with the only choice in the post-auction stage being the investment decision $\bar{q}$ (without additional information regarding v). Moreover, as in our main model, the contingent payment $S(v;\hat{z})$ depends on the project value and the reported type. The agent's interim payoff is then

$$U(\hat{z}, z, \bar{q}^*) = P(\hat{z})\,(z - \mathbb{E}\left[S(v;\hat{z})|z\right] - c)\,\bar{q}^* - X(\hat{z}). \tag{OA.3}$$

[1] The general security $S_i(\lambda; c)$ implies an investment threshold without additional information that differs from $z_0^P(c_i) = c_i + \frac{F(c_i)}{f(c_i)}$ due to its steepness. However, the winning bidder invests without the additional information only when $z_i > z_I(c_i) \geq z_0(c_i)$. Hence, the exact position of $z_0(c_i)$ under the contract $S_i(\lambda; c)$ generates no important impact on the virtual surplus. See expression (A.2). It is clear that $z_0(c_i)$ does not affect the seller's payoff.

We now differentiate $V(z) = \max_{\hat{z}} U(\hat{z}, z, \bar{q}^*)$ and obtain the integral equation

$$V(z) = V(\underline{z}) + \int_{\underline{z}}^{z} P(\tilde{z}) \left(1 - \int_{-\infty}^{\infty} S(v;\tilde{z}) \frac{\partial g(v|\tilde{z})}{\partial z} dv\right) \bar{q}^* d\tilde{z}. \tag{OA.4}$$

The seller's expected revenue is then $\mathbb{E}\left[X(z) + P(z)\mathbb{E}\left[\bar{q}^* S(v;z)|z\right]\right] = \mathbb{E}\left[P(z)(z-c)\bar{q}^*\right] - \mathbb{E}\left[V(z)\right]$, where the equality uses (OA.3) and the expectation is taken under the seller's belief F about the bidder's signal. Using the integral equation (OA.4), the seller's expected revenue can be written as the following virtual surplus as the objective of the relaxed program.

$$\mathbb{E}\left[P(z)\left(z - \frac{1-F(z)}{f(z)} - c + \int_{-\infty}^{\infty} S(v;z)\frac{\partial g(v|z)}{\partial z} dv \cdot \frac{1-F(z)}{f(z)}\right)\bar{q}^*\right] - V(\underline{z}). \tag{OA.5}$$

Obviously, the solution includes $V(\underline{z}) = 0$. However, the key conceptual difference between (OA.5) and (11) is that in the latter without the ex-ante private signal, contingent payment affects the virtual surplus only via its effect on the winning bidder's ex-post policy. In contrast, the contingent payment does not only affect the ex-post investment decision $\bar{q}^*$ in (OA.5), but it also produces a direct impact via the term $\int_{-\infty}^{\infty} S(v;z)\frac{\partial g(v|z)}{\partial z} dv$. This term essentially captures the changes in the seller's estimate of the contingent payment to be received when the bidder reports an ex-ante private signal.

To gain further insight, suppose the contingent payment is differentiable with respect to v almost everywhere. Then we can integrate the term by parts and write (OA.5) as

$$\mathbb{E}\left[P(z)\left(z - \frac{1-F(z)}{f(z)} - c + \mathbb{E}\left[\frac{\partial S(v;z)}{\partial v} \cdot \frac{-\partial G(v|z)/\partial z}{g(v|z)}|z\right] \cdot \frac{1-F(z)}{f(z)}\right)\bar{q}^*\right]. \tag{OA.6}$$

Note that $-\partial G(v|z)/\partial z \geq 0$ because the family of conditional distributions $\{G(\cdot|z)\}$ satisfies FOSD. From the seller's point of view, as a high signal z could imply a large project value v, a steeper security (high $\frac{\partial S(v;z)}{\partial v}$) will then increase surplus extraction. Nevertheless, this goes against the bidder's incentive to report the true type (the same term $\int_{-\infty}^{\infty} S(v;z)\frac{\partial g(v|z)}{\partial z} dv$ appears in (OA.4), especially for the high-type bidder.)

In fact, there are both the level and slope effects even only for the incentive alignment between the seller and the bidder. In particular, the contingent payment that equates the bidder's ex-post payoff (OA.2) and the seller's surplus (OA.6) must satisfy

$$\frac{1-F(z)}{f(z)} = \mathbb{E}\left[S(v;z)|z\right] + \mathbb{E}\left[\frac{\partial S(v;z)}{\partial v} \cdot \frac{-\partial G(v|z)/\partial z}{g(v|z)}|z\right] \cdot \frac{1-F(z)}{f(z)}, \tag{OA.7}$$

where the first term on the right-hand side represents the level effect and the second term captures the slope effect. Equation (OA.7) then implies a trade off: the seller must lower the average level of contingent

payment if she uses a steep security to extract surplus. This has an important implication: the consideration of incentive alignment alone, as discussed in Section 4.1, is insufficient to determine the optimal mechanism;[2] and the characterization of the optimal mechanism awaits future research. We believe that the trade-off of the slope and level of the contingent security carries to other environments with an ex-ante private signal.

References

Board, Simon, 2007a, Selling options, *Journal of Economic Theory* 136, 324–340.

[2]Roughly speaking, there are two related degrees of freedom, $S(v;z)$ and $\frac{\partial S(v;z)}{\partial v}$, in (OA.7). Without the ex-ante signal, that is, in the main model or Board (2007a), $\partial G(v|z)/\partial z = 0$. As a result, only the level matters and we can choose $S(v;z) = \frac{1-F(z)}{f(z)}$ for incentive provisions.