Supplemental Paper: Monetary Policy Trade-Offs in a Portfolio Model with Endogenous Asset Supply

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10 November 2011

This supplemental paper shows in detail the solving of the open economy portfolio balance model developed in the paper 'Monetary Policy Trade-Offs in a Portfolio Model with Endogenous Asset Supply'.

Domestic producers choose an optimal capital structure and finance capital goods through credit, bonds and equity assets. Private households hold a portfolio of domestic and foreign assets, shift balances depending on risk-return considerations, and maximise real consumption in accordance with the law of one price.

Within this general equilibrium model, it will be shown that central bank interventions may promote an inefficient international allocation of real capital. The application of expansive monetary interventions throughout the course of economic crises maintains the domestic stock of real capital at the cost of inflation, currency devaluation, distortions of interest rates and asset prices, and risk clusters on the central bank's balance sheet. Exchange rate stabilising interventions have the result that the central bank can also stabilise the domestic stock of real capital. However, such interventions produce either risk clusters on the central bank's balance sheet or changes in the domestic price level.

JEL: E10, E44, E52

Keywords: portfolio balance, monetary policy, real capital, macroeconomic risk, exchange rate

This supplemental paper shows in detail, and is limited to, the solving of the open economy portfolio balance model developed in the paper 'Monetary Policy Trade-Offs in a Portfolio Model with Endogenous Asset Supply', which can be downloaded from:

http://ssrn.com/abstract=1873482 or
http://dvn.iq.harvard.edu/dvn/dv/schueder

The paper is structured as follows; the first section deals with the partial deviations, followed by the representation of the short term and the long term system of linear equations. These are solved respectively. Following on from here, both systems are successfully tested for true dynamic stability.

Please see the main paper for the economic analysis and references.

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Mathematical Appendix

Partial Deviations

Supply of Domestic Bonds

The long term supply of domestic bonds is given by equation 32. The partial deviations are:

$$\frac{\partial (n^B)^s}{\partial i^B} = -\frac{\widehat{K}}{\overline{q^B}} \tag{65}$$

$$\frac{\partial (n^B)^s}{\partial \widehat{K}} = \frac{\overline{dc} \cdot \overline{v} - i^B}{\overline{q^B}} \tag{66}$$

$$\frac{\partial (n^B)^s}{\partial \widehat{n^B_{CB}}} = \frac{\overline{dc} \cdot \overline{v}}{i^B} \tag{67}$$

$$\frac{\partial (n^B)^s}{\partial \widehat{n^F_{CB}}} = \frac{s \cdot \overline{q^F} \cdot \overline{dc} \cdot \overline{v}}{\widehat{i^F} \cdot \overline{q^B}} \tag{68}$$

The short term and long term supply of domestic bonds vis-à-vis the private households is given by equations 27 and 33. Excluding the effects of 65 to 68 with regard to the long term, the only partial deviation is:

$$\frac{\partial (n^B_P)^s}{\partial \widehat{n^B_{CB}}} = -1 \tag{69}$$

Dividend Payments

The short term amount of dividend payments is given by equation 37. The partial deviations are:

$$\frac{\partial Div}{\partial i^K} = -\widehat{K} \tag{70}$$

$$\frac{\partial Div}{\partial \widehat{n^B_{CB}}} = \frac{\overline{q^B}}{i^B} \cdot \overline{v} \tag{71}$$

$$\frac{\partial Div}{\partial \widehat{n^F_{CB}}} = \frac{s \cdot \overline{q^F}}{\widehat{i^F}} \cdot \overline{v} \tag{72}$$

The long term amount of dividend payments is given by equation 38. The partial deviations are:

$$\frac{\partial Div}{\partial \widehat{K}} = \overline{v} \cdot (1 - \overline{dc}) \tag{73}$$

$$\frac{\partial Div}{\partial \widehat{n^B_{CB}}} = \frac{\overline{q^B}}{i^B} \cdot \overline{v} \cdot (1 - \overline{dc}) \tag{74}$$

$$\frac{\partial Div}{\partial \widehat{n^F_{CB}}} = \frac{s \cdot \overline{q^F}}{\widehat{i^F}} \cdot \overline{v} \cdot (1 - \overline{dc}) \tag{75}$$

Supply of Foreign Assets and Purchasing Power Parity

The long term supply of foreign assets changes with dn^F (see equation 44). dn^F is different from zero when deviations from purchasing power parity occur (follows from equations 41, 43, and 44). Changes in the long term supply of foreign assets are therefore derived through the partial deviations of the purchasing power parity condition (equation 64).

The purchasing power parity condition determines the nominal exchange rate (s) in the long term. By solving the partial deviations, the determinants of the money amount (equation 4), the determinants of the stock of real capital goods (equation 15), and the changes in the amount of foreign assets (equation 44) need to be considered:

$$\frac{\partial s}{\partial n^F} = \frac{s \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}} \tag{76}$$

$$\frac{\partial s}{\partial \widehat{p^*}} = -\frac{s}{\widehat{p^*}} \tag{77}$$

$$\frac{\partial s}{\partial \widehat{K}} = \frac{\overline{v}}{\overline{a} \cdot n^{CG} \cdot \widehat{p^*}} \tag{78}$$

$$\frac{\partial s}{\partial \widehat{n^B_{CB}}} = \frac{\overline{q^B} \cdot \overline{v}}{i^B \cdot \overline{a} \cdot n^{CG} \cdot \widehat{p^*}} \tag{79}$$

$$\frac{\partial s}{\partial \widehat{n^F_{CB}}} = \frac{s \cdot \overline{q^F} \cdot \overline{v}}{\widehat{i^F} \cdot \overline{a} \cdot n^{CG} \cdot \widehat{p^*}} \tag{80}$$

The short term and long term supply of foreign assets vis-à-vis the private households is given by equations 45 and 46. Excluding the effects of 76 to 80 with regard to the long term, the only partial deviation is:

$$\frac{\partial (n^F_P)^s}{\partial \widehat{n^F_{CB}}} = -1 \tag{81}$$

Money Demand

Equation 48 shows private households' demand for money. By rearranging it, we obtain:

$$M^d = m(i^B, i^E, i^{\widehat{F}}, \widehat{\sigma}, \frac{p \cdot Y^r}{\overline{v}}) \cdot (\underbrace{M + \frac{n_P^B \cdot \overline{q^B}}{i^B} + \frac{Div}{i^E} + \frac{n_P^F \cdot s \cdot \overline{q^F}}{i^{\widehat{F}}}}_{=W}) \tag{82}$$

The partial deviations are:

$$\frac{\partial M^d}{\partial i^B} = \frac{\partial m}{\partial i^B} \cdot W - m \cdot \frac{B}{i^B} \qquad < 0 \tag{83}$$

$$\frac{\partial M^d}{\partial i^E} = \frac{\partial m}{\partial i^E} \cdot W - m \cdot \frac{E}{i^E} \qquad < 0 \tag{84}$$

$$\frac{\partial M^d}{\partial i^{\widehat{F}}} = \frac{\partial m}{\partial i^{\widehat{F}}} \cdot W - m \cdot \frac{sF_P}{i^{\widehat{F}}} \qquad < 0 \tag{85}$$

$$\frac{\partial M^d}{\partial \widehat{\sigma}} = \frac{\partial m}{\partial \widehat{\sigma}} \cdot W \qquad < 0 \tag{86}$$

$$\frac{\partial M^d}{\partial M} = \frac{\partial m}{\partial (\frac{p \cdot Y^r}{\overline{v}})} \frac{\partial (\frac{p \cdot Y^r}{\overline{v}})}{\partial M} \cdot W + m = 1^* \tag{87}$$

$$\frac{\partial M^d}{\partial n_P^B} = m \cdot \frac{\overline{q^B}}{i^B} \qquad > 0 \tag{88}$$

$$\frac{\partial M^d}{\partial Div} = m \cdot \frac{1}{i^E} \qquad > 0 \tag{89}$$

$$\frac{\partial M^d}{\partial n_P^F} = m \cdot \frac{s \cdot \overline{q^F}}{i^{\widehat{F}}} \qquad > 0 \tag{90}$$

$$\frac{\partial M^d}{\partial s} = m \cdot \frac{n_P^F \cdot \overline{q^F}}{i^{\widehat{F}}} \qquad > 0 \tag{91}$$

* From Quantity Equation 24 follows $\frac{\partial (\frac{p \cdot Y^r}{\overline{v}})}{\partial M} = 1$. As money market equilibrium implies $M = \frac{p \cdot Y^r}{\overline{v}} = M^d$, it consequently results that $1 = \frac{\partial (\frac{p \cdot Y^r}{\overline{v}})}{\partial M} = \frac{\partial M^d}{\partial M}$. In turn, it results $\frac{\partial B_P^d}{\partial M} = \frac{\partial E^d}{\partial M} = \frac{\partial s F_P^d}{\partial M} = 0$, and respectively $\frac{\partial (n_P^B)^d}{\partial M} = \frac{\partial Div^d}{\partial M} = \frac{\partial (n_P^F)^d}{\partial M} = 0$ due to private households' balance restriction 10. The mathematics here expresses the economic relationship that if additional money increases the domestic price level proportionally, the money demand increases by exactly dM, thus allowing for the same amount of transactions as before. Consequently, the absolute demand for alternative assets is (considering the increase in the amount of money on its own) not affected.

Demand for Domestic Bonds

Equation 52 shows private households' demand for quantities of domestic bonds. By rearranging it, we obtain:

$$(92) \qquad (n_P^B)^d = b(i^B, i^E, \widehat{i^F}, \widehat{\sigma}, \frac{p \cdot Y^r}{\overline{v}}) \cdot \underbrace{(\frac{M \cdot i^B}{\overline{q^B}} + n_P^B + \frac{Div \cdot i^B}{i^E \cdot \overline{q^B}} + \frac{n_P^F \cdot s \cdot \overline{q^F} \cdot i^B}{\widehat{i^F} \cdot \overline{q^B}})}_{= W \cdot \frac{i^B}{\overline{q^B}}}$$

The partial deviations are:

$$(93) \qquad \frac{\partial (n_P^B)^d}{\partial i^B} = \frac{\partial b}{\partial i^B} \cdot W \cdot \frac{i^B}{\overline{q^B}} + b \cdot \frac{M + E + sF_P}{\overline{q^B}} \qquad > 0$$

$$(94) \qquad \frac{\partial (n_P^B)^d}{\partial i^E} = \frac{\partial b}{\partial i^E} \cdot W \cdot \frac{i^B}{\overline{q^B}} - b \cdot \frac{E \cdot i^B}{i^E \cdot \overline{q^B}} \qquad < 0$$

$$(95) \qquad \frac{\partial (n_P^B)^d}{\partial \widehat{i^F}} = \frac{\partial b}{\partial \widehat{i^F}} \cdot W \cdot \frac{i^B}{\overline{q^B}} - b \cdot \frac{sF_P \cdot i^B}{\widehat{i^F} \cdot \overline{q^B}} \qquad < 0$$

$$(96) \qquad \frac{\partial (n_P^B)^d}{\partial \widehat{\sigma}} = \frac{\partial b}{\partial \widehat{\sigma}} \cdot W \cdot \frac{i^B}{\overline{q^B}} \qquad < 0$$

$$(97) \qquad \frac{\partial (n_P^B)^d}{\partial M} = \frac{\partial b}{\partial (\frac{p \cdot Y^r}{\overline{v}})} \frac{\partial (\frac{p \cdot Y^r}{\overline{v}})}{\partial M} \cdot \frac{W \cdot i^B}{\overline{q^B}} + b \cdot \frac{i^B}{\overline{q^B}} = 0^*$$

$$(98) \qquad \frac{\partial (n_P^B)^d}{\partial n_P^B} = b \qquad > 0$$

$$(99) \qquad \frac{\partial (n_P^B)^d}{\partial Div} = b \cdot \frac{i^B}{i^E \cdot \overline{q^B}} \qquad > 0$$

$$(100) \qquad \frac{\partial (n_P^B)^d}{\partial n_P^F} = b \cdot \frac{s \cdot \overline{q^F} \cdot i^B}{\widehat{i^F} \cdot \overline{q^B}} \qquad > 0$$

$$(101) \qquad \frac{\partial (n_P^B)^d}{\partial s} = b \cdot \frac{\overline{q^F} \cdot n_P^F \cdot i^B}{\widehat{i^F} \cdot \overline{q^B}} \qquad > 0$$

* See explanation in money demand section.

Demand for Dividend Payments

Equation 53 shows private households' demand for domestic dividend payments. By rearranging it, we obtain:

$$Div^d = e(i^B, i^E, \widehat{i^F}, \widehat{\sigma}, \frac{p \cdot Y^r}{\overline{v}}) \cdot \underbrace{(M \cdot i^E + \frac{n_P^B \cdot \overline{q^B} \cdot i^E}{i^B} + Div + \frac{n_P^F \cdot s \cdot \overline{q^F} \cdot i^E}{\widehat{i^F}})}_{= W \cdot i^E} \tag{102}$$

The partial deviations are:

$$\frac{\partial Div^d}{\partial i^B} = \frac{\partial e}{\partial i^B} \cdot W \cdot i^E - e \cdot \frac{B_P \cdot i^E}{i^B} \qquad < 0 \tag{103}$$

$$\frac{\partial Div^d}{\partial i^E} = \frac{\partial e}{\partial i^E} \cdot W \cdot i^E + e \cdot (M + B_P + sF_P) > 0 \tag{104}$$

$$\frac{\partial Div^d}{\partial \widehat{i^F}} = \frac{\partial e}{\partial \widehat{i^F}} \cdot W \cdot i^E - e \cdot \frac{sF_P \cdot i^E}{\widehat{i^F}} \qquad < 0 \tag{105}$$

$$\frac{\partial Div^d}{\partial \widehat{\sigma}} = \frac{\partial e}{\partial \widehat{\sigma}} \cdot W \cdot i^E \qquad < 0 \tag{106}$$

$$\frac{\partial Div^d}{\partial M} = \frac{\partial e}{\partial (\frac{p \cdot Y^r}{\overline{v}})} \frac{\partial (\frac{p \cdot Y^r}{\overline{v}})}{\partial M} \cdot W \cdot i^E + e \cdot i^E \qquad = 0^* \tag{107}$$

$$\frac{\partial Div^d}{\partial n_P^B} = e \cdot \frac{\overline{q^B} \cdot i^E}{i^B} \qquad > 0 \tag{108}$$

$$\frac{\partial Div^d}{\partial Div} = e \qquad > 0 \tag{109}$$

$$\frac{\partial Div^d}{\partial n_P^F} = e \cdot \frac{s \cdot \overline{q^F} \cdot i^E}{\widehat{i^F}} \qquad > 0 \tag{110}$$

$$\frac{\partial Div^d}{\partial s} = e \cdot \frac{\overline{q^F} \cdot n_P^F \cdot i^E}{\widehat{i^F}} \qquad > 0 \tag{111}$$

* See explanation in money demand section.

Demand for Foreign Assets

Equation 54 shows private households' demand for quantities of foreign assets. By rearranging it, we obtain:

$$(n_P^F)^d = f(i^B, i^E, \widehat{i^F}, \widehat{\sigma}, \frac{p \cdot Y^r}{\overline{v}}) \cdot \underbrace{(\frac{M \cdot \widehat{i^F}}{s \cdot \overline{q^F}} + \frac{n_P^B \cdot \overline{q^B} \cdot \widehat{i^F}}{i^B \cdot s \cdot \overline{q^F}} + \frac{Div \cdot \widehat{i^F}}{i^E \cdot s \cdot \overline{q^F}} + n_P^F)}_{= W \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}}} \tag{112}$$

The partial deviations are:

$$\frac{\partial (n_P^F)^d}{\partial i^B} = \frac{\partial f}{\partial i^B} \cdot W \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}} - f \cdot \frac{B_P \cdot \widehat{i^F}}{i^B \cdot s \cdot \overline{q^F}} \qquad < 0 \tag{113}$$

$$\frac{\partial (n_P^F)^d}{\partial i^E} = \frac{\partial f}{\partial i^E} \cdot W \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}} - f \cdot \frac{E \cdot \widehat{i^F}}{i^E \cdot s \cdot \overline{q^F}} \qquad < 0 \tag{114}$$

$$\frac{\partial (n_P^F)^d}{\partial \widehat{i^F}} = \frac{\partial f}{\partial \widehat{i^F}} \cdot W \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}} + f \cdot \frac{M + B_P + E}{s \cdot \overline{q^F}} \qquad > 0 \tag{115}$$

$$\frac{\partial (n_P^F)^d}{\partial \widehat{\sigma}} = \frac{\partial f}{\partial \widehat{\sigma}} \cdot W \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}} \qquad > 0 \tag{116}$$

$$\frac{\partial (n_P^F)^d}{\partial M} = \frac{\partial f}{\partial (\frac{p \cdot Y^r}{\overline{v}})} \frac{\partial (\frac{p \cdot Y^r}{\overline{v}})}{\partial M} \cdot W \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}} + f \cdot \frac{\widehat{i^F}}{s \cdot \overline{q^F}} = 0^* \tag{117}$$

$$\frac{\partial (n_P^F)^d}{\partial n_P^B} = f \cdot \frac{\overline{q^B} \cdot \widehat{i^F}}{i^B \cdot s \cdot \overline{q^F}} \qquad > 0 \tag{118}$$

$$\frac{\partial (n_P^F)^d}{\partial Div} = f \cdot \frac{\widehat{i^F}}{i^E \cdot s \cdot \overline{q^F}} \qquad > 0 \tag{119}$$

$$\frac{\partial (n_P^F)^d}{\partial n_P^F} = f \qquad > 0 \tag{120}$$

$$\frac{\partial (n_P^F)^d}{\partial s} = -f \cdot \frac{(M + B_P + E) \cdot \widehat{i^F}}{s^2 \cdot \overline{q^F}} \qquad < 0 \tag{121}$$

* See explanation in money demand section.

Asset Fractions

It follows from equation 47 ($m + b + e + f = 1$), in connection with the propositions of the asset demand functions 48, 49, 50, and 51, that:

$$\frac{\partial m}{\partial i^B} + \frac{\partial b}{\partial i^B} + \frac{\partial e}{\partial i^B} + \frac{\partial f}{\partial i^B} = 0 \tag{122}$$
$$\frac{\partial m}{\partial i^B} < 0,\ \frac{\partial b}{\partial i^B} > 0,\ \frac{\partial e}{\partial i^B} < 0,\ \frac{\partial f}{\partial i^B} < 0$$

$$\frac{\partial m}{\partial i^E} + \frac{\partial b}{\partial i^E} + \frac{\partial e}{\partial i^E} + \frac{\partial f}{\partial i^E} = 0 \tag{123}$$
$$\frac{\partial m}{\partial i^E} < 0,\ \frac{\partial b}{\partial i^E} < 0,\ \frac{\partial e}{\partial i^E} > 0,\ \frac{\partial f}{\partial i^E} < 0$$

$$\frac{\partial m}{\partial i^{\widehat{F}}} + \frac{\partial b}{\partial i^{\widehat{F}}} + \frac{\partial e}{\partial i^{\widehat{F}}} + \frac{\partial f}{\partial i^{\widehat{F}}} = 0 \tag{124}$$
$$\tfrac{\partial m}{\partial i^{\widehat{F}}} < 0,\ \tfrac{\partial b}{\partial i^{\widehat{F}}} < 0,\ \tfrac{\partial e}{\partial i^{\widehat{F}}} < 0,\ \tfrac{\partial f}{\partial i^{\widehat{F}}} > 0$$

$$\frac{\partial m}{\partial \widehat{\sigma}} + \frac{\partial b}{\partial \widehat{\sigma}} + \frac{\partial e}{\partial \widehat{\sigma}} + \frac{\partial f}{\partial \widehat{\sigma}} = 0 \tag{125}$$
$$\tfrac{\partial m}{\partial \widehat{\sigma}} < 0,\ \tfrac{\partial b}{\partial \widehat{\sigma}} < 0,\ \tfrac{\partial e}{\partial \widehat{\sigma}} < 0,\ \tfrac{\partial f}{\partial \widehat{\sigma}} > 0$$

$$\frac{\partial m}{\partial \frac{p \cdot Y^r}{\overline{v}}} + \frac{\partial b}{\partial \frac{p \cdot Y^r}{\overline{v}}} + \frac{\partial e}{\partial \frac{p \cdot Y^r}{\overline{v}}} + \frac{\partial f}{\partial \frac{p \cdot Y^r}{\overline{v}}} = 0 \tag{126}$$
$$\tfrac{\partial m}{\partial \frac{p \cdot Y^r}{\overline{v}}} > 0,\ \tfrac{\partial b}{\partial \frac{p \cdot Y^r}{\overline{v}}} < 0,\ \tfrac{\partial e}{\partial \frac{p \cdot Y^r}{\overline{v}}} < 0,\ \tfrac{\partial f}{\partial \frac{p \cdot Y^r}{\overline{v}}} < 0$$

The elasticities of the asset fractions, with regard to interest rates (i^B, i^E, $i^{\widehat{F}}$) and relative macroeconomic risk ($\widehat{\sigma}$), are subsequently written as:

$$\eta_{m,i^B} = \frac{\partial m}{\partial i^B}\frac{i^B}{m} \qquad \eta_{m,i^E} = \frac{\partial m}{\partial i^E}\frac{i^E}{m} \qquad \eta_{m,i^{\widehat{F}}} = \frac{\partial m}{\partial i^{\widehat{F}}}\frac{i^{\widehat{F}}}{m} \qquad \eta_{m,\widehat{\sigma}} = \frac{\partial m}{\partial \widehat{\sigma}}\frac{\widehat{\sigma}}{m} \tag{127}$$

$$\eta_{b,i^B} = \frac{\partial b}{\partial i^B}\frac{i^B}{b} \qquad \eta_{b,i^E} = \frac{\partial b}{\partial i^E}\frac{i^E}{b} \qquad \eta_{b,i^{\widehat{F}}} = \frac{\partial b}{\partial i^{\widehat{F}}}\frac{i^{\widehat{F}}}{b} \qquad \eta_{b,\widehat{\sigma}} = \frac{\partial b}{\partial \widehat{\sigma}}\frac{\widehat{\sigma}}{b} \tag{128}$$

$$\eta_{e,i^B} = \frac{\partial e}{\partial i^B}\frac{i^B}{e} \qquad \eta_{e,i^E} = \frac{\partial e}{\partial i^E}\frac{i^E}{e} \qquad \eta_{e,i^{\widehat{F}}} = \frac{\partial e}{\partial i^{\widehat{F}}}\frac{i^{\widehat{F}}}{e} \qquad \eta_{e,\widehat{\sigma}} = \frac{\partial e}{\partial \widehat{\sigma}}\frac{\widehat{\sigma}}{e} \tag{129}$$

$$\eta_{f,i^B} = \frac{\partial f}{\partial i^B}\frac{i^B}{f} \qquad \eta_{f,i^E} = \frac{\partial f}{\partial i^E}\frac{i^E}{f} \qquad \eta_{f,i^{\widehat{F}}} = \frac{\partial f}{\partial i^{\widehat{F}}}\frac{i^{\widehat{F}}}{f} \qquad \eta_{f,\widehat{\sigma}} = \frac{\partial f}{\partial \widehat{\sigma}}\frac{\widehat{\sigma}}{f} \tag{130}$$

Solving for Short Term Reactions

System of Equations

Changes in the domestic bond interest rate (i^B), the equity discount rate (i^E) and the exchange rate (s) need to be simultaneously determined in the short term. Their reaction to changes in the (short term) exogenous variables ($i^{\widehat{F}}$, $\widehat{\sigma}$, i^K, $n^{\widehat{B}}_{CB}$, $n^{\widehat{F}}_{CB}$) can be identified through the total differentiation of the equilibrium conditions 56, 57 and 58, and by solving the respective system of linear equations:

$$
(131)\quad \underbrace{\begin{bmatrix} \frac{\partial (n_P^B)^d}{\partial i^B} & \frac{\partial (n_P^B)^d}{\partial i^E} & \frac{\partial (n_P^B)^d}{\partial s} \\ \frac{\partial Div^d}{\partial i^B} & \frac{\partial Div^d}{\partial i^E} & \frac{\partial Div^d}{\partial s} \\ \frac{\partial (n_P^F)^d}{\partial i^B} & \frac{\partial (n_P^F)^d}{\partial i^E} & \frac{\partial (n_P^F)^d}{\partial s} \end{bmatrix}}_{=A_{st}} \cdot \begin{bmatrix} di^B \\ di^E \\ ds \end{bmatrix} = \begin{bmatrix} -\frac{\partial (n_P^B)^d}{\partial \widehat{i^F}} \cdot d\widehat{i^F} & -\frac{\partial (n_P^B)^d}{\partial \widehat{\sigma}} \cdot d\widehat{\sigma} \\ -\frac{\partial Div^d}{\partial \widehat{i^F}} \cdot d\widehat{i^F} & -\frac{\partial Div^d}{\partial \widehat{\sigma}} \cdot d\widehat{\sigma} \\ -\frac{\partial (n_P^F)^d}{\partial \widehat{i^F}} \cdot d\widehat{i^F} & -\frac{\partial (n_P^F)^d}{\partial \widehat{\sigma}} \cdot d\widehat{\sigma} \end{bmatrix}
$$

$$
\begin{bmatrix} -\frac{\partial (n_P^B)^d}{\partial Div} \frac{\partial Div}{\partial i^K} \cdot di^K & +(-\frac{\partial (n_P^B)^d}{\partial n_P^B} \frac{\partial n_P^B}{\partial \widehat{n_{CB}^B}} + \frac{\partial (n_P^B)^s}{\partial \widehat{n_{CB}^B}} - \frac{\partial (n_P^B)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n_{CB}^B}}) \cdot d\widehat{n_{CB}^B} \\ +(-\frac{\partial Div^d}{\partial Div} \frac{\partial Div}{\partial i^K} + \frac{\partial Div}{\partial i^K}) \cdot di^K & +(-\frac{\partial Div^d}{\partial n_P^B} \frac{\partial n_P^B}{\partial \widehat{n_{CB}^B}} - \frac{\partial Div^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n_{CB}^B}} + \frac{\partial Div}{\partial \widehat{n_{CB}^B}}) \cdot d\widehat{n_{CB}^B} \\ -\frac{\partial (n_P^F)^d}{\partial Div} \frac{\partial Div}{\partial i^K} \cdot di^K & +(-\frac{\partial (n_P^F)^d}{\partial n_P^B} \frac{\partial n_P^B}{\partial \widehat{n_{CB}^B}} - \frac{\partial (n_P^F)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n_{CB}^B}}) \cdot d\widehat{n_{CB}^B} \\ & +(-\frac{\partial (n_P^B)^d}{\partial n_P^F} \frac{\partial n_P^F}{\partial \widehat{n_{CB}^F}} - \frac{\partial (n_P^B)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n_{CB}^F}}) \cdot d\widehat{n_{CB}^F} \\ & +(-\frac{\partial Div^d}{\partial n_P^F} \frac{\partial n_P^F}{\partial \widehat{n_{CB}^F}} - \frac{\partial Div^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n_{CB}^F}} + \frac{\partial Div}{\partial \widehat{n_{CB}^F}}) \cdot d\widehat{n_{CB}^F} \\ & +(-\frac{\partial (n_P^F)^d}{\partial n_P^F} \frac{\partial n_P^F}{\partial \widehat{n_{CB}^F}} + \frac{\partial (n_P^F)^s}{\partial \widehat{n_{CB}^F}} - \frac{\partial (n_P^F)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n_{CB}^F}}) \cdot d\widehat{n_{CB}^F} \end{bmatrix}
$$

Regarding the abbreviations for the asset fractions elasticities, the determinant of matrice A_{st} is:

$$
\begin{aligned}
(132)\quad detA_{st} = &-\frac{m \cdot n_P^B \cdot Div \cdot n_P^F}{i^B \cdot i^E \cdot s}(1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} \\
&- \eta_{m,i^B} \cdot \eta_{e,i^E} - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B}) \\
&< 0
\end{aligned}
$$

Only $-\eta_{b,i^E} \cdot \eta_{e,i^B}$ is negative. However, added together with $+\eta_{b,i^B} \cdot \eta_{e,i^E}$, this results in $\eta_{e,i^B} \cdot \frac{i^E}{b} \cdot (\frac{\partial m}{\partial i^E} + \frac{\partial f}{\partial i^E}) - \eta_{e,i^E} \cdot \frac{i^B}{b} \cdot (\frac{\partial m}{\partial i^B} + \frac{\partial f}{\partial i^B})$ definitely being positive. Thus, it always holds that $detA_{st} < 0$. Since $detA_{st} \neq 0$, the changes in endogenous variables can be definitely determined by Cramer's rule given all parameters of the model.

Impact of Changes in the Foreign Interest Rate

$$
(133)\quad \frac{di^B}{d\widehat{i^F}} = \frac{i^B}{\widehat{i^F}} \cdot \frac{\begin{matrix} -\eta_{b,\widehat{i^F}} + \eta_{m,i^E} \cdot \eta_{b,\widehat{i^F}} + \eta_{b,i^E} \cdot \eta_{e,\widehat{i^F}} - \eta_{b,\widehat{i^F}} \cdot \eta_{e,i^E} \\ + \eta_{m,\widehat{i^F}} - \eta_{m,i^E} \cdot \eta_{e,\widehat{i^F}} - \eta_{m,\widehat{i^F}} \cdot \eta_{b,i^E} + \eta_{m,\widehat{i^F}} \cdot \eta_{e,i^E} \end{matrix}}{\begin{matrix} 1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} \\ - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B} \end{matrix}}
$$

$$
\begin{cases} > 0 & \text{if } -\eta_{b,\widehat{i^F}} + \eta_{m,i^E} \cdot \eta_{b,\widehat{i^F}} + \eta_{b,i^E} \cdot \eta_{e,\widehat{i^F}} - \eta_{b,\widehat{i^F}} \cdot \eta_{e,i^E} > \\ & \qquad -(\eta_{m,\widehat{i^F}} - \eta_{m,i^E} \cdot \eta_{e,\widehat{i^F}} - \eta_{m,\widehat{i^F}} \cdot \eta_{b,i^E} + \eta_{m,\widehat{i^F}} \cdot \eta_{e,i^E}) \\ \leq 0 & \text{else} \end{cases}
$$

$$(134)\qquad \frac{di^E}{d\widehat{i^F}} = \frac{i^E}{\widehat{i^F}} \cdot \frac{\begin{matrix}-\eta_{e,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{e,\widehat{i^F}}-\eta_{b,i^B}\cdot\eta_{e,\widehat{i^F}}+\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^B}\\ +\eta_{m,\widehat{i^F}}-\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}+\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B}-\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^B}\end{matrix}}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}}$$

$$\begin{cases} > 0 & \text{if } -\eta_{e,\widehat{i^F}} + \eta_{m,i^B}\cdot\eta_{e,\widehat{i^F}} - \eta_{b,i^B}\cdot\eta_{e,\widehat{i^F}} + \eta_{b,\widehat{i^F}}\cdot\eta_{e,i^B} > \\ & \qquad -\left(\eta_{m,\widehat{i^F}} - \eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}} + \eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B} - \eta_{m,\widehat{i^F}}\cdot\eta_{e,i^B}\right) \\ \leq 0 & \text{else} \end{cases}$$

$$(135)\qquad \frac{ds}{d\widehat{i^F}} = \frac{s}{\widehat{i^F}} \cdot \left(1 + \frac{\begin{matrix}-\eta_{m,\widehat{i^F}}+\eta_{f,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}-\eta_{m,i^B}\cdot\eta_{f,\widehat{i^F}}+\eta_{m,i^E}\cdot\eta_{e,\widehat{i^F}}-\eta_{m,i^E}\cdot\eta_{f,\widehat{i^F}}-\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B}\\ -\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^E}+\eta_{m,\widehat{i^F}}\cdot\eta_{f,i^B}+\eta_{m,\widehat{i^F}}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,\widehat{i^F}}-\eta_{b,\widehat{i^F}}\cdot\eta_{f,i^B}\\ +\eta_{e,i^E}\cdot\eta_{f,\widehat{i^F}}-\eta_{e,\widehat{i^F}}\cdot\eta_{f,i^E}+\frac{1}{f}\cdot(-\eta_{m,i^B}\cdot\eta_{b,i^E}\cdot\eta_{e,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^E}\\ +\eta_{m,i^E}\cdot\eta_{b,i^B}\cdot\eta_{e,\widehat{i^F}}-\eta_{m,i^E}\cdot\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^B}-\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B}\cdot\eta_{e,i^E}+\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^E}\cdot\eta_{e,i^B})\end{matrix}}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}}\right)$$

$$> 0$$

Impact of Changes in the Domestic Excess Risk

$$(136)\qquad \frac{di^B}{d\widehat{\sigma}} = \frac{i^B}{\widehat{\sigma}} \cdot \frac{\begin{matrix}-\eta_{b,\widehat{\sigma}}+\eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}}+\eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E}\\ +\eta_{m,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}\end{matrix}}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}}$$

$$\begin{cases} > 0 & \text{if } -\eta_{b,\widehat{\sigma}} + \eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}} + \eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}} - \eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E} > \\ & \qquad -\left(\eta_{m,\widehat{\sigma}} - \eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}} - \eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E} + \eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}\right) \\ \leq 0 & \text{else} \end{cases}$$

$$(137)\qquad \frac{di^E}{d\widehat{\sigma}} = \frac{i^E}{\widehat{\sigma}} \cdot \frac{\begin{matrix}-\eta_{e,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{e,\widehat{\sigma}}-\eta_{b,i^B}\cdot\eta_{e,\widehat{\sigma}}+\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^B}\\ +\eta_{m,\widehat{\sigma}}-\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}+\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B}-\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^B}\end{matrix}}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}}$$

$$\begin{cases} > 0 & \text{if } -\eta_{e,\widehat{\sigma}} + \eta_{m,i^B}\cdot\eta_{e,\widehat{\sigma}} - \eta_{b,i^B}\cdot\eta_{e,\widehat{\sigma}} + \eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^B} > \\ & \qquad -\left(\eta_{m,\widehat{\sigma}} - \eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}} + \eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B} - \eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^B}\right) \\ \leq 0 & \text{else} \end{cases}$$

$$(138)\qquad \frac{ds}{d\widehat{\sigma}} = \frac{s}{\widehat{\sigma}} \cdot \frac{\begin{matrix}-\eta_{m,\widehat{\sigma}}+\eta_{f,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}-\eta_{m,i^B}\cdot\eta_{f,\widehat{\sigma}}+\eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{f,\widehat{\sigma}}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B}\\ -\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{f,i^B}+\eta_{m,\widehat{\sigma}}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,\widehat{\sigma}}-\eta_{b,\widehat{\sigma}}\cdot\eta_{f,i^B}\\ +\eta_{e,i^E}\cdot\eta_{f,\widehat{\sigma}}-\eta_{e,\widehat{\sigma}}\cdot\eta_{f,i^E}+\frac{1}{f}\cdot(-\eta_{m,i^B}\cdot\eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E}\\ +\eta_{m,i^E}\cdot\eta_{b,i^B}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^B}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B}\cdot\eta_{e,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E}\cdot\eta_{e,i^B})\end{matrix}}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}}$$

$$> 0$$

Impact of Changes in the Credit Interest Rate

$$\frac{di^B}{di^K} = \frac{i^B \cdot \widehat{K}}{Div} \cdot \frac{-\eta_{m,i^E} + \eta_{b,i^E}}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} \tag{139}$$

$$\begin{cases} > 0 & \text{if} \quad -\eta_{m,i^E} > -\eta_{b,i^E} \\ \leq 0 & \text{else} \end{cases}$$

$$\frac{di^E}{di^K} = -\frac{i^E \cdot \widehat{K}}{Div} \cdot \frac{1 - \eta_{m,i^B} + \eta_{b,i^B}}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} \tag{140}$$

$$< 0$$

$$\frac{ds}{di^K} = \frac{s \cdot \widehat{K}}{Div} \cdot \frac{\substack{-\eta_{f,i^E}+\eta_{m,i^B}\cdot\eta_{f,i^E}-\eta_{b,i^B}\cdot\eta_{f,i^E}+\eta_{b,i^E}\cdot\eta_{f,i^B} \\ +\eta_{m,i^E}-\eta_{m,i^B}\cdot\eta_{b,i^E}+\eta_{m,i^E}\cdot\eta_{b,i^B}-\eta_{m,i^E}\cdot\eta_{f,i^B}}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} \tag{141}$$

$$\begin{cases} > 0 & \text{if} \quad -\eta_{f,i^E} + \eta_{m,i^B} \cdot \eta_{f,i^E} - \eta_{b,i^B} \cdot \eta_{f,i^E} + \eta_{b,i^E} \cdot \eta_{f,i^B} > \\ & \qquad -(\eta_{m,i^E} - \eta_{m,i^B} \cdot \eta_{b,i^E} + \eta_{m,i^E} \cdot \eta_{b,i^B} - \eta_{m,i^E} \cdot \eta_{f,i^B}) \\ \leq 0 & \text{else} \end{cases}$$

Impact of Changes in Central Bank's Amount of Domestic Bonds

$$\frac{di^B}{d\widehat{n^B_{CB}}} = -\frac{i^B}{n^B_P} \cdot \frac{1 - \eta_{m,i^E} + \eta_{e,i^E} + \frac{b\cdot\overline{v}}{e\cdot i^E} \cdot (-\eta_{m,i^E} + \eta_{b,i^E})}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} \tag{142}$$

$$\begin{cases} < 0 & \text{if} \quad 1 - \eta_{m,i^E} + \eta_{e,i^E} - \frac{b\cdot\overline{v}}{e\cdot i^E} \cdot \eta_{m,i^E} > -\frac{b\cdot\overline{v}}{e\cdot i^E} \cdot \eta_{b,i^E} \\ \geq 0 & \text{else} \end{cases}$$

$$\frac{di^E}{d\widehat{n^B_{CB}}} = \frac{i^E}{n^B_P} \cdot \frac{-\eta_{m,i^B} + \frac{b\cdot\overline{v}}{e\cdot i^E} \cdot (1 - \eta_{m,i^B} + \eta_{b,i^B}) + \eta_{e,i^B}}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} \tag{143}$$

$$\begin{cases} > 0 & \text{if} \quad -\eta_{m,i^B} + \frac{b\cdot\overline{v}}{e\cdot i^E} \cdot (1 - \eta_{m,i^B} + \eta_{b,i^B}) > -\eta_{e,i^B} \\ \leq 0 & \text{else} \end{cases}$$

$$(144)\qquad \frac{ds}{d\widehat{n^B_{CB}}} = \frac{s}{n^B_P}\cdot\frac{\begin{array}{c}-\eta_{f,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}+\eta_{e,i^B}\cdot\eta_{f,i^E}-\eta_{e,i^E}\cdot\eta_{f,i^B}\\ +\frac{b\cdot\overline{v}}{e\cdot i^E}\cdot(-\eta_{m,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B})\\ +\eta_{m,i^B}+\eta_{m,i^B}\cdot\eta_{e,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}-\eta_{m,i^E}\cdot\eta_{e,i^B}\end{array}}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array}}$$

$$\begin{cases} >0 & \text{if } \begin{array}{c}-\eta_{f,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}+\eta_{e,i^B}\cdot\eta_{f,i^E}-\eta_{e,i^E}\cdot\eta_{f,i^B}\\ +\frac{b\cdot\overline{v}}{e\cdot i^E}\cdot(-\eta_{m,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B})\end{array} > \\ & \qquad -(\eta_{m,i^B}+\eta_{m,i^B}\cdot\eta_{e,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}-\eta_{m,i^E}\cdot\eta_{e,i^B} \\ & \qquad +\frac{b\cdot\overline{v}}{e\cdot i^E}\cdot(+\eta_{f,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,i^E}-\eta_{b,i^E}\cdot\eta_{f,i^B}))\\ \leq 0 & \text{else}\end{cases}$$

Impact of Changes in Central Bank's Amount of Foreign Assets

$$(145)\qquad \frac{di^B}{d\widehat{n^F_{CB}}} = \frac{i^B}{n^F_P}\cdot\frac{\frac{f\cdot\overline{v}}{e\cdot i^E}\cdot\left(-\eta_{b,i^E}+\eta_{m,i^E}\right)}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array}}$$

$$\begin{cases} >0 & \text{if } -\eta_{b,i^E} > -\eta_{m,i^E}\\ \leq 0 & \text{else}\end{cases}$$

$$(146)\qquad \frac{di^E}{d\widehat{n^F_{CB}}} = \frac{i^E}{n^F_P}\cdot\frac{\frac{f\cdot\overline{v}}{e\cdot i^E}\cdot\left(1-\eta_{m,i^B}+\eta_{b,i^B}\right)}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array}}$$

$$>0$$

$$(147)\qquad \frac{ds}{d\widehat{n^F_{CB}}} = \frac{s}{n^F_P}\cdot\left(1+\frac{\begin{array}{c}\frac{f\cdot\overline{v}}{e\cdot i^E}\cdot(-\eta_{m,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}\\ +\eta_{f,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,i^E}-\eta_{b,i^E}\cdot\eta_{f,i^B})\end{array}}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array}}\right)$$

$$\begin{cases} >0 & \text{if } 1+\frac{\frac{f\cdot\overline{v}}{e\cdot i^E}\cdot(-\eta_{m,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B})}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array}} > \\ & \qquad -\frac{\frac{f\cdot\overline{v}}{e\cdot i^E}(\eta_{f,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,i^E}-\eta_{b,i^E}\cdot\eta_{f,i^B})}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array}}\\ \leq 0 & \text{else}\end{cases}$$

System of Equations

Changes in the domestic bond interest rate (i^B), the equity discount rate (i^E), the total amount of foreign bonds held domestically (n^F), the total amount of domestic bonds (n^B), and the exchange rate (s) need to be simultaneously determined in the long term. Their reaction to changes in the (long term) exogenous variables ($\widehat{i^F}$, $\widehat{\sigma}$, $\widehat{p^*}$, $\widehat{K}$, $\widehat{n^B_{CB}}$, $\widehat{n^F_{CB}}$) can be identified through the total differentiation of the equilibrium conditions 60, 61, 62 and 64, and by solving the respective system of linear equations:

$$
\underbrace{\begin{bmatrix}
\frac{\partial (n_P^B)^d}{\partial i^B} & \frac{\partial (n_P^B)^d}{\partial i^E} & \frac{\partial (n_P^B)^d}{\partial n_P^F} & \frac{\partial (n_P^B)^d}{\partial n_P^B} - 1 & \frac{\partial (n_P^B)^d}{\partial s} \\
\frac{\partial Div^d}{\partial i^B} & \frac{\partial Div^d}{\partial i^E} & \frac{\partial Div^d}{\partial n_P^F} & \frac{\partial Div^d}{\partial n_P^B} & \frac{\partial Div^d}{\partial s} \\
\frac{\partial (n_P^F)^d}{\partial i^B} & \frac{\partial (n_P^F)^d}{\partial i^E} & \frac{\partial (n_P^F)^d}{\partial n_P^F} - 1 & \frac{\partial (n_P^F)^d}{\partial n_P^B} & \frac{\partial (n_P^F)^d}{\partial s} \\
-\frac{\partial (n^B)^s}{\partial i^B} & 0 & 0 & 1 & 0 \\
0 & 0 & \frac{\partial s}{\partial n^F} & 0 & -1
\end{bmatrix}}_{=A_{lt}} \cdot \begin{bmatrix} di^B \\ di^E \\ dn^F \\ dn^B \\ ds \end{bmatrix} = \tag{148}
$$

$$
\begin{bmatrix}
-\frac{\partial (n_P^B)^d}{\partial \widehat{i^F}} \cdot d\widehat{i^F} & -\frac{\partial (n_P^B)^d}{\partial \widehat{\sigma}} \cdot d\widehat{\sigma} & -\frac{\partial (n_P^B)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{K}} \cdot d\widehat{K} \\
-\frac{\partial Div^d}{\partial \widehat{i^F}} \cdot d\widehat{i^F} & -\frac{\partial Div^d}{\partial \widehat{\sigma}} \cdot d\widehat{\sigma} & +\left(-\frac{\partial Div^d}{\partial Div} \frac{\partial Div}{\partial \widehat{K}} + \frac{\partial Div}{\partial \widehat{K}}\right) \cdot d\widehat{K} \\
-\frac{\partial (n_P^F)^d}{\partial \widehat{i^F}} \cdot d\widehat{i^F} & -\frac{\partial (n_P^F)^d}{\partial \widehat{\sigma}} \cdot d\widehat{\sigma} & -\frac{\partial (n_P^F)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{K}} \cdot d\widehat{K} \\
 & & +\frac{\partial (n^B)^s}{\partial \widehat{K}} \cdot d\widehat{K} \\
 & -\frac{\partial s}{\partial \widehat{p^*}} \cdot d\widehat{p^*} & -\frac{\partial s}{\partial \widehat{K}} \cdot d\widehat{K}
\end{bmatrix}
$$

$$
\begin{bmatrix}
+\left(-\frac{\partial (n_P^B)^d}{\partial n_P^B} \frac{\partial n_P^B}{\partial \widehat{n^B_{CB}}} + \frac{\partial (n_P^B)^s}{\partial \widehat{n^B_{CB}}} - \frac{\partial (n_P^B)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n^B_{CB}}}\right) \cdot d\widehat{n^B_{CB}} \\
+\left(-\frac{\partial Div^d}{\partial n_P^B} \frac{\partial n_P^B}{\partial \widehat{n^B_{CB}}} - \frac{\partial Div^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n^B_{CB}}} + \frac{\partial Div}{\partial \widehat{n^B_{CB}}}\right) \cdot d\widehat{n^B_{CB}} \\
+\left(-\frac{\partial (n_P^F)^d}{\partial n_P^B} \frac{\partial n_P^B}{\partial \widehat{n^B_{CB}}} - \frac{\partial (n_P^F)^d}{\partial Div} \frac{\partial Div}{\partial \widehat{n^B_{CB}}}\right) \cdot d\widehat{n^B_{CB}} \\
+\frac{\partial (n^B)^s}{\partial \widehat{n^B_{CB}}} \cdot d\widehat{n^B_{CB}} \\
-\frac{\partial s}{\partial \widehat{n^B_{CB}}} \cdot d\widehat{n^B_{CB}}
\end{bmatrix}
$$

$$
\begin{aligned}
&+\left(-\frac{\partial (n_P^B)^d}{\partial n_P^F}\frac{\partial n_P^F}{\partial \widehat{n_{CB}^F}}-\frac{\partial (n_P^B)^d}{\partial Div}\frac{\partial Div}{\partial \widehat{n_{CB}^F}}\right)\cdot d\widehat{n_{CB}^F}\\
&+\left(-\frac{\partial Div^d}{\partial n_P^F}\frac{\partial n_P^F}{\partial \widehat{n_{CB}^F}}-\frac{\partial Div^d}{\partial Div}\frac{\partial Div}{\partial \widehat{n_{CB}^F}}+\frac{\partial Div}{\partial \widehat{n_{CB}^F}}\right)\cdot d\widehat{n_{CB}^F}\\
&+\left(-\frac{\partial (n_P^F)^d}{\partial n_P^F}\frac{\partial n_P^F}{\partial \widehat{n_{CB}^F}}+\frac{\partial (n_P^F)^s}{\partial \widehat{n_{CB}^F}}-\frac{\partial (n_P^F)^d}{\partial Div}\frac{\partial Div}{\partial \widehat{n_{CB}^F}}\right)\cdot d\widehat{n_{CB}^F}\\
&+\frac{\partial (n^B)^s}{\partial \widehat{n_{CB}^F}}\cdot d\widehat{n_{CB}^F}\\
&-\frac{\partial s}{\partial \widehat{n_{CB}^F}}\cdot d\widehat{n_{CB}^F}
\end{aligned}
\Bigg]
$$

The determinant of matrice A_{lt} is:

$$
\begin{aligned}
detA_{lt} =& \frac{m\cdot n_P^B\cdot Div}{i^B\cdot i^E}\cdot\left(1+\frac{n_P^F\cdot \overline{q^F}}{\widehat{i^F}\cdot n^{CG}\cdot \widehat{p^*}}\right)\\
&\cdot\Big[\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}\\
&\quad+\frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)\Big]\\
>&\,0
\end{aligned}
\tag{149}
$$

Impact of Changes in the Foreign Interest Rate

(150)

$$
\frac{di^B}{d\widehat{i^F}}=\frac{i^B}{\widehat{i^F}}\cdot\frac{\begin{matrix}-\eta_{b,\widehat{i^F}}+\eta_{m,i^E}\cdot\eta_{b,\widehat{i^F}}+\eta_{b,i^E}\cdot\eta_{e,\widehat{i^F}}-\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^E}\\ +\eta_{m,\widehat{i^F}}-\eta_{m,i^E}\cdot\eta_{e,\widehat{i^F}}-\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^E}+\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^E}\end{matrix}}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}+\frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}
$$

$$
\begin{cases}
>0 & \text{if } -\eta_{b,\widehat{i^F}}+\eta_{m,i^E}\cdot\eta_{b,\widehat{i^F}}+\eta_{b,i^E}\cdot\eta_{e,\widehat{i^F}}-\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^E}>\\
& \qquad -\left(\eta_{m,\widehat{i^F}}-\eta_{m,i^E}\cdot\eta_{e,\widehat{i^F}}-\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^E}+\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^E}\right)\\
\le 0 & \text{else}
\end{cases}
$$

(151)

$$
\frac{di^E}{d\widehat{i^F}}=\frac{i^E}{\widehat{i^F}}\cdot\frac{\begin{matrix}-\eta_{e,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{e,\widehat{i^F}}-\eta_{b,i^B}\cdot\eta_{e,\widehat{i^F}}+\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^B}\\ +\eta_{m,\widehat{i^F}}-\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}+\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B}-\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^B}\end{matrix}+\frac{\widehat{K}}{B_P}\cdot\left(-\eta_{e,\widehat{i^F}}+\eta_{m,\widehat{i^F}}\right)}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}\\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}+\frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}
$$

$$
\begin{cases}
>0 & \text{if } -\eta_{e,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{e,\widehat{i^F}}-\eta_{b,i^B}\cdot\eta_{e,\widehat{i^F}}+\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^B}-\frac{\widehat{K}}{B_P}\cdot\eta_{e,\widehat{i^F}}>\\
& \qquad -\left(\eta_{m,\widehat{i^F}}-\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}+\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B}-\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^B}+\frac{\widehat{K}}{B_P}\cdot\eta_{m,\widehat{i^F}}\right)\\
\le 0 & \text{else}
\end{cases}
$$

(152)

$$\frac{dn^F}{d\widehat{i^F}} = \frac{n_P^F}{\widehat{i^F}} \cdot \frac{1}{(1+\frac{n_P^F \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}})} \cdot (1 + \frac{\alpha + \frac{\widehat{K}}{B_P} \cdot \beta}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} + \frac{\widehat{K}}{B_P} \cdot (1-\eta_{m,i^E}+\eta_{e,i^E})})$$

$$> 0$$

$$\alpha := \begin{array}{l} -\eta_{m,\widehat{i^F}}+\eta_{f,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}-\eta_{m,i^B}\cdot\eta_{f,\widehat{i^F}}+\eta_{m,i^E}\cdot\eta_{e,\widehat{i^F}}-\eta_{m,i^E}\cdot\eta_{f,\widehat{i^F}}-\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B} \\ -\eta_{m,\widehat{i^F}}\cdot\eta_{e,i^E}+\eta_{m,\widehat{i^F}}\cdot\eta_{f,i^B}+\eta_{m,\widehat{i^F}}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,\widehat{i^F}}-\eta_{b,\widehat{i^F}}\cdot\eta_{f,i^B} \\ +\eta_{e,i^E}\cdot\eta_{f,\widehat{i^F}}-\eta_{e,\widehat{i^F}}\cdot\eta_{f,i^E}+\frac{1}{f}\cdot(-\eta_{m,i^B}\cdot\eta_{b,i^E}\cdot\eta_{e,\widehat{i^F}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^E} \\ +\eta_{m,i^E}\cdot\eta_{b,i^B}\cdot\eta_{e,\widehat{i^F}}-\eta_{m,i^E}\cdot\eta_{b,\widehat{i^F}}\cdot\eta_{e,i^B}-\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^B}\cdot\eta_{e,i^E}+\eta_{m,\widehat{i^F}}\cdot\eta_{b,i^E}\cdot\eta_{e,i^B}) \end{array}$$

$$\beta := \begin{array}{l} -\eta_{m,\widehat{i^F}} + \eta_{f,\widehat{i^F}} + \eta_{m,i^E}\cdot\eta_{e,\widehat{i^F}} - \eta_{m,i^E}\cdot\eta_{f,\widehat{i^F}} - \eta_{m,\widehat{i^F}}\cdot\eta_{e,i^E} \\ +\eta_{m,\widehat{i^F}}\cdot\eta_{f,i^E} + \eta_{e,i^E}\cdot\eta_{f,\widehat{i^F}} - \eta_{e,\widehat{i^F}}\cdot\eta_{f,i^E} \end{array}$$

(153)

$$\frac{dn^B}{d\widehat{i^F}} = -\frac{di^B}{d\widehat{i^F}} \cdot \frac{\widehat{K}}{\overline{q^B}}$$

$$\begin{cases} < 0 & \text{if } -\eta_{b,\widehat{i^F}} + \eta_{m,i^E}\cdot\eta_{b,\widehat{i^F}} + \eta_{b,i^E}\cdot\eta_{e,\widehat{i^F}} - \eta_{b,\widehat{i^F}}\cdot\eta_{e,i^E} > \\ & \qquad -(\eta_{m,\widehat{i^F}} - \eta_{m,i^E}\cdot\eta_{e,\widehat{i^F}} - \eta_{m,\widehat{i^F}}\cdot\eta_{b,i^E} + \eta_{m,\widehat{i^F}}\cdot\eta_{e,i^E}) \\ \geq 0 & \text{else} \end{cases}$$

(154)

$$\frac{ds}{d\widehat{i^F}} = \frac{dn^F}{d\widehat{i^F}} \cdot \frac{s \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}}$$

$$> 0$$

Impact of Changes in the Domestic Excess Risk

(155)

$$\frac{di^B}{d\widehat{\sigma}} = \frac{i^B}{\widehat{\sigma}} \cdot \frac{\substack{-\eta_{b,\widehat{\sigma}}+\eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}}+\eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E} \\ +\eta_{m,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}}{\substack{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}} + \frac{\widehat{K}}{B_P} \cdot (1-\eta_{m,i^E}+\eta_{e,i^E})}$$

$$\begin{cases} > 0 & \text{if } -\eta_{b,\widehat{\sigma}} + \eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}} + \eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}} - \eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E} > \\ & \qquad -(\eta_{m,\widehat{\sigma}} - \eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}} - \eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E} + \eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}) \\ \leq 0 & \text{else} \end{cases}$$

(156)

$$\frac{di^E}{d\widehat{\sigma}} = \frac{i^E}{\widehat{\sigma}} \cdot \frac{\begin{array}{c}-\eta_{e,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{e,\widehat{\sigma}}-\eta_{b,i^B}\cdot\eta_{e,\widehat{\sigma}}+\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^B} \\ +\eta_{m,\widehat{\sigma}}-\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}+\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B}-\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^B}\end{array} + \frac{\widehat{K}}{B_P}\cdot\left(-\eta_{e,\widehat{\sigma}}+\eta_{m,\widehat{\sigma}}\right)}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array} + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}$$

$$\begin{cases} > 0 & \text{if} \quad -\eta_{e,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{e,\widehat{\sigma}}-\eta_{b,i^B}\cdot\eta_{e,\widehat{\sigma}}+\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^B}-\frac{\widehat{K}}{B_P}\cdot\eta_{e,\widehat{\sigma}} > \\ & \qquad\qquad -\left(\eta_{m,\widehat{\sigma}}-\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}+\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B}-\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^B}+\frac{\widehat{K}}{B_P}\cdot\eta_{m,\widehat{\sigma}}\right) \\ \leq 0 & \text{else} \end{cases}$$

(157)

$$\frac{dn^F}{d\widehat{\sigma}} = \frac{n_P^F}{\widehat{\sigma}} \cdot \frac{1}{\left(1+\frac{n_P^F\cdot\overline{q^F}}{\widehat{i^F}\cdot n^{CG}\cdot\widehat{p^*}}\right)} \cdot \frac{\alpha+\frac{\widehat{K}}{B_P}\cdot\beta}{\begin{array}{c}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{array} + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}$$

$$> 0$$

$$\alpha := \begin{array}{c}-\eta_{m,\widehat{\sigma}}+\eta_{f,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}-\eta_{m,i^B}\cdot\eta_{f,\widehat{\sigma}}+\eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{f,\widehat{\sigma}}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B} \\ -\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{f,i^B}+\eta_{m,\widehat{\sigma}}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,\widehat{\sigma}}-\eta_{b,\widehat{\sigma}}\cdot\eta_{f,i^B} \\ +\eta_{e,i^E}\cdot\eta_{f,\widehat{\sigma}}-\eta_{e,\widehat{\sigma}}\cdot\eta_{f,i^E}+\frac{1}{f}\cdot(-\eta_{m,i^B}\cdot\eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}}+\eta_{m,i^B}\cdot\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E} \\ +\eta_{m,i^E}\cdot\eta_{b,i^B}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^B}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^B}\cdot\eta_{e,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E}\cdot\eta_{e,i^B})\end{array}$$

$$\beta := \begin{array}{c}-\eta_{m,\widehat{\sigma}}+\eta_{f,\widehat{\sigma}}+\eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{f,\widehat{\sigma}}-\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E} \\ +\eta_{m,\widehat{\sigma}}\cdot\eta_{f,i^E}+\eta_{e,i^E}\cdot\eta_{f,\widehat{\sigma}}-\eta_{e,\widehat{\sigma}}\cdot\eta_{f,i^E}\end{array}$$

(158)

$$\frac{dn^B}{d\widehat{\sigma}} = -\frac{di^B}{d\widehat{\sigma}}\cdot\frac{\widehat{K}}{\overline{q^B}}$$

$$\begin{cases} < 0 & \text{if} \quad -\eta_{b,\widehat{\sigma}}+\eta_{m,i^E}\cdot\eta_{b,\widehat{\sigma}}+\eta_{b,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{b,\widehat{\sigma}}\cdot\eta_{e,i^E} > \\ & \qquad\qquad -\left(\eta_{m,\widehat{\sigma}}-\eta_{m,i^E}\cdot\eta_{e,\widehat{\sigma}}-\eta_{m,\widehat{\sigma}}\cdot\eta_{b,i^E}+\eta_{m,\widehat{\sigma}}\cdot\eta_{e,i^E}\right) \\ \geq 0 & \text{else} \end{cases}$$

(159)

$$\frac{ds}{d\widehat{\sigma}} = \frac{dn^F}{d\widehat{\sigma}}\cdot\frac{s\cdot\overline{q^F}}{\widehat{i^F}\cdot n^{CG}\cdot\widehat{p^*}}$$

$$> 0$$

Impact of Changes in the Foreign Price Level

$$\frac{di^B}{d\widehat{p^*}} = 0 \tag{160}$$

$$\frac{di^E}{d\widehat{p^*}} = 0 \tag{161}$$

$$\begin{aligned}\frac{dn^F}{d\widehat{p^*}} &= \frac{n_P^F}{\widehat{p^*}} \cdot \frac{1}{(1 + \frac{n_P^F \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}})} \\ &> 0\end{aligned} \tag{162}$$

$$\frac{dn^B}{d\widehat{p^*}} = 0 \tag{163}$$

$$\begin{aligned}\frac{ds}{d\widehat{p^*}} &= -\frac{s}{\widehat{p^*}} \cdot \frac{1}{(1 + \frac{n_P^F \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}})} \\ &< 0\end{aligned} \tag{164}$$

Impact of Changes in the Credit Amount

(165)

$$\frac{di^B}{d\widehat{K}} = \frac{i^B}{\overline{q^B}} \cdot \frac{i^B}{n_P^B} \cdot \frac{(\frac{\overline{dc} \cdot \overline{v}}{i^B} - 1) \cdot (1 - \eta_{m,i^E} + \eta_{e,i^E}) + \frac{b \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot (-\eta_{b,i^E} + \eta_{m,i^E})}{\begin{array}{c}1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} \\ - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B}\end{array} + \frac{\widehat{K}}{B_P} \cdot (1 - \eta_{m,i^E} + \eta_{e,i^E})}$$

$$\begin{cases} > 0 & \text{if } \frac{\overline{dc} \cdot \overline{v}}{i^B} \cdot (1 - \eta_{m,i^E} + \eta_{e,i^E}) - \frac{b \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \eta_{b,i^E} > \\ & \qquad 1 - \eta_{m,i^E} + \eta_{e,i^E} - \frac{b \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \eta_{m,i^E} \\ \leq 0 & \text{else} \end{cases}$$

(166)

$$\frac{di^E}{d\widehat{K}} = \frac{i^B}{\overline{q^B}} \cdot \frac{i^E}{n_P^B} \cdot \frac{\frac{\widehat{K}\cdot(\overline{v}-\overline{dc})}{E\cdot i^E} + \frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E} \cdot \left(1-\eta_{m,i^B}+\eta_{b,i^B}\right) + \left(\frac{\overline{dc}\cdot\overline{v}}{i^B}-1\right)\left(-\eta_{e,i^B}+\eta_{m,i^B}\right)}{\begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix} + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}$$

$$\begin{cases} > 0 & \text{if } \frac{\widehat{K}\cdot(\overline{v}-\overline{dc})}{E\cdot i^E} + \frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(1-\eta_{m,i^B}+\eta_{b,i^B}\right) - \frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\eta_{e,i^B} - \eta_{m,i^B} > \\ & \qquad -\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\eta_{m,i^B} - \eta_{e,i^B} \\ \leq 0 & \text{else} \end{cases}$$

(167) $$\frac{dn^F}{d\widehat{K}} = \frac{i^B}{\overline{q^B}} \cdot \frac{n_P^F}{n_P^B} \cdot \frac{1}{\left(1+\frac{n_P^F\cdot\overline{q^F}}{\overline{i^F}\cdot n^{CG}\cdot\widehat{p^\star}}\right)} \cdot \left(-\frac{n_P^B\cdot\overline{q^B}\cdot\overline{v}}{i^B\cdot s\cdot\overline{a}\cdot n^{CG}\cdot\widehat{p^\star}} + \frac{\alpha+\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot(\delta+\gamma)}{\omega}\right)$$

$$\begin{cases} < 0 & \text{if } \frac{n_P^B\cdot\overline{q^B}\cdot\overline{v}}{i^B\cdot s\cdot\overline{a}\cdot n^{CG}\cdot\widehat{p^\star}} - \frac{\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\gamma}{\omega} > \frac{\alpha+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\delta}{\omega} \\ \geq 0 & \text{else} \end{cases}$$

$$\begin{aligned}
\alpha :=\ & -\eta_{f,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}+\eta_{e,i^B}\cdot\eta_{f,i^E}-\eta_{e,i^E}\cdot\eta_{f,i^B} \\
& +\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(-\eta_{m,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}-\frac{\widehat{K}}{B_P}\cdot\eta_{m,i^E}\right) \\
\beta :=\ & +\eta_{m,i^B}+\eta_{m,i^B}\cdot\eta_{e,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}-\eta_{m,i^E}\cdot\eta_{e,i^B} \\
& +\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(+\eta_{f,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,i^E}-\eta_{b,i^E}\cdot\eta_{f,i^B}+\frac{\widehat{K}}{B_P}\cdot\eta_{f,i^E}\right) \\
\delta :=\ & -\eta_{m,i^B}-\eta_{m,i^B}\cdot\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{m,i^E}\cdot\eta_{e,i^B} \\
\gamma :=\ & +\eta_{f,i^B}-\eta_{m,i^E}\cdot\eta_{f,i^B}-\eta_{e,i^B}\cdot\eta_{f,i^E}+\eta_{e,i^E}\cdot\eta_{f,i^B} \\
\omega :=\ & \begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix} + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)
\end{aligned}$$

(168) $$\frac{dn^B}{d\widehat{K}} = \frac{i^B}{\overline{q^B}} \cdot \frac{\left(\frac{\overline{dc}\cdot\overline{v}}{i^B}-1\right)\cdot\alpha + \frac{\widehat{K}}{B_P}\cdot\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(-\eta_{m,i^E}+\eta_{b,i^E}\right)}{\alpha+\frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}$$

$$\begin{cases} > 0 & \text{if } \frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\alpha - \frac{\widehat{K}}{B_P}\cdot\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\eta_{m,i^E} > \alpha - \frac{\widehat{K}}{B_P}\cdot\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\eta_{b,i^E} \\ \leq 0 & \text{else} \end{cases}$$

$$\alpha := \begin{matrix}1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}\end{matrix}$$

(169) $$\frac{ds}{d\widehat{K}} = \frac{1}{n^{CG}\cdot\widehat{p^*}}\cdot\frac{1}{(1+\frac{n_P^F\cdot\overline{q^F}}{\widehat{i^F}\cdot n^{CG}\cdot\widehat{p^*}})}\cdot\left(\frac{\overline{v}}{\overline{a}}+\frac{f}{b}\cdot\frac{\alpha+\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot(\delta+\gamma)}{\omega}\right)$$

$$\begin{cases} >0 & \text{if } \ \frac{\overline{v}}{\overline{a}}+\frac{f}{b}\cdot\left(\frac{\alpha+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\delta}{\omega}\right) > -\frac{f}{b}\cdot\left(\frac{\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\gamma}{\omega}\right) \\ \leq 0 & \text{else} \end{cases}$$

$$\begin{aligned}
\alpha :=\ & -\eta_{f,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}+\eta_{e,i^B}\cdot\eta_{f,i^E}-\eta_{e,i^E}\cdot\eta_{f,i^B} \\
& +\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(-\eta_{m,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{f,i^B}-\frac{\widehat{K}}{B_P}\cdot\eta_{m,i^E}\right) \\
\beta :=\ & +\eta_{m,i^B}+\eta_{m,i^B}\cdot\eta_{e,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}-\eta_{m,i^E}\cdot\eta_{e,i^B} \\
& +\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(+\eta_{f,i^E}-\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{b,i^B}\cdot\eta_{f,i^E}-\eta_{b,i^E}\cdot\eta_{f,i^B}+\frac{\widehat{K}}{B_P}\cdot\eta_{f,i^E}\right) \\
\delta :=\ & -\eta_{m,i^B}-\eta_{m,i^B}\cdot\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{f,i^E}+\eta_{m,i^E}\cdot\eta_{e,i^B} \\
\gamma :=\ & +\eta_{f,i^B}-\eta_{m,i^E}\cdot\eta_{f,i^B}-\eta_{e,i^B}\cdot\eta_{f,i^E}+\eta_{e,i^E}\cdot\eta_{f,i^B} \\
\omega :=\ & \frac{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}}{-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}+\frac{\widehat{K}}{B_P}\cdot(1-\eta_{m,i^E}+\eta_{e,i^E})
\end{aligned}$$

Impact of Changes in the Amount of Central Bank's Domestic Bonds

(170)

$$\frac{di^B}{d\widehat{n^B_{CB}}} = \frac{i^B}{n_P^B}\cdot\frac{\left(\frac{\overline{dc}\cdot\overline{v}}{i^B}-1\right)\cdot(1-\eta_{m,i^E}+\eta_{e,i^E})+\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot(-\eta_{b,i^E}+\eta_{m,i^E})}{\frac{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}}{-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}+\frac{\widehat{K}}{B_P}\cdot(1-\eta_{m,i^E}+\eta_{e,i^E})}$$

$$\begin{cases} >0 & \text{if } \ \frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot(1-\eta_{m,i^E}+\eta_{e,i^E})-\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\eta_{b,i^E} > \\ & \qquad\qquad 1-\eta_{m,i^E}+\eta_{e,i^E}-\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\eta_{m,i^E} \\ \leq 0 & \text{else} \end{cases}$$

(171)

$$\frac{di^E}{d\widehat{n^B_{CB}}} = \frac{i^E}{n_P^B}\cdot\frac{\frac{\widehat{K}\cdot(\overline{v}-\overline{dc})}{E\cdot i^E}+\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot(1-\eta_{m,i^B}+\eta_{b,i^B})+\left(\frac{\overline{dc}\cdot\overline{v}}{i^B}-1\right)(-\eta_{e,i^B}+\eta_{m,i^B})}{\frac{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}}{-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}}+\frac{\widehat{K}}{B_P}\cdot(1-\eta_{m,i^E}+\eta_{e,i^E})}$$

$$\begin{cases} >0 & \text{if } \ \frac{\widehat{K}\cdot(\overline{v}-\overline{dc})}{E\cdot i^E}+\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot(1-\eta_{m,i^B}+\eta_{b,i^B})-\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\eta_{e,i^B}-\eta_{m,i^B} > \\ & \qquad\qquad -\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\eta_{m,i^B}-\eta_{e,i^B} \\ \leq 0 & \text{else} \end{cases}$$

(172)

$$\frac{dn^F}{d\widehat{n^B_{CB}}} = \frac{n^F_P}{n^B_P} \cdot \frac{1}{(1+\frac{n^F_P \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}})} \cdot \left(-\frac{n^B_P \cdot \overline{q^B} \cdot \overline{v}}{i^B \cdot s \cdot \overline{a} \cdot n^{CG} \cdot \widehat{p^*}} + \frac{\alpha+\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot(\delta+\gamma)}{\omega}\right)$$

$$\begin{cases} < 0 & \text{if } \frac{n^B_P \cdot \overline{q^B} \cdot \overline{v}}{i^B \cdot s \cdot \overline{a} \cdot n^{CG} \cdot \widehat{p^*}} - \frac{\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\gamma}{\omega} > \frac{\alpha+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\delta}{\omega} \\ \geq 0 & \text{else} \end{cases}$$

$$\begin{aligned}
\alpha :=\ & -\eta_{f,i^B} + \eta_{m,i^E}\cdot\eta_{f,i^B} + \eta_{e,i^B}\cdot\eta_{f,i^E} - \eta_{e,i^E}\cdot\eta_{f,i^B} \\
& + \tfrac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(-\eta_{m,i^E} + \eta_{m,i^B}\cdot\eta_{b,i^E} - \eta_{m,i^E}\cdot\eta_{b,i^B} + \eta_{m,i^E}\cdot\eta_{f,i^B} - \tfrac{\widehat{K}}{B_P}\cdot\eta_{m,i^E}\right) \\
\beta :=\ & +\eta_{m,i^B} + \eta_{m,i^B}\cdot\eta_{e,i^E} - \eta_{m,i^B}\cdot\eta_{f,i^E} - \eta_{m,i^E}\cdot\eta_{e,i^B} \\
& + \tfrac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(+\eta_{f,i^E} - \eta_{m,i^B}\cdot\eta_{f,i^E} + \eta_{b,i^B}\cdot\eta_{f,i^E} - \eta_{b,i^E}\cdot\eta_{f,i^B} + \tfrac{\widehat{K}}{B_P}\cdot\eta_{f,i^E}\right) \\
\delta :=\ & -\eta_{m,i^B} - \eta_{m,i^B}\cdot\eta_{e,i^E} + \eta_{m,i^B}\cdot\eta_{f,i^E} + \eta_{m,i^E}\cdot\eta_{e,i^B} \\
\gamma :=\ & +\eta_{f,i^B} - \eta_{m,i^E}\cdot\eta_{f,i^B} - \eta_{e,i^B}\cdot\eta_{f,i^E} + \eta_{e,i^E}\cdot\eta_{f,i^B} \\
\omega :=\ & \tfrac{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}}{-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}} + \tfrac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)
\end{aligned}$$

(173)

$$\frac{dn^B}{d\widehat{n^B_{CB}}} = \frac{\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\alpha + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}+\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(-\eta_{m,i^E}+\eta_{b,i^E}\right)\right)}{\alpha + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}\right)}$$

$$\begin{cases} > 0 & \text{if } \frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\alpha + \frac{\widehat{K}}{B_P}\cdot\left(1-\eta_{m,i^E}+\eta_{e,i^E}-\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\eta_{m,i^E}\right) > \\ & \qquad -\frac{\widehat{K}}{B_P}\cdot\frac{b\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\eta_{b,i^E} \\ \leq 0 & \text{else} \end{cases}$$

$$\alpha := \begin{array}{c} 1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^B}-\eta_{m,i^B}\cdot\eta_{e,i^E} \\ -\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B} \end{array}$$

(174)

$$\frac{ds}{d\widehat{n^B_{CB}}} = \frac{\overline{q^B}}{i^B\cdot n^{CG}\cdot\widehat{p^*}} \cdot \frac{1}{(1+\frac{n^F_P\cdot\overline{q^F}}{\widehat{i^F}\cdot n^{CG}\cdot\widehat{p^*}})} \cdot \left(\frac{\overline{v}}{\overline{a}} + \frac{f}{b}\cdot\frac{\alpha+\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot(\delta+\gamma)}{\omega}\right)$$

$$\begin{cases} > 0 & \text{if } \frac{\overline{v}}{\overline{a}} + \frac{f}{b}\cdot\left(\frac{\alpha+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\delta}{\omega}\right) > -\frac{f}{b}\cdot\left(\frac{\beta+\frac{\overline{dc}\cdot\overline{v}}{i^B}\cdot\gamma}{\omega}\right) \\ \leq 0 & \text{else} \end{cases}$$

$$\alpha := \begin{array}{c} -\eta_{f,i^B} + \eta_{m,i^E} \cdot \eta_{f,i^B} + \eta_{e,i^B} \cdot \eta_{f,i^E} - \eta_{e,i^E} \cdot \eta_{f,i^B} \\ + \frac{b \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(-\eta_{m,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{f,i^B} - \frac{\widehat{K}}{B_P} \cdot \eta_{m,i^E} \right) \end{array}$$

$$\beta := \begin{array}{c} +\eta_{m,i^B} + \eta_{m,i^B} \cdot \eta_{e,i^E} - \eta_{m,i^B} \cdot \eta_{f,i^E} - \eta_{m,i^E} \cdot \eta_{e,i^B} \\ + \frac{b \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(+\eta_{f,i^E} - \eta_{m,i^B} \cdot \eta_{f,i^E} + \eta_{b,i^B} \cdot \eta_{f,i^E} - \eta_{b,i^E} \cdot \eta_{f,i^B} + \frac{\widehat{K}}{B_P} \cdot \eta_{f,i^E} \right) \end{array}$$

$$\delta := -\eta_{m,i^B} - \eta_{m,i^B} \cdot \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{f,i^E} + \eta_{m,i^E} \cdot \eta_{e,i^B}$$

$$\gamma := +\eta_{f,i^B} - \eta_{m,i^E} \cdot \eta_{f,i^B} - \eta_{e,i^B} \cdot \eta_{f,i^E} + \eta_{e,i^E} \cdot \eta_{f,i^B}$$

$$\omega := \begin{array}{c} 1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} \\ - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B} \end{array} + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E} \right)$$

Impact of Changes in the Amount of Central Bank's Foreign Assets

(175)

$$\frac{di^B}{\widehat{dn^F_{CB}}} = \frac{i^B}{n^F_P} \cdot \frac{\frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E} \right) + \frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(-\eta_{b,i^E} + \eta_{m,i^E} \right)}{\begin{array}{c} 1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} \\ - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B} \end{array} + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E} \right)}$$

$$\begin{cases} > 0 & \text{if} \quad \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E} \right) - \frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \eta_{b,i^E} > - \frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \eta_{m,i^E} \\ \leq 0 & \text{else} \end{cases}$$

(176)

$$\frac{di^E}{\widehat{dn^F_{CB}}} = \frac{i^E}{n^F_P} \cdot \frac{\frac{f \cdot \widehat{K} \cdot (\overline{v} - \overline{dc})}{b \cdot E \cdot i^E} + \frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(1 - \eta_{m,i^B} + \eta_{b,i^B} \right) + \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \left(-\eta_{e,i^B} + \eta_{m,i^B} \right)}{\begin{array}{c} 1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} \\ - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B} \end{array} + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E} \right)}$$

$$\begin{cases} > 0 & \text{if} \quad \frac{f \cdot \widehat{K} \cdot (\overline{v} - \overline{dc})}{b \cdot E \cdot i^E} + \frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(1 - \eta_{m,i^B} + \eta_{b,i^B} \right) - \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \eta_{e,i^B} > - \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \eta_{m,i^B} \\ \leq 0 & \text{else} \end{cases}$$

(177)

$$\frac{dn^F}{\widehat{dn^F_{CB}}} = \frac{1}{\left(1 + \frac{n^F_P \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}} \right)} \cdot \left(1 - \frac{n^F_P \cdot \overline{q^F} \cdot \overline{v}}{\widehat{i^F} \cdot \overline{a} \cdot n^{CG} \cdot \widehat{p^*}} + \frac{\frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(\alpha + \beta + \frac{\widehat{K}}{B_P} \cdot \left(-\eta_{m,i^E} + \eta_{f,i^E} \right) \right) + \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \left(\delta + \gamma \right)}{\omega} \right)$$

$$\begin{cases} > 0 & \text{if} \quad 1 + \frac{\frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(\alpha - \frac{\widehat{K}}{B_P} \cdot \eta_{m,i^E} \right) + \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \delta}{\omega} > \\ & \qquad \frac{n^F_P \cdot \overline{q^F} \cdot \overline{v}}{\widehat{i^F} \cdot \overline{a} \cdot n^{CG} \cdot \widehat{p^*}} - \frac{\frac{f \cdot (\overline{v} - \overline{dc})}{e \cdot i^E} \cdot \left(\beta + \frac{\widehat{K}}{B_P} \cdot \eta_{f,i^E} \right) + \frac{f \cdot \overline{dc} \cdot \overline{v}}{b \cdot i^B} \cdot \gamma}{\omega} \\ \leq 0 & \text{else} \end{cases}$$

$$\alpha := -\eta_{m,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{f,i^B}$$

$$\beta := +\eta_{f,i^E} - \eta_{m,i^B} \cdot \eta_{f,i^E} + \eta_{b,i^B} \cdot \eta_{f,i^E} - \eta_{b,i^E} \cdot \eta_{f,i^B}$$

$$\delta := -\eta_{m,i^B} - \eta_{m,i^B} \cdot \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{f,i^E} + \eta_{m,i^E} \cdot \eta_{e,i^B}$$

$$\gamma := +\eta_{f,i^B} - \eta_{m,i^E} \cdot \eta_{f,i^B} - \eta_{e,i^B} \cdot \eta_{f,i^E} + \eta_{e,i^E} \cdot \eta_{f,i^B}$$

$$\omega := \frac{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}}{-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}} + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E}\right)$$

(178)
$$\frac{dn^B}{d\widehat{n^F_{CB}}} = \frac{n^B_P}{n^F_P} \cdot \frac{\frac{f\cdot\overline{dc}\cdot\overline{v}}{b\cdot i^B} \cdot \alpha + \frac{\widehat{K}}{B_P} \cdot \frac{f\cdot(\overline{v}-\overline{dc})}{e\cdot i^E} \cdot \left(-\eta_{m,i^E} + \eta_{b,i^E}\right)}{\alpha + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E}\right)}$$

$$\begin{cases} > 0 & \text{if} \quad \frac{f\cdot\overline{dc}\cdot\overline{v}}{b\cdot i^B} \cdot \alpha - \frac{\widehat{K}}{B_P} \cdot \frac{f\cdot(\overline{v}-\overline{dc})}{e\cdot i^E} \cdot \eta_{m,i^E} > -\frac{\widehat{K}}{B_P} \cdot \frac{f\cdot(\overline{v}-\overline{dc})}{e\cdot i^E} \cdot \eta_{b,i^E} \\ \leq 0 & \text{else} \end{cases}$$

$$\alpha := \frac{1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E}}{-\eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B}}$$

(179)
$$\frac{ds}{d\widehat{n^F_{CB}}} = \frac{s \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}} \cdot \frac{1}{\left(1 + \frac{n^F_P \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}}\right)} \cdot \left(1 + \frac{\overline{v}}{\overline{a}} + \frac{\frac{f\cdot(\overline{v}-\overline{dc})}{e\cdot i^E} \cdot \left(\alpha + \beta + \frac{\widehat{K}}{B_P} \cdot \left(-\eta_{m,i^E} + \eta_{f,i^E}\right)\right) + \frac{f\cdot\overline{dc}\cdot\overline{v}}{b\cdot i^B} \cdot \left(\delta + \gamma\right)}{\omega}\right)$$

$$\begin{cases} > 0 & \text{if} \quad 1 + \frac{\overline{v}}{\overline{a}} + \frac{\frac{f\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(\alpha - \frac{\widehat{K}}{B_P}\cdot\eta_{m,i^E}\right) + \frac{f\cdot\overline{dc}\cdot\overline{v}}{b\cdot i^B}\cdot\delta}{\omega} > -\frac{\frac{f\cdot(\overline{v}-\overline{dc})}{e\cdot i^E}\cdot\left(\beta + \frac{\widehat{K}}{B_P}\cdot\eta_{f,i^E}\right) + \frac{f\cdot\overline{dc}\cdot\overline{v}}{b\cdot i^B}\cdot\gamma}{\omega} \\ \leq 0 & \text{else} \end{cases}$$

$$\alpha := -\eta_{m,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{f,i^B}$$

$$\beta := +\eta_{f,i^E} - \eta_{m,i^B} \cdot \eta_{f,i^E} + \eta_{b,i^B} \cdot \eta_{f,i^E} - \eta_{b,i^E} \cdot \eta_{f,i^B}$$

$$\delta := -\eta_{m,i^B} - \eta_{m,i^B} \cdot \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{f,i^E} + \eta_{m,i^E} \cdot \eta_{e,i^B}$$

$$\gamma := +\eta_{f,i^B} - \eta_{m,i^E} \cdot \eta_{f,i^B} - \eta_{e,i^B} \cdot \eta_{f,i^E} + \eta_{e,i^E} \cdot \eta_{f,i^B}$$

$$\omega := \frac{1-\eta_{m,i^B}+\eta_{b,i^B}-\eta_{m,i^E}+\eta_{e,i^E}+\eta_{m,i^B}\cdot\eta_{b,i^E}-\eta_{m,i^B}\cdot\eta_{e,i^E}}{-\eta_{m,i^E}\cdot\eta_{b,i^B}+\eta_{m,i^E}\cdot\eta_{e,i^B}+\eta_{b,i^B}\cdot\eta_{e,i^E}-\eta_{b,i^E}\cdot\eta_{e,i^B}} + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E}\right)$$

Stability Test

General Procedure

Subsequently, it is demonstrated that both the short term and the long term system exhibit true dynamic stability. The proof is based on Metzler (1945), who shows that Hicksian perfect stability is a necessary and sufficient condition for true dynamic stability provided that the portfolio assets are gross substitutes, as is the case within the current model.

To test for Hicksian perfect stability, the matrices of both the short term and the long term system are transposed into Hicksian matrices, which are also called NP-matrices within contemporary mathematics (Hands, 2010). Afterwards, the n principal minors of the Hicksian matrices are solved. If they alternate in sign like $(-1)^n$, the systems exhibit Hicksian perfect stability.

Short Term System

The short term system, transposed into a Hicksian matrix, is:

$$\underbrace{\begin{bmatrix} -\frac{\partial (n_P^B)^d}{\partial i^B} & -\frac{\partial (n_P^B)^d}{\partial i^E} & \frac{\partial (n_P^B)^d}{\partial s} \\ -\frac{\partial Div^d}{\partial i^B} & -\frac{\partial Div^d}{\partial i^E} & \frac{\partial Div^d}{\partial s} \\ -\frac{\partial (n_P^F)^d}{\partial i^B} & -\frac{\partial (n_P^F)^d}{\partial i^E} & \frac{\partial (n_P^F)^d}{\partial s} \end{bmatrix}}_{=H_{st}} \cdot \begin{bmatrix} -di^B \\ -di^E \\ ds \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \tag{180}$$

The first principal minor of H_{st} is:

$$-\frac{\partial (n_P^B)^d}{\partial i^B} = -\frac{1}{\overline{q^B}} \cdot \left(\frac{\partial b}{\partial i^B} \cdot W \cdot i^B + B_P \cdot (1-b)\right) \tag{181}$$
$$< 0$$

The second principal minor of H_{st} is:

$$det \begin{bmatrix} -\frac{\partial (n_P^B)^d}{\partial i^B} & -\frac{\partial (n_P^B)^d}{\partial i^E} \\ -\frac{\partial Div^d}{\partial i^B} & -\frac{\partial Div^d}{\partial i^E} \end{bmatrix} = \frac{n_P^B \cdot Div}{i^B \cdot i^E} \cdot \left[m \cdot \begin{pmatrix} 1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} \\ + \frac{1}{b} \cdot (-\eta_{m,i^B} \cdot \eta_{e,i^E} + \eta_{m,i^E} \cdot \eta_{e,i^B}) \end{pmatrix} \right] \tag{182}$$

$$+ f \cdot \begin{pmatrix} 1 + \eta_{b,i^B} - \eta_{f,i^B} + \eta_{e,i^E} - \eta_{f,i^E} \\ + \frac{1}{b} \cdot (\eta_{e,i^B} \cdot \eta_{f,i^E} - \eta_{e,i^E} \cdot \eta_{f,i^B}) \end{pmatrix}]$$

$$> 0$$

The third principal minor of H_{st} is:

$$(183) \quad detH_{st} = -\frac{m \cdot n_P^B \cdot Div \cdot n_P^F}{i^B \cdot i^E \cdot s} \cdot (1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} - \eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B})$$

$$< 0$$

Long Term System

The long term system, transposed into a Hicksian matrix, is:

$$(184) \quad \underbrace{\begin{bmatrix} -\frac{\partial (n_P^B)^d}{\partial i^B} - (\frac{\partial (n_P^B)^d}{\partial n_P^B} - 1) \cdot \frac{\partial (n^B)^s}{\partial i^B} & -\frac{\partial (n_P^B)^d}{\partial i^E} & \frac{\partial (n_P^B)^d}{\partial n_P^F} + \frac{\partial (n_P^B)^d}{\partial s} \cdot \frac{\partial s}{\partial n^F} \\ -\frac{\partial Div^d}{\partial i^B} - \frac{\partial Div^d}{\partial n_P^B} \cdot \frac{\partial (n^B)^s}{\partial i^B} & -\frac{\partial Div^d}{\partial i^E} & \frac{\partial Div^d}{\partial n_P^F} + \frac{\partial Div^d}{\partial s} \cdot \frac{\partial s}{\partial n^F} \\ -\frac{\partial (n_P^F)^d}{\partial i^B} - \frac{\partial (n_P^F)^d}{\partial n_P^B} \cdot \frac{\partial (n^B)^s}{\partial i^B} & -\frac{\partial (n_P^F)^d}{\partial i^E} & \frac{\partial (n_P^F)^d}{\partial n_P^F} - 1 + \frac{\partial (n_P^F)^d}{\partial s} \cdot \frac{\partial s}{\partial n^F} \end{bmatrix}}_{=H_{lt}} \cdot \begin{bmatrix} -di^B \\ -di^E \\ dn^F \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

The first principal minor of H_{lt} is:

$$(185) \quad -\frac{\partial (n_P^B)^d}{\partial i^B} - (\frac{\partial (n_P^B)^d}{\partial n_P^B} - 1) \cdot \frac{\partial (n^B)^s}{\partial i^B} = -\frac{1}{\overline{q^B}} \cdot (\frac{\partial b}{\partial i^B} \cdot W \cdot i^B + (B_P + \widehat{K}) \cdot (1 - b))$$

$$< 0$$

The second principal minor of H_{lt} is:

$$
\begin{aligned}
(186)\quad det & \begin{bmatrix} -\frac{\partial (n_P^B)^d}{\partial i^B} - \left(\frac{\partial (n_P^B)^d}{\partial n_P^B} - 1\right) \cdot \frac{\partial (n^B)^s}{\partial i^B} & -\frac{\partial (n_P^B)^d}{\partial i^E} \\ -\frac{\partial Div^d}{\partial i^B} - \frac{\partial Div^d}{\partial n_P^B} \cdot \frac{\partial (n^B)^s}{\partial i^B} & -\frac{\partial Div^d}{\partial i^E} \end{bmatrix} \\
& = \frac{n_P^B \cdot Div}{i^B \cdot i^E} \cdot \Big[m \cdot \begin{pmatrix} -\eta_{m,i^B} + \eta_{b,i^B} + \left(1 + \frac{\widehat{K}}{B_P}\right) \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E}\right) \\ + \frac{1}{b} \cdot \left(-\eta_{m,i^B} \cdot \eta_{e,i^E} + \eta_{m,i^E} \cdot \eta_{e,i^B}\right) \end{pmatrix} \\
& \qquad + f \cdot \begin{pmatrix} \eta_{b,i^B} - \eta_{f,i^B} + \left(1 + \frac{\widehat{K}}{B_P}\right) \cdot \left(1 + \eta_{e,i^E} - \eta_{f,i^E}\right) \\ + \frac{1}{b} \cdot \left(\eta_{e,i^B} \cdot \eta_{f,i^E} - \eta_{e,i^E} \cdot \eta_{f,i^B}\right) \end{pmatrix} \Big] \\
& > 0
\end{aligned}
$$

The third principal minor of H_{lt} is:

$$
\begin{aligned}
(187)\qquad detH_{lt} = & -\frac{m \cdot n_P^B \cdot Div}{i^B \cdot i^E} \cdot \left(1 + \frac{n_P^F \cdot \overline{q^F}}{\widehat{i^F} \cdot n^{CG} \cdot \widehat{p^*}}\right) \\
& \cdot \Big[\begin{matrix} 1 - \eta_{m,i^B} + \eta_{b,i^B} - \eta_{m,i^E} + \eta_{e,i^E} + \eta_{m,i^B} \cdot \eta_{b,i^E} - \eta_{m,i^B} \cdot \eta_{e,i^E} \\ -\eta_{m,i^E} \cdot \eta_{b,i^B} + \eta_{m,i^E} \cdot \eta_{e,i^B} + \eta_{b,i^B} \cdot \eta_{e,i^E} - \eta_{b,i^E} \cdot \eta_{e,i^B} \\ + \frac{\widehat{K}}{B_P} \cdot \left(1 - \eta_{m,i^E} + \eta_{e,i^E}\right) \Big] \end{matrix} \\
& < 0
\end{aligned}
$$

Summary Stability Test

The $n = 3$ principal minors of H_{st} and H_{lt} alternate in sign in the way of $(-1)^n$. Thus, both the short term system and the long term system exhibit Hicksian perfect stability. Consequently, both systems exhibit true dynamic stability due to the fact that the portfolio assets are gross substitutes (Metzler, 1945).