# Competition and Collaboration in Aid-for-Policy Deals: Online Appendix

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December 2, 2015

**Abstract**

Despite the extensive empirical and theoretical research into foreign aid, there is little or no formal analysis of aid giving in a competitive donor environment. We endeavor to fill this lacuna with both a model and empirical analysis of aid-for-policy deals with rival aid donors. The model indicates that a dominant donor captures all the surplus from any deal. We test several hypotheses that follow from the model, demonstrating that the US paid less (in constant dollars) and gained more in policy terms through aid before the Soviet Union became a significant aid player and that once the Soviet's were in the aid picture, the US paid more for aid and got less by way of security concessions from recipients.

# 1 Appendix

**Proof of lemma 2.** From lemma 1 leader T accepts any offer that gives him higher utility than rejecting all aid offers or accepting any other offer. Let $V = \max\{v_T(y_B, r_B), v_T(x_T, 0)\}$. Suppose $v_T(y_A, r_A) > V$. A can reduce its aid transfer by some $\delta > 0$ and make T indifferent between A's offer and an alternative: $v_T(y_A, r_A - \delta) = V$. However, if A proposes the aid-for-policy deal $(y_A, r_A - \frac{\delta}{2})$, then T strictly accepts A's new offer since $v_T(y_A, r_A - \frac{\delta}{2}) > V$ and this new offer improves A's coalition welfare: $v_A(y_A, r_A - \frac{\delta}{2}) > v_T(y_A, r_A)$. Therefore $(y_A, r_A)$ such that $v_T(y_A, r_A) > V$ can not be part of an equilibrium strategy profile. ■

**Proof of proposition 1.** The proof proceeds by standard constrained maximization techniques. From the program in equation 1, the Lagrangian equation is $L = v_A(y, r) + \lambda(v_T(y, r) - v_T(x_T, 0))$. Differentiation yields the following first order conditions $L_y = \frac{dL}{dy} = -2\sigma_A(y - x_A) - \lambda 2\sigma_T(y - x_T) = 0$, $L_r = -\frac{1}{W_A}u'(\frac{R_A - r - k}{W_A}) + \lambda\frac{1}{W_T}u'(\frac{R_T + r}{W_T}) = 0$, $L_\lambda = v_T(y, r) - v_T(x_T, 0) = 0$ and second order conditions: $L_{yy} = \frac{d^2L}{dy^2} = -2\sigma_A - \lambda 2\sigma_T < 0$, $L_{yr} = \frac{d^2L}{dydr} = 0$, $L_{y\lambda} = \frac{d^2L}{dyd\lambda} = -2\sigma_T(y - x_T) > 0$, $L_{rr} = \frac{d^2L}{dr^2} = \frac{1}{W_A^2}u''(\frac{R_A - r - k}{W_A}) + \lambda\frac{1}{W_T^2}u''(\frac{R_T + r}{W_T}) < 0$, $L_{r\lambda} = \frac{d^2L}{drd\lambda} = \frac{1}{W_T}u'(\frac{R_T + r}{W_T}) > 0$ and $L_{\lambda\lambda} = \frac{d^2L}{d\lambda^2} = 0$ Since $L_{rr}$ and $L_{yy}$ are negative and the bordered Hessian matrix $\begin{vmatrix} L_{yy} & 0 & L_{y\lambda} \\ 0 & L_{rr} & L_{r\lambda} \\ L_{y\lambda} & L_{r\lambda} & 0 \end{vmatrix} = -L_{rr}L_{y\lambda}^2 - L_{yy}L_{r\lambda}^2 > 0$, the program is concave and the first order conditions correspond to a unique maximum. Rearranging the FOCs yields $\frac{\sigma_A(y - x_A)}{\sigma_T(x_T - y)} = \lambda = \frac{W_T}{W_A}\frac{u'(\frac{R_A - r}{W_A})}{u'(\frac{R_T + r}{W_T})} > 0$ and $v_T(y, r) - v_T(x_T, 0) = u(\frac{R_T + r}{W_T}) - u(\frac{R_T}{W_T}) - \sigma_T(y - x_T)^2 = 0$, which lead to equations 2 and 3. ■

**Proof of proposition 2.** We proceed as follows: we rule out characteristics of strategy profiles that cannot be part of an equilibrium. We then show that the strategies characterized above are mutual best responses.

1) First suppose leader B offers the deal $(y_B, r_B)$ such that $v_T(y_B, r_B) \leq v_T(x_T, 0)$. In this setting, A's best response is identical to that described in the proof of proposition 1 and such a response would make T indifferent between the deal with A and implementing $x_T$ and receiving no aid. There exists a sufficiently small $\delta > 0$ such that $(y_B, r_B) = (y_B^\dagger, r_B^\dagger + \delta)$ where $(y_B^\dagger, r_B^\dagger)$ are the policies characterized in proposition 1, T strictly prefers to accept this offer and $v_B(y_B^\dagger, r_B^\dagger + \delta) > v_B(x_T, 0) > v_B(y_A^\dagger, 0)$. Hence the conjectured policies $(y_B, r_B)$ cannot be part of an equilibrium. Therefore, in any equilibria, $v_T(y_B, r_B) > v_T(x_T, 0)$ and (by repeating the argument for A) $v_T(y_A, r_A) > v_T(x_T, 0)$.

2) By lemma 2, in equilibrium $v_T(y_A, r_A) = v_T(y_B, r_B)$.

3) Next we show that one leader must be exhausted: suppose $v_T(y_A, r_A) = v_T(y_B, r_B) > v_T(x_T, 0)$ and given this indifference T accepts A's offer with probability $\alpha$ and accepts B's offer with probability $\beta = 1 - \alpha$. Suppose that neither leader is exhausted. If $\alpha < 1$ and $v_A(y_A, r_A) > v_A(y_B, 0)$ (that is A is not exhausted), then there exists some $\delta > 0$ such that $v_A(y_A, r_A) > v_A(y_A, r_A + \delta) > v_A(y_B, 0)$ and T strictly prefers to accept $(y_A, r_A + \delta)$ rather than $(y_B, r_B)$. Leader A prefers to offer $\delta$ more to ensure her offer is accepted. Similarly, suppose $\beta < 1$ and $v_B(y_B, r_B) > v_B(y_A, 0)$. Then there exists some $\zeta > 0$ such that $v_B(y_B, r_B) > v_B(y_B, r_B + \zeta) > v_B(y_A, 0)$ and T strictly prefers to accept $(y_B, r_B + \zeta)$ rather than $(y_A, r_A)$. Leader B prefers to offer $\zeta$ more to ensure her offer is accepted. Hence if neither leader is exhausted (and $\alpha$ and $\beta$ cannot both be 1), then at least one leader wants to increase her offer by some infinitesimal $\delta$ or $\zeta$. Hence, at least one leader must be exhausted in equilibrium.

4) Suppose both leaders are strictly exhausted: $v_T(y_A, r_A) = v_T(y_B, r_B) > v_T(x_T, 0)$ and $v_A(y_A, r_A) < v_A(y_B, 0)$ and $v_B(y_B, r_B) < v_B(y_A, 0)$. Given T's indifference between A and B's offers, suppose T accepts A's offer with probability $\alpha$ and accepts B's offer with probability $\beta = 1 - \alpha$. If $\alpha > 0$, then A strictly prefers to offer $(x_A, 0)$ which is rejected by T in favor of $(y_B, r_B)$. But this improves A's payoff because A is strictly exhausted. If $\beta > 0$ then B strictly prefers to offer $(x_B, 0)$ which is rejected by T in favor of $(y_A, r_A)$. But this improves B's payoff because B is strictly exhausted. Therefore, both A and B cannot be strictly exhausted.

5) We now characterize the equilibrium conditions, focusing on the case where B is exhausted (the analysis is analogous if A is exhausted) and show that no player can improve its payoff from those described by the proposition. Given the results above, any strategy that is a candidate for an equilibrium has the properties that $v_T(y_A, r_A) = v_T(y_B, r_B) > v_T(x_T, 0)$ and (at least) one leader is exhausted. Suppose that leader B is exhausted. A picks policies such that $(y_A, r_A) = \arg\max_{(y_A, r_A)} v_A(y_A, r_A)$

subject to $v_T(y_A, r_A) \geq v_T(y_B, r_B)$. This program can be solved by standard constrained maximization techniques. In particular, we form a Lagrangian equation $L = -\sigma_A(y_A - x_A)^2 + u(\frac{R_A - r_A}{W_A}) + \lambda(-\sigma_T(y_A - x_T)^2 + u(\frac{R_T + r_A}{W_T}) + \sigma_T(y_B - x_T)^2 - u(\frac{R_T + r_B}{W_T}))$, where $\lambda$ is the Lagrangian multiplier. Since $L_{y_A} = -2\sigma_A(y_A - x_A) + \lambda(-2\sigma_T(y_A - x_T)) = 0$, $L_{r_A} = -\frac{1}{W_A}u'(\frac{R_A - r_A}{W_A}) + \lambda(\frac{1}{W_T}u'(\frac{R_T + r_A}{W_T})) = 0$, $L_\lambda = v_T(y_A, r_A) - v_T(y_B, r_B) = 0$ this implies $\frac{W_T}{W_A}\frac{u'(\frac{R_A - r_A}{W_A})}{u'(\frac{R_T + r_A}{W_T})} = \lambda = \frac{\sigma_A(y_A - x_A)}{\sigma_T(y_A - x_T)}$(equation 4) and $v_T(y_A, r_A) = v_T(y_B, r_B)$ (equation 6); the second order conditions are identical to proposition 1.

There are no deviations that improve A's payoff. If A offers more to T, then she reduces her payoff. If she offer less then T accepts B's offer, and by non-exhaustion of A, this makes A worse off.

Next consider B's optimal aid-for-policy offer $(y_B, r_B) = \arg\max_{(y_B, r_B)} v_B(y_B, r_B)$ subject to $v_T(y_B, r_B) \geq v_T(y_A, r_A)$. This program is analogous to that for nation A above and so satisfies equation 5. On the equilibrium path, B's payoff is $v_B(y_A^\ddagger, 0)$. By exhaustion, any offer that is acceptable to T makes B worse off since for all $(y_B, r_B)$ such that $v_T(y_B, r_B) \geq v_T(y_A^\ddagger, r_A^\ddagger)$, $v_B(y_B, r_B) \leq v_B(y_A^\ddagger, 0)$. If B offers $(y_B, r_B)$ such that $v_T(y_B, r_B) \leq v_T(y_A^\ddagger, r_A^\ddagger)$, then T accepts A's offer so such alternative strategies are not utility improving. As per lemma **??**, T accepts any strictly preferred policy. On the equilibrium path, T accepts the offer of the non-exhausted leader, which is a best response since T is indifferent. ■

**The uniqueness of the equilibrium characterized in proposition 2**

The equilibrium is not unique as there are multiple equilibria where B is exhausted and makes offers that, if accepted, would make her strictly worse off: $v_B(y_B, r_B) < v_B(y_A^\ddagger, 0)$ and $v_A(y_A^\ddagger, r_A^\ddagger) \geq v_A(y_B, 0)$. However these equilibria rely on B making non-credible offers that she would never want to implement. This is to say, by offering more than she is willing to pay, leader B might force leader A to bid more. However, we restrict attention to cases where the losing state's bid must be credible (i.e. its leader would want to make the trade if its proposed deal were accepted). In this case the competitive bidding equilibrium is unique (except for the degenerate case when both nations are simultaneously exhausted).

**Proof of proposition 3.** From lemma 2, A and C never spend more than is necessary. Therefore either $(v_T(y_A, r_A) = v_T(x_T, 0)$ and $v_T(y_C, r_C) \leq v_T(x_T, 0))$ or $(v_T(y_A, r_A) \leq v_T(x_T, 0)$ and $v_T(y_C, r_C) = v_T(x_T, 0))$. From the constrained maximization in proposition 1 any bid that is accepted must therefore be either $(y_A^\dagger, r_A^\dagger)$ or $(y_C^\dagger, r_C^\dagger)$. Hence A and C offer either of these deals or one that is unacceptable.

Dominant Buyer of Policy: If $v_A(y_A^\dagger, r_A^\dagger) > v_A(y_C^\dagger, 0)$, then on the equilibrium path A offers $(y_A, r_A) = (y_A^\dagger, r_A^\dagger)$ which is accepted. T is indifferent between accepting A's offer and rejecting and so accepting is a best response. $v_A(y_A^\dagger, r_A^\dagger) > v_A(y_C^\dagger, 0)$ implies that $y_A^\dagger < y_C^\dagger$ so C strictly prefers that A buys policy (since it buys more) than having C buy policy. Therefore C's best response is to make an unacceptable offer. Given that C makes an unacceptable offer, A's program is identical to that characterized in the single bidder case.

Free-rider: $v_A(y_C^\dagger, 0) \geq v_A(y_A^\dagger, r_A^\dagger) > v_A(x_T, 0)$ and $v_C(y_A^\dagger, 0) \geq v_C(y_C^\dagger, r_C^\dagger) > v_A(x_T, 0)$. If C's bid is unacceptable, then A's best response is $v_A(y_A^\dagger, r_A^\dagger)$. If C's bid is $(y_C^\dagger, r_C^\dagger)$, then A's best response is to make an unacceptable offer. Hence the first two strategy profiles are equilibria.

Next we examine the mixed strategy. With probability $\rho_A$ A offers $(y_A, r_A) = (y_A^\dagger, r_A^\dagger)$ and with probability $(1 - \rho_A)$ A offers $(y_A, r_A) \in U_T$. With probability $\rho_C$ A offers $(y_C, r_C) = (y_C^\dagger, r_C^\dagger)$ and with probability $(1 - \rho_C)$ A offers $(y_C, r_C) \in U_T$. If $(y_C, r_C) \in U_T$ and $v_T(y_A, r_A) \geq v_T(x_T, 0)$, then T accepts $(y_A, r_A)$; if $(y_A, r_A) \in U_T$ and $v_T(y_C, r_C) \geq v_T(x_T, 0)$, then T accepts $(y_C, r_C)$; if $v_T(y_C, r_C) > v_T(y_A, r_A) \geq v_T(x_T, 0)$ then T accepts $(y_C, r_C)$; if $v_T(y_A, r_A) > v_T(y_C, r_C) \geq v_T(x_T, 0)$, then T accepts $(y_A, r_A)$; if $v_T(y_A, r_A) = v_T(y_C, r_C) \geq v_T(x_T, 0)$ then T accepts $(y_A, r_A)$ with probability $\alpha$ and T accepts $(y_C, r_C)$ with probability $(1 - \alpha)$, where $\alpha \in (0, 1)$, $\rho_A = \frac{v_C(y_C^\dagger, r_C^\dagger) - v_C(x_T, 0)}{v_C(y_A^\dagger, 0) - v_C(x_T, 0) + \alpha(v_C(y_C^\dagger, r_C^\dagger) - v_C(y_A^\dagger, 0))}$ and $\rho_C = \frac{v_A(y_C^\dagger, 0) - v_A(x_T, 0)}{v_A(y_A^\dagger, r_A^\dagger) - v_A(x_T, 0) + \alpha(v_A(y_C^\dagger, 0) - v_A(y_A^\dagger, r_A^\dagger))}$.

Given $\alpha$ and $\rho_C$, A is indifferent between offering $(y_A^\dagger, r_A^\dagger)$ and an unacceptable deal and so randomizing is a best response for A. C's best responses are analogous. ■

**Proof of proposition 5.** M's deals are characterized by M's commission. To ensure that her deal is accepted, leader B must offer $(y_B, r_B)$ that T accepts and exhausts M. Hence leader B's program is $\max_{(y_B, r_B)} v_B(y_B, r_B)$ subject to $v_T(y_B, r_B) \geq v_T(y_M, \mu)$. This constrained maximization problem is similar to those examined earlier. It yields the first order condition (equation 9) and indifference condition (equation 10). The deal $(y_B^\diamond, r_B^\diamond)$ is thus B's best deal that outbids M. If $v_B(y_B^\diamond, r_B^\diamond) > v_B(y_M, 0)$, then B prefers to outbid M and does so with the minimal possible offer that T will accept, $(y_B^\diamond, r_B^\diamond)$. If

$v_B(y_B^\diamond, r_B^\diamond) \leq v_B(y_M, 0)$, then B prefers the policy position $y_M$ rather than making an offer that T accepts. Hence making an offer that T rejects is a best response. ■