2025

The fundamental equation of the physics theory of time travel and cosmology

The physics theory of time travel

It is important to understand that time travel is enabled by the physical system of hyperspace and normal space, which manifests in reality as the cosmological expansion of the universe.

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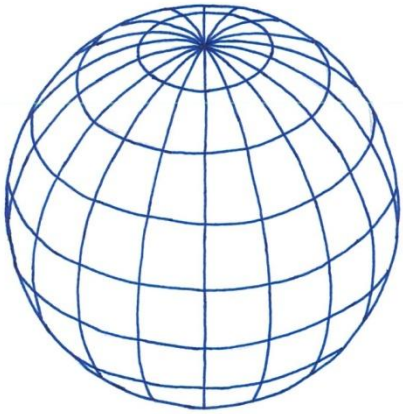
2025



Company: MLK Technology and Science Ltd

Date and location: February 2025, Tallinn, Estonia (EE), European Union (EU).

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NOTE: This is the second part, but the first version (1)(3).

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General introduction

Marek-Lars Kruusen's time travel physics theory and technology is extremely voluminous, containing a lot of details (1)(3). This paper presents only a small part of this theory. Since this is the third part of the series, the following parts will present more comprehensive and detailed reviews of the physics of time travel (1)(3). This paper describes the physical nature of wormholes and its relation to the physics of time travel. The physics of time travel is actually much more voluminous and detailed than it appears in this work. It is important to understand that time travel is enabled by the physical system of hyperspace and normal space, which manifests in reality as the cosmological expansion of the universe. The mathematical parameters of a tunnel in spacetime, or wormhole, such as length and direction, follow from the above. The physics theory of time travel shows very strongly that the universe must be expanding at a constant speed of light c, but as we know, it doesn't look like that. Therefore, time must have slowed down throughout the universe, which is speeding up over time. As a result, the rate of expansion of the universe increases over time, which we know in cosmology as the concept of "dark energy".

It can be argued that "dark energy" as a term does not correspond to its physical content. For example, the expansion rate of the universe is increasing over time, so it is believed that something is causing such a phenomenon. Any movement is related to energy, but in case of dark energy, it is more related to time and space itself. Throughout the universe, time has slowed down billions of times, which is why we see the expansion rate of the universe much slower than the speed of light c. However, it becomes faster in time due to the movement of normal space K with respect to hyperspace K', whereas the speed of movement of normal space is constant in time. According to such a theory, the change in the rate of expansion of the universe over time is related to time and space itself, and not to an unknown form of energy. However, the concept of "dark energy" continues to be used, because the concept is already so established in physics, cosmology, and also in the wider audience that there is no point in changing it. Similarly, it is the same with the concept of the "Big Bang" of the universe, where there was no real big bang or explosion, but the concept is still in circulation. The Big Bang of the universe marks the beginning of the universe, or its birth.

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1 The fundamental equation of the physics theory of time travel

As the universe expands, there are actually two forms of time. Firstly the fact that one represents the lifetime of the universe (that is, the duration of the existence of the universe itself), and, secondly, time also occurs at the rate of expansion of the universe (that is, how fast the universe expands). There is also a physical connection between these two times - namely, the longer the universe exists (i.e. the longer the life of the universe increases), the faster the universe expands (i.e. the expansion of the universe accelerates).

One of the fundamental foundations of the theory of time travel is the statement that different moments in time are also different points in space (3). Such regularity manifests itself in nature as the expansion of the universe. For example, if the universe expands (i.e. the volume of the universe increases over time), then at different (cosmological) moments in time, the volume of the universe is different, and thus the coordinates of the spatial points of the universe are also different. The expansion of the universe is often imagined as the expansion of a sphere or balloon. Then it can be seen very clearly that the spherical coordinates of a sphere (i.e. the coordinates of spatial points) and the radius of the sphere are different at different moments in time.

The fundamental connection between time and space emerges from the phenomenon of time dilation, which consists in the fact that the closer the speed of movement of a body gets to the speed of light c in vacuum, the slower the clock moves relative to a stationary observer:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or there is a time-slowing effect. If the body moves exactly at the speed of light c, then time has slowed down to infinity, i.e. time itself no longer exists:

$$t' = \frac{t}{\sqrt{1 - \frac{c^2}{c^2}}} = \infty$$

This means that the body reaches any space point in the universe in "its own time" t in 0 seconds. Time no longer exists for it. The same is the case with the length of the body, in which case the length of the body l' contracts or shortens to zero when the body moves at the speed of light c:

$$\frac{l'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{l'}{\sqrt{1 - \frac{c^2}{c^2}}} = l = \infty$$

The length l' of the body is equal to zero with respect to a stationary external observer. Since time is equal to infinity $t' = \infty$ and also the length of the body (i.e. "space") is equal to infinity $l = \infty$, so the two in this case are equal to each other:

$$t' = l$$

Such an equation is valid only under the condition that time and space no longer exist, i.e. time has slowed down to infinity and the length of "space" has shortened to an infinitesimally small, i.e. zero. From the latter equation we get:

$$1 = \frac{l}{t'}$$

in which we do not directly consider infinities, i.e. $\frac{l}{t'} = \frac{\infty}{\infty} = 1$. We know from classical mechanics that the quotient of the road section l and the time t defines the speed v, i.e. $\frac{l}{t} = v$, so the latter equation must actually show some kind of speed v. Since the strict condition of this situation is that time and space do not exist, then the speed v must be exactly equal to the speed of light c, because when light moves at the speed c, time and space no longer exist:

$$v = c$$

This now means that if we multiply both sides of the equation

$$1 = \frac{l}{t'}$$

by the speed of light c, we obtain as a result:

$$c = c\frac{l}{t'}$$

or

$$ct' = cl = d$$

The obtained result shows the speed of light c, which indicates that some kind of hitherto unknown space "moves" with respect to something at the speed of light c. This is actually described by the equation for the definition of speed c:

$$ct' = d$$

in which case it can be clearly seen that the longer the time period t' (time is like "moving"), the longer the distance d, i.e. the "more" the space moves. It can also be said that the longer the distance d, i.e. the "more" the space moves, the longer the time period t'. The speed of light c is constant, so the only variables are time and space. The speed of light c shows a "measurement": for example, after one second, space has "moved" a distance of 300 000 kilometers. Such an expression very clearly shows the fundamental connection between time, space and movement, which manifests itself in nature as the movement of ordinary space K in relation to hyperspace K', i.e. as the cosmological expansion of the universe.

Let us model the expansion of the universe as the expansion of a sphere that does not rotate.

In this case, the two-dimensional surface of the three-dimensional sphere is a three-dimensional version of our three-dimensional universe. The sphere expands and the body m moves along the surface of the sphere, i.e. on the surface of the sphere. The body always moves perpendicular to the radius of the sphere. For simplicity, a body moves along the circumference of a sphere with a length of 2πR. The expansion of the sphere illustrates the expansion of the universe, but the movement of the body m on the surface of the sphere illustrates the course of events and processes in the universe. Nothing can travel faster than light. The speed of light in vacuum and the rate of expansion of a sphere are both equal to c. The closer the speed of movement of the body gets to the speed of light c, i.e. the speed of expansion of the sphere, the slower the body m moves relative to the surface of the expanding sphere. This illustrates the slowing down of events and processes in the universe. If the speed of expansion of the sphere and the speed of movement of the body on the surface of the sphere match each other, then the body m will no longer move at all and thus time has stopped. It should be noted again that the sphere expands, not rotates.

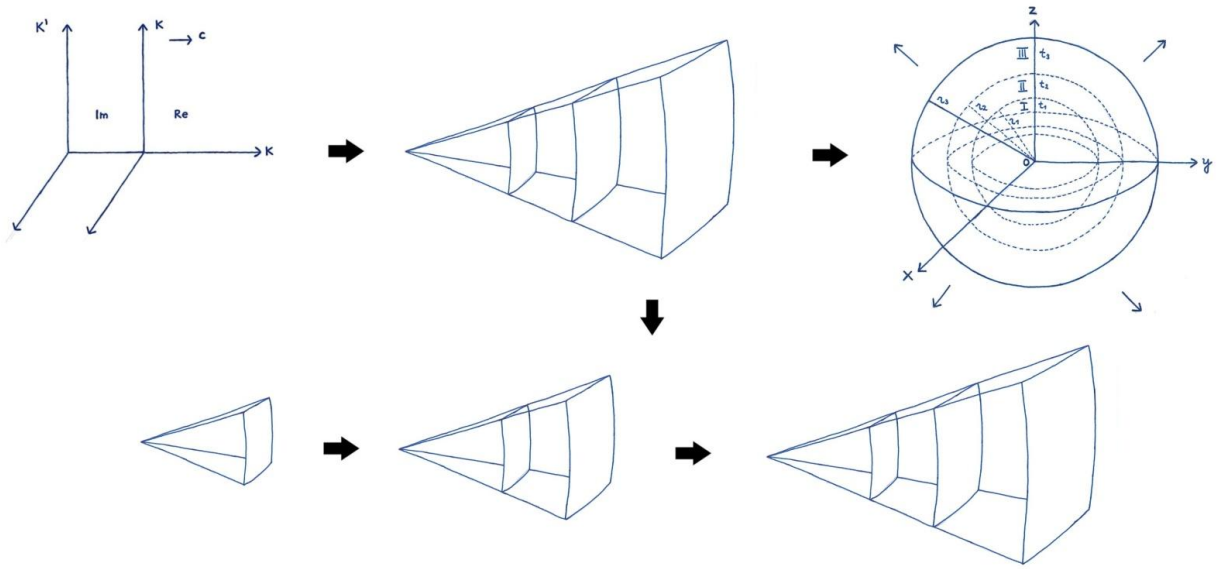


Figure: The physical system of ordinary space K ja hyperspace K´ manifests itself in nature as the cosmological expansion of the universe.

Since the body m always moves perpendicular to the radius of the sphere and at the same time always along with the expansion of the sphere, the movement of the body on the surface of the expanding sphere can be described by the Pythagorean theorem as follows:

$$d^2 = l^2 + (vt')^2$$

or

$$d = \sqrt{(ct + vt')^2 + (vt')^2}$$

where c is the speed of the expansion of the sphere (which matches the speed of light in vacuum), d is the (resultant) distance due to the movement of the body m and the expansion of the sphere, vt´ is the distance due to the movement of the body only, ct is the distance due to the expansion of the sphere, $l = ct + vt'$ is the distance of the body m on the surface of the expanding sphere, at the same time taking into account the increase in the distance due to the

expansion of the sphere, i.e. ct, t and t´ are different moments of time, i.e. the moments of the non-expanding and expanding states of the sphere, respectively. Nothing can move faster than light, i.e. faster than the expansion of the sphere, and therefore d = ct´, which means that the body m had to travel the distance d at speed c (not faster than that).

It is worth noting here that the derived relation:

$$ct' = d$$

was actually derived above: $ct' = cl = d$, in which case it was derived from the time dilation equation. This shows that the current and subsequent mathematical and physical analysis is based on the existence of moving space, i.e. the physical system of ordinary space K and hyperspace K', and not on the theory of special relativity.

Next, we perform a series of mathematical transformations to obtain the final equation that describes the given system mathematically. Since d = ct´, we express the rate of expansion c of the sphere from the Pythagorean theorem as follows:

$$\frac{\sqrt{l^2+(vt')^2}}{t'} = c,$$

l is the distance of the body m on the surface of the expanding sphere, taking into account also the increase in the distance due to the expansion of the sphere, i.e. ct:

$$l = ct + vt'$$

and so we can rewrite the latter equation as follows:

$$\frac{\sqrt{(ct+vt')^2+(vt')^2}}{t'} = c$$

Let's move t' to the other side of the equation, square both sides of the equation and write the expression of the quadratic equation:

$$(ct)^2+2(ct)(vt') + (vt')^2+(vt')^2 = (ct')^2$$

Let's move one member $(vt')^2$ to the other side and we get

$$(ct)^2+2(ct)(vt') + (vt')^2 = (ct')^2-(vt')^2$$

or

$$(ct')^2-(vt')^2 = (c^2-v^2)t'^2,$$

divide both sides of the latter obtained equation by c^2:

$$\frac{(ct)^2+2(ct)(vt')+(vt')^2}{c^2}=\frac{(c^2-v^2)t'^2}{c^2}$$

or

$$\frac{(c^2-v^2)t'^2}{c^2}=\frac{c^2-v^2}{c^2}t'^2$$

or

$$\frac{c^2-v^2}{c^2}t'^2=\left[1-\frac{v^2}{c^2}\right]t'^2.$$

Since the mathematical relationship of the quadratic equation holds

$$(ct)^2+2(ct)(vt')+(vt')^2=(ct+vt')^2,$$

then we obtain the latter equation in the form of the following expression:

$$(ct+vt')^2=\left[1-\frac{v^2}{c^2}\right]t'^2c^2$$

that is, by taking a square root of both sides of the equation:

$$ct+vt'=\sqrt{1-\frac{v^2}{c^2}}t'c.$$

The latter expression is the final equation we are looking for, which describes the given physical system. It is actually a general equation from which a whole series of very important fundamental physical and mathematical connections and conclusions can be derived. It can also be said that it is one of the basic equations, the conclusions of which are in good agreement with the basic principles of the theory of time travel. These conclusions have been further studied and analyzed in terms of relativity and quantum mechanics.

> The theory of relativity did not disprove Newtonian mechanics, but showed that Newtonian mechanics is derived from relativistic mechanics. In this sense, relativity complemented Newtonian mechanics, not overturned it. The same principle actually applies to the physics theory of time travel (3). The theory of time travel does not disprove relativity or quantum mechanics, it simply shows that relativity and quantum mechanics derive from the theory of time travel. In this sense, the theory of time travel complements the theory of relativity and quantum mechanics, rather than rejecting them. If Newtonian mechanics was derived from the theory of relativity, then the theory of relativity (and also quantum mechanics) itself derives from the theory of time travel.

However, in the following we will nevertheless do a small mathematical analysis, which is otherwise presented in the section on special and general relativity. This is necessary to finally reach an understanding of the kinematics of the expansion of the universe. For example, in the

previously mathematically derived general time travel equation

$$ct + vt' = \sqrt{1 - \frac{v^2}{c^2}}t'c$$

A member in it equaled as follows:

$$ct + vt` = l$$

In the equation thus obtained:

$$l = \sqrt{1 - \frac{v^2}{c^2}}t'c$$

or

$$l = ct`\sqrt{1 - \frac{v^2}{c^2}}$$

there are only four possible exact solutions. For example, if:

$$l = vt`$$

Then, accordingly, we get

$$vt` = ct`\sqrt{1 - \frac{v^2}{c^2}}$$

or

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

However, we will analyze such an equation much more thoroughly in the future. However, if $l = ct`$, then we get:

$$ct` = ct`\sqrt{1 - \frac{v^2}{c^2}}$$

The resulting equation must reduce to only one:

$$1 = \frac{ct`}{ct`} = \sqrt{1 - \frac{v^2}{c^2}}$$

where $v = 0$. However, if $l = vt$, we will obtain the relation:

$$vt = ct`\sqrt{1 - \frac{v^2}{c^2}}$$

or

$$\frac{t}{t`\sqrt{1-\frac{v^2}{c^2}}}=\frac{c}{v}$$

Since the equation for kinematic time dilation was expressed in the special theory of relativity in the following form:

$$t`=\frac{t}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{t}{\sqrt{1-\frac{c^2}{c^2}}}=\frac{t}{\sqrt{1-1}}=\frac{t}{0}=\infty$$

where v = c, then therefore the equation can only be equal to one:

$$\frac{t}{t`\sqrt{1-\frac{v^2}{c^2}}}=\frac{1}{t`}\frac{t}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{\infty}{\infty}=\frac{c}{v}=1$$

where $v=c$. However, if $l=ct$, then we obtain the equation in the following form:

$$ct=t`c\sqrt{1-\frac{v^2}{c^2}}$$

2 Spacetime interval

The length or the distance between two spatial points l in three-dimensional space is described by an equation known to us from mathematics:

$$l^2=\Delta x^2+\Delta y^2+\Delta z^2$$

in which the three-dimensional space coordinates appear as follows:

$$\Delta x=x_2-x_1$$

$$\Delta y=y_2-y_1$$

$$\Delta z=z_2-z_1$$

Consequently, we can write out the time coordinate as follows:

$$t'=\Delta t=t_2-t_1$$

Considering the previous relations, we get the definition of velocity v in classical mechanics:

$$v = \frac{l}{\Delta t}$$

Let's square the equation for velocity v

$$v^2 = \frac{l^2}{\Delta t^2} = \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\Delta t^2}$$

and we also square the general time travel equation in which $vt' = 0$:

$$(ct)^2 = \left(1 - \frac{v^2}{c^2}\right)(t'c)^2$$

From the latter expression, we write down the definition of velocity v through the previously derived relations:

$$(ct)^2 = \left(1 - \frac{\Delta x^2 + \Delta y^2 + \Delta z^2}{\Delta t^2 c^2}\right)(\Delta tc)^2 = \Delta t^2 c^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

The obtained metric equation

$$c^2 t^2 = \Delta t^2 c^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

is called the "spacetime interval" in the theory of relativity, which shows the distance in spacetime between two events or two points in space in the theory of relativity. It is clearly seen in this derived equation that the time t

$$t = \tau$$

is directly related to the speed of light c

$$c\tau = s$$

or

$$c = \frac{s}{\tau}$$

However, according to the interpretation of the physics theory of time travel, it can be related to the physical system of ordinary space and hyperspace, where it shows the "movement" of ordinary space K with respect to hyperspace K' at the speed of light c. It is also possible to interpret it in this way, since the equation for the spacetime interval

$$s^2 = \Delta t^2 c^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

can also be directly derived from the general time travel equation:

$$ct + vt' = \sqrt{1 - \frac{v^2}{c^2}}\, t'c$$

not only from the equations for time and space transformations in the theory of relativity. However, in Albert Einstein's theory of relativity, the metric equation of the spacetime interval is

used for a four-dimensional coordinate system in which there are three space coordinates and one time coordinate. The moment of time is multiplied by the speed of light c to make it the fourth dimension of space, but it is not associated with the physical system of normal space and hyperspace. In relativity, the fourth spatial dimension is necessary to obtain four coordinates (x, y, z and ct) that describe physical phenomena in spacetime, not outside of it.

The metric equation of a space-time interval:

$$c^2t^2 = \Delta t^2c^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

or

$$c^2t^2 = \Delta t^2c^2 - l^2$$

shows the distance between two points in time and space in Albert Einstein's special theory of relativity. The equation also describes the interval between the two observed events A and B. However, at the same time, this equation also describes the physical system of ordinary space K and hyperspace K', which is the absolute basis of the physics theory of time travel. Thus, the spacetime interval equation is important both in relativity and in the physical theory of time travel. We will show this briefly in the following. For example, when we divide both sides of the latter equation by the expression Δt^2c^2, the result is:

$$\frac{t^2}{\Delta t^2} = 1 - \frac{l^2}{\Delta t^2c^2} = 1 - \frac{v^2}{c^2}$$

This means that the time dilation equation can be directly derived from the metric equation of the spacetime interval:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

which shows the slowing down of time relative to an outside observer as it approaches the speed of light c. This is the basis of the special theory of relativity. However, at the same time, the spacetime interval equation also describes the system of normal space and hyperspace, which actually causes the phenomenon of time dilation to occur in the universe. For example, we can write the expression derived above in the following form:

$$\frac{t^2}{\Delta t^2} = 1 - \frac{l^2}{\Delta t^2c^2} = 1 - \frac{l^2}{\Delta l^2}$$

Let's move one term to the other side of the equation:

$$\frac{t^2}{\Delta t^2} + \frac{l^2}{\Delta l^2} = 1$$

The expression for the resulting amount can be equal to the following:

$$\frac{1}{2} + \frac{1}{2} = 1$$

which means that the following equations may apply:

$$\frac{t^2}{\Delta t^2} = \frac{l^2}{\Delta l^2}$$

$$\frac{t^2}{\Delta t^2} = \frac{1}{2}$$

$$\frac{l^2}{\Delta l^2} = \frac{1}{2}$$

From these we can obtain the connection:

$$\frac{t^2}{2t^2} = \frac{l^2}{2l^2}$$

The latter can result in fractures:

$$\frac{1}{2} = \frac{1}{2}$$

but at the same time the following equality is also seen: $t^2 = l^2$ ehk $t = l$. In more detail, its form would be as follows:

$$t = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

The physical meaning of such an expression would overlap with the nature of hyperspace. The equation shows that the time period is equal to the spatial extent, that is, when moving in space, movement also occurs in time. It must be a space outside of spacetime, because moving in ordinary space does not mean traveling in time, for example to the past. In classical mechanics, such an expression also includes the speed v:

$$vt = l$$

which in turn coincides with the equation for velocity v:

$$v = \frac{l}{t}$$

Since time is "moving" and time t exists in normal space, then space outside spacetime must be "stationary" and normal space must "move" relative to it at speed c. This can be briefly shown as follows. For example, in the equation derived above:

$$\frac{t^2}{\Delta t^2} = 1 - \frac{v^2}{c^2}$$

we multiply both sides of the equation by c^2:

$$\frac{c^2 t^2}{\Delta t^2} = c^2 - v^2$$

As a result, we get:

$$\frac{l^2}{\Delta t^2} = \Delta v^2 = c^2 - v^2$$

or

$$\Delta v^2 = c^2 - v^2$$

The resulting expression can be interpreted as follows. If v = 0, i.e. the body m is stationary in normal space, then relative to hyperspace it moves at the speed of light c:

$$\Delta v^2 = c^2$$

In this case, time dilation does not occur, i.e. time does not cease to exist:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t}{\sqrt{1 - \frac{0}{c^2}}} = t$$

because in a regular room you are standing still. Time and space exist in ordinary space. However, if v = c, i.e. the body m moves in ordinary space at the speed of light c (for example, the movement of light in vacuum), then in relation to hyperspace it is instead stationary:

$$\Delta v^2 = 0$$

In this case, time dilation equals infinity, i.e. time has ceased to exist:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t}{\sqrt{1 - \frac{c^2}{c^2}}} = \infty$$

because one is stationary relative to hyperspace. Time and space do not exist in hyperspace. Such an analysis shows that the system of ordinary space and hyperspace can also be directly derived from the theory of relativity, and that the transformations of time and space described in the theory of relativity derive from the system of ordinary space and hyperspace, and not the other way around. In the theory of relativity, there are connections and hints to the system of normal space and hyperspace, but they are mostly half-hidden or completely hidden, so you have to know how to properly discover and present them.

It follows from the above that two different interpretations can be obtained from the metric equation of the spacetime interval, which are closely related to each other. According to the theory of relativity, a spacetime interval indicates the distance in spacetime between two events or two points in space. This is correct. However, according to the physics theory of time travel, the spacetime interval also shows that ordinary space K moves with respect to hyperspace K' exactly at the speed of light c. This means that the physics theory of time travel shows from the metric equation of the spacetime interval what the theory of relativity does not show. The physical system of ordinary space and hyperspace remains hidden or invisible in the theory of relativity. The physical theory of time travel makes it visible to us.

3 Energy equations

The law of conservation of energy is mathematically derived from the homogeneity of time. Since time exists in ordinary space K (due to its "movement" in relation to hyperspace K), the law of conservation of energy also applies to it. Therefore, we will see below that all the subsequent mathematical and physical analysis show that the homogeneity of time resulting from the law of conservation of energy can also be shown through a long analysis of the previously derived general equation of time travel, in which case it is possible to derive from it the basic equation of cosmology, which is actually the law of conservation of energy in essence.

For example, from the previously derived general time travel equation:

$$ct + vt' = \sqrt{1 - \frac{v^2}{c^2}}\, t'c$$

the following mathematical transformations are possible. In case ct = 0, we get the following equation

$$vt' = \sqrt{1 - \frac{v^2}{c^2}}\, t'c$$

We divide both sides of the resulting equation by t´:

$$\frac{vt'}{t'} = \frac{\sqrt{1 - \frac{v^2}{c^2}}\, t'c}{t'} = \sqrt{1 - \frac{v^2}{c^2}}\, \frac{ct'}{t'}$$

and hence we finally get the following very important equation:

$$v = \sqrt{1 - \frac{v^2}{c^2}}\, c$$

or, as visually better presented:

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

Since the time dilation equation, which can also be derived from the general time travel equation, is in the following form

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and therefore we can express the kinematic coefficient as:

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{t}{t'}$$

Therefore, we can transform the equation derived from the general time travel equation

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

into the following mathematical form:

$$v = c\sqrt{1 - \frac{v^2}{c^2}} = \frac{ct}{t'}$$

or

$$v = c\frac{t}{t'}$$

We take t´ to the other side

$$vt' = ct$$

and transform he latter equation into the following form:

$$v \triangle t = ct$$

where $\triangle t = t'$.

Since normal space K moves at speed c relative to hyperspace K', theoretically the universe should also expand at speed c. However, in reality, the rate of expansion of the universe is many times less than the speed of light c. The only way to rationally explain such an apparent dilemma is that the difference in speeds is due to the transformation of time and space throughout the universe. For example, in the previously derived equation

$$v\Delta t = ct$$

it can be seen quite clearly that the difference in speeds can be caused by the fact that time has been altered:

$$\frac{\Delta t}{t} = \frac{c}{v} = y$$

where y is the known kinematic coefficient from the theory of special relativity:

$$y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or the theory of general relativity

$$y = \frac{1}{\sqrt{1 - \frac{R}{r}}}$$

Since in the relativistic equation for the transformation of velocities derived above

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

the kinematic factor y shows the ratio of different speeds

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c}{v}$$

then, according to the equation of time dilation

$$\Delta t = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the coefficient y shows the transformation of time:

$$\frac{\Delta t}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = y$$

Physically, this means that normal space K "moves" with respect to hyperspace K' at a constant speed c, which causes the transformation of time, i.e. the decrease of the value of y. However, at the same time the decrease in y causes the expansion rate of the universe to increase for the observer inside the expanding universe.

From the previously derived equation

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

it is possible to mathematically derive the basic equation of cosmology, i.e. the Friedmann equation, and also the equation for the static energy of a body $E = mc^2$. For example, the kinetic energy E of a body m is expressed by the equation:

$$E = \frac{mv^2}{2}$$

If the velocities are small compared to the speed of light in vacuum, approximate equations can be used:

$$\frac{1}{y} \approx 1 - \frac{1}{2}\beta^2$$

The expression for the kinematic factor y:

$$y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

can be presented as follows

$$y = \frac{1}{\sqrt{1 - \beta^2}}$$

where β^2 is expressed like this:

$$\beta^2 = \frac{v^2}{c^2}$$

Considering all the above, the kinematic factor y can be replaced with an approximate formula:

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)^2 - \frac{1}{4}\frac{v^4}{c^4}}$$

since $\left(\frac{v}{c} \ll 1\right)$. Since the member

$$-\frac{1}{4}\frac{v^4}{c^4}$$

is very small, then we can write the latter expression as follows:

$$\sqrt{1 - \frac{v^2}{c^2}} \approx \sqrt{\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)^2} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

Consequently, we can express y as follows:

$$y \approx 1 + \frac{v^2}{2c^2}$$

or

$$\frac{1}{y} \approx 1 - \frac{v^2}{2c^2}$$

and perform the following mathematical transformations:

$$v = c\sqrt{1-\frac{v^2}{c^2}} = c\frac{1}{y} \approx c\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

If we multiply both sides of the equation by mc, we get the following "analysis of operations":

$$v \approx c\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

or

$$mcv \approx mc^2\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

in which we eliminate parentheses:

$$mcv \approx mc^2 - \frac{mv^2}{2}$$

and express kinetic energy:

$$\frac{mv^2}{2} = mc^2 - mcv$$

The physical essence of the latter derived expression

$$\left(\frac{mv'^2}{2}\right)' = (mc^2 - mcv)$$

lies in the following analysis. For example, if the speed of movement of a body m is zero in relation to ordinary space K, i.e. v = 0, then in relation to hyperspace K´, the kinetic energy of the body is equal to the static energy known from the theory of relativity, i.e.

$$\frac{mv^2}{2} = mc^2$$

NOTE: Above we saw that a negative number could appear under the square root of the time dilation equation:

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t}{\sqrt{1-\frac{2c^2}{c^2}}} = \frac{t}{\sqrt{1-2}} = \frac{t}{\sqrt{-1}} = \frac{t}{i}$$

if the speed is greater than the speed of light: $v^2 = 2c^2$. Now we see that if in the derived equation:

$$\frac{mv^2}{2} = mc^2$$

we cancel out masses:

$$\frac{v^2}{2} = c^2$$

we get the expression for the square of the speed presented above:

$$v^2 = 2c^2$$

It is probably a coincidence, but nevertheless a rather remarkable coincidence, which can show that the mathematical derivations and expressions presented in this work are all connected and harmoniously consistent. In this way, a complex number is created, which contains an imaginary unit: $i = \sqrt{-1}$.

However, if the speed of movement of a body relative to normal space is equal to the speed of light c, i.e. v = c, then relative to hyperspace the kinetic energy of the body is zero, i.e.

$$\frac{mv^2}{2} = mc^2 - mc^2 = 0$$

In all the previous cases, $\frac{mv'^2}{2}$ is the kinetic energy of a body relative to hyperspace, and therefore this expression only acts as a mathematical definition, i.e.

$$\frac{mc^2}{2} \neq mc^2$$

but

$$E'_k = mc^2 - mcv$$

or

$$E'_k = E_k$$

This means that the kinetic energy of a body m relative to hyperspace E'_k

$$E'_k = \frac{mv'^2}{2}$$

depends on what the body's speed of movement v is relative to normal space K:

$$\frac{mv^2}{2} = mc^2 - mcv = mc(c - v)$$

or

$$\frac{mv^2}{2} = mc(c - v)$$

or

$$E'_k = mc(c - v)$$

But at the same time in the expression of total energy of a body

$$E = mc^2 + \frac{mv^2}{2}$$

is a member of the equation $\frac{mv^2}{2}$ only when viewed relative to normal space. It shows that if the body's speed of movement v is zero relative to normal space, i.e. v = 0 (and therefore $\frac{mv^2}{2}$ = 0), then the body's kinetic energy E relative to hyperspace is $E = mc^2$, which we call the "state energy" of a body.

In the previously derived equation

$$\frac{mv^2}{2} = mc^2 - mcv$$

we take the member $-mcv$ to the other side of the equals sign:

$$\frac{mv^2}{2} + mcv = mc^2$$

Next, we consider that if v=0 in the member +mcv, then we get as the equation

$$\frac{mv^2}{2} = mc^2$$

and due to this relation $v^2 = -c^2$:

$$-\frac{mc^2}{2} = mc^2$$

The latter relation $v^2 = -c^2$ derives from the formal geometry of the theory of special relativity, in which the square of the four-dimensional momentum $p_\mu p_\mu$ can be defined together with the potential energy as follows:

$$-{m_0}^2c^2 = p^2 - \frac{E^2}{c^2} = -{m_0}^2c^2\frac{1-\beta^2}{1-\beta^2} = \frac{{m_0}^2(v^2-c^2)}{1-\beta^2} = p_\mu p_\mu$$

in which the multiplier term

$$\frac{v^2-c^2}{\sqrt{1-\beta^2}} = v_\mu v_\mu = v^2 = -c^2\frac{1-\beta^2}{1-\beta^2} = -c^2$$

is a time-like vector and is a constant. However, $v_\mu v_\mu$ here is the square of the four-dimensional speed, which actually shows that all bodies in the universe move at the speed of light c. Consequently, the (four-dimensional velocity) vector v_μ can be expressed as follows

$$v_\mu = \frac{dx_\mu}{d\tau}$$

in which the quotient member

$$\tau = t\sqrt{1-\beta^2}$$

is the eigentime of a moving body, and thus we get the final relation of a velocity vector:

$$v_\mu = \frac{dx_\mu}{t\sqrt{1-\beta^2}}$$

According to the mathematical definition of the four-dimensional velocity vector, the form of the impulse p will be

$$p = m_0 v$$

with different designations as follows

$$p_\mu = m_0 v_\mu$$

The equation derived above

$$\frac{mv^2}{2} = mc^2 - mcv$$

or

$$\frac{mv^2}{2} + mcv = mc^2$$

is very important, because Einstein's equation can be derived from it. For example, if mcv has v = c and mc^2 = E, we get the expression for the total energy of a body:

$$mc^2 = mcv + \frac{mv^2}{2}$$

or

$$E = mc^2 + \frac{mv^2}{2}$$

where $\frac{mv^2}{2}$ is the kinetic energy of a body and mc^2 is the static energy of a body. From the latter equation we can derive the relationship between mass and energy of a body in relativistic mechanics (Einstein's equation) as follows:

$$E = mc^2 + \frac{mv^2}{2} = mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = mc^2 y = mc^2 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

and indeed we get that $E = mc^2$ when a body is stationary, i.e. v = 0.

However, in the previously derived equation

$$\frac{mv^2}{2} = mc^2 - mcv$$

we took the member $-mcv$ to the other side of the equal sign as well:

$$\frac{mv^2}{2} + mcv = mc^2$$

and if this member of the equation is equal to zero:

$$mcv = 0$$

i.e. the speed is zero $v = 0$, then we got the following very important expression:

$$\frac{mv^2}{2} = mc^2$$

But now we show that the latter expression is actually equivalent to the following equation:

$$\frac{mv^2}{2} = mcv$$

if in the latter equation the mcv term in the equation has the velocity v = c. But if v = 0, then we get kinetic energy, which equals to zero:

$$\frac{mv^2}{2} = 0$$

According to the law of conservation of mechanical energy, the potential energy of a body must be maximum when its "energy of motion" or kinetic energy equals zero:

$$\frac{mv^2}{2} = -\frac{GMm}{R}$$

Consequently, we get the following relation:

$$-\frac{GMm}{R} = mcv$$

where $v = 0$. We will immediately justify the entire previous reasoning with the following physical analysis. For example, if v = 0 in the mcv term of the equation, it physically means that a body m is stationary with respect to normal space K, but the body m moves at speed c with respect to hyperspace K´. Therefore, the kinetic energy $\frac{mv^2}{2} = E_k$ of the body m is equal to $E = mc^2$ and this is relative to hyperspace K´. In this case, this is the definition of kinetic energy: E_k. Since v = 0 in relation to mcv, which means that the body m is at rest with respect to the normal space K, i.e. the kinetic energy of the body is zero with respect to normal space K, so the rest or potential energy of the body m with respect to normal space K is equal to the gravitational potential energy as follows:

$$mcv = -\frac{GMm}{R}$$

and this potential energy is no longer equal to zero in the equation.

At this point, it should be noted that in the theory of special relativity, the potential energy of

a body due to the presence of an external field is not taken into account in the expression of the body's total energy and static energy. The change in the body's potential energy in the external force field is not taken into account. Mass and energy are equivalent quantities. The relativistic mass of a body is also a measure of the body's total energy.

All bodies exist in ordinary space, in which time (and space) exists. Time as duration is constantly 'moving'. This means that time never "stands still". Moving bodies have kinetic energy. Absolutely all bodies in the universe also move in relation to time (i.e. we all move in time towards the future), but time is not an object of any kind. This is where the static energy $E = mc^2$ for all bodies in the universe can come from. This means that the energy mc^2 is still the "kinetic energy of the body with respect to time". After all, all bodies move relative to hyperspace K', because ordinary space K moves relative to hyperspace K' at speed c. Consequently, all bodies have kinetic energy and therefore mass. In this way, the energy mc^2 can be kinetic energy "relative to moving hyperspace", i.e. $E = mc^2$ is the energy of a body relative to time.

It can be seen from the physical system of normal space and hyperspace that all bodies in the universe actually move at the speed of light c. This follows directly from the fact that, according to the physical model, ordinary space moves incessantly with respect to hyperspace at the speed of light c. All bodies and phenomena exist in spacetime, which according to the model is denoted as normal space moving relative to hyperspace. One of the best proofs of this is the expression of static energy, which is possessed by all bodies in the universe (for light it is equal to zero):

$$E = mc^2$$

According to this, the impulse p is manifested as follows:

$$\frac{E}{c} = mc = p$$

Momentum is the product of velocity and mass, and due to rest energy, all bodies with rest mass in the universe possess it. The speed of light c actually shows the speed relative to hyperspace, which is the result of the movement of normal space relative to hyperspace. Bodies "standing" in spacetime also have such momentum and energy. This is one of the most direct manifestations of the physical system of normal space and hyperspace in the entire universe.

Rest energy $E = mc^2$ actually has several meanings in physics. For example, in Albert Einstein's special theory of relativity, it simply means the energy E of a physical body with a rest mass, which does not include the fact that any macroscopic body consists of indivisible particles. In this case, the static energy is a fairly abstract physical quantity. However, nuclear physics shows that the nature of static energy is related to the binding energy of the atomic nucleus, which makes the nature of static energy much more concrete. The binding energy of a nucleus is equal to the work that must be done to move the nucleons of the nucleus to such a distance from

each other that they no longer affect each other. The binding energy per nucleon is called the specific binding energy in physics. The binding energy of a nucleus can also be characterized in terms of a mass defect. When a nucleus is completely fissioned, the mass of its component particles, or nucleons, differs from the mass of the nucleus as a whole by a certain amount. This difference is called the mass defect:

$$\Delta m = Zm_p + Nm_n - m$$

where m is the mass of a nucleus, m_p and m_n are the mass of the proton and neutron, respectively, and Z and N are their number in the nucleus. With the help of the mass defect, we can find the energy released during the formation of the nucleus, if we use the famous equation for the rest energy:

$$\Delta E = \Delta mc^2$$

This equation expresses the relationship between the mass defect and the binding energy of the nucleus. Based on the law of conservation of energy, exactly the same amount of energy should also be spent for complete fission of the nucleus. According to the physics theory of time travel, the nature of static energy results from the movement of ordinary space K with respect to hyperspace K', in which case all bodies in the universe move at the speed of light c. In this case, it would be a form of kinetic energy that cannot be derived from classical mechanics. This can only be seen from the physical system of normal space and hyperspace, i.e. the further development of the theory of relativity. In this case, the physical nature of the static energy is even more concrete compared to the previously mentioned ones.

In the theory of special relativity, the potential energy of a body caused by the presence of an external field is not taken into account in the expression for the body's total energy and static energy. However, considering all the previous mathematical relationships, we still get the following very important equation:

$$\frac{mv^2}{2} + mcv = mc^2$$

or

$$\frac{mv^2}{2} - \frac{GMm}{R} = -\frac{mc^2}{2}$$

where

$$+mcv = mc^2 = \frac{mv^2}{2} = -\frac{GMm}{R}$$

and

$$mc^2 = \frac{mv^2}{2} = -\frac{mc^2}{2}$$

All the masses m cancel out:

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

The latter derived relation actually directly expresses the law of conservation of mechanical energy:

$$E = E_k + E_p = const$$

where

$$E = E_k + E_p = \frac{v^2}{2} + \left(-\frac{GM}{R}\right) = \frac{v^2}{2} - \frac{GM}{R} = const = -\frac{c^2}{2}$$

The constant $-\frac{c^2}{2}$ in the latter equation does not derive mathematically from the law of conservation of energy known in classical mechanics. The form of such a law of conservation of energy can only be derived by relativistic mechanics as we have done here before. Later we will see that the constant term $-\frac{c^2}{2}$ in the latter equation can also be equal to zero:

$$\frac{v^2}{2} - \frac{GM}{R} = 0$$

from which, in turn, we get the law of conservation of mechanical energy in its classical form:

$$\frac{v^2}{2} = \frac{GM}{R}$$

4 Robertson-Walker Metric

Based on the spacetime interval equation derived from the general time travel equation:

$$ds^2 = {dx_0}^2 - {dx_1}^2 - {dx_2}^2 - {dx_3}^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = c^2dt^2 - dl^2$$

one can mathematically derive the well-known Robertson-Walker metric, mathematically describing the expansion of the spacetime of the Universe. To do this, instead of using the Cartesian rectangular coordinate system

$$dl^2 = dx^2 + dy^2 + dz^2$$

we will present spherical coordinates

$$dl^2 = dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

as the expansion of the Universe is usually modelled in cosmology as the expansion of a sphere in space. As a result, we get the following equation:

$$ds^2 = c^2dt^2 - (dr^2 + r^2d\theta^2 + r^2sin^2\theta d\varphi^2)$$

Since the sphere is expanding, the quantity r in this last equation is the comoving radial coordinate moving along with the coordinate system:

$$R = r = a(t)\chi$$

The radial distance R = r = a(t)χ could also be the distance between two galaxies in the Universe. Consequently, we can write down the spherical coordinates as follows:

$$(dl)^2 = a^2(t)\{(d\chi)^2 + K[(d\theta)^2 + sin^2\theta(d\varphi)^2]\}$$

and the Robertson-Walker metric will take the following form:

$$ds^2 = c^2dt^2 - a^2(t)\{(d\chi)^2 + K[(d\theta)^2 + sin^2\theta(d\varphi)^2]\}$$

where K could have the following values:

$$K = \begin{cases} sin^2\chi \\ \chi^2 \\ sh^2\chi \end{cases}$$

The Robertson-Walker metric is sometimes also presented using the following form:

$$ds^2 = -dt^2 + a(t)^2\left\{\frac{dr^2}{1-kr^2} + r^2[(d\theta)^2 + sin^2\theta(d\varphi)^2]\right\}$$

where k could have the following values:

$$k = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

In the Robertson-Walker metric

$$(ds)^2 = c^2dt^2 - (dl)^2$$

where

$$(dl)^2 = a^2(t)\{(d\chi)^2 + K[(d\theta)^2 + sin^2\theta(d\varphi)^2]\}$$

the coefficient K has three possibilities. This is due to the fact that any space of the Universe can either have a positive or negative curvature or be flat. For positive curvature space, the value of K equals:

$$K = sin^2\chi$$

and thus k = +1. Its regions of variation are as follows: $\chi \in (0, \pi)$, $\theta \in (0, \pi)$, $\varphi \in (0, 2\pi)$. For negative curvature space, the value of K equals:

$$K = (sh\chi)^2$$

and thus k = –1. Its regions of variation are as follows: $\chi \in (0, \infty)$, $\theta \in (0, \pi)$, $\varphi \in (0, 2\pi)$. For flat space, the value of K equals:

$$K = \chi^2$$

and thus k = 0. Its regions of variation are as follows: $\chi \in (0, \infty)$, $\theta \in (0, \pi)$, $\varphi \in (0, 2\pi)$.
The quantity r is the comoving radial coordinate moving along with the coordinate system:

$$R = r = a(t)\chi$$

The radial distance R = r = a(t)χ could also be the distance between two galaxies in the Universe. As this distance is increasing over time due to the metric expansion of the Universe, we can take its derivative in respect of time as follows:

$$v = \frac{dR}{dt} = \frac{D}{dt}(a(t)\chi) = \chi\frac{da(t)}{dt} = \chi\dot{a}$$

where the time derivative is denoted by the dot above 'a':

$$\dot{a} = \frac{da(t)}{dt}$$

The radial distance R = r = a(t)χ gives us the following relation:

$$\chi = \frac{R}{a}$$

and based on this we get Hubble's law, commonly known in cosmology:

$$v = \frac{R}{a}\dot{a} = \frac{\dot{a}}{a}R = HR$$

where the Hubble constant H is

$$H = \frac{\dot{a}}{a}$$

or

$$\dot{a} = Ha$$

The Hubble constant H depends on time:

$$H \sim \frac{1}{t}$$

Simplified, the preceding analysis could be understood to mean that the radial coordinate R in Hubble's law $v = HR$:

$$R = a(t)\chi$$

will give us velocity $v = \dot{a}\chi$ as its derivative in respect of time, and acceleration $a = \ddot{a}\chi$ as its second-order derivative in respect of time. As ä equals the following:

$$\ddot{a} = \frac{D^2a}{dt^2}$$

we can obtain the acceleration a as follows:

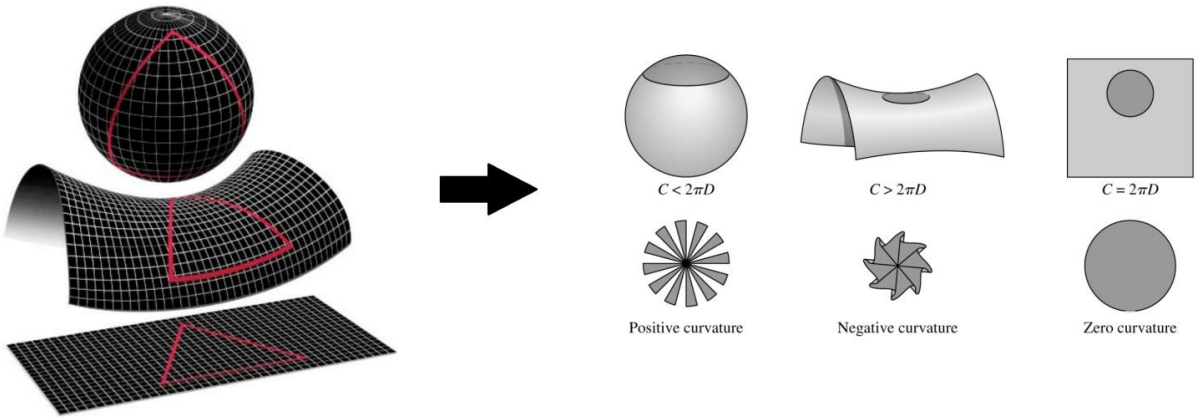
$$a = \frac{D^2a}{dt^2}\chi = \frac{D^2R}{dt^2}$$

The velocity v is the first-order derivative with respect to time:

$$v = \frac{dR}{dt}$$

and if there is no derivative at all then $R = R$.

The time coordinate t in the Robertson-Walker metric is the lifetime of the Universe; K is a constant related to curved space, and a(t) is a function of time depending on the expansion or possible contraction of the Universe. The distance between two points in space (i.e. also the 'size' of the Universe) is indicated by the quantity s whose value changes with time t. The metric also depends on the value of the constant K, i.e. the curvature of space – whether we are dealing with flat, negative or positive curvature space of the Universe. The last equation, known as the Robertson-Walker metric, shows us the cosmological future of the expansion of the Universe. This depends on whether the space of the Universe is generally flat or has positive or negative curvature. Figure:



Photos from: *https://people.ast.cam.ac.uk/~pettini/Intro%20Cosmology/Lecture03.pdf*

http://popia.ft.uam.es/Cosmology/files/02FriedmannModels.pdf

5 Friedmann equations

In traditional cosmology, curved spacetime, and first of all curved space, is used as the model for the expansion of the Universe. Albert Einstein's theory of general relativity is the only physical theory describing these curved spacetimes; therefore, Einstein's theory of general relativity forms the basis for all modern cosmology. The metric formalism is the most common mathematical form for describing the curvature of space. Based on the previously derived Robertson-Walker metric

$$(ds)^2 = c^2dt^2 - (dL_k)^2$$

but not on the Riemannian metric:

$$(ds)^2 = \sum_{i=0}^{3}\sum_{k=0}^{3} g_{ik}(x)dx_i dx_k$$

where $g_{ik} = g_{ki}$

$$g_{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 sin^2\theta \end{pmatrix}$$

the components of Albert Einstein's tensor G_{il} known from the theory of general relativity are calculated from Einstein's equation:

$$G_{ik}(g(x)) = -\frac{8\pi G}{c^4} T_{ik}$$

where g(x) is:

$$g(x) = g_{ik}(x)$$

Consequently, the components of the Einstein tensor comprise the function a(t) that depends on time:

$$g_{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a(t) & 0 & 0 \\ 0 & 0 & a(t)sin\chi & 0 \\ 0 & 0 & 0 & a(t)sin\chi\ sin\,\theta \end{pmatrix}$$

This unknown function must be found from Einstein's equations. In cosmology, a(t) is the scale factor of the Universe. Taking into account the cosmological principle, the components of the energy-momentum tensor should be expressed as follows for the given Robertson-Walker metric:

$$T_{00} = \rho c^2$$

$$T_{\alpha\alpha} = -p g_{\alpha\alpha}$$

where α = 1, 2, 3, $T_{il} = 0$, when i ≠ l, i, l = 0, 1, 2, 3. $g_{\alpha\alpha}$ are the diagonal spatial components of the metric tensor, ρc^2 is the density of matter and energy in the Universe, and p is the pressure. Since collisions between galaxies are extremely rare, the pressure p is zero. By placing the components of the Einstein tensor G_{il} and the energy-momentum tensor T_{il} found as described above into Einstein's equations, we get the well-known Friedmann equations that form the basis of all modern cosmology:

$$\ddot{a} = -\frac{4\pi G a}{3}\left(\rho + \frac{3p}{c^2}\right)$$

$$\frac{1}{2}(\dot{a})^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{kc^2}{2}$$

where k = 1, 0, –1 and

$$\dot{a} = \frac{D}{dt}a(t)$$

$$\ddot{a} = \frac{D^2 a}{Dt^2}$$

These last mathematical equations show that the relation obtained (if p = 0)

$$\ddot{a} = -\frac{4\pi G a}{3}\rho$$

is actually also present in the equation

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{kc^2}{2}$$

and therefore the last equation can be written in the following form:

$$\frac{1}{2}\dot{a}^2 + \ddot{a}a = -\frac{kc^2}{2}$$

In the equations presented above, a(t) is the scale factor, $c^2\rho(t)$ is the matter-energy density and p(t) is the pressure.

Next, we will analytically show that the previously derived fundamental equation of cosmology, i.e. the Friedmann equation, cannot be derived based on Newtonian mechanics. To do this, we will start from the formula for mass density M, which could also describe the expansion of the Universe, i.e. masses moving away from each other:

$$M = \frac{4\pi\rho r^3}{3}$$

Here, we have an expanding ball or sphere with the radius r and mass m; ρ is the energy density in this case. Masses exert Newton's gravitational interaction force F upon each other:

$$F = \frac{GMm}{r^2}$$

and due to that objects have gravitational potential energy:

$$U = -\int_r^\infty \vec{F} D\vec{r} = -\frac{GMm}{r} = -\frac{4\pi G m \rho r^2}{3}$$

The kinetic energy of a moving object is

$$T = \frac{m\dot{r}^2}{2}$$

where $\dot{r}$ denotes the derivative with respect to time. We get the formula for the total energy of an object from the law of conservation of energy as the following expression:

$$E = T + U = \frac{m\dot{r}^2}{2} - \frac{4\pi G m \rho r^2}{3}$$

The expression for the total energy of object E must remain constant in time. Next, we switch to a coordinate system with comoving coordinates as we need to describe the expansion of the Universe. Therefore, the actual distance $\vec{r}$ between the objects and the initial fixed distance $\vec{x}$ are:

$$\vec{r} = a(t)\vec{x}$$

a(t) is a scale factor known from cosmology, describing "a change in the relative distance between two points in space over time". Consequently, we can express the total energy E of an object as follows:

$$E = \frac{m\dot{a}^2 x^2}{2} - \frac{4\pi G m \rho a^2 x^2}{3}$$

We will multiply both sides of the last equation by the expression

$$\frac{2}{ma^2x^2}$$

and set the value of k as

$$k \equiv -\frac{2E}{mx^2}$$

and set the value of k squared as

$$k^2 \equiv 8\pi G$$

as a result, we get the following equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{k^2\rho}{3} - \frac{k}{a^2}$$

where the expression

$$H = \frac{\dot{a}}{a}$$

is the Hubble parameter describing the expansion rate of spacetime. Next, we analyse the equality of H^2, which shows us that the Friedmann equation cannot actually be derived based on Newtonian mechanics. For example, the last equation obtained for H^2 is expressed as follows:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{k^2\rho}{3} - \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{2E}{ma^2x^2}$$

and this is due to the terms of the equation equalling $k^2 \equiv 8\pi G$, $k \equiv -\frac{2E}{mx^2}$ and $\vec{r} = a(t)\vec{x}$. Thus, the term of the last equation could also equal:

$$+\frac{2E}{ma^2x^2} = \frac{2E}{mr^2}$$

We will now assume that if the term of the last equation $\frac{8\pi G}{3}\rho$ is equal to zero, we will get the

formula

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{a}^2}{a^2} = \frac{2E}{ma^2x^2}$$

By moving the term a^2x^2 to the other side of the equality sign in the equation:

$$\dot{a}^2x^2 = \frac{2E}{m}$$

we see that the resulting equation equals the following:

$$\dot{r}^2 = \frac{2E}{m}$$

i.e. we have the expression for kinetic energy E known from classical mechanics:

$$\frac{m\dot{r}^2}{2} = E$$

where the square of velocity v is expressed as follows:

$$v^2 = \dot{r}^2 = \dot{a}^2x^2$$

Assuming now that the term $+\frac{2E}{ma^2x^2}$ of the equation for H^2 is equal to zero, we get the following expression:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$$

or

$$H^2 = \frac{8\pi G}{3}\rho$$

When dividing both sides of the last equation by 2, we get:

$$\frac{H^2}{2} = \frac{4\pi G}{3}\rho = \frac{GM}{R^3}$$

where mass M is expressed as mass density ρ:

$$M = \frac{4\pi r^3}{3}\rho$$

The last equation obtained also equals the following:

$$\frac{v^2}{2} = \frac{GM}{R}$$

where velocity v is Hubble's law $v = HR$, or $v^2 = H^2R^2$. The resulting equation is the law of conservation of mechanical energy known from classical mechanics:

$$\frac{v^2}{2} - \frac{GM}{R} = 0$$

where the difference between kinetic and potential energies is equal to zero. The last equation is actually also present in the previously derived equation:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{2E}{ma^2x^2}$$

For example, by dividing both sides of the equation H^2 by 2, we get:

$$\frac{H^2}{2} = \frac{4\pi G}{3}\rho + \frac{E}{ma^2x^2}$$

Let us move the term $\frac{4\pi G}{3}\rho$ of the last equation to the other side of the equality sign; the result will then be:

$$\frac{H^2}{2} - \frac{4\pi G}{3}\rho = + \frac{E}{ma^2x^2}$$

We can see from this last equation that it is not equal to the Friedmann equation derived above:

$$\frac{H^2}{2} - \frac{4\pi G}{3}\rho = -\frac{kc^2}{2}$$

and this is due to the fact that Newtonian mechanics (i.e. the law of conservation of energy arising from classical mechanics) does not yield the term containing the constant c for the speed of light:

$$+\frac{E}{ma^2x^2} \neq -\frac{kc^2}{2} = const$$

It can therefore be concluded that it is not possible to derive Friedmann equations based on Newtonian mechanics. Friedmann equations can only be derived from the equations of relativistic mechanics (together with the Robertson-Walker metric) – in this case from the equations of Albert Einstein's theory of general relativity.

In cosmology, the Milne model has also been described, similarly showing the derivation of the fundamental equation of cosmology, i.e. the Friedmann equation, by following the classical path based on Newtonian mechanics. This process of mathematical derivation differs somewhat from the one outlined above. First, we start with a coordinate system of comoving coordinates as we need to describe the cosmological expansion of the Universe. Therefore, the actual distance $\vec{r}$ between the objects and the initial fixed distance $\vec{r}_0$ are:

$$\vec{r} = R(t)\vec{r}_0$$

The time-dependent function R(t) is a scale factor known from cosmology, describing "a change in the relative distance between two points in space over time". The last equation also yields the following relation:

$$R(t_0) = 1$$

The following formula describes the dependence of the matter density of the Universe as a function of time ρ(t):

$$\rho(t) = \rho(t_0)\frac{r^3(t_0)}{r^3(t)} = \frac{\rho(t_0)r^3(t_0)}{R^3(t)r^3(t_0)} = \frac{\rho(t_0)}{R^3(t)}$$

As the Universe is expanding, the galaxies are moving away from each other; therefore, we can use Newton's well-known second law:

$$a = \frac{F}{m}$$

The force F affecting the galaxy can in turn be found using Newton's law of gravitation:

$$F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2(t)}\frac{4\pi r^3(t)}{3}\rho(t) = -\frac{4\pi G}{3}m\rho(t)r(t) = -\frac{4\pi G}{3}mr(t)\frac{\rho(t_0)}{R^3(t)}$$

meaning the force F affecting the galaxy can be expressed as follows:

$$F = -\frac{4\pi G}{3}mR(t)r_0\frac{\rho(t_0)}{R^3(t)}$$

Let us convert Newton's second law to the following differential form:

$$F = ma = m\ddot{r} = mr_0\ddot{R}$$

and, as a result, we get the differential equation for the force affecting the galaxy:

$$mr_0\ddot{R} = -\frac{4\pi G}{3}mr_0\frac{\rho(t_0)}{R^2(t)}$$

or

$$R^2\ddot{R} = -\frac{4\pi G}{3}\rho(t_0)$$

The two dots denote the second-order derivative with respect to time:

$$\ddot{R} = \frac{D^2R}{dt^2} = \frac{D}{dt}\left(\frac{dR}{dt}\right) = \frac{dv}{dt}$$

and we can in turn express acceleration a as follows:

$$\frac{dv}{dt} = \frac{dv}{dt}\frac{dR}{dR} = \frac{dR}{dt}\frac{dv}{dR} = v\frac{dv}{dR}$$

We can obtain the equation of motion for the galaxy based on the preceding relations as follows:

$$R^2 v\frac{dv}{dR} = -\frac{4\pi G}{3}\rho_0$$

Let us move the functions of the radius R to the other side of the equality sign in the last equation:

$$vdv = -\frac{4\pi G}{3}\rho_0\frac{dR}{R^2}$$

and integrate, obtaining the following equation as a result:

$$v^2 = (\dot{R})^2 = +\frac{4\pi G}{3}\rho_0\frac{1}{R} + k$$

describing the cosmological expansion of the entire Universe, i.e. we were 'supposedly' able to derive the fundamental equation of cosmology. For example, when comparing the last equation with the Friedmann equation derived before, we see that the constant of integration +k in the last equation must have the following value:

$$v^2 - \frac{4\pi G}{3}\rho_0\frac{1}{R} = +k = -\frac{c^2}{2}$$

or

$$+k = -\frac{c^2}{2}$$

In the present case, the last equation lacks the constant k, which determines the curvature of the Universe in the Friedmann equation, in front of the velocity c^2. The reason for this will be explained later. The previously derived Milne model is, however, physically unworkable, meaning that the fundamental equation of cosmology cannot be derived based on Newtonian mechanics. One reason for this is the fact that from the Milne model

$$v^2 = \frac{4\pi G}{3}\rho_0\frac{1}{R} + k$$

or

$$v^2 - \frac{4\pi G}{3}\rho_0\frac{1}{R} = +k = 0$$

which is supposed to express the law of conservation of mechanical energy, one cannot in fact mathematically derive the equation of the law of conservation of energy:

$$\frac{v^2}{2} = \frac{GM}{R}$$

This cannot be mathematically derived, and this is why the Milne model is in fact physically unworkable. The following representation of the formula for changing of the matter density in the Universe was used to derive the Milne model:

$$\rho(t) = \rho(t_0)\frac{r^3(t_0)}{r^3(t)} = \frac{\rho(t_0)r^3(t_0)}{R^3(t)r^3(t_0)} = \frac{\rho(t_0)}{R^3(t)}$$

and this is, in fact, why the final result does not correspond to the law of conservation of energy.

6 Law of conservation of energy

The law of conservation of energy is mathematically derivable from the homogeneity of time. In short, it lies in the following. Let us assume we have a physical system of some kind, consisting of n objects. The physical quantities of these objects at the instant of time t are the position vectors

$$\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N$$

inertial masses

$$m_{10}, m_{20}, \dots, m_{N0}$$

velocities

$$\vec{v}_1, \vec{v}_2, \dots, \vec{v}_N$$

momentums

$$\vec{p}_1, \vec{p}_2, \dots, \vec{p}_N$$

and moments of momentum

$$\vec{L}_1, \vec{L}_2, \dots, \vec{L}_N$$

Let us next observe this system at a different instant of time, from a different point in space and from a different direction, but leave everything else the same. From now on, we will treat all this ‘stuff’ here in scalar form. However, if something were to happen to this system, the physical states (i.e. the quantities) of the objects would change. Work was done for this purpose though, and this work is obtained by summing up the works A done by each object belonging to the system. If something were to happen to the system, this would be mathematically expressed as follows:

$$dA = dA_1 + dA_2 + \cdots + dA_N = F_1 ds_1 + F_2 ds_2 + \cdots + F_N ds_N$$

However, if nothing happens to the system, this is shown by the following expression:

$$dA = F_1 ds_1 + F_2 ds_2 + \cdots + F_N ds_N = 0$$

Now let us observe a similar system at a different instant of time:

$$t' = t + dt$$

leaving everything else the same. Since all instants of time are equal, nothing will happen to this system. Considering the equality dA = 0, the definition of force from physics and the derivation rules for composite functions from mathematics, we mathematically obtain the following:

$$0 = \frac{m_{10}}{\left(1 - \frac{{v_1}^2}{c^2}\right)^{\frac{3}{2}}} \frac{dv_1}{dt} ds_1 + \frac{m_{20}}{\left(1 - \frac{{v_2}^2}{c^2}\right)^{\frac{3}{2}}} \frac{dv_2}{dt} ds_2 + \cdots + \frac{m_{N0}}{\left(1 - \frac{{v_N}^2}{c^2}\right)^{\frac{3}{2}}} \frac{dv_N}{dt} ds_N =$$

$$= \frac{m_{10}}{\left(1 - \frac{{v_1}^2}{c^2}\right)^{\frac{3}{2}}} \frac{ds_1}{dt} dv_1 + \frac{m_{20}}{\left(1 - \frac{{v_2}^2}{c^2}\right)^{\frac{3}{2}}} \frac{ds_2}{dt} dv_2 + \cdots + \frac{m_{N0}}{\left(1 - \frac{{v_N}^2}{c^2}\right)^{\frac{3}{2}}} \frac{ds_N}{dt} dv_N =$$

$$= \frac{m_{10} v_1 dv_1}{\left(1 - \frac{{v_1}^2}{c^2}\right)^{\frac{3}{2}}} + \frac{m_{20} v_2 dv_2}{\left(1 - \frac{{v_2}^2}{c^2}\right)^{\frac{3}{2}}} + \cdots + \frac{m_{N0} v_N dv_N}{\left(1 - \frac{{v_N}^2}{c^2}\right)^{\frac{3}{2}}} =$$

$$= D\left[\frac{m_{10} c^2}{\sqrt{1 - \frac{{v_1}^2}{c^2}}}\right] + D\left[\frac{m_{20} c^2}{\sqrt{1 - \frac{{v_2}^2}{c^2}}}\right] + \cdots + D\left[\frac{m_{N0} c^2}{\sqrt{1 - \frac{{v_N}^2}{c^2}}}\right] =$$

$$= dA_1 + dA_2 + \cdots + dA_N = D(A_1 + A_2 + \cdots + A_N) = dA$$

From the above, we can see that dA = 0 or dE = 0, which in turn means the law of conservation of energy:

$$\int dE = E = const.$$

This means that the law of conservation of energy derives from the equivalence of instants of time, i.e. the homogeneity of time.

7 Fundamental equation of cosmology

The expansion rate of the Universe is increasing over time, not decreasing. This regularity does not follow from the Robertson-Walker metric for any curvature of space in the Universe, and it can therefore be concluded that the geometric curvature of the Universe implied from the Robertson-Walker metric is not decisive for the kinematics of the expansion of the Universe, especially when one wants to describe the accelerating expansion of the Universe, i.e. dark energy.

From now on, we will see that the kinematics of the expansion rate of the Universe does not depend on the curvature of space in the Universe but depends instead on the law of conservation of mechanical energy and the variation of a mysterious coefficient y in time. This means that the entire subsequent mathematical and physical analysis, following from the previously derived general time travel equation, shows us quite clearly that the variation of the expansion rate of the Universe over time does not really depend on the curvature of space in the Universe as the Robertson-Walker metric tries to show us.

The kinematics of the expansion rate of the Universe depends on the law of conservation of mechanical energy. The law of conservation of energy derives from the homogeneity of time. It follows from the physical systems of ordinary space K and hyperspace K` that time and space exist in ordinary space K, but time and space no longer exist in hyperspace K`, i.e. outside spacetime. As time and space exist in ordinary space K (due to its movement relative to hyperspace K`), the law of conservation of energy also applies to it. Different conservation laws derive precisely from the different properties of time and space, which, in turn, require the existence of time and space. Time and space do not exist outside spacetime, i.e. in hyperspace K`.

We have previously seen that the law of conservation of energy is mathematically derivable from the homogeneity of time. As time exists in ordinary space K (due to its movement relative to hyperspace K`), the law of conservation of energy also applies in it. Therefore, we are next going to see that the entire subsequent mathematical and physical analysis will show that the possibility to derive the law of conservation of energy from the homogeneity of time can also be demonstrated by an extended analysis of the previously derived general time travel equation, allowing us to derive from it the fundamental equation of cosmology, which is, in essence, the law of conservation of energy.

For example, the previously derived general time travel equation:

$$ct + vt' = \sqrt{1 - \frac{v^2}{c^2}}\, t'c$$

allows us to make the following mathematical transformations. If ct = 0, we get the following equation

$$vt' = \sqrt{1 - \frac{v^2}{c^2}}\, t'c$$

Let us divide both sides of the resulting equation by t´:

$$\frac{vt'}{t'} = \frac{\sqrt{1 - \frac{v^2}{c^2}}\, t'c}{t'} = \sqrt{1 - \frac{v^2}{c^2}}\frac{ct'}{t'}$$

and, as a result, we finally get the following very important equation:

$$v = \sqrt{1 - \frac{v^2}{c^2}}\, c$$

or, using visually better presentation:

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

As the time dilation equation, also derivable from the general time travel equation, has the following form

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

we can express the kinematic factor as follows:

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{t}{t'}$$

The formula derived from the general time travel equation

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

can therefore be presented in the following mathematical form:

$$v = c\sqrt{1 - \frac{v^2}{c^2}} = \frac{ct}{t'}$$

or

$$v = c\frac{t}{t'}$$

Let us move t´ to the other side

$$vt' = ct$$

and transform the last equation into the following form:

$$v \triangle t = ct$$

where $\triangle t = t'$.

As ordinary space K is moving relative to hyperspace K´ at the velocity c, the expansion rate of the Universe should theoretically also equal c. In reality, however, the expansion rate of the Universe is many times lower than the speed of light c. The only possibility to rationally explain such an apparent dilemma is to say that the difference in velocities is due to the transformation of time and space over the whole Universe. For example, the previously derived equation

$$v\Delta t = ct$$

quite clearly shows that the difference in velocities may be due to the fact that time has been transformed:

$$\frac{\Delta t}{t} = \frac{c}{v} = y$$

where y is the well-known kinematic factor from the theory of special relativity:

$$y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or from the theory of general relativity

$$y = \frac{1}{\sqrt{1 - \frac{R}{r}}}$$

As in the previously derived relativistic formula for the transformation of velocities

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

the relation between the different velocities is indicated by the kinematic factor y

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c}{v}$$

then according to the time dilation formula

$$\Delta t = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the y-factor indeed indicates the transformation of time:

$$\frac{\Delta t}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = y$$

In physical terms, this means that ordinary space K 'is moving' relative to hyperspace K′ at a constant velocity of c, causing the transformation of time, i.e. decreasing of the value of y. The decreasing of y, however, causes the expansion rate of the Universe to increase for the observer located inside the expanding Universe.

The previously derived equation

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

allows the mathematical derivation of the fundamental equation of cosmology, the Friedmann equation, and the formula for the rest energy of an object $E = mc^2$. For example, the kinetic energy E of an object m is expressed by the formula:

$$E = \frac{mv^2}{2}$$

When the velocities are small compared to the speed of light in a vacuum, the approximate formulae can be used:

$$\frac{1}{y} \approx 1 - \frac{1}{2}\beta^2$$

The expression for the kinematic factor y:

$$y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

can also be presented as follows

$$y = \frac{1}{\sqrt{1 - \beta^2}}$$

where β^2 is expressed as:

$$\beta^2 = \frac{v^2}{c^2}$$

Considering all of the above, the kinematic factor y can be substituted by the approximate formula:

$$\sqrt{1-\frac{v^2}{c^2}}=\sqrt{\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)^2-\frac{1}{4}\frac{v^4}{c^4}}$$

because $\left(\frac{v}{c}\ll 1\right)$. As the term

$$-\frac{1}{4}\frac{v^4}{c^4}$$

is very small, the last expression can be written as follows:

$$\sqrt{1-\frac{v^2}{c^2}}\approx\sqrt{\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)^2}\ \approx\ 1-\frac{1}{2}\frac{v^2}{c^2}$$

Consequently, we can express y as follows:

$$y\approx\ 1+\frac{v^2}{2c^2}$$

or

$$\frac{1}{y}\approx\ 1-\frac{v^2}{2c^2}$$

and perform the following mathematical transformations:

$$v\ =c\sqrt{1-\frac{v^2}{c^2}}\ =c\frac{1}{y}\approx c\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

When multiplying both sides of this equation by mc, we get the following ‘analysis of operations’:

$$v\ \approx c\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

or

$$mcv\ \approx mc^2\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

where we get rid of the parentheses:

$$mcv\ \approx mc^2-\frac{mv^2}{2}$$

and express the kinetic energy:

$$\frac{mv^2}{2}=mc^2-mcv$$

The last equation derived

$$\left(\frac{mv'^2}{2}\right)'=(\,mc^2-mcv\,)$$

has the physical content explained by the following analysis. For example, if the velocity of an object m relative to ordinary space K is zero, i.e. v = 0, then the kinetic energy of this object relative to hyperspace K´ is equal to the rest energy known from the theory of relativity, i.e.

$$\frac{mv^2}{2} = mc^2$$

NOTE: As we have seen hereinabove, a negative number could appear under the square root sign in the time dilation equation:

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t}{\sqrt{1-\frac{2c^2}{c^2}}} = \frac{t}{\sqrt{1-2}} = \frac{t}{\sqrt{-1}} = \frac{t}{i}$$

if the velocity is greater than the speed of light: $v^2 = 2c^2$. Now, we can see that by taking the derived equation:

$$\frac{mv^2}{2} = mc^2$$

and cancelling out the masses:

$$\frac{v^2}{2} = c^2$$

we get the expression for the square of the velocity presented above:

$$v^2 = 2c^2$$

This is obviously a coincidence, but nevertheless quite a remarkable one, which could indicate the fact that the mathematical derivations and expressions presented in this paper are all related and harmonically consistent. This creates a complex number comprising the imaginary unit: $i = \sqrt{-1}$.

If the velocity of an object relative to ordinary space K equals the speed of light c, i.e. v = c, then the kinetic energy of this object relative to hyperspace is zero, i.e.

$$\frac{mv^2}{2} = mc^2 - mc^2 = 0$$

$\frac{mv'^2}{2}$ is the kinetic energy of the object relative to hyperspace for the whole case presented above, and thus this expression only acts as a mathematical definition, i.e.

$$\frac{mc^2}{2} \neq mc^2$$

but instead

$$E'_k = mc^2 - mcv$$

or

$$E'_k = E_k$$

This means that the kinetic energy E'_k of the object m relative to hyperspace

$$E'_k = \frac{mv'^2}{2}$$

depends on the velocity v of the object relative to ordinary space K:

$$\frac{mv^2}{2} = mc^2 - mcv = mc(c - v)$$

or

$$\frac{mv^2}{2} = mc(c - v)$$

or

$$E'_k = mc(c - v)$$

But at the same time, the expression for the total energy of an object

$$E = mc^2 + \frac{mv^2}{2}$$

has $\frac{mv^2}{2}$ as the term of the equation only for ordinary space. This shows that if the velocity v of an object relative to ordinary space is zero, i.e. v = 0 (and thus $\frac{mv^2}{2}$ = 0), then the kinetic energy E of the object relative to hyperspace is $E = mc^2$, and this is called the 'rest energy' of the object.
In the previously derived equation

$$\frac{mv^2}{2} = mc^2 - mcv$$

we will move the term $-mcv$ to the other side of the equality sign:

$$\frac{mv^2}{2} + mcv = mc^2$$

We will now take into account that if $v = 0$ is in the term $+mcv$, the equation will be

$$\frac{mv^2}{2} = mc^2$$

and based on this relation $v^2 = -c^2$:

$$-\frac{mc^2}{2} = mc^2$$

The last relation $v^2 = -c^2$ derives from the formal geometry of the theory of special relativity, where the square of the four-dimensional momentum $p_\mu p_\mu$ can be defined with the rest energy as follows:

$$-{m_0}^2c^2 = p^2 - \frac{E^2}{c^2} = -{m_0}^2c^2\frac{1-\beta^2}{1-\beta^2} = \frac{{m_0}^2(v^2-c^2)}{1-\beta^2} = p_\mu p_\mu$$

where the coefficient term

$$\frac{v^2-c^2}{\sqrt{1-\beta^2}} = v_\mu v_\mu = v^2 = -c^2\frac{1-\beta^2}{1-\beta^2} = -c^2$$

is a time-like vector, and this is a constant. $v_\mu v_\mu$ is the square of the four-dimensional velocity here, which in effect shows that all objects in the Universe are moving at the speed of light c. Consequently, the (four-dimensional velocity) vector v_μ can be expressed as follows

$$v_\mu = \frac{dx_\mu}{d\tau}$$

where the quotient term

$$\tau = t\sqrt{1-\beta^2}$$

is the proper time of a moving object, and thus we will get the following as the final relation for the velocity vector:

$$v_\mu = \frac{dx_\mu}{t\sqrt{1-\beta^2}}$$

According to the mathematical definition of a four-dimensional velocity vector, the form of momentum p

$$p = m_0 v$$

will be as follows when using a different denotation

$$p_\mu = m_0 v_\mu$$

The previously derived equation

$$\frac{mv^2}{2} = mc^2 - mcv$$

or

$$\frac{mv^2}{2} + mcv = mc^2$$

is very important because the Einstein equation can be derived from it. For example, if in the case of mcv we have v = c and $mc^2 = E$, we get the expression for the total energy of an object:

$$mc^2 = mcv + \frac{mv^2}{2}$$

or

$$E = mc^2 + \frac{mv^2}{2}$$

where $\frac{mv^2}{2}$ is the kinetic energy of the object and mc^2 is the rest energy of the object. From the last formula, we can derive the relation between the mass and energy of an object in relativistic mechanics (the Einstein formula) as follows:

$$E = mc^2 + \frac{mv^2}{2} = mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) = mc^2 y = mc^2 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and indeed we get $E = mc^2$ when the object is stationary, i.e. v = 0.

But in the previously derived equation

$$\frac{mv^2}{2} = mc^2 - mcv$$

we moved the term $-mcv$ to the other side of the equality sign:

$$\frac{mv^2}{2} + mcv = mc^2$$

and if this term of the equation is equal to zero:

$$mcv = 0$$

i.e. the speed is zero $v = 0$, we get the following very important expression:

$$\frac{mv^2}{2} = mc^2$$

Let us now show that the last expression is in fact equivalent to the following formula:

$$\frac{mv^2}{2} = mcv$$

when the velocity $v = c$ in the term of the equation is mcv in the last formula. If, however, $v = 0$, we get the kinetic energy that is equal to zero:

$$\frac{mv^2}{2} = 0$$

According to the law of conservation of mechanical energy, the potential energy of an object must be at its maximum when its ‘motion energy’, or kinetic energy, is equal to zero:

$$\frac{mv^2}{2} = -\frac{GMm}{R}$$

As a result, we get the following relation:

$$-\frac{GMm}{R} = mcv$$

where $v = 0$. We will justify the entire previous reasoning right away with the following physical analysis. For example, when $v = 0$ is in the term of the equation mcv, it means in physical terms that the object m is stationary in relation to ordinary space K, but the object m is moving in relation to hyperspace K´ at the velocity of c. The kinetic energy $\frac{mv^2}{2} = E_k$ of the object m equals $E = mc^2$ and this is valid relative to hyperspace K´. In this case, we have the definition of kinetic energy: E_k. As $v = 0$ in the relation mcv, meaning that the object m is stationary relative to ordinary space K, i.e. the kinetic energy of this object is zero relative to ordinary space K, the state of rest or potential energy of the object m relative to ordinary space K is equal to the gravitational potential energy as follows:

$$mcv = -\frac{GMm}{R}$$

and this potential energy is no longer equal to zero in the equation.

It should be noted here that in the theory of special relativity, the expressions for the total and rest energy of an object do not consider the potential energy of the object due to the existence of an external field. The changing of the potential energy of an object in an external force field is not considered. Mass and energy are equivalent quantities. The relativistic mass of an object is also the measure of the object's total energy.

> All objects exist in ordinary space where time (and space) exist. Time as a duration is constantly 'moving'. This means that time will never 'stand still'. Moving objects have kinetic energy. Absolutely all objects in the Universe are moving relative to time (i.e. we are all moving towards the future in time), but time is not an object of any kind. This could be the source of the rest energy $E = mc^2$ for all objects in the Universe. This means that the energy mc^2 is essentially still an object's "kinetic energy relative to time". All objects are after all moving relative to hyperspace K´ as ordinary space K is moving relative to hyperspace K´ at the velocity of c. Consequently, all objects have kinetic energy and therefore mass. This way, the energy mc^2 could be the kinetic energy "relative to moving hyperspace", i.e. $E = mc^2$ is the energy existing relative to the time of the object.
>
> The physical system of ordinary space and hyperspace shows that all objects in the Universe are in effect moving at the speed of light c. This is directly due to the fact that according to the physical model, ordinary space is constantly moving relative to hyperspace at the speed of light c. All objects and phenomena exist in

spacetime, which according to the model is denoted as ordinary space moving relative to hyperspace. One of the best proofs for this is the expression for rest energy, carried by all objects in the Universe (it is equal to zero for light):

$$E = mc^2$$

The momentum p is then expressed as follows:

$$\frac{E}{c} = mc = p$$

Momentum is the product of velocity and mass, and due to the rest energy, all objects in the Universe with inertial mass have it. The speed of light c in fact represents the velocity relative to hyperspace, which follows from the moving of ordinary space relative to hyperspace. Objects 'standing' in spacetime also have this momentum and energy. It is one of the most direct manifestations of the physical system of ordinary space and hyperspace in the entire Universe.

In the theory of special relativity, the expressions for the total and rest energy of an object do not consider the potential energy of the object due to the existence of an external field. However, considering all the previous mathematical relations, we nevertheless obtain the following very important equation:

$$\frac{mv^2}{2} + mcv = mc^2$$

or

$$\frac{mv^2}{2} - \frac{GMm}{R} = -\frac{mc^2}{2}$$

where

$$+mcv = mc^2 = \frac{mv^2}{2} = -\frac{GMm}{R}$$

and

$$mc^2 = \frac{mv^2}{2} = -\frac{mc^2}{2}$$

The masses m all cancel out:

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

The relation derived last in fact directly expresses the law of conservation of mechanical energy:

$$E = E_k + E_p = const.$$

where

$$E = E_k + E_p = \frac{v^2}{2} + \left(-\frac{GM}{R}\right) = \frac{v^2}{2} - \frac{GM}{R} = const. = -\frac{c^2}{2}$$

The constant $-\frac{c^2}{2}$ in the last relation does not mathematically follow from the law of

conservation of energy known in classical mechanics. This form of the law of conservation of energy can only be derived via relativistic mechanics, as we have previously done here. We will later see that the constant term $-\frac{c^2}{2}$ present in the last equation can also be equal to zero:

$$\frac{v^2}{2} - \frac{GM}{R} = 0$$

that in turn will yield the law of conservation of mechanical energy in its classical form:

$$\frac{v^2}{2} = \frac{GM}{R}$$

The last equation obtained

$$\frac{v^2}{2} - \frac{GM}{R} = 0$$

or

$$\frac{v^2}{2} = \frac{GM}{R}$$

is well suited to describe the cosmological expansion of the Universe. The velocity v in the last equation could be the expansion rate of the Universe, i.e. the well-known Hubble's law v = HR:

$$\frac{H^2}{2} = \frac{GM}{R^3}$$

From the Robertson-Walker metric we know that the Hubble constant H equals:

$$H = \frac{\dot{a}}{a}$$

and therefore we can convert the last equation to the following form

$$\frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{GM}{R^3}$$

or

$$\frac{1}{2}\frac{\dot{a}^2}{a^2} = \frac{GM}{R^3}$$

We will move the term a^2 to the other side of the equality sign:

$$\frac{1}{2}\dot{a}^2 = \frac{GM}{R^3}a^2$$

The mass M can be expressed via the density:

$$M = \frac{4\pi R^3}{3}\rho$$

and so the equation will acquire the following form:

$$\frac{1}{2}\dot{a}^2 = \frac{4\pi G}{3}\rho a^2$$

We will move all the terms of the equation to one side of the equality sign and get:

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = 0$$

As this last equation was obtained from the equation derived above:

$$\frac{v^2}{2} - \frac{GM}{R} = 0$$

that in fact does not equal zero:

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

we therefore get the following:

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = \frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

or

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{c^2}{2}$$

This last equation is almost identical to the Friedmann equation obtained from the Robertson-Walker metric:

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{kc^2}{2}$$

where k = 1, 0, –1. This means that if the value of k in the last equation is +1, meaning we have positive curvature space of the Universe, we get a completely identical relation to the previously derived equation:

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{c^2}{2}$$

Physically, this means that according to the Robertson-Walker metric, the previously derived equation represents positive curvature space, and thus all other spatial curvatures of the Universe (i.e. flat space or negative curvature) are excluded. We are subsequently going to see that in fact the kinematics of the expansion rate of the Universe does not depend on positive spatial curvature either, it depends instead on the law of conservation of mechanical energy and the variation of a mysterious coefficient y over time.

7.1 Mathematical note

Newton's second law is expressed in relativistic mechanics as follows:

$$mya + my^3(\vec{\beta}a)\vec{\beta} = F$$

We will here multiply with β:

$$mya + my^3a\beta^2 = F$$

and factor out mya:

$$mya(1 + y^2\beta^2) = F$$

The expression in parentheses can be mathematically transformed as follows:

$$mya(1 + y^2\beta^2) = mya\left(1 + \frac{\beta^2}{1 - \beta^2}\right) = mya\left(1 + \frac{v^2}{\left(1 - \frac{v^2}{c^2}\right)c^2}\right) = mya\left(1 + \frac{v^2}{c^2 - v^2}\right)$$

The result is the expression for the relativistic force F:

$$mya\left(1 + \frac{v^2}{c^2 - v^2}\right) = F$$

or

$$\frac{F}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{F}{1 + \frac{v^2}{c^2 - v^2}}$$

We can cancel out the forces F:

$$1 + \frac{v^2}{c^2 - v^2} = \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{\frac{c^2 - v^2}{c^2}}$$

and move certain terms to the other side of the equality sign:

$$\frac{v^2}{c^2 - v^2} = \sqrt{\frac{c^2 - v^2}{c^2}} - 1$$

Let us set the equation to be equal to one:

$$1 = \left(\sqrt{\frac{c^2 - v^2}{c^2}} - 1\right)\frac{c^2 - v^2}{v^2}$$

and remove the parentheses:

$$1 = \sqrt{\frac{c^2 - v^2}{c^2}}\frac{c^2 - v^2}{v^2} - \frac{c^2 - v^2}{v^2}$$

We will move the last term and the square root to the other side of the equality sign in the equation:

$$\frac{c^2 - v^2}{v^2}\frac{1}{\sqrt{\frac{c^2 - v^2}{c^2}}} = \frac{c^2 - v^2}{v^2}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c^2 - v^2}{v^2}y = \frac{c^2 - v^2}{v^2}$$

As a result:

$$\frac{c^2 - v^2}{v^2}y = \frac{c^2 - v^2}{v^2}$$

or

$$\frac{v^2}{v^2}y = \frac{c^2 - v^2}{c^2 - v^2}$$

The resulting equation shows that $y = 1$ and:

$$v^2 = c^2 - v^2$$

By transforming the last equation:

$$v^2 + v^2 = 2v^2 = c^2$$

we get the expression for the square of the velocity:

$$v^2 = \frac{c^2}{2}$$

By multiplying both sides of the equation by mass m, and as v = c, we get the expression for the rest energy E:

$$E = mc^2 = \frac{mc^2}{2}$$

that is equivalent to the previously derived equation. This shows the correctness of all the analyses hereinabove as well as the correlation with the physics of time travel.

8 De Sitter Universe

The following relation must apply when dealing with a "vacuum energy dominated Universe":

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_\Lambda = \frac{\Lambda c^2}{3}$$

This means that the expansion rate of the Universe, or the rate of change of the scale factor, is proportional to the scale factor of the Universe: $\dot{a} \sim a$.

> Before discovering the dark energy of the Universe, the expansion of the Universe was thought to be slowing down due to the gravitational pull of matter-energy: $\ddot{a} < 0$. The following relation would apply in a radiation-dominated Universe: $a \sim \sqrt{t}$ and $\dot{a} \sim t^{-\frac{1}{2}}$; in a matter-dominated Universe, however: $a \sim t^{\frac{2}{3}}$ and $\dot{a} \sim t^{-\frac{1}{3}}$. In these, a stands for the scale factor of the Universe.

This leads to 'exponential expansion':

$$a(t) = a(t_1) * e^{\frac{t-t_1}{\Delta\tau}}$$

where

$$\Delta\tau = \sqrt{\frac{3}{\Lambda c^2}} = \sqrt{\frac{3}{8\pi G\rho}}$$

Since the Hubble constant can also be expressed like this in the Robertson-Walker metric:

$$H = \frac{\dot{a}}{a} = \frac{\dot{R}}{R}$$

we can write the exponential equation using this form:

$$R(t) = R(t_1) * e^{\frac{t-t_1}{\Delta\tau}}$$

This kind of a Universe is called the 'de Sitter Universe'. This shows that the larger the Universe, the faster it expands. The higher the negative vacuum pressure and ρ_Λ, the faster the Universe expands as ρ_Λ is a constant. This last relation is actually not quite accurate, as the relation between the Hubble constant H and the age of the Universe t can also be represented as follows:

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho a^2} * \frac{1}{\sqrt{2ca_m t}} = \sqrt{\frac{8\pi G}{3}a^2 * \frac{3}{32\pi G t^2} * \frac{1}{2ca_m t}} =$$

$$= \sqrt{\frac{8\pi G}{3} * 2ca_m t * \frac{3}{32\pi G t^2} * \frac{1}{2ca_m t}} = \sqrt{\frac{1}{4t^2}} = \frac{1}{2t}$$

We can therefore write:

$$H^2 = \frac{1}{4t^2} = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_\Lambda = \frac{\Lambda c^2}{3}$$

Next, we will analyse the relation between the age and the density of the Universe:

$$\frac{1}{4t^2} = \frac{8\pi G}{3}\rho_\Lambda = \frac{8\pi G}{3}\frac{M}{V} = \frac{8\pi G}{3}M\frac{3}{4\pi R^3} = \frac{2GM}{R^3}$$

or

$$\frac{1}{4t^2} = \frac{2GM}{R^3}$$

We can see the expressions for kinetic and potential energy in the relation obtained:

$$\frac{1}{4}\frac{R^2}{t^2} = \frac{1}{4}v^2 = \frac{2GM}{R}$$

But, in this case, we must consider the square of the speed of light:

$$\frac{2GM}{R} = c^2$$

We will justify its use a little later. This course of action will give us the following expression:

$$\frac{v^2}{4} = c^2$$

Let us multiply both sides of the last expression by the mass m:

$$\frac{mv^2}{4} = mc^2$$

and transform accordingly:

$$\frac{mv^2}{2} = 2mc^2 = mc^2 + mc^2$$

The resulting expression:

$$\frac{mv^2}{2} = mc^2 + mc^2$$

can be written using the form below due to the relation $v^2 = -c^2$:

$$\frac{mv^2}{2} = mc^2 - mv^2$$

or

$$\frac{mv^2}{2} = mc^2 - mcv$$

This result coincides completely with the expression obtained above, which clearly shows that the fundamental equation of cosmology, i.e. the Friedmann equation, can be derived from the general time travel equation. For further analysis, let us move one term of the equation:

$$\frac{mv^2}{2} = mc^2 + mc^2$$

to the other side of the equality sign:

$$\frac{mv^2}{2} - mc^2 = mc^2 = E$$

As a result we now have the following equation:

$$E = \frac{mv^2}{2} - mc^2$$

which is almost identical to the expression for the total energy of a physical object, known in the theory of special relativity:

$$E = \frac{mv^2}{2} + mc^2$$

The only difference is the sign, as one expression involves subtraction and the other addition. Thus, in this case, we must only consider that if v = 0, we will get:

$$E = -mc^2$$

and we will get a positive result by first squaring this equation and then taking its square root:

$$E = mc^2$$

The final result obtained coincides with the rest energy equation for an object, known from the theory of special relativity, and this strongly suggests some sort of relation between the expansion of the Universe and the rest energy of an object.

However, at this point, we need to justify the method used above:

$$\frac{2GM}{R} = c^2$$

This is basically the expression for the Schwarzschild radius R, but let us justify it with the following analysis. For example, let us take from the relation provided above:

$$H^2 = \frac{1}{4t^2} = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_\Lambda = \frac{\Lambda c^2}{3}$$

the following equality:

$$\frac{1}{4t^2} = \frac{\Lambda c^2}{3}$$

Let us transform the last expression as follows:

$$\frac{3}{4\Lambda} = c^2t^2 = L^2$$

and consider the relation that we will derive later:

$$\frac{3}{\Lambda} = -R^2$$

As a result:

$$-\frac{R^2}{4} = c^2t^2$$

or

$$-\frac{1}{4}\frac{R^2}{t^2} = c^2$$

where we can see the square of the speed of light:

$$-\frac{v^2}{4} = c^2$$

Let us multiply both sides of this equation by the mass m:

$$-\frac{mv^2}{4} = mc^2$$

and transform accordingly:

$$-\frac{mv^2}{2} = 2mc^2 = mc^2{+}mc^2$$

In the resulting expression:

$$-\frac{mv^2}{2} = mc^2{+}mc^2$$

we must get rid of the minus sign. To do this, we square both sides and then take the square root:

$$\frac{mv^2}{2} = mc^2{+}mc^2$$

We can again apply the relation $v^2 = -c^2$ for this last one and write the equation in the following form as a result:

$$\frac{mv^2}{2} = mc^2 - mv^2$$

This coincides fully with the expression obtained above:

$$\frac{mv^2}{2} = mc^2 - mcv$$

This analysis also resulted in the expression that is otherwise derived from the general time travel equation. For further analysis, let us move one term of the equation:

$$\frac{mv^2}{2} = mc^2 + mc^2$$

to the other side of the equality sign:

$$E = \frac{mv^2}{2} - mc^2$$

If v = 0, then the final result will fully coincide with the expressions obtained above:

$$E = - mc^2$$

or

$$E = mc^2$$

which demonstrates the accuracy and validity of the whole preceding analysis.

9 Expansion rate of the Universe

Before proceeding to the deep analysis of the fundamental equation of cosmology, we must prove the preceding analysis of ideas. This means that the whole preceding reasoning can be proved by the following mathematical analysis, showing the correctness of the previously shown derivations.

For further analysis, we will first take the fundamental equation of cosmology derived above

$$\frac{H^2}{2} - \frac{GM}{R^3} = -\frac{c^2}{2}$$

and set H to be equal to zero:

$$-\frac{GM}{R^3} = -\frac{c^2}{2}$$

Let us substitute the mass M with the density of matter and energy in the Universe:

$$\frac{G4\pi R^3}{R^3 3}\rho = \frac{c^2}{2}$$

and perform some simple mathematical transformations. The result will be:

$$\frac{8\pi G}{3}\rho = c^2$$

We can express the equation for the density of the Universe from this last relation:

$$\rho = \frac{M}{V} = \frac{3c^2}{8\pi G}$$

Let us keep this in mind, and for further analysis let us set the constant c in the fundamental equation of cosmology:

$$\frac{H^2}{2} - \frac{GM}{R^3} = -\frac{c^2}{2}$$

to be equal to zero instead, the result will be:

$$\frac{H^2}{2} - \frac{GM}{R^3} = 0$$

or

$$\frac{v^2}{2} = \frac{GM}{R}$$

where Hubble's law is v = HR and M is the density of the Universe. Hence, we get the equation:

$$\frac{H^2R^2}{2} = \frac{4\pi GR^3}{3R}\rho_0$$

where we can see that the constant 8πG is equal to:

$$\frac{H^2R^2 3R}{R^3\rho_0} = 8\pi G = \frac{H^2}{\rho_0}3$$

or

$$8\pi G = \frac{c^2 3}{\rho}$$

Since the matter-energy density of the Universe was extremely high at the initial moment of its expansion, the pressure of the Universe was also decisive during this relatively short cosmological period. The pressure p of the Universe was first presented in the first Friedmann equation:

$$\ddot{a} = -\frac{4\pi G a}{3}\left(\rho + \frac{3p}{c^2}\right)$$

For the following analysis, we will only consider the second half of the equation:

$$\ddot{a} = -\frac{4\pi G a}{3}\rho = -\frac{4\pi G a}{3}\frac{3p}{c^2} = -\frac{4\pi G a}{c^2}p$$

where p is the pressure of the Universe. From the last equation, we also get the Hubble constant H:

$$\frac{\ddot{a}}{a} = -\frac{H^2}{2} = -\frac{4\pi G}{c^2}p$$

or

$$-\frac{H^2}{2} = -\frac{4\pi G}{c^2}p$$

and this shows that the expansion of the Universe also depended on the matter-energy pressure p of the Universe. Let us multiply both sides of the last equation by 2:

$$-\frac{2H^2}{2} = -\frac{8\pi G}{c^2}p$$

since above we were able to derive the equality:

$$8\pi G = \frac{H^2}{\rho}3$$

When we now use this to express 8πG, we get the following relation:

$$-\frac{2H^2}{2} = -\frac{H^2 3}{\rho c^2}p$$

coinciding with the relation between the pressure p and the density ρ of the Universe in the Friedmann equation:

$$p = \frac{\rho c^2}{3}$$

However, in addition to the above, the previously derived equation:

$$-\frac{2H^2}{2} = -\frac{8\pi G}{c^2}p$$

allows us to use the following equality:

$$8\pi G = \frac{c^2}{\rho}3$$

where H = c. We get the following relation by moving c^2 to the other side of the equality sign:

$$\frac{8\pi G}{c^2} = \frac{3}{\rho}$$

The pressure p and density ρ of the Universe were related as follows in the Friedmann equation:

$$p = \frac{\rho c^2}{3}$$

which in turn will yield the equality for c^2:

$$\frac{p3}{\rho} = c^2$$

Considering these last simple relations, we can derive the following well-known equation:

$$-\frac{2H^2}{2} = -H^2 = -\frac{8\pi G}{c^2}p = -\frac{3}{\rho}p = -c^2$$

or

$$-H^2 = -c^2$$

showing that H = c. Such a coincidence cannot be accidental. The physical and mathematical analysis of the equation for the pressure of the Universe shows that in cosmology, all fundamental equations boil down to the fact that the Universe is actually expanding at the speed of light c.

For example, if in the cosmology equation derived before

$$\frac{1}{2}(\dot{a})^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{c^2}{2}$$

the constant is equal to zero

$$\frac{1}{2}(\dot{a})^2 - \frac{4\pi G}{3}\rho a^2 = 0$$

and we will move one term to the other side of the equality sign so both sides of the equation are negative:

$$-\frac{1}{2}(\dot{a})^2 = -\frac{4\pi G}{3}\rho a^2$$

we will get the following equality, in this case, presented as a postulate:

$$-\frac{1}{2}(\dot{a})^2\frac{1}{a} = -\frac{4\pi G}{3}\rho a \; = \ddot{a} = \frac{D^2 a}{dt^2}$$

or

$$\ddot{a} = -\frac{1}{2}\frac{(\dot{a})^2}{a}$$

This latter relation is known in cosmology as the Friedmann equation if the pressure of the

Universe is equal to zero, i.e. p = 0. We will prove this later on. Based on the Robertson-Walker metric, the Hubble constant H can be represented as follows:

$$H = \frac{\dot{a}}{a}$$

where

$$\dot{a} = Ha$$

Hence, we get the following form for this last equation:

$$\ddot{a} = -\frac{1}{2}H^2 a$$

or

$$\frac{\ddot{a}}{a} = -\frac{1}{2}H^2 = -\frac{H^2}{2}$$

By taking for the Hubble constant H

$$H = \frac{\dot{a}}{a}$$

the first order derivative with respect to time, we get

$$\dot{H} = \frac{D}{dt}\left(\frac{\dot{a}}{a}\right) = \frac{\ddot{a}a - \ddot{a}\dot{a}}{a^2} = \frac{\ddot{a}a}{a^2} - \frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = \frac{\ddot{a}}{a} - H^2$$

or

$$\dot{H} = \frac{\ddot{a}}{a} - H^2$$

As the term present in the last equality can be represented as follows based on the previously derived relation

$$\frac{\ddot{a}}{a} = -\frac{H^2}{2}$$

we obtain the following very important equation:

$$\dot{H} = -\frac{H^2}{2} - H^2$$

In the following, we will take into account that the Universe is actually expanding at the speed of light c:

$$H = v = c$$

and thus we get

$$\dot{c} = -\frac{c^2}{2} - c^2$$

As the derivative of the constant c is zero

$$0 = -\frac{c^2}{2} - c^2$$

multiplying both sides of the last equation by the mass m gives us:

$$0 = -\frac{mc^2}{2} - mc^2$$

We will now move the term $-mc^2$ of this equation to the other side of the equality sign:

$$mc^2 = -\frac{mc^2}{2}$$

and take into consideration the relation known from the theory of special relativity

$$-c^2 = v^2$$

and finally, we get the equation

$$\frac{mv^2}{2} = mc^2$$

Since the classical formula for kinetic energy can be expressed as follows:

$$\frac{mv^2}{2} = E$$

we can present the equation for rest energy:

$$E = mc^2$$

that can be understood as the definition of kinetic energy. This means that the Universe is actually expanding over time at the speed of light, and because of that, all objects in the Universe have kinetic energy, which in this case is expressed as the formula for rest energy. Consequently, all objects existing in the Universe also have the momentum p:

$$\frac{E}{c} = mc$$

or

$$mc = p$$

where v = c. This also shows that all objects in the Universe are in effect moving at the speed of light because the expansion rate of the Universe is equal to the speed of light c, and all the objects are moving together with it at the speed of light c. The last equation obtained

$$\frac{mv^2}{2} = mc^2$$

is actually equivalent to the equation

$$\frac{mv^2}{2} = mc^2 - 0$$

or

$$\frac{mv^2}{2} = mc^2 - mcv$$

when the velocity of the object m in relation to ordinary space K is equal to zero: $mcv = 0$ or $v = 0$. Since we are now back to the equation used to derive the fundamental equation of cosmology in the theory of time travel, we have proved the entire preceding reasoning and analysis of ideas.

The cosmological expansion of the Universe results in an inertial force F_{in} affecting all objects; in short, the formula describing it is derived from the above formulas as follows. As we know, the well-known Hubble's law is expressed as follows:

$$v = HR$$

or

$$v = Hl$$

where $l = R$. After multiplying both sides of the last equation by the mass m, the definition of momentum p will obtain the following form:

$$mv = p = mlH$$

Next, we will divide both sides of the equation by time t, getting as a result the inertial force F_{in}, which is due to the expanding of the Universe:

$$\frac{p}{t} = \frac{mv}{t} = m\frac{v}{t} = ma = F_{in} = \frac{mlH}{t} = ml\frac{H}{t}$$

A relation describing the dependence of the Hubble constant H on time has been proven in cosmology:

$$\frac{H}{t} = \dot{H} = \frac{\ddot{a}}{a} - H^2$$

and we will therefore get the following form for the formula for inertial force:

$$F_{in} = ml\dot{H}$$

or

$$F_{in} = ml\left(\frac{\ddot{a}}{a} - H^2\right)$$

The following relation has also been proven in cosmology:

$$\frac{\ddot{a}}{a} = -\frac{H^2}{2}$$

and thus we can also write the formula for the inertial force F_{in} using the following form:

$$F_{in} = ml\left(-\frac{H^2}{2} - H^2\right)$$

In the last equation, H is the expansion rate of the Universe, m is the mass of an object or galaxy, and l is the distance between objects or galaxies.

For the following, it is interesting to note that in some other sources of literature on the theory of special relativity, the rest energy equation is derived from the classical expression for kinetic energy E:

$$E = \frac{mv^2}{2} = mk$$

or

$$E_{kin} = m_{kin}k$$

As the mass of an object increases in proportion to the kinematic factor y, i.e. the mass of a moving object increases y times compared to its stationary mass, the kinetic mass can be expressed as follows:

$$m_{kin} = m - m_0 = ym_0 - m_0 = m_0(y - 1)$$

where

$$m = ym_0 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m_0$$

We can thus transform the formula for kinetic energy into the following form:

$$E_{kin} = m_0(y - 1)k$$

For low velocities, the following approximate formula is used for y:

$$y = 1 + \frac{v^2}{2c^2}$$

and, as a result, we get the classical formula for kinetic energy:

$$E_{kin} \approx m_0 \frac{v^2}{2c^2} k$$

and the equation for rest energy

$$E_{kin} = m_{kin}k$$

when $k = c^2$.

The equation for rest energy derived in the theory of special relativity describes an energy that is in essence the energy of the existence of matter in the Universe. A remarkable fact about this equation is that it is in principle also directly derivable from classical mechanics. For example, from the law of conservation of energy:

$$\frac{mv^2}{2} = E_k = \frac{GMm}{R} = U$$

we immediately get the expression for the rest energy E:

$$mv^2 = \frac{2GMm}{R}$$

when the velocity v of the object is equal to the speed of light c:

$$E = mc^2 = \frac{2GMm}{R} = 2U = 2E_k$$

From the last equation, we can see that the rest energy is equal to twice the gravitational potential. However, when considering the equation derived in cosmology:

$$\frac{mc^2}{2} = mc^2$$

we get the following interesting relations:

$$E = mc^2 = \frac{GMm}{R} = U = E_k$$

It is worth noting that the gravitational field is not an energy field, and therefore the relationship:

$$E = U = \varphi$$

could instead express the potential of an energy field (for example, the potential of an electric field). The expression for rest energy is also mathematically derivable from Newton's second law:

$$F = ma = mv\frac{1}{t}$$

For example, when putting Newton's second law into the formula describing the definition of work A:

$$A = E = Fx$$

we get the expression for the rest energy

$$E = mv\frac{x}{t} = mv^2$$

when the velocity v of the object is also equal to the speed of light c:

$$E = mc^2$$

Although the strict mathematical derivation of the equation for rest energy is primarily performed in relativistic mechanics, in principle, the equation for rest energy can nevertheless be

expressed to some extent also in Newtonian mechanics.

10 Solutions to Friedmann equations

In the following, we will explore and present for further analysis the solutions to the previously derived Friedmann equations for the flat space of the Universe. The flat space of the Universe is described by the following Robertson-Walker metric:

$$ds^2 = c^2dt^2 - a^2(t)\{(d\chi)^2 + K[(d\theta)^2 + sin^2\theta(d\varphi)^2]\}$$

where $K = \chi^2$, i.e. the space of the Universe is flat. Its regions of variation are the following: $\chi \in (0, \infty)$, $\theta \in (0, \pi)$, $\varphi \in (0, 2\pi)$. The Robertson-Walker metric can also be presented in the following format:

$$ds^2 = -dt^2 + a(t)^2\left\{\frac{dr^2}{1-kr^2} + r^2[(d\theta)^2 + sin^2\theta(d\varphi)^2]\right\}$$

As the space of the Universe is flat, i.e. k = 0, the last equation will get the following form

$$ds^2 = -dt^2 + a(t)^2\{dr^2 + r^2[(d\theta)^2 + sin^2\theta(d\varphi)^2]\}$$

where the quantity r is the comoving radial coordinate moving along with the coordinate system. The solutions to the Friedman equations in the parametric form for p = 0 and flat space (k = 0) are, however, as follows:

$$t = \frac{a_m}{12c}\eta^3\,, \qquad a = \frac{a_m}{4}\eta^2\,, \qquad H = \frac{8c}{a_m}\eta^{-3}\,,$$

where

$$\eta = 0 \leqslant \eta \leqslant \infty\,.$$

Next, we will study and analyse the changing of Hubble's constant H over time, or the expansion rate of the Universe over time. For example, let us take the following limit:

$$\lim_{\eta\to 0} H = \lim_{\eta\to 0}\frac{8c}{a_m}\eta^{-3} = \lim_{\eta\to 0}\frac{8}{a_m}\frac{c}{\eta^3} = \frac{8}{a_m}\frac{c}{0} = \infty$$

The last calculation of the limit shows that at the initial moment of the expansion of the Universe, i.e. at t = 0, the expansion rate of the Universe was infinitely high, i.e. $H(t) \longrightarrow \infty$. This, however, contradicts astronomical observations. But let us find the following limit of the Hubble constant H:

$$\lim_{\eta\to\infty} H = \lim_{\eta\to\infty}\frac{8c}{a_m}\eta^{-3} = \lim_{\eta\to\infty}\frac{8}{a_m}\frac{c}{\eta^3} = \lim_{\eta\to\infty}\frac{\infty c}{\infty} = c$$

The last limit is obtained because a_m is zero, as it shows the curvature of the space of the Universe and has the value of zero when space is flat:

$$\frac{8}{a_m} = \frac{8}{0} = \infty$$

Infinity cubed is still equal to infinity:

$$\infty^3 = \infty$$

and the quotient of two infinities equals one

$$\frac{\infty}{\infty} = 1$$

because the result of their subtraction equals zero:

$$\infty - \infty = 0$$

c is the speed of light in a vacuum and the constant of integration a_m indicates how curved space is (in this case, it is zero or flat: $a_m = 0$). The last limit shows us that in the infinitely distant future, i.e. at $t \longrightarrow \infty$, the expansion rate of the Universe has decreased to the velocity c. The Hubble constant H

$$H = \frac{8c}{a_m}\eta^{-3}$$

can be written using the following form

$$H = yc$$

where the coefficient y is

$$y = \left(\frac{8}{a_m}\frac{1}{\eta^3}\right)$$

In this case, the range of variation of this coefficient y is

$$y = 1 \leqslant y \leqslant \infty$$

and because of this, the regularity of variation of the Hubble constant H over time

$$H = yc$$

contradicts empirical observations as the expansion rate of the Universe is increasing over time, not decreasing. It should be noted that the above regularity was derived from the Robertson-Walker metric for the flat space of the Universe. The following law of change would, however, be more correct

$$H = \frac{c}{y}$$

where the range of variation of the coefficient y is also

$$y = 1 \leqslant y \leqslant \infty$$

and this is already well in line with astronomic observations because the expansion rate of the Universe is increasing over time, not decreasing. This regularity does not follow from the Robertson-Walker metric for any curvature of space in the Universe, and it can therefore be concluded that the geometric curvature of the Universe implied from the Robertson-Walker metric is not decisive for the kinematics of the expansion of the Universe, especially when one wants to describe the accelerating expansion of the Universe, i.e. dark energy.

The Hubble constant H

$$H = \frac{c}{y}$$

indicates the expansion rate of the Universe v

$$v = \frac{c}{y}$$

and here we can see that the expansion rate of the Universe actually depends on the transformation of time over the whole Universe:

$$\frac{\Delta t}{t} = \frac{c}{v} = y$$

where y is the well-known kinematic factor from the theory of special relativity:

$$y = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or from the theory of general relativity:

$$y = \frac{1}{\sqrt{1 - \frac{R}{r}}}$$

This means that when the kinematic factor y shows the relation between different velocities

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c}{v}$$

then according to the time dilation formula known from the theory of special relativity

$$\Delta t = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the y-factor indeed indicates the transformation of time:

$$\frac{\Delta t}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = y$$

In physical terms, this means that ordinary space K 'is moving' relative to hyperspace K´ at a constant velocity of c, causing the transformation of time, i.e. decreasing of the value of y. The decreasing of y, however, causes the expansion rate of the Universe to increase for the observer located inside the expanding Universe. The changing of y shows the temporal progress of the cosmology of the Universe. The magnitude of y indicates the number of times time and space have been transformed for a real observer located inside the Universe or the number of times time and space have been transformed for a hypothetical observer located outside of the Universe. y is not a constant. Its value decreases over time, and due to that, for a real observer located inside the Universe, the expansion rate of the Universe increases over time.

11 Cosmological constant

The gravitational field is the curvature of the spacetime mathematically described by Albert Einstein's differential equation:

$$G_{\mu\nu}\left(g_{\rho\delta}(x)\right) = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

We will analyse this equation at much greater length in the section dealing with the theory of general relativity, and so we are not going to do it here. However, the last equation was actually presented by Albert Einstein at first as comprising the 'cosmological constant' Λ:

$$G_{\mu\nu}\left(g_{\rho\delta}(x)\right) = -\frac{8\pi G}{c^4} T_{\mu\nu} + g_{\mu\nu}\Lambda$$

because he wanted to describe a static, i.e. non-expanding, Universe. The spacetime of the Universe is described by the Robertson-Walker metric:

$$ds^2 = c^2 dt^2 - a^2(t) dl_k^2$$

where the curvature of space may be either positive or negative, or the space may be flat:

$$dl_k^2 = \begin{cases} d\chi^2 + sin^2\chi(d\theta^2 + sin^2\theta d\varphi^2), & k = +1 \\ d\chi^2 + \chi^2(d\theta^2 + sin^2\theta d\varphi^2), & k = 0 \\ d\chi^2 + incl.^2\,\chi(d\theta^2 + sin^2\theta d\varphi^2), & k = -1 \end{cases}$$

The energy-momentum tensor is expressed in a general form as follows:

$$T^{\mu v} = \begin{pmatrix} \rho c^2 & \rho c v^i \\ \rho c v^j & \rho v^i v^j + p\delta^{ij} \end{pmatrix}$$

but in a coordinate system comoving with matter, it acquires the following form:

$$T^{\mu v} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

If Albert Einstein's differential equation for the gravitational field included the Robertson-Walker metric, we could derive the Friedmann equations with the cosmological constant Λ:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{c^2}\left[(p_M + p_\Lambda) + \frac{1}{3}(\rho_{AE} + \rho_\Lambda)^2\right]$$

which, due to the cosmological constant, comprise the energy density

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$$

and 'negative pressure':

$$p_\Lambda = -\rho_\Lambda c^2 < 0$$

With the cosmological constant, positive energy density leads to negative pressure.
Friedmann equations:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{c^2}\left[(p_M + p_\Lambda) + \frac{1}{3}(\rho_M + \rho_\Lambda)c^2\right]$$

contain the matter-energy density ρ_M and pressure p_M subject to gravitational attraction, and the energy density ρ_Λ and pressure p_Λ due to the cosmological constant. As 'negative pressure' is expressed as

$$p_\Lambda = -\rho_\Lambda c^2$$

and the pressure of matter in the Universe is zero at the present moment:

$$p = 0$$

we get the following expression from the second Friedmann equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_M - 2\rho_\Lambda)$$

Since the original intention was to describe a static Universe:

$$\dot{a} = 0$$

$$\ddot{a} = 0$$

then following from the last equation:

$$\rho_M = 2\rho_\Lambda = \frac{\Lambda c^2}{4\pi G}$$

If we now insert the resulting expression in the first Friedmann equation:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda)$$

we get the following result:

$$\frac{kc^2}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda) = \frac{8\pi G}{\rho_{\ \Lambda}} = \Lambda c^2$$

In mathematical terms, this can be expressed in longer form as follows:

$$\frac{kc^2}{a^2} = \frac{8\pi G}{3}\left(\frac{\Lambda c^2}{4\pi G} + \frac{\Lambda c^2}{4\pi G}\frac{1}{2}\right) = 2\frac{\Lambda c^2}{3} + 2\frac{\Lambda c^2}{3}\frac{1}{2} = 2\frac{\Lambda c^2}{3} + \frac{\Lambda c^2}{3} =$$

$$= 2\frac{\Lambda c^2}{3} + \frac{\Lambda c^2}{3} = \frac{\Lambda c^2}{3} + \frac{\Lambda c^2}{3} + \frac{\Lambda c^2}{3} = 3\frac{\Lambda c^2}{3} = \Lambda c^2$$

Since the cosmological constant is usually positive:

$$\Lambda > 0$$

the space of the Universe must also have a positive curvature:

$$k = +1$$

The scale factor a (i.e. the radius) of the Universe and the cosmological constant Λ are related:

$$a = \frac{1}{\sqrt{\Lambda}}$$

This means that for a static Universe, the cosmological constant determines the matter-energy density and the radius of the Universe, i.e. its size.

The existence of this kind of a repelling force that is manifested only when the distance between objects increases has previously been interpreted in cosmological calculations as ‘dark

energy'. However, this essence of 'dark energy' has been interpreted primarily as vacuum energy, creating such a known repelling force. It is calculated as follows. Using Poisson's equation, it can be written in the form of the gravitational potential:

$$\nabla^2\Phi = 4\pi G\left(\rho + \frac{3p}{c^3}\right) = 4\pi G p(1+3\omega)$$

where the pressure also shows the source of the gravitational force, and the density ρ and pressure p are expressed, respectively as

$$\rho = \rho_A + \rho_\Lambda$$

$$p = p_A + p_\Lambda$$

where p is the pressure and ρ is the density, and respectively their A-indices indicate the total density (total pressure) of ordinary matter, energy and dark matter. The equation describes matter that is subject to gravity. If we take, however,

$$p_A = 0$$

$$p_\Lambda = -\rho_\Lambda c^2$$

where

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$$

we obtain the following expression from the first equation

$$\nabla^2\Phi = 4\pi G\rho_M - \Lambda c^2$$

Assuming that the 'vacuum energy' is very high

$$\Lambda c^2 >> 4\pi G\rho_A$$

we get

$$\Phi_\lambda = -\frac{\Lambda c^2}{6} r^2$$

and thus the mass is affected by the force

$$\vec{g}_r = -\vec{\nabla}\Phi_\Lambda = \frac{\Lambda c^2}{3}\vec{r}.$$

The repelling force increasing as the distance between objects increases becomes evident from this last equation. This means the vacuum energy induces a repelling force that starts to manifest especially well on the scale of an extremely large space.

We will subsequently analyse in more detail the Friedmann equation containing the cosmological constant:

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G}{3}(\rho_M + \rho_\Lambda)$$

or

$$\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2}=\frac{8\pi G}{3}(\rho_{AE}+\rho_{\Lambda})$$

For this, we remove the parentheses:

$$\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2}=\frac{8\pi G}{3}\rho_{AE}+\frac{8\pi G}{3}\rho_{\Lambda}$$

and since the matter-energy density containing the cosmological constant is expressed as:

$$\rho_{\Lambda}=\frac{\Lambda c^2}{8\pi G}$$

we obtain the following final form for the Friedmann equation:

$$\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2}=\frac{8\pi G}{3}\rho_{AE}+\frac{\Lambda c^2}{3}$$

Let us perform some mathematical transformations with the last equation:

$$\frac{\dot{a}^2}{a^2}-\frac{8\pi G}{3}\rho_{AE}=-\frac{kc^2}{a^2}+\frac{\Lambda c^2}{3}$$

and divide both sides of the equation by the following expression:

$$\frac{a^2}{2}$$

As a result, we get this expression:

$$\frac{\dot{a}^2}{2}-\frac{8\pi G}{6}\rho_{AE}a^2=-\frac{kc^2}{2}+\frac{\Lambda c^2}{6}a^2$$

As in the first Friedmann equation:

$$\frac{\dot{a}^2}{2}-\frac{8\pi G}{6}\rho_{AE}a^2=-\frac{kc^2}{2}+\frac{\Lambda c^2}{6}a^2$$

both sides are equally equivalent to each other, then this equality should also apply:

$$\frac{\dot{a}^2}{2}=-\frac{kc^2}{2}$$

or

$$\frac{\dot{a}^2}{a^2}=-\frac{kc^2}{a^2}$$

According to the Robertson-Walker metric, it can be used to express the Hubble constant H:

$$\frac{\dot{a}^2}{a^2} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = -\frac{kc^2}{a^2}$$

and from this, we get:

$$H^2 a^2 = -c^2$$

The relation we obtained lacks the constant k because we are not considering it (i.e. the curvature of Robertson-Walker space). Since the Hubble constant H can be expressed in the Robertson-Walker metric:

$$H \equiv \frac{\dot{R}}{R} = \frac{\dot{a}}{a}$$

we thus end up with Hubble's law:

$$H^2 R^2 = -c^2$$

where the Universe expands at the speed of light c:

$$v^2 = -c^2$$

This last relation is similar to the equality derived in the theory of special relativity:

$$\frac{v^2 - c^2}{\sqrt{1-\beta^2}} = v_\mu v_\mu = v^2 = -c^2 \frac{1-\beta^2}{1-\beta^2} = -c^2$$

In physical terms, this means that according to Hubble's law v = HR, in infinity, the expansion rate of the Universe v (not H) is equal to the speed of light c, and therefore the spacetime of the Universe has been transformed to infinity. We are only considering the positive speed of light c here.

> As the negative velocity $v^2 = -c^2$ does not exist physically, by squaring the last expression and then taking its square root, we still get a positive velocity: $v^2 = c^2$ or $v = c$, which is already in accordance with the physical reality.

On an extremely large spatial scale (for example, even on a larger spatial scale than superclusters of galaxies), the expansion rate of the Universe v is approaching the speed of light in the vacuum. If z > 1 in the formula $v_r = cz$, then the separation velocity v_r of galaxies moving away from each other is greater than the speed of light in the vacuum. In this case, the change in wavelength or the shift of spectral line at relative velocities close to the speed of light is found with the help of the theory of relativity. For example, the frequency of a wave decreases, i.e. the wavelength increases, as the source of light moves away from us, the observer:

$$\frac{v}{c} = \frac{(z\ +\ 1)^2 - 1}{(z\ +\ 1)^2 + 1}$$

and

$$z = \sqrt{\frac{c+v}{c-v}} - 1$$

Here, the redshift z depends on the velocity of separation v in the relativistic form or for $z = \frac{\triangle\lambda}{\lambda} > 1$, which is derived and analysed in more detail in the cosmology section above.

It should be noted here that if the expansion rate of the Universe v in Hubble's law:

$$v = HR$$

is exactly equal to the speed of light c:

$$c = HR$$

then the volume of the Universe must be infinitely large, as the expansion rate of the Universe v can only be exactly equal to the speed of light c in infinity. The infinite volume of the Universe would mean that the number of galaxies is infinite.

If based on the first Friedmann equation:

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

the following equality applies:

$$\frac{\dot{a}^2}{2} = +\frac{\Lambda c^2}{6}a^2$$

as a result, we get the following relation:

$$\frac{\dot{a}^2}{a^2} = H^2 = +\Lambda\frac{c^2}{3}$$

As the cosmological constant Λ is related to the square of the radius R (we will prove that a little later):

$$-1 = +\Lambda\frac{R^2}{3}$$

or

$$-\frac{1}{R^2} = +\frac{\Lambda}{3}$$

we get the following solution from the equation for the square of the Hubble constant H:

$$H^2 = -\frac{c^2}{R^2}$$

or

$$H^2R^2 = v^2 = -c^2$$

which shows us that in infinity, the expansion rate of the Universe v is equal to the speed of light.

Since based on the first Friedmann equation:

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

we will prove (albeit a little later) this equality:

$$-\frac{8\pi G}{6}\rho_{AE}a^2 = +\frac{\Lambda c^2}{6}a^2$$

and we have already seen that the following relation also applies:

$$\frac{\dot{a}^2}{2} = +\frac{\Lambda c^2}{6}a^2$$

then the following equation must thus also apply:

$$\frac{\dot{a}^2}{2} = -\frac{8\pi G}{6}\rho_{AE}a^2$$

This last relation is very easy to prove. For example, we will perform the following mathematical transformations on it:

$$\frac{\dot{a}^2}{2a^2} = \frac{H^2}{2} = -\frac{4\pi G}{3}\frac{E}{Vc^2} = -\frac{GE}{R^3c^2} = -\frac{GM}{R^3}$$

Given the relations obtained above, we again get the square of the speed of light c:

$$H^2R^2 = v^2 = -\frac{2GM}{R} = -c^2$$

or

$$H^2R^2 = v^2 = -c^2$$

which shows us that in infinity, the expansion rate of the Universe v is equal to the speed of light.

If in the fundamental equation of cosmology derived previously

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

the following equality applies:

$$\frac{1}{2}(\dot{a})^2 = -\frac{4\pi G}{3}\rho a^2$$

we would also get this equality, in this case, presented as a postulate:

$$\frac{1}{2}(\dot{a})^2\frac{1}{a} = -\frac{4\pi G}{3}\rho a \; = \ddot{a} = \frac{D^2 a}{dt^2}$$

or

$$\ddot{a} = \frac{1}{2}\frac{(\dot{a})^2}{a}$$

This latter relation is known in cosmology as the Friedmann equation if the pressure of the Universe is equal to zero, i.e. p = 0. We will prove this later on. Based on the Robertson-Walker metric, the Hubble constant H can be represented as follows:

$$H = \frac{\dot{a}}{a}$$

where

$$\dot{a} = Ha$$

Hence, we get the following form for this last equation:

$$\ddot{a} = \frac{1}{2}H^2 a$$

or

$$\frac{\ddot{a}}{a} = \frac{1}{2}H^2 = \frac{H^2}{2}$$

By taking for the Hubble constant H

$$H = \frac{\dot{a}}{a}$$

the first order derivative with respect to time, we get

$$\dot{H} = \frac{D}{dt}\left(\frac{\dot{a}}{a}\right) = \frac{\ddot{a}a - \ddot{a}\dot{a}}{a^2} = \frac{\ddot{a}a}{a^2} - \frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = \frac{\ddot{a}}{a} - H^2$$

or

$$\dot{H} = \frac{\ddot{a}}{a} - H^2$$

As the term present in the last equality can be represented as follows based on the previously derived relation

$$\frac{\ddot{a}}{a} = \frac{H^2}{2}$$

we obtain the following very important equation:

$$\dot{H} = \frac{H^2}{2} - H^2$$

In the following, we will take into account that the Universe is actually expanding at the speed of light c:

$$H = v = c$$

and thus we get

$$\dot{c} = \frac{c^2}{2} - c^2$$

The previously presented physical and mathematical analysis of the equation for the pressure of the Universe p showed that in cosmology, all fundamental equations boil down to the fact that the Universe is actually expanding at the speed of light c:

$$-\frac{2H^2}{2} = -H^2 = -\frac{8\pi G}{c^2}p = -\frac{3}{\rho}p = -c^2$$

or

$$-H^2 = -c^2$$

showing that H = c. This kind of coincidence could not have been accidental, and this is what we are actually seeing now when applying the equality of H = c.

As the derivative of the constant c is zero

$$0 = \frac{c^2}{2} - c^2$$

multiplying both sides of the last equation by the mass m gives us:

$$0 = \frac{mc^2}{2} - mc^2$$

We will now move the term $-mc^2$ of this equation to the other side of the equality sign:

$$mc^2 = \frac{mc^2}{2}$$

and take into consideration the relation known from the theory of special relativity

$$c^2 = v^2$$

and finally, we get the equation

$$\frac{mv^2}{2} = mc^2$$

Since the classical formula for kinetic energy can be expressed as follows:

$$\frac{mv^2}{2} = E$$

we can present the equation for rest energy:

$$E = mc^2$$

which can be understood as the ‘definition’ of kinetic energy. This means that the Universe is actually expanding over time at the speed of light, and because of that, all objects in the Universe have kinetic energy, which in this case is expressed as the formula for rest energy.

> One could think that the energy $E = mc^2$ results from the fact that all objects in the Universe with ‘inertial mass’ are ‘moving’ at the speed c in relation to light, as the light itself is moving at the speed c in a vacuum. Motion is, after all, relative, and the speed of light is absolute. Actually, this is not the case, as the Universe is expanding over time at the speed of light, and because of that, all objects in the Universe have kinetic energy, which in this case is expressed as the formula for rest energy.
>
> Since the Universe is actually expanding at the speed of light c, which is why all objects in the Universe have energy $E = mc^2$, the cosmological expansion of the Universe cannot stop. The cessation of the expansion of the Universe would cause the immediate cessation of the matter in the Universe, which would clearly be contrary to the law of conservation of energy. It also means that the cessation of time in the Universe as a whole is impossible, although it is possible locally (for example, in the centres of black holes).
>
> It is known that all energies can be classified as potential or kinetic energies. This raises the question: which type is rest energy $E = mc^2$? What kind of energy is it actually? All objects exist in ordinary space where time (and space) exist. Time as a duration is constantly ‘moving’. This means that time will never ‘stand still’. Moving objects have kinetic energy. Absolutely all objects in the Universe are moving relative to time (i.e. we are all moving towards the future in time), but time is not an object of any kind. This could be the source of the rest energy $E = mc^2$ for all objects in the Universe. This means that the energy mc^2 is essentially still an object’s “kinetic energy relative to time”. All objects are after all moving relative to hyperspace K´ as ordinary space K is moving relative to hyperspace K´ at the velocity of c. Consequently, all objects have kinetic energy and therefore mass. This way, the energy mc^2 could be the kinetic energy “relative to moving hyperspace”, i.e. $E = mc^2$ is the energy existing relative to the time of the object.

Our previous equations imply formulas for kinetic energy. Based on the last equalities, in the equation:

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

the following equality should also apply:

$$-\frac{8\pi G}{6}\rho_{AE}a^2 = \frac{\Lambda c^2}{6}a^2$$

which, in that case, plays the role of potential energy. Consequently, the cosmological constant is equal to:

$$-\frac{8\pi G}{c^2}\rho = \Lambda$$

It is worth noting here that for the cosmological constant Λ:

$$-\frac{8\pi G}{c^2}\rho = \Lambda$$

the density is not so much related to energy:

$$-\frac{8\pi G}{c^4}\frac{E}{V} = \Lambda$$

but the density is rather related to the curvature of spacetime, or gravity:

$$-\frac{8\pi G}{c^4}T_{\mu v}\frac{1}{V} = \frac{G_{\mu v}}{V} = \Lambda$$

where the energy E is described by the energy-momentum tensor $T_{\mu v}$, and the Albert Einstein tensor $G_{\mu v}$ is related to the curvature of spacetime as follows:

$$G_{\mu v} = \Lambda * V = -\frac{2GM}{c^2 R} * 4\pi R = \left(\frac{t^2}{t'^2} - 1\right)4\pi R$$

In this case, the curvature of spacetime is presented by the example of gravitational time dilation.

Since the mass density is expressed by the relation:

$$\rho = \frac{M}{V} = \frac{M3}{4\pi R^3}$$

we can write the expression for the cosmological constant Λ in the following form:

$$-\frac{2G}{c^2}\frac{M3}{R^3} = \Lambda$$

If we now transform the last equation as follows:

$$-\frac{2GM}{Rc^2} = +\Lambda\frac{R^2}{3}$$

we obtain this relation:

$$-\frac{2GM}{Rc^2} = -\frac{R}{R} = -\frac{c^2}{c^2} = -1 = +\Lambda\frac{R^2}{3}$$

or

$$-1 = \Lambda\frac{R^2}{3}$$

As we have previously established the expansion rate of the Universe as $v^2 = c^2$, we must only consider the relation $\frac{c^2}{c^2} = 1$, not $\frac{v^2}{c^2} \neq 1$. The mathematical expression obtained:

$$-\frac{c^2}{c^2} = -1 = +\Lambda\frac{R^2}{3}$$

is noteworthy in that if it did not equal –1:

$$-\frac{v^2}{c^2} = +\Lambda\frac{R^2}{3}$$

or

$$\frac{v^2}{c^2} = -\Lambda\frac{R^2}{3}$$

then we could apply Hubble's law:

$$v^2 = H^2R^2$$

expressing the square of the Hubble constant H as a result:

$$\frac{H^2R^2}{c^2} = -\Lambda\frac{R^2}{3}$$

or

$$H^2 = -\Lambda\frac{c^2}{3}$$

The equation obtained can be interpreted in two ways. First, if we now apply the relation derived above as follows:

$$-1 = +\Lambda\frac{R^2}{3}$$

or

$$\frac{1}{R^2} = -\frac{\Lambda}{3}$$

we get the following result:

$$H^2 = \frac{c^2}{R^2}$$

or

$$H^2R^2 = v^2 = c^2$$

telling us that in infinity, the expansion rate of the Universe v is equal to the speed of light c. In

this case, we have obtained the positive speed of light, which is physical, i.e. real. But the previously derived equation

$$\frac{H^2}{c^2} = -\frac{\Lambda}{3}$$

could also be interpreted in another way. For example, let us mathematically transform the last expression as follows:

$$-\frac{3}{\Lambda} = \frac{c^2}{H^2}$$

or

$$\sqrt{-\frac{3}{\Lambda}} = \frac{c}{H}$$

In this case, we can see the 'quotient of velocities' that is related to the cosmological constant Λ. The quotient of velocities was also present in the previously derived relativistic formula:

$$v = c\sqrt{1 - \frac{v^2}{c^2}}$$

or

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c}{v}$$

which, in turn, shows the phenomenon of time dilation:

$$\Delta t = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or the y-factor:

$$\frac{\Delta t}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = y$$

Consequently, we can write as follows:

$$\sqrt{-\frac{3}{\Lambda}} = \frac{c}{H} = y = \frac{\Delta t}{t}$$

showing that the cosmological constant Λ is related specifically to the y-factor. Now this means that the Universe is in fact expanding at the speed of light c, but we perceive this as a much lower velocity because time has been transformed or has slowed down y times over the whole Universe:

$$H = \frac{c}{y}$$

Only such a notion could explain the previously derived energy equation:

$$E = \frac{mv^2}{2} = mc^2$$

where we obtained H = c. In this case, the cosmological constant Λ is related to the y-factor known from time dilation:

$$-\frac{3}{y^2} = \Lambda$$

but the relation between the cosmological constant Λ and the radius R:

$$-\frac{3}{R^2} = \Lambda$$

is simply Hubble's law with our Universe expanding at the speed of light c on an infinitely large spatial scale (due to the increasing radius R). The last two expressions cannot therefore be equal: $y \neq R$.

It is interesting to note here that from Hubble's law:

$$HR = v$$

one can also 'derive' the equation for the Universe's y-factor:

$$R = \frac{v}{H} = \frac{c}{H}$$

or

$$y = \frac{c}{H}$$

where R in this case represents the radius/size of the Universe.

When the equation expressing the relation of the Hubble constant H and the y-factor:

$$\sqrt{-\frac{3}{\Lambda}} = \frac{c}{H} = y = \frac{\Delta t}{t} = \frac{t'}{t}$$

has y equal to infinity, i.e. time has slowed down/transformed to infinity across the whole Universe:

$$y = \infty$$

then the expansion rate of the Universe H is equal to zero:

$$H = 0$$

and the cosmological constant is also equal to zero:

$$\Lambda = 0$$

But during the current cosmological epoch, the y-factor has the following value:

$$y = \frac{t'}{t} = \frac{3 * 10^8 \left(\frac{m}{s}\right)}{2{,}3979 * 10^{-18} \left(\frac{m}{s}\right)} = 1{,}25109 * 10^{26} \approx 1{,}25 * 10^{26}$$

because Hubble's constant H is not absolutely equal to zero:

$$\frac{v}{R} = H = 2{,}3979 * 10^{-18} \left(\frac{m}{s}\right)$$

and the value of the speed of light has been measured to be:

$$c = 3 * 10^8 \left(\frac{m}{s}\right)$$

Therefore, the cosmological constant Λ cannot actually be equal to zero. For further analysis, however, we can consider the cosmological constant Λ to be zero, since instead we can take into account only the transformation of time, encompassing the entire Universe at once.

However, after using the previously derived equation:

$$-\frac{2GM}{Rc^2} = +\Lambda\frac{R^2}{3}$$

to perform the following mathematical transformations:

$$\frac{GM}{R} = -\Lambda\frac{R^2c^2}{6}$$

we get the equation for the gravitational potential U:

$$U = -\Lambda\frac{c^2R^2}{6}$$

which is exactly identical to the gravitational potential derivable from Poisson's equation:

$$\Phi_\lambda = -\frac{\Lambda c^2}{6}r^2$$

We have already presented the analysis of Poisson's equation, but we can now see that the

equation:

$$U = -\Lambda \frac{c^2 R^2}{6}$$

or

$$U = -\Lambda \frac{R^2}{3} \frac{c^2}{2}$$

comprises the previously derived equality:

$$-1 = \Lambda \frac{R^2}{3}$$

or

$$1 = -\Lambda \frac{R^2}{3}$$

This gives us the following ‘value’ for gravitational potential:

$$U = \frac{c^2}{2}$$

which, as we know, is the maximum possible potential in the whole Universe. The maximum potential U is expressed e.g. in the formula for gravitational time dilation:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t}{\sqrt{1 - \frac{2GM}{c^2 R}}} = \frac{t}{\sqrt{1 - \frac{2U}{c^2}}} = \frac{t}{\sqrt{1 - 1}} = \infty$$

or

$$t' = \frac{t}{\sqrt{1 - \frac{2GM}{c^2 R}}}$$

which is the physical basis of curved spacetime. One can see from this that time transforms or ‘curves’ to infinity. If we now use the time dilation formula:

$$\sqrt{1 - \frac{2GM}{c^2 R}} = \frac{t}{t'}$$

to perform the following transformations:

$$-\frac{2GM}{c^2 R} = \frac{t^2}{t'^2} - 1$$

we see that the resulting expression coincides with the previously derived equation:

$$-\frac{2GM}{c^2R} = +\Lambda\frac{R^2}{3}$$

comprising the cosmological constant Λ. We can therefore write:

$$\frac{t^2}{t'^2} - 1 = +\Lambda\frac{R^2}{3}$$

or

$$\frac{t^2}{t'^2} = +\Lambda\frac{R^2}{3} + 1$$

As we have also obtained this relation before:

$$1 = -\Lambda\frac{R^2}{3}$$

we can therefore use parentheses:

$$\frac{t^2}{t'^2} = \Lambda\frac{R^2}{3} - \Lambda\frac{R^2}{3} = \Lambda\frac{R^2}{3}(1-1)$$

or

$$\frac{t^2}{t'^2} = \Lambda\frac{R^2}{3}(1-1)$$

We will move the parentheses to the other side of the equality sign in the equation:

$$\frac{t^2}{t'^2(1-1)} = \Lambda\frac{R^2}{3}$$

and we can do the same with the radius R:

$$\frac{t^2 3}{t'^2(1-1)R^2} = \Lambda$$

or the cosmological constant Λ:

$$\frac{t^2 3}{t'^2(1-1)\Lambda} = R^2$$

or

$$\sqrt{\frac{t^2 3}{t'^2(1-1)\Lambda}} = R$$

As the time dilation t' was equal to infinity:

$$t' = \frac{t}{\sqrt{1-\frac{R}{R}}} = \frac{t}{\sqrt{1-\frac{c^2}{c^2}}} = \infty$$

we get the value of the cosmological constant Λ to be zero:

$$\frac{t^2 3}{t'^2(1-1)R^2} = \frac{t^2}{t'^2}\frac{3}{(1-1)R^2} = \frac{t^2}{\infty}\frac{3}{(1-1)R^2} = 0 * \frac{3}{0 * R^2} = \frac{0}{0} = 0 = \Lambda$$

or the radius R to be zero:

$$\sqrt{\frac{t^2 3}{t'^2(1-1)\Lambda}} = \sqrt{\frac{t^2}{t'^2}\frac{3}{(1-1)\Lambda}} = \sqrt{\frac{t^2}{\infty}\frac{3}{(1-1)\Lambda}} = \sqrt{0 * \frac{3}{0 * \Lambda}} = \sqrt{\frac{0}{0}} = \sqrt{0} = 0 = R$$

or

$$\begin{cases} \Lambda = 0 \\ R = 0 \end{cases}$$

> At this point, it must be mentioned that the cosmological constant is not really equal to zero: $\Lambda \neq 0$. In the course of the preceding mathematical analysis, we cancelled it out of the given system, that is, we (artificially) set it equal to zero because it will simplify further physical and mathematical analysis of the cosmological expansion of the Universe. The cosmological constant is not actually equal to zero. We now know that it (i.e. dark energy) is related to the spacetime of the Universe rather than to vacuum energy. We will simply take this knowledge into account for further analysis.

As the cosmological constant Λ is equal to zero, based on this we would get the value of the scale factor (i.e. the radius) of the Universe to be infinity ∞:

$$a = \frac{1}{\sqrt{\Lambda}} = \frac{1}{\sqrt{0}} = \frac{1}{0} = \infty$$

showing that our Universe is actually infinitely large. We will present the real possibility for this and the mathematical-physical analysis later. For now, we must note that the first Friedmann equation:

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

acquires the following form:

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2}$$

as the cosmological constant Λ is equal to zero. This in turn means that all subsequent mathematical and physical analysis will be carried out using the form of the last Friedmann equation that does not comprise the cosmological constant Λ.

As both the pressure of the Universe and the cosmological constant are both equal to zero, we can also take the second Friedmann equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{c^2}\left[(p_M + p_\Lambda) + \frac{1}{3}(\rho_M + \rho_\Lambda)c^2\right]$$

and transform it to the following form:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{c^2}\left[\frac{1}{3}(\rho_M + \rho_\Lambda)c^2\right]$$

or

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{c^2}\left[\frac{1}{3}(\rho_M)c^2\right]$$

In the resulting equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_M$$

we will move the term a to the other side of the equality sign in the equation:

$$\ddot{a} = -\frac{4\pi G}{3}\rho_M a$$

and see that this relation coincides with the previously used form of the second Friedmann equation:

$$\ddot{a} = -\frac{4\pi G a}{3}\left(\rho + \frac{3p}{c^2}\right)$$

where the pressure of the Universe is equal to zero:

$$\ddot{a} = -\frac{4\pi G a}{3}\rho$$

We have previously proved that the first Friedmann equation:

$$\frac{\dot{a}^2}{2} - \frac{8\pi G}{6}\rho_{AE}a^2 = -\frac{kc^2}{2}$$

that does not comprise the cosmological constant or constant k:

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = -\frac{c^2}{2}$$

can be transformed into the following form to simplify mathematical and physical analysis:

$$\frac{1}{2}\dot{a}^2 - \frac{4\pi G}{3}\rho a^2 = \frac{H^2}{2} - \frac{GM}{R^3} = -\frac{c^2}{2}$$

or

$$\frac{H^2}{2} - \frac{GM}{R^3} = -\frac{c^2}{2}$$

However, when beginning to study/analyse the changing of the Hubble constant H over time, i.e. the expansion rate of the Universe H over time (not in space), which is due to the transformation

or curving of time and space over the whole Universe, we will use this relation:

$$\frac{H^2}{2} - \frac{GM}{R^3} = -\frac{c^2}{2}$$

and when beginning to study/analyse the changing of Hubble's law v = HR in space, i.e. the expansion rate of the Universe v in space (not in time), we will use this relation:

$$\frac{H^2R^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

or

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

For example, if in the given equation:

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

R is equal to infinity, i.e. we are dealing with the infinitely large spatial scale of the Universe:

$$R = \infty$$

we get the expansion rate of the Universe as exactly equal to the speed of light c:

$$\frac{v^2}{2} = -\frac{c^2}{2}$$

or

$$v^2 = -c^2$$

which is well in line with the notions presented earlier. If, however, looking at the infinitely small spatial scale of the Universe:

$$R = 0$$

according to Hubble's law

$$v = HR$$

we get the expansion rate of the Universe to be zero: v = 0. However, when analysing the equation:

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

where R = 0, we obtain a physically and mathematically unrealistic relation as a result:

$$\frac{v^2}{2} - \infty \neq -\frac{c^2}{2}$$

For example, if the radius of the Universe $R = \infty$ equals infinity in Hubble's law $v = HR$, we get the value of the expansion rate of the Universe to be infinitely

large $v = \infty$, which would also be an unrealistic result. The highest speed in nature can only be the speed of light c.

To avoid such unrealistic relations, we will in this equation:

$$\frac{v^2}{2} - \frac{GM}{R} = -\frac{c^2}{2}$$

only consider the expression for the gravitational potential U:

$$\frac{v^2}{2} - U = -\frac{c^2}{2}$$

The essence of such analysis stands in that the infinitely small spatial scale does not actually exist, as the smallest length in space can only be Planck's length. The infinite curvature of spacetime is manifested on the spatial scale of Planck's length, and the maximum possible potential U in the Universe that we presented earlier corresponds to the infinite curvature of spacetime. We can therefore write as follows:

$$\frac{v^2}{2} - \frac{c^2}{2} = -\frac{c^2}{2}$$

or

$$\frac{v^2}{2} = \frac{c^2}{2} - \frac{c^2}{2} = 0$$

showing that, on the smallest possible spatial scale, the expansion rate of the Universe is equal to zero: $v = 0$.

11.1 Mathematical note

We must note here that if in the Friedmann equation:

$$\frac{\dot{a}^2}{2} - \frac{4\pi G}{3}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

the equality is expressed as follows:

$$\frac{\dot{a}^2}{2} = -\frac{4\pi G}{3}\rho_{AE}a^2$$

we get a 'positive' result:

$$\frac{\dot{a}^2}{2}\frac{1}{a} = -\frac{4\pi G}{3}\rho_{AE}a = \ddot{a} = \frac{D^2 a}{dt^2}$$

or

$$\ddot{a} = \frac{\dot{a}^2}{2}\frac{1}{a}$$

The time dependence of the Hubble constant H is obtained based on this relation:

$$\dot{H} = \frac{H^2}{2} - H^2$$

However, if in the Friedmann equation:

$$\frac{\dot{a}^2}{2} - \frac{4\pi G}{3}\rho_{AE}a^2 = -\frac{kc^2}{2} + \frac{\Lambda c^2}{6}a^2$$

one side is equal to zero:

$$\frac{\dot{a}^2}{2} - \frac{4\pi G}{3}\rho_{AE}a^2 = 0$$

we get this equality:

$$-\frac{\dot{a}^2}{2} = -\frac{4\pi G}{3}\rho_{AE}a^2$$

giving us a ‘negative’ result:

$$\ddot{a} = -\frac{\dot{a}^2}{2}\frac{1}{a}$$

Because of this we will also have the ‘-’ sign in this equation:

$$\dot{H} = -\frac{H^2}{2} - H^2$$

We also get exactly the same difference in possibilities for the previously derived energy equation:

$$\frac{mv^2}{2} = mc^2 - mcv$$

or

$$\frac{mv^2}{2} + mcv = mc^2$$

where the velocity is equal to zero $+mcv$ in the term $v = 0$:

$$\frac{mv^2}{2} = mc^2$$

Based on this, we can now see that when $v^2 = -c^2$, we get:

$$-\frac{mc^2}{2} = mc^2$$

However, we get a ‘positive’ result by squaring both sides of the equation obtained and then taking the square root:

$$\frac{mc^2}{2} = mc^2$$

We in fact also get a ‘positive’ result when the equation:

$$\frac{mv^2}{2} = mc^2$$

expresses the square of velocity $v^2 = c^2$:

$$\frac{mc^2}{2} = mc^2$$

This kind of mathematical analysis shows that the equations obtained can be presented as negative or positive depending on the need and context.

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Author's declaration

The author have declared him have no conflict of interest with regard to this content and ethics committee/IRB approval is not relevant to this content.

Data availability statement

Data sharing not applicable to this content as no datasets were generated or analysed during the current study.

About the company

"*MLK Technology and Science Ltd*" is a startup company primarily engaged in scientific research on wormholes and technology development. The official data of the company can be seen on the websites:

1) https://ariregister.rik.ee/eng/company/17008425/

2) https://orcid.org/0000-0002-3223-6099

3) Company homepage: https://www.technologyandscience.eu

4) See more here: https://zenodo.org/communities/time_travel/

5) OSF project: https://doi.org/10.17605/OSF.IO/H96YM

Area of activity: scientific research and development, research and experimental development on natural sciences and engineering, other research and experimental development on natural sciences and engineering. The company is registered in the Republic of Estonia (EE), which is a member state of the European Union (EU).

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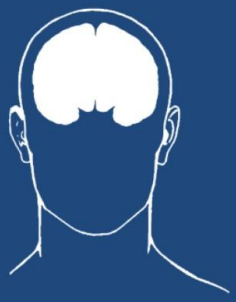

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