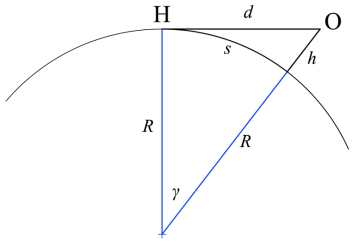
Technical Appendix

This appendix, or extended technical footnote, elaborates some statements in the body of the paper, *Future Warfare in the Western Pacific* by Stephen Biddle and Ivan Oelrich, concerning weapon and sensor performance and provides brief descriptions of some of the supporting calculations.

Detection ranges of airborne radars.

A key assertion of the paper is that airborne radars, looking out to sea, will be able to detect large ships out to the horizon but, of course, no further. Calculation of the horizon distance is based on simple geometry. The horizon is the point where the line of sight just grazes the surface of the Earth, so the Earth radius and the line of sight at that point form a right angle, as in point H in the diagram.



Calculation of the horizon distance using Pythagoras's Theorem, from Jeff Conrad via Wikimedia Commons.

Pythagoras's Theorem then yields, for the straight-line distance, d, from an observer at a height h, to the horizon, the equation

$$d = \sqrt{h \cdot (2R + h)}$$

where R is the radius of the sphere, which, in the case of the Earth, is 6371 kilometers. Note the effect of the square root: as altitude increases, the distance to the horizon increases more slowly. A doubling of altitude results in an increase in the horizon by square root of 2, or a factor of 1.4 or a 40% increase.

Normally, one is not so much interested in straight line distances, that is, d, but instead the distance measured along the surface of the Earth, that is, s in the diagram above. So long as $h \ll R$, which, in our case, is 10-20 km compared to 6000, the difference between d and s is negligible. In the cases considered here, even for low Earth satellites, the difference between d and s never amounts to as much as one percent so the simpler expression for d is used and the difference ignored.

A somewhat more significant complication is the refraction of a radar beam by the atmosphere. The purely geometric treatment above ignores the atmosphere. The atmosphere is not homogeneous; it grows thinner at higher altitude. Electromagnetic radiation, for example, radar beams, will travel slightly slower in denser air. The speed difference will tend to bend the beam

slightly toward greater density, that is, toward the Earth. This bending of light around the curve of the Earth effectively increases the distance to the visible horizon. The degree of bending depends on barometric pressure, temperature, even rainfall density. In discussions of radar horizon, one often sees the "4/3 Earth approximation," that is, taking the effective radius of the Earth to be 4/3 of the actual value in horizon calculations. A 4/3 R adjustment increases the calculated horizon distance by 15% and is a useful rule-of-thumb for ground-based or mast-mounted radars under typical weather conditions and for much closer horizons than we consider here. When calculating long horizons from high altitude aircraft, the density of the air is much less over most of the line-of-sight path, so differences in density are less, and the bending effect over most of the radar beam's path is much less. Careful calculation might show effective horizons a few percent greater than we calculate but the effect is neglected here because it is much smaller than other uncertainties.

At several points in our discussion of radar performance, the width, or angular resolution, of the radar beam is important. The beamwidth is a function of two fundamental characteristics of any radar: the wavelength, λ, of the electromagnetic radiation used by the radar and the width of the antenna, D. (Note that, for any waves, the wavelength times the frequency equals the propagation speed of the waves; in the case of radar, this is the speed of light, always denoted by c. So specifying either wavelength or frequency characterizes the radiation and both are used throughout descriptions of radar.) This limitation on beam width is a fundamental result of the wave nature of light and there are no clever engineering work-arounds. The beamwidth, typically denoted by Θ, is usually defined as the angular width where the signal strength falls to one half the maximum signal found at the center point of the beam and $\Theta = \lambda/D$. (Noting that angles are measured in radians, not degrees, where one radian is equal to 57.3°.) Thus, a narrow beam requires either a short wavelength or a large antenna.

Our analysis shows that airborne radars will be able to detect large warships off to the horizon. Warships typically have large radar cross sections, creating large radar returns in comparisons to the system noise, that is, the signal-to-noise ratio is good, but that is only half the problem when looking at an object on the ocean's surface. The ocean surface also reflects radar signals so the question is whether the signal from the ship is large enough to stand out from the background or "clutter" of the ocean itself. For that, one must calculate the signal-to-clutter ratio. For any given beamwidth, as the beam looks further out over the ocean, it will illuminate a larger patch on the surface, both because the beam is wider at greater ranges and because the beam is hitting the surface at an ever smaller grazing angle. Countering this increased scattering area, the scattering coefficient, or radar cross section in square meters per square meter of illuminated area, decreases as grazing angle decreases. These two effects trade off in a way that creates a maximum in clutter echo at intermediate range. This means that, for some combinations of ship radar cross section and radar, a ship may be visible at close range and at the horizon but lost in clutter at some ranges in between.

Putting all these factors together, we used Equation 6.10 from Mahafza and Elsherbeni[1]:

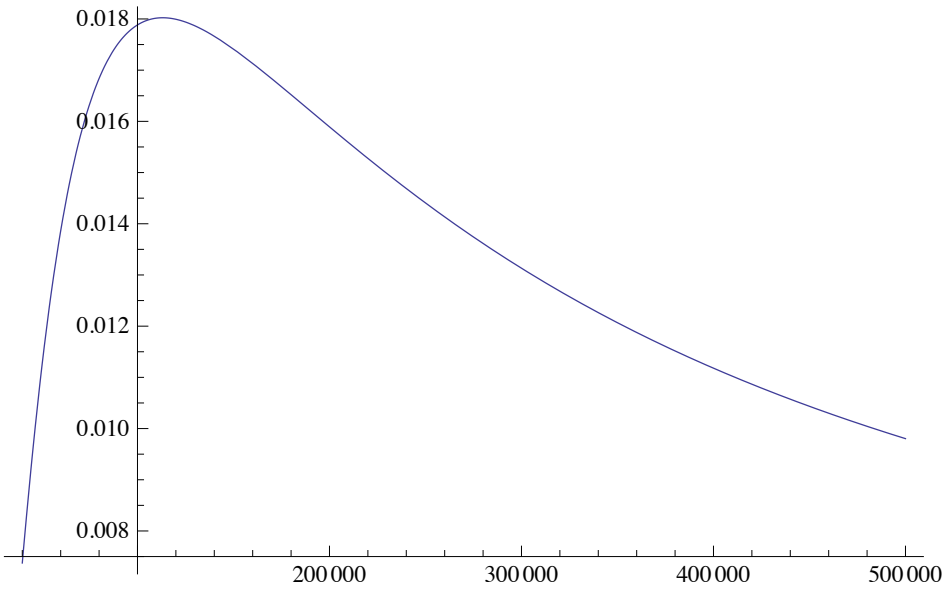
[1] Bassem R. Mahafza and Atef Z. Elsherbeni, *MATLAB Simulations for Radar System Design* (New York: CRC Press, 2004), esp. equation 6.10. Available at http://staff.on.br/puxiu/MatLab_Pack/MATLAB%20Simulations%20for%20Radar%20Systems%20Design%20-%20Bassem%20R.%20Mahafza%20&%20Atef%20Z.%20Elsherbeni.pdf accessed 25 November 2014.

$$\text{Signal-to-scatter ratio} = \frac{2 \cdot \sigma_{target} \cdot cos\psi}{\sigma_{ocean} \cdot \theta \cdot R \cdot c \cdot \tau}$$

Where σ_{target} is the cross section of the target, Ψ is the grazing angle, σ_{ocean} is the cross section coefficient of the ocean, that is, the cross section in square meters per square meter and is a function of Ψ, R is the range, c is the speed of light, and τ is the length of the radar pulse. Note that the power of the radar does not come into the equation at all because, while a stronger outgoing signal does, indeed, increase the return echo signal, greater power also increases the clutter return proportionally, so the *ratio* remains the same. The antenna size does not appear explicitly but determines, along with the wavelength, the beam width. The equation suggests that the trick is to get a tight radar beam, not simply to focus a more intense beam of radiation at the target to create a stronger echo signal, but to illuminate as little as possible of the ocean around the target, thereby reducing the clutter signal.

We assumed a JSTARS-like radar with a wavelength of 3 cm (X-band) and an antenna width of 7 m and a pulse width of one microsecond.[2] The relationship between ocean cross section and grazing angle is very complex, depends on the roughness of the sea, is impossible to calculate, and best derived from experimental measurements taken in the field. We used values for an average sea state.[3] Solving using the equation-solving computer language, Mathematica, we produced this graph, normalized to a 1 m^2 radar cross section target:

Signal to clutter ratio.



Range in meters.

[2] For JSTARS details see Robert M. O'Donnell, "Radar Systems Engineering, Lecture 14: Airborne Pulse-Doppler Radar," (IEEE New Hampshire Section, 1 January 2010), slides 18, 48-50 (http://ece.wpi.edu/radarcourse/Radar%202010%20PDFs/Radar%202009%20A_14%20Airborne%20Pulse%20Doppler%20Radar.pdf) accessed 30 June 2015.

[3] Merril Skolnik, "Sea Echo," Chap. 26 in *Radar Handbook*, Merril Skolnik (ed.), (New York: McGraw-Hill, 1970), in particular, Fig. 3, p. 26-7

Note that at 500 km, the signal to clutter ratio is 0.01, that is, the clutter is 100 times stronger than the signal from a 1 m^2 target. The ratio scales linearly with target cross section so a 100 m^2 target would return a signal equal to clutter. A confirmed radar hit usually requires a signal to noise ratio of at least 20, better, 100, so the return would clearly appear as a legitimate target if it had a cross section of 2,000 to 10,000 m^2. We can find in the literature no actual measurements of radar cross sections of large warships, such values are probably considered classified, but rules of thumb and comparison to large cargo ships suggests that aircraft carriers might have radar cross sections of a million square meters.[4] We estimate that large cargo ships and warships larger that destroyers should be easily detectible at the horizon. The clutter of the ocean suggests that reducing the radar cross section of ships might have payoff even if the radar cross section remains huge compared to that of a stealth aircraft. A surface radar will see an aircraft against the blackness of space so the target aircraft must produce extremely small radar echos. A ship designer must reduce the radar cross section of a warship only enough for it to become lost in the clutter of the ocean.

If airborne radars are good, would space-based radars be better? Exploiting the higher altitude (hence longer horizon distances) in orbit is costly but has obvious benefits. Recall that the key to getting good signal-to-clutter ratios is a tight beam meaning short wavelengths or large antennas. Wavelengths less than ~3cm are not practical at long range because atmospheric absorption increases so the narrow beam width has to come from large antenna size. Large antennas and solar panels create drag that will make very low orbits unstable but putting the radar in a higher orbit creates the need for an ever larger antenna. Russian ocean-surveillance RORSAT satellites were nuclear powered, presumably to allow operation at lower altitudes where solar panels would produce too much drag for a stable orbit.[5] RORSAT has, however, demonstrated that ships can be detected from space and we discuss the need to counter satellite radars.

Dynamics of SAM defense of airborne radars against ARM attack

Our analysis of the ultimate range limits of A2AD rests in part on the dynamics of defending high-altitude airborne radars from anti-radiation missile (ARM) attack. As noted in the text, SAM-based defense would be complex. A prudent planner would most likely want to allow for at least two independent defending shots at each incoming ARM, that is, shooting one or two missiles at the attacker, assessing the result, and firing at least one more salvo if necessary before the attacker reaches the airborne radar.

The size of the protected envelope created by the SAMs depends on the relative speeds of the SAM and attacking missile, and the range at which the defending radar can detect the attacker. Let us assume the attacker to be a stealthy ramjet-powered radar-homing cruise missile that can sustain Mach 2 at the airborne radar's altitude, a system the U.S. does not field today but could readily develop with current technology. We can take the Russian S-400 Triumf system,

---

[4] Merril Skolnik, "An Empirical Formula for the Radar Cross Sections of Ships at Grazing Incidence," *IEEE Transactions on Aerospace and Electronic Systems*, March, 1974, p. 292

[5] For RORSAT details, see Asif Siddiqi, "Staring at the Sea: The Soviet RORSAT and EOSAT Programmes," *Journal of the British Interplanetary Society,* Vol. 52 (No. 11/12), November/December 1999, pp. 397-416

known to NATO as the Growler, to be the current state of the art of SAM defense. If the radar is on a U-2-like aircraft at 20 kilometers altitude, then even a SAM directly below is still at least 20 kilometers away and flight times from the surface to altitude must be accommodated. Western analysis based on Russian open sources concludes that Triumf missiles could reach a 20 km altitude to defend a U-2 type aircraft but only if the missile fires almost straight up.[6] Thus, if a second SAM were fired the instant the first SAM missed, and assuming that SAM speeds average Mach 3, then the second line of SAMs must be set back 12 kilometers from the first to protect the U-2, and the U-2 would have to stay behind that line. If the SAM had a bit of slant range to work with, the U-2 could defend a bit further forward against a cruise missile heading directly overhead but if the slant range is needed to defend a different angle of approach, the set back would have to increase.

If the airborne radar operated at a lower altitude, say 13 kilometers more typical of airborne radars, then current high performance SAMs can more easily reach forward, perhaps enough to extend the protected envelope somewhat beyond the coast. Reducing the radar's altitude from 20 to 13 kilometers reduces the horizon from 500 to 400 kilometers but some of that loss could be made up if the airplane carrying the radar can extend out to sea.

An airborne radar may be able to see a giant aircraft carrier hundreds of kilometers away and still not be sensitive enough to detect a very stealthy attacker in time. So, the defender is forced to depend on the SAM's big powerful surface-based mobile air defense radars. The performance details of most military radars are classified but reasonable engineering constraints allow useful calculation of performance. The Russian S-400 radar, known to NATO as Big Bird, is sometimes compared to a land-based Aegis. It has a larger antenna but probably lower power.[7] The US deploys a large, truck-transportable radar, the AN/TPY-2, as part of the THAAD theater missile defense system. A similar system could serve for air defense.[8] In the absence of jamming, a radar of that type should be able to detect targets with RCSs of only a few square centimeters out to 190 kilometers, where a radar looking up just five degrees to avoid ground interference would first see an attacker flying 20 kilometers high.[9] The first SAM would then be launched. If the attacking cruise missile penetrates at Mach 2, the SAMs average Mach 3, and the first SAM fails to intercept, triggering the launch of a second, then the second intercept will occur about 60 kilometers from the SAM launcher. If the SAM launcher were on the beach, then the airborne radar could be protected 60 kilometers out to sea.

---

[6] Carlo Kopp, *Almaz S-300, Air Power Australia Technical Report APA-TR-2006-1201*, 27 January 2014 http://www.ausairpower.net/APA-Grumble-Gargoyle.html accessed 25 November 2014.

[7] Carlo Kopp, "Search and Acquisition Radars (S-Band, X-Band), Technical Report APA-TR-2009-0101," *Air Power Australia*, January 2009, http://www.ausairpower.net/APA-Acquisition-GCI.html (accessed 29 May 2015)

[8] Missile Defense Agency, *Fact Sheet Army Navy/Transportable Radar Surveillance (AN/TPY-2)*, undated, http://www.mda.mil/global/documents/pdf/an_tpy2.pdf accessed 25 November 2014.

[9] Based on the authors' calculations (see below) using performance parameters of the Aegis SPY-1 radar from George Lewis and Theodore Postol, "Ballistic Missile Defense: Estimating the Range of an Aegis Radar against a Missile Warhead Target," mostlymissiledefense.com (October 23, 2012) http://mostlymissiledefense.com/2012/10/23/438/ accessed 24 November 2014.

Of course, the exact answers depend on the input assumptions. If the attacking cruise missile is a bit faster, then the airborne radar will be forced somewhat further back. If the SAMs were a bit faster, the airborne radar could move forward. For example, if the Chinese developed a more powerful SAM than we assume, they may be able to have a U-2-altitude airborne radar *and* protect it perhaps several tens of kilometers over the sea. But there are limits on the SAM's size (and thus its speed and range) because its survival requires mobility, hence any missile must be transportable by truck; the achievable improvement is nonzero but modest. Perhaps the most important assumption in the example above is that defenders would want time for two independent shots. If the Chinese were willing to accept much higher risk, they could operate at the edge of the SAM engagement envelope. (Recall, however, that during the Cold War, US airborne radars, like JSTARS, used much of their nominal radar range to allow the aircraft to stay far back behind many layers of air defense. JSTARS was never intended to fly along the front lines.) Conversely, in our example the defending radar was not jammed. Because surface radars can pump out so much power and have such large antennas, they are quite effective even against very stealthy targets but the *combination* of stealth and jamming is extremely challenging. Jamming could significantly push back airborne radar's safe operating range.

Although there is uncertainty around the edges, this implies a safe sanctuary for airborne radar of at most several tens of kilometers from land-based SAM defenders; given the radar horizon for airborne radars at plausible altitudes, this in turn implies a likely limit on the effective reach for survivable radar of up to 400-600 kilometers from a controlled coastline.

Energy and cost advantage of anti-satellites vs. satellites.

We state that anti-satellite systems will have a strong cost advantage over their satellite targets. This was not based on detailed cost analysis of launchers, rather a comparison of the relevant energetics. In a low circular Earth orbit, say 300 km above the surface, a satellite is moving at 7.7 km/sec, or with an energy of about 30 million joules/kg. To simply hoist a kilogram to that height, without the orbital speed, is about 3 million joules/kg. Although this advantage alone is important, the greater advantage comes because the target satellite probably weighs hundreds of times more than the interceptor. The interceptor's energy advantage could thus be in total a factor of a thousand and the relative launcher cost roughly proportional. The relative energy per kilogram advantage of the interceptor decreases as the orbital altitude increases because the speed of the satellite goes down but at 1000 km the interceptor still requires only one third as much energy and the mass differences will still obtain. If we think of rockets not as energy transfer devices, rather as momentum transfer devices, the interceptor still has a relative momentum advantage but not as great. The real advantage is the mass advantage. A final note: the kinetic energy of one kilogram of interceptor hitting a satellite at 300 km altitude is 30 million joules and the energy of a kilogram of TNT is about 4 million joules so adding an explosive warhead is superfluous for a hit-to-kill interceptor.

Secondary damage to satellites created by debris from antisatellite attacks.

We point out the critical importance to the US of blinding Chinese satellites, by direct attack if need be, and argue that even a two-sided anti-satellite battle would ultimately benefit the US. Two major objections are raised to antisatellite warfare by the US: First, that the US depends more than any other nation on satellites and thus anti-satellite warfare is self-defeating. We argue this is not actually true. The US makes greater use of satellites but has alternatives for, for example, ocean surveillance that other nations do not have. Use does not necessarily imply dependence. Second, it is argued that attacks on satellites will create debris that will, in turn, create unacceptable levels of collateral damage to other satellites, both civilian and military. Debris is certainly a problem but one that we believe the US would accept in the context of a contest over control of the western Pacific. We use data presented by David Wright of the Union of Concerned Scientists. Empirical data about debris from destruction of satellites is, fortunately, limited but collisions in space can create millions of debris particles. By far the largest number are the smallest particles, less than a millimeter diameter, but even these can degrade a satellite through collision, for example, by damaging solar panels. The tens of thousands of particles larger than a centimeter can cause catastrophic damage through impact on the body of a satellite.

In January 2007, China tested a direct ascent anti-satellite weapon against one of their defunct weather satellites. The test was successful and created an estimated 42,000 bits of debris larger than a centimeter in diameter. Wright estimates that before the Chinese test, losing an LEO satellite through a debris collision would occur about twice per decade and, after the test, three times per decade. This is the basis for the statement that one additional satellite per decade would be lost per satellite attacked. We did not estimate how many satellites might be attacked during a Pacific war but it could easily be high enough to lead to one or more collateral satellite losses per year. Note that this loss rate would not continue indefinitely in LEO because small particles have orbital lifetimes of years to a few decades depending on altitude and solar activity. Average cost of a commercial satellite is approximately $150M.[10]

Surveillance range of ground-based radar

The paper argues that large, powerful ground-based radar can detect even very stealthy targets at considerable range, and uses the specific example of a mobile ground-based surveillance radar trying to identify long-range ramjet-powered anti-radiation missiles.

The surveillance radar range equation varies in important ways from the more familiar tracking equation. Low resolution is not an important handicap; with low resolution, the search cells will be larger, requiring the radar to stare longer at each cell but the number of search cells is reduced proportionally so the total search time remains constant. (Of course, the surveillance radar has to have good enough resolution to hand the target off to engagement radars.) Thus, the only details of the radar that are important to surveillance are the average power transmitted by the radar and the size of the antenna that is collecting the radar echoes. Because the lower resolution of lower

[10] Charles Q. Choi, "Space Forecast Predicts Satellite Production Boom," *Space.com*, 15 June 2009, available at http://www.space.com/6839-space-forecast-predicts-satellite-production-boom.html accessed 5 may 2016. The $150M value reflects satellite costs of $100M and launch costs of $50M.

frequency is not penalized, surveillance radars tend to operate at lower frequency where output power is cheaper to produce, allowing higher total power. In addition, stealthy targets will tend to have larger radar cross sections when illuminated by lower frequency radiation. Using Skolnik, eq 2.57, for the surveillance range equation, we have

$$R = \sqrt[4]{\frac{P \cdot A \cdot \sigma \cdot t}{4\pi \cdot k \cdot T \cdot \frac{S}{N} \cdot L \cdot \Omega}}$$

Where R is the range, P average power, A antenna area, σ target radar cross section, t the search time available, k Boltzman's constant, T effective noise temperature, S/N the signal-to-noise threshold, L internal losses in the receiver, and Ω is the solid angle that must be scanned.

Many of the performance details of most military radars are unavailable but we can make some reasonable assumptions. For example, the power of the Russian Big Bird radar may be lower than the US Navy Aegis radar but the Big Bird antenna is larger.[11] It is reasonable to take the power-aperture product to be the same. At the longer wavelengths of the Big Bird, even a stealthy missile will have a radar cross section of around 10 cm^2. The radar sweeps out an arc 60° wide. The height of the swept arc cannot be less than the width of the beam and has to be large enough such that intruders cannot slip past the search arc in a time smaller than the sweep time. If we take a 3° high arc (yielding Ω of 0.025), even supersonic targets at high altitude will take minutes to cross the searched volume and we have set a sweep time of 20 seconds. If we take noise temperature to be 400° K, S/N of 20, L of 6, the calculated surveillance range is 496 km. Geometry shows that a radar looking up 5° at a 20 km altitude target will see it come over the horizon only at 190km away, so even with somewhat different choices for the radar performance parameters, the limit for this notional radar is the horizon, not the radar return from a 10 cm^2 target.

[11] We base our assumptions for the Big Bird on Carlo Kopp, *Almaz S-300, Air Power Australia Technical Report APA-TR-2006-1201*, 27 January 2014 http://www.ausairpower.net/APA-Grumble-Gargoyle.html accessed 25 November 2014. Details on the Aegis are drawn from Norman Friedman, *The Naval Institute Guide to World Naval Systems* (Annapolis: US Naval Institute, 1997), pp. 374-5.