Supporting Information for
Lobbyists as Gatekeepers: Theory and Evidence

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Note: The following notation differs between the main text and this Appendix:

- The politician's interim and maximum costs of review $(c_P, \bar{c}_P)$ are denoted $(r, \bar{r})$
- The politician's posture on each channel α_P^c is denoted α^c
- The politician's review cutpoint on each channel ϕ_P^c is denoted ϕ^c
- The lobbying cutpoints $(\underline{\omega}_\ell, \underline{\omega}_d)$ are denoted $(\underline{\omega}^\ell, \underline{\omega}^d)$
- The SIG's type-specific willingness to pay F^ω is denoted F_ω

A Appendix Tables and Figures

Table A1: Lobbying Firm Characteristics by the LDA Registration

	LDA & FARA		FARA Only	
	Mean	SD	Mean	SD
Annual revenues† ($thousand)	539.1	652.9	420.1	675.7
Number of government clients†	2.29	2.01	1.28	0.70
Number of contacted members	45.6	59.8	31.02	44.68
FARA registration year	2002.0	9.23	2006.0	3.46
Number of lobbyists				
All	15.38	18.98	4.17	4.53
Former member of Congress	0.58	1.10	0.11	0.32
Former congressional staff	2.02	2.62	0.26	0.59
Executive branch experience	1.06	1.23	0.19	0.48
Number of observations	72		36	

Notes: As for time-varying variables, the summary statistics are over the average value of each variable across multiple filings for each lobbying firm. †: For these two variables, we consider the lobbying reports included in this paper only. Therefore, the total annual revenues and the total number of foreign government clients are larger than the counterparts included here.

Table A2: Report-Level Summary Statistics

Contact-level Lobbyist Information:	Retrieved		Not Retrieved
Source:	Inferred, Single Lobbyist	Observed from the Report	
Number of reports	87	139	214
Number of FARA Clients	1.08	1.60	1.55
Fees ($K)	158.6	405.8	383.6
FARA registration year	2003.8	2003.1	1999.2
Involved in the domestic Lobbying (LDA)	0.41	0.82	0.75
Number of lobbyists	2.92	11.20	15.08
Number of total contacts	12.22	44.59	27.50

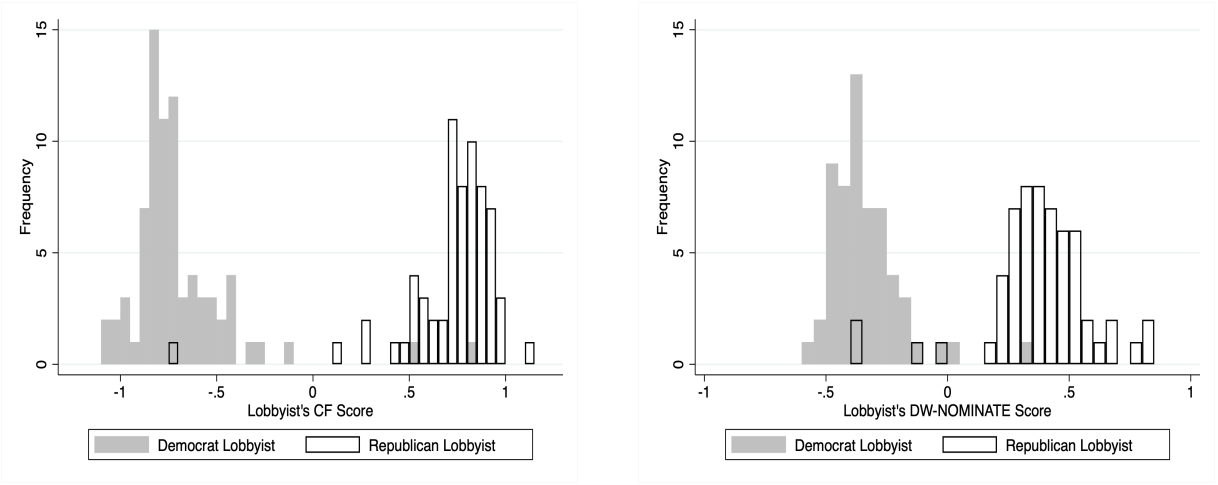
Notes: The unit of observation is a semi-annual FARA report. a. The average DW-NOMINATE scores of the contacted members, weighted by the contact frequency.

Table A3: Lobbyist's Issue Specialization

Dependent variable	Lobbied on both trade and security issues			
	(1)	(2)	(3)	(4)
CF score difference	-0.0068	-0.0066		
	(0.0043)	(0.0092)		
Ideologically aligned†			0.0074	0.0063
			(0.0076)	(0.0087)
Lobbyist FE	Yes	Yes	Yes	Yes
Politician FE	No	Yes	No	Yes
Number of observations	1,704	1,704	1,704	1,704
R^2	0.125	0.361	0.125	0.361

Notes: The unit of observation is politician × lobbyist. Based on the lobbying issues in the reports, we categorize lobbying issues into trade, security, budget, and others. We include all politician-lobbyist pairs with any contacts on either trade or security issues, the main two issues, during the period of study. Standard errors clustered at the lobbyist level are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. † The CF score difference between the lobbyist and the politician is less than the median value among the pairs with any contacts, 0.37.

Figure A1: Distribution of Ideology Measures by Party Affiliation



(a) CF Scores

(b) DW-NOMINATE Scores

Figure A2: A FARA Supplemental Statement: Contacts

Name of Registered Individual: Hal S. Shapiro

Name of Client: Embassy of United Arab Emirates			
Activity/Contact (name, title and office/agency)	**Type of Activity/ Method of contact**	**Topic**	**Date**
Rep. H. Berman, Chairman, House Committee on Foreign Affairs	Meeting	US-UAE relations, status of pending arms sale, international nuclear fuel bank regional issues	7/31/08
Alan Makovsky - Senior Staff, House Committee on Foreign Affairs	Meeting	US-UAE relations, status of pending arms sale, international nuclear fuel bank regional issues	7/31/08
Alan Makovsky - Senior Staff, House Committee on Foreign Affairs	Phone call and email message	Status of pending arms sale and satellite sale	8/5/08
Alan Makovsky - Senior Staff, House Committee on Foreign Affairs	Email message	Status of pending arms sale and satellite sale	8/6/08
Alan Makovsky - Senior Staff, House Committee on Foreign Affairs	Email message	Status of pending arms sale	9/8/08
Alan Makovsky - Senior Staff, House Committee on Foreign Affairs	Email message	Status of pending arms sale	9/19/08
Rep. C. Rangel, Chairman, House Ways & Means Committee	Meeting	Regional issues, bilateral relationship, international nuclear fuel bank	9/23/08
Tim Reif, Majority Chef Trade Counsel, House Ways & Means Committee	Meeting	Regional issues, bilateral relationship, international nuclear fuel bank	9/23/08
Jennifer McCadney, Majority Trade Staff, House Ways & Means Subcommittee on Trade	Meeting	Regional issues, bilateral relationship, international nuclear fuel bank	9/23/08
Tim Reif, Majority Chief Trade Counsel, House Ways & Means Committee	Email message	Meeting request on regional issues	11/11/08

Notes: This is an excerpt from the report for the six-month period ending December 31, 2008, by Akin Gump Strauss Hauer & Feld, LLP. During this period, the embassy of the United Arab Emirates was one of its clients, and this page reports detailed information on the contacts made by a lobbyist of the firm, Hal S. Shapiro, on behalf of the embassy.

B Deriving Empirical Predictions

This section derives the comparative statics (E1) and (E2). We first formally state the verbal assumptions described in the main text:

Assumption B.1. *Each lobbyist-politician pair randomly draws a large number of potential clients.*

Assumption B.2. $0 < L \leq P < \overline{\omega}$.

Consider a pair of a lobbyist and a politician. Denoting by N the total number of clients that the lobbyist represented to the politician, the following corollary shows that the probability that there is at least one contact between the pair, $\Pr(N > 0)$, and the expected number of clients on behalf of whom the lobbyist contacts the politician conditioning on having any contacts between them, $\mathbb{E}(N|N > 0)$, are weakly increasing in the pair's ideological difference, $P - L \equiv D$.

Corollary B.1. *Suppose Assumptions B.1 and B.2 hold. Then, (i)* $\Pr(N > 0)$ *converges to one if* $D \leq \bar{\omega} - P$ *and to zero otherwise as the number of potential clients goes infinity; (ii)* $\mathbb{E}(N|N > 0)$ *are nondecreasing in* D.

Proof. Under the two assumptions, Proposition 2 implies that the probability that the lobbyist contacts the politician for a random client, ρ, is:

$$\rho = \begin{cases} \Pr(\omega > \underline{\omega}_\ell) = \min\left\{\overline{\omega} - L + \frac{\pi}{\delta}, 2(\overline{\omega} - P)\right\}/\overline{\omega} & \text{if } L \geq 2P - \bar{\omega}, \\ 0 & \text{otherwise.} \end{cases}$$

Denoting $\tilde{P} \equiv \overline{\omega} - P$, the probability can be rewritten as:

$$\rho = \begin{cases} \min\left\{\tilde{P} + D + \frac{\pi}{\delta}, 2\tilde{P}\right\}/\overline{\omega} & \text{if } D \leq \tilde{P}, \\ 0 & \text{otherwise.} \end{cases} \tag{B.1}$$

Now, denoting the number of potential draws by T, Assumption B.1 implies that N follows a Binomial distribution with T draws and ρ probability of success, and therefore the following holds:

$$\Pr(N > 0) = 1 - (1 - \rho)^T, \tag{B.2}$$

$$\mathbb{E}(N|N > 0) = \rho T \Big/ \left[1 - (1 - \rho)^T\right]. \tag{B.3}$$

Thus, (i) follows from (B.2). From (B.1), ρ is nondecreasing in D if $D \leq \tilde{P}$, and from (B.3), $\mathbb{E}(N|N > 0)$ are increasing in ρ. Thus (ii) holds. □

C Spatial Model Foundation

Consider a policy space $X \subset R$, and suppose that the favor is interpreted as a request to change a policy q in this space to an exogenous alternative $y < q$. Each player $i \in \{S, L, P\}$ has a state-contingent personal ideal policy $x_i(\omega) = \hat{I} - \beta_i \omega$, where the state ω is interpreted as a sufficient statistic for the information unknown to the politician that affects his utility for policies in this space, and is normalized such that the politician prefers granting the favor more for higher values of ω. Further suppose that in addition to their own preferences, each player i places a weight λ_i^j on player j's (unweighted) ideological preferences, so that player i's overall state-contingent utility over policies is $\sum_{j \in N} -\frac{\lambda_i^j}{2} \cdot (x_j(\omega) - x)^2$. Given these assumptions, player i's net benefit for the favor being granted can be written as:

$$\sum_{j \in N} (q - y) \lambda_i^j \left(\beta_j \omega - \left(\hat{J} - \frac{y+q}{2} \right) \right)$$

Letting $d = q - y$ denote the *magnitude* of the favor and $m = \frac{y+q}{2}$ the *location* of the favor, the net benefit is then $\sum_{j \in N} d\lambda_i^j \left(\beta_j \omega - \left(\hat{J} - m \right) \right)$.

Now in the reduced-form model, the SIG lacks state contingent preferences; a necessary and sufficient for this property is that $\sum_{j \in N} \lambda_S^j \beta_j = 0$, and then the SIG's utility is $d \sum_{j \in N} \lambda_i^j \left(m - \hat{J} \right) = \pi$.In the reduced-form model the politician and the lobbyist also have state contingent preferences, requiring that $\sum_{j \in N} \lambda_i^j \beta_j > 0 \ \forall i \in \{L, P\}$, so then their net benefit for the favor being granted may be rewritten as:

$$\delta_i (\omega - I) \text{ where } \delta_i = \sum_{j \in N} \lambda_i^j \beta_j \quad \text{and} \quad I = \frac{\sum_{j \in N} \lambda_i^j \beta_j \left(\hat{J} - m \right)}{\sum_{j \in N} \lambda_i^j \beta_j}$$

Now to derive an example that generates the specific properties stated in the main text, suppose that $\beta_L = \beta_P = \beta$ (the lobbyist and politician's ideal points are equally responsive to the state) and $\lambda_L^S = \lambda_P^S$ (neither the lobbyist nor the politician care intrinsically about the SIG's preferences). Then

$$P - L = \left(\frac{\lambda_P^P \lambda_L^L - \lambda_L^P \lambda_P^L}{(\lambda_P^P + \lambda_P^L)(\lambda_L^L + \lambda_L^P)} \right) \left(\hat{P} - \hat{L} \right)$$

Thus, we immediately have the result that closer ideal points in the original "ideology space" (smaller $\hat{P} - \hat{L}$) implies closer thresholds in "SIG merit space" (smaller $P - L$).

With respect connections, suppose that a stronger connection has the effect of making the lobbyist care more about whether policy matching that politician's ideal (higher λ_L^P). This change will then result in *both* a higher value of $\delta_L = \beta \left(\lambda_L^L + \lambda_L^P \right)$ (it is as if the lobbyist cares more about the merits) and smaller $P - L$ (the lobbyist and politician have closer effective thresholds in "SIG merit space").

D Preliminary Analysis of Model

D.1 The politician's calculus

The politician seeks to grant the favor to a worthy SIG ($\omega \geq P$) and decline the favor to an unworthy one ($\omega < P$) while minimizing review costs. She can't observe the details of the prior contact between the SIG and the lobbyist (or prices), and so bases her decisions only on the observed channel of contact $c \in \{\ell, d, \emptyset\}$. The CDF of the politician's interim beliefs about the SIG's case ω given the strategies of the other players is denoted $H^c(\omega)$. For each observed channel and realized cost of review r, the politician makes two decisions – whether or not to review to learn ω (and decide accordingly), and if she does not review, whether or not to grant the favor.

We first characterize the probability $\alpha^c \in [0,1]$ that the politician grants the favor absent a review for each channel c (her posture). Because the politician's preferences are linear, her posture depends only on whether the interim expectation $E^c[\omega]$ about the state ω is above or below her threshold P. Specifically, if $E^c[\omega] > P$ then she must always grant ($\alpha^c = 1$), if $E^c[\omega] < P$ she must always deny ($\alpha^c = 0$), and if $E^c[\omega] = P$ any α^c is optimal.

We next characterize the politician's review cutpoint. Her value of review given each channel c derives from the possibility that a review might alter her default decision. In a best response she will review if and only if her realized cost r is below this value, which is therefore ϕ^c. When $E^c[\omega] \geq P$ and the politician weakly prefers to grant absent additional information, a review is only pivotal for changing her decision when it reveals negative evidence that the SIG is unworthy, which she believes will occur with probability $H^c(P)$. In this event, the expected net benefit of changing her decision from granting to denying the favor is $P - E^c[\omega|\omega < P]$, and the overall value of review is thus $\phi^c_- = H^c(P) \cdot (P - E^c[\omega|\omega < P])$. Similarly, when $E^c[\omega] \leq P$ and the politician weakly prefers to deny the favor absent additional information, a review is only pivotal for changing her decision when it reveals positive evidence that the SIG is worthy. The value of review is thus $\phi^c_+ = (1 - H^c(P)) \cdot (E^c[\omega|\omega > P] - P)$. Collecting the above observations yields the politician's best-response behavior.

Observation D.1. *Let $H^c(\omega)$ denote the CDF of the politician's beliefs after channel c. The politician's strategy is a best response i.f.f she reviews when $r \leq \phi^c$ and absent a review grants with probability α^c, where*

- $\alpha^c = 1$ *and* $\phi^c = \phi^c_- = H^c(P) \cdot (P - E^c[\omega|\omega < P])$ *if* $E^c[\omega] > P$
- $\alpha^c = 0$ *and* $\phi^c = \phi^c_+ = (1 - H^c(P)) \cdot (E^c[\omega|\omega > P] - P)$ *if* $E^c[\omega] < P$
- $\alpha^c \in [0,1]$ *and* $\phi^c = \phi^c_- = \phi^c_+$ *if* $E^c[\omega] = P$

D.2 The SIG's calculus

The probability the SIG expects the favor from pursuing channel $c \in \{\ell, d, \emptyset\}$ depends on the politician's strategy and its type, and is equal to $\Delta^c_\omega = \left(1 - \frac{\phi^c}{\bar{r}}\right)\alpha^c + \mathbf{1}_{\omega \geq P}\frac{\phi^c}{\bar{r}}$. The favor is granted with probability equal to the politician's posture α^c when she fails to investigate, and if and only if the SIG is worthy when she does.

The SIG will weakly prefer pursuing the direct channel to the null channel if and only if $\Delta^d_\omega \pi - k \geq \Delta^\emptyset_\omega \pi \iff \left(\Delta^d_\omega - \Delta^\emptyset_\omega\right)\pi \geq k$; if the lobbyist is unavailable it will be willing to lobby directly i.f.f. this inequality is satisfied. If it finds the lobbyist available, it will be willing to pay up to $F_\omega = \Delta^\ell_\omega \pi - \max\left\{\Delta^d_\omega \pi - k, \Delta^\emptyset_\omega \pi\right\}$ for the lobbyist's representation. Collecting the above yields the following.

Observation D.2. *The SIG's strategy is a best response i.f.f.*

- *it always (never) accepts a representation offer that is* $< (>) F_\omega$
- *following a history h that resulted in a lack of representation, it always (never) lobbies directly when* $\left(\Delta^d_\omega - \Delta^\emptyset_\omega\right)\pi > (<) k$

D.3 Equilibrium without the lobbyist (as a player)

To both aid in the analysis of the full model and clarify implications of the preceding best response behavior, we first characterize the unique cutpoint equilibrium of a variant of the model without the lobbyist. We consider a general atomless prior over $[0, \bar{\omega}]$ with CDF $H(\omega)$ that satisfies $H(P) \in (0,1)$ (with a strictly positive probability the SIG is both unworthy or worthy) and $E[\omega] < P$ (absent more information the politician prefers to deny the favor). The rationale for restricting attention to a cutpoint strategy for direct lobbying decisions is that the assumption of state-independent preferences for the SIG is simplifying rather than substantive – were the SIG to have even slightly state-dependent preferences matching the form of the other players, then the cutpoint equilibrium would be unique.

To define the equilibrium, let $\underline{\hat{\omega}}^d$ denote the unique cutpoint $< P$ satisfying $E\left[\omega | \omega \geq \underline{\hat{\omega}}^d\right] = P$, and further let

$$\phi_-(\underline{\omega}) = \left(\frac{H(P) - H(\underline{\omega})}{1 - H(\underline{\omega})}\right) \cdot \left(P - E[\omega | \omega \in [\underline{\omega}, P]]\right).$$

$\underline{\hat{\omega}}^d$ is the unique direct lobbying cutpoint above which the politician will have a strictly favorable posture in a best response, and below which she will have a strictly adversarial one. $\phi_-(\underline{\omega})$ is the value of reviewing for negative evidence after the politician update his priors with the information that $\omega \geq \underline{\omega}$, and is decreasing in $\underline{\omega}$. Observe that in the two player game, the value of reviewing for negative evidence ϕ^d_- after direct lobbying when the

SIG employs a cutpoint strategy $\underline{\omega}^d \in \left[\hat{\underline{\omega}}^d, P\right]$ is exactly equal to $\phi_-\left(\underline{\omega}^d\right)$. These quantities yield the following.

Lemma D.1. *In the game without the lobbyist there is a unique cutpoint equilibrium.*

- *If* $1 - \frac{\phi_-\left(\hat{\underline{\omega}}^d\right)}{\bar{r}} > \frac{k}{\pi}$ *then* $\underline{\omega}^d = \hat{\underline{\omega}}^d$*; otherwise* $\underline{\omega}^d$ *solves* $1 - \frac{\phi_-\left(\underline{\omega}^d\right)}{\bar{r}} = \frac{k}{\pi}$
- $\phi^d = \phi_-\left(\underline{\omega}^d\right)$, $\Delta_U^d = \alpha^d\left(1 - \frac{\phi^d}{\bar{r}}\right) = \frac{k}{\pi}$, *and* $\alpha^\emptyset = \phi^\emptyset = \Delta_\omega^\emptyset = 0$

The unique cutpoint equilibrium has a simple structure; all worthy SIGs lobby alongside a strictly positive measure of the "best" unworthy SIGs. The unique cutpoint generates sufficiently unfavorable treatment by the politician (either through more reviews, or a lower posture) to make all unworthy SIGs indifferent to lobbying, while all worthy SIGs strictly prefer to lobby.

E Equilibrium with the lobbyist

We now consider the model when the lobbyist is present ($\lambda \in (0,1)$), and characterize equilibria in which the lobbyist represents the SIG with strictly positive probability.[28]

E.1 Form of Equilibria

We begin by justifying attention to the form of strategy profiles described in Remark 1. The first step is to impose a key substantive assumption.

Assumption E.1. *The probability that a SIG of type* ω *lobbies directly absent representation does not depend on the history that led to a lack of representation.*

Assumption E.1 states that an unrepresented SIGs direct lobbying decision is invariant to *exactly how* it found itself unrepresented – that is, whether it was after finding the lobbyist busy, whether it was after finding the lobbyist available and rejecting his price, and if the latter the exact price he rejected. The rationale for the assumption is to rule out an indifferent unrepresented SIG conditioning its direct lobbying decision on a payoff-irrelevant history. Absent it, it is possible to artificially sustain equilibria of a variety of forms by exploiting a combination of an unworthy SIGs' indifference to lobbying directly off the equilibrium path and the lobbyist's policy motivations.

Imposing history independence on the SIG's direct lobbying strategy allows us further simplify the space of strategy profiles considered as follows.

Lemma E.1. *Given Assumption E.1 the following restrictions are w.l.o.g.*

[28] Contact authors for a characterization of equilibria in which lobbyist representation is off path.

- *the lobbyist proposes the SIG's willingness to pay F_ω with probability ρ_ω^L, and a price $F_\omega + \varepsilon$ strictly above with probability $1 - \rho_\omega^L$*
- *the SIG always accepts (rejects) offers of representation $\leq (>) F_\omega$. Absent representation, it lobbies directly with probability ρ_ω^S*

In the simplified strategy profiles the lobbyist mixes over at most two prices for each ω – one at the SIG's maximum willingness to pay F_ω – that is always accepted – and one strictly above – that is always rejected. We thus term charging the SIG's willingness to pay "accepting the SIG" and charging above "rejecting it." In these profiles an interior probability that a SIG of type ω acquires representation can be achieved only via the lobbyist mixing between accepting and rejecting the SIG.

We next restrict attention to cutpoint direct lobbying strategies for the SIG based on the justification in Section D.3.

Assumption E.2. *The probability that a SIG of type ω lobbies directly absent representation is $\rho_\omega^S = \mathbf{1}_{\omega > \underline{\omega}^d}$ for some $\underline{\omega}^d$.*

Assuming that the SIG's direct lobbying strategy is both history-independent and follows a cutpoint allows us to connect equilibrium of the three player game to the equilibrium of the two player game characterized in Lemma D.1 as follows.

Observation E.1. *In an equilibrium strategy profile satisfying Assumptions E.1-E.2 and the restrictions in Lemma E.1, $\underline{\omega}_d$ and (ϕ^c, α^c) $\forall c \in \{\emptyset, d\}$ must be the unique cutpoint equilibrium strategies of the two-player game with a prior $H(\omega)$ equal to the politician's posterior $H^\ell(\omega)$ after observing a lack of representation.*

In words, equilibrium requires "leftover" SIG's lacking representation to sort themselves between direct lobbying and staying home *as if* it is a game absent the lobbyist, but with a distribution over the SIG's case equal to the politician's posterior after observing only a lack of representation. Consequently, the equilibrium that would prevail after the lobbyist has "taken" his desired share of the market determines the value of the SIGs "outside options" (direct lobbying and staying home) if she rejects the lobbyist, pinning down both the price the lobbyist can charge and his beliefs about what the SIG will do absent representation.

Our final assumption further restricts attention to strategy profiles in which the lobbyist's representation strategy can also be described by a cutpoint.

Assumption E.3. *The probability that the lobbyist offers an acceptable price to a SIG of type ω is $\rho_\omega^L = \mathbf{1}_{\omega > \underline{\omega}^\ell}$ for some $\underline{\omega}^\ell$.*

The justification for restricting attention to a cutpoint strategy for the lobbyist is somewhat weaker than for restricting attention to a cutpoint strategy for the SIG.

Lemma E.2. *If $\Delta^{\ell}_{\omega} > \Delta^{d}_{\omega}$ $\forall \omega$, then the lobbyist's behavior is described by a cutpoint.*

That the lobbyist would strictly help SIGs who otherwise lobby directly is a natural property, and one that holds in all of the equilibria described in the main text with representation. However, it is not one that applies to all equilibria with representation, or even equilibria with representation in which some clients are strictly helped. We do not fully characterize the set of representation equilibria that do not have a cutpoint structure, and the set may be potentially large.[29]

We now characterize the equilibria in the main text (as well as some others). To do so we subdivide the class into three cases, and characterize conditions under which an equilibrium in each case holds: **Case A** ($\underline{\omega}^{\ell} > (2P - \bar{\omega}, \bar{\omega})$) involves representation with a fully favorable posture. **Case B** ($\underline{\omega}^{\ell} = 2P - \bar{\omega}$) involves representation with a weakly favorable posture. **Case C** ($\underline{\omega}^{\ell} < 2P - \bar{\omega}$) involves representation with an adversarial posture, and we show that such equilibria cannot exist.

E.2 Preliminary Properties of Equilibria

We first describe some equilibrium properties in each case.

Properties of the Lobbyist Channel

(Case A: $\underline{\omega}^{\ell} > (2P - \bar{\omega}, \bar{\omega})$): This implies $E^{\ell}[\omega] > P$, which then requires $\phi^{\ell} = \phi^{\ell}_{-}$ and $\alpha^{\ell} = 1$. Applying that $H(\cdot)$ is uniform we have that $\phi^{\ell}_{-} = \frac{\left(P - \underline{\omega}^{\ell}\right)^2}{2\left(\bar{\omega} - \underline{\omega}^{\ell}\right)}$. These jointly imply that $\Delta^{\ell}_{U} = 1 - \left(\frac{1}{\bar{r}}\right) \frac{\left(P - \underline{\omega}^{\ell}\right)^2}{2\left(\bar{\omega} - \underline{\omega}^{\ell}\right)}$ and $\Delta^{\ell}_{W} = 1$.

(Case B: $\underline{\omega}^{\ell} = 2P - \bar{\omega}$): This implies $E^{\ell}[\omega] = P$, so then $\phi^{\ell}_{+} = \phi^{\ell}_{-} = \frac{\bar{\omega} - P}{4}$ and any $\alpha^{\ell} \in [0, 1]$ is a best-response, so $\Delta^{\ell}_{U} = \alpha^{\ell}\left(1 - \left(\frac{1}{\bar{r}}\right)\frac{\bar{\omega} - P}{4}\right)$ and $\Delta^{\ell}_{W} = \Delta^{\ell}_{U} + \left(\frac{1}{\bar{r}}\right)\frac{\bar{\omega} - P}{4}$.

(Case C: $\underline{\omega}^{\ell} < 2P - \bar{\omega}$): This implies $E^{\ell}[\omega] < P$, so then $\phi^{\ell} = \phi^{\ell}_{+}$ and $\alpha^{\ell} = 0$. Applying that $H(\cdot)$ is uniform we have that $\phi^{\ell}_{+} = \frac{(\bar{\omega} - P)^2}{2(\bar{\omega} - \underline{\omega}_{\ell})}$. These jointly imply that $\Delta^{\ell}_{U} = 0$ and $\Delta^{\ell}_{W} = \left(\frac{1}{\bar{r}}\right)\frac{(\bar{\omega} - P)^2}{2(\bar{\omega} - \underline{\omega}_{\ell})}$.

Properties of the Direct and Null Channel

As previously described, the direct lobbying cutpoint and politician strategies toward $\{d, \emptyset\}$ when the SIG's direct lobbying strategy satisfies Assumption E.1 must be an equilibrium of the two player game when $H(\omega) = H^{\neg \ell}(\omega)$. Next observe that a cutpoint strategy by the lobbyist implies that $E^{\neg \ell}[\omega] < P$, so by Lemma D.1 for each $\underline{\omega}^{\ell} < \bar{\omega}$ there is a unique

[29] Contact authors for example of a non-cutpoint equilibrium in which representation strictly increases the chance that some but not all clients get the favor.

cutpoint equilibrium of the two player game that pins down the direct lobbying cutpoint and politician strategies toward $\{d, \emptyset\}$.

We now apply Lemma D.1 to derive properties of the direct and null channels as a function of $\underline{\omega}^\ell$ in each of the three cases (A,B,C). Immediately we have that $\underline{\omega}^d \in (0, P)$, $\phi^d = \phi^d_-$, $\Delta^d_U = \frac{k}{\pi}$, $\Delta^d_W = \frac{k}{\pi} + \frac{\phi^d_-}{\bar{r}}$, and $\alpha^\emptyset = \phi^\emptyset = \Delta^\emptyset_\omega = 0$. Next we derive the precise values of $\underline{\omega}^d$ and ϕ^d_- for each $\underline{\omega}_\ell < \bar{\omega}$.

(Case A: $\underline{\omega}^\ell \in (2P - \bar{\omega}, \bar{\omega})$): Using the equilibrium characterization in Lemma D.1, we first derive the unique cutpoint $\hat{\underline{\omega}}^d$ satisfying $E\left[\omega | \omega \geq \hat{\underline{\omega}}^d\right] = P$. It is straightforward that we must have $\hat{\underline{\omega}}^d < \underline{\omega}^\ell$ for the equality to be satisfied. Next, observe that since the prior over ω is uniform over $[0, \bar{\omega}]$, for $\underline{\omega}^d < \underline{\omega}^\ell$ we have that

$$\begin{aligned} E\left[\omega | \omega \geq \underline{\omega}^d\right] &= \left(\frac{\left(\underline{\omega}^\ell - \underline{\omega}^d\right)}{\left(\underline{\omega}^\ell - \underline{\omega}^d\right) + (1-\lambda)\left(\bar{\omega} - \underline{\omega}^\ell\right)}\right)\left(\frac{\underline{\omega}^d + \underline{\omega}^\ell}{2}\right) \\ &\quad + \left(\frac{(1-\lambda)\left(\bar{\omega} - \underline{\omega}^\ell\right)}{\left(\underline{\omega}^\ell - \underline{\omega}^d\right) + (1-\lambda)\left(\bar{\omega} - \underline{\omega}^\ell\right)}\right)\left(\frac{\underline{\omega}^\ell + \bar{\omega}}{2}\right). \end{aligned}$$

Algebraic manipulation then yields that $\hat{\underline{\omega}}^d$ uniquely satisfies the equality:

$$\left(P - \hat{\underline{\omega}}^d\right)^2 = (1-\lambda)\left(\bar{\omega} - P\right)^2 + \lambda\left(P - \underline{\omega}^\ell\right)^2$$

which yields $\hat{\omega}^d = P - \sqrt{(1-\lambda)\left(\bar{\omega} - P\right)^2 + \lambda\left(P - \underline{\omega}^\ell\right)^2}$ which is $\in \left(2P - \bar{\omega}, \min\left\{\underline{\omega}^\ell, P\right\}\right)$ unless $\underline{\omega}^\ell = 2P - \bar{\omega}$ or $\underline{\omega}^\ell = \bar{\omega}$ in which case $\hat{\underline{\omega}}^d = 2P - \bar{\omega}$ and $\phi^d_- = \phi^\ell_- = \frac{\bar{\omega} - P}{4}$. Further, observe that $\hat{\underline{\omega}}^d$ is only affected by $\underline{\omega}^\ell$ via its *distance* $\left(P - \underline{\omega}^\ell\right)^2$ from P. Thus, two representation cutpoints $\underline{\omega}^\ell$ and $\tilde{\underline{\omega}}^\ell = 2P - \underline{\omega}^\ell$ yield the same $\hat{\underline{\omega}}^d$. Next, the following lemma is proved in Appendix F.

Lemma E.3. *If $\underline{\omega}^\ell \in (2P - \bar{\omega}, \bar{\omega})$ and $\underline{\omega}^d = \hat{\underline{\omega}}^d$ then $\frac{\bar{\omega} - P}{4} > \phi^d_-$.*

Finally Lemma E.3 implies that $1 - \frac{\phi^d_-}{\bar{r}} > 1 - \frac{\bar{\omega} - P}{4\bar{r}} > \frac{k}{\pi}$, so that by Lemma D.1 the equilibrium cutpoint on the direct channel $\underline{\omega}^d$ is indeed equal to $\hat{\underline{\omega}}^d$.

To derive the explicit expression for ϕ^d_- in this case, recall from Observation D.1 that when $\underline{\omega}^d = \hat{\underline{\omega}}^d$ we have $\phi^d_- = \phi^d_+$ by definition, and we may thus employ either expression. The expression for ϕ^d_+ is simpler to write when $\underline{\omega}^\ell \leq P$, and the expression for ϕ^d_+ is simpler to write when $\underline{\omega}^\ell \geq P$. For $\underline{\omega}^\ell \leq P$ we have

$$\begin{aligned} \phi^d_+ &= \left(\frac{(1-\lambda)\left(\bar{\omega} - P\right)}{\lambda\left(\underline{\omega}^\ell - \hat{\underline{\omega}}^d\right) + (1-\lambda)\left(\bar{\omega} - \hat{\underline{\omega}}^d\right)}\right)\left(\frac{P + \bar{\omega}}{2} - P\right) \\ &= \frac{1}{2}\left(\frac{(1-\lambda)\left(\bar{\omega} - P\right)^2}{\lambda\left(\underline{\omega}^\ell - \hat{\underline{\omega}}^d\right) + (1-\lambda)\left(\bar{\omega} - \hat{\underline{\omega}}^d\right)}\right), \end{aligned}$$

where the denominator of the second term is the unconditional probability of direct lobbying.

For $\underline{\omega}^\ell \geq P$ we have

$$\begin{aligned}\phi_-^d &= \left(\frac{P-\hat{\underline{\omega}}^d}{\lambda\left(\underline{\omega}^\ell-\hat{\underline{\omega}}^d\right)+(1-\lambda)\left(\bar{\omega}-\hat{\underline{\omega}}^d\right)}\right)\left(P-\frac{\hat{\underline{\omega}}^d+P}{2}\right)\\ &= \frac{1}{2}\left(\frac{\left(P-\hat{\underline{\omega}}^d\right)^2}{\lambda\left(\underline{\omega}^\ell-\hat{\underline{\omega}}^d\right)+(1-\lambda)\left(\bar{\omega}-\hat{\underline{\omega}}^d\right)}\right)\end{aligned}$$

Now using that $\left(P-\hat{\underline{\omega}}^d\right)^2=(1-\lambda)(\bar{\omega}-P)^2+\lambda\left(\underline{\omega}^\ell-P\right)^2$, $\phi_+^d=\phi_-^d$ at $\hat{\underline{\omega}}^d$, and combining yields that for any $\underline{\omega}^\ell\in(2P-\bar{\omega},\bar{\omega})$ we have

$$\phi_-^d=\frac{1}{2}\left(\frac{(1-\lambda)(\bar{\omega}-P)^2+\lambda\left(\max\left\{\underline{\omega}^\ell-P,0\right\}\right)^2}{\lambda\left(\underline{\omega}^\ell-\hat{\underline{\omega}}^d\right)+(1-\lambda)\left(\bar{\omega}-\hat{\underline{\omega}}^d\right)}\right).$$

(Cases B and C: $\underline{\omega}^\ell \leq 2P-\bar{\omega}$): We have that $\underline{\omega}^d=2P-\bar{\omega}$ and $\phi_-^d=\frac{\bar{\omega}-P}{4}$, and the only SIGs who lobby directly are those that found the lobbyist unavailable.

The Lobbyist's Incentives

If the lobbyist accepts the SIG, he expects it to acquire the favor with probability Δ_ω^ℓ, while if he rejects it he expects it to acquire the favor with probability $\mathbf{1}_{\omega>\underline{\omega}_d}\cdot\Delta_\omega^d+\left(1-\mathbf{1}_{\omega>\underline{\omega}_d}\right)\Delta_\omega^\emptyset$ (the probability the rejected SIG lobbies directly times the probability direct lobbying yields the favor, plus the probability it stays home times the probability staying home yields the favor). The net benefit to the lobbyist of accepting the SIG is thus:

$$(F_\omega-k)+\left(\Delta_\omega^\ell-\left(\mathbf{1}_{\omega>\underline{\omega}_d}\cdot\Delta_\omega^d+\left(1-\mathbf{1}_{\omega>\underline{\omega}_d}\right)\Delta_\omega^\emptyset\right)\right)\cdot\delta(\omega-L) \tag{B.4}$$

$F_\omega-k$ is the net profit from representation at the SIG's maximum willingness to pay, $\Delta_\omega^\ell-\left(\mathbf{1}_{\omega>\underline{\omega}_d}\cdot\Delta_\omega^d+\left(1-\mathbf{1}_{\omega>\underline{\omega}_d}\right)\Delta_\omega^\emptyset\right)$ is the net change in the probability the SIG acquires the favor, and $\delta(\omega-L)$ is the net policy benefit to the lobbyist of the SIG going from being denied to being granted the favor.

Now, equilibrium on the direct and null channels imply that all types of SIGs weakly prefer lobbying directly to staying home, so $\forall\omega$ we have $F_\omega=\left(\Delta_\omega^\ell-\Delta_\omega^d\right)\pi+k$. Further, unworthy SIGs are indifferent to lobbying directly and staying home, so $F_U=\left(\Delta_U^\ell-\Delta_U^d\right)\pi+k=\Delta_U^\ell\pi$. Applying these properties to the net benefit expression and rearranging yields the following best response behavior.

Observation E.2. *Among strategy profiles of the form in Remark 1, the lobbyist's strategy is a best response i.f.f.* $\left(\Delta_\omega^\ell-\Delta_\omega^d\right)(\pi+\delta(\omega-L))+\mathbf{1}_{\omega<\underline{\omega}^d}\cdot\Delta_\omega^d\delta(\omega-L)>(<)\,0$ *implies that* $\omega>(\leq)\,\underline{\omega}_\ell$.

E.3 Representation Equilibria

We now characterize the three classes of equilibria with representation. Observe that $\pi+\delta(\omega-L)$ is the net benefit of representation if the lobbyist is *pivotal* for the favor being granted, and let $\hat{\underline{\omega}}^\ell(L)=L-\frac{\pi}{\delta}$ denote the unique value of ω s.t. this is $=0$. This quantity

will be crucial in the equilibrium characterization that follows.

Case A: Representation with a fully-favorable posture

When $\underline{\omega}^\ell \in (2P-\bar{\omega},\bar{\omega})$, the remaining strategies are pinned down to unique values as characterized above. We argue that such a $\underline{\omega}^\ell$ is an equilibrium i.f.f. $\underline{\omega}^\ell = \hat{\underline{\omega}}^\ell(L)$, and thus such equilibria exist $\iff \hat{\underline{\omega}}^\ell(L) \in (2P-\bar{\omega},\bar{\omega})$.

We begin by arguing that $\Delta^\ell_\omega - \Delta^d_\omega > 0\ \forall\omega$. For $\omega < P$ we have that $\Delta^\ell_\omega = 1 - \frac{\phi^\ell_-}{\bar{r}} = 1 - \frac{\left(P-\underline{\omega}^\ell\right)^2}{2\left(\bar{\omega}-\underline{\omega}^\ell\right)} > 1 - \frac{\bar{\omega}-P}{4\bar{r}} > \frac{k}{\pi} = \Delta^d_\omega$. For $\omega > P$ we have $\Delta^d_\omega = \frac{k}{\pi} + \phi^d_-$, which has already been shown to be $< 1 = \Delta^\ell_\omega$.

Now recall that $\underline{\omega}^d < \underline{\omega}^\ell$ when $\underline{\omega}^\ell \in (2P-\bar{\omega},\bar{\omega})$. Thus, for $\omega \geq \underline{\omega}^d$ (SIGs who would lobby directly absent representation) eqn. E.2 is $> (<) 0 \iff \pi + \delta(\omega - L) > (<) 0$. A necessary condition for $\underline{\omega}^\ell$ to be a best response for the lobbyist is thus $\underline{\omega}^\ell = \hat{\underline{\omega}}^\ell(L)$. To argue that this is also sufficient and therefore an equilibrium we must also show the lobbyist would not wish to represent SIGs who would not lobby directly absent representation, i.e. $\omega < \underline{\omega}^d$. But this is straightforward since $\omega < \underline{\omega}^d < \underline{\omega}^\ell \to \pi + \delta(\omega - L) < 0 \to \delta(\omega - L) < 0$.

Case B: Representation with a partially-favorable posture

When $\underline{\omega}^\ell = 2P - \bar{\omega}$, all remaining strategies *except* the politician's posture α^ℓ toward the lobbyist's client are pinned down. We derive conditions under which $\underline{\omega}^\ell = 2P-\bar{\omega}$ for some value(s) of α^ℓ. From the preceding analysis we have that $\underline{\omega}^d = \underline{\omega}^\ell = 2P-\bar{\omega}$ and $\phi^d = \phi^\ell = \frac{\bar{\omega}-P}{4}$, which further implies that $\Delta^\ell_U - \Delta^d_U = \Delta^\ell_U - \frac{k}{\pi} = \Delta^\ell_W - \Delta^d_W$, where $\Delta^\ell_U = \alpha^\ell\left(1 - \frac{\bar{\omega}-P}{4\bar{r}}\right)$ may take any value in $\left[0, 1 - \frac{\bar{\omega}-P}{4\bar{r}}\right]$ and has a one to one relationship with implied $\alpha^\ell \in [0,1]$.

Now using eqn. E.2 and exploiting the preceding observations, for this behavior to be a best response for the lobbyist the following two conditions are necessary and sufficient; (i) she prefers to decline unworthy SIGs $\omega \in [0, 2P-\bar{\omega}]$ who would otherwise stay home, i.e. $\Delta^\ell_U(\pi + \delta(\omega - L)) \leq k\ \forall\omega \in [0, 2P-\bar{\omega}]$ (ii) she prefers to represent SIGs $\omega \geq 2P-\bar{\omega}$ who would otherwise lobby directly (which is a mixture of worthy and unworthy clients), i.e. $\left(\Delta^\ell_U - \frac{k}{\pi}\right)(\pi + \delta(\omega - L)) \geq 0\ \forall\omega \geq 2P-\bar{\omega}$ Condition (i) may be checked only at $\omega = 2P-\bar{\omega}$ since the l.h.s is strictly increasing in ω when $\Delta^\ell_U > 0$. Thus to be satisfied it is necessary and sufficient that either $\hat{\underline{\omega}}^\ell(L) \geq 2P-\bar{\omega}$ or $\hat{\underline{\omega}}^\ell(L) < 2P-\bar{\omega}$ and

$$\Delta^\ell_U \leq \tilde{\Delta}^\ell_U(L) = \frac{k}{\pi + \delta\left((2P-\bar{\omega}) - L\right)}$$

where $\tilde{\Delta}^\ell_U(L)$ is a strictly increasing function that $\to \infty$ as $\hat{\underline{\omega}}^\ell(L) \to 2P-\bar{\omega}$.

With conditions (i) and (ii) , we walk through when $\underline{\omega}^\ell = 2P-\bar{\omega}$ is an equilibrium for all possible values of $\hat{\underline{\omega}}^\ell(L) = L - \frac{\pi}{\delta} \in (-\infty,\infty)$. There are three subcases.

(Subcase B.1: $\hat{\underline{\omega}}^\ell(L) \leq 2P-\bar{\omega}$): Condition (i) is satisfied i.f.f. $\Delta^\ell_U \leq \tilde{\Delta}^\ell_U(L)$. Now since $\pi + \delta(\omega - L) > 0\ \forall\omega > 2P-\bar{\omega}$, condition (ii) is satisfied $\iff \Delta^\ell_U \geq \frac{k}{\pi}$. Thus, this is

an equilibrium i.f.f.

$$\Delta_U^\ell \in \left[\frac{k}{\pi}, \min\left\{\tilde{\Delta}_U^\ell(L), \frac{\bar{\omega}-P}{4\bar{r}}\right\}\right].$$

This set is nonempty i.f.f. $L \geq 2P - \bar{\omega}$; when it is empty ($L < 2P - \bar{\omega}$) we later argue that an equilibrium with lobbyist exit will prevail.

(Subcase B.2: $\underline{\hat{\omega}}^\ell(L) \in (2P - \bar{\omega}, \bar{\omega})$): Condition (i) is always satisfied. Since $\underline{\hat{\omega}}^\ell(L)$ is interior to $(2P - \bar{\omega})$, condition (ii) is satisfied i.f.f. $\Delta_U^\ell = \frac{k}{\pi}$.

(Subcase B.3: $\underline{\hat{\omega}}^\ell(L) \geq \bar{\omega}$): Condition (i) is always satisfied. Since $\pi + \delta(\omega - L) < 0$ $\forall \omega < \bar{\omega}$, condition (ii) is satisfied i.f.f. representation weakly hurts the chances that a SIG who would otherwise lobby directly gets the favor, i.e. $\Delta_U^\ell \in \left[0, \frac{k}{\pi}\right]$.

We assume that equilibria in subcase $(B.1)$ prevail when $\underline{\hat{\omega}}^\ell(L) \leq 2P - \bar{\omega}$, which is precisely when Case A equilibria with a fully favorable posture ($\underline{\omega}^\ell > 2P - \bar{\omega}$) do not exist. We further assume that the equilibrium lobbyist exit on the extensive margin will prevail when $L < 2P - \bar{\omega}$ and neither equilibria in subcase $(B.1)$ nor any other case with representation exist.

Equilibria in subcase $(B.2)$ co-exist exactly with Case A equilibria $\underline{\omega}_\ell \in (2P - \bar{\omega})$ with a fully favorable posture. We assume that in this case, the equilibrium with a fully favorable posture will prevail, as opposed to subcase $(B.2)$ equilibrium with a somewhat favorable posture that are sustained by a knife-edge condition that the lobbyist is exactly no better at securing the favor than the SIG is on her own.

Subcase $(B.3)$ equilibria prevail under conditions that we state the main text will lead to an equilibrium with lobbyist exit because she is too ideologically opposed to the SIG, i.e. $\underline{\hat{\omega}}^\ell(L) \geq \bar{\omega}$. In these equilibria, a policy-motivated lobbyist wishes to *harm* the SIG's ability to acquire the favor regardless of the value of ω, and is willing to take a loss on lobbying in order to do it. She thus charges a price below the mutual cost of access, and the SIG accepts because it is willing to have its prospects harmed in exchange for a reduced cost of access. We consider these equilibria empirically implausible and omit their consideration from the main text.

Case C: Representation with an adversarial posture

We argue that equilibria of the form in Remark 1 with $\underline{\omega}^\ell < 2P - \bar{\omega}$ (so the politician's posture toward the lobbyist is adversarial) do not exist.

From the preceding observe $\underline{\omega}^\ell < 2P - \bar{\omega} = \underline{\omega}^d$ and $\Delta_U^\ell = 0$. Thus, the net benefit of representing a SIG $\omega < \underline{\omega}^d$ who would not lobby directly (and are also all unworthy) is $\Delta_U^\ell(\pi + \delta(\omega - L)) - k = -k < 0$. Thus it cannot be an equilibrium for the lobbyist to represent unworthy SIGs in $\left(\underline{\omega}^\ell, \underline{\omega}^d\right)$. More generally, when the lobbyist has no chance of acquiring the favor for an unworthy SIG, she will never be willing to represent such a SIG if

it does not otherwise intend to lobby directly; she will both lose money and have *no effect* on the likelihood the SIG acquires the favor.

F Proofs

Proof of Lemma D.1 We begin by characterizing properties that apply to all PBEs in which the SIG sometimes lobbies directly.

First, we argue that in any such PBE, unworthy SIGs must stay home with strictly positive probability, and worthy SIGs must lobby with strictly positive probability. If all unworthy SIG's lobbied or only unworthy SIG's lobbied, then $E^d[\omega] < P \rightarrow \alpha^d = 0 \rightarrow \Delta_U^d = 0 \rightarrow \left(\Delta_U^d - \Delta_U^\emptyset\right)\pi \leq 0 < k$, so all unworthy SIGs would want to deviate to not lobbying. Next, we argue that unworthy SIGs lobby with strictly positive probability; if not then (a) lobbying would be a perfect signal that the SIG is worthy, and (b) $E^\emptyset[\omega] < P$. Then (a) would imply $\alpha^\ell = 1$ and $\phi^\ell = 0 \rightarrow \Delta_U^d = 1$, while (b) would imply $\alpha^\emptyset = 0 \rightarrow \Delta_U^\emptyset = 0$, together implying $\left(\Delta_U^d - \Delta_U^\emptyset\right)\pi = \pi > k$, implying all unworthy SIGs would want to deviate to lobbying.

Now observe that

$$\begin{aligned} & E[\omega] = \Pr(c = d) \cdot E^d[\omega] + \Pr(c = \emptyset) \cdot E^\emptyset[\omega] \\ \iff \quad & \Pr(c = d) \cdot \left(E^d[\omega] - E[\omega]\right) = \Pr(c = \emptyset)\left(E[\omega] - E^\emptyset[\omega]\right) \end{aligned}$$

Since $E[\omega]$ is $< P$ by assumption, the preceding expression implies that $E^c[\omega] \geq P$ (and so $\alpha^c > 0$) for at most one $c \in \{d, \emptyset\}$. Now a strictly interior probability of lobbying for unworthy SIG's implies that they must be indifferent, i.e. $\left(\Delta_\omega^d - \Delta_\omega^\emptyset\right)\pi = k$. This clearly requires $\alpha^d > 0$, in turn implying $E^d[\omega] \geq P$, $E^\emptyset[\omega] < P$, and $\alpha^\emptyset = 0$, further implying $\Delta_U^\emptyset = 0$ and $\Delta_U^d = \frac{k}{\pi}$.

We now examine the additional implications of the SIG using a cutpoint strategy $\underline{\omega}_d$. First, $\alpha^d > 0$ requires that $E^d[\omega] = E\left[\omega | \omega \geq \hat{\underline{\omega}}^d\right] \geq P \rightarrow \underline{\omega}^d \geq P$. Further we must have $\underline{\omega}^d < P$ since $\underline{\omega}^d \geq P \rightarrow \alpha^d = 1$ and $\phi_-\left(\underline{\omega}^d\right) = 0 \rightarrow \Delta_U^d = 1 > \frac{k}{\pi}$. Finally, for every value of $\Delta_U^d \in (0,1)$ there is a unique combination of lobbyist cutpoint $\underline{\omega}^d$ and politician best responses $\left(\alpha^d, \phi^d\right)$ that achieve it. For $\Delta_U^d \in \left(0, 1 - \frac{\phi_-\left(\hat{\underline{\omega}}^d\right)}{\bar{r}}\right)$ it is $\underline{\omega}^d = \hat{\underline{\omega}}^d$ $(\rightarrow E^d[\omega] = P)$ and $\alpha^d = \frac{\Delta_U^d}{1 - \phi_-\left(\hat{\underline{\omega}}^d\right)/\bar{r}}$. For $\Delta_U^d \in \left[1 - \frac{\phi_-\left(\hat{\underline{\omega}}^d\right)}{\bar{r}}, 1\right]$ it is the unique value satisfying $1 - \frac{\phi_-\left(\underline{\omega}^d\right)}{\bar{r}} = \Delta_U^d$, implying $\underline{\omega}^d > \hat{\underline{\omega}}^d$ and $\alpha^d = 1$. The preceding ensures that the behavior of unworthy SIGs and the politician are mutual best responses. To verify the behavior of worthy SIGs (always lobby) is also a best response observe $\underline{\omega}^d < P \rightarrow \phi_+^\emptyset = 0 < \phi_-\left(\underline{\omega}^d\right)$. **QED**

Proof of Lemma E.1 Consider an equilibrium strategy profile satisfying Assumption E.1, let p_A be the probability that the SIG accepts price F_W, and let ρ_ω^L denote the probability

a SIG of type ω acquires lobbyist representation through the bargaining process. Now let $U_\omega^\ell(F)$ denote the lobbyist's utility from offering price F in the original strategy profile; it is easily verified that

$$U_\omega^\ell(F) = \begin{cases} U_\omega^A - (F_\omega - F) & \text{if } F < F_\omega \\ p_A \cdot U_\omega^A + (1-p_A) U_\omega^R & \text{if } F = F_\omega \\ U_\omega^R & \text{if } F > F_\omega \end{cases}$$

where U_ω^A is the lobbyist's utility from "accepting" the SIG at its willingness to pay and U_ω^R is the lobbyist's utility from "rejecting" the SIG (which is unaffected by the price due to assumption E.1). From this it is clear that the lobbyst must only mix over prices $F \geq F_\omega$, and that the lobbyist's equilibrium utility must be $\rho_\omega^L U_\omega^A + \left(1-\rho_\omega^L\right) U_\omega^R$.

Now, jointly perturbing p_A to $p'_A = 1$ and the lobbyist's pricing strategy to F_ω with probability ρ_ω^L and $F_\omega + \varepsilon$ with probability $1-\rho_\omega^L$ keeps the SIG's strategy a best response, and does not perturb the lobbyist's utility. We further argue that perturbing the SIG's strategy did not change the maximum utility $\max_F \left\{U_\omega^\ell(F)\right\}$ achievable by the lobbyist, and thus his strategy in the perturbed profile must also be a best response. If $U_\omega^A \leq U_\omega^R$ this is straightforward, and if $U_\omega^A > U_\omega^R$ then p_A must have already been 1 (otherwise the lobbyist would not have had a best response in the original profile). Lastly, the politician's best response set is only affected by the probability the SIG pursues each channel, which was unaffected. **QED**

Proof of Lemma E.2 First suppose that the lobbyist only represents types $\omega \geq \underline{\omega}^d$ who would otherwise lobby directly (for whom the net benefit of representation is $\left(\Delta_\omega^\ell - \Delta_\omega^d\right)\left(\pi + \delta\left(\underline{\omega}^\ell - L\right)\right)$). Then she represents the SIG i.f.f. $\pi + \delta(\omega - L) \geq 0$, and her behavior must follow a cutpoint.

Suppose next that the lobbyist represents some types $\omega < \underline{\omega}^d$ that do not lobby directly. The net benefit of representing such types is $\Delta_U^\ell(\pi + \delta(\omega - L)) \geq k$. Thus, *among* $\omega \leq \underline{\omega}^d$ her behavior is described by a cutpoint. In addition, if she represents some $\omega \leq \underline{\omega}^d$ then $\pi + \delta(\omega - L) > 0 \ \forall \omega > \underline{\omega}^d$ so she also represent types who would otherwise lobby directly her behavior is described by a cutpoint overall. **QED**

Proof of Lemma E.3 First, recall from Observation D.1 that at $\underline{\omega}^d = \hat{\underline{\omega}}^d$ we have $\phi_-^d = \phi_+^d$. Next observe that when $\underline{\omega}^\ell \leq P$, the definitions yield that:

$$\phi_-^d = \phi_+^d = \left(\frac{1}{2}\right) \frac{(1-\lambda)(\bar{\omega} - P)^2}{\left(\lambda\left(\underline{\omega}^\ell - \hat{\underline{\omega}}^d\right) + (1-\lambda)(\bar{\omega} - P) + (1-\lambda)\left(P - \hat{\underline{\omega}}^d\right)\right)}$$

where the denominator of the second term is the unconditional probability of direct lobbying, which we denote $\rho^S\left(\underline{\omega}^d; \underline{\omega}^\ell\right)$. Next we argue $\underline{\omega}^\ell \in (2P - \bar{\omega}, \bar{\omega}) \rightarrow \lambda\left(\underline{\omega}^\ell - \hat{\underline{\omega}}^d\right) > (1-\lambda)\left(\hat{\underline{\omega}}^d - (2P - \bar{\omega})\right)$, which further implies that $\rho^S\left(\hat{\underline{\omega}}^d; \underline{\omega}^\ell\right) > 2(1-\lambda)(\bar{\omega} - P)$. Ob-

serve that

$$\begin{aligned}\left(P-\underline{\hat{\omega}}^d\right)^2 &= (1-\lambda)(\bar{\omega}-P)^2+\lambda\left(P-\underline{\omega}^\ell\right)^2 \\ &> \left((1-\lambda)(\bar{\omega}-P)+\lambda\left(P-\underline{\omega}^\ell\right)\right)^2 \text{ (since } x^2 \text{ convex)}\end{aligned}$$

which implies $P-\underline{\hat{\omega}}^d > (1-\lambda)(\bar{\omega}-P)+\lambda\left(P-\underline{\omega}^\ell\right)$, which implies $\lambda\left(\underline{\omega}^\ell-\underline{\hat{\omega}}^d\right) > (1-\lambda)\left(\underline{\hat{\omega}}^d-(2P-\bar{\omega})\right)$. Finally, using this we have

$$\frac{(1-\lambda)(\bar{\omega}-P)^2}{2\rho^S\left(\underline{\hat{\omega}}^d;\underline{\omega}^\ell\right)} < \frac{(1-\lambda)(\bar{\omega}-P)^2}{2\cdot 2(1-\lambda)(\bar{\omega}-P)} = \frac{\bar{\omega}-P}{4}$$

proving the desired property for $\underline{\omega}^\ell \leq P$.

To prove the desired property for $\underline{\omega}^\ell \geq P$, observe that $\underline{\hat{\omega}}^d$ is identical for the reflection point $\underline{\tilde{\omega}}^\ell = 2P-\underline{\omega}^\ell \leq P$ about P. We already know from the preceding that $\frac{\bar{\omega}-P}{4} > \phi_+^d = \phi_-^d$ at $\underline{\tilde{\omega}}^\ell \leq P$. We now wish to show that ϕ_-^d at $\underline{\tilde{\omega}}^\ell \leq P$ is strictly greater than ϕ_-^d at $\underline{\omega}^\ell = 2P-\underline{\tilde{\omega}}^\ell \geq P$, which yields the desired property. First observe that ϕ_-^d at $\underline{\tilde{\omega}}^\ell$ is $W\cdot X$, where

$$W = \frac{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)}{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)(\bar{\omega}-P)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)}$$

$$\begin{aligned}X &= P-\left(\frac{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)}{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)}\right)\left(\frac{\underline{\hat{\omega}}^d+\underline{\tilde{\omega}}^\ell}{2}\right) \\ &\quad -\left(\frac{(1-\lambda)(P-\underline{\hat{\omega}}_d)}{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}_d\right)+(1-\lambda)(P-\underline{\hat{\omega}}_d)}\right)\left(\frac{\underline{\hat{\omega}}_d+P}{2}\right).\end{aligned}$$

Next observe that ϕ_-^d at $\underline{\omega}^\ell$ is $Y\cdot Z$, where

$$Y = \frac{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)+\lambda\left(P-\underline{\tilde{\omega}}^\ell\right)}{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)(\bar{\omega}-P)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)+2\lambda\left(P-\underline{\tilde{\omega}}^\ell\right)}$$

and $Z = P-\left(\frac{\underline{\hat{\omega}}^d+P}{2}\right)$. Clearly, $X > Z$. We wish to show that also $W > Y$, proving the desired property. Write W as $\frac{a}{b}$ which yields $Y = \frac{a+c}{b+2c}$, where $c = \lambda\left(P-\underline{\tilde{\omega}}^\ell\right)$; taking the difference $W-Y = \frac{a}{b}-\frac{a+c}{b+2c}$ yields $\left(\frac{2c}{b+2c}\right)\left(\frac{a}{b}-\frac{1}{2}\right)$, which is >0 provided $W = \frac{a}{b} > \frac{1}{2}$. This is simple again using $\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right) > (1-\lambda)\left(\underline{\hat{\omega}}^d-(2P-\bar{\omega})\right)$ since

$$\begin{aligned}W &= \frac{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)}{\lambda\left(\underline{\tilde{\omega}}^\ell-\underline{\hat{\omega}}^d\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)+(1-\lambda)(\bar{\omega}-P)} \\ &> \frac{(1-\lambda)\left(\underline{\hat{\omega}}^d-(2P-\bar{\omega})\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)}{(1-\lambda)\left(\underline{\hat{\omega}}^d-(2P-\bar{\omega})\right)+(1-\lambda)\left(P-\underline{\hat{\omega}}^d\right)+(1-\lambda)(\bar{\omega}-P)} = \frac{1}{2}\end{aligned}$$ **.QED**

Supplemental Author Appendix for *Lobbyists as Gatekeepers: Theory and Evidence*

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G Equilibria with lobbyist exit

We next consider equilibria when the lobbyist is present as a player, but chooses to exit the market. For simplicity we restrict attention to strategy profiles of the form in Remark 1, so that exit is described by $\underline{\omega}^{\ell} = \bar{\omega}$.

It is straightforward that in strategy profiles of the form in Remark 1, $\underline{\omega}^{d} = 2P - \bar{\omega}$ and $\phi^{d} = \frac{\bar{\omega}-P}{4}$, which further implies that $\Delta_U^{\ell} - \Delta_U^{d} = \Delta_U^{\ell} - \frac{k}{\pi}$. Thus, necessary and sufficient conditions to sustain lobbyist exit are that (i)

$$\Delta_U^{\ell}\left(\pi + \delta\left((2P - \omega) - L\right)\right) \leq k$$

(to reject $\omega \leq \underline{\omega}^{d} = 2P - \bar{\omega}$), and (ii)

$$\left(\Delta_{\omega}^{\ell} - \Delta_{\omega}^{d}\right)\left(\pi + \delta\left(\omega - L\right)\right) \leq 0 \ \forall \omega \in [2P - \bar{\omega}, \bar{\omega}]$$

(to reject $\omega \in (2P - \bar{\omega}, \bar{\omega}]$). The (off-path) probabilities that the SIG acquires the favor through the lobbyist $\left(\Delta_U^{\ell}, \Delta_W^{\ell}\right)$ must then arise from a politician strategy $\left(\phi^{\ell}, \alpha^{\ell}\right)$ toward the lobbyist that is optimal given the politician's off-path beliefs.

The model is not a straightforward signaling game since the "signals" that the politician receives result from the choices by *two* privately informed players – the SIG and the lobbyist – rather than just one. It is thus necessary to somehow structure expectations of what sorts of off-path beliefs and best responses for the politician might be reasonable.

To do so we begin with the supposition that off-the-equilibrium path, the politician will ascribe deviations only to the lobbyist in her pricing strategy, rather than the SIG and its acceptance strategy. For the purposes of refinement we therefore treat the model *as if* it is a standard two-player signaling game, in which the lobbyist's payoffs from deviating to different representation decisions are computed as if the SIG will accept representation if and only if it is weakly profitable. With this assumption we apply universal divinity (Banks and Sobel 1987) and then heuristic arguments to restrict the set off-path beliefs.

First, it is straightforward that any value of $\Delta_U^{\ell} \in [0, 1]$ may be constructed from a strategy toward the lobbyist $\left(\phi^{\ell}, \alpha^{\ell}\right)$ that is a best responses to *some* feasible beliefs by the politician. This observation can be used to "prune" some types by applying D1, in the sense that the politician should place zero probability weight on them.

Lemma G.1. *In any equilibrium satisfying universal divinity with lobbyist representation off the equilibrium path, the politician's interim-belief may only place positive probability on types* $[2P - \bar{\omega}, \bar{\omega}]$*, all of whom except* $\omega = 2P - \bar{\omega}$ *lobby directly absent representation. In addition, if* $L > 2P - \bar{\omega}$*, then type* $\omega = \underline{\omega}^{d} = 2P - \bar{\omega}$ *may also be pruned.*

Proof: The net benefit of representing SIGs $\omega \leq 2P - \bar{\omega}$ is

$$\Delta_U^{\ell}\left(\pi + \delta\left(\omega - L\right)\right) = k$$

It is clear that types $\omega < 2P - \bar{\omega} = \underline{\omega}^d$ may be pruned; either no feasible value of Δ_U^ℓ will invite a deviation, or if some value Δ_U^ℓ makes type ω indifferent to deviating then types $\omega' \in (\omega, 2P - \bar{\omega})$ strictly prefer to deviate.

We next argue the highest type $\omega = \underline{\omega}^d = 2P - \bar{\omega}$ that does not lobby directly may also be pruned if $L > 2P - \bar{\omega}$. If $\pi + \delta\left((2P - \bar{\omega}) - L\right) < k$ then this is straightforward since no value of Δ_U^ℓ invites deviation, so suppose $\pi + \delta\left((2P - \bar{\omega}) - L\right) \geq k$ but $L > 2P - \bar{\omega}$. The probability makes type $\omega = 2P - \bar{\omega}$ indifferent to deviating is $\frac{k}{\pi - \delta(L - (2P - \bar{\omega}))} > \frac{k}{\pi}$ but since $\pi + \delta\left(\omega - L\right) > k > 0 \ \forall \omega \in (2P - \bar{\omega}, P)$, this probability makes all unworthy types who lobby directly strictly prefer to deviate. QED

Using the preceding we now propose an equilibrium with lobbyist exist satisfying universal divinity for every possible value of L. The lemma is divided into several subcases. The first two subcases ($L < 2P - \bar{\omega}$ and $\hat{\underline{\omega}}^\ell(L) \geq 2P - \bar{\omega}$) are the ones for which we argue that lobbyist exit will indeed prevail as the equilibrium. The remaining subcases are those in which we have already characterized another equilibrium with representation, and argue that equilibrium will prevail instead of lobbyist exit. In several subcases, our equilibrium is not the unique one sustaining lobbyist exit that satisfies universal divinity, but in the proof we provide a heuristic argument for our choice.

Lemma G.2. *The following equilibria with lobbyist exit ($\underline{\omega}^\ell = \bar{\omega}$) satisfy universal divinity for each possible value of L.*

(G.1) $L < 2P - \bar{\omega}$*: off-path the politician believes* $\omega = 2P - \bar{\omega}$ *with probability* 1*, and* $\Delta_U^\ell = \Delta_W^\ell = 0$.

(G.2) $\hat{\underline{\omega}}^\ell(L) \geq \bar{\omega}$*: off-path the politicain believes* $\omega \sim U\left[2P - \bar{\omega}, \bar{\omega}\right]$*, and* $\Delta_\omega^\ell = \Delta_\omega^d \ \forall \omega$.

(G.3) $L \geq 2P - \bar{\omega}$ *and* $\hat{\underline{\omega}}^\ell(L) < \bar{\omega}$*: off-path the politician believes* $\omega \sim U\left[2P - \bar{\omega}, \bar{\omega}\right]$*, and* $\Delta_\omega^\ell = \Delta_\omega^d \ \forall \omega$.

Proof:

(Case G.1): We first argue that iterating refinement D1 implies that unworthy types who will lobby directly $\omega \in \left(\underline{\omega}^d, P\right)$ may be pruned, and so the set of off-equilibrium path beliefs for the politician must be concentrated on $\{2P - \bar{\omega}\} \cup [P, \bar{\omega}]$. For such types $\pi + \delta\left(\omega - L\right) > 0$. To make all such types exactly indifferent to deviation then requires that $\Delta_U^\ell = \Delta_U^d = \frac{k}{\pi}$, but then $\frac{k}{\pi}\left(\pi + \delta\left((2P - \omega) - L\right)\right) = k\left(1 + \frac{\delta}{\pi}\left((2P - \omega) - L\right)\right) > k$ so a type $\omega = \underline{\omega}^d$ would strictly prefer to deviate.

Next we argue that universal divinity places *no further restriction on beliefs*. Observe that to make worthy types $\omega \in [P, \bar{\omega}]$ indifferent to deviating requires that $\Delta_W^\ell = \Delta_W^d \iff \Delta_U^\ell = \frac{k}{\pi} + \left(\frac{1}{\bar{r}}\right)\left(\frac{\bar{\omega}-P}{4} - \frac{\phi^\ell}{\bar{r}}\right)$; given such a best response the net benefit to a type $\omega = 2P - \bar{\omega}$ of deviating is

$$\frac{1}{\bar{r}}\left(\frac{\bar{\omega}-P}{4} - \phi^\ell\right) + \Delta_U^\ell \delta\left((2P-\omega) - L\right)$$

For any $L < 2P - \bar{\omega}$ this expression can be both strictly negative given some best responses by the politician to beliefs concentrated on $\{2P - \bar{\omega}\} \cup [P, \bar{\omega}]$ (for example, if the politician places equal weight on $2P - \bar{\omega}$ and $\bar{\omega}$ then $\Delta_U^\ell = 0$ and $\Delta_W^\ell = \phi^\ell = \frac{\bar{\omega}-P}{2}$ is a best response) and strictly positive (for example, if the politician believes $\omega \in [P, \bar{\omega}]$ so $\Delta_U^\ell = 1$ and $\phi^\ell = 0$).

Although universal divinity places no further restrictions on beliefs, we further argue heuristically that off-equilibrium path, we should consider only beliefs that are a point mass on $\omega = 2P - \bar{\omega}$ ($\to \Delta_U^\ell = \Delta_W^\ell = 0$) or uniform on $\omega \in [P, \bar{\omega}]$ ($\to \Delta_U^\ell = \Delta_W^\ell = 1$). Our logic is as follows. First, if a *given* worthy type $\omega \in [P, \bar{\omega}]$ is indifferent to (strictly prefers) to deviate, then so too do *all* worthy types – so there is such no reason for the politician to believe that some worthier types are likelier to have deviated than others. Second, the ex-ante probability that ω is exactly $2P - \bar{\omega}$ is 0 since ω is distributed uniformly; thus, the politician can only believe off path that $\omega = 2P - \bar{\omega}$ if she places *no weight* on $\omega \in [P, \bar{\omega}]$.

Among these two possible off-path beliefs, only one supports lobbyist exit as an equilibrium – if the politician believes that $\omega = 2P - \bar{\omega}$ off equilibrium path. We thus argue that this equilibrium will prevail when $L < 2P - \bar{\omega}$.

(Case G.2). By Lemma G.1 type $\omega = 2P - \bar{\omega}$ may also be pruned. We argue that universal divinity places no further restriction on the politician's beliefs beyond Lemma G.1 by arguing that there exists a best response to beliefs concentrated on $(2P - \bar{\omega}, \omega]$ that makes unworthy types indifferent to deviation worthy types strictly prefer to deviate, and another best response that makes worthy types indifferent and unworthy types strictly prefer to deviate.

For the former, suppose the politician believes exactly that $\omega = P$ so she is indifferent to granting or denying the favor. Then she never reviews ($\phi^\ell = 0$), so $\Delta_U^\ell = \frac{k}{\pi}$ and $\Delta_W^\ell = \frac{k}{\pi} < \Delta_W^d = \frac{k}{\pi} + \frac{1}{\bar{r}}\left(\frac{\bar{\omega}-P}{4}\right)$ is a best response. For the latter, suppose the politician places equal weight on only $(2P - \bar{\omega}) + 2\varepsilon$ and $\bar{\omega} - 2\varepsilon$, so that $\phi^\ell = \frac{\bar{\omega}-P}{2} - \varepsilon > \frac{\bar{\omega}-P}{4}$. Then $\Delta_W^\ell = \frac{k}{\pi} + \left(\frac{1}{\bar{r}}\right)\left(\frac{\bar{\omega}-P}{4}\right) = \Delta_W^d$ and $\Delta_U^\ell = \frac{k}{\pi} - \left(\frac{1}{\bar{r}}\right)\left(\phi^\ell - \frac{\bar{\omega}-P}{4}\right) < \frac{k}{\pi} = \Delta_U^d$ is a best response.

Although universal divinity places no further restrictions on beliefs, we argue heuristically that off-equilibrium path, we should consider only beliefs that place probability γ on $\omega \sim U[2P - \bar{\omega}, P]$ and $1 - \gamma$ on $\omega \sim U[P, \bar{\omega}]$. Our logic is similar to the previous case; if a

given worthy type $\omega \in [P, \bar{\omega}]$ is indifferent to (strictly prefers) to deviate then so too do *all* worthy types, and if a given unworthy type $\omega \in [2P - \bar{\omega}, P]$ is indifferent to (strictly prefers) to deviate then so too do *all* unworthy types. Thus, there is no reason for the politicain to believe that unworthy (worthy) types do not all deviate together.

It is straightforward to show that the set of politician best responses to this restricted set of beliefs yields feasible pairs $\left(\Delta_U^\ell, \Delta_W^\ell\right)$ satisfying

$$\left\{\Delta_U^\ell \in [0,1]\,,\ \Delta_W^\ell = \min\left\{\Delta_U^\ell + \frac{\phi^\ell}{\bar{r}}\right\}, 1\right\}$$

with $\phi^\ell = \frac{\bar{\omega}-P}{4}$. The set of best responses to this restricted set of beliefs that invite or make a SIG indifferent to deviation is *identical* across worthy and unworthy types. Iterating our previous heuristic argument then suggests that upon observing a deviation, it is reasonable to suppose that the politician should infer that ω is uniform over the set $[2P - \bar{\omega}, \bar{\omega}]$ who would otherwise lobby directly. With these beliefs, $\Delta_U^\ell = \frac{k}{\pi} = \Delta_U^d$ and $\Delta_W^\ell = \frac{k}{\pi} + \left(\frac{1}{\bar{r}}\right)\left(\frac{\bar{\omega}-P}{4}\right) = \Delta_W^d$ is the unique best response that sustains an equilibrium with lobbyist exit. We thus argue that this is equilibrium will prevail when $\underline{\hat{\omega}}^\ell(L) \geq \bar{\omega}$.

(Case G.3): In Case G.3, the equilibrium with exit coexists with the equilibrium with representation, and we argue that the latter will prevail. In this case, by Lemma G.1 type $\omega = 2P - \bar{\omega}$ may also be pruned. We now examine three subcases.

Subcase G.3.1 ($\underline{\hat{\omega}}^\ell(L) \in (2P - \bar{\omega}, \bar{\omega})$): We argue universal divinity places no further restriction on beliefs. Consider first $\underline{\hat{\omega}}^\ell(L) \in (P, \bar{\omega})$. By previous arguments we know that some best responses will invite deviation from $\omega \in (2P - \bar{\omega}, P]$ but not $\left[P, \underline{\hat{\omega}}^\ell(L)\right]$, and otherwise will invite deviation from $\left[P, \underline{\hat{\omega}}^\ell(L)\right]$ but not $\omega \in (2P - \bar{\omega}, P]$. In addition, types $\omega \in \left(\underline{\hat{\omega}}^\ell(L), \bar{\omega}\right)$ with $\pi + \delta(\omega - L) < 0$ prefer to deviate exactly when types $\omega \in \left(P, \underline{\hat{\omega}}^\ell(L)\right)$ with $\pi + \delta(\omega - L) > 0$ do not, and visa versa.

Although universal divinity places no further restrictions on beliefs, we argue heuristically that off-equilibrium path, we should consider only beliefs that place probability γ on $\omega \sim U[2P - \bar{\omega}, P]$ and $1 - \gamma$ on *either* (i) only $U\left[P, \underline{\hat{\omega}}^\ell(L)\right]$, (ii) only $U\left[\underline{\hat{\omega}}^\ell(L), \bar{\omega}\right]$, or (iii) $U[2P - \bar{\omega}, \bar{\omega}]$. The argument is similar to before; when one type within these sets wishes to deviate so to do all types, and in addition types $\omega \in \left(P, \underline{\hat{\omega}}^\ell(L)\right)$ strictly prefer deviate precisely exactly when types $\omega \in \left(\underline{\hat{\omega}}^\ell(L), \bar{\omega}\right)$ strictly prefer not to (and visa versa). An equilibrium with lobbyist exit then requires that $\Delta_\omega^\ell = \Delta_\omega^d\ \forall \omega$, and it is easily verified that the only beliefs among this restricted set generating such a best response is $\omega \sim U[2P - \bar{\omega}, \bar{\omega}]$ as in Case G.2. We thus argue that this is the only reasonable equilibrium with lobbyist exit to consider; and further recall that under these conditions we argue that the equilibrium with lobbyist entry will instead prevail.

Consider next $\underline{\hat{\omega}}^{\ell}(L) \in (2P-\bar{\omega}, P]$. An effectively identical argument implies that the same equilibrium with lobbyist exit is the only reasonable equilibrium with exit to consider.

Subcase G.3.2 ($L \geq 2P-\bar{\omega}$ and $\underline{\hat{\omega}}^{\ell}(L) \leq 2P-\bar{\omega}$): By a similar argument as case G.2 ($\underline{\hat{\omega}}^{\ell}(L) \geq \bar{\omega}$) we argue that the only reasonable beliefs are $\omega \sim U[2P-\bar{\omega}, \bar{\omega}]$, and (if $L = 2P-\bar{\omega}$) also that $\omega = 2P-\bar{\omega}$. The latter belief does not support lobbyist exit in a best response, and the former belief does only when $\Delta^{\ell}_{\omega} = \Delta^{d}_{\omega}\ \forall \omega$.

H Non-cutpoint equilibrium with representation

Suppose $\underline{\hat{\omega}}(L) \in (P, \bar{\omega})$. Consider the lobbyist representing $\left[2P-\bar{\omega}, \underline{\omega}^{\ell}\right] \cup \left[\bar{\omega}^{\ell}, \bar{\omega}\right]$ with $\bar{\omega}^{\ell} = \underline{\hat{\omega}}(L)$ and $\underline{\omega}^{\ell} = 2P - \underline{\hat{\omega}}(L)$, and suppose the SIG lobbies directly i.f.f. $\underline{\omega}^{d} = 2P-\bar{\omega}$. It is easily verified that both $E^{\ell}[\omega] = P$ and $E^{d}[\omega] = P$. Further it is easily verified that:

$$\phi^{\ell} = \frac{(\bar{\omega}-P) + (\underline{\hat{\omega}}(L)-P)}{4} \quad \text{and} \quad \phi^{d} = \frac{(1-\lambda)(\bar{\omega}-P) + \lambda(\underline{\hat{\omega}}(L)-P)}{4}$$

and observe that $\phi^{\ell} > \frac{\bar{\omega}-P}{4} > \phi^{d}$.

Now for both $c \in \{\ell, d\}$ we argue that $\exists \alpha^{c} \leq 1$ s.t. $\left(1-\frac{\phi^{c}}{\bar{r}}\right)\alpha^{c} = \frac{k}{\pi}$. This requires that $\frac{\phi^{c}}{\bar{r}} \leq 1-\frac{k}{\pi}\ \forall c \in \{\ell, d\}$ which will hold for $\underline{\hat{\omega}}(L)$ sufficiently close to P. We last argue that the lobbyist's strategy is indeed a best response. She is indifferent over representing $\omega \in [2P-\bar{\omega}, P]$ since $\Delta^{\ell}_{U} = \Delta^{d}_{U}$ and so is willing to carry out her strategy among this set, and since $\Delta^{\ell}_{W} > \Delta^{d}_{W}$ prefers to represent worthy SIGs i.f.f. $\omega \geq \underline{\hat{\omega}}(L)$ which is exactly her strategy among this set. QED.