**Figure 5:**

```
\begin{figure}[ht]
\caption{Filling Out Space}\label{fig: segments_hexagons_in_square}
\setlength{\tabcolsep}{25pt}
\begin{tabular}{cc}
\begin{tikzpicture}[line width=1pt,line cap=round, x = 0.6cm, y = 0.7cm]
   \filldraw [white] (0,0) -- (0,0);
   \begin{scope}[shift={(0, 0.5)}]
      \draw (-6, 4) -- (6, 4);
      \draw (-6, 3.5) -- (-6, 4.5);
      \draw (-3, 3.5) -- (-3, 4.5);
      \draw (0, 3.5) -- (0, 4.5);
      \draw (3, 3.5) -- (3, 4.5);
      \draw (6, 3.5) -- (6, 4.5);
      \filldraw [black] (-4.5,4) circle (1pt);
      \filldraw [black] (-1.5,4) circle (1pt);
      \filldraw [black] (1.5, 4) circle (1pt);
      \filldraw [black] (4.5, 4) circle (1pt);
      \draw[decorate, decoration={brace,amplitude=3pt,mirror}] (-6, 3.2) {} -- (-3, 3.2)
      node[midway, below = 0.2 cm] {$\frac{|\mathcal S|}{N}$};
   \end{scope}
\end{tikzpicture}
& \begin{tikzpicture}

        \tikzset{box/.style={regular polygon,
                    regular polygon sides=6,
                    minimum size=2*0.233107cm,
                    inner sep=0mm,
```

```
                    outer sep=0mm,
                    rotate=0,
                    draw
                    }
            }
        \draw [thick, fill=gray!10] (0,0) -- (0,6) -- (6,6) -- (6,0) -- (0,0);
        \foreach \i in {0,...,7}
          \foreach \j in {0,...,14} {
              \node[box] at (0.233107+3*0.233107*\i,1.732*0.233107*\j) {};
              \filldraw [black] (0.233107+3*0.233107*\i,1.732*0.233107*\j) circle (.5pt);
             \node[box] at
(0.233107+3*0.233107*\i+1.5*0.233107,1.732*0.233107*\j+0.233107*0.866) {};
             \filldraw [black]
(0.233107+3*0.233107*\i+1.5*0.233107,1.732*0.233107*\j+0.233107*0.866) circle (.5pt);
        }
        \foreach \j in {0,...,14} {
          \node[box] at (0.233107+3*0.233107*8,1.732*0.233107*\j) {};
          \filldraw [black] (0.233107+3*0.233107*8,1.732*0.233107*\j) circle (.5pt);
        }
        %\node(A) at (-.5,6) {$\sqrt{A}$};
        %\node(B) at (6,-.5) {$\sqrt{A}$};

    \end{tikzpicture}
\end{tabular}
\vskip.2cm\parbox[c]{18cm}{\scriptsize \textbf{Notes:} The left panel in the figure shows a line of length $\mathcal{S}$ divided by $N=4$ line segments of length $\mathcal{S}/N$. The right panel shows a square of area $|\mathcal S|$ divided by $N$ hexagons of area $|\mathcal S|/N$.}
\end{figure}
```