Remark (Appendices A, B and C)**.** *Appendices A, B and C are available online at Dataverse:* https://doi.org/10.7910/DVN/FYH5WG

AppendixA. Marginal effects of district magnitude

On the basis of the model in Table 6, Figures A.10 and A.11 show, *ceteris paribus*:

- the expected relative **decrease** (in fractions) in disproportionality (GHI) if district magnitude increases by **3** seats - for all configurations of moderating variables (the number of parties (NP) and *an allocation method*);
- the expected relative **increase** (in fractions) in disproportionality (GHI) if district magnitude decreases by **3** seats - for all configurations of moderating variables (the number of parties (NP) and *an allocation method*).

Also, Figures A.12 and A.13 show:

- the expected relative (in fractions) change of disproportionality (GHI) for different apportionment methods with reference to the Hamilton formula (the baseline category).
- the expected relative (in fractions) change of disproportionality (GHI) for different party system formats with reference to the 3-party system (the baseline category).

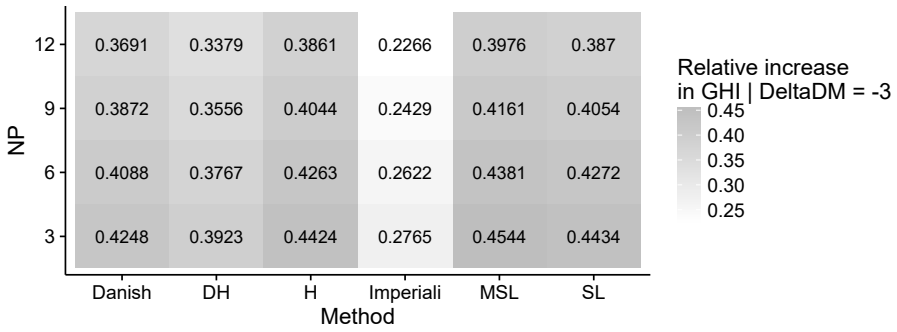

Figure A.10: The expected relative increase (in fractions) in disproportionality (GHI) if district magnitude **decreases** by **3** seats - for all configurations of moderating variables (the number of parties (NP) and *an allocation method*) - other variables are held constant.

The log-level regression model presented in Table 6 suggests that, *ceteris paribus*, e.g. for d'Hondt and the 6-party system ($\mathbb{E}[GHI|DM, method = \text{Jefferson-d'Hondt (DH)}, NP =$

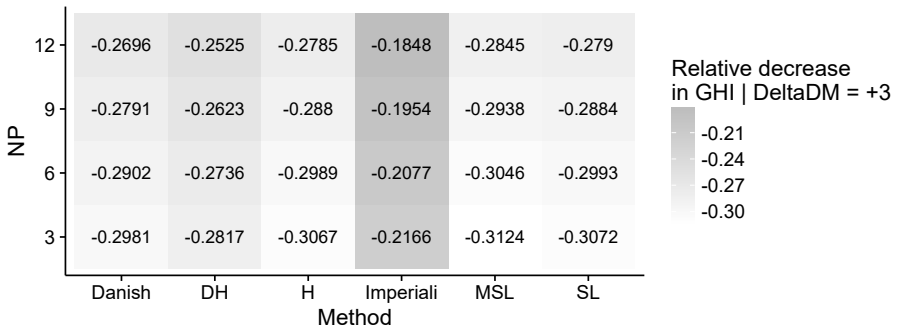

Figure A.11: The expected relative decrease (in fractions) in disproportionality (GHI) if district magnitude **increases** by **3** seats - for all configurations of moderating variables (the number of parties (NP) and *an allocation method*) - other variables are held constant.

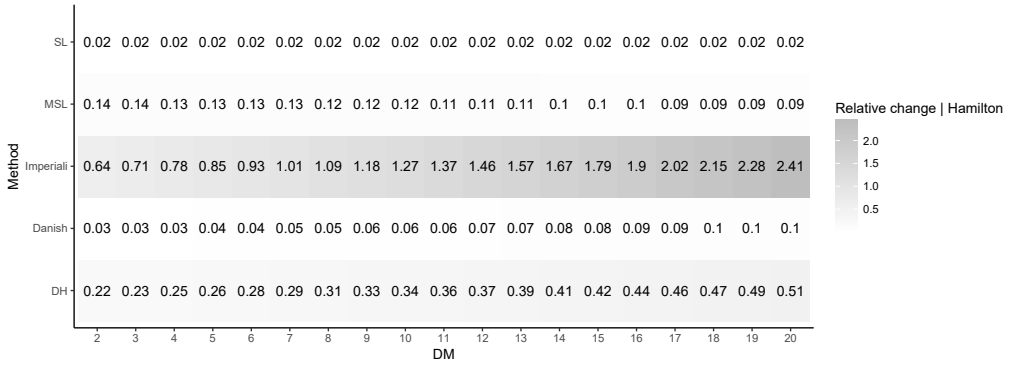

Figure A.12: The expected relative (in fractions) change of disproportionality (GHI) for various apportionment methods with reference to the Hamilton/Hare formula (the baseline category) - *ceteris paribus*.

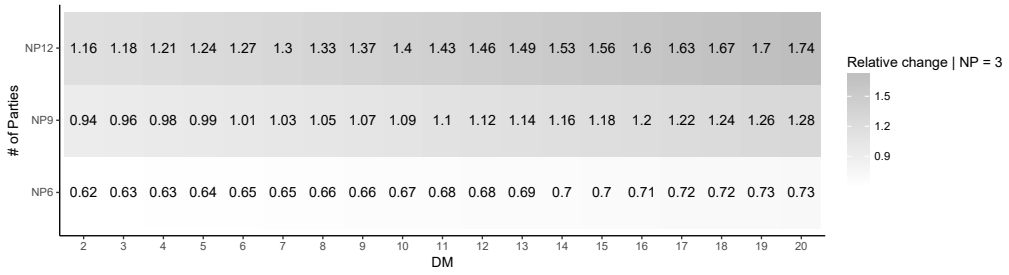

Figure A.13: The expected relative (in fractions) change of disproportionality (GHI) for different party system formats with reference to the 3-party system (the baseline category) - *ceteris paribus*.

6]), if DM decreases by 3 seats ($\Delta DM = DM_0 - DM_1 = -3$), aggregate disproportionality is expected to increase by around **38**%. This value can be calculated as follows:

$$\frac{\mathbb{E}[GHI|DM_0, method = \text{DH}, NP = 6] - \mathbb{E}[GHI|DM_1, method = \text{DH}, NP = 6]}{\mathbb{E}[GHI|DM_1, method = \text{DH}, NP = 6]} =$$

$$= \Delta GHI \approx 0.38$$

and

$$\mathbb{E}[GHI|DM_0, method = \text{DH}, NP = 6] =$$

$$= \exp(\hat{\beta}_0 + \hat{\beta}_1 \times DM_0 + \hat{\beta}_2 \times \text{DH} + \hat{\beta}_3 \times 6 + \hat{\beta}_4 \times DM_0 \times DH + \hat{\beta}_5 \times DM_0 \times 6)$$

$$\mathbb{E}[GHI|DM_1, method = \text{DH}, NP = 6] =$$

$$= \exp(\hat{\beta}_0 + \hat{\beta}_1 \times DM_1 + \hat{\beta}_2 \times \text{DH} + \hat{\beta}_3 \times 6 + \hat{\beta}_4 \times DM_1 \times DH + \hat{\beta}_5 \times DM_1 \times 6)$$

Also, switching from e.g. **Hamilton to d'Hondt** results, on average, in the approx. **36**% increase in disproportionality, in the range of 2-20 seats per a district. For example, for 8-member districts it equals 0.31 (31%). This value is computed as follows:

$$\frac{\mathbb{E}[GHI|DM = 8, method = \text{d'Hondt}, NP] - \mathbb{E}[GHI|DM = 8, method = \text{Hamilton}, NP]}{\mathbb{E}[GHI|DM = 8, method = \text{Hamilton}, NP]} =$$

$$= \Delta GHI \approx 0.31$$

AppendixB. Seat excesses and seat biases

Figures B.14-B.19 visualize seat excesses and seat biases and indicate that the disproportionality profiles significantly varies for different voting system parameter configurations. For example, for the 6-party system, if we compare the most commonly used apportionment algorithms, i.e. d'Hondt and S-L, we can see that the situation of political parties is rad-

ically different. Sainte-Lague produces low deviations from the 'ideal' shares of seats and, what is especially important, treats all political parties equally. It is not the case as for d'Hondt, because we can observe that the largest party, which is coded: YRQ for 3-party systems; MSZ for 6-party systems and GKA for 9-party systems, is highly overrepresented for low-magnitude districts. On average, in particular for the parties ranging from 3 to 6-8 seats.

For low-magnitude districts, Figures B.14-B.19 indicate that the situation of the winning political party significantly depends on the apportionment method and also on the number of parties in the system. Comparing d'Hondt and S-L, we can conclude that if we use low-magnitude districts ranging from around 4 to around 8 seats, elections employing d'Hondt suffer not only from the relatively high level of overrepresentation in favor of the largest party and from the underrepresentation to the disadvantage of small parties, but also from the substantial degree of dispersion (the inter-quartile range) as for the values of seat excesses. When it comes to S-L, in low-magnitude districts, dispersion of seat excesses around the mean is also high, but in contrast to d'Hondt, the method does not benefit large parties at the expense of small ones.

However, if we employ districts of slightly greater sizes, e.g. 10 seats, we end up with a much smaller bias in favor of the largest party in d'Hondt. Also, what is very important, standard deviations and also inter-quartile ranges (IQR), which measure a dispersion of seat excesses, become smaller if we move up beyond low district magnitudes (4-8). Thus, elections in moderate district magnitudes, ranging from around 8 to around 12, generate much more 'stable' outcomes when it comes to deviations from the proportionality - the larger the DM, the less variation in the seat excesses.

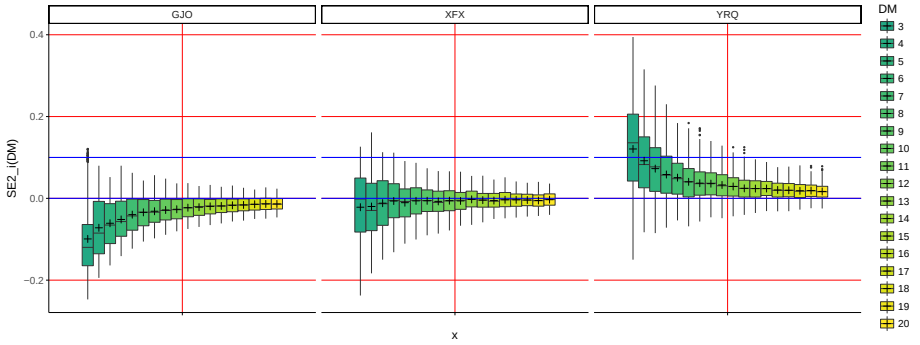

Figure B.14: Seat excesses and district magnitude for a 3-party system and Jefferson-d'Hondt method. The ranking of parties (coded using 3-letter strings): $YRQ \succ XFX \succ GJO$

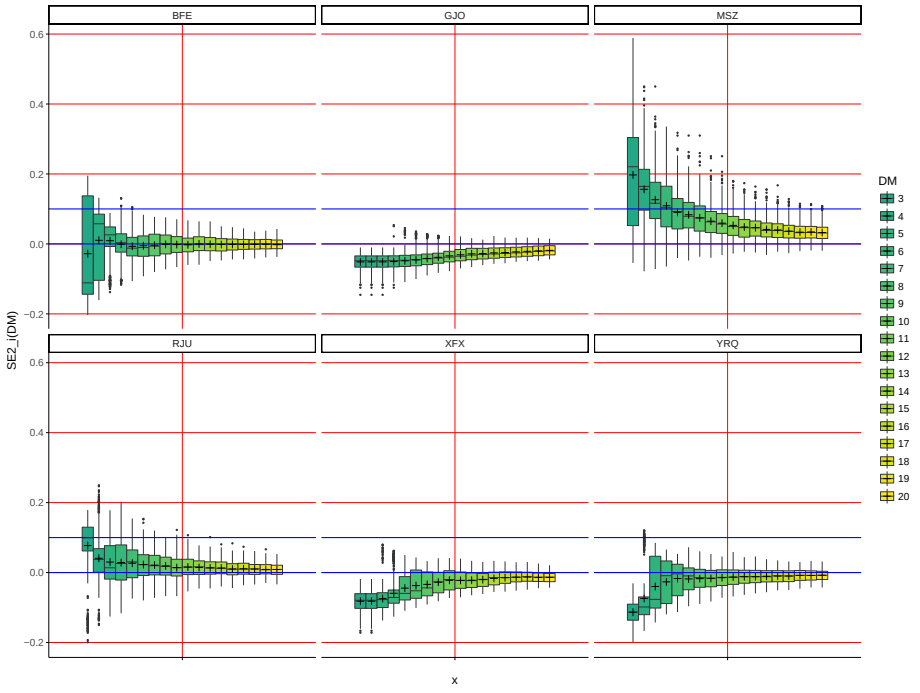

Figure B.15: Seat excesses and district magnitude for a 6-party system and Jefferson-d'Hondt method. The ranking of parties (coded using 3-letter strings): $MSZ \succ RJU \succ BFE \succ YRQ \succ XFX \succ GJO$

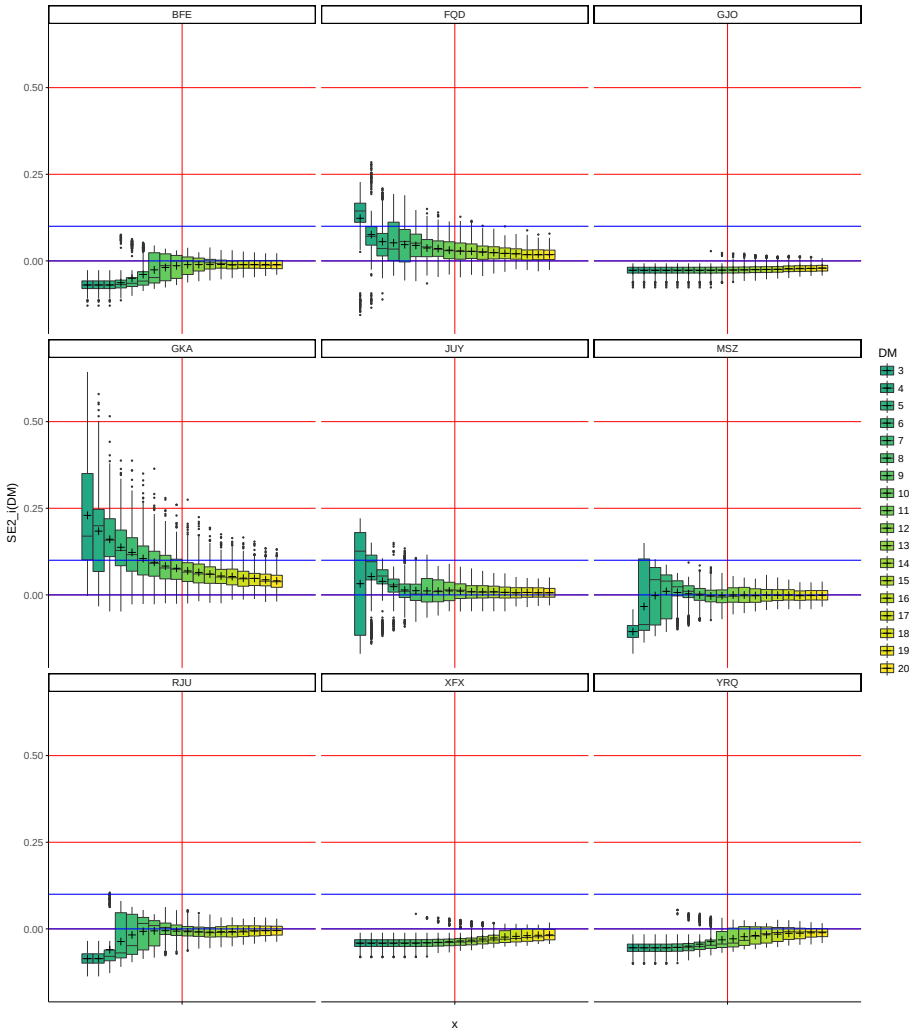

Figure B.16: Seat excesses and district magnitude for a 9-party system and Jefferson-d'Hondt method. The ranking of parties (coded using 3-letter strings): $GKA \succ FQD \succ JUY \succ MSZ \succ RJU \succ BFE \succ YRQ \succ XFX \succ GJO$

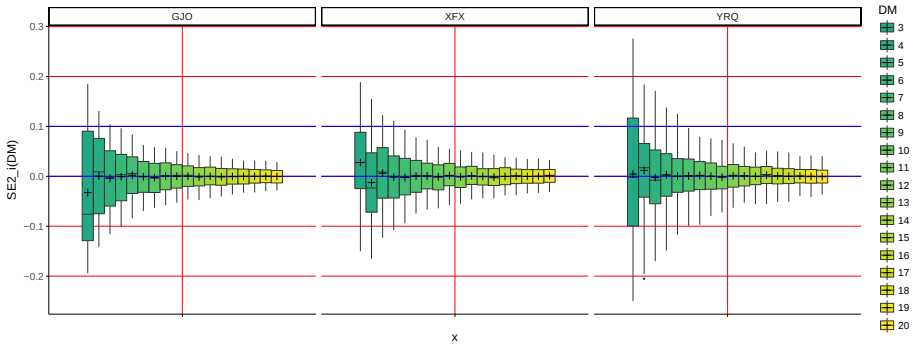

Figure B.17: Seat excesses and district magnitude for a 3-party system and Webster / Sainte-Lague method

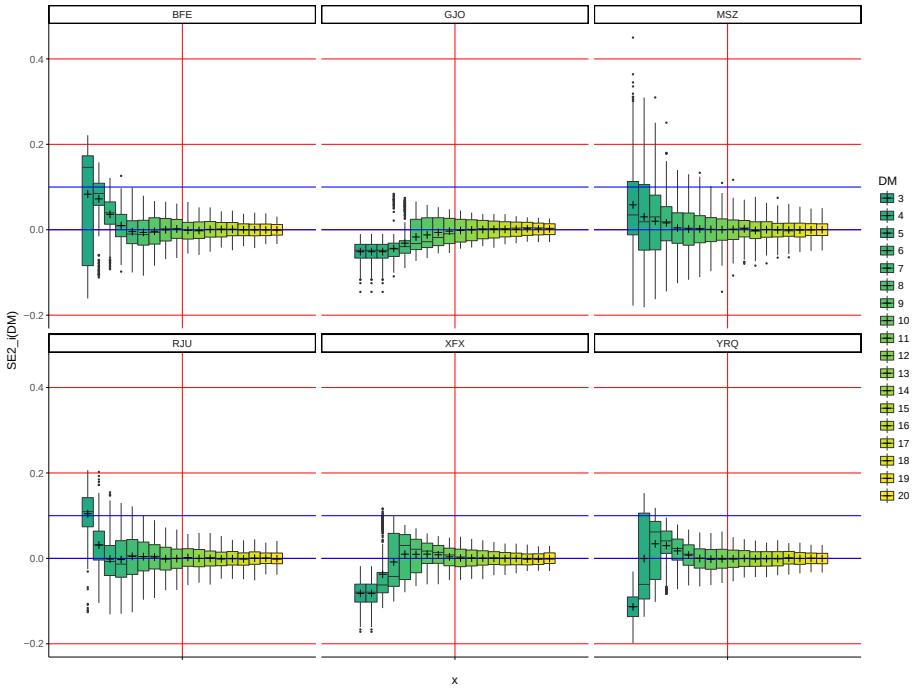

Figure B.18: Seat excesses and district magnitude for a 6-party system and Webster / Sainte-Lague method

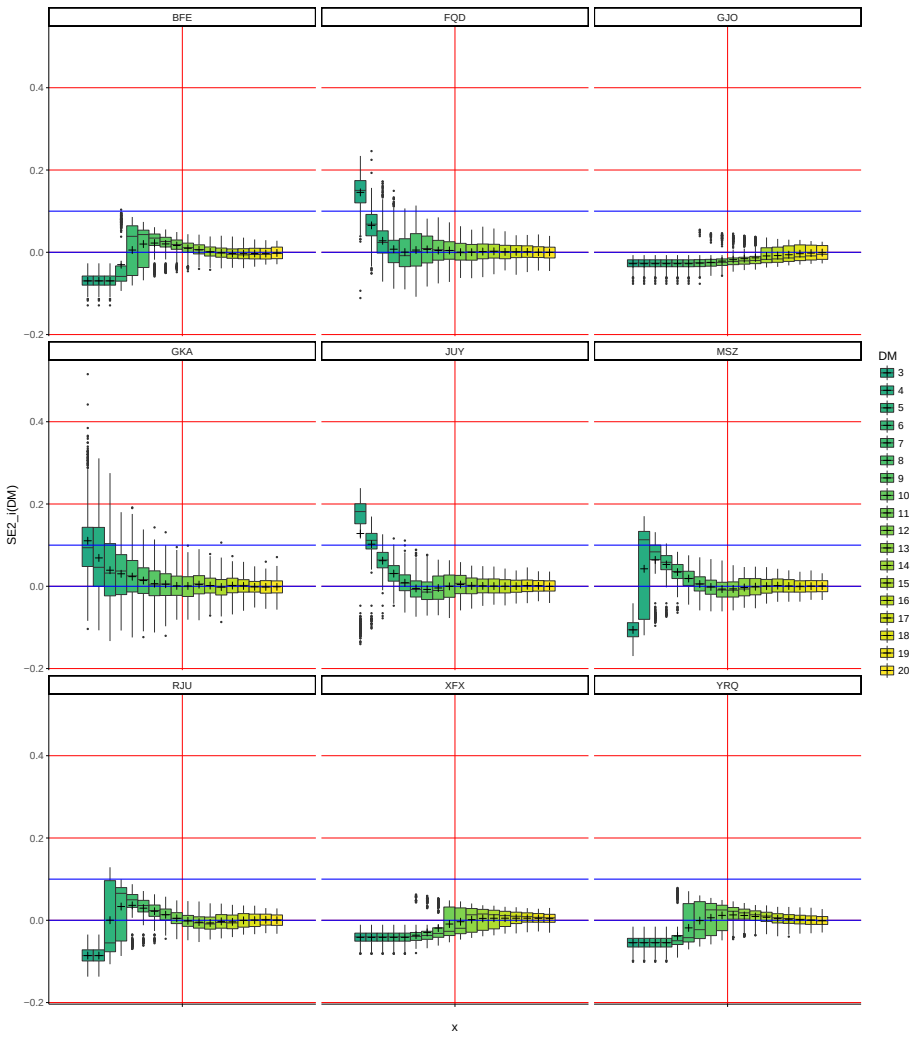

Figure B.19: Seat excesses and district magnitude for a 9-party system and Webster / Sainte-Lague method

AppendixC. District magnitude and a parliamentary party system format

DH - Jefferson/d'Hondt; H - Hamilton/Hare Largest Remainders; Imperiali - Imperiali Highest Averages; MSL - The Modified Sainte-Lague; SL - Webster/Sainte-Lague.

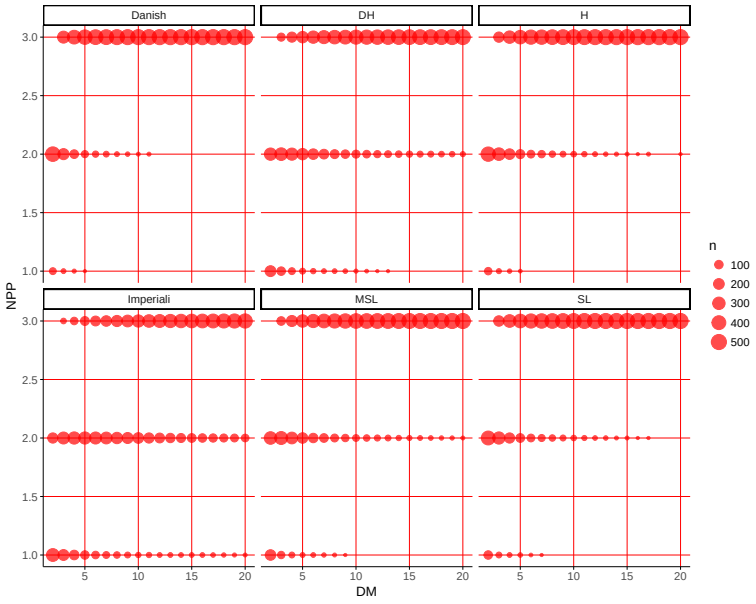

Figure C.20: The correlation between district magnitude (DM ranges from 2 to 20) and the number of parliamentary parties (NPP) - for a **3-party system**. The size of a circle refers to the number of observations at a location.

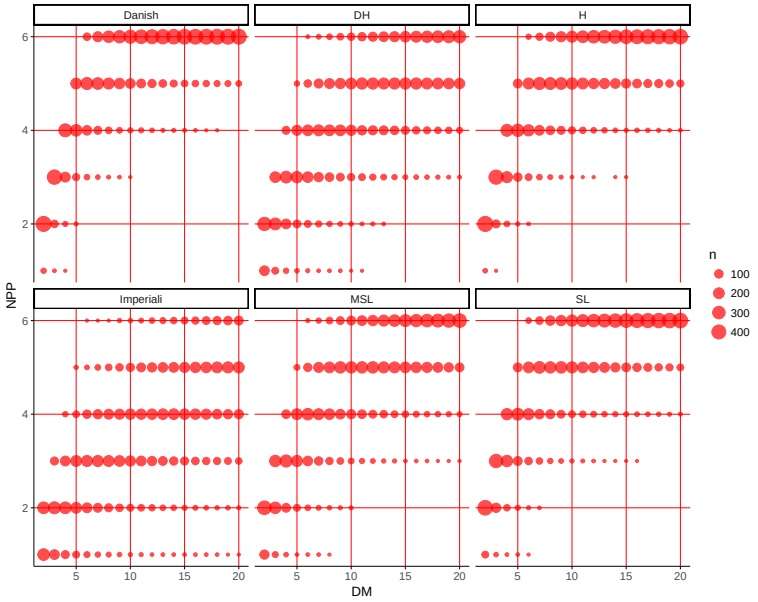

Figure C.21: The correlation between district magnitude (DM ranges from 2 to 20) and the number of parliamentary parties (NPP) - for a **6-party system**

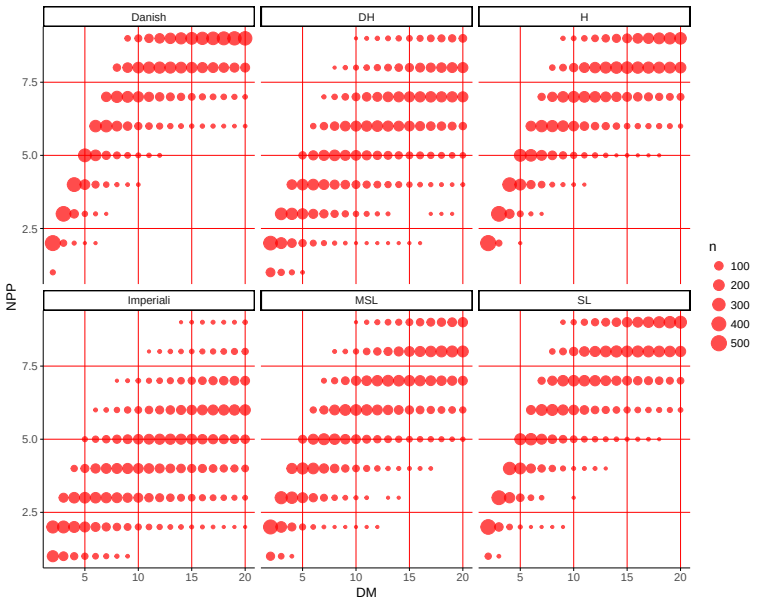

Figure C.22: The correlation between district magnitude (DM ranges from 2 to 20) and the number of parliamentary parties (NPP) - for a **9-party system**

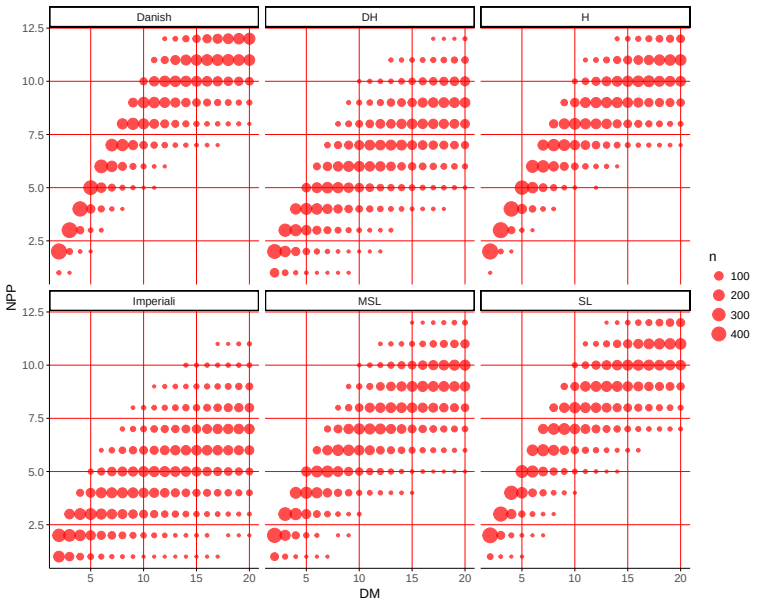

Figure C.23: The correlation between district magnitude (DM ranges from 2 to 20) and the number of parliamentary parties (NPP) - for a **12-party system**

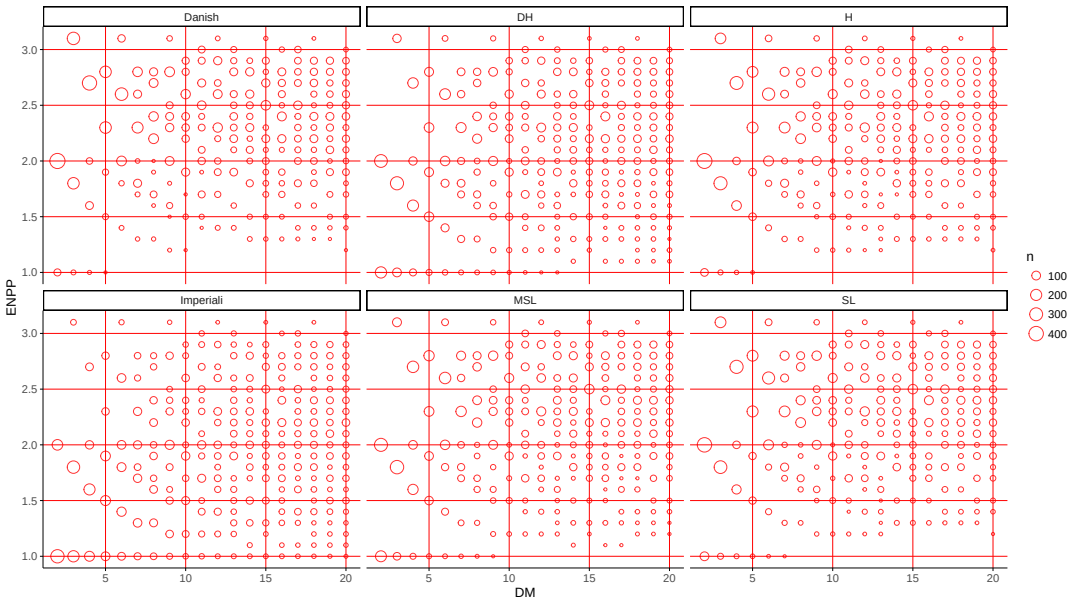

Figure C.24: The correlation between district magnitude (DM ranges from 2 to 20) and the effective number of parliamentary parties (ENPP) - for a **3-party system**. The size of a circle refers to the number of observations at a location.

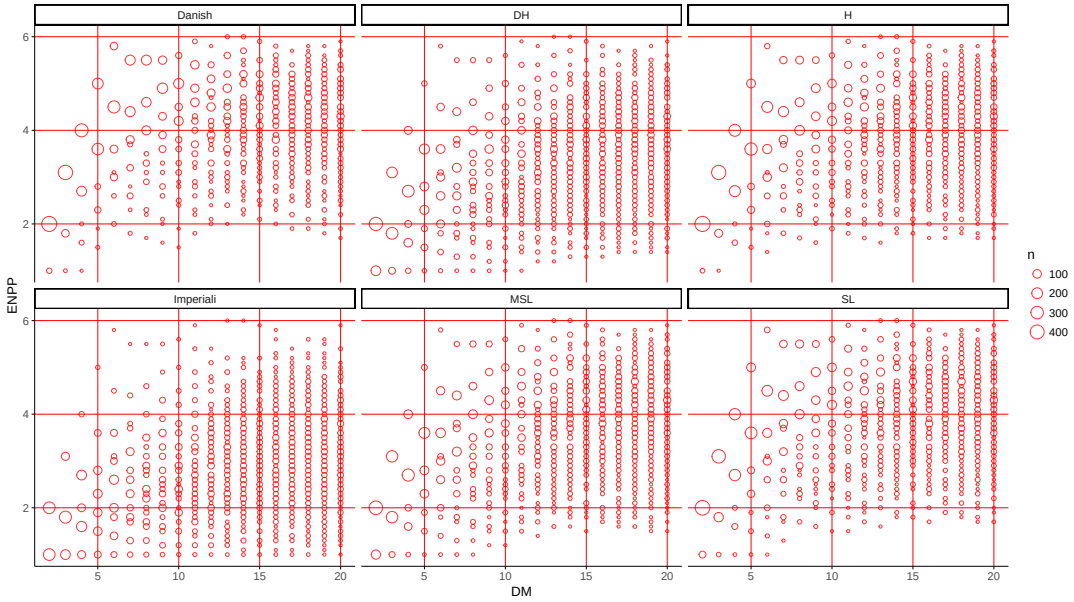

Figure C.25: The correlation between district magnitude (DM ranges from 2 to 20) and the effective number of parliamentary parties (ENPP) - for a **6-party system**

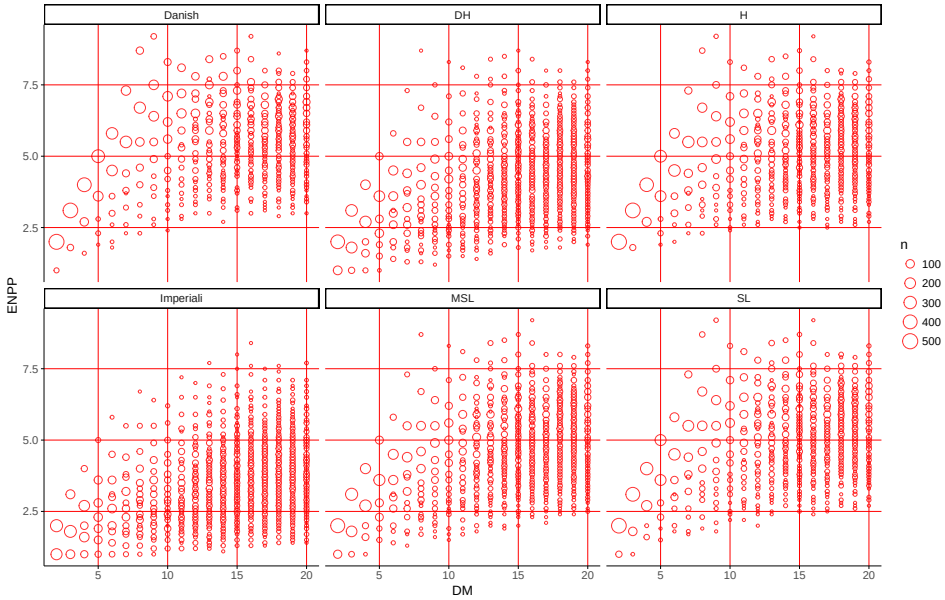

Figure C.26: The correlation between district magnitude (DM ranges from 2 to 20) and the effective number of parliamentary parties (ENPP) - for a **9-party system**

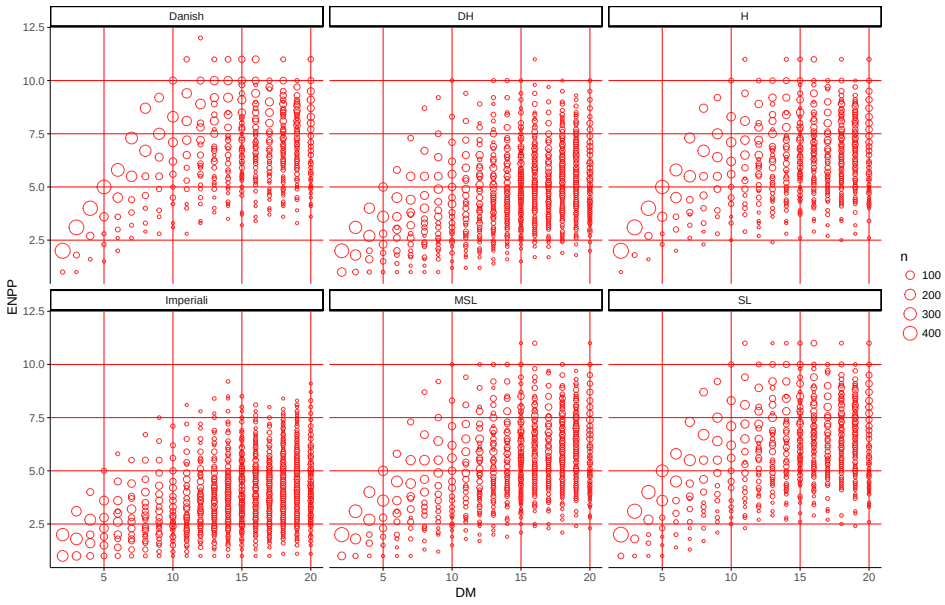

Figure C.27: The correlation between district magnitude (DM ranges from 2 to 20) and the effective number of parliamentary parties (ENPP) - for a **12-party system**