Supplementary Material: On the Directional Nature of Celestial Object's Fall on the Earth (Part 1: Distribution of Fireball Shower, Meteor Fall & Crater on Earth's Surface)

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1 Introduction

The statistical nature of meteor falls on the Earth's Surface is affected by the intricate interplay of Earth's rotation and other diverse factors, raising an inquiry of whether the overall meteor fall distribution on the Earth's surface is truly random or follows some more specific distributions. Very little literature exists concerning the statistical nature of meteor showers on the Earth, primarily focused on investigating the randomness of meteor fall using only linear statistical tests. In this paper, we contest the dependency of the existing literature on linear statistical analysis for meteor fall distribution. [1]

We used circular statistics to validate the Distributional pattern of the meteor fall. [3]

2 Fireball Dataset

Fireballs and bolides are astronomical terms for exceptionally bright meteors that are spectacular enough to be seen over a wide area. A world map visually represents the data table that provides a chronological data summary of fireball and bolide events provided by U.S. Government sensors. Ground-based observers sometimes witness these events at night, or much more rarely in daylight, as impressive atmospheric light displays. This website is not meant to be a complete list of all fireball events. Only the brightest fireballs are noted.

The accompanying table provides information on the date and time of each reported fireball event with its approximate total optical radiated energy and its calculated total impact energy. When reported, the event's geographic location, altitude and velocity at peak brightness are also provided. Note that data are not provided in real-time and not all fireballs are reported. A blank (empty) field in the table indicates the associated value was not reported.

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[4]This dataset is taken from `https://cneos.jpl.nasa.gov/fireballs/intro.html`

```
> head(fireball)   # For the fireball Dataset

  fireball.Peak.Brightness.Date.Time..UT. fireball.Altitude..km.
1                       12-10-2007 09:14                   37.0
2                       13-12-2014 02:53                   30.7
3                       17-07-2018 15:17                     NA
4                       23-11-1998 20:20                     NA
5                       01-02-1999 14:24                     NA
6                       04-10-1991 09:22                     NA
  fireball.Velocity..km.s. fireball.vx fireball.vy fireball.vz
1                     18.4        -4.5       -14.1       -10.9
2                     21.7        15.3       -13.3        -7.8
3                       NA          NA          NA          NA
4                       NA          NA          NA          NA
5                       NA          NA          NA          NA
6                       NA          NA          NA          NA
  fireball.Total.Radiated.Energy..J.
1                          3.30e+10
2                          4.40e+10
3                          2.11e+11
4                          2.50e+10
5                          2.80e+10
6                          5.50e+11
  fireball.Calculated.Total.Impact.Energy..kt. fireball.Latitude
1                                        0.110              88.5
2                                        0.150              86.7
3                                        0.590              82.5
4                                        0.089              82.3
5                                        0.098              80.0
6                                        1.400              78.7
  fireball.Longitude
1              116.6
2             -162.1
3             -136.7
4              160.1
5              154.2
6                6.3


fire = cbind(fireball$fireball.Latitude , fireball$fireball.Longitude)
```

3 Detail of Von Mises Fisher Mixture Model Fit for the Two Datasets

A mixture model is a probabilistic model for representing the presence of sub-populations within an overall population without requiring that an observed data set should identify the sub-population to which an individual observation belongs. However, while problems associated with "mixture distributions" relate to deriving the properties of the overall population from those of the sub-populations, "mixture models" are used to make statistical inferences about the properties of the sub-populations given only observations on the pooled population, without sub-population identity information.
the density for directional parametric mixture distribution $p(x|\boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_K)$ represent appropriate parameter set, as follows.

$$p(\boldsymbol{\theta}) = \sum_{i=1}^{K} \phi_i f(\vec{x}; \vec{\mu}, \kappa) \tag{1}$$

- K = number of mixture components
- N = number of observations
- $\theta_{i=1\ldots K}$ = parameter of distribution of observation associated with component i
- $\phi_{i=1\ldots K}$ = mixture weight, i.e., prior probability of a particular component i
- Φ = K-dimensional vector set $\phi_{1\cdots K}$; sum to 1
- $z_{i=1\ldots N}$ = component of observation i
- $x_{i=1\ldots N}$ = observation i
- $F(x|\theta)$ = probability distribution of an observation, parametrized on θ
- $z_{i=1\ldots N}$ = Categorical(ϕ)
- $x_{i=1\ldots N}|z_{i=1\ldots N} = F(x_{i=1\ldots N}|\theta_{z_i})$

In our paper, the i th vector component is characterized by $f(\vec{x}; \vec{\mu}, \kappa) = C_3(\kappa) \exp(\kappa \vec{x} \cdot \vec{\mu})$ as von Mises-Fisher distribution with weights ϕ_i, parameter θ_i as means $\boldsymbol{\mu_i}$ and concentration matrices $\boldsymbol{\kappa_i}$. Here, $C_3(\kappa)$ is the normalization coefficient which can be shown to be dependent only on κ and the dimension (3, in the case of the sphere). For details, we refer to [3], [5].

For fitting the mixture Von Mises Fisher distribution to both datasets, we take the help of R package **movMF** of [2]. We treat the circular uniform distribution as Von Mises Fisher with large means $\boldsymbol{\mu}$ and large (diluted) concentration matrices $\boldsymbol{\kappa}$. In the "Statistical Analysis for Directional Mixture Distribution Model Fitting for Dataset 1" section in the original paper, we used those tables and gave the interpretation of how it is used.

The codes used in the below table gives us α value which are the weights used in the below parameter results. The vMF (Von Mises Fisher) distribution is commonly parameterized as

$$\theta = \kappa\mu \tag{2}$$

where $\kappa = ——\theta——$ and we have μ are the concentration and mean direction parameters. respectively (if θ not equal to 0, μ is uniquely determined as $\frac{\theta}{||\theta||}$ `https://cran.r-project.org/web/packages/movMF/vignettes/movMF.pdf`. According to the notation used in the package movMF $\alpha = \phi$ and $\theta = \mu, \kappa$

3.1 Fit mixtures of vonMises-Fisher distributions for Firefall Dataset

```
set.seed(2022)
EvMFs <-
  function(K){
    movMF(fire, k = K, control= list(nruns = 20))
  }

Esd = lapply(1:10, EvMFs)

> Esd
[[1]]
theta:
          [,1]       [,2]
1 -0.02301825 0.05171485
alpha:
[1] 1
L:
[1] 0.6156502

[[2]]
theta:
         [,1]      [,2]
1 -0.1246680  3.457741
2  0.0326093 -3.295590
alpha:
[1] 0.5128561 0.4871439
L:
[1] 224.0846

[[3]]
theta:
          [,1]       [,2]
1  0.41486109 -9.0440480
2 -0.06441305 -0.2225257
3 -0.30622873  6.9874133
alpha:
[1] 0.3161662 0.2981029 0.3857309
L:
[1] 247.5455

[[4]]
theta:
         [,1]       [,2]
1 -0.4296112  6.6869419
2  1.1622817  0.3928193
3 -1.1744491 -0.8237735
4  0.5566761 -8.3582845
```

```
alpha:
[1] 0.4064265 0.1133998 0.1367193 0.3434545
L:
[1] 248.6265

[[5]]
theta:
        [,1]       [,2]
1 -1.4339346 -0.7184444
2  2.3808775  0.5864099
3  0.5842880 -9.3916108
4 -0.3892404  6.4809937
5  0.4468795 -5.2463977
alpha:
[1] 0.1262765 0.0785648 0.2394671 0.4248161 0.1308755
L:
[1] 248.8488

[[6]]
theta:
        [,1]       [,2]
1 -1.2846318 -0.7912272
2  3.0263758  0.7683888
3  0.6343880 -5.6827432
4  0.6629851  8.0949562
5  0.5367309 -9.5483969
6 -1.1735678  6.4729036
alpha:
[1] 0.13604383 0.07114722 0.14703040 0.18795955 0.21790267
0.23991632
L:
[1] 249.0676

[[7]]
theta:
        [,1]       [,2]
1 -2.8528986 -0.8134857
2  0.1214515 -3.6005301
3 -1.2701723  5.5957956
4  0.4131727 -3.9838242
5  0.1169006  6.9679743
6  4.9971910  1.1571691
7  0.5939341 -9.7887243
alpha:
[1] 0.07841870 0.09243114 0.14853128 0.08839363 0.29901511
0.05793826 0.23527188
L:
[1] 249.2897
```

```
[[8]]
theta:
        [,1]        [,2]
1 -2.4810620   9.6366723
2 -2.7799818  -0.3022653
3  0.5040613 -11.5331593
4 -1.0159734  -3.8634881
5  0.7609175   1.0677088
6  1.2672515  -5.2727801
7  3.2225078  13.4586047
8  5.4078836   1.3973760
alpha:
[1] 0.25492123 0.07417556 0.18994194 0.09436699 0.03958970
0.13782744 0.16077133 0.04840581
L:
[1] 249.7812

[[9]]
theta:
        [,1]        [,2]
1  8.7211373   1.9023967
2 -1.3617404  -1.0415985
3  1.3080193 -14.3199033
4  1.0286913  -4.2052242
5 -0.4332869  -6.9121178
6  0.6972171  -2.9720434
7 -2.2951788  -0.4650225
8 -0.4850963   5.9976137
9  0.2575544   6.4216911
alpha:
[1] 0.04396518 0.05291199 0.13762500 0.08052704 0.12511789
0.05871790 0.05345628 0.31464951
[9] 0.13302922
L:
[1] 249.6644

[[10]]
theta:
         [,1]          [,2]
1   1.1224697  -4.920107112
2   5.5944007 -33.019608703
3  -1.0198770   6.378721941
4  -1.5849011  -1.410398833
5   0.9822943  -5.193342951
6  -0.5113603   6.629862615
7   0.2571067   7.727787312
8  -2.0786757 -11.578492142
9   2.0117019   0.735481700
```

```
10 -1.9442946   0.003771334
alpha:
 [1] 0.07921971 0.09718048 0.14645449 0.06944716 0.08597624
 0.12122824 0.14949662
 [8] 0.11345780 0.08693686 0.05060239
L:
[1] 250.4978


> Esd[[1]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
      1     100

$logLiks
 [1] 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502
 [8] 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502
[15] 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502 0.6156502

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[2]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     49     100

$logLiks
 [1] 224.0846 224.0846 224.0846 224.0846 224.0846 224.0846 224.0846 224.0846
 [9] 224.0846 224.0846 224.0846 224.0846 224.0846 224.0846 224.0846 224.0846
[17] 224.0846 224.0846 224.0846 224.0846

$E
```

```
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[3]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     97     100

$logLiks
 [1] 247.5268 247.5401 247.5400 247.5415 247.5436 247.5451 247.5443 247.5308
 [9] 247.5429 247.5426 247.5455 247.5411 247.5429 247.5436 247.5453 247.5424
[17] 247.5453 247.5374 247.5440 247.5449

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[4]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 248.6231 248.6172 247.8915 247.5773 248.6265 247.6785 248.1661 248.4675
 [9] 247.5437 248.4537 248.2631 248.4513 248.0161 248.4980 247.8866 248.4067
[17] 247.5503 248.2981 247.9054 248.3808
```

```
$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[5]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 248.2026 247.8247 248.8488 248.2331 248.4626 248.2479 248.3031 248.5703
 [9] 247.6217 248.3412 248.2196 248.5517 248.4962 248.4835 248.7570 248.7404
[17] 248.4129 248.4537 248.3895 248.6450

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[6]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 248.4498 248.5986 247.9372 248.3565 248.6406 248.1298 248.2852 248.1145
 [9] 248.1962 248.6274 248.0414 248.3713 248.3358 248.3629 248.4222 249.0676
```

```
[17] 248.1555 248.3403 248.8163 248.5312

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[7]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 248.5034 248.5123 248.2799 248.7617 248.4889 248.5163 249.0519 248.1055
 [9] 249.2786 248.4488 249.2897 248.5923 249.1525 248.4866 248.7718 248.3646
[17] 248.3869 248.3463 248.2675 248.5823

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[8]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
```

```
 [1] 248.4395 248.6643 248.4424 249.4515 248.3820 248.9050 248.6911 248.4640
 [9] 248.2520 248.7217 249.2896 248.6714 247.9189 249.7812 248.1046 248.6778
[17] 248.3825 248.1610 248.9340 248.5338

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[9]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 248.4104 248.6220 249.0821 249.1000 249.6644 248.3475 249.1778 248.9196
 [9] 249.4842 249.1617 249.2474 248.5203 249.3378 248.7425 248.6184 249.0169
[17] 248.5225 248.4493 248.5426 248.7604

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> Esd[[10]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100
```

```
$logLiks
 [1] 248.8886 248.5159 249.7179 249.8781 249.0789 250.4978 249.4535 249.3066
 [9] 249.8514 249.5385 249.4601 249.1731 249.5602 248.9601 249.9757 249.3587
[17] 249.3764 248.5059 249.2330 248.2493

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE
```

4 Fit mixtures of vonMises-Fisher distributions for Crater Dataset

This list of impact craters on Earth contains a selection of the 190 confirmed craters given in the Earth Impact Database as of 2017. To keep the lists manageable, only the largest craters within a time period are included. Alphabetical lists for different continents can be found under Craters by continent below. These features were caused by the collision of meteors (consisting of large fragments of asteroids) or comets (consisting of ice, dust particles and rocky fragments) with the Earth. For eroded or buried craters, the stated diameter typically refers to the best available estimate of the original rim diameter, and may not correspond to present surface features. Time units are either in ka (thousands) or Ma (millions) of years.

[6]This dataset is taken form `https://en.wikipedia.org/wiki/List_of_impact_craters_on_Earth`

```
> head(dataset_imp)   # Crater Data set

  Impact_dataset.Name Impact_dataset.Location Impact_dataset.Country
2               Wabar    Rub' al Khali desert           Saudi Arabia
3               Kaali                Saaremaa                Estonia
4    Campo del Cielo                   Chaco              Argentina
5             Henbury      Northern Territory              Australia
6             Morasko                  Pozna?                 Poland
7             Boxhole      Northern Territory              Australia
  Impact_dataset.Diameter Impact_dataset.Age Impact_dataset.Coordinates
2                     0.1                0.2   21\xb030?N\xa050\xb028?E
3                     0.1                3.5   58\xb024?N\xa022\xb040?E
4                  0.1[7]                4.5   27\xb038?S\xa061\xb042?W
5                     0.2                4.7   24\xb034?S\xa0133\xb08?E
6                     0.1             5.0[9]   52\xb029?N\xa016\xb054?E
```

```
7                     0.2                 5.4  22\xb037?S\xa0135\xb012?E
  Impact_dataset.LAT...9 Impact_dataset.LON...13
2               21.50000                50.46667
3               58.40000                22.66667
4              -27.63333               -61.70000
5              -24.56667               133.13333
6               52.48333                16.90000
7              -22.61667               135.20000

cra = cbind(dataset_imp$Impact_dataset.LAT...9 ,
dataset_imp$Impact_dataset.LON...13)
set.seed(2023)
vMFs <-
  function(K){
    movMF(cra, k = K, control= list(nruns = 20))
  }

sd = lapply(1:10, vMFs)

> sd
[[1]]
theta:
       [,1]      [,2]
1 0.8233394 0.2588236
alpha:
[1] 1
L:
[1] 11.4361

[[2]]
theta:
      [,1]      [,2]
1 1.213879  2.146970
2 3.092024 -6.857282
alpha:
[1] 0.64005 0.35995
L:
[1] 26.70414

[[3]]
theta:
        [,1]        [,2]
1   1.261988    2.105457
2  34.357958  -60.109357
3 -86.050607 -212.680528
alpha:
[1] 0.65013161 0.29355402 0.05631438
L:
```

```
[1] 42.92726

[[4]]
theta:
        [,1]        [,2]
1  -2.295965    9.315376
2   2.817050    2.438691
3  33.740509  -59.102981
4 -85.857900 -212.161891
alpha:
[1] 0.18757966 0.45801245 0.29734990 0.05705798
L:
[1] 44.77042

[[5]]
theta:
         [,1]       [,2]
1   33.843761  73.921349
2    2.773714  -5.757212
3 -146.117300 786.258380
4   10.207982   5.111339
5 -353.685021 353.784542
alpha:
[1] 0.16073749 0.38623223 0.15712161 0.25305186 0.04285681
L:
[1] 50.72216

[[6]]
theta:
         [,1]        [,2]
1   35.071575   77.224700
2    6.358697    2.819464
3   33.517582  -58.692484
4 -146.476601  787.973601
5  -85.836374 -212.110625
6 -353.690102  353.788954
alpha:
[1] 0.14895435 0.29469552 0.29951201 0.15684338 0.05714227
0.04285247
L:
[1] 66.68767

[[7]]
theta:
        [,1]        [,2]
1 -353.68437  353.783834
2  -85.83622 -212.110353
3   17.00487    8.798395
```

```
4 -146.08600  786.110403
5   53.32954  -13.486480
6   33.48865  -58.618889
7   31.87041   68.172838
alpha:
[1] 0.04285714 0.05714286 0.22403020 0.15714211 0.04526040
0.29999990
[7] 0.17356739
L:
[1] 68.096

[[8]]
theta:
        [,1]       [,2]
1   33.48922  -58.62041
2   35.53114   -6.84091
3  194.55241  152.37027
4 -353.68437  353.78383
5   29.06961   60.23797
6 -146.08769  786.11838
7  -85.83622 -212.11035
8  179.21693   63.02367
alpha:
[1] 0.29999340 0.05828186 0.08506522 0.04285714 0.18620399
0.15714149
[7] 0.05714286 0.11331404
L:
[1] 70.99241

[[9]]
theta:
        [,1]        [,2]
1  -85.83622 -212.110353
2  180.20129   63.279677
3   49.97494  -83.466197
4  169.34770  132.857290
5 -146.08674  786.113921
6   29.47465   61.250687
7  200.63592 -744.830088
8 -353.68437  353.783834
9   35.70035   -6.887298
alpha:
[1] 0.05714286 0.11294983 0.27765675 0.08628970 0.15714186
0.18539458
[7] 0.02233579 0.04285714 0.05823149
L:
[1] 71.84604
```

```
[[10]]
theta:
         [,1]        [,2]
1   183.79336 -252.409204
2  -353.68437  353.783834
3   194.25988 -381.027970
4   141.13138 -495.060910
5  -146.08769  786.118382
6   -85.83622 -212.110353
7   179.20468   63.018481
8   194.55337  152.371000
9    35.55315   -6.846432
10   29.06947   60.237628
alpha:
 [1] 0.12798765 0.04285714 0.14114677 0.03085872 0.15714148
 0.05714286
 [7] 0.11331929 0.08506502 0.05827696 0.18620412
L:
[1] 72.75372


> sd[[1]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
      1     100

$logLiks
 [1] 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361
[10] 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361 11.4361
[19] 11.4361 11.4361

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> sd[[2]]$details
$reltol
```

```
[1] 1.490116e-08

$iter
   iter maxiter
     34     100

$logLiks
 [1] 26.70414 26.70414 26.70414 26.70414 26.70414 26.70414 26.70414 26.70414
 [9] 26.70414 26.70414 26.70414 26.70414 26.70414 26.70414 26.70414 26.70414
[17] 26.70414 26.70414 26.70414 26.70414

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> sd[[3]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     26     100

$logLiks
 [1] 42.92726 28.22887 28.35157 28.18133 28.26496 28.26692 28.21603 28.40256
 [9] 42.92726 28.27178 28.37575 28.20023 42.92726 42.92726 28.28745 28.28155
[17] 28.32708 28.23797 28.33061 28.26974

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE
```

```
> sd[[4]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 31.72765 31.72795 31.67932 31.72831 44.76890 44.77042 44.73656 31.72831
 [9] 31.72831 44.71463 44.75464 31.72822 44.60149 28.44022 44.74384 31.72829
[17] 31.06065 44.70743 28.41111 44.69264

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> sd[[5]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     98     100

$logLiks
 [1] 44.75130 45.60028 50.72049 45.57443 50.72216 31.72839 44.85985 44.95580
 [9] 44.96682 45.25074 45.29629 31.72191 45.59429 44.79651 31.77713 47.63394
[17] 45.49387 50.72216 45.63970 45.59355

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
```

```
[1] TRUE

> sd[[6]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     51     100

$logLiks
 [1] 47.91183 66.68767 45.69277 62.94387 49.10667 33.05385 31.91968 44.78288
 [9] 31.93611 48.40423 47.83921 31.78284 48.30001 66.68759 45.33498 49.10500
[17] 45.13397 48.17650 49.10667 66.68767

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> sd[[7]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     65     100

$logLiks
 [1] 48.32306 67.30767 31.92606 67.67047 48.13082 68.09600 48.50506 48.73271
 [9] 48.37984 48.97670       NA 67.11857 47.89965 52.78326 45.28335       NA
[17] 49.13434 68.09600 67.87424 49.11879

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0
```

```
$converge
[1] TRUE

> sd[[8]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
     86     100

$logLiks
 [1] 48.72121 46.36950       NA 70.99241 68.09654 48.77905 49.11687 52.89086
 [9]       NA       NA 48.74434 52.41160 70.99241 68.10801 67.75607 48.35983
[17] 67.42862 46.19374 70.99241 67.92510

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE

> sd[[9]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 68.07246       NA 71.84604 68.53528 69.46478 49.08524 49.83509 52.43713
 [9]       NA 68.50117 70.16362 70.92864       NA 50.45183 48.73714 68.16020
[17] 68.97512 71.39398       NA 52.77255

$E
[1] "softmax"

$kappa
NULL

$minalpha
```

```
[1] 0

$converge
[1] TRUE

> sd[[10]]$details
$reltol
[1] 1.490116e-08

$iter
   iter maxiter
    100     100

$logLiks
 [1] 72.75372       NA 72.64503 68.97416 71.17256 67.88252 70.99292 69.03350
 [9] 71.39413 68.50075 72.35900 69.66331 55.50873 52.44256 52.44150 70.99578
[17] 71.45313 68.98221 71.15115 68.50654

$E
[1] "softmax"

$kappa
NULL

$minalpha
[1] 0

$converge
[1] TRUE
```

5 For Meteor Landing Dataset

Now for the meteor landing or the third dataset observations we used the dataset collected from `https://data.nasa.gov/Space-Science/Meteorite-Landings/gh4g-9sfh`. Here, the Meteoritical Society collects data on meteorites that have fallen to Earth from outer space. This dataset includes the location, mass, composition, and fall year for over 45,000 meteorites that have struck our planet. We have many parameters in this dataset but used "reclat" , "reclong" as our primary parameter. Note that a few column names start with "rec" (e.g., recclass, reclat, reclon). These are the recommended values of these variables, according to The Meteoritical Society. In some cases, there were historical reclassification of a meteorite, or small changes in the data on where it was recovered; this dataset gives the currently recommended values.

```
> head(dataset_landing)
      name  id nametype    recclass   mass fall year    reclat    reclong
1   Aachen   1    Valid          L5     21 Fell 1880  50.77500    6.08333
2   Aarhus   2    Valid          H6    720 Fell 1951  56.18333   10.23333
3     Abee   6    Valid         EH4 107000 Fell 1952  54.21667 -113.00000
```

```
4 Acapulco  10    Valid Acapulcoite   1914 Fell 1976  16.88333  -99.90000
5  Achiras 370    Valid          L6    780 Fell 1902 -33.16667  -64.95000
6 Adhi Kot 379    Valid         EH4   4239 Fell 1919  32.10000   71.80000
              GeoLocation
1    (50.775000, 6.083330)
2   (56.183330, 10.233330)
3 (54.216670, -113.000000)
4  (16.883330, -99.900000)
5 (-33.166670, -64.950000)
6   (32.100000, 71.800000)
```

```
    > sdl
[[1]]
theta:
        [,1]     [,2]
1 -0.7828266 1.382048
alpha:
[1] 1
L:
[1] 13569.39

[[2]]
theta:
        [,1]      [,2]
1  0.1616651 0.5268991
2 -6.4366094 8.1819952
alpha:
[1] 0.4260062 0.5739938
L:
[1] 17997.56

[[3]]
theta:
        [,1]      [,2]
1  2.1865286  2.785070
2 -6.2933655  7.723079
3 -0.4569421 -3.184853
alpha:
[1] 0.2126370 0.6695268 0.1178362
L:
[1] 19254.35

[[4]]
theta:
           [,1]        [,2]
1 -2105.5202963 4237.126773
2    -0.4586867   -3.257561
3     1.8878450    2.755783
```

```
4   -30.5465243   18.109305
alpha:
[1] 0.3730054 0.1171300 0.2333750 0.2764897
L:
[1] 44700.54

[[5]]
theta:
           [,1]          [,2]
1 -4.501622e-01     -3.279424
2  2.101589e+00      2.945275
3 -6.831470e+05 705952.685277
4 -3.673616e+02    171.084855
5 -2.098401e+03   4222.956739
alpha:
[1] 0.11748459 0.22867639 0.07857406 0.20162686 0.37363811
L:
[1] 59753.27

[[6]]
theta:
           [,1]          [,2]
1 -6.814015e+05 704148.633766
2  4.485645e+03  12763.484654
3 -5.441114e-01     -3.390858
4 -3.666452e+02    170.762906
5 -2.080927e+03   4188.286710
6  2.011788e+00      1.568547
alpha:
[1] 0.07860791 0.09300235 0.11319773 0.20173970 0.37498337 0.13846892
L:
[1] 66962.02

[[7]]
theta:
           [,1]          [,2]
1  4.488833e+03  12772.621439
2 -2.080218e+03   4186.878644
3  1.094557e+02   -311.309501
4 -3.653342e+02    170.126519
5  2.076524e+00      1.611166
6 -1.097342e+01    -13.641403
7 -6.810709e+05 703806.947186
alpha:
[1] 0.09297792 0.37504062 0.05504244 0.20211492 0.13950016 0.05670991 0.07861404
L:
[1] 69816.07
```

```
[[8]]
theta:
           [,1]          [,2]
1    -362.98555    169.036312
2 -681124.95257 703862.834145
3      -0.75980     10.078525
4      11.07496      3.973479
5   -2096.78424   4219.737099
6     -10.63500    -13.292902
7    4188.64085  11913.785461
8      89.14631   -251.077939
alpha:
[1] 0.20257802 0.07861314 0.03869292 0.09688460 0.37378563 0.05744819 0.09502427 0.05697323
L:
[1] 70684.41

[[9]]
theta:
           [,1]          [,2]
1 -2.141810e+01    -43.059320
2 -3.655572e+02    170.230218
3 -7.999132e+04 -64307.350388
4 -6.812207e+05 703961.861537
5  4.488437e+03  12771.478584
6  2.026376e+00      1.570489
7  4.488437e+03  12771.478584
8  1.099974e+02   -312.999755
9 -2.080734e+03   4187.903586
alpha:
[1] 0.02837704 0.20206750 0.02788596 0.07861128 0.04649020 0.14011821 0.04649020 0.05495995 0.374
L:
[1] 72813.5

[[10]]
theta:
            [,1]          [,2]
1  -1.066314e+01    -13.331899
2   1.380058e+01      2.724934
3   8.920868e+01   -251.264996
4  -2.076421e+03   4179.334275
5   1.417487e+04   8448.704323
6   4.204240e+03  11958.801731
7  -6.797681e+05 702460.577720
8   1.225632e+00      4.904589
9  -3.624384e+02    168.795103
10 -5.175302e+03  21851.633891
alpha:
 [1] 0.05743564 0.05924524 0.05697004 0.37533818 0.03134464 0.09493903 0.07863730 0.02739575 0.2
```

```
[10] 0.01612172
L:
[1] 73648.02

[[11]]
theta:
            [,1]          [,2]
1   5.579367e-01     -6.062989
2  -3.623596e+02    168.759967
3  -8.087339e+04 -65017.091197
4   1.652920e+01      3.462594
5  -2.076220e+03   4178.934480
6  -3.623596e+02    168.759967
7   1.407912e+04   8391.526598
8   4.209987e+03  11975.210581
9  -5.124759e+03  21638.774092
10 -6.797230e+05 702414.037290
11  1.286884e+00      4.886620
alpha:
 [1] 0.08832366 0.10128852 0.02779521 0.05693304 0.37535377 0.10128852 0.03142160 0.09490510 0.0
[10] 0.07863808 0.02790779
L:
[1] 74235.55

[[12]]
theta:
            [,1]          [,2]
1  -6.800958e+05  7.027993e+05
2  -3.644207e+02  1.697107e+02
3   4.668392e+03  1.328563e+04
4   8.584592e+01  2.532413e+02
5  -1.318627e+07  2.899805e+07
6   9.720341e+01 -2.748612e+02
7  -2.793270e+03  1.182239e+04
8   1.006378e+00  1.546649e+00
9   1.419629e+04  8.461564e+03
10  1.576917e+01  3.267000e+00
11 -3.160559e+03  6.323578e+03
12 -1.079825e+01 -1.347361e+01
alpha:
 [1] 0.07863154 0.20224593 0.09117703 0.01284470 0.02455196 0.05635250 0.01752452 0.02384267 0.0
[10] 0.05338471 0.35108159 0.05703693
L:
[1] 76696.57

[[13]]
theta:
            [,1]          [,2]
```

```
1   1.341468e+01  5.043036e+00
2   7.275767e+01  2.138653e+02
3  -2.960212e+07  3.063376e+07
4  -7.998725e+04 -6.430408e+04
5  -1.819490e+00  8.235368e+00
6  -4.473864e+03  1.890047e+04
7  -2.175581e+01 -4.378518e+01
8  -3.635114e+02  1.692826e+02
9   1.057130e+02 -2.999688e+02
10  1.131854e+00 -5.186441e-01
11 -2.638570e+06  2.719827e+06
12  4.664158e+03  1.327362e+04
13 -2.078120e+03  4.182717e+03
alpha:
 [1] 0.091833521 0.014696090 0.050146550 0.027886240 0.009501326 0.016462429 0.028159021 0.20246
 [9] 0.055398416 0.008716203 0.028304857 0.091244203 0.375189413
L:
[1] 77019.44

[[14]]
theta:
            [,1]          [,2]
1  -1.756010e+01    -34.906854
2   1.458003e+04   8691.162531
3  -8.018307e+04 -64461.499501
4   4.674269e+03  13302.609318
5  -1.612510e+02    141.021162
6  -3.645526e+02    169.740638
7   1.194162e+01      2.686715
8  -3.217466e+03  13608.511924
9  -1.612510e+02    141.021162
10  8.350217e+01    244.345392
11  8.813067e+01   -248.470091
12  6.092291e-01      5.509207
13 -6.809744e+05 703707.343364
14 -2.072896e+03   4172.337168
alpha:
 [1] 0.0294547770 0.0310607125 0.0278693828 0.0911388308 0.0005283248
 0.2025123387 0.0631698243 0.0171962783
 [9] 0.0005283248 0.0132798527 0.0570368153 0.0120155387 0.0786154923 0.3755935069
L:
[1] 76811.44

[[15]]
theta:
            [,1]          [,2]
1  -1.969395e+04  8.318424e+04
2   1.485113e+01  3.266628e+00
```

```
3  -6.797311e+05  7.024224e+05
4   4.660874e+03  1.326413e+04
5  -3.703393e+02  1.600568e+03
6   6.354293e-01  2.934376e-01
7   1.008231e+02 -2.855751e+02
8  -1.969395e+04  8.318424e+04
9  -3.639903e+02  1.695178e+02
10 -2.227609e+01 -4.507064e+01
11  1.439762e+04  8.582080e+03
12  1.171407e+00  2.110523e+00
13 -2.071114e+03  4.168803e+03
14 -7.999828e+04 -6.431297e+04
15  7.613934e+01  2.248571e+02
alpha:
 [1] 0.005756607 0.055729601 0.078637901 0.091265680 0.008030976 0.009363491
 0.055947592 0.005756607
 [9] 0.202287720 0.028145336 0.031191470 0.010497501 0.375711351 0.027885156 0.013793013
L:
[1] 76960.77
```

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