SUPPLEMENTARY MATERIAL

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This document offers detailed results from a number of robustness checks that were not included in the main text or in the online appendix. To maintain coherence, we have preserved the section numbering to align with the corresponding sections in the main article. Consequently, Section A.x.y complements Section x.y in the main text and so forth. For ease of reference, we also included a table of contents.

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6. BASELINE PARAMETRIZATION AND ROBUSTNESS ANALYSIS

6.1. *Increasing the number of latent factors*

We augment the dimensionality by increasing the number of latent factors from 2 to 5 and proceed to examine the implications on our main outcomes of interest. Consistent with our earlier analytical framework and for the sake of maintaining comparability, we assume that the algorithm possesses a priori knowledge of the true number of latent factors. In appendix 10 we relax this assumption and allow the number of latent factors to be estimated through cross-validation.

With only two latent factors, it is easy to evenly position both products and consumers along the section of the unit circle located in the first orthant. In more than two dimensions, however, no general solution is known for the same problem, and one needs to resort to some kind of approximation. In this paper, we position 801 products and 24030 consumers on the surface $\mathcal{S}$ of the positive half-segment of the 5-dimensional hypersphere in the following way. We first generated a large number R of random uniformly distributed points on $\mathcal{S}$, and grouped them in G subsets using the k-means algorithm, a well-known clustering technique. This iterative algorithm aims at minimizing the ratio of the total variance within the subsets and the total variance between them. Upon convergence, the algorithm delivers G points on $\mathcal{S}$, the "centroids" of each subset, whose coordinates are the averages of the coordinates of the point in the same subset (normalized so that they belong to $\mathcal{S}$). Our implementation of the k-means approach pre-selects G_0 centroids, and lets the algorithm determine the position of the remaining $G - G_0$. The products' position is determined in two steps. In the first, we set $G_0 = 6$, fixing the central product (located at $(\sqrt{5})^{-1}$) and the 5 products situated at the corners of $\mathcal{S}$, and find the position of $G - G_0 = 19 - 6 = 13$ additional products; this delivers $m_{\mathcal{M}} = 19$ approximately evenly spaced points on $\mathcal{S}$, while keeping the central and the corner products. In the second step, we fix the $G_0 = 19$ points obtained in the first step and let the algorithm determine the position of the remaining $G - G0 = 801 - 19 = 782$. Finally, consumers are positioned in groups of 24030/801 = 30 at the same point as the products. The k-means algorithm is only guaranteed to converge to a local minimum. To avoid spurious solutions, we tried 10^6 random initial allocations of the R points to G groups and we picked the final allocation that minimizes the random search benchmark's HHI index at zero prices. We also experimented with this approach in two dimensions, obtaining a products' positioning very close to the optimal deterministic one.

Table 1 shows that when prices are set to zero, market concentration significantly increases. However, this increase is limited compared to the baseline case with two factors ($H = 2$). In Table 2, we confirm that equilibrium prices also rise relative to the benchmark. This price increase is significantly larger than that of the baseline case and thus has a larger impact on consumer surplus, as indicated in Table 3.

However, the effect of higher prices does not outweigh reduced search costs, leading to a positive impact on consumer surplus, though somewhat lower compared to the baseline for small values of α. We omit the analysis based on product location in this 5-dimensional extension product as user location changes from session to session and therefore averaging results across sessions is not useful.

α	0	0.25	0.5	0.75	1
Benchmark mean	624.1146	904.9350	2620.8662	5981.9177	8828.5837
Benchmark sd	4.0595	12.3004	35.5777	53.7795	46.3047
RS mean	719.5800	1787.9373	5249.1539	8980.8898	9966.9714
RS sd	6.4692	26.5780	38.1902	18.3508	2.3191
Perc. Diff. mean	15.26	97.94	102.65	51.27	13.22
Perc. Diff. sd	0.65	1.83	2.14	1.32	0.63

TABLE 1. Herfindahl-Hirschman Index (market concentration)

α	0	0.25	0.5	0.75	1
Benchmark mean	0.0774	0.0705	0.0564	0.0377	0.0200
Benchmark sd	0.0005	0.0004	0.0003	0.0002	0.0001
RS mean	0.0935	0.0859	0.0710	0.0471	0.0231
RS sd	0.0008	0.0007	0.0006	0.0003	0.0001
Perc. Diff. mean	20.67	21.73	25.66	24.89	15.28
Perc. Diff. sd	0.49	0.46	0.38	0.29	0.17

TABLE 2. Equilibrium prices.

α	0	0.25	0.5	0.75	1
Full information CS	0.9433	0.8534	0.7923	0.7654	0.7614
Random Choice CS	0.5914	0.5322	0.4730	0.4137	0.3545
Benchmark mean	0.7993	0.7184	0.6560	0.6216	0.6160
Benchmark sd	0.0006	0.0005	0.0005	0.0004	0.0002
RS mean	0.8044	0.7270	0.6773	0.6631	0.6783
RS sd	0.0007	0.0005	0.0004	0.0003	0.0001
Perc. Diff. mean	0.63	1.19	3.25	6.67	10.11
Perc. Diff. sd	0.03	0.04	0.05	0.06	0.04

TABLE 3. Equilibrium consumer surplus.

6.2. *Robustness checks*

6.2.1. *Density.*

We report Herfindahl-Hirschman Index, equilibrium prices and equilibrium consumer surplus when we let the density d of the observe matrix $\tilde{\mathbf{R}}$ vary. In particular, we let d vary from 0.6% to 2.4% in increments of 0.3%.

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	950.6389	1477.9730	2905.4322	4884.6005	7005.0397	8937.8364
RS sd	11.2169	35.6656	60.5713	49.9707	47.7075	41.4883
Perc. Diff. mean	76.50	142.91	241.62	266.27	227.56	168.96
Perc. Diff. sd	2.08	5.86	7.12	3.75	2.23	1.25

TABLE 4. Herfindahl-Hirschman Index ($d = 0.6\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0392	0.0378	0.0355	0.0329	0.0294	0.0244
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	2.41	4.34	5.72	6.99	7.78	7.42
Perc. Diff. sd	0.37	0.26	0.13	0.05	0.03	0.01

TABLE 5. Equilibrium prices ($d = 0.6\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9341	0.8982	0.8685	0.8452	0.8287	0.8202
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.30	0.45	0.89	1.62	2.64	4.00
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 6. Equilibrium Consumer surplus ($d = 0.6\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1073.3042	1417.3413	2473.6887	4219.3915	6270.4629	8351.6888
RS sd	11.8508	31.7424	61.6614	75.5232	80.9414	74.6452
Perc. Diff. mean	99.28	132.94	190.85	216.39	193.21	151.32
Perc. Diff. sd	2.20	5.22	7.25	5.66	3.78	2.25

TABLE 7. Herfindahl-Hirschman Index ($d = 0.9\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0431	0.0403	0.0367	0.0332	0.0294	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	12.43	11.46	9.38	7.87	7.59	7.18
Perc. Diff. sd	0.44	0.32	0.21	0.13	0.08	0.03

TABLE 8. Equilibrium prices ($d = 0.9\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9368	0.9007	0.8705	0.8463	0.8291	0.8202
RS sd	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
Perc. Diff. mean	0.59	0.73	1.12	1.75	2.68	4.00
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.00	0.00

TABLE 9. Equilibrium Consumer surplus ($d = 0.9\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1078.0114	1396.8593	2519.6441	4263.4955	6317.3898	8410.1934
RS sd	11.4180	30.2396	52.3189	59.9991	61.4467	56.8680
Perc. Diff. mean	100.15	129.58	196.26	219.70	195.40	153.08
Perc. Diff. sd	2.12	4.97	6.15	4.50	2.87	1.71

TABLE 10. Herfindahl-Hirschman Index ($d = 1.2\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0441	0.0412	0.0370	0.0332	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	15.10	13.74	10.45	8.09	7.36	7.13
Perc. Diff. sd	0.41	0.31	0.18	0.12	0.07	0.03

TABLE 11. Equilibrium prices ($d = 1.2\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9371	0.9011	0.8711	0.8468	0.8293	0.8202
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.62	0.77	1.18	1.81	2.71	4.00
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 12. Equilibrium Consumer surplus ($d = 1.2\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1033.3880	1388.1419	2560.1793	4328.9196	6430.7585	8526.3288
RS sd	11.2155	29.4850	51.3811	61.6083	61.8095	57.5242
Perc. Diff. mean	91.87	128.14	201.02	224.60	200.70	156.57
Perc. Diff. sd	2.08	4.85	6.04	4.62	2.89	1.73

TABLE 13. Herfindahl-Hirschman Index ($d = 1.5\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0448	0.0416	0.0373	0.0333	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	16.98	14.94	11.11	8.22	7.33	7.16
Perc. Diff. sd	0.49	0.34	0.22	0.14	0.07	0.03

TABLE 14. Equilibrium prices ($d = 1.5\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9368	0.9011	0.8712	0.8469	0.8294	0.8203
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.60	0.78	1.19	1.83	2.72	4.02
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 15. Equilibrium Consumer surplus ($d = 1.5\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1017.4631	1371.0047	2591.7375	4375.2156	6475.3324	8586.1493
RS sd	12.0860	28.9124	48.4404	55.0257	54.7588	49.1929
Perc. Diff. mean	88.91	125.33	204.73	228.07	202.79	158.37
Perc. Diff. sd	2.24	4.75	5.70	4.13	2.56	1.48

TABLE 16. Herfindahl-Hirschman Index ($d = 1.8\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0455	0.0419	0.0374	0.0333	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	18.79	15.87	11.40	8.37	7.32	7.16
Perc. Diff. sd	0.51	0.33	0.21	0.13	0.07	0.03

TABLE 17. Equilibrium prices ($d = 1.8\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9366	0.9010	0.8712	0.8470	0.8295	0.8203
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.57	0.77	1.20	1.83	2.73	4.02
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 18. Equilibrium Consumer surplus ($d = 1.8\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1002.8917	1351.6685	2581.2375	4393.7375	6511.9962	8634.8346
RS sd	12.9556	30.2390	48.8115	55.3923	54.3984	47.9265
Perc. Diff. mean	86.20	122.15	203.50	229.46	204.50	159.84
Perc. Diff. sd	2.41	4.97	5.74	4.15	2.54	1.44

TABLE 19. Herfindahl-Hirschman Index ($d = 2.1\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0460	0.0422	0.0375	0.0334	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	20.03	16.65	11.91	8.61	7.40	7.15
Perc. Diff. sd	0.57	0.35	0.20	0.14	0.07	0.04

TABLE 20. Equilibrium prices ($d = 2.1\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9364	0.9010	0.8713	0.8470	0.8295	0.8203
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.55	0.76	1.21	1.84	2.74	4.02
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 21. Equilibrium Consumer surplus ($d = 2.1\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	993.6107	1355.6169	2592.0474	4417.4695	6546.2168	8679.7671
RS sd	11.4711	28.7454	48.0191	52.7452	50.7113	43.9675
Perc. Diff. mean	84.48	122.80	204.77	231.24	206.10	161.19
Perc. Diff. sd	2.13	4.72	5.65	3.96	2.37	1.32

TABLE 22. Herfindahl-Hirschman Index ($d = 2.4\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0464	0.0424	0.0376	0.0334	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	21.06	17.17	12.10	8.60	7.30	7.09
Perc. Diff. sd	0.57	0.30	0.19	0.13	0.07	0.04

TABLE 23. Equilibrium prices ($d = 2.4\%$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9362	0.9009	0.8713	0.8471	0.8295	0.8204
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.53	0.76	1.21	1.85	2.74	4.02
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 24. Equilibrium Consumer surplus ($d = 2.4\%$)

6.2.2. *Number of consumers and products.*

Another way to modify the amount of information is to change the numbers of consumers I and products J while keeping the density d fixed. We report four variations to the baseline specification. We begin by doubling and then halving the levels of I and J, all the while keeping their ratio I/J constant[1]. The results are respectively in Tables 25, 26, 27 and Tables 28, 29, 30.

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5342	608.3878	850.3649	1335.0679	2146.6318	3347.3619
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	991.5968	1325.4986	2543.8020	4377.8090	6524.0203	8662.4783
RS sd	11.0799	28.0635	50.2521	57.7166	56.8383	50.9778
Perc. Diff. mean	84.13	117.87	199.14	227.91	203.92	158.79
Perc. Diff. sd	2.06	4.61	5.91	4.32	2.65	1.52

TABLE 25. Herfindahl-Hirschman Index ($I = 48060$, $J = 1601$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0464	0.0426	0.0377	0.0334	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	21.13	17.65	12.41	8.75	7.38	7.12
Perc. Diff. sd	0.51	0.35	0.21	0.14	0.08	0.04

TABLE 26. Equilibrium prices ($I = 48060$, $J = 1601$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8608	0.8316	0.8072	0.7883
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9363	0.9009	0.8713	0.8471	0.8295	0.8202
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.54	0.75	1.21	1.86	2.76	4.05
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 27. Equilibrium consumer surplus ($I = 48060$, $J = 1601$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5944	608.2762	849.3557	1330.9985	2134.6734	3318.1692
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	968.0434	1490.7210	2758.1026	4662.8626	6817.2688	8837.7313
RS sd	14.0054	32.6130	67.0700	72.6138	68.4101	50.3510
Perc. Diff. mean	79.74	145.07	224.73	250.33	219.36	166.34
Perc. Diff. sd	2.60	5.36	7.90	5.46	3.20	1.52

TABLE 28. Herfindahl-Hirschman Index ($I = 12030$, $J = 401$)

[1]We still keep the number of the J products odd so that their distribution can be symmetric around the central product

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0336	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0397	0.0381	0.0357	0.0329	0.0294	0.0244
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	3.65	5.10	6.34	7.14	7.75	7.38
Perc. Diff. sd	0.34	0.25	0.15	0.07	0.03	0.02

TABLE 29. Equilibrium prices ($I = 12030$, $J = 401$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9312	0.8941	0.8609	0.8317	0.8074	0.7887
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9344	0.8985	0.8687	0.8453	0.8287	0.8201
RS sd	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
Perc. Diff. mean	0.34	0.49	0.92	1.63	2.63	3.99
Perc. Diff. sd	0.02	0.01	0.01	0.01	0.01	0.00

TABLE 30. Equilibrium consumer surplus ($I = 12030$, $J = 401$)

We now decrease the I/J ratio first to 15 and then to 3 while adjust the levels to maintain a constant ratio of observations to the number of parameters to be estimated[2]. We present the results in Tables 31, 32, 33 for I/J ratio of 15 and in Tables 34, 35, 36 for I/J ratio of 3.

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5241	608.2999	850.4220	1335.9728	2149.9786	3356.0296
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1126.8599	1357.3711	2275.2335	3877.9478	5900.0229	7989.6420
RS sd	12.9518	25.7847	56.2532	76.8880	80.1187	75.0702
Perc. Diff. mean	109.25	123.14	167.54	190.27	174.42	138.07
Perc. Diff. sd	2.41	4.24	6.61	5.76	3.73	2.24

TABLE 31. Herfindahl-Hirschman Index (ratio $I/J = 15$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0434	0.0408	0.0370	0.0333	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	13.26	12.84	10.39	8.29	7.59	7.14
Perc. Diff. sd	0.45	0.31	0.18	0.14	0.10	0.04

TABLE 32. Equilibrium prices (ratio $I/J = 15$)

[2]Recall that this is done to match the densities of other datasets

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8608	0.8315	0.8071	0.7882
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9374	0.9010	0.8708	0.8465	0.8290	0.8199
RS sd	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
Perc. Diff. mean	0.66	0.78	1.17	1.80	2.71	4.03
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.00	0.00

TABLE 33. Equilibrium consumer surplus (ratio $I/J = 15$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5349	608.2807	850.0765	1333.1192	2138.8601	3326.2951
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1176.0483	1414.5418	2274.6561	3711.1470	5642.1602	7739.3479
RS sd	13.7638	29.2090	64.5726	94.9614	118.7593	117.6780
Perc. Diff. mean	118.38	132.55	167.58	178.38	163.79	132.67
Perc. Diff. sd	2.56	4.80	7.60	7.12	5.55	3.54

TABLE 34. Herfindahl-Hirschman Index (ratio $I/J = 3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0414	0.0391	0.0362	0.0332	0.0294	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	8.12	8.07	8.03	7.92	7.74	7.00
Perc. Diff. sd	0.43	0.34	0.22	0.16	0.13	0.05

TABLE 35. Equilibrium prices (ratio $I/J = 3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8073	0.7885
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9369	0.9003	0.8698	0.8455	0.8283	0.8195
RS sd	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001
Perc. Diff. mean	0.60	0.70	1.04	1.67	2.60	3.93
Perc. Diff. sd	0.01	0.02	0.01	0.02	0.02	0.02

TABLE 36. Equilibrium consumer surplus (ratio $I/J = 3$)

6.2.3. *Reporting noise I: idiosyncratic shocks.*

We study how changing the variance of the idiosyncratic shocks ϵ_{ij} affects the performance of the recommendation system. In the baseline, σ_ϵ was set at 20% of the expected utility across the J products. We now present estimates for the Herfindahl-Hirschman Index, the equilibrium prices and the equilibrium consumer surplus for σ_ϵ ranging from 0 to 40% of the aforementioned benchmark.

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	920.9533	1348.5485	2667.0094	4579.3303	6755.1980	8910.1278
RS sd	9.9414	24.5993	44.4845	46.6179	44.3108	36.2665
Perc. Diff. mean	70.99	121.64	213.58	243.38	215.88	168.12
Perc. Diff. sd	1.85	4.04	5.23	3.50	2.07	1.09

TABLE 37. Herfindahl-Hirschman Index ($\sigma = 0$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0481	0.0432	0.0380	0.0334	0.0292	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	25.51	19.47	13.22	8.82	6.94	6.89
Perc. Diff. sd	0.53	0.30	0.18	0.12	0.06	0.05

TABLE 38. Equilibrium prices ($\sigma = 0$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9353	0.9007	0.8714	0.8472	0.8296	0.8204
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.44	0.74	1.22	1.87	2.75	4.03
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 39. Equilibrium Consumer Surplus ($\sigma = 0$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	932.6290	1353.8145	2691.4944	4589.3228	6771.3384	8911.5482
RS sd	9.7212	24.9756	43.3438	47.4894	43.2100	35.2830
Perc. Diff. mean	73.16	122.50	216.46	244.13	216.63	168.16
Perc. Diff. sd	1.80	4.10	5.10	3.56	2.02	1.06

TABLE 40. Herfindahl-Hirschman Index ($\sigma = 0.025$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0477	0.0431	0.0379	0.0334	0.0292	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	24.48	18.97	12.94	8.82	6.94	6.93
Perc. Diff. sd	0.52	0.31	0.18	0.12	0.06	0.05

TABLE 41. Equilibrium prices ($\sigma = 0.025$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9356	0.9008	0.8714	0.8472	0.8296	0.8204
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.46	0.74	1.22	1.86	2.75	4.03
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 42. Equilibrium Consumer Surplus ($\sigma = 0.025$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	954.2841	1360.5180	2671.8228	4570.0100	6739.5959	8886.9167
RS sd	9.9547	26.2155	44.6314	47.6821	42.7920	35.3154
Perc. Diff. mean	77.18	123.60	214.15	242.68	215.15	167.42
Perc. Diff. sd	1.85	4.31	5.25	3.58	2.00	1.06

TABLE 43. Herfindahl-Hirschman Index ($\sigma = 0.05$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0472	0.0428	0.0378	0.0334	0.0292	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	23.22	18.36	12.71	8.69	7.01	6.96
Perc. Diff. sd	0.52	0.32	0.18	0.13	0.06	0.05

TABLE 44. Equilibrium prices ($\sigma = 0.05$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9358	0.9009	0.8714	0.8472	0.8296	0.8204
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.49	0.75	1.22	1.86	2.75	4.03
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 45. Equilibrium Consumer Surplus ($\sigma = 0.05$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	977.8090	1368.9376	2661.3247	4537.1694	6696.5415	8836.9023
RS sd	9.9249	27.4372	46.3644	49.1393	44.7775	37.0609
Perc. Diff. mean	81.55	124.99	212.91	240.22	213.13	165.92
Perc. Diff. sd	1.84	4.51	5.45	3.68	2.09	1.12

TABLE 46. Herfindahl-Hirschman Index ($\sigma = 0.075$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0468	0.0426	0.0377	0.0334	0.0292	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	22.00	17.72	12.26	8.57	7.13	7.00
Perc. Diff. sd	0.49	0.32	0.19	0.12	0.06	0.05

TABLE 47. Equilibrium prices ($\sigma = 0.075$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9360	0.9009	0.8713	0.8472	0.8296	0.8204
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.51	0.75	1.22	1.86	2.75	4.03
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 48. Equilibrium Consumer Surplus ($\sigma = 0.075$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1002.6943	1370.6305	2641.2573	4488.3497	6641.1056	8765.6877
RS sd	10.1040	28.6513	48.0892	49.9916	46.5962	40.0192
Perc. Diff. mean	86.17	125.26	210.55	236.56	210.54	163.78
Perc. Diff. sd	1.88	4.71	5.65	3.75	2.18	1.20

TABLE 49. Herfindahl-Hirschman Index ($\sigma = 0.1$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0461	0.0424	0.0375	0.0333	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	20.42	17.00	11.81	8.49	7.20	7.05
Perc. Diff. sd	0.45	0.32	0.20	0.12	0.06	0.05

TABLE 50. Equilibrium prices ($\sigma = 0.1$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9363	0.9009	0.8713	0.8471	0.8295	0.8204
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.54	0.76	1.21	1.85	2.74	4.03
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 51. Equilibrium Consumer Surplus ($\sigma = 0.1$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1027.6255	1384.0977	2600.6092	4435.4879	6566.8754	8688.3486
RS sd	10.5860	29.6446	48.4746	51.7128	50.0280	44.6341
Perc. Diff. mean	90.80	127.48	205.78	232.59	207.07	161.45
Perc. Diff. sd	1.97	4.87	5.70	3.88	2.34	1.34

TABLE 52. Herfindahl-Hirschman Index ($\sigma = 0.125$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0456	0.0420	0.0374	0.0333	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	18.90	16.05	11.55	8.43	7.28	7.10
Perc. Diff. sd	0.44	0.33	0.19	0.12	0.06	0.04

TABLE 53. Equilibrium prices ($\sigma = 0.125$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9366	0.9010	0.8713	0.8470	0.8295	0.8203
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.57	0.77	1.21	1.84	2.73	4.02
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 54. Equilibrium Consumer Surplus ($\sigma = 0.125$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1051.2923	1380.6310	2578.1805	4390.4308	6485.1083	8587.3019
RS sd	10.9468	30.1442	49.7873	54.3002	54.4537	49.5426
Perc. Diff. mean	95.19	126.91	203.14	229.22	203.25	158.41
Perc. Diff. sd	2.03	4.95	5.85	4.07	2.55	1.49

TABLE 55. Herfindahl-Hirschman Index ($\sigma = 0.15$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0450	0.0418	0.0373	0.0333	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	17.32	15.55	11.15	8.29	7.33	7.13
Perc. Diff. sd	0.47	0.33	0.19	0.12	0.07	0.04

TABLE 56. Equilibrium prices ($\sigma = 0.15$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9368	0.9010	0.8712	0.8469	0.8294	0.8203
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.59	0.76	1.20	1.83	2.73	4.02
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 57. Equilibrium Consumer Surplus ($\sigma = 0.15$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1063.0687	1391.7886	2548.8080	4341.0726	6407.1588	8494.4084
RS sd	11.1539	30.2498	51.1808	56.8608	57.8399	54.4617
Perc. Diff. mean	97.38	128.74	199.68	225.51	199.60	155.61
Perc. Diff. sd	2.07	4.97	6.02	4.26	2.70	1.64

TABLE 58. Herfindahl-Hirschman Index ($\sigma = 0.175$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0446	0.0415	0.0372	0.0332	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	16.42	14.60	10.80	8.14	7.38	7.14
Perc. Diff. sd	0.43	0.32	0.19	0.13	0.07	0.03

TABLE 59. Equilibrium prices ($\sigma = 0.175$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9369	0.9010	0.8711	0.8468	0.8294	0.8203
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.61	0.77	1.19	1.82	2.72	4.01
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00	0.00

TABLE 60. Equilibrium Consumer Surplus ($\sigma = 0.175$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1078.0114	1396.8593	2519.6441	4263.4955	6317.3898	8410.1934
RS sd	11.4180	30.2396	52.3189	59.9991	61.4467	56.8680
Perc. Diff. mean	100.15	129.58	196.26	219.70	195.40	153.08
Perc. Diff. sd	2.12	4.97	6.15	4.50	2.87	1.71

TABLE 61. Herfindahl-Hirschman Index ($\sigma = 0.2$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0441	0.0412	0.0370	0.0332	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	15.10	13.74	10.45	8.09	7.36	7.13
Perc. Diff. sd	0.41	0.31	0.18	0.12	0.07	0.03

TABLE 62. Equilibrium prices ($\sigma = 0.2$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9371	0.9011	0.8711	0.8468	0.8293	0.8202
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Perc. Diff. mean	0.62	0.77	1.18	1.81	2.71	4.00
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 63. Equilibrium Consumer Surplus ($\sigma = 0.2$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1076.9626	1395.7686	2492.7084	4202.0038	6245.8396	8317.0582
RS sd	11.7821	28.9922	52.8801	62.9441	65.3122	63.0273
Perc. Diff. mean	99.96	129.40	193.09	215.09	192.06	150.28
Perc. Diff. sd	2.19	4.76	6.22	4.72	3.05	1.90

TABLE 64. Herfindahl-Hirschman Index ($\sigma = 0.225$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0437	0.0409	0.0369	0.0332	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	14.11	13.06	10.02	7.97	7.35	7.12
Perc. Diff. sd	0.43	0.28	0.18	0.12	0.07	0.03

TABLE 65. Equilibrium prices ($\sigma = 0.225$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9371	0.9010	0.8710	0.8467	0.8292	0.8201
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.63	0.77	1.17	1.79	2.70	3.99
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 66. Equilibrium Consumer Surplus ($\sigma = 0.225$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1081.3506	1404.0147	2456.5851	4125.6787	6174.2628	8222.3974
RS sd	11.2505	29.2846	54.4521	64.1379	69.0044	66.5455
Perc. Diff. mean	100.77	130.75	188.84	209.36	188.71	147.43
Perc. Diff. sd	2.09	4.81	6.40	4.81	3.23	2.00

TABLE 67. Herfindahl-Hirschman Index ($\sigma = 0.25$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0434	0.0407	0.0368	0.0332	0.0293	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	13.36	12.33	9.68	7.90	7.35	7.08
Perc. Diff. sd	0.42	0.27	0.18	0.12	0.07	0.03

TABLE 68. Equilibrium prices ($\sigma = 0.25$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9371	0.9011	0.8709	0.8466	0.8291	0.8201
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.63	0.77	1.16	1.79	2.69	3.99
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 69. Equilibrium Consumer Surplus ($\sigma = 0.25$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1095.5524	1399.2223	2400.5753	4073.7902	6101.2704	8129.5738
RS sd	11.1893	29.6186	55.3104	67.4571	73.4747	71.4973
Perc. Diff. mean	103.41	129.96	182.26	205.47	185.30	144.63
Perc. Diff. sd	2.08	4.87	6.50	5.06	3.44	2.15

TABLE 70. Herfindahl-Hirschman Index ($\sigma = 0.275$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0431	0.0404	0.0367	0.0331	0.0293	0.0243
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	12.40	11.53	9.48	7.83	7.30	7.05
Perc. Diff. sd	0.37	0.28	0.17	0.12	0.07	0.03

TABLE 71. Equilibrium prices ($\sigma = 0.275$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9373	0.9010	0.8708	0.8464	0.8290	0.8200
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.65	0.77	1.15	1.77	2.68	3.98
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.00

TABLE 72. Equilibrium Consumer Surplus ($\sigma = 0.275$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1097.8392	1397.9191	2407.2800	4038.0073	6027.1939	8059.9187
RS sd	11.1005	29.6149	55.0839	70.7939	74.8596	75.7687
Perc. Diff. mean	103.83	129.75	183.04	202.79	181.83	142.54
Perc. Diff. sd	2.06	4.87	6.48	5.31	3.50	2.28

TABLE 73. Herfindahl-Hirschman Index ($\sigma = 0.3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0427	0.0402	0.0366	0.0331	0.0293	0.0243
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	11.42	11.07	9.09	7.69	7.22	6.99
Perc. Diff. sd	0.35	0.29	0.18	0.12	0.08	0.03

TABLE 74. Equilibrium prices ($\sigma = 0.3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9373	0.9010	0.8707	0.8463	0.8289	0.8199
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.64	0.77	1.14	1.76	2.67	3.96
Perc. Diff. sd	0.01	0.01	0.01	0.00	0.00	0.01

TABLE 75. Equilibrium Consumer Surplus ($\sigma = 0.3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1102.1033	1392.7936	2371.2492	4001.5794	5975.7170	7986.4220
RS sd	10.8209	29.2617	55.7340	71.5175	78.0159	78.1510
Perc. Diff. mean	104.62	128.91	178.81	200.06	179.43	140.33
Perc. Diff. sd	2.01	4.81	6.55	5.36	3.65	2.35

TABLE 76. Herfindahl-Hirschman Index ($\sigma = 0.325$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0424	0.0400	0.0365	0.0331	0.0293	0.0243
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	10.71	10.42	8.93	7.54	7.18	6.92
Perc. Diff. sd	0.33	0.29	0.17	0.12	0.07	0.03

TABLE 77. Equilibrium prices ($\sigma = 0.325$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9373	0.9009	0.8706	0.8462	0.8289	0.8198
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.65	0.75	1.13	1.74	2.66	3.95
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.00	0.01

TABLE 78. Equilibrium Consumer Surplus ($\sigma = 0.325$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1100.0411	1387.0383	2355.1872	3942.3220	5944.3470	7946.7357
RS sd	10.6027	29.3051	54.9968	73.9802	83.5290	79.3231
Perc. Diff. mean	104.24	127.96	176.92	195.61	177.96	139.13
Perc. Diff. sd	1.97	4.82	6.47	5.55	3.91	2.39

TABLE 79. Herfindahl-Hirschman Index ($\sigma = 0.35$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0421	0.0398	0.0365	0.0330	0.0292	0.0243
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	9.98	10.00	8.67	7.50	7.16	6.88
Perc. Diff. sd	0.34	0.26	0.17	0.11	0.07	0.03

TABLE 80. Equilibrium prices ($\sigma = 0.35$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9373	0.9009	0.8705	0.8461	0.8287	0.8197
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001
Perc. Diff. mean	0.65	0.75	1.11	1.73	2.64	3.94
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 81. Equilibrium Consumer Surplus ($\sigma = 0.35$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1089.2094	1383.7508	2347.1756	3929.1995	5881.8936	7919.3099
RS sd	11.2626	28.5171	55.2013	76.0760	83.6275	81.8351
Perc. Diff. mean	102.23	127.42	175.98	194.63	175.04	138.31
Perc. Diff. sd	2.09	4.69	6.49	5.70	3.91	2.46

TABLE 82. Herfindahl-Hirschman Index ($\sigma = 0.375$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0419	0.0396	0.0364	0.0330	0.0293	0.0243
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	9.23	9.42	8.39	7.41	7.18	6.85
Perc. Diff. sd	0.35	0.27	0.17	0.11	0.08	0.03

TABLE 83. Equilibrium prices ($\sigma = 0.375$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9372	0.9008	0.8703	0.8460	0.8286	0.8196
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001
Perc. Diff. mean	0.64	0.74	1.09	1.72	2.63	3.93
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 84. Equilibrium Consumer Surplus ($\sigma = 0.375$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1089.7285	1374.1183	2344.4928	3898.9942	5856.1474	7896.8267
RS sd	10.7198	28.5479	58.1253	78.3522	87.5598	85.0419
Perc. Diff. mean	102.33	125.84	175.66	192.37	173.84	137.63
Perc. Diff. sd	1.99	4.69	6.83	5.88	4.09	2.56

TABLE 85. Herfindahl-Hirschman Index ($\sigma = 0.4$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0418	0.0395	0.0363	0.0330	0.0292	0.0243
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	9.01	9.09	8.16	7.38	7.15	6.84
Perc. Diff. sd	0.32	0.25	0.16	0.10	0.08	0.04

TABLE 86. Equilibrium prices ($\sigma = 0.4$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9372	0.9007	0.8702	0.8459	0.8285	0.8195
RS sd	0.0001	0.0001	0.0001	0.0000	0.0001	0.0001
Perc. Diff. mean	0.64	0.74	1.08	1.70	2.61	3.92
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 87. Equilibrium Consumer Surplus ($\sigma = 0.4$)

6.2.4. *Reporting noise II: Likert scale.*

Partitioning the range of the ratings $\tilde{u}_{ij}$ into ℓ equally sized intervals is an alternative way to degrade information. We present the results for ℓ ranging from 2 to 10.

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	921.8214	1250.2694	1535.2794	1906.8560	2492.8100	3515.8061
RS sd	11.9993	34.5765	54.2922	65.8041	80.4124	69.1533
Perc. Diff. mean	71.15	105.48	80.52	42.99	16.56	5.80
Perc. Diff. sd	2.23	5.68	6.38	4.93	3.76	2.08

TABLE 88. Herfindahl-Hirschman Index ($\ell = 2$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0386	0.0373	0.0342	0.0306	0.0268	0.0224
RS sd	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000
Perc. Diff. mean	0.68	3.17	2.05	-0.29	-1.64	-1.18
Perc. Diff. sd	0.15	0.13	0.17	0.20	0.19	0.13

TABLE 89. Equilibrium prices ($\ell = 2$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9362	0.9003	0.8685	0.8402	0.8153	0.7961
RS sd	0.0002	0.0002	0.0002	0.0003	0.0004	0.0006
Perc. Diff. mean	0.53	0.69	0.88	1.02	0.98	0.94
Perc. Diff. sd	0.02	0.02	0.02	0.03	0.05	0.08

TABLE 90. Equilibrium Consumer Surplus ($\ell = 2$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	956.1759	1202.0704	1475.5953	1984.7847	2899.6806	4284.0050
RS sd	11.6182	25.4280	41.3912	59.7957	78.5323	114.2503
Perc. Diff. mean	77.53	97.56	73.50	48.83	35.59	28.91
Perc. Diff. sd	2.16	4.18	4.87	4.48	3.67	3.44

TABLE 91. Herfindahl-Hirschman Index ($\ell = 3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0404	0.0391	0.0355	0.0316	0.0278	0.0233
RS sd	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
Perc. Diff. mean	5.40	7.91	5.69	2.93	1.92	2.44
Perc. Diff. sd	0.30	0.25	0.19	0.16	0.14	0.13

TABLE 92. Equilibrium prices ($\ell = 3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9377	0.9015	0.8701	0.8427	0.8214	0.8093
RS sd	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004
Perc. Diff. mean	0.69	0.82	1.07	1.32	1.73	2.62
Perc. Diff. sd	0.01	0.01	0.01	0.02	0.03	0.05

TABLE 93. Equilibrium Consumer Surplus ($\ell = 3$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1000.7191	1152.3332	1434.9465	1996.3932	3160.3713	4760.5557
RS sd	11.9904	22.1674	35.6220	51.0193	90.3426	130.8348
Perc. Diff. mean	85.80	89.39	68.72	49.70	47.78	43.25
Perc. Diff. sd	2.23	3.64	4.19	3.83	4.22	3.94

TABLE 94. Herfindahl-Hirschman Index ($\ell = 4$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0417	0.0402	0.0365	0.0329	0.0290	0.0239
RS sd	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	8.89	11.18	8.82	7.10	6.11	5.42
Perc. Diff. sd	0.39	0.33	0.17	0.14	0.11	0.10

TABLE 95. Equilibrium prices ($\ell = 4$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9378	0.9014	0.8700	0.8436	0.8244	0.8146
RS sd	0.0001	0.0001	0.0001	0.0001	0.0002	0.0004
Perc. Diff. mean	0.70	0.81	1.06	1.42	2.10	3.29
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.03	0.05

TABLE 96. Equilibrium Consumer Surplus ($\ell = 4$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1019.7522	1148.1481	1491.6813	2307.4944	3683.7748	5425.7943
RS sd	12.8870	22.2243	40.3042	63.9959	92.4479	115.0040
Perc. Diff. mean	89.33	88.70	75.39	73.03	72.25	63.27
Perc. Diff. sd	2.39	3.65	4.74	4.80	4.32	3.46

TABLE 97. Herfindahl-Hirschman Index ($\ell = 5$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0428	0.0410	0.0371	0.0335	0.0296	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	11.57	13.36	10.70	9.02	8.55	6.99
Perc. Diff. sd	0.40	0.35	0.22	0.14	0.10	0.07

TABLE 98. Equilibrium prices ($\ell = 5$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9377	0.9013	0.8700	0.8444	0.8265	0.8178
RS sd	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
Perc. Diff. mean	0.69	0.80	1.05	1.53	2.36	3.70
Perc. Diff. sd	0.01	0.01	0.01	0.02	0.02	0.03

TABLE 99. Equilibrium Consumer Surplus ($\ell = 5$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	996.8698	1162.4068	1613.4179	2539.3879	3858.6504	5689.4196
RS sd	11.8780	17.5168	44.0658	68.0820	101.8932	130.3804
Perc. Diff. mean	85.09	91.04	89.70	90.42	80.43	71.21
Perc. Diff. sd	2.21	2.88	5.18	5.11	4.76	3.92

TABLE 100. Herfindahl-Hirschman Index ($\ell = 6$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0438	0.0413	0.0375	0.0338	0.0297	0.0244
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	14.21	13.98	11.64	10.14	8.97	7.35
Perc. Diff. sd	0.43	0.29	0.19	0.15	0.10	0.06

TABLE 101. Equilibrium prices ($\ell = 6$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9374	0.9012	0.8702	0.8450	0.8275	0.8187
RS sd	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Perc. Diff. mean	0.65	0.79	1.08	1.60	2.49	3.81
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.02	0.02

TABLE 102. Equilibrium Consumer Surplus ($\ell = 6$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1005.8933	1154.3671	1674.1855	2671.8836	4174.0061	6161.5585
RS sd	11.4077	19.5486	44.0565	78.3076	107.3967	121.0794
Perc. Diff. mean	86.76	89.72	96.85	100.35	95.18	85.41
Perc. Diff. sd	2.12	3.21	5.18	5.87	5.02	3.64

TABLE 103. Herfindahl-Hirschman Index ($\ell = 7$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0439	0.0416	0.0377	0.0338	0.0296	0.0244
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	14.55	15.03	12.39	10.10	8.58	7.34
Perc. Diff. sd	0.46	0.32	0.21	0.13	0.12	0.05

TABLE 104. Equilibrium prices ($\ell = 7$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9374	0.9010	0.8703	0.8455	0.8282	0.8193
RS sd	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Perc. Diff. mean	0.66	0.77	1.09	1.65	2.57	3.89
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 105. Equilibrium Consumer Surplus ($\ell = 7$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1010.7945	1177.8217	1686.2146	2793.4593	4435.4694	6597.1872
RS sd	10.8187	22.0336	42.9359	71.4394	101.3052	121.0265
Perc. Diff. mean	87.67	93.58	98.26	109.47	107.40	98.52
Perc. Diff. sd	2.01	3.62	5.05	5.36	4.74	3.64

TABLE 106. Herfindahl-Hirschman Index ($\ell = 8$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0440	0.0417	0.0378	0.0338	0.0296	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	14.70	15.11	12.62	9.86	8.28	7.26
Perc. Diff. sd	0.46	0.30	0.19	0.13	0.10	0.05

TABLE 107. Equilibrium prices ($\ell = 8$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9373	0.9010	0.8704	0.8459	0.8284	0.8195
RS sd	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Perc. Diff. mean	0.64	0.76	1.10	1.70	2.60	3.91
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 108. Equilibrium Consumer Surplus ($\ell = 8$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1012.6690	1196.9160	1765.2128	2907.5538	4684.8693	6879.1092
RS sd	12.6014	21.6844	47.0050	71.9118	104.7133	111.4706
Perc. Diff. mean	88.02	96.71	107.55	118.02	119.07	107.01
Perc. Diff. sd	2.34	3.56	5.53	5.39	4.90	3.35

TABLE 109. Herfindahl-Hirschman Index ($\ell = 9$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0443	0.0417	0.0378	0.0337	0.0295	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	15.69	15.16	12.64	9.70	8.25	7.17
Perc. Diff. sd	0.48	0.30	0.20	0.13	0.12	0.05

TABLE 110. Equilibrium prices ($\ell = 9$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9372	0.9010	0.8704	0.8461	0.8286	0.8197
RS sd	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Perc. Diff. mean	0.63	0.77	1.11	1.72	2.62	3.94
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 111. Equilibrium Consumer Surplus ($\ell = 9$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	538.5979	608.4532	850.4961	1333.6043	2138.5663	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1031.5170	1211.9948	1792.5608	3098.1953	4877.4719	7077.4606
RS sd	11.2818	23.1466	44.5658	78.9504	97.0882	100.6384
Perc. Diff. mean	91.52	99.19	110.77	132.32	128.07	112.97
Perc. Diff. sd	2.09	3.80	5.24	5.92	4.54	3.03

TABLE 112. Herfindahl-Hirschman Index ($\ell = 10$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.0383	0.0362	0.0335	0.0307	0.0273	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0441	0.0418	0.0377	0.0336	0.0295	0.0243
RS sd	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	15.07	15.57	12.41	9.34	8.23	7.12
Perc. Diff. sd	0.46	0.32	0.20	0.14	0.11	0.05

TABLE 113. Equilibrium prices ($\ell = 10$)

α	0	0.2	0.4	0.6	0.8	1
Benchmark mean	0.9313	0.8941	0.8609	0.8317	0.8074	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9373	0.9009	0.8705	0.8462	0.8287	0.8198
RS sd	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Perc. Diff. mean	0.64	0.76	1.12	1.74	2.64	3.95
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01	0.01

TABLE 114. Equilibrium Consumer Surplus ($\ell = 10$)

7. MARKET CONCENTRATION

7.5. *The feedback loop hypothesis*

As discussed in the main text, in our baseline analysis we abstract from the feedback loop effect extensively discussed in the computer science literature, which arises when the recommendations made by the RS generate the data that the system uses to make future recommendations. But what would be the impact of the feedback loop mechanism on our results?

To answer this question, we consider a different data-generating process. In the new process, the matrix $\tilde{\mathbf{R}}$ is filled in gradually, in a number of successive steps indexed by $\tau = 0, 1, 2, ...$ Initially, $\tilde{\mathbf{R}}_0$ is the empty matrix; that is, a matrix with all entries missing. At step $\tau = 0$, each consumer then inspects a randomly drawn product resulting in ratings $\tilde{u}_{ij}$ for the RS. The matrix $\tilde{\mathbf{R}}_1$ therefore has a density of $1/J$.

At each subsequent step $\tau \geq 1$, the RS uses $\tilde{\mathbf{R}}_\tau$ to estimate $\hat{\mathbf{R}}_\tau$ according to the estimation procedure described in section 2 and recommends to each consumer i the product with the highest rating:

$$j^\star(i, \tau) = \arg\max_j \{\hat{r}_{ij\tau}\}. \tag{1}$$

This product is then tried by the consumer. Thus, $\tilde{\mathbf{R}}_{\tau+1}$ is obtained by adding to $\tilde{\mathbf{R}}_\tau$ one and only one entry $\tilde{u}_{ij^\star(i,\tau)}$ for each of the I rows (i.e. for each consumer). If $\tilde{u}_{ij}$ is not missing in $\tilde{\mathbf{R}}_\tau$, its value is overwritten. All other elements of $\tilde{\mathbf{R}}_\tau$ are unchanged. Thus, $\tilde{\mathbf{R}}_{\tau+1}$ is basically equal to $\tilde{\mathbf{R}}_\tau$ plus one new entry per consumer, which could be either a previous value overwritten or a brand new one.

As this process proceeds, the density of $\tilde{\mathbf{R}}_\tau$ increases. The algorithm's learning phase ends when the matrix $\tilde{\mathbf{R}}_\tau$ achieves the same level of density d as in the baseline.[3]

We now investigate how the endogeneity of the data impacts our findings regarding market concentration, equilibrium prices, and consumer surplus. First, let us consider the case of subscription platforms, where the prices of individual products are set to zero. Table 115 shows that with endogenous data, RSs increase market concentration. The increase is smaller than in the baseline case when α is low, larger when α is high, but in any case the differences with the baseline case are modest.

α	0	0.25	0.5	0.75	1
Benchmark mean	538.5979	650.1194	1057.3305	1902.0176	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1006.8629	1558.5147	3235.7893	5838.3269	8484.1710
RS sd	14.2009	39.1034	85.6464	127.9772	91.9598
Perc. Diff. mean	86.94	139.73	206.03	206.95	155.30
Perc. Diff. sd	2.64	6.01	8.10	6.73	2.77

TABLE 115. HHI with endogenous data and zero prices for different values of α.

[3]To achieve the target density level of $d = 1.2\%$, we need to assume that the algorithm operates in "exploration mode" during a certain percentage of the periods; otherwise, the process would get stuck in a steady state with a lower density. During exploration mode, the algorithm randomly recommends a product. An exploration rate of 10% is sufficient for our purposes. However, by adjusting the probability of exploration mode, we can create data that lie between purely exogenous and endogenous.

The analysis of the superstar effect and the reverse long-tail effect show a similar pattern: the effects of the RS identified in the baseline case are amplified when α is low, reduced when it is high.

α	0	0.25	0.5	0.75	1
Benchmark mean	0.0325	0.0263	0.0189	0.0103	0.0029
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0247	0.0240	0.0180	0.0088	0.0013
RS sd	0.0008	0.0004	0.0002	0.0001	0.0000
Perc. Diff. mean	-23.93	-8.99	-4.63	-14.21	-53.25
Perc. Diff. sd	2.62	1.70	0.84	0.70	0.94

TABLE 116. Market share of peripheral products with endogenous data for different values of α .

α	0	0.25	0.5	0.75	1
Benchmark mean	0.0565	0.1291	0.2302	0.3650	0.5312
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0776	0.2987	0.5379	0.7476	0.9177
RS sd	0.0079	0.0128	0.0109	0.0115	0.0058
Perc. Diff. mean	37.28	131.32	133.72	104.82	72.75
Perc. Diff. sd	13.98	9.94	4.73	3.16	1.09

TABLE 117. Market share of central product with endogenous data for different values of α .

As for the equilibrium prices, Table 118, confirms that they increase with the RS. Data endogeneity slightly reduces the price increase when α is low. The price increase is identical to the baseline case when α is high.

α	0	0.25	0.5	0.75	1
Benchmark mean	0.0383	0.0356	0.0322	0.0283	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0413	0.0375	0.0341	0.0303	0.0243
RS sd	0.0002	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	7.73	5.57	6.09	7.08	7.10
Perc. Diff. sd	0.45	0.21	0.10	0.10	0.08

TABLE 118. Equilibrium prices with endogenous data, for different values of α.

The effect on the prices of individual products are also similar to that reported in the paper for the case of random data, as can be seen by comparing Figure 1 with Figure 5 in the main text.

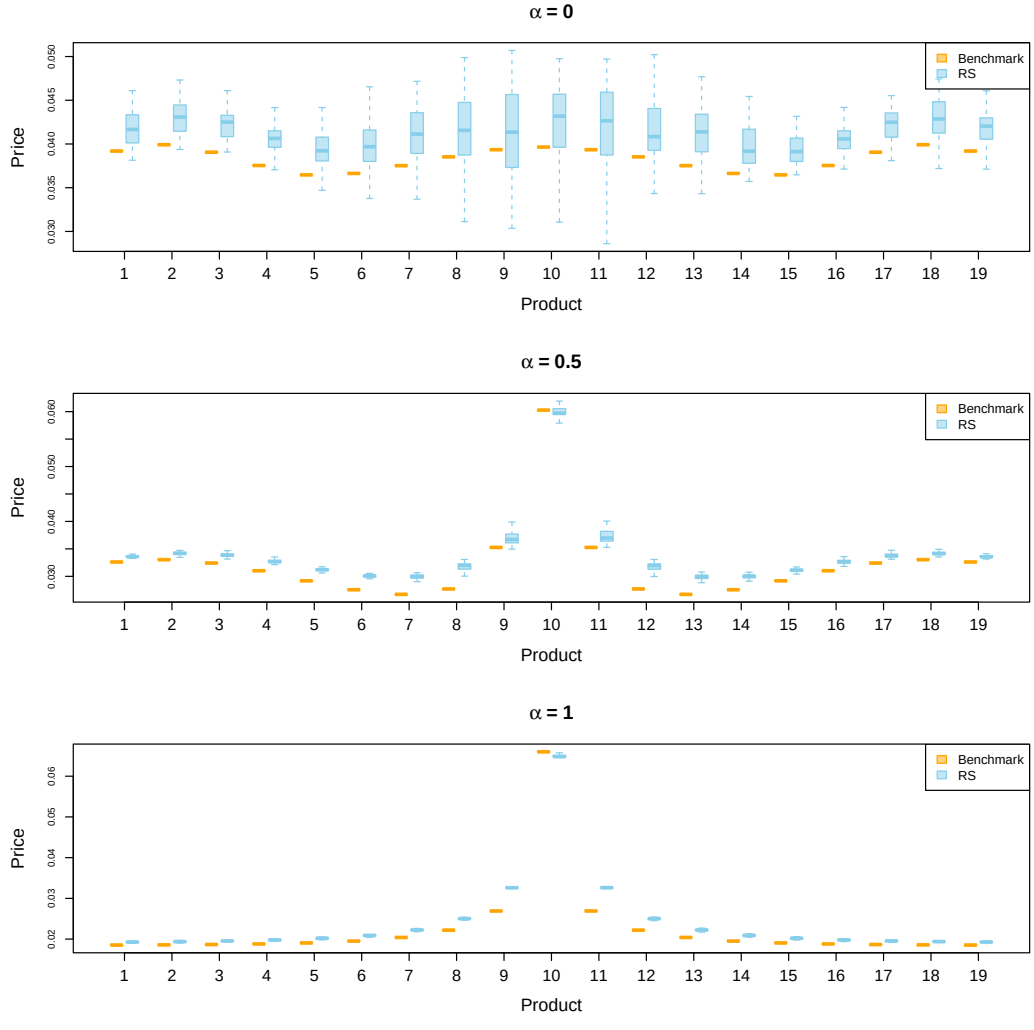


FIGURE 1.—The prices of the different products ranked from 1 to 19 according to their location in the space of products: products 1 and 19 are peripheral, product 10 is central. The figure shows median, inter-quartile range (IQR), and the values obtained by subtracting 1.5 times the IQR range from the first quartile (Q1), and obtained by adding 1.5 times the IQR to the third quartile (Q3). That is the highest and lowest value excluding outliers.

Table 119 illustrates the impact of the RS on consumer surplus with endogenous data. Even though prices increase less than in the benchmark, consumer surplus is slightly lower (except for $\alpha = 0.25$). This is because the user/item pairs that the algorithm observes with endogenous data are somewhat correlated, which reduces the quality of the information available to the algorithm. This means that the precision of the estimates and the quality of the recommendations decrease compared to the case of completely random data.

α	0	0.25	0.5	0.75	1
Benchmark mean	0.9313	0.8855	0.8457	0.8130	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9358	0.8902	0.8551	0.8310	0.8196
RS sd	0.0001	0.0001	0.0001	0.0002	0.0002
Perc. Diff. mean	0.49	0.54	1.10	2.21	3.93
Perc. Diff. sd	0.01	0.01	0.02	0.03	0.03

TABLE 119. Consumer surplus net of search costs with equilibrium prices and endogenous data, for different values of α.

7.6. *The uniformity effect*

A noteworthy difference of endogeneity of the data is that the uniformity effect discussed in section 7.6 of the main text applies not only to consumers but also to products, as illustrates in Figure 2.

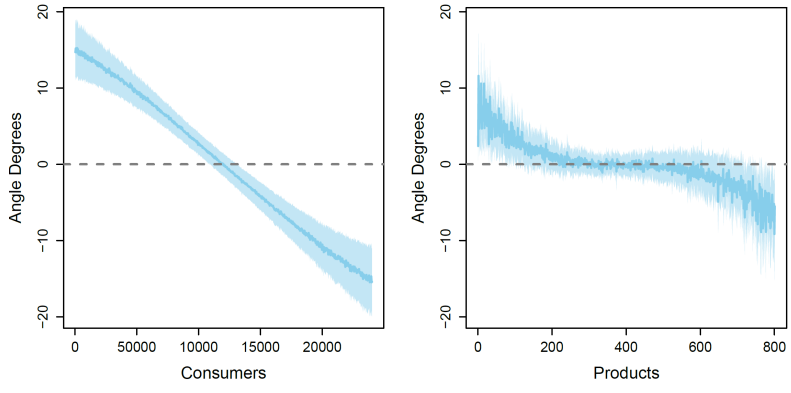
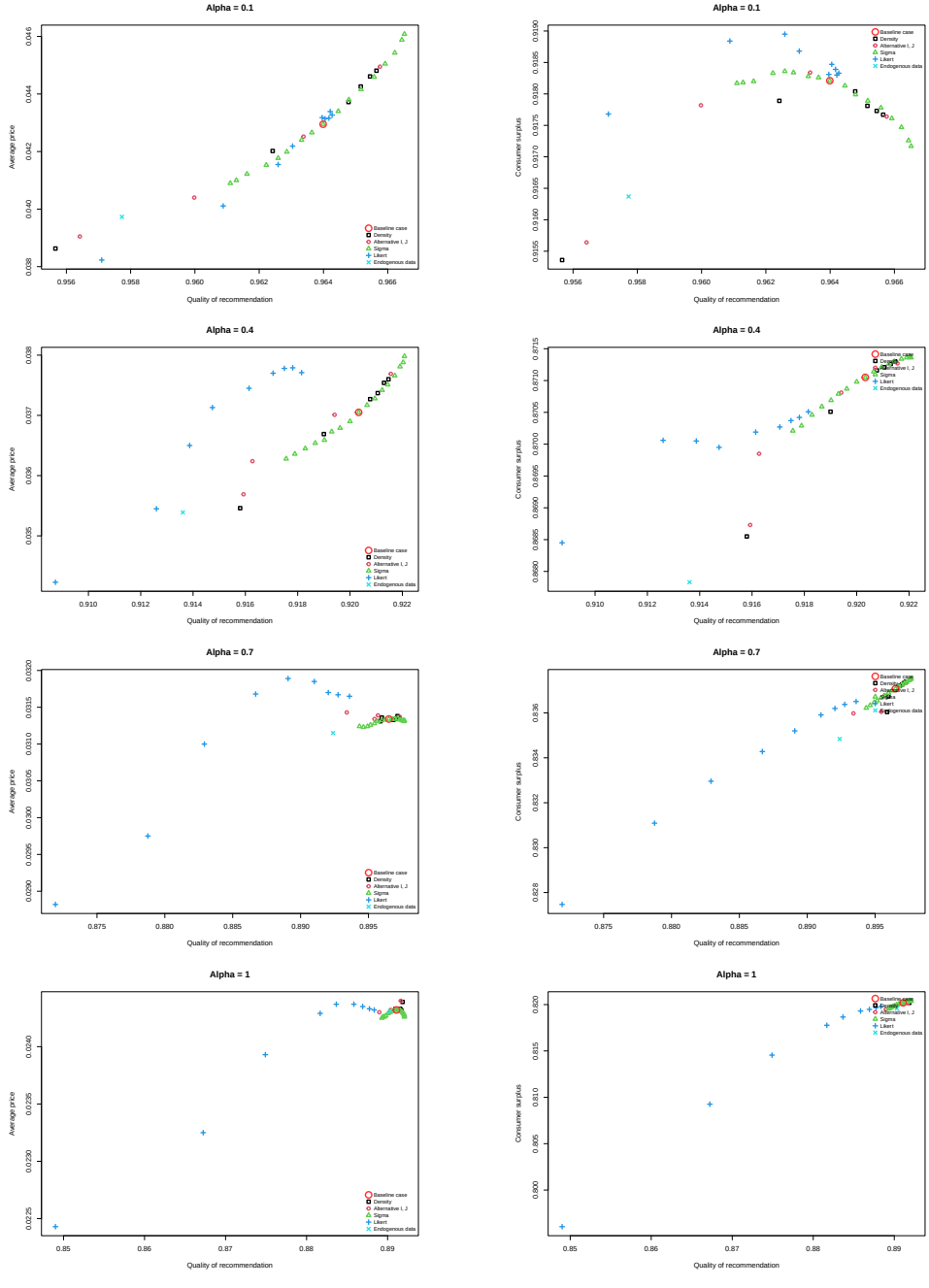


FIGURE 2.—Estimation biases (with $\alpha = 0$) with endogenous data: consumers (left) and products (right).

9. IMPACT OF INFORMATION ON PRICES AND SURPLUS

In order to gain further insights into the impact of information on prices, we conducted additional analyses by varying the parameter governing product differentiation levels and we present our result in Figure 3. These additional analyses confirm the significant role that information quality plays in price determination. The analysis consistently reaffirms that having more information leads to higher prices. The relationship between information and consumer



(a) The effect of information on prices

(b) The effect of information on Consumer Surplus

FIGURE 3.—Effect of more information

surplus continues to exhibit an inverted-U pattern, even when the value of α is positive but not excessively high. However, this pattern vanishes for non-small values of α.

10. ADDITIONAL ROBUSTNESS

10.1. *Estimating the number of latent factors*

In this exercise we let the algorithm autonomously estimate the number of latent factors H, which keeping the true number of factors equal to 2. The choice is made by means of a cross-validation procedure (see section A2 for a detailed description of the procedure). In this case, holding the true number of latent factors equal to $H = 2$, we let the algorithm choose the estimate $\hat{H}$ from a grid $1, 2, 3, 4, 5$. The cross-validation routine almost systematically identifies the correct number of factors, $H = 2$. In less than 5% of the cases, the algorithm chooses $\hat{H} = 3$ instead (see table 1 in section A2 for detail). In light of this, it is unsurprising that our findings remain mostly unchanged even when the algorithm independently determines the number of latent factors. The tables below substantiate this assertion for several selected exercises.

α	0	0.25	0.5	0.75	1
Benchmark mean	538.5979	650.1194	1057.3305	1902.0176	3323.1593
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	1064.7200	1583.2785	3304.5746	5774.7341	8406.9044
RS sd	11.9891	36.6897	58.0629	60.9698	57.2163
Perc. Diff. mean	97.68	143.54	212.54	203.61	152.98
Perc. Diff. sd	2.23	5.64	5.49	3.21	1.72

TABLE 120. The HHI when the number of latent factors is determined by cross-validation.

α	0	0.25	0.5	0.75	1
Full information	0.9957	0.9510	0.9199	0.9020	0.8963
Random Choice CS	0.7969	0.7563	0.7158	0.6753	0.6347
Benchmark mean	0.9694	0.9225	0.8844	0.8569	0.8411
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9834	0.9396	0.9102	0.8943	0.8911
RS sd	0.0001	0.0001	0.0001	0.0001	0.0001
Perc. Diff. mean	1.44	1.85	2.91	4.37	5.94
Perc. Diff. sd	0.01	0.01	0.01	0.01	0.01

TABLE 121. Consumer surplus at zero prices, and net of search costs, when the number of factors is determined with cross-validation.

α	0	0.25	0.5	0.75	1
Benchmark mean	0.0383	0.0356	0.0322	0.0283	0.0227
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.0442	0.0402	0.0351	0.0304	0.0243
RS sd	0.0002	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	15.28	13.10	9.15	7.41	7.12
Perc. Diff. sd	0.42	0.27	0.15	0.09	0.03

TABLE 122. Equilibrium prices when the number of latent factors is determined by cross-validation.

α	0	0.25	0.5	0.75	1
Full information CS	0.9957	0.9510	0.9199	0.9020	0.8963
Benchmark mean	0.9313	0.8855	0.8457	0.8130	0.7886
Benchmark sd	0.0000	0.0000	0.0000	0.0000	0.0000
RS mean	0.9371	0.8930	0.8581	0.8329	0.8202
RS sd	0.0001	0.0001	0.0000	0.0000	0.0000
Perc. Diff. mean	0.62	0.85	1.46	2.45	4.00
Perc. Diff. sd	0.01	0.01	0.00	0.00	0.00

TABLE 123. Consumer surplus when the number of latent factors is determined by cross-validation.