STADL Up! The Spatio-Temporal Autoregressive Distributed Lag Model for TSCS Data Analysis*

Scott J. Cook† Jude C. Hays‡ Robert J. Franzese, Jr.§

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Abstract

Time-series cross-section (TSCS) data are prevalent in political science, yet many distinct challenges presented by TSCS data remain under-addressed. We focus on how dependence in *both* space and time complicates estimating *either* spatial or temporal dependence, dynamics, and effects. Little is known about how modeling one of temporal or cross-sectional dependence well while neglecting the other affects results in TSCS analysis. We demonstrate analytically and through simulations how misspecification of either temporal or spatial dependence inflates estimates of the other dimension's dependence and thereby induces biased estimates and tests of other covariate effects. We therefore recommend the spatiotemporal autoregressive distributed lag (STADL) model with distributed lags in both space and time as an effective general starting-point for TSCS model-specification. We illustrate with two example reanalyses, and provide R code to facilitate researchers' implementation—from automation of common spatial-weights matrices (**W**) through estimated spatiotemporal effects/response calculations—for their own TSCS analyses.

Keywords: time-series cross-section (TSCS) or panel data, spatial dependence, temporal dependence, spatiotemporal dependence, autoregressive distributed lag models, spatial lag, lagged dependent variable, model specification, model selection

*Corresponding author: Scott J. Cook.

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†Associate Professor of Political Science, Department of Political Science, Texas A&M University, College Station, TX 77843. Email: sjcook@tamu.edu

‡Professor of Political Science, Department of Political Science, University of Pittsburgh, Pittsburgh, PA 15260. Email: jch61@pitt.edu

§Professor of Political Science, Department of Political Science, University of Michigan, Ann Arbor, MI 48109. Email: franzese@umich.edu

Introduction

Stimson (1985) introduced political science to the promise and peril of 'regression in space and time,' heralding a boom in research utilizing space-time data. In the 35+ years since, panel and time-series cross-section (TSCS) data have come to dominate quantitative empirical analyses in political science. Figure 1 illustrates with yearly counts of keyword text-identified TSCS articles appearing in *American Political Science Review*, *American Journal of Political Science*, and *Journal of Politics* from 1980 to 2019.[1] In recent years, 2012-2019, at least 201 articles, nearly 1 of every 8, contained TSCS data-analysis, yet very few of these analyses of data in 'space *and* time' seem to meaningfully consider *both* temporal and spatial dependence.

Our manual review of these 201 TSCS articles found only 94 model temporal dependence directly, via the inclusion of time-lags;[2] only about 23 model spatial dependence directly, using spatial lags;[3] and merely 12, less than 6%, model *both* temporal and spatial dependence directly. The dearth of TSCS studies directly addressing space *and* time is especially problematic because, as we will demonstrate, proper accounting of both is crucial for accurate estimation of, and valid inference regarding, spatiotemporal dynamics and covariate effects as well.

Applied researchers are not alone in neglecting to address spatial and temporal dependence jointly; many of the unique statistical challenges of TSCS data-analysis remain un- or under-addressed in methodological research too. In particular, inadequate attention has been paid to the methodological challenges presented in modeling *spatial-temporal* dependence in TSCS data. Current understandings of the problems arising from neglecting temporal or spatial/cross-unit dependence derive from considerations of one-way time-serial or cross-sectional data or stylized two-way TSCS data, with dependence assumed in only one dimension. As a result, researchers have largely borrowed strategies from time-serial or spatial-statistical methods designed for uni-

1. Of the 7336 total articles, we counted those containing keyword roots *time-series cross-section*, *panel-data*, or *TSCS*. The Appendix details these and further textual analyses (Cook, Hays, and Franzese 2022).

2. Of the rest, 75 use time-indicator, time-trend, and/or differencing strategies, 16 use some other strategy (e.g., time-period random-effects), with some overlap, and 16 seem to employ no address of temporal correlation at all.

3. Of the rest, 118 use only some unit fixed-effect strategy, 6 use some spatial random-effect strategy, 23 apply clustered or panel-corrected standard-error adjustment, leaving 31 with no apparent address of spatial association.

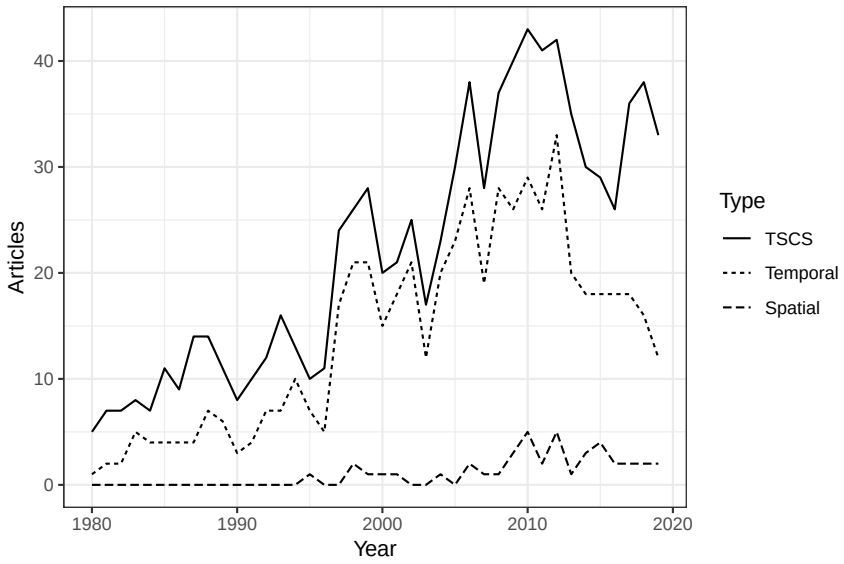


Figure 1: Count of articles using TSCS data in the 'Top-3' general PS journals, 1980-2019

dimensional data, with little consideration to two-dimensional *spatial-temporal* dependence, and its implications for diagnostics, specification, estimation, and inference.[4]

We show that when both spatial and temporal dependence are present, as in most real-world TSCS data, a more complex set of relationships manifests because temporal and spatial dependence are *necessarily* related, and, therefore, cannot generally be safely considered separately. Consequently, one cannot simply import strategies from time-series or cross-sectional analysis, but instead must directly confront the time-series-cross-sectional nature of the data, i.e., address *both* time and space. Failing to do so will bias estimates of *all* dependence parameters, which in turn will induce biases also in estimates of the other covariates' coefficients and their dynamic and total effects; and such mismodeled spatiotemporal dependence thereby also compromises standard diagnostic tests used to guide model specification. To list briefly our conclusions, as demonstrated analytically and in simulation below: inadequate address of spatial *or* temporal dependence (1) biases the estimated coefficients on both temporal *and* spatial lags, with the better-specified process

4. Some do briefly mention issues of two-dimensional dependence (e.g., Beck and Katz 1995, Wilson and Butler 2007, Franzese and Hays 2007, Beck and Katz 2011), but previous analyses generally consider only one-way models that address time-serial *or* cross-sectional dependence in data that assume away the other dimension of dependence or that assume one dimension can be adequately addressed orthogonally to the emphasized other dimension, which we show does not generally hold, and none explore the issues emphasized here.

This paper emphasizes proper simultaneous specification of temporal *and* spatial dependence, i.e., on the *dimension*, space and/or time, of (inter)dependence in TSCS data. Cook, Hays, and Franzese (2020) instead emphasizes proper specification of the *source* of (primarily) spatial interdependence, i.e., in y, $\mathbf{X}$, and/or ε.

overestimated and the less-well-specified underestimated; which (2) induces omitted-variable bias in other covariates' coefficient-estimates; and these biased dependence parameters and covariate coefficients combine to bias (3) the estimated spatiotemporal effects and (4) the diagnostic and specification tests derived from them. In sum, therefore, careful specification of *both* temporal *and* spatial dependence is essential for accurate estimates and valid inferences in TSCS data-analysis.

Furthermore, far from being a purely methodological exercise, crucial political-science substance is at stake in thusly modeling well *both* the temporal and spatial processes inherent in TSCS data. Consider, for instance, the well-known 'development and democracy' (Lipset 1959) and 'democratic dominoes' (Starr 1991) propositions. We know that more-developed political-economies are more likely to become, and far more likely to *remain*, democracies (Przeworski et al. 2000; Robinson 2006), indicating temporal dependence. We also know that democracy clusters spatially: "...the probability that a randomly chosen nation would be a democracy is about 0.75 if a majority of its neighbors are democracies, but only 0.14 if a majority of its neighbors are non-democracies" (Gleditsch and Ward 2006), suggesting spatial dependence. However, simply knowing outcomes persist in time or cluster in space is insufficient for model specification, as one must determine the *source* of the observed spatial or temporal dependence. Spatiotemporal patterns may arise from spatiotemporal dependence in the outcome itself, spatial spillovers or temporal 'spillforwards' in the observed covariates, spatiotemporal dependence in the unobserved/unmodeled residual, or any combination thereof (see Cook, Hays, and Franzese 2020). These different sources correspond to very different theories of our phenomena of interest, motivate distinct empirical models, and imply substantively importantly different *effects*. Consider again our development-and-democracy example; temporal dependence may be present because: (1) democracies persist because accumulating experience with democracy reinforces its institutionalization (i.e., autodependence in the outcome); (2) economic development causes democracy contemporaneously and economic development persists or (3) past economic development produces present democracy (i.e., spill-forwards in covariates); and/or (4) some unobserved/unmodeled covariate of democracy, culture perhaps, persists or has a persistent effect on democracy (i.e., serial dependence in the residuals). Analogously, the observed spatial association or clustering

of democracy may arise because: (1) economic development causes democracy and development clusters spatially (i.e., spatial clustering in observed covariates); (2) developed or underdeveloped neighbors spur democracy at home (i.e., spillovers in covariates); (3) clustering through unobserved/unmodeled external or foreign factors (i.e., clustered unobservables or spatially correlated errors);[5] and/or (4) foreign democracy directly influences domestic democracy, as argued in the democratic dominoes literature (i.e., contagion or interdependence).

Empirically distinguishing between these theoretically competing sources of spatial or temporal dependence is difficult, even confining attention to a single dimension, time *or* space, because spatial or temporal lags of one type–e.g., spatial lags of the outcome–have power against alternative spatial processes–e.g., spatial spillovers in covariates.[6] The empirical distinction is further complicated in TSCS data, as dependence processes modeled in one dimension, say: spatial, will have power against dependence processes in the other dimension, temporal, and *vice versa*. For example, if democracies are both serially autodependent (persist in time) and spatially interdependent (contagious) for the reasons given above, then any omitted spatial lag of the outcome would also be serially autodependent, and any omitted temporal lag would also be spatially interdependent. As a result, the included or better-modeled one of the dependence processes would have power against the other omitted or poorer-modeled process, and so suffer omitted-variable bias. This is why researchers must jointly model the spatiotemporal process: when both spatial and temporal dependence are present, obtaining accurate estimates of one dimension of dependence while neglecting the other is impossible.

Accurately distinguishing and estimating *both* spatial and temporal dependence is also essential for obtaining good estimates and tests of 'the effect of x on y', even if researchers have little interest in theoretically understanding the spatiotemporal process itself. First, since the spatiotemporal processes of the outcome y and of the included x's are generally related, any failure to sufficiently

5. Conceptually, spatial dependence in unobservables or errors can also arise from clustering, spillovers, and/or contagion, but the specific modality of unobserved dynamics in unobserved factors is difficult to discern empirically.

6. The order of the dependencies, i.e., the number of temporal or spatial lags, is also important but not a focus here (nor in (Cook, Hays, and Franzese 2020)). Time periodization of most TSCS data-analyses in political science is annual, and at that coarse temporal granularity first-order time-lags seem to suffice in most applications. Multiple spatial-lag models bring greater complications, also not discussed here (see Hays, Kachi, and Franzese 2010).

model spatiotemporal autodependence will bias covariate coefficient-estimates on which tests of causal-effect existence rely. Second, in the presence of spatiotemporal lags of outcomes and/or covariates, the coefficients on x alone are not the sought 'effect of x on y'. As Franzese and Hays (2007, 2008a, 2008b) and Cook, Hays, and Franzese (2020) show, different forms of spatiotemporal dependence imply substantively importantly different *effects*, meaning how outcomes, $\mathbf{y}$, respond across units over time to hypothetical or counterfactual 'shocks' in covariates, $d\mathbf{x}$.[7] Temporal and/or spatial dependence in outcomes $\mathbf{y}$ are autoregressive processes, which imply geometrically fading dynamics and long-run steady-state multipliers. For example, a democratization event in one country at some time propagates forward in time infinitely, fading geometrically, and reverberates around through neighboring countries, and then neighbors of neighbors, and neighbors of neighbors' neighbors, and so on infinitely, again fading geometrically. This is the distinct from the effects of $\mathbf{x}$ on $\mathbf{y}$ if the spatiotemporal dependence arises instead from spatial spillovers or temporal 'spill-forwards' in $\mathbf{x}$, where shocks only affect directly adjacent neighbors or periods, or spatiotemporal autoregression in the unobserved or unmodeled errors, where shocks in $\mathbf{x}$ only affect the specific unit-time shocked. Thus, good estimates of how development affects democracy, to continue our example, will require proper specification of *both* spatial and temporal dependencies.

Our suggested Spatio-Temporal Autoregressive Distributed Lag (STADL) model, which follows on and builds from Elhorst (2001, 2014), spans these dependence source and dimension possibilities—i.e., the STADL nests within it most common spatial, temporal, and spatiotemporal specifications—enabling proper address of *both* spatial and temporal dependence and therefore valid statistical tests and good substantive estimates of spatiotemporal dynamic effects, making the STADL an effective starting point for researchers' TSCS data-analyses.

Spatial, Temporal, and Spatiotemporal Dependence

Spatial and temporal dependence have received considerable attention elsewhere, *separately*, including by political scientists (e.g., Box-Steffensmeier et al. 2014, Franzese and Hays 2007,

7. As in Franzese (2020), we define *effects* $\equiv \frac{d\mathbf{y}}{d\mathbf{x}}$, and not just as *impulses* $\equiv \frac{\partial y}{\partial x} = \beta_x$.

2008a), so readers likely have some familiarity with both the statistical importance and the practical challenges of accounting for dependence in political-science TSCS data. Accordingly, this section is brief in reviewing these conventional *separate* understandings of spatial and temporal dependence, focusing primarily on identifying and modeling the source(s) of this dependence. We then demonstrate that spatial and temporal dependencies are *necessarily* intertwined and therefore spatiotemporal dependence is best considered jointly simultaneously. The next section offers the STADL as a practical and effective strategy for doing so.

Spatial Dependence

Cross-sectional or spatial dependence—meaning 'nearby' units have more- or less-similar realizations than expected by chance alone—will manifest whenever multiple units are observed in non-random samples. For instance, with nearby defined geographically, mappings of almost any variable, e.g. the level of democracy, will exhibit geospatial clustering of so-called 'hotspots' or 'coldspots'. As previously noted, however, such spatial dependence can arise by several subtly but substantively importantly distinct reasons: common traits among spatially-proximate units, exogenous spillovers in covariates across units, interdependence/contagion in outcomes between units; and/or due to clustering, spillovers, or interdependence in unobservables.[8] Whether by clustering, spillovers, or contagion, we can expect spatial (cross-unit) dependence to manifest across the entire substantive range of political science—intergovernmental diffusion of policies and institutions among nations or subnational jurisdictions (e.g., Graham, Shipan, and Volden 2013); international diffusion of democracy (e.g., Starr 1991); parties', representatives', and citizens' votes and other behaviors in legislatures and elections (e.g., Kirkland 2011; Tam Cho and Fowler 2010; Baybeck and Huckfeldt 2002); interdependence in globalization studies (e.g., Simmons and Elkins 2004); contextual/neighborhood effects in microbehavioral research (e.g., Huckfeldt and Sprague

8. We sidestep here issues of spatial-unit aggregation, i.e. the *MAUP: Modifiable Areal Unit Problem* (Fotheringham and Wong 1991), which are similar to, but more complex than, the more-familiar issue of temporal granularity/aggregation affecting time-serial dependence (Stram and Wei 1986; Freeman 1989). We also do not emphasize crucial specification issues regarding $\mathbf{W}$, the matrix of relative connectivity or distance between the units, i.e. the network, by which spatial association manifests (see, e.g., Franzese and Hays 2008b; Neumayer and Plümper 2016).

1987); wars, coups, riots, civil wars, revolutions, terrorism (e.g., Buhaug and Gleditsch 2008)—and many more. Indeed, interdependence across units is a defining characteristic of the *social* sciences, where its study is prominent also in geography and environmental sciences; in regional, urban, and real-estate economics; in public health and epidemiology; in education, psychology, sociology, and social-psychology; and beyond.

Spatial dependence, in short, is everywhere, empirically and theoretically. Applied researchers almost always, perhaps unknowingly, account for some clustering in regression models simply through the inclusion of exogenous covariates, which are also often spatially clustered. We call this *clustering in observed covariates* and note that its corresponding model is nonspatial (NON): $y_{it}=\mathbf{x}_{it}\boldsymbol{\beta}+\varepsilon_{it}$.[9] Insofar as these spatially clustered $\mathbf{x}$ are omitted or are inadequate to account all of the spatial dependence in the dependent variable, the remainder will manifest as spatially correlated errors, as anything omitted from the systematic component is shunted to the residual component. As shown elsewhere (Franzese and Hays 2007, 2008a), left unaddressed, such spatial dependence risks inefficiency at best and typically bias as well.

Often, though, additional sources of spatial correlation—correlated unobservables, exogenous spillovers, and/or outcome interdependence—are also present. When other manifest sources are omitted, including spatially correlated $\mathbf{x}$ regressors not only fails to fully address spatial dependence, but can actually further compromise our understanding of the data-generating process. These included $\mathbf{x}$ have power against the unmodeled spatial processes, which biases their coefficient estimates following the familiar omitted-variable bias (OVB) formula and logic (Franzese and Hays 2007, 2008a). Accordingly, political scientists have increasingly sought to model these other spatial processes directly also, using the workhorse models of spatial econometrics—spatial-error model (SEM), spatially-lagged $\mathbf{x}$ model (SLX), and spatial-lag (of y) model (SAR)—each of which assumes and reflects a single additional source of cross-unit dependence—correlated unobservables, exogenous spillovers, and outcome interdependence, respectively—via an additional modeling device, the *spatial lag*, to bring 'neighboring' values of $\boldsymbol{\varepsilon}$, $\mathbf{x}$, or $\mathbf{y}$ into the model. A brief

9. We assume linear-additive separable mean and stochastic component here solely for ease of exposition.

summary of these models will help familiarize concepts and notation.

Each of these spatial models, and indeed any spatial analysis whatsoever, must begin with specifying the connectivity or spatial-*w*eights matrix, $\mathbf{W}$, a $N \times N$ matrix with elements w_{ij} reflecting the relative connection, tie, distance, or potential influence, from unit j to unit i. This pre-specification of $\mathbf{W}$ is primary to any spatial analysis (Neumayer and Plümper 2016), being essential for everything from preliminary descriptives and diagnostics through model specification and estimation to effects calculation. Any relational data (e.g., trade, alliance co-membership) can undergird $\mathbf{W}$, and theory and substance should always be paramount in this indispensable foundational step of spatial analysis. Absent strong theory, though, researchers often use geographic proximity since geography correlates with so many other potential bases for interconnection (e.g., economic interchange, cultural and linguistic similarities, and flows of people and information).[10]

Different specifications of $\mathbf{W}$ allow researchers to study alternative substantive/theoretical bases of cross-unit relations.[11] The researcher defines the relevant concept of space and metric of distance for her application—again, geographic distance or contiguity is convenient and powerful default, and will be ours here—and then usually *normalizes* this $\mathbf{W}$ in some way to ease interpretation, reduce dependence on scale factors, ensure the invertibility of the spatial multiplier, etc. The most-common *row normalization* divides each w_{ij} by the row-sum $\sum_j w_{ij}$, which makes spatial lags equal to weighted averages and thereby facilitates direct interpretation of lag coefficients.[12] With $\mathbf{W}$ specified and normalized, it then pre-multiplies a vector—$\boldsymbol{\varepsilon}$, $\mathbf{x}$, or $\mathbf{y}$—to produce so-called spatial lags—$\mathbf{W}\boldsymbol{\varepsilon}$, $\mathbf{Wx}$, or $\mathbf{Wy}$, which are weighted averages of neighbors' errors, covariates, or outcomes—for use in preliminary measures and tests of spatial correlation, in specification and estimation of spatial models, and in interpretation of spatial effects.

10. Given uncertainty over the relevant ties/network, a Bayesian Model-Averaging approach to *estimating* $\mathbf{W}$ simultaneously with a model of its effect seems promising (Juhl 2020).

11. While misspecified $\mathbf{W}$ necessarily reduces the accuracy and power of spatial-association tests/measurements and spatial-model estimates, research shows the consequences of errors in $\mathbf{W}$ are often less severe than feared (LeSage and Pace 2014) and clearly better than ignoring spatial dependence outright (Betz, Cook, and Hollenbach 2020).

12. Some other common normalizations include spectral or min-max, which have other convenient properties. Neumayer and Plümper (2016) further discuss $\mathbf{W}$ specification and normalization issues, the most important being that the choice of whether and how to normalize crucially affects the substantive interpretation of the lag variable and its coefficient.

Quickly reviewing the baseline spatial models: the spatial error model (SEM) assumes spatially autocorrelated residuals, which are orthogonal to the included regressors. Spatial-error processes result in non-spherical error variance-covariance matrices and consequently inefficient OLS estimates with incorrect standard errors, but coefficient estimates remain unbiased. In the democracy-development example, spatial error dependence may occur due to unmodeled country-specific determinants of democracy, e.g., cultural/historical legacies (Acemoglu et al. 2008), or from unmodeled spatially-correlated heterogeneity across countries in the effect of development on democracy. Formally, the SEM model is:[13]

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{u}, \text{ with } \mathbf{u} = \lambda \mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon} \tag{1}$$

with $\mathbf{W}$ the $N \times N$ connectivity matrix defined above and λ the strength of spatial autocorrelation propagated in this predetermined pattern, $\mathbf{W}$.

Next, cross-unit spillovers or externalities in exogenous observed factors (regressors, $\mathbf{x}$) can also produce spatial dependence in outcomes. In our democracy-development example, exogenous spillovers occur if economic development in a country influences, not only its own democracy, but that of neighboring countries as well, perhaps via development spurring the emergence of transnational advocacy networks as Keck and Sikkink (1998) propose. Alternatively, conflict or public health in neighboring countries, x_j, may influence democratic emergence or stability at home, y_i. The spatial-lag $\mathbf{x}$ or SLX model captures exogenous spillovers like these:

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta + \boldsymbol{\varepsilon}. \tag{2}$$

Here, the spatial lag of regressor, $\mathbf{W}\mathbf{x}$, introduces neighboring (as per $\mathbf{W}$) values of $x_{j \neq i}$ into the model for y_i.[14] With $\mathbf{x}$ exogenous, $\mathbf{W}\mathbf{x}$ is too, so SLX models can be estimated efficiently by OLS, with $\hat{\theta}$ giving the strength of these exogenous spatial spillovers. Halleck Vega and Elhorst (2015) and Wimpy, Whitten, and Williams (2021) offer further discussions of SLX.

Finally, where theory and/or substance indicate interdependence or contagion in outcomes, a

13. This SEM assumes spatial-autoregressive errors, though moving-average (spillover) or spatial-hierarchical (clustered) versions also exist. The distinctions are not easily discerned empirically, being unobserved processes in unobserved factors. Fortunately, the distinctions are also immaterial to *effects* as defined here.

14. To simplify exposition, we use a single covariate and lag; the generalization to multiple covariates and lags is straightforward. Wimpy, Whitten, and Williams (2021) discuss several advantages of this general SLX model.

process autoregressive in y like the increasingly widely-used SAR model is called for:

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{x}\beta + \boldsymbol{\varepsilon}. \tag{3}$$

This spatial-lag y model may be most familiar to readers, having quickly become the dominant model of applied spatial work. In the democracy-development example, Starr's 'Democratic Dominoes' notion implicates such spatial autoregression directly: democracy is contagious; neighboring democracies *cause* democracy at home. Mechanisms for causal contagion could be suasion, diplomacy, foreign policy, or demonstration effects: being surrounded by democracies could reveal much to domestic actors about the workings, requisites, benefits and costs of democracy (Elkink 2011).

The key substantive differences of spatial-autoregressive processes compared to the alternative sources are its aforementioned exponentially reverberating dynamic and steady-state effects. The key methodological difference is that the spatial-lag, $\mathbf{W}\mathbf{y}$, being other units' *outcomes*, is an endogenous regressor. Thus, consistent estimation of SAR models requires instrumental variables (spatial two-stage least-squares or generalized method-of-moments) or systems maximum-likelihood (spatial-ML). We suspect SAR's popularity among these single-source spatial models owes, first, to its substantive resonance in political science, where outcomes are often social and/or strategic behaviors wherein some units' outcomes/choices directly influence others' outcomes/choices. Second, the other single-source models imply that clustering or spillovers occur only in observed/modeled or only in unobserved/unmodeled components, which seems generally less plausible than that dependence would operate in both as in SAR.[15]

In any case, these single-source models can be combined in whatever pairs may be substantively/theoretically implicated.[16] If one expects spillovers in observed covariates (SLX) and in unobserved features (SEM), but not necessarily to the same extent or autoregressively as SAR implies, this SLX+SEM combination gives the so-called Spatial Durbin Error Model (SDEM):

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta + \mathbf{u}, \text{ with } \mathbf{u} = \lambda\mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon}. \tag{4}$$

15. SAR does impose equal, autoregressive processes in observed and unobserved components, though, which may seem restrictive; multiple-source models relax this restriction.

16. Even the three-source model is estimable, albeit with great fragility, being identified by functional-form differences across the lag-y, lag-x, and lag-ε processes (Elhorst 2014; Cook, Hays, and Franzese 2020).

These multi-source models are advantageous in that they allow researchers to simultaneously account for alternative spatial processes, here exogenous spatial spillovers and spatial error autocorrelation. This is significant because spatial-model specifications often have power against incorrect alternative spatial processes: SAR, SLX, or SEM lag-coefficients or tests will pick up unmodeled SLX, SEM, or SAR processes.[17] As a consequence, modeling one source of spatial dependence (e.g., SAR) while neglecting others (e.g., SEM) risks inaccurate estimates of the included dependence parameter. Researchers are therefore advised to condition on these potential alternative processes when performing diagnostic tests (Anselin et al. 1996) or specifying their empirical models (Cook, Hays, and Franzese 2020). Below we build on this, demonstrating that in TSCS data not only do different spatial models have power against alternative spatial processes, but alternative *temporal* processes as well. This motivates our suggested STADL model, which combines multiple dependence sources across both spatial and temporal dimensions.

Temporal Dependence

Many readers may be more familiar with the time-series analogs to the spatial processes/models just described, owing to discussions in Keele and Kelly (2006) and elsewhere, so we will be very brief here. As with space, temporal dependence or serial correlation may arise from four sources: y_t may correlate with y_{t-1} simply because exogenous covariates $\mathbf{x}$ correlate over time, because unobserved/unmodeled factors ε exhibit serial correlation, because past values of $\mathbf{x}_{t-s}$ have lagged effects on current outcomes y_t, and/or because past outcomes y_{t-s} themselves continue to shape current outcomes y_t, i.e. outcomes are persistent, exhibit inertia. Also as with space, these alternative sources correspond to distinct substantive/theoretical processes and model specifications.

Identical to the nonspatial model is the static model, $y_t{=}x_t\beta{+}\varepsilon_t$; here, democracy exhibits serial correlation simply because the exogenous-covariate development does. The SEM analog is the classic serially correlated errors (SCE) model, $y_t{=}x_t\beta{+}u_t$ with $u_t{=}\delta u_{t-1}{+}\varepsilon_t$,[18] which reflects per-

17. Rüttenauer (2019) and Cook, Hays, and Franzese (2020) explore the similarities and differences among these alternative specifications in the purely spatial (cross-sectional) context.

18. As before, we proceed with first-order, i.e. one-period, lags for expositional simplicity (see note 6).

sistence in unobserved/unmodeled factors, such as cultural-historical legacies, perhaps. The finite distributed-lag (FDL) model, $y_t{=}x_t\beta{+}x_{t-1}\gamma{+}\varepsilon_t$, corresponds to the SLX model; substantively this would reflect that past development directly affects present democracy, perhaps through long-term structural changes whose impact materializes later. Finally, in the temporal autoregressive outcome process, i.e. the lagged-dependent-variable (LDV) model, $y_t{=}\phi y_{t-1}{+}x_t\beta{+}\varepsilon_t$, past democracy directly influences present democracy, a persistent or inertial process, which here substantively may reflect democratic institutionalization wherein experience with democracy itself yields increasingly entrenched or consolidated democracy (Alexander 2001; Diamond 1994). Again in parallel with the spatial context, effects of x on y in the static or SCE model are static: $\frac{dy_t}{dx_t}{=}\beta$ and $\frac{dy_s}{dx_t}{=}0\ \forall\ s{\neq}t$; whereas effects are dynamic in the FDL and LDV models, decaying discretely and persisting only to the lag-length order in FDL models but persisting infinitely with exponential/geometric decay, implying long-run steady-state (LRSS) multipliers, $\frac{1}{1-\phi}$, and cumulative LRSS effects, $\frac{1}{1-\phi}\cdot dx\cdot\beta$, in the autoregressive LDV model.[19]

Spatiotemporal Dependence

With readers (re)familiarized with the base temporal and spatial dependence models/processes, we turn next to illustrating how these spatial and temporal dependencies are *necessarily* related. Start with the simple static/nonspatial linear-regression model, now indexed by unit i and time t:

$$y_{it} = \beta_0 + \beta_1 x_{it} + u_{it}, \tag{5}$$

except here assume that some residual dependence may result from omitted $y_{i,t-1}$, $y_{j,t}$, or both:

$$u_{it} = \phi_y y_{i,t-1} + \rho_y \sum_{n=1}^{N} w_{ij} y_{j\neq i,t} + \varepsilon_{it}, \text{ with } \varepsilon_{it} \sim N(0, \sigma^2), \tag{6}$$

19. Also analogously (see note 21), the question of the 'effect of x on y' in temporally dynamic contexts requires more precise statement of both the hypothetical/counterfactual, dx, and the effect, dy, refining to specify dx *when?*, in what period(s) is x 'shocked'?, and dy *when?*, in what period(s) do we want to know the response of y thereto?

where $\sum_{n=1}^{N} w_{ij} y_{j \neq i,t}$ is the scalar representation of spatial-lag $\mathbf{y}$ presented above in matrix form: $\mathbf{Wy}$. Furthermore, let x be stochastic, exogenous, and follow its own spatiotemporal process:

$$x_{it} = \phi_x x_{i,t-1} + \rho_x \sum_{n=1}^{N} w_{ij} x_{j \neq i,t} + e_{it}, \text{ with } e_{it} \sim N(0, \sigma^2). \tag{7}$$

Given all standard regression assumptions otherwise, we now walk through the relationship between spatial and temporal dependence (also depicted visually in Figures 2-5).

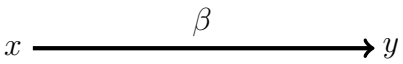


Figure 2: Static Relationship

First, restricting both ϕ_y=0 and ρ_y=0 produces *i.i.d.* residuals u_{it}, so the nonspatial, static equation (5) depicted in Figure 2 fully accurately models the relationship of x to y. Relaxing one restriction, say $\phi_y \neq 0$, but keeping the other, ρ_y=0, induces time-serial dependence in the residuals u, which biases $\hat{\beta}$ in the static model if $\phi_x \neq 0$. This situation, depicted in Figure 3, is textbook omitted-variable bias (OVB)—with $\text{Cov}(x_t, y_{t-1})$ increasing in ρ_x—and is easily remedied by including time-lagged y (LDV model) as commonly done. Similarly, freeing $\rho_y \neq 0$ while keeping ϕ_y=0 also threatens OVB in the static model. Again, OVB arises if x has dependence in the same dimension as y, here if $\rho_x \neq 0$ as depicted in Figure 4, so that $\text{Cov}(x_i, y_j) \neq 0$, and the simple remedy, increasingly common in applied work, adds spatial-lag y to form the spatially dynamic SAR model.

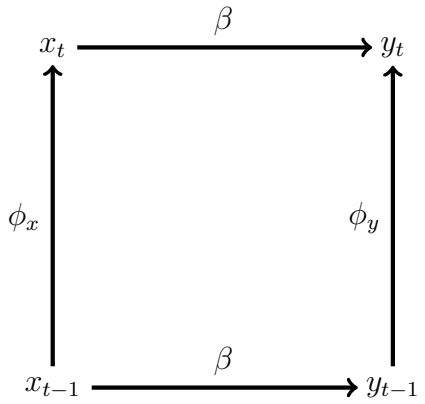


Figure 3: Time-serial Dependence

This is all familiar: with single-dimensional dependence, purely cross-sectional/spatial or time-serial modeling suffices. However, if *both* $\phi_y \neq 0$ *and* $\rho_y \neq 0$ as in Figure 5 so that *both* temporal and spatial dependence manifest, researchers must model dependence in *both* dimensions adequately.

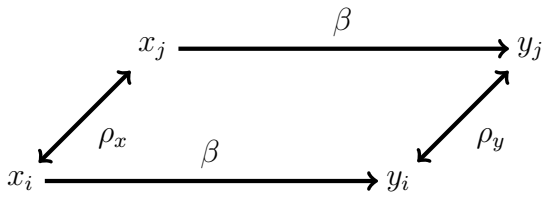


Figure 4: Cross-Sectional Dependence

Omitting/mismodeling spatial dynamics, e.g., will leave residual time-serial correlation because the omitted/mismodeled spatial-lag y_{jt} is serially correlated with $y_{j,t-1}$ which in turn exhibits that same omitted/mismodeled spatial relation to the included time-lag $y_{i,t-1}$. Symmetrically, failing to model temporal dynamics adequately will leave spatial autocorrelation, as the missed aspect of the past, $y_{i,t-1}$, has the same spatial relation to $y_{j,t-1}$ as does y_{it} to the included spatial-lag, y_{jt}.

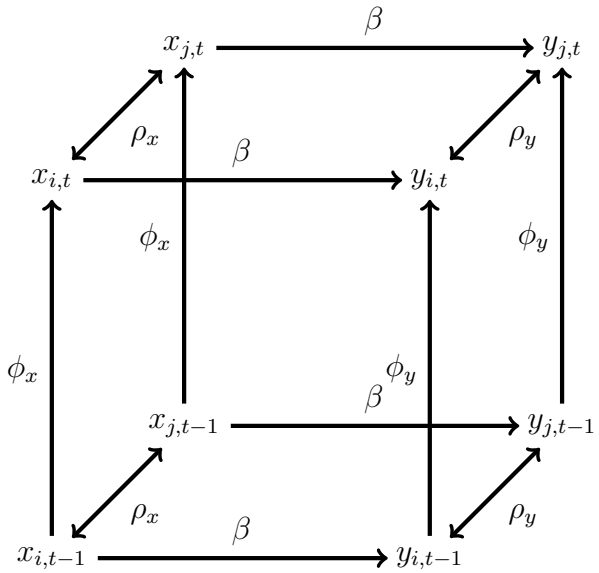


Figure 5: Space-Time Dependence

To see that spatiotemporal dependence causes bias (OVB) when only one of spatial or temporal dependence is modeled, consider the spatiotemporal-lag model: $\mathbf{y}_t{=}\beta\mathbf{x}_t{+}\rho\mathbf{W}\mathbf{y}_t{+}\phi\mathbf{y}_{t-1}{+}\boldsymbol{\varepsilon}_t$ (which is also equation (20)), as depicted in Figure 5. If the truth is (20), but one omits $\phi\mathbf{y}_{t-1}$ to estimate SAR or omits $\rho\mathbf{W}\mathbf{y}_t$ to estimate LDV, then OVB arises if $\rho\phi\text{Cov}(\mathbf{W}\mathbf{y}_t, \mathbf{y}_{t-1}){\neq}0$. This covariance is necessarily nonzero because spatial dependence implies $\mathbf{W}\mathbf{y}_\mathbf{t}{\longleftrightarrow}\mathbf{y}_\mathbf{t}$ and temporal dependence implies $\mathbf{y}_\mathbf{t-1}{\longrightarrow}\mathbf{y}_\mathbf{t}$, so $\mathbf{y}_\mathbf{t-1}{\longrightarrow}\mathbf{y}_\mathbf{t}{\longleftrightarrow}\mathbf{W}\mathbf{y}_\mathbf{t}$, meaning that $\text{Cov}(\mathbf{W}\mathbf{y}_t, \mathbf{y}_{t-1}){=}\text{Cov}(f(\mathbf{y}_t), \mathbf{y}_{t-1}){\neq}0$. The sign and magnitude of these OVB can be seen in Achen (2000)'s derivation of the biased $\hat{\phi}_y$

and $\hat{\beta}$ in an LDV model when additional, unmodeled dynamics ϕ_e remain in the residual:

$$\operatorname{plim} \hat{\phi}_y = \phi_y + \left[\frac{\phi_e \sigma^2}{(1 - \phi_e \phi_y) s^2} \right], \tag{8}$$

$$\operatorname{plim} \hat{\beta} = \left[1 - \frac{\phi_x g}{1 - \phi_x \phi_y} \right] \beta, \tag{9}$$

where $s^2 = \sigma^2_{y_{t-1},x}$ and $g = \operatorname{plim}(\hat{\phi}_y) - \phi_y$ (see also Keele and Kelly 2006). As Achen (2000) notes, any $\phi_e > 0$ inflates $\hat{\phi}_y$ and attenuates $\hat{\beta}$ estimates; and, as just proven, any unmodeled spatial dependence *necessarily* produces precisely these same conditions since $y_{i,t} = \phi_y y_{i,t-1} + x_{i,t}\beta + u_{i,t} \implies y_{j,t} = \phi_y y_{j,t-1} + x_{j,t}\beta + u_{j,t}$. Therefore, any $\rho_y \neq 0$ produces $\phi_e > 0$ and 'Achen's LDV-bias' manifests. By the usual OVB logic, it follows also that omission or underestimation of ρ_y induces primarily overestimation (inflation bias) of $\hat{\phi}_y$, being the coefficient on the included regressor most related to the omitted/mismeasured $\mathbf{Wy}$, and those two biases in turn induce partially compensatory bias of $\hat{\beta}_x$, with the resultant direction depending on whether the spatial or temporal dependence in x is stronger and resembles more-closely those dependencies in y.

These biases arise because, in TSCS analysis with temporal dependence modeled but spatial dependence excluded, for instance, the factor among the included that is most like the omitted "today's democracy abroad"—say German democracy today, $y_{j,t}$, as omitted explanator of French democracy today, $y_{i,t}$—is "yesterday's democracy at home", i.e. French democracy yesterday, *time*-lagged $y_{i,t-1}$. Intuitively, this is because insofar as, "Germany yesterday" relates to "France yesterday"—spatial dependence is present—and "Germany yesterday" relates to "Germany today"—temporal dependence is also present, so the omitted "Germany today" relates to the included "France yesterday". Of course, all of the analogous holds also in the other direction, regarding the omission or relatively inadequate address of temporal dependence.[20]

In sum, even if Stimson's (1985) 'inherent' temporal autocorrelation is accurately modeled, misspecification of the spatial dynamics sets off a chain of biases: the primary attenuation bias (or

20. In practice, given the typically greater strength of temporal than of spatial dependence, omitted spatial factors' relation to included temporal factors is usually by far the strongest of the OVB formula's partial correlations, meaning inadequate address of spatial dependence induces largest inflation biases on temporal-dependence parameters and secondary induced biases in other covariate coefficients. Conversely, included or better specified spatial dependence being typically considerably weaker than omitted or more-poorly specified temporal dependence, and the temporal persistence of other exogenous covariates being likewise stronger than their spatial association, the OVB biases of temporal-dependence misspecification tend to be more-evenly distributed across included parameters.

'zeroing' if omitted) of ρ_y, induces overestimation/inflation bias (OVB) of ϕ_y, which biases combine to induce misestimation of β_x, usually attenuation since temporal dependence is generally stronger than spatial. Naturally, these biased coefficient estimates mean any related causal-inference tests are biased too, as are estimates of the dynamic and total causal effects of x on y. Specifically regarding effect estimates: typically, the initial 'impulses' from x to y (β_x) are underestimated and the spatiotemporal dynamics misconstrued as 'too-persistent' if spatial dependence is the mismodeled process, and conversely initial impulses overestimated and spatiotemporal dynamics 'too-contagious' if temporal dependence is the mismodeled process. In either case, long-run steady-state effect estimates are biased also.

Given that inadequate address of spatiotemporal dependence will bias inferential tests and estimates of coefficients, dynamics, and steady-state effects, even researchers for whom spatiotemporal dynamics and dependencies are nuisance cannot neglect their careful attention. Furthermore, these biases induced by relative neglect of spatial or, less commonly, temporal dependence are of central substantive-theoretical importance as well. In our *development-and-democracy* terms, relative inadequacy in addressing spatial dependence—inadequate account in the model that, and by what process, democracy clusters—yields estimates that imply inaccurately greater temporal persistence of democracy, e.g. an overestimate of democratic-institutionalization and -consolidation effects. If democratic persistence derives from a temporally autoregressive process as such arguments imply, this overestimated temporal dependence will mean smaller immediate-impact estimates, i.e. smaller $\hat{\beta}_x$, with slower decay and so larger long-run-steady-state multipliers in other covariates' effects on democracy. Beyond these misestimated dynamic and steady-state effects, the biased $\hat{\beta}_x$ means that hypothesis tests about the effects of x on y will be biased too, likely increasing false negatives (failure to reject when should).[21]

Applied researchers also commonly deploy unit and/or period fixed-effects to 'account for' spatial or temporal dependence. Unit or period dummies or random effects do address particular

21. Notice also that, in spatiotemporally dynamic contexts, the usual statement of the causal estimand, 'the effect of x on y' is itself *underspecified*. For the question to be fully enunciated given spatiotemporal interdependence, we need to ask about 'the effect of x *when? and where?* on y *when? and where?*'.

forms of spatiotemporal dependence (Elhorst 2014), but often fail to adequately capture the spatiotemporal dependence typically present in TSCS data. Unit indicators absorb long-run, fixed or constant, spatial clustering in outcomes, plus any other time-invariant unmodeled unit-specific factors. However, these captured 'effects' are additive mean-shifts, time-invariant clustering, and *not* autoregressive or distributed-lag in form. Unit-specific 'effects' also cannot account time-varying unmodeled effects, such as evolving spatially clustered sociocultural or institutional factors. Analogously, period fixed-effects/time-dummies account for spatially-invariant, uniform shocks common across all units. These too are fixed, additive mean-shifts, and so cannot well-account autoregressive or distributed-lag processes, or unit or regional variation in clustered additive shocks, such as influences diffusing among members of regional organizations.

Finally, given the substantively and statistically critical importance of adequately addressing spatiotemporal dependence, researchers will want to conduct appropriate and effective specification testing. In principle, one can conduct specification searches from 'specific-to-general', starting with sparse spatiotemporal models and determining, using Lagrange-Multiplier (LM) tests, whether to add spatiotemporal-lag terms, or 'general-to-specific' (Hendry 1995), starting with a more-general specification and using Wald or likelihood-ratio (LR) tests to decide whether some spatiotemporal-lag terms may safely be omitted. However, in practice in this context, LM tests of underspecified models will have power against incorrect alternatives (Anselin 1988): for example, LM tests may reject the nonspatial model indicating missing spatial-lag y (SAR) or spatial-lag error (SEM) when actually temporal dependence is the missing/poorly-specified process. LM tests can be adjusted using cross-partial gradients of the fuller-specification likelihood to prevent such false/misleading rejections, but these robust-LM tests (Anselin et al. 1996) as-yet exist for very few combinations of spatiotemporal processes. Instead, we suggest the (first-order) Spatio-Temporal Autoregressive-Distributed-Lag (STADL) model, as a convenient and effective more-general starting point (see note 6), and 'testing down'. Researchers can 'test down' using either Wald tests or fit statistics and loss-of-fit tests, such as likelihood and LR tests (Juhl 2021), or R^2 and F-tests of ΔR^2, which fit-testing may be preferred given that Wald-testing can be sensitive to reparameterization.[22]

22. Juhl (2021) shows Wald tests of alternative equivalent specifications can yield different results, and so argues

Perhaps better still, researchers can use Akaike or Bayesian-Schwartz Information Criteria (AIC or BIC) fit statistics, which penalize more appropriately for degrees of freedom consumed/excess paramterization than do LR or F and also enable non-nested model comparison, the latter being particularly important given the many alternative models contained within STADL to compare.

In summary, as we will further demonstrate by simulation and in reanalyses of real-world applications below, TSCS analyses that relatively neglect spatial (temporal) dependence will estimate greater temporal (spatial) dependence than actually present, and correspondingly misestimate *spatiotemporal* dynamic and cumulative effects, and so yield biased tests and erroneous inferences regarding substantive-theoretical propositions. The more-general STADL model offers effective alternative for applied TSCS analyses.

The STADL Model

The workhorse cross-sectional and time-serial models from spatial and time-series econometrics were introduced above. To review compactly, the baseline spatial-econometric models correspond to the different potential sources for observed spatial association: nonspatial models (NON) for spatially clustered exogenous covariates (including fixed-effects), spatial error (SEM) for clustering in unobservables, spatially lagged covariates (SLX) for exogenous spillovers/externalities, and spatial-lag/spatial-autoregressive (SAR) models for endogenous contagion/interdependence:

$$\text{Clustered Covariates} = NON : \mathbf{y}_t = \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t, \text{ with } \mathbf{x}_t \text{ spatially correlated} \tag{10}$$

$$\text{Clustered Unobservables} = SEM : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{u}_t, \text{ with } \mathbf{u}_t = \lambda\mathbf{W}\mathbf{u}_t + \boldsymbol{\varepsilon}_t \tag{11}$$

$$\text{Spillovers/Externalities} = SLX : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{W}\mathbf{x}_t\theta + \boldsymbol{\varepsilon}_t \tag{12}$$

$$\text{Interdependence/Contagion} = SAR : \mathbf{y}_t = \rho\mathbf{W}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t \tag{13}$$

strongly for LR testing which is insensitive to such reparameterization. We agree, but would also note that Wald and LR tests rely on different estimates: parameters and standard errors versus likelihood fit. Researchers may have greater confidence in one or the other, such as when using 'robust standard errors', which usage indicates lesser confidence in likelihood or fit. In any case, LM testing upward is clearly dominated.

Vectors $\mathbf{y}_t$, $\mathbf{x}_t$, and $\boldsymbol{\varepsilon}_t$ are $N\times1$; the matrix $\mathbf{W}$ is $N\times N$. Notice, crucially, that 'the effect of x' differs importantly across these models/sources/processes. With clustered exogenous covariates (*NON*), $\frac{dy_{it}}{dx_{it}}=\beta$, and $\frac{dy_{js}}{dx_{it}}=0\ \forall\ j\neq i, s\neq t$. Likewise with spatial dependence confined to the *orthogonal* unobserved component (*SEM*), the effect of x on y is merely $\frac{dy_{it}}{dx_{it}}=\beta$, and $\frac{dy_{js}}{dx_{it}}=0\ \forall\ j\neq i, s\neq t$. In both of these models, with respect to the effect of x on y: "What happens in France stays in France". With exogenous externalities, i.e. in the spatial distributed-lag model (*SLX*), "What happens in France spills over into Germany (and France's other first-order neighbors according to $\mathbf{W}$)," and the story ends there: $d\mathbf{y}=\mathbf{W}\cdot d\mathbf{x}_t\cdot\theta+d\mathbf{x}_t\cdot\beta$. Notice that both the hypothetical/counterfactual, $d\mathbf{x}_t$, and the effect, $d\mathbf{y}_t$, are vectors, not scalars: with spatial spillovers, the effect of x differs depending on which units are 'shocked' and these effects manifest not only in y_i of the shocked unit(s) but also in their first-order neighbors.[23] In the spatial-autoregressive (SAR) model corresponding to interdependent/contagious contexts, "What happens in France also influences Germany and France's other neighbors, which in turn influence their neighbors, including France, which influences those neighbors' neighbors' neighbors, including Germany again, and so on," with the effect of $d\mathbf{x}_t$ on $\mathbf{y}_t$ reverberating outward and back thusly in an exponentiating series:

$$\begin{aligned} d\mathbf{y}_t &= (\underbrace{\mathbf{I}}_{\text{self}}+\underbrace{\rho\mathbf{W}}_{\text{neighbors}}+\underbrace{\rho^2\mathbf{W}^2}_{\text{neighbors' neighbors}}+\underbrace{\rho^3\mathbf{W}^3}_{\text{neighs' neighs' neighs}}+\underbrace{\rho^4\mathbf{W}^4}_{\text{neighbors}^4}+\ldots)\cdot d\mathbf{x}_t\cdot\beta \\ &= \left(\sum_{m=0}^{\infty}\rho^m\mathbf{W}^m\right)\cdot d\mathbf{x}_t\cdot\beta=\underbrace{(\mathbf{I}-\rho\mathbf{W})^{-1}}_{\text{spatial multiplier}}\cdot\underbrace{d\mathbf{x}_t}_{\text{shock}}\cdot\underbrace{\beta}_{\text{impulse}} \end{aligned} \tag{14}$$

Again, insofar as researchers omit or misspecify the spatial-dependence process, say $\mathbf{Wy}$, ρ is underestimated ($\hat{\rho}=0$ if omitted), and the OVB formula and intuition implies inflated $\hat{\boldsymbol{\beta}}$, with those biases distributed proportionately across the $\hat{\beta}_x$ according to those x's partial association with the misspecified/omitted $\mathbf{Wy}$, meaning larger induced biases will accrue to the $\hat{\beta}_x$ whose x cluster spatially more similarly to the pattern implied by $\mathbf{W}$.

The time-series analogs, also first-order, are compactly expressed using the lag operator, $L^s y_t\equiv y_{t-s}\ \forall t>s$, or the matrix equivalent, $\mathbf{Ly_t}$ (see note 26), as the serially correlated errors (SCE), finite

23. Higher-order SLX models capture neighbor-of-neighbor, etc., effects (Whitten, Williams, and Wimpy 2021).

distributed lag (FDL), and lagged dependent variable (LDV) models, along with the static model (StM) with serially correlated exogenous covariates, including time-period fixed-effects:

$$StM : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{u}_t, \text{ with } \mathbf{x}_t \text{ serially correlated} \tag{15}$$

$$SCE : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{u}_t, \text{ with } \mathbf{u}_t = \delta\mathbf{L}\mathbf{u}_t + \boldsymbol{\varepsilon}_t \tag{16}$$

$$FDL : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{L}\mathbf{x}_t\gamma + \boldsymbol{\varepsilon}_t \tag{17}$$

$$LDV : \mathbf{y}_t = \phi\mathbf{L}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t \tag{18}$$

Notice again the dynamics, or lack thereof, of the effects of x on y in these time-series models. In the static and serially correlated errors models, the effect of x_t is confined to y_t; there are no temporal dynamics: $\frac{dy_t}{dx_t}=\beta$ and $\frac{dy_s}{dx_t}=0 \; \forall \; s \neq t$. In distributed-lag or autoregressive processes, contrarily, and again paralleling the spatial case, we need first specify *dx when?* and expand our question about the effect on *y when?*. In the distributed-lag case, the effects of x simply spill forward the number of periods equal to the lag-order, $d\mathbf{y}_t = \mathbf{L} \cdot d\mathbf{x}_t \cdot \beta$, and are completely dissipated beyond that: $\frac{d\mathbf{y}_{t+s}}{d\mathbf{x}_t}=0 \; \forall \; s>p$. Temporally autoregressive processes, finally, imply exponentiating decay for temporary shocks, or decaying accumulation for permanent shocks, of long-run steady-state effects going forward infinitely in time, like so:

$$\underbrace{dy_\infty}_{\substack{\text{LRSS}\\ \text{response}}} = \underbrace{\beta dx}_{\text{period 0}} + \underbrace{\phi\beta dx}_{\text{period 1}} + \underbrace{\phi^2\beta dx}_{\text{period 2}} + \underbrace{\phi^3\beta dx}_{\text{period 3}} + ... = \underbrace{\sum_{s=0}^{\infty}\phi^s\beta dx}_{\text{if } 0<\phi<1, \Rightarrow} = \underbrace{\left(\frac{1}{1-\phi}\right)}_{\text{LR multiplier}} \times \underbrace{\beta}_{\text{impulse}} \times \underbrace{dx}_{\substack{\text{perm.}\\ \text{shock}}} \tag{19}$$

From these differing expressions of the 'effects of x on y' implied by the range of possible spatial and temporal processes, one can see how omissions or misspecifications of either temporal or spatial dependence will yield inaccurate tests and estimates of the substantive effects of interest.

Given this critical substantive and statistical importance of allowing the estimation model to express the spatiotemporal dependence inherent to TSCS data in the manner that it manifests, we suggest a combined spatiotemporal autoregressive-distributed-lag (STADL) model of order $(sy^0, sx^0, se^0; ty^1, tx^1, te^1)$, where the s or t indicate spatial or temporal lag, the y, x, e indicate which terms are lagged, and the superscript indicates the *temporal* order of the lag.[24] We rec-

24. For multiple spatial-weights matrices, $\mathbf{W}$, the s can be subscripted numerically or mnemonically; likewise, x can be subscripted in cases where only some regressors $\mathbf{X}$ are lagged. Researchers writing for audiences more-familiar with ADL and/or spatial notation, could use SAR/SLX/SEM+ADL(p,q) notation instead.

ommend labeling only the terms actually used, so STADL(sy^0, ty^1) would indicate a first-order spatiotemporal-lag model:

$$\mathbf{y}_t = \rho\mathbf{W}\mathbf{y}_t + \phi\mathbf{L}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t, \tag{20}$$

while the general version of the STADL($ty^p, tx^q, te^r, sy^P, sx^Q, sx^R$) is

$$\mathbf{M}\mathbf{y}_t = \mathbf{F}\mathbf{x}_t + \mathbf{A}\boldsymbol{\varepsilon}_t, \tag{21a}$$

$$\mathbf{M} \equiv \left(\mathbf{I} - \phi_1\mathbf{L} - ... - \phi_p\mathbf{L}^p - \rho_0\mathbf{W} - ... - \rho_{P-1}\mathbf{W}^{P-1}\right), \tag{21b}$$

$$\mathbf{F} \equiv \left(\mathbf{I}\beta + \mathbf{L}\gamma_1 + ... + \mathbf{L}^q\gamma_q + \mathbf{W}\theta_0 + ... + \mathbf{W}^{Q-1}\theta_{Q-1}\right), \tag{21c}$$

$$\mathbf{A} \equiv \left(\mathbf{I} - \delta_1\mathbf{L} - ... - \delta_r\mathbf{L}^r - \lambda_0\mathbf{W} - ... - \lambda_{R-1}\mathbf{W}^{R-1}\right)^{-1}. \tag{21d}$$

where $\mathbf{M}$, $\mathbf{F}$, $\mathbf{A}$ are the space-time filters of the outcome, predictors, and residuals, respectively.[25]

We express a first-order STADL conveniently for interpretation of spatiotemporal effects as:

$$\mathbf{y} = \phi\mathbf{L}\mathbf{y} + \rho\mathbf{W}\mathbf{y} + \mathbf{x}\beta + \mathbf{L}\mathbf{x}\gamma + \mathbf{W}\mathbf{x}\theta + (\mathbf{I} - \delta\mathbf{L} - \lambda\mathbf{W})^{-1}\boldsymbol{\varepsilon}, \tag{22a}$$

$$\mathbf{y} = (\mathbf{I} - \phi\mathbf{L} - \rho\mathbf{W})^{-1}\left(\mathbf{x}\beta + \mathbf{L}\mathbf{x}\gamma + \mathbf{W}\mathbf{x}\theta + (\mathbf{I} - \delta\mathbf{L} - \lambda\mathbf{W})^{-1}\boldsymbol{\varepsilon}\right). \tag{22b}$$

where $\mathbf{I}$, $\mathbf{L}$, and $\mathbf{W}$ are $NT\times NT$; $\mathbf{y}$, $\mathbf{x}$, and $\boldsymbol{\varepsilon}$ are $NT\times 1$; and $\mathbf{L}$ creates a one-period time-lag of variables it premultiplies.[26] Recall that in spatiotemporal analyses of 'the effect of x', one must specify which units are shocked and when—this is the 'treatment'—and, correspondingly, the responses or 'effects' will be across all units over all time-periods as determined by the spatiotemporal process given in $\mathbf{W}$ and $\mathbf{L}$. In time-series analysis, one must specify *when* shock dx occurs, and the default shocks are *temporary*, a one-period shock—dx=+1 in period t_0 with x reverting to its previous level thereafter—and *permanent*—dx=+1 in time t_0 with the higher x persisting infinitely. In spatial analysis, one must specify *where* dx occurs, and the analogous defaults are dx=+1 in one unit and dx=+1 in all units. In space-time analysis, the combined spatial and

25. Although not a focus here, the STADL model can also easily incorporate recursive spatial processes (Anselin 2001) with time-lagged spatial lags (TLSL) (Drolc, Gandrud, and Williams 2021). Like $y_{i,t-1}$ or $\mathbf{W}\mathbf{x}$, TLSLs are pre-determined in the system of equations, meaning they can be treated as exogenous regressors. For interpretation, the $\mathbf{W}$ in equation (22) for TLSL will have its nonzero w_{ij} in the off-diagonal elements of the lower block first-minor.

26. Spatiotemporal TSCS analyses order the data as all N units in period 1, all N units in period 2, ... $\mathbf{L}$'s $N\times N$ first block has all-0 elements, reflecting the omitted N first-period observations. All other elements are 0 too, except the diagonal of the lower block first-minor (the $N\times N$ blocks immediately below the $N\times N$ block prime-diagonal); those prime-diagonal elements of the lower block-minor are all 1.

temporal defaults yield four default shocks: {unit-i or all-units}×{1-period or permanent}.[27]

A convenient way to express equation (22) for differentiation by $\mathbf{x}$ to track responses over time across all units to some series of hypothetical/counterfactual *shocks* in N units over T periods, is to create a $NT\times1$ vector, $d\mathbf{x}$, containing the desired spatiotemporal series of shocks:

$$d\mathbf{y} = (\mathbf{I} - \phi\mathbf{L} - \rho\mathbf{W})^{-1} \cdot (\mathbf{I}\beta + \mathbf{L}\gamma + \mathbf{W}\theta) \cdot d\mathbf{x}. \tag{23}$$

The $NT\times1$ vector $d\mathbf{y}$ gives the response across all N units period-by-period to this series of shocks. So, for instance, the 1-period, 1-unit default shock is a 1 in that unit's row of the first $N\times1$ vector and 0 elsewhere. The 1-unit permanent shock repeats this $N\times1$ vector down T periods. The all-units 1-period shock starts with an $N\times1$ vector of ones, and all subsequent elements are 0. The all-units permanent shock is an $NT\times1$ vector of ones. Beyond these defaults, any set of substantive hypothetical/counterfactual shocks across units over time that researchers may wish to consider may simply be entered as that $d\mathbf{x}$. Indeed, multiple columns of hypothetical shocks can be offered and responses calculated at once, with say $NT\times N$ matrices $d\mathbf{X}$ and $d\mathbf{Y}$. We suggest N columns here to facilitate the scalar summaries of spatial effects that LeSage and Pace (2009) call *impacts*. Working in a single-period cross-section, they suggest a set of shocks across units given by the $N\times N$ identity matrix, which corresponds to shocking each unit alone, i.e., the 1-unit default shock, one at a time, column-by-column. For TSCS data, the other T-1 $N\times N$ blocks are all-zeros for the 1-period shock, and the $\mathbf{I}_N$ repeats over all T for the permanent shock. In a cross-section, the average of the diagonal elements of the $N\times N$ $d\mathbf{Y}$ gives LeSage and Pace's *average direct effect* (ADE) of unit on itself, inclusive of spatiotemporal feedback, and the sum of the off-diagonal elements divided by N gives their *average indirect effects* (AIE), a summary of the spillover effects. With $d\mathbf{X}$ just the identity matrix $\mathbf{I}_N$, it may be dropped from equation

27. Notice the ambiguity that arises here surrounding comparable treatments and effects in static models versus in temporally, spatially, and spatiotemporally dynamic models/processes: in static-nonspatial models, all of x's effects incur exclusively in the unit-time shocked; in spatiotemporal models, contrarily, x_{it} has effects also in unit-times $j\neq i, s\neq t$. Thus, in comparison to the static case, and to empirical realism, the hypothetical of a one-unit or one-period $d\mathbf{X}$ shock radically understates both a typical 'treatment' and a typical 'effect' of x because the spatiotemporal model *correctly* distributes the total impact of $d\mathbf{X}$ on $d\mathbf{Y}$ across space and time, whereas the static model *incorrectly* aggregates those unit-time differentiated relationships into one summary relation. By the same token, hypothetical all-unit permanent $d\mathbf{X}$ shocks overstate both a realistic $d\mathbf{X}$ and the static-model dx effect estimates. Perhaps most realistic, and what static-model estimates would be approximating, with bias due to misspecifcation, would be a $d\mathbf{X}$ that followed its empirical spatiotemporal average in the sample.

(23) and the elements of the multiplier times coefficients matrices may simply be summed and averaged in these ways as LeSage and Pace (2009) do.

Long-run steady-state (LRSS) responses in all N units to some *permanent* $N\times1$ set of shocks, $d\mathbf{x}$, is found by returning to equation (22a), setting $\mathbf{y}_{t-1}=\mathbf{y}_t$ and $\mathbf{x}_{t-1}=\mathbf{x}_t$ by definition of LRSS, to obtain:

$$d\mathbf{y} = (\mathbf{I} - \phi\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{I}\beta + \mathbf{I}\gamma + \mathbf{W}\theta) \cdot d\mathbf{x}. \tag{24}$$

Note both the shocks/treatments $d\mathbf{x}$ and responses/effects $d\mathbf{y}$ reference all $N\times1$ units i.[28]

STADL models can be estimated via maximum-likelihood (or Bayesian) methods, with likelihoods (posteriors) given in Elhorst (2001) (and LeSage and Pace 2009) and maximization detailed in Anselin (1988).[29,30] Even previous works that discuss spatiotemporal models and their estimation have neither discussed or derived analytically as above, nor evaluated through simulation as next, the biases from omitting or mismodeling one of the dependence dimensions in estimates of the other dependence parameters, the covariate coefficients, and the dynamic and total effects.

Monte Carlo Analysis of Dynamic TSCS Models

Our Monte Carlo analyses demonstrate that the biases shown analytically above are of substantively important magnitudes in spatiotemporal TSCS data with properties designed to be representative of common political-science application contexts.

Given the combinatorically vast number of STADL-model variations—$2^6 = 64$ first-order models alone—we focus on evaluating the two currently most-widely used in political science: LDV

28. $\mathbf{I}$ and $\mathbf{W}$ in equation 24 are $N\times N$; implicitly, some long-run, permanently operant, $\mathbf{W}$ must be set.

29. Instrumental-variables estimators exist for some, but not all, STADL model variations as well.

30. As with the 3-source spatial model (see note 16), the 3-source temporal and STADL models are identified, but frailly, when all 3 sources are included (e.g., an unrestricted first-order STADL model). Given this, researchers will want to use design (Gibbons and Overman 2012) or theory (Cook, Hays, and Franzese 2020) to restrict some spatial and temporal parameters *ex ante*. In Cook, Hays, and Franzese (2020), we suggest that researchers should generally consider including terms capturing spillovers in the mean component (either $\mathbf{Wy}$ or $\mathbf{Wx}$) plus spatial error autocorrelation. Similarly, a time-series model including a time-lagged outcome and a correction for serially correlated errors would be robust to the concerns of Achen (2000). Taken together, we believe applied researchers with TSCS will be well served by including outcome lags ($\mathbf{Wy}$ and $\mathbf{Ly}$) *or* covariate lags ($\mathbf{Wx}$ and $\mathbf{Lx}$)—whichever is best motivated by their theory—*and* spatial and temporal error lags ($\mathbf{W}\boldsymbol{\varepsilon}$ and $\mathbf{L}\boldsymbol{\varepsilon}$).

and SAR, i.e. STADL(ty^1) and STADL(sy^0). LDV and SAR model performance under various forms of temporal *or* spatial dependence is well known, but we know little as-yet about how either one-way model performs under *spatiotemporal* dependence in both dimensions. We therefore generate data from a STADL(sy^0, ty^1), i.e., the first-order spatiotemporal autoregressive model, with the exogenous covariate $\mathbf{x}$ also following a STADL(sy^0, ty^1) process, for realism and so induced biases will manifest in entirely $\hat{\beta}$:

$$\mathbf{y}_t = \phi_y \mathbf{y}_{t-1} + \rho_y \mathbf{W}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_\mathbf{y}, \tag{25a}$$

$$\mathbf{x}_t = \phi_x \mathbf{x}_{t-1} + \rho_x \mathbf{W}\mathbf{x}_t + \boldsymbol{\varepsilon}_\mathbf{x}, \tag{25b}$$

with $\mathbf{x}_0$, $\boldsymbol{\varepsilon}_\mathbf{y}$, and $\boldsymbol{\varepsilon}_\mathbf{x}$ drawn independent standard-normal. To focus comparisons, we fix several conditions across all simulations: N=50 and T=20, giving balanced panels with common sampling dimensions (e.g., U.S. states over 20 years); $\mathbf{W}$ generated as k-nearest-neighbor binary-contiguity (k=5), row-normalized based on xy-coordinate locations for each unit drawn $U(0, 100)$; and parameters β=2, ϕ_x=0.6, and ρ_x=0.3. We vary the strength of temporal, ϕ_y, and spatial, ρ_y, dependence in the outcome $\mathbf{y}$ (further design details in the Appendix).[31]

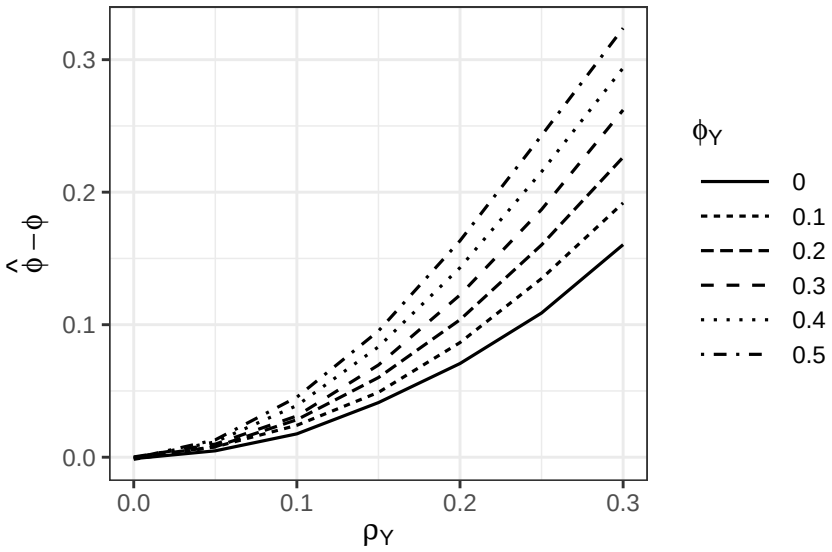


Figure 6: LDV Performance with Spatial Dependence — Bias in $\hat{\phi}_y$

31. The Appendix reports results with a first-order spatiotemporal autoregressive *error*, i.e. STADL(se^0, te^1), DGP, so as to explore the performance of commonly used *outcome*-lag models, LDV and SAR, and our STADL model, under spatiotemporal *error* autocorrelation. Results indicate the LDV and SAR models produce biased estimates under spatiotemporal error autocorrelation, whereas STADL performs well under all conditions.

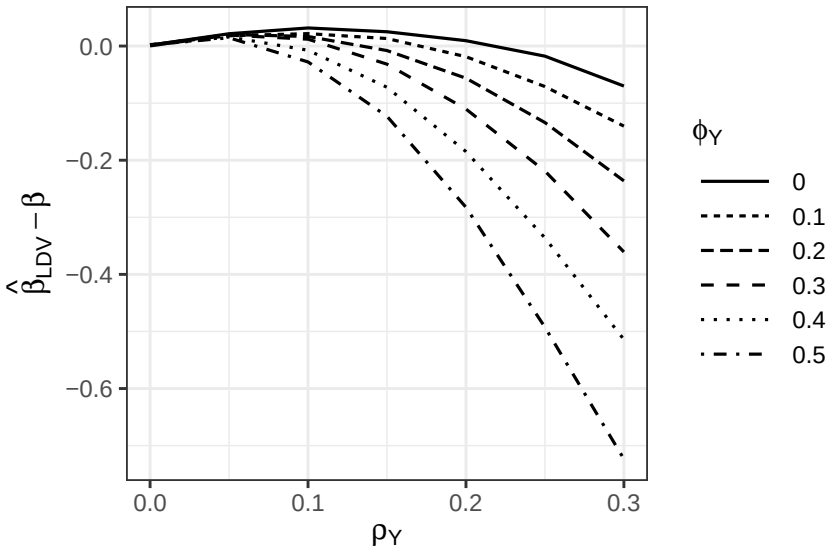


Figure 7: LDV Performance with Spatial Dependence — Bias in $\hat{\beta}$

Figures 6 and 7 present simulation results for the LDV-model estimates. Figure 6 shows that temporal-lag coefficient-estimate $\hat{\phi}_y$ suffers inflation bias for all true spatial-lag coefficients ρ_y>0, with the bias magnitude increasing in both ρ_y and ϕ_y. Even when ϕ_y=0, substantial bias obtains—$\hat{\phi}_y$ reaches 0.18 at the modest maximum spatial dependence considered here: ρ_y=.3—and this bias grows as ϕ_y increases, the very condition making account of temporal dependence more important. The intuition is simple: the modeled temporal dependence can partially compensate for the missing (or, by extension, mismodeled) spatial dependence, in omitted-variable-bias fashion.

While the strength of temporal dependence is important in itself, researchers often have greater interest in $\hat{\beta}$, for testing and for estimating the 'effects' of covariates $\mathbf{x}$. Figure 7 shows how the inflated $\hat{\phi}_y$ estimate generally attenuates the $\hat{\beta}$ estimate, with this induced attenuation bias also quite sizable and increasing in ρ_y and ϕ_y. This is striking given that, with ρ_y>0 and Cov($\mathbf{x}, \mathbf{Wy}$)>0, textbook discussion on omitting the spatial lag indicates *inflationary* OVB in $\hat{\beta}$. The opposite obtains here because that textbook inflationary bias manifests so strongly in $\hat{\phi}_y$ as to induce a countervailing deflationary bias in $\hat{\beta}$, underscoring how conventional understandings from single-dimensional analyses do not extend straightforwardly to TSCS contexts.[32]

32. The nearly monotonically negative bias in $\hat{\beta}$ seen in Figure 7 is the resultant of a direct positive OVB of $\rho_y \times Reg(\mathbf{Wy}$ on $\mathbf{x} \mid \mathbf{Ly})$ and a negative bias induced in compensation for the direct OVB in $\hat{\phi}_y$. The net of these competing biases in $\hat{\beta}$ can manifest differently than seen in 7 depending on $\mathbf{W}$ and parameter values (See Appendix). Beyond these biases in $\hat{\beta}$ and $\hat{\phi}_y$, their standard errors are also notably off: average reported s.e. for $\hat{\beta}$ considerably

Furthermore, the unmodeled spatial dependence also undermines standard LM tests for serial correlation in the LDV-model estimation residuals, producing an unacceptably high false-positive rate, meaning that using 'remaining residual autocorrelation' to assess the adequacy of the LDV in addressing dependence will fail to guide specification properly (see Appendix).

In sum, with spatiotemporal dependence, LDV underestimates the 'impulse' effect of x_t, $\frac{\partial y_t}{\partial x_t}{=}\beta$, but overestimates ϕ_y. As such, researchers may wonder how well these biases offset in long-run steady-state effect-estimates. In the LDV, the LRSS effect on unit i of permanent dx_i, is:

$$\frac{dy_{i,ss}}{dx_i} = \frac{\beta}{1-\phi_y} \text{ and } \frac{dy_{i,ss}}{dx_j} = 0 \; \forall j{\neq}i, \tag{26}$$

while the contemporaneous spatial steady-state effect[s] of one-unit $d\mathbf{x}$ on $\mathbf{y}$ in the SAR model are:

$$\frac{d\mathbf{y}}{d\mathbf{x}} = (\mathbf{I}-\rho\mathbf{W})^{-1}\beta, \tag{27}$$

which is an $N{\times}N$ matrix of the effects, column-by-column, of dx_i in that column-unit i on $\mathbf{y}$ *in all units*. Thus, the *single* estimated LRSS effect of x on y from the LDV, or any other nonspatial model, is not even in the correct dimensionality of the spatial effects (plural) of $d\mathbf{x}$ on $d\mathbf{y}$. As emphasized above, spatial dynamics imply changes in x in any unit have effects across all connected units, and changes in x in different units have different effects because units are differently connected to each other. LeSage and Pace's (2009) *average direct effects* (ADE) scalar summary of the average across i of $\frac{dx_i}{dy_i}$, inclusive of spatial dynamics, can be obtained, but even the ADE will not compare closely to the LDV's LRSS, because the LDV's temporal dynamics are quite imperfect substitutes for SAR's spatial dynamics.

The correctly spatiotemporal dynamic and LRSS effects of $d\mathbf{X}$ in the general first-order STADL, inclusive of both spatial and temporal dynamics and feedback, are given in Equations 23 and 24. Their simplifications to this STADL(sy^0, ty^1) model are:

$$\text{STADL}(sy^0, ty^1) \text{ LRSS Effects: } d\mathbf{Y} = (\mathbf{I}-\phi\mathbf{I}-\rho\mathbf{W})^{-1}\cdot d\mathbf{X}\cdot\beta, \tag{28a}$$

$$\text{STADL}(sy^0, ty^1) \text{ Dynamic Effects: } d\mathbf{Y} = (\mathbf{I}-\phi\mathbf{L}-\rho\mathbf{W})^{-1}\cdot d\mathbf{X}\cdot\beta. \tag{28b}$$

As explained above, for shocks to one unit at a time, column-by-column, $d\mathbf{X}$ is the $N{\times}N$ identity

overstate the standard deviation of $\hat{\beta}$, and yet, given the large biases in $\hat{\beta}$, the estimated 95% confidence intervals *never* bound the true value in our simulations, i.e. coverage is zero.

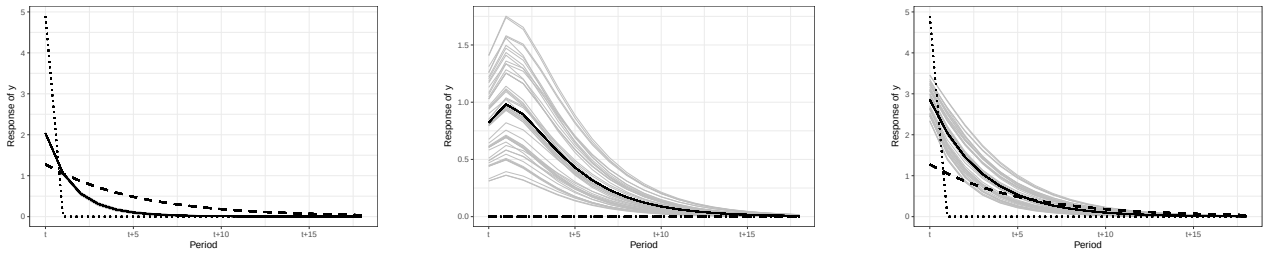
matrix, $\mathbf{I}_N$, in the LRSS-effects equation (28a), and, in the dynamic-effects equation (28b), $d\mathbf{X}$ is that $\mathbf{I}_N$ stacked vertically T times for permanent shocks, and only in the first $N \times N$ block for 1-period shocks. Each $(N \times 1)$ column of the resulting $d\mathbf{Y}$ in (28a) gives the LRSS effects in all N units to shocking the column-unit. In (28b), these $N \times N$ blocks of effects in $d\mathbf{Y}$ evolve period-by-period T times vertically. The scalar summaries of LRSS or period-by-period ADE and AIE are found by averaging across the $N \times N$ effects-block's diagonals or over all its off-diagonal elements as before. Given all this, clearly, even if the LDV model accurately recovered the LRSS ADE—we will show it does not—it would still produce biased estimates of these unit-specific responses.

Figures 8 and 9 illustrate all this, for single-unit shocks under one set of conditions: $\phi_y = 0.5$ and $\rho_y = 0.3$. Figure 8 compares the N marginal period-by-period incremental response paths, i.e., impulse-response functions, a.k.a. the responses to temporary (1-period) shocks, using (28b) of (*i*) the true STADL model: N grey thinner response-lines, and heavier black response-line average; (*ii*) the estimated LDV model: *one* dashed response-line; and (*iii*) the estimated static-model: *one* dotted response-line.[33] Labeled 'Direct' are effects of shocks to unit i on outcomes in unit i; 'Indirect' are summed responses in units $j \neq i$ to shocks in unit i; and 'Total' sums Direct and Indirect. Figure 9 plots the analogous cumulative response paths to permanent single-unit shocks.[34] As shown analytically above, the LDV substantially underestimates the contemporaneous effect for all N units, and it overestimates the temporal persistence, giving incorrectly slower decay. Thus, the LDV estimates *one* smaller, but more-persistent, effect, than the true STADL's *heterogeneous*, larger, quicker-decaying correct effect*s*. The LDV also overestimates (underestimates) the cumulative LRSS direct (total) effect at 6.41, to which it arrives more slowly, compared to the *average* cumulative LRSS direct effect of 4.39 and total effect of 10.0 from the correct STADL, to which the STADL responses arrive more quickly. The static model, meanwhile, radically overstates direct (and total) contemporaneous effect, and badly mischaracterizes (and understates) the

33. To compare the LDV and static models against the true results, we use the STADL response paths generated from the generating process (i.e., $\beta = 2$, $\phi = 0.5$, and $\rho = 0.3$). However, using the *estimated* STADL response paths would produce effectively identical results as it is an unbiased estimator of these paramaters (i.e., $\mathbb{E}[\hat{\beta}] = 2.0009$, $\mathbb{E}[\hat{\phi}] = 0.5003$, $\mathbb{E}[\hat{\rho}] = 0.2996$). See the Appendix for greater detail.

34. The responses to all-unit shocks differ only for spatially aware models, and follow the same patterns as seen in Figures 8 and 9, at roughly N times greater scale.

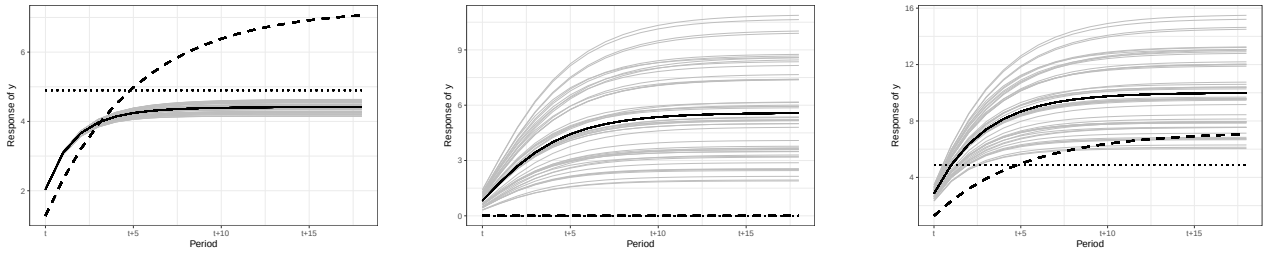
direct (and total) cumulative effects. In sum, even on average—i.e., disregarding the unit-specific variation—the LDV model performs poorly, and the static nonspatial model very poorly.



(a) Direct **(b)** Indirect **(c)** Total

Figure 8: Response-Path Estimates of LDV Model with Spatial Dependence

Note: dotted line = Static, dashed line = LDV, grey lines = STADL unit-by-unit, black line = STADL average



(a) Direct **(b)** Indirect **(c)** Total

Figure 9: Cum. Response-Path Estimates of LDV Model with Spatial Dependence

Note: dotted line = Static, dashed line = LDV, grey lines = STADL unit-by-unit, black line = STADL average

The analogous explorations of SAR-model estimation-performance shows (Figure 10) the expected inflation bias in $\hat{\rho}_y$ when temporal dependence is present but unmodeled. When ϕ_y=0.5, for example, the $\hat{\rho}_y$ estimates from the SAR model average more than 2 times(!) the true value of ρ_y. As researchers more-commonly attach theoretic importance to their spatial-dependence specifications than to temporal dependence—selecting connectivity matrices to test competing theories of diffusion, e.g.—this is *per se* more substantively concerning than in the LDV case. Researchers interested in evaluating spatial theories must attend equally highly carefully to accurately modeling temporal dynamics. Even with spatial processes well-specified, the **W** and spatial-lag terms correct and well-measured in the model, failing to adequately address temporal dependence can produce wildly inaccurate understandings of the spatial processes actually operating in the data.[35]

35. The Appendix shows that, as with the LDV estimation-model case above, average reported standard errors in SAR applied to STADL(s^0y, t^1y) exceed actual standard deviations of coefficient estimates across simulation trials and yet 95% confidence intervals rarely contain true coefficient values; whenever $\rho_y \neq 0$ and $\phi_y \neq 0$, coverage probabilities are far below 95%, due to the coefficient-estimate bias.

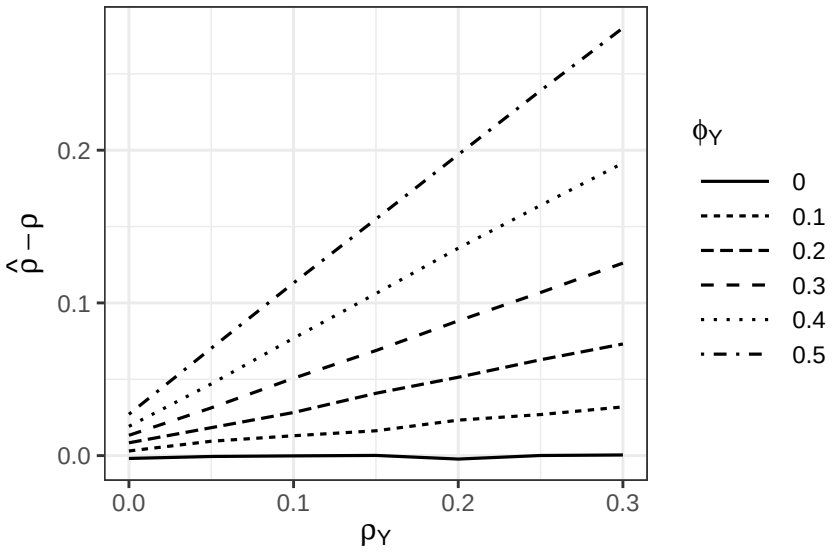


Figure 10: SAR Performance with Temporal Dependence — Bias in ρ_y

Regarding $\hat{\beta}$, Figure 11 reveals the expected inflationary bias from failing to model temporal dependence, and this bias increases in the unmodeled ϕ_y. However, this bias does not also increase with ρ_y. Why? First, temporal dependence often, as in our simulation, far exceeds spatial dependence. As such, the inflationary bias in $\hat{\beta}$ from the unmodeled temporal dynamics weighs more heavily against the downward bias from overestimated $\hat{\rho}_y$ than in the reverse scenario. Second, our simulation parameters, again realistically, set x also to manifest greater temporal than spatial dependence: ϕ_x=0.6 vs ρ_x=0.3. Thus, the correlation between $x_{i,t}$ and $y_{i,t-1}$, and so the bias from omitting the latter, is greater than the bias induced by overestimating $\hat{\rho}_y$ and the correlation of $x_{i,t}$ and $y_{j,t}$.[36]

Although the $\hat{\beta}$ estimate is inflated in proportion solely to the temporal-dependence misspecification, that bias plus the inflation bias also in $\hat{\rho}_y$ seriously compromises the effects estimates. Recall that in spatial-autoregressive models, as in all models beyond the purely linear-additive and separable, the effect of x on y is not β, which is merely the pre-spatial impulse, but instead the effect*s* are given by equations (23) and (24). For scalar summaries of these multidimensional effects, we use the average-direct and average-indirect effects, ADE and AIE, described above. Comparing the values estimated by the incorrect SAR to those from the true STADL, we find

36. The Appendix verifies this: reversing the strengths of the dependencies in x to ϕ_x=0.3 and ρ_x=0.6, the relative magnitude of the bias in $\hat{\beta}$ is reduced, and the extent of the bias is affected more acutely by the level of ρ.

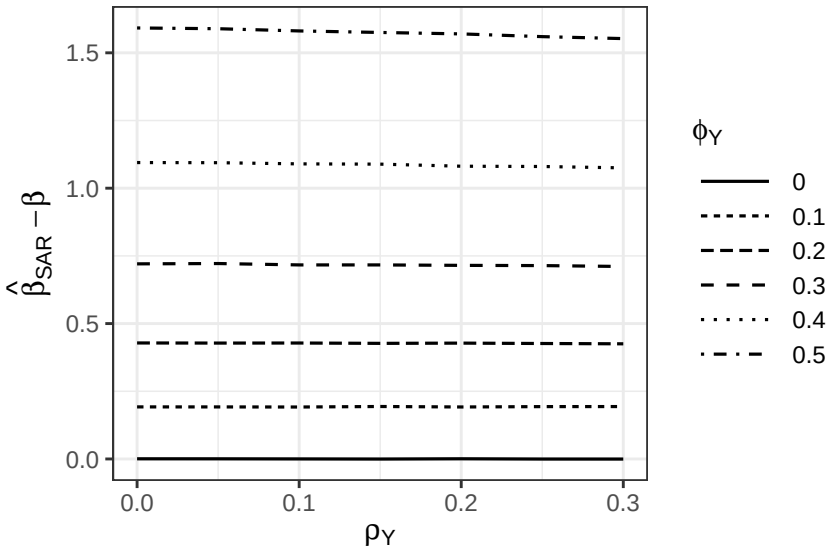


Figure 11: SAR Performance with Temporal Dependence — Bias in β

that SAR overestimates the single-unit-shock responses, SAR ADE=3.96 vs. STADL ADE=2.03, and radically overestimates the AIE, SAR AIE=6.76 vs. STADL AIE=0.82, and so the average total effects (ATE), SAR ATE=10.72 vs STADL ATE=2.85. Furthermore, despite the absence of temporal dynamics from the estimation model, the long-run steady-state (LRSS) total effects are also overestimated: SAR LRSS ATE=10.72 vs. STADL LRSS ATE=10.

Empirical Reanalyses

To demonstrate the importance of modeling spatiotemporal dependence appropriately in applied TSCS data-analysis, we conduct two brief reanalyses of prominent recent studies. First, we revisit Acemoglu et al. (2008) to evaluate empirically the development–democracy connection, our running illustration. Particularly apt for our purposes, they include temporal autoregressive dependence, and fixed unit and period effects, but otherwise neglect spatial dependence. Second, we analyze data from Lührmann, Marquardt, and Mechkova (2020), who develop several new country-year indices of vertical, horizontal, and diagonal political accountability, plus an overall accountability index. The article focuses on demonstrating the content, convergent, and construct validity of these measures, but includes analyses of accountability and infant morality in which

they account for spatial dependence and include fixed unit and period effects while omitting autoregressive temporal dependence. These reanalyses also nicely parallel our simulation studies: one models temporal dependence while relatively neglecting spatial dependence; the other models spatial dependence while relatively neglecting temporal dependence.

Reanalysis of Acemoglu et al. (2008) on Development and Democracy

Acemoglu et al.'s main finding is that the otherwise robust positive relation of economic development with democratization disappears with country fixed-effects included. Table 1 reports five models using their data to regress Polity IV Democracy on lagged real GDP per capita (RGDPpc), plus various combinations of fixed effects and autoregressive lags, with column (4) replicating their main two-way-fixed-effects regression. The results starkly highlight how these specification choices affect one's analysis and inferences.

Table 1: Reanalysis of Development and Democracy in Acemoglu et al. (2008)

	Dependent variable: Democracy (Polity IV)				
	(1)	(2)	(3)	(4)	(5)
Lagged RGDP Per Capita	0.186^{***}	0.199^{***}	-0.002	-0.007	0.037^{***}
	(0.01)	(0.01)	(0.027)	(0.027)	(0.009)
Temporal Lag				0.449^{***}	0.743^{***}
				(0.034)	(0.021)
Spatial Lag	0.315^{***}	0.197^{***}	0.163^{**}		0.115^{***}
	(0.045)	(0.050)	(0.065)		(0.035)
Observations	854	854	854	854	854
Fixed Country Effects	No	No	Yes	Yes	No
Fixed Year Effects	No	Yes	Yes	Yes	Yes
LL	-141.84	-125.88	255.85	344.85	250.28
DoF (Parameters)	850 (4)	842 (12)	709 (145)	709 (145)	841 (13)
BIC	310.69	332.76	467.05	289.04	-412.81

Note: $^{*}p < 0.1;^{**}p < 0.05;^{***}p < 0.01$

The column (1) model includes a spatial lag created with a row-standardized nearest-neighbor weights matrix (auto-generated by our `tscsdep` R package). With this spatial autoregression the only spatiotemporal dependence in model (1), the positive and significant spatial-lag coefficient may be an overestimate. Column (2) adds fixed year-effects. Since democracy trends globally

over the sample period, this addition greatly improves model fit: log-likelihood increases only 11% (-141.84 to -125.88), but BIC *increases* from 310.69 to 332.76, reflecting little purchase from the 8 degrees of freedom consumed. The $\hat{\rho}$ estimate declines by about $\frac{1}{3}$ (and the RGDPpc coefficient-estimate compensatorily increases slightly: about 7%), but remains highly significant.

Column (3) adds country fixed-effects, and column (4) replaces the spatial lag with a temporal lag, the latter replicating Acemoglu et al.'s Table 3, column 2. These results reflect the main takeaway from their analysis: the statistical significance of the RGDPpc coefficient-estimate disappears with country fixed-effects added. The spatial-lag coefficient becomes less, but still quite, significant in column (3), with the country fixed-effects apparently absorbing noticeable time-invariant spatial clustering in both RGDPpc and democracy. The impact on model fit, however, is less clear. LL improves greatly, but the BIC fit statistic, which penalizes for over-parameterizing the model and over-fitting the sample, gets much worse, increasing 40% (332.76 to 467.05). Scholars can reasonably disagree about the model-selection implications of these comparisons, but BIC strongly indicates that the LL improvement from the country dummies does not merit the 133 degrees of freedom they consume.

Acemoglu et al.'s column (4) model improves on (2) and (3) in terms of both coefficient significance and model fit, underscoring the crucial importance of temporally autoregressive dynamics in democratic development. The final column (5) presents results from the model with by far the best BIC (-412.81) and LL close to model (3) despite 132 fewer estimated parameters. The Likelihood Ratio test of Model (5), which includes both temporal and spatial lag, and period fixed-effects, versus Model (3) yields a Chi-Squared statistic of 11.14, which with 132 degrees of freedom overwhelmingly fails to reject: $p \approx 1.0$. The STADL(s_y^0, t_y^1) plus time dummies model is therefore overwhelmingly preferred by our model-selection criteria. The coefficients on RGDPpc, the temporal lag, and the spatial lag are all statistically significant. The comparison of the STADL(s_y^0, t_y^1) plus time dummies (5) with Acemoglu et al.'s temporally autoregressive two-way fixed-effects model (4) is similar to the comparison of the models in columns (3) and (2): the inclusion of country fixed-effects leads to a significant improvement in LL, but if we penalize for

over-parameterizing and over-fitting, the BIC concludes emphatically that the LL increase does not remotely justify the cost in parsimony terms (consumed degrees of freedom).

Comparing the coefficient estimates across these models also nicely illustrates in real-data application the conclusions drawn above in our analytics and simulations. Failing to include a temporal lag in model (2) results in larger estimates of $\hat{\rho}$ and $\hat{\beta}$ relative to model (5), where temporal dependence is directly accounted. This parallels our simulations' findings that SAR (here model 2) suffers inflation bias in $\hat{\rho}$ and $\hat{\beta}$ when temporal dependence is present but unmodeled. In the simulations, the SAR model $\hat{\rho}$ averaged about twice the true STADL ρ; similarly here, $\hat{\rho}$ from model (5) is 0.115, whereas model (2) $\hat{\rho}$ is roughly 1.7 times larger at 0.197.

This also has substantively important implications for how development is estimated to affect democracy across space and time. The positive, significant temporal-lag and spatial-lag coefficients in our preferred model (5) indicate that development effects on democracy in one country at one time reverberate autoregressively both forward in time and across countries in space. We can calculate summaries of these average short-run (first-period) and long-run direct, indirect, and total effects (ADE, AIE, ATE) as shown in equations 28a and 28b. Using model (5) coefficient-estimates, a one-unit single-country shock to RGDPpc has contemporaneous ADE of +0.037 on democracy in that same country, AIE of +0.005 on democracy in other countries, for a combined ATE of +0.042. The respective long-run cumulative estimates are LRSS ADE=+0.144, LRSS AIE=+0.018, LRSS ATE=+0.162. These differ considerably from model (2), where the estimated effects have no temporal dynamics and are instead instantaneous at ADE=+0.200, AIE=+0.048, and ATE=+0.248. These latter static SAR-model estimated effects might seem dissimilar enough from the long-run effects from spatiotemporally dynamic STADL model (5)—differences of ADE +.056≈39%, AIE +.030≈167%, and ATE=+.086≈53%—but the effects from model (2) incur immediately and fully, so the same-country, same-period effects from a change in development on democracy is ADE=+0.037 in model (5) versus ADE=+0.200 in model (2), more than a four-fold (≈+441%) difference.

In short, the choice of spatiotemporal model dramatically affects one's conclusions both of

whether and of how development relates to democracy.

Reanalysis of Lührmann et al. (2020) on Accountability and Infant Mortality

In their recent *APSR* article, Lührmann, Marquardt, and Mechkova (2020) demonstrate construct validity for their overall index of political accountability (in part) by verifying its negative correlation with infant-mortality rates. They estimate four time-series-cross-sectional regressions, both in isolation and in combination with alternative measures of accountability taken from the World Bank and Freedom House. We reanalyze one of their primary regressions: MODEL 1 in Figure 8. The model includes the new overall accountability index and a large set of controls, including country and year fixed-effects as some account of spatial and temporal dependence, plus a regional-average infant-mortality variable. This regional-average variable is actually a kind of spatial lag, being the average dependent-variable among regional neighbors, but it is treated as an exogenous regressor. Beyond the time-period indicators, temporal dependence and dynamics are not modeled.

The country indicators account for fixed (long-run) additive spatial clustering in the outcome, infant mortality rates. *Fixed* here means constant over the entire sample period (1960-2010). *Additive* means the clustering manifests as a single mean-shift, as opposed to a multiplicative effect on some observed or unobserved covariate or an autoregressive spatially dynamic process. The regional-average variable, which instead gives a spatial-autoregressive process, accounts for potential time-varying (long-run) spatial clustering. If there are multiple regional equilibria over time (e.g., Southeast Asia 1961-1980; Southeast Asia 1981-2000; Southeast Asia 2001-2010), though, the regional-average spatial-lag cannot account for this. Country fixed-effects cannot either.

The year fixed-effects can account for 'short-run' (unique year-by-year) common shocks that are global in scope. Again, these are additive: some mean-shift each year that is common across all countries. The same infant-mortality shock, equal to that year's single time-dummy coefficient, hits every country. Year fixed-effects cannot account for common shocks that are regional or otherwise sub-global in nature: e.g., an infant-mortality shock specific to Southeast Asia. If the

relevant regions or groups of countries were known pre-analysis, regional-period shock indicators (e.g., Southeast Asia 1987) could be included in regression models, but the relevant spatiotemporal units are rarely known, and this strategy quickly overloads degrees of freedom.

An alternative strategy to account for regional common shocks is to add spatial lags in first differences to regression models. Because spatial lags represent autoregression in space—countries influence first, second and third (etc.) order neighbors with geometrically decaying impact—they provide a certain flexibility with respect to identifying the geographical boundaries of shocks that regional indicators do not. The spatial-weights matrix could connect k-nearest neighbors, e.g., around each country (automatically generated using `tscsdep`), whereas regions must be pre-identified for indicator-variable strategies. Spatial lags are also more parsimonious than regional-period indicators because a single spatial-lag defines a 'neighborhood' for every sample-unit.

More generally, in STADL models, time-differenced right-hand-side variables produce only 'short-run' effects in left-hand-side outcomes, whereas regressors in levels produce long-run effects through temporal multipliers. Lührmann et al.'s MODEL 1 includes a *de facto* endogenous spatial lag in the regional averages, which, for our reanalysis purposes, we will retain as an (erroneously) exogenous regressor and will assume to reasonably proxy the true spatial-dependence process. MODEL 1 also includes country and year fixed-effects, but no temporal dynamics, a stark omission given that infant-mortality rates are highly persistent temporally. We also think heterogeneous regional infant-mortality shocks are highly plausible. Therefore, we add to our reanalysis model a temporal lag and a time-differenced nearest-neighbor spatial-lag:

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \phi y_{it-1} + \rho \mathbf{w}_i \Delta \mathbf{y}_t + f_i + g_t + \varepsilon_{it}, \tag{29}$$

with y_{it} being infant mortality in unit i in year t, $\mathbf{x}_{it}$ a $1\times k$ vector of exogenous covariates for unit-year it, $\boldsymbol{\beta}$ a $k\times 1$ vector of coefficients, ρ the spatial-lag coefficient, $\mathbf{w}_i$ unit-i's $1\times N$ vector of spatial weights on units j, $\Delta\mathbf{y}_t$ the time-t $N\times 1$ vector of differenced outcomes, f_i a fixed unit-effect, g_t a fixed period-effect, and ε_{it} an *i.i.d.* disturbance for unit-time it. Some algebraic manipulation rewrites this with a differenced outcome (more-convenient for expressing the likelihood):

$$\Delta y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + (\phi - 1) y_{it-1} + \rho \mathbf{w}_i \Delta \mathbf{y}_t + f_i + g_t + \varepsilon_{it}. \tag{30}$$

While the regional-average variable (treated as exogenous still for comparability) incorporates some spatial dependence, its coefficient is likely overestimated because temporal dependence, likely very high in infant mortality, is omitted, beyond the year-effects—which year-effects, due to regional concentration in infant-mortality shocks, likely miss considerable spatiotemporal dependence also. Our analyses above suggest that the unfortunate consequence of this mis-estimation of the spatiotemporal dependence is that Lührmann, Marquardt, and Mechkova may have *underestimated* the strength of their political-accountability measure's relationship to infant mortality.

Table 2 column 1 replicates their original results. Then, with `tscsdep`, we create a nearest-neighbor spatial weights matrix and estimate the spatiotemporal-autoregressive STADL(sy^0, ty^1) model incorporating spatially and temporally lagged dependent-variable regressors, reported in the second column. The LRSS effects[37] of each covariate x in $\mathbf{x}_{it}$ are given in the third column. Comparing the implicit spatial steady-state implied by the regional-average variable in the original regression, which ignores temporal (autoregressive) dynamics, with our estimate of the spatial steady-state effect, we estimate that the former overstates the extent of spatial dependence by nearly 12% in this comparison. More simply and starkly, comparing the first and third columns, we estimate that the spatiotemporal LRSS effect of Lührmann et al.'s political accountability on infant mortality rates (-9.842) is more than double the 'effect' they reported ($\hat{\beta} = -4.339$), which mostly ignores these important spatial and temporal dynamic dependencies. In column 4 (model 3), we drop the country dummies from model 2. In contrast with the analysis in Table 1, we find that the inclusion of country fixed-effects here leads to unambiguous improvement in model-fit by either LL or BIC. Model 2 is clearly the preferred model among this set.

Conclusion

This paper considers the implications of the multidimensional dependence, dynamics in both space and time, that typically manifests in TSCS data. With both spatial and temporal depen-

37. These LRSS use only the temporal multiplier as Lührmann, Marquardt, and Mechkova do not interpret their implicit spatial-lag regional-average as such and our added spatial lag is in changes, not levels.

Table 2: Reanalysis of the Accountability / Infant Mortality Regression in Lührmann, Marquardt, and Mechkova (2020)

	Dependent variable: Infant Mortality				
	(1)	(2)	LRSS	(3)	LRSS
Accountability	-4.339^{***}	-0.187^{***}	-9.842	0.083^{***}	4.368
	(0.350)	(0.038)		(0.029)	
Foreign aid	-0.048	0.013^{***}	0.684	0.007^{***}	0.368
	(0.031)	(0.003)		(0.003)	
GDP/capita (ln)	-10.551^{***}	0.805^{***}	42.37	0.038	2.000
	(0.771)	(0.085)		(.036)	
Economic Growth	0.035	-0.019^{***}	-1.000	-0.022^{***}	-1.158
	(0.023)	(0.003)		(0.003)	
Resource dependence	0.040^{*}	0.013^{***}	0.684	0.007^{***}	0.368
	(0.022)	(0.002)		(0.002)	
Economic inequality	-0.072^{**}	0.005	0.263	-0.001	-0.053
	(0.031)	(0.003)		(0.002)	
Population (ln)	-17.743^{***}	0.806^{***}	42.42	0.001	0.053
	(1.606)	(0.175)		(.015)	
Urbanization	-0.125^{***}	0.023^{***}	1.211	-0.005^{***}	-0.263
	(0.023)	(0.003)		(.001)	
Political violence	0.332^{***}	-0.009	-0.474	0.000	0.000
	(0.128)	(0.014)		(.011)	
Communist	0.387	-0.741	-39.00	0.158	8.316
	(1.620)	(0.173)		(0.114)	
Infant mortality	0.646^{***}	0.011^{***}	0.579	0.011^{***}	0.579
(regional average)	(0.020)	(0.002)		(.001)	
Political corruption index	-3.400^{*}	-0.147	-7.737	-0.393^{***}	-20.68
	(1.902)	(0.203)		(.094)	
Temporal Lag (Level)		-0.019^{***}		-0.019^{***}	
		(0.002)		(0.001)	
Spatial Lag (Difference)		0.195^{***}		0.251^{***}	
		(0.021)		(0.020)	
Observations	4,354	4,311		4,311	
Dependent Variable	Level	Diff.		Diff.	
Fixed Country Effects	Yes	Yes		No	
Fixed Year Effects	Yes	Yes		Yes	
LL	-15466.54	-5763.80		-6644.56	
DoF (Parameters)	4149 (205)	4104 (207)		4250 (61)	
BIC	32659.1	13260.0		13799.6	

Note: $^{*}p<0.1$;$^{**}p < 0.05$;$^{***}p < 0.01$

dence present, we have shown that modeling dependence in only one dimension while neglecting the other biases estimates of all dependence parameters, usually resulting in inflation bias in the parameter(s) of the included or better-specified dimension. Not only does this risk misunderstandings about whether the observed spatiotemporal patterns are due to temporal or spatial dependence, it can also threaten researchers' ability to effectively discriminate between competing sources of dependence—dependent outcomes, covariates, or unobservables—along either dimension. These biases in the dependence-parameter estimates also induce biases in the covariate coefficient estimates. With these dependence-parameter and covariate-coefficient estimates both biased, spatiotemporal-dynamic and long-run-steady-state effects are biased too, as are hypothesis tests and inferences made using these estimates. Furthermore, we demonstrate that these estimation biases with dependence in both space *and* time differ from those considered heretofore in textbook treatments of temporal *or* spatial dependence alone. For example, usually omitted spatial interdependence results in partially offsetting inflationary bias in the covariate coefficients. However, we showed that in TSCS analyses, if a time-lagged outcome is included in the model, then the inflationary bias from the omitted spatial lag will primarily manifest in this temporal lag, and that in turn induces partially offsetting *attenuation* in the covariate coefficients. Thus, researchers' previous methodological understandings, which derived from single-dimensional studies, can sometimes provide poor guide when analyzing TSCS data.

Given these concerns, we propose that researchers instead use a spatiotemporal model, the first-order STADL, as an effective starting point that nests many of the most-widely used space-time specifications in political science (e.g., the first-order LDV, ADL, SAR, SDM) and combinations thereof. We suggested that beginning with this more-general STADL specification and using model-restriction tests and goodness-of-fit statistics to guide model refinement reduces the risk of unmodeled dynamics, and so of biased estimation and invalid inferences. While estimating these models is not significantly more difficult than single-dimensional analogs, interpretation of the results raises considerable challenges, because the meanings of both β_x and the calculation of $\frac{d\mathbf{y}}{d\mathbf{x}}$ changes for different STADL models. Therefore, we have discussed at length the varieties of spatiotemporally dynamic effects different STADL specifications entail. To better enable re-

searchers to adopt the strategies presented here, we developed R package `tscsdep` (see Appendix for detail; GitHub to download https://github.com/judechays/STADL) to automate construction of common weights matrices, $\mathbf{W}$, including for unbalanced panels, estimation of the first-order STADL model, and calculation of STADL dynamic and LRSS effects.

We see several priorities for expanding upon our recommended STADL approach for TSCS data analysis. First, we have not addressed the topic of order specification, focusing instead on source specification. Second, we did not raise the possibility of overfitting STADL models to sample idiosyncrasies. Finally, we have not discussed the implications of measurement error, particularly with respect to $\mathbf{W}$, in spatiotemporally interdependent data. We believe effective approaches to these challenges extend naturally from time-series and spatial econometrics. For instance, autocorrelation and partial autocorrelation (AC, PAC) functions used to guide time-series order specification might be extended to *spatiotemporal* AC and PAC functions. Likewise, cross-validation and out-of-sample performance are the gold-standard safeguards against overfitting and are similarly extendable to spatiotemporal TSCS contexts. On measurement error, various Bayesian strategies of model averaging (Juhl 2020) and/or combining measurement and estimation models seem most promising to us. These projects head our research agenda going forward.

Human subjects

The authors affirm this research did not involve human subjects.

Ethics & conflicts of interest

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Data availability statement

Research documentation and data that support the findings of this study are openly available at the APSR Dataverse at https://doi.org/10.7910/DVN/DBMJQZ

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