Cost configurations

Backup for Mulligan "Prices and Policies in Opioid Markets" *JPE* Appendix

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Setup

Load Economicreasoning package only if it is not already loaded

In[ ]:=
```
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

```
Remove@QEengine;
QEengine[expr_] := Resolve[expr /. C[1, pR, pI, 0, 0] → tmpvar, Reals] /.
   tmpvar → C[1, pR, pI, 0, 0] // Simplify
```

{Q,z} allocations that are outside the various single-price budget sets

In[•]:=
```
outside[Rx] = y < z + fR + Q pR;
outside[im] = y < z + fI + Q pI;(* Im is reserved by the Wolram Language *)
outside[mixed] = y < z + fI + fR + Q C[1, pR, pI, 0, 0];
```

In[•]:=
```
SignConditions =
  {pR > 0, pI > 0, C[1, pR, pI, 0, 0] > 0, fI > fR ≥ 0, y > 0, 0 < Q10 < 1 < Q01, y > fR};
```

Parameters for which Rx-only is part of the upper envelope

The parameters are y, fR, fI, pR, pI, C[1,pR,pI,0,0]. What follows are necessary and sufficient conditions.

In[•]:=
```
dominated[segment_] := Not[notdominated@segment]
```

In[•]:=
```
notdominated[Rx] = QEengine@Exists[{z, Q}, z ≥ 0 ∧ Q ≥ 0,
    Not@outside@Rx ∧
     outside@im ∧ outside@mixed];
```

In[•]:=
```
notdominated[Rx] = FullSimplify[notdominated[Rx], And @@ Most@SignConditions]
```

Out[•]=
```
fR ≤ y || (C[1, pR, pI, 0, 0] < pR &&
    ((pI < pR && ((fI pI + (-fI + fR) C[1, pR, pI, 0, 0] ≤ fR pR && fI pR + pI y ≤ fR pI + pR y) ||
          (fR pR + (fI - fR) C[1, pR, pI, 0, 0] ≤ fI pI &&
            pR (fI + fR - y) + (-fR + y) C[1, pR, pI, 0, 0] ≤ 0))) ||
      (fR pR + (fI - fR) C[1, pR, pI, 0, 0] < fI pI && pR ≤ pI &&
        pR (fI + fR - y) + (-fR + y) C[1, pR, pI, 0, 0] ≤ 0))) ||
  (pI < pR && fI pI + (-fI + fR) C[1, pR, pI, 0, 0] < fR pR &&
    pR ≤ C[1, pR, pI, 0, 0] &&
    fI pR + pI y ≤ fR pI + pR y)
```

Parameters for which Im-only is part of the upper envelope

In[•]:=
```
MixedisIm = fR == 0 == C[1, pR, pI, 0, 0] - pI;
```

In[•]:=
```
notdominated[im] = QEengine@Exists[{z, Q}, z ≥ 0 ∧ Q ≥ 0,
    Not@outside@im ∧
     outside@Rx ∧ (outside@mixed ∨ MixedisIm)];
```

In[•]:=
```
notdominated[im] = FullSimplify[notdominated[im], And @@ Most@SignConditions]
```

Out[•]=
```
(fI pR + pI y < fR pI + pR y && fI ≤ y && ((fR == 0 && pI == C[1, pR, pI, 0, 0]) ||
      pI (fI + fR - y) + (-fI + y) C[1, pR, pI, 0, 0] > 0)) ||
 ((fI pI + (-fI + fR) C[1, pR, pI, 0, 0] < fR pR || pI > pR) && fR > 0 &&
    (pI ≤ pR || fI pI + (-fI + fR) C[1, pR, pI, 0, 0] ≤ fR pR) &&
    pI > C[1, pR, pI, 0, 0] && pI (fI + fR - y) + (-fI + y) C[1, pR, pI, 0, 0] ≤ 0)
```

Parameters for which mixed is part of the upper envelope

In[•]:=
```
notdominated[mixed] = QEengine@Exists[{z, Q}, z ≥ 0 ∧ Q ≥ 0,
     Not@outside@mixed ∧
      outside@im ∧ outside@Rx];
```

In[•]:=
```
notdominated[mixed] =
 FullSimplify[notdominated[mixed], And @@ Most@SignConditions]
```

Out[•]=
```
fI + fR ≤ y && pI (fI + fR - y) + (-fI + y) C[1, pR, pI, 0, 0] < 0 &&
 pR (fI + fR - y) + (-fR + y) C[1, pR, pI, 0, 0] < 0
```

Check whether mixed makes sense even without fixed costs

In[•]:=
```
mixedstatus =
   {pR / pI ≤ Q10 ⇒ C[1, pR, pI, 0, 0] == pR(* Rx-only corner solution *),
    Q10 < pR / pI < Q01 ⇒ (C[1, pR, pI, 0, 0] < pR ∧ C[1, pR, pI, 0, 0] < pI),
    pR / pI ≥ Q01 ⇒ C[1, pR, pI, 0, 0] == pI(* Im-only corner solution *)};
```

```
TheoryDifficulty = 10;
```

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Four zones for pR/pI

four possible configurations are possible depending how pR/pI fits into the interval 0 < Q10 < 1 < Q01 < ∞

```
In[•]:= TheoryGuru[{SignConditions, mixedstatus},
         notdominated@Rx]
Out[•]= True
```

```
In[•]:= TheoryGuru[{SignConditions, mixedstatus},
          (0 < pR/pI ≤ Q10 ∧ dominated@mixed ∧ dominated@im) ∨
           (Q10 < pR/pI ≤ 1 ∧ dominated@im) ∨
           1 < pR/pI < Q01 ∨
           (Q01 ≤ pR/pI ∧ dominated@mixed)]
Out[•]= True

In[•]:= TheoryGuru[{SignConditions[[4]]},
          (dominated@im ∧ dominated@mixed) ∨
           (notdominated@im ∧ dominated@mixed) ∨
           (dominated@im ∧ notdominated@mixed) ∨
           (notdominated@im ∧ notdominated@mixed)]
Out[•]= True
```

Shorthand for critical values

```
In[•]:= yRxmixed = fR + fI pR/(pR - C[1, pR, pI, 0, 0]);

In[•]:= yImmixed = fI + fR pI/(pI - C[1, pR, pI, 0, 0]);

In[•]:= yRxIm = pR/(pR - pI) fI - pI/(pR - pI) fR;

In[•]:= fRcritical = fI (pI - C[1, pR, pI, 0, 0])/(pR - C[1, pR, pI, 0, 0]);

In[•]:= TheoryGuru[{SignConditions[[1 ;; 4]], mixedstatus, 1 < pR/pI < Q01},
          SameSign[fRcritical - fR, yRxmixed - yImmixed]]
Out[•]= True
```

```
In[•]:= TheoryGuru[{pR > pI, SignConditions, mixedstatus},
  (C[1, pR, pI, 0, 0] == pR ∨ yRxmixed > fR + fI) ∧
   (C[1, pR, pI, 0, 0] == pI ∨ yImmixed ≥ fR + fI) ∧
   yRxIm > fI]
Out[•]= True
```

More detail on Q10 $< \frac{pR}{pI} \leq 1$

```
In[•]:= TheoryGuru[{Most@SignConditions, mixedstatus, Q10 < pR/pI ≤ 1},
  (y > yRxmixed ∧ notdominated@mixed) ∨
   (y ≤ yRxmixed ∧ dominated@mixed)]
Out[•]= True
```

More detail on 1 $< \frac{pR}{pI} <$ Q01

The optional evaluations at the end **// Normal // Simplify // Grid** help display the formulas more concisely.

```
In[•]:= TheoryOverlap[{Most@SignConditions, mixedstatus, 1 < pR/pI < Q01},
    fR > fRcritical ∧ y > yRxIm,
    notdominated@im] // Normal // Simplify // Grid
```

```
Out[•]=
{fR > (fI (pI - C[1,pR,pI,0,0])) / (pR - C[1,pR,pI,0,0]) ∧ y > (fR pI - fI pR) / (pI - pR),   are equivalent
 (fI pR + pI y < fR pI + pR y ∧ fI ≤ y ∧ ((fR == 0 ∧ pI == C[1, pR, pI, 0, 0]) ∨
      (y - fI) C[1, pR, pI, 0, 0] + pI (fI + fR - y) > 0)) ∨
  (((fR - fI) C[1, pR, pI, 0, 0] + fI pI < fR pR ∨ pI > pR) ∧ fR > 0 ∧
    (pI ≤ pR ∨ (fR - fI) C[1, pR, pI, 0, 0] + fI pI ≤ fR pR) ∧ pI >
     C[1, pR, pI, 0, 0] ∧ (y - fI) C[1, pR, pI, 0, 0] + pI (fI + fR - y) ≤ 0)}
```

```
In[•]:= TheoryOverlap[{Most@SignConditions, mixedstatus, 1 < pR/pI < Q01},
    y > yRxmixed ∧ y > yImmixed,
    notdominated@mixed]
```

```
Out[•]=
{y > (fR pI) / (pI - C[1,pR,pI,0,0]) + fI ∧ y > (fI pR) / (pR - C[1,pR,pI,0,0]) + fR,   are equivalent
 (y - fI) C[1, pR, pI, 0, 0] + pI (fI + fR - y) < 0 ∧
  (y - fR) C[1, pR, pI, 0, 0] + pR (fI + fR - y) < 0 ∧ fI + fR ≤ y}
```

In[•]:=
```
TheoryOverlap[{Most@SignConditions, mixedstatus, 1 < pR/pI < Q01},
  y > yRxIm ∧ y > yImmixed ∧ fR > fRcritical,
  notdominated@mixed ∧ notdominated@im] // Normal // Simplify // Grid
```

Out[•]=

$\left\{fR > \frac{fI\,(pI - C[1,pR,pI,0,0])}{pR - C[1,pR,pI,0,0]} \wedge y > \frac{fR\,pI - fI\,pR}{pI - pR} \wedge y > \frac{fR\,pI}{pI - C[1,pR,pI,0,0]} + fI,\right.$ are equivalent

$(y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) < 0 \wedge$
$(y - fR)\,C[1, pR, pI, 0, 0] + pR\,(fI + fR - y) < 0 \wedge fI + fR \le y \wedge$
$((fR = 0 \wedge pI = C[1, pR, pI, 0, 0] \wedge fI\,pR + pI\,y < fR\,pI + pR\,y \wedge fI \le y) \vee$
$(fR > 0 \wedge pI > C[1, pR, pI, 0, 0] \wedge$
$((fR - fI)\,C[1, pR, pI, 0, 0] + fI\,pI < fR\,pR \vee$
$(pI > pR \wedge (fR - fI)\,C[1, pR, pI, 0, 0] + fI\,pI \le fR\,pR))))\Big\}$

More detail on Q01 $\le \frac{pR}{pI}$

In[•]:=
```
TheoryOverlap[{SignConditions, mixedstatus, Q01 ≤ pR/pI},
  y > yRxIm,
  notdominated@im ∧ dominated@mixed] // Normal // Simplify // Grid
```

Out[•]=

$\left\{y > \frac{fR\,pI - fI\,pR}{pI - pR},\right.$ are equivalent

$(fI\,pR + pI\,y < fR\,pI + pR\,y \wedge fI \le y \wedge ((fR = 0 \wedge pI = C[1, pR, pI, 0, 0] \wedge$
$((y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) \ge 0 \vee$
$(y - fR)\,C[1, pR, pI, 0, 0] + pR\,(fI + fR - y) \ge 0 \vee fI + fR > y)) \vee$
$(y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) > 0)) \vee$
$(((y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) = 0 \vee$
$(y - fR)\,C[1, pR, pI, 0, 0] + pR\,(fI + fR - y) \ge 0 \vee fI + fR > y) \wedge$
$(y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) \le 0 \wedge$
$((fR - fI)\,C[1, pR, pI, 0, 0] + fI\,pI < fR\,pR \vee pI > pR) \wedge$
$(fR - fI)\,C[1, pR, pI, 0, 0] + fI\,pI \le fR\,pR \wedge$
$fR > 0 \wedge pI > C[1, pR, pI, 0, 0])\Big\}$

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The upper envelope is only one piece if pR/pI < Q10

More specifically, both Im and mixed are dominated.

```
In[•]:= TheoryGuru[{{SignConditions, mixedstatus, pR/pI < Q10},
  notdominated@Rx ∧ dominated@im ∧ dominated@mixed}]
Out[•]= True
```

If income is great enough and fR>0, the upper envelope consists of two pieces (as in Figure 1b) if pR/pI ≥ Q01 or Q10 < pR/pI ≤ 1,

```
In[•]:= upperenvisRxonlyandImonly = notdominated@Rx ∧ notdominated@im ∧ dominated@mixed;
(* one version of "two pieces" *)
```

```
In[•]:= upperenvisRxonlyandmixed = notdominated@Rx ∧ dominated@im ∧ notdominated@mixed;
(* another version of "two pieces" *)
```

```
In[•]:= TheoryGuru[{{fR > 0, y > yRxmixed, pR/pI > Q01 ∨ Q10 < pR/pI ≤ 1,
   mixedstatus, Most@SignConditions},
  upperenvisRxonlyandmixed ∨ upperenvisRxonlyandImonly}]
Out[•]= True
```

The mixed (Im-only) constraint is dominated by the other two in the former (latter) case, respectively.

```
In[•]:= TheoryGuru[{{fR > 0, y > yRxmixed, pR/pI > Q01,
   mixedstatus, SignConditions⟦{1, 2, 4, 6}⟧},
  dominated@mixed}]
Out[•]= True
```

```
In[•]:= TheoryGuru[{{fR > 0, y > yRxmixed, Q10 < pR/pI ≤ 1,
   mixedstatus, Most@SignConditions},
  dominated@im}]
Out[•]= True
```

The remaining interval is where three pieces are possible when fR > 0, with the mixed piece forming the upper envelope at the highest quantities

In[•]:=
```
TheoryGuru[{SignConditions, mixedstatus,
  notdominated@Rx, notdominated@im, notdominated@mixed},
 1 < pR/pI < Q01 ∧ fR > 0]
```

Out[•]= True

When fR is sufficiently close to zero, the only difference is that the interval 1 < pR/pI < Q01 cannot have three pieces because the Im-only piece is dominated by mixed consumption.

In[•]:=
```
TheoryGuru[{0 ≤ fR ≤ fRcritical, SignConditions, mixedstatus,
  1 < pR/pI < Q01},
 dominated@im]
```

Out[•]= True

The case aggregating to equation (2) is $Q_{01} \le \frac{pR}{pI} \wedge y > \frac{fI\,pR}{pR-pI} \wedge fR == 0$.

In[•]:=
```
Eqn2 = fR == 0 ∧ dominated@mixed ∧ notdominated@im // Simplify;
```

In[•]:=
```
TheoryOverlap[{SignConditions, mixedstatus},
 Eqn2,
 Q01 ≤ pR/pI ∧ y > fI pR/(pR - pI) ∧ fR == 0] // Normal // Simplify // Grid
```

Out[•]=

$\{fR = 0 \wedge fI\,pR + pI\,y < fR\,pI + pR\,y \wedge fI \le y \wedge ((pI = C[1, pR, pI, 0, 0] \wedge$	are equivalent
$((y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) \ge 0 \vee$	
$(y - fR)\,C[1, pR, pI, 0, 0] + pR\,(fI + fR - y) \ge 0 \vee fI + fR > y)) \vee$	
$(y - fI)\,C[1, pR, pI, 0, 0] + pI\,(fI + fR - y) > 0)$,	
$fR = 0 \wedge y > \frac{fI\,pR}{pR-pI} \wedge Q01 \le \frac{pR}{pI}\}$	

A second case would differ only in replacing pI with C(1,pR,pI;0,0).

In[•]:=
```
Eqn2alt = fR == 0 ∧ notdominated@mixed ∧ dominated@im // Simplify;
```

In[•]:=
```
TheoryOverlap[{SignConditions, mixedstatus},
  Eqn2alt,
  Q10 < pR/pI < Q01 ∧ y > yRxmixed ∧ fR == 0] // Normal // Simplify // Grid
```

Out[•]=

$$\left\{\begin{array}{l} \mathrm{fR} = 0 \wedge (\mathrm{pI} \neq C[1, \mathrm{pR}, \mathrm{pI}, 0, 0] \vee \mathrm{fI}\,\mathrm{pR} + \mathrm{pI}\,y \geq \mathrm{fR}\,\mathrm{pI} + \mathrm{pR}\,y \vee \mathrm{fI} > y) \wedge \\ \quad (y - \mathrm{fI})\,C[1, \mathrm{pR}, \mathrm{pI}, 0, 0] + \mathrm{pI}\,(\mathrm{fI} + \mathrm{fR} - y) < 0 \wedge \\ \quad (y - \mathrm{fR})\,C[1, \mathrm{pR}, \mathrm{pI}, 0, 0] + \mathrm{pR}\,(\mathrm{fI} + \mathrm{fR} - y) < 0 \wedge \mathrm{fI} + \mathrm{fR} \leq y, \\ \mathrm{fR} = 0 \wedge y > \frac{\mathrm{fI}\,\mathrm{pR}}{\mathrm{pR} - C[1,\mathrm{pR},\mathrm{pI},0,0]} + \mathrm{fR} \wedge \frac{\mathrm{pR}}{\mathrm{pI}} < \mathrm{Q01} \wedge \mathrm{Q10} < \frac{\mathrm{pR}}{\mathrm{pI}} \end{array}\right\} \quad \text{are equivalent}$$