Theory Backup for Mulligan "Prices and Policies in Opioid Markets" *JPE*

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Setup

In[ ]:=
```
(* Load Economicreasoning package only if it is not already loaded *)
If[Length@Names["PLTools`*"] < 10,
 Get["http://economicreasoning.com"]]
```

Proof & Logic Tools 6.4

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Type ERCommands for a list of commands in the package.

Introduction to Automated Economic Reasoning		
Tutorials:	Entering calculus	General Mathematica tips
Get started	Load extras	Browse examples

```
f[pR_, pI_] := v[pI, y] - v[pR, y] (* in the text, f*[pR,pI] *)
```

```
d[pR_, pI_, θ_] := θ H[pR] + θ F[θ f[pR, pI]] × (H[pI] - H[pR])
```

```
rRoysId = {v^(1,0)[p_, _] :> -H[p], v^(0,1)[_, _] :> constant};
```

```
RoySign[p1_, p2_] := SameSign[p2 - p1, v[p1, y] - v[p2, y]]
```

```
commonpricechange = Dt@pR == Dt@pI ≠ 0 == Dt@θ;
```

```
Impricechange = Dt@pI ≠ 0 == Dt@pR == Dt@θ;
```

```
Prefchange = Dt@θ ≠ 0 == Dt@pR == Dt@pI;
```

```
SignConditions =
  {H[pI] > 0, H'[pI] < 0 , H[pR] > 0, H'[pR] < 0, θ > 0, 0 ≤ F[θ f[pR, pI]] ≤ 1,
   RoySign[pR, pI], SameSign[pR - pI, H[pI] - H[pR]], F'[θ f[pR, pI]] ≥ 0};
(* SameSign[pR-pI,v[pI,y]-v[pR,y]] follows from Roy's
 Identity and the reamining conditions, each which appears in the
 paragraph beginning "Let there be a continuum of consumers...." *)
```

Let there be a continuum of consumers who differ only in terms of their Im fixed cost f_I. All consumers face the same marginal prices $\{p_R,p_I\}$ and have the same preferences for Q versus other goods. $F(x) \in [0,1]$ is the fraction of consumers with $f_I \leq x$ and the corresponding density function is $F'(x) \geq 0$. Its upper support is denoted $\bar{x} > 0$. Let $\theta f^*(p_R,p_I) \equiv \theta[v(p_I,y) - v(p_R,y)]$ denote the critical value of the Im fixed cost that leaves the consumer indifferent between sourcing from Rx and Im. Without income effects on opioid demand, both $f^*(p_R,p_I)$ and the price derivative of v are independent of y. The latter is $-H(p) < 0$ with $H'(p) < 0$, so that the consumer's Hicksian demand function is $\theta H(p)$. It follows that $\theta f^*(p_R,p_I)$ is the area under $\theta H(p)$ between the prices p_I and p_R. The fraction of consumers sourcing from Im rather than Rx is therefore $F(\theta f^*(p_R,p_I))$, with each demanding $Q = \theta H(p_I)$ individually and $D_I = F(\theta f^*(p_R,p_I))\theta H(p_I)$ in aggregate. The remaining consumers demand $Q = \theta H(p_R)$ individually and $D_R = [1-F(\theta f^*(p_R,p_I))]\theta H(p_R)$ in aggregate. Each proposition that follows assumes that $H(p_I)$ and $H(p_R)$ are strictly positive.

```
PositivePrices = {pR > 0, pI > 0};(* See setup of choice problem (1) *)
```

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Lemma 2

LEMMA 2 (Market-level comparative statics). Let $D(p_R,p_I,\theta)$ denote aggregate opioid consumption as a function of the two marginal prices and a common demand parameter θ

$$D(p_R,p_I,\theta) \equiv F\big(\theta f^*(p_R,p_I)\big)\theta H(p_I) + \big[1 - F\big(\theta f^*(p_R,p_I)\big)\big]\theta H(p_R) \tag{2}$$

In the neighborhood of $\theta = 1$, the comparative statics of aggregate opioid demand are

$$\begin{aligned} dD(p_R,p_I,\theta)|_{\theta=1} &= \big[1 - F\big(f^*(p_R,p_I)\big)\big]H'(p_R)dp_R + F\big(f^*(p_R,p_I)\big)H'(p_I)dp_I + D(p_R,p_I,1)d\theta \\ &+ [H(p_I) - H(p_R)]F'\big(f^*(p_R,p_I)\big)[H(p_R)dp_R - H(p_I)dp_I + f^*(p_R,p_I)d\theta] \end{aligned} \tag{3}$$

Proof. Totally differentiate (2) and evaluate at $\theta = 1$ to arrive at equation (3). QED

In[•]:=
```
Lemma2TotalDeriv = (1 - F[θ f[pR, pI]]) H'[pR] × Dt@pR +
   F[θ f[pR, pI]] × H'[pI] × Dt@pI + d[pR, pI, θ] × Dt@θ +
   (H[pI] - H[pR]) × F'[θ f[pR, pI]] ×
    (H[pR] × Dt@pR - H[pI] × Dt@pI + f[pR, pI] × Dt@θ);
```

In[•]:=
```
Dt@d[pR, pI, θ] == Lemma2TotalDeriv /. rRoysId /. {Dt@θ → Dt@z, θ → 1} /. Dt@z → Dt@θ //
 Simplify
```

Out[•]= True

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Lemma 2

Proposition 1

In[•]:=
```
qI[pR_, pI_, θ_] := F[θ f[pR, pI]] θ H[pI]
```

In[•]:=
```
qR[pR_, pI_, θ_] := (1 - F[θ f[pR, pI]]) θ H[pR]
```

In[•]:=
```
d[pR, pI, θ] == qI[pR, pI, θ] + qR[pR, pI, θ] // Simplify
```

Out[•]=
```
True
```

PROPOSITION 1 (Equivalence across price changes). Assuming a common demand parameter normalized to one,

```
In[•]:= qI^(1,0,0)[pR, pI, 1] + qI^(0,1,0)[pR, pI, 1] == d^(0,1,0)[pR, pI, 1] /. rRoysId // Simplify
Out[•]= True

In[•]:= qR^(1,0,0)[pR, pI, 1] + qR^(0,1,0)[pR, pI, 1] == d^(1,0,0)[pR, pI, 1] /. rRoysId // Simplify
Out[•]= True
```

The switching term from the Im-price change has the same magnitude as the switching term from a Rx-price change multiplied by the ratio of Hicksian demands H(pI)/H(pR).

```
In[•]:= switchingtermfrom[p_] := Last@# × Fold[Coefficient, Lemma2TotalDeriv, #] &@
          {Dt@p, F'[θ (v[pI, y] - v[pR, y])](* switching term iff F' appears *)};

In[•]:= switchingtermfrom[pI] == -switchingtermfrom[pR] H[pI]/H[pR] // Simplify
Out[•]= True
```

From equation (3), the switching term for a common price change is
$-(H[pI] - H[pR])^2 F'[v[pI, y] - v[pR, y]]\, dpI$.
... The switching term for a demand shift,
$(H[pI] - H[pR]) \times (v[pI, y] - v[pR, y])\, F'[v[pI, y] - v[pR, y]]$

Recall that equation (3) has $\theta == 1$

```
In[•]:= switchingtermfrom[pR] + switchingtermfrom[pI] ==
          -(H[pI] - H[pR])^2 F'[f[pR, pI]] /. θ → 1 // Simplify
Out[•]= True

In[•]:= switchingtermfrom[θ] == (H[pI] - H[pR]) × (v[pI, y] - v[pR, y]) × F'[f[pR, pI]] /. θ → 1 //
        Simplify
Out[•]= True
```

Both switching terms are zero when the two prices are equal (pR = pI) or no consumers are on the source margin (F' = 0)

```
In[•]:= TheoryGuru[{pR == pI ∨ F'[θ f[pR, pI]] == 0,
          SignConditions⟦-3 ;; -2⟧},
        switchingtermfrom[θ] == 0 == switchingtermfrom[pR] + switchingtermfrom[pI]]
Out[•]= True
```

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In[•]:=
```
qR[pR, pI, θ]/d[pR, pI, θ] == (1 - F[θ f[pR, pI]])/d[pR, pI, θ] θ H[pR](* confirm the formula for r *) // Simplify
```

Out[•]=
```
True
```

In[•]:=
```
Prop2assume = {pR > pI, F'[θ f[pR, pI]] > 0,
   H'[pI]/H[pI] - H'[pR]/H[pR] ≤ 0 (* weaker assumption than "close enough to 0" *)};
```

In[•]:=
```
TheoryDifficulty = 5;
```

PROPOSITION 2 (Overall consumption and its Rx share change in opposite directions). Assume $p_R > p_I$ and $F' > 0$. The comparative statics for opioid consumption $D(p_R,p_I,\theta)$ and the Rx quantity share $r \equiv \frac{1-F(\theta f^*(p_R,p_I))}{D(p_R,p_I,\theta)}\theta H(p_R)$ have opposite signs if (a) $H'(p_I)/H(p_I) - H'(p_R)/H(p_R)$ is sufficiently close to zero and (b) the impulse is any one of: $dp_I = dp_R \neq 0 = d\theta$ (common price change), $dp_I \neq dp_R = 0 = d\theta$ (Im price change), or $dp_I = dp_R = 0 \neq d\theta$ (preference change).

Proof. From Lemma 2 and $p_R > p_I$, a common price reduction, an Im price reduction, or an increase in the demand parameter must each increase $D(p_R,p_I,\theta)$. Totally differentiating the definition of r (see the on-line appendix) shows that the Im price decrease and θ increase also reduce r. The semi-elasticity restriction (a) yields the same for a common price increase. QED

In[•]:=
```
TheoryGuru[{Prop2assume, SignConditions[[3 ;; -2]],
  commonpricechange ∨ Impricechange ∨ Prefchange},
 SameSign[-Lemma2TotalDeriv, Dt[qR[pR, pI, θ]/d[pR, pI, θ]]] /. rRoysId]
```

Out[•]=
```
True
```

θ f[pR,pI] can be increased by raising θ, reducing pI alone, or by reducing both prices together.

In[•]:=
```
Impricereduction = Dt@pI < 0 == Dt@pR == Dt@θ;
```

In[•]:=
```
commonpricereduction = Dt@pR == Dt@pI < 0 == Dt@θ;
```

In[•]:=
```
Prefincrease = Dt@θ > 0 == Dt@pR == Dt@pI;
```

In[•]:=
```
TheoryGuru[{Prefincrease ∨ Impricereduction ∨ commonpricereduction,
  SignConditions[[{3, 5, 7, 8}]], First@Prop2assume},
 Dt[θ f[pR, pI]] > 0 /. rRoysId,
 showdrops → "Unilateral"]
```

```
All assumptions are needed.
```

Out[•]=
```
True
```

From Lemma 2 and pR > pI, a common price reduction, an Im price reduction, or an increase in the

demand parameter must each increase D(pR,pI,).

```
In[•]:= TheoryGuru[{Prefincrease ∨ Impricereduction ∨ commonpricereduction,
   SignConditions[[2 ;; -2]], Most@Prop2assume},
  Dt@d[pR, pI, θ] > 0 /. rRoysId,
  showdrops → "Unilateral"]
```

```
All assumptions are needed.
```

```
Out[•]= True
```

the Im price decrease and θ increase also reduce r.

```
In[•]:= TheoryGuru[{Prefincrease ∨ Impricereduction,
   SignConditions[[2 ;; -2]], Most@Prop2assume},
  Dt[qR[pR, pI, θ] / d[pR, pI, θ]] < 0 /. rRoysId]
```

```
Out[•]= True
```

```
In[•]:= TheoryGuru[{commonpricereduction,
   SignConditions[[2 ;; -2]], Prop2assume(*last of these is semi-elas condition *)},
  Dt[qR[pR, pI, θ] / d[pR, pI, θ]] < 0 /. rRoysId]
```

```
Out[•]= True
```

Neither common price changes nor changes in the demand parameter are neutral with respect to the composition of opioid consumption.

The Proposition 2 assumptions guarantee this, but even without those assumptions non-neutrality is possible (if not likely).

```
In[•]:= TheoryGuru[{Prop2assume, SignConditions[[3 ;; -2]],
   commonpricechange ∨ Prefchange},
  Dt[qR[pR, pI, θ] / d[pR, pI, θ]] ≠ 0 /. rRoysId]
```

```
Out[•]= True
```

```
In[•]:= TheoryGuru[{Most@Prop2assume, SignConditions[[3 ;; -2]],
   Prefchange},
  Dt[qR[pR, pI, θ] / d[pR, pI, θ]] ≠ 0 /. rRoysId]
```

```
Out[•]= True
```

In[•]:=
```
Not@TheoryEmpty[{SignConditions,
    commonpricechange ∨ Prefchange,
    Dt[qR[pR, pI, θ] / d[pR, pI, θ]] ≠ 0} /. rRoysId]
```

Out[•]=
```
True
```

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```
In[•]:= prop2appendixterm1 = -1/(1 - F[θ f[pR, pI]]) θ H[pI]/d[pR, pI, θ]
          F'[θ f[pR, pI]] × (f[pR, pI] × Dt@θ - θ (H[pI] × Dt@pI - H[pR] × Dt@pR));
```

```
In[•]:= prop2appendixterm2 = (1 - qR[pR, pI, θ]/d[pR, pI, θ]) H'[pR]/H[pR] (Dt@pR - H'[pI] / H[pI] / (H'[pR] / H[pR]) Dt@pI);
```

```
In[•]:= Dt@Log[qR[pR, pI, θ]/d[pR, pI, θ]] == prop2appendixterm1 + prop2appendixterm2 /. rRoysId // Simplify
Out[•]= True
```

The final expression has two terms on the RHS, each with negative coefficients

```
In[•]:= Delete[#, Position[#, Dt@pR][[1, 1]]] &@prop2appendixterm1
        (* this is the coefficient *)
        TheoryGuru[{SignConditions[[{1, 3, 6}]], Prop2assume[[2]]},
         % < 0]
```

$$\text{Out[•]}= -\frac{\theta\, H[pI]\, F'[\theta\,(v[pI, y] - v[pR, y])]}{(1 - F[\theta\,(v[pI, y] - v[pR, y])])\,(\theta\, F[\theta\,(v[pI, y] - v[pR, y])] \times (H[pI] - H[pR]) + \theta\, H[pR])}$$

```
Out[•]= True
```

```
In[•]:= Delete[#, Position[#, Dt@pR][[1, 1]]] &@prop2appendixterm2
        (* this is the coefficient *)
        TheoryGuru[SignConditions[[{1, 3, 4, 6}]],
         % < 0 ∨
           (% == 0 ∧ F[θ f[pR, pI]] == 0)]
```

$$\text{Out[•]}= \frac{\left(1 - \frac{\theta\,(1-F[\theta\,(v[pI,y]-v[pR,y])])\,H[pR]}{\theta\, F[\theta\,(v[pI,y]-v[pR,y])] \times (H[pI]-H[pR]) + \theta\, H[pR]}\right) H'[pR]}{H[pR]}$$

```
Out[•]= True
```

Everywhere the Im price change appears inside in both terms with its own negative coefficient to multiply the aforementioned negative coefficients.

```
In[•]:= And @@ Thread[
          Coefficient[#, Dt@pI]
        ------------------------------ & /@ {prop2appendixterm1, prop2appendixterm2} <
        Delete[#, Position[#, Dt@pI][[1, 1]]]
         0] (* Dt@pI coefficient inside each *)
      TheoryGuru[SignConditions[[1 ;; 5]], %]
```

$$\text{Out[•]=}\quad -\ominus H[pI] < 0 \;\&\&\; -\frac{H[pR]\,H'[pI]}{H[pI]\,H'[pR]} < 0$$

```
Out[•]= True
```

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PROPOSITION 3 (Price interactions in demand). Evaluated at $\theta = 1$, the cross-price derivative of the aggregate demand function $D(p_R,p_I,1)$ is:

$$\frac{\partial^2 D(p_R,p_I,1)}{\partial p_R \partial p_I} = \left\{ F'\left(f^*(p_R,p_I)\right)\left[\frac{H'(p_R)}{H(p_R)} + \frac{H'(p_I)}{H(p_I)}\right] - F''\left(f^*(p_R,p_I)\right)[H(p_I) - H(p_R)]\right\} H(p_R)H(p_I) \tag{4}$$

Proof. Calculate $\frac{\partial D(p_R,p_I,1)}{\partial p_I}$ from (3) and then partially differentiate with respect to p_R. QED

```
In[•]:= d^(1,1,0)[pR, pI, 1] == H[pR] × H[pI] × ( F'[f[pR, pI]] ( H'[pR]/H[pR] + H'[pI]/H[pI] ) -
          F''[f[pR, pI]] × (H[pI] - H[pR]) ) /. rRoysId // Simplify
Out[•]= True
```

The cross derivative (4) is negative unless the density changes sufficiently in the right direction to offset equation (4)'s first term in braces.

Here are two definitions of not "chang[ing] sufficiently":

```
In[•]:= TheoryGuru[{SignConditions[{1, 2, 3, 4, 8, 9}] /. θ → 1,
          F''[f[pR, pI]] > (H'[pR]/H[pR] + H'[pI]/H[pI])/(H[pI] - H[pR]) F'[f[pR, pI]] ∧ pR ≥ pI ∨
          F''[f[pR, pI]] < (H'[pR]/H[pR] + H'[pI]/H[pI])/(H[pI] - H[pR]) F'[f[pR, pI]] ∧ pR < pI},
        d^(1,1,0)[pR, pI, 1] < 0 ∧
          ( H'[pR]/H[pR] + H'[pI]/H[pI] ) F'[f[pR, pI]] ≤ 0 /. rRoysId]
Out[•]= True
```

```
In[•]:= TheoryGuru[{SignConditions[{1, 2, 3, 4}], F'[f[pR, pI]] > 0,
          F''[f[pR, pI]] == 0},
        d^(1,1,0)[pR, pI, 1] < 0 /. rRoysId]
Out[•]= True
```

Corollary

In[•]:=
```
pointsr[p_] := H'[p]/H[p] p
```

COROLLARY (Bounding price-effect differentials). Let *POINTSR* denote the price elasticity of H evaluated at p_R. If $F'' \geq 0$, $POINTSR \geq p_I \frac{H'(p_I)}{H(p_I)}$ and $\frac{\partial D(p_R,p_I,1)}{\partial p_R} \geq 0$, then the effect of p_R on the Im-price effect is bounded by (5)

$$\frac{\partial^2 D(p_R,p_I,1)}{\partial p_R \partial \ln p_I} \Big/ \frac{\partial D_R}{\partial p_R} \geq -\left(1+\frac{p_I}{p_R}\right) POINTSR > 0 \qquad (5)$$

In[•]:=
```
TheoryGuru[{SignConditions[[{3, 4, 6, 9}]] /. θ → 1, PositivePrices,
    F''[f[pR, pI]] ≥ 0, pointsr[pR] ≥ pointsr[pI], d^(1,0,0)[pR, pI, 1] ≥ 0} /. rRoysId /.
  θ → 1,
 d^(1,1,0)[pR, pI, 1] pI / qR^(1,0,0)[pR, pI, 1] ≥ -(1 + pI/pR) pointsr[pR] > 0 /. rRoysId]
```

Out[•]=
```
True
```

Long-run definition

Proposition 4

Short-run definition

Proposition 5

Proposition 6

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On-line Appendix: Short- and long-run demand with a wider range of price changes

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Long-run definition

Demand is more price elastic in the long run by a factor of $\left[1-\frac{\theta'[h[p]]\times h[p]}{\theta[h[p]]}\right]^{-1}\geq 1$, with equality only when θ is constant.

In[•]:=
```
DefineLR[p_] := h[p] == Θ[h[p]] × H[p]
```

In[•]:=
```
pointlr[p_] := h'[p]/h[p] p
```

In[•]:=
```
AssumeΘ[p_] := {Θ[h[p]] > 0, 0 ≤ Θ'[h[p]] × h[p]/Θ[h[p]] < 1, 0 < Θ'[h[p]] × h[p]/Θ[h[p]] < 1};
(* "function Θ() > 0 has elasticity in the interval [0,1)";
in context, the strict version is used for some results *)
```

In[•]:=
```
AssumeΘdiscrete[pR_, pI_] := {Θ[h[pR]] > 0, Θ[h[pI]] > 0,
   SameSign[Θ[h[pI]] - Θ[h[pR]], h[pI] - h[pR]],
   SameSign[Θ[h[pI]]/h[pI] - Θ[h[pR]]/h[pR], h[pR] - h[pI]]};
 (* discrete version of AssumeΘ *)
```

In[•]:=
```
TheoryGuru[{DefineLR[p], DefineLR'[p], AssumeΘ[p][[2]], H'[p] < 0, H[p] > 0, p > 0},
 pointlr[p] == pointsr[p]/(1 - Θ'[h[p]]×h[p]/Θ[h[p]]) ≤ pointsr[p] < 0]
```

Out[•]= True

In[•]:=
```
TheoryOverlap[{DefineLR[p], DefineLR'[p], AssumeΘ[p][[2]], H'[p] < 0, H[p] > 0, p > 0},
 Θ'[h[p]] == 0,
 pointlr[p] == pointsr[p]]
```

Out[•]= $\left\{\Theta'(h(p)) = 0, \frac{p\,H'(p)}{H(p)} = \frac{p\,h'(p)}{h(p)}\right\}$ are equivalent

In[•]:=
```
criticalfixedcostat[pR_, pI_] := Θ[h[pR]] × f[pR, pI]
```

In[•]:=
```
dss[pR_, pI_] := h[pR] + F[criticalfixedcostat[pR, pI]] × (h[pI] - h[pR]);
```

Proposition 4

Short-run definition

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Proposition 6

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In[•]:=
```
DefineUpper[x_] = x ≥ upper ⇔ F[x] == 1;(* define upper support *)
```

In[•]:=
```
Prop4assume =
  {upper/Θ[h[pHI]] < v[0, y] - v[pHI, y], upper/Θ[h[pHI]] < Θ[h[pLO]] (v[pLO, y] - v[pHI, y])/(Θ[h[pLO]] - Θ[h[pHI]])};
```

In[•]:=
```
SignConditions4a =
  {pI ≥ pHI ⇒ F[criticalfixedcostat[pHI, pI]] == 0(* lower support > 0 *),
   pI ≥ pLO ⇒ F[criticalfixedcostat[pLO, pI]] == 0(* lower support > 0 *),
   SameSign[pHI - pLO, h[pLO] - h[pHI]](* LR demand slopes down *)};
SignConditions4b =
  {SameSign[pI - pLO, h[pLO] - h[pI]](* LR demand slopes down *),
   RoySign[pLO, pI](* v slopes down in price *),
   DefineUpper[criticalfixedcostat[pHI, pI]],
   DefineUpper[criticalfixedcostat[pLO, pI]],
   0 ≤ F[criticalfixedcostat[pLO, pI]] ≤ 1};
```

In[•]:=
```
SignConditions4c = {RoySign[0, pI], RoySign[pLO, pI], RoySign[pLO, pHI], upper > 0};
```

PROPOSITION 4 (Group ranks reversed by Im price changes). Fix the Rx prices paid by each group. If $\frac{\bar{x}}{\theta(h(p_{HI}))} < \min\left\{\lim_{p_I \to 0} v(p_I, y) - v(p_{HI}, y), \theta\big(h(p_{LO})\big)\frac{v(p_{LO},y)-v(p_{HI},y)}{\theta(h(p_{LO}))-\theta(h(p_{HI}))}\right\}$, then

(a) At any common Im price no less than p_{HI}, the steady-state average consumption gap between the groups is $h(p_{LO})$–$h(p_{HI}) > 0$, with the low-cost group consuming more.

(b) There exists another common Im price in the interval $(0,p_{LO})$ such that the high-cost group consumes more opioids than the low-cost group in the steady state.

Proposition 4a

In[•]:=
```
TheoryGuru[{SignConditions4a, pI ≥ pHI > pLO(* "Im price no less than pHI *)},
 dss[pHI, pI] == h[pHI] < h[pLO] == dss[pLO, pI]]
```

Out[•]=
```
True
```

Proposition 4b

In[•]:=
```
ValidLowpI =
  {v[pI, y] > v[pLO, y], upper/Θ[h[pLO]] + v[pLO, y] > v[pI, y] > upper/Θ[h[pHI]] + v[pHI, y]};
 (* example pI is low enough to switch all Group 1 and be below pLO,
but not switching all of Group 0 *)
```

Average steady-state consumption is h(pI) for the high-cost group and in the interval [h[pLO],h[pI]) for the low-cost group.

In[•]:=
```
TheoryGuru[{SignConditions4b, ValidLowpI,
  Θ[h[pLO]] > 0, Θ[h[pHI]] > 0},
 h[pI] == dss[pHI, pI] > dss[pLO, pI] ∧
  h[pLO] ≤ dss[pLO, pI] < h[pI]]
```

Out[•]= True

Proposition 4's assumption guarantees that required interval ValidLowpI is not empty

In[•]:=
```
TheoryGuru[{Prop4assume, SignConditions4c[[3 ;; 4]], pLO < pHI, First@AssumeΘ[pLO]},
  upper/Θ[h[pLO]] + v[pLO, y] > upper/Θ[h[pHI]] + v[pHI, y] ∧
   upper/Θ[h[pLO]] + v[pLO, y] > v[pLO, y]]
```

Out[•]= True

For all valid {pHI,pLO,upper,h[pHI],h[pLO],v[0,y],v[pHI,y],v[pLO,y],Θ[h[pHI]],Θ[h[pLO]]}, there must be a pI in the interval ValidLowpI that is also in (0,pLO)

In[•]:=
```
crosswalk = Thread[{v[0, y], v[pHI, y], v[pI, y],
     v[pLO, y], Θ[h[pHI]], Θ[h[pLO]]} → {z1, z2, z3, z4, z5, z6}];
```

In[•]:=
```
Remove@QEengine;
QEengine[expr_] :=
 Resolve[expr /. crosswalk, Reals] /. (Reverse /@ crosswalk) // Simplify
```

In[•]:=
```
ExistsPositivepI = Exists[{pI, v[pI, y]}, And @@ ValidLowpI,
    0 < pI < pLO] // QEengine;
```

```
In[•]:= TheoryGuru[{Prop4assume, SignConditions4c[[3 ;; 4]], Assume©discrete[pLO, pHI],
          SameSign[pHI - pLO, h[pLO] - h[pHI]], h[pLO] > 0, h[pHI] > 0, pHI > pLO > 0},
         ExistsPositivepI]
Out[•]= True
```

Short-run definition

Proposition 5

Proposition 6

Main Appendix

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Short-run definition

{pR,pI} are the new prices.
{pR0,pI0} are the lagged prices, which were in a steady state.
With f[pR,pI] ≥ f[pR0,pI0], nobody switch from Im to Rx.

```
In[•]:= shRxtoRx = 1 - F[Θ[h[pR0]] × f[pR, pI]];
        shRxtoIm = F[Θ[h[pR0]] × f[pR, pI]] - F[criticalfixedcostat[pR0, pI0]];
        shImtoIm = F[criticalfixedcostat[pR0, pI0]];
```

```
In[•]:= shRxtoRx + shRxtoIm + shImtoIm == 1
Out[•]= True
```

```
In[•]:= Δ[pR_, pI_, pR0_, pI0_] = shRxtoRx Θ[h[pR0]] × H[pR] + shRxtoIm Θ[h[pR0]] × H[pI] +
          shImtoIm Θ[h[pI0]] × H[pI];
```

```
In[•]:= ΔI[pR_, pI_, pR0_, pI0_] = shRxtoIm Θ[h[pR0]] × H[pI] + shImtoIm Θ[h[pI0]] × H[pI];
```

Confirm equation (7)

```
In[•]:= Δ[pR, pI, pR0, pI0] ==
          Θ[h[pR0]] × H[pR] + F[Θ[h[pR0]] × f[pR, pI]] × Θ[h[pR0]] × (H[pI] - H[pR]) +
           F[criticalfixedcostat[pR0, pI0]] × (Θ[h[pI0]] - Θ[h[pR0]]) × H[pI] // Simplify
Out[•]= True
```

The steady-state version of equation (7) is Δ[pR,pI,pR,pI], which is identical to the expression (6) because long- and short-run demand are related according to h[p]==Θ[h[p]]H[p]

```
In[•]:= TheoryGuru[{DefineLR /@ {pR, pI}},
         Δ[pR, pI, pR, pI] == dss[pR, pI]]
Out[•]= True
```

d[pR,pI,θ] (equation (2)) is itself the special case of Δ[pR,pI,pR,pI] in which the taste parameter is independent of past prices and constant across consumers.

```
In[•]:= rconstantθ = {Θ[_] :→ θ};
```

```
In[•]:= Δ[pR, pI, pR, pI] == d[pR, pI, θ] /. rconstantθ // Simplify
Out[•]= True
```

Proposition 5

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In[•]:=
```
crosssr = ΔI^(1,0,0,0)[pR, pI, pR, pI] pR / ΔI[pR, pI, pR, pI];
```

In[•]:=
```
arcsr = (1 - H[pR]/H[pI]) / (1 - pR/pI);
```

In[•]:=
```
pointsr[p_] := H'[p] p / H[p]
```

In[•]:=
```
CDFConditions =
  {0 ≤ F[criticalfixedcostat[pR, pI]] ≤ 1, F'[criticalfixedcostat[pR, pI]] ≥ 0};
```

In[•]:=
```
TheoryGuru[{SignConditions[[{1, 3}]], First@PositivePrices, ΔI[pR, pI, pR, pI] > 0,
   Last@CDFConditions},
 crosssr ≥ 0 /. rRoysId]
```

Out[•]= True

In[•]:=
```
TheoryGuru[{pR ≠ pI, SignConditions[[{1, 3, 4, 8}]], PositivePrices},
 arcsr < 0 ∧
  pointsr[pR] < 0]
```

Out[•]= True

PROPOSITION 5 (Sufficient statistics sign the short-run Rx-price effect). Defining th short-run cross-price elasticity of Im demand $CROSSSR \equiv \left.\frac{\partial \ln \Delta_I(p_R,p_I,\hat{p}_R,\hat{p}_I)}{\partial \ln p_R}\right|_{\hat{p}_R=p_R,\hat{p}_I=p_I} \geq 0$, th short-run arc elasticity $ARCSR \equiv \frac{1-\frac{H(p_R)}{H(p_I)}}{1-\frac{p_R}{p_I}} < 0$, and the short-run point elasticity $POINTSR \equiv \frac{H'(p_R)}{H(p_R)} p_R < 0$, the short-run effect of the Rx price on opioid consumption can be signed as:

$$Sign\left[\left.\frac{\partial \ln \Delta(p_R,p_I,\hat{p}_R,\hat{p}_I)}{\partial \ln p_R}\right|_{\hat{p}_R=p_R,\hat{p}_I=p_I}\right] = Sign\left[\frac{1-r}{r} CROSSSR \frac{ARCSR}{POINTSR}\left(\frac{p_R}{p_I}-1\right)-1\right] \quad (8$$

Proof. Evaluate the partial derivative of (7), eliminating $F()$, $F'()$, $H(p_R)$, $H'(p_R)$, and th price derivative of v using Roy's identity and the definitions of r, CROSSSR, ARCSR, an POINTSR. Factoring out the positive factor ($-r$ POINTSR) yields (8). QED

```
In[•]:= TheoryGuru[{r == 1 - Δ[pR, pI, pR, pI] / Δ[pR, pI, pR, pI], Δ[pR, pI, pR, pI] > 0, 0 < r < 1,
  SignConditions[[{3, 4}]], PositivePrices},
 SameSign[Δ^(1,0,0,0)[pR, pI, pR, pI] pR / Δ[pR, pI, pR, pI],
  (1 - r)/r crosssr arcsr/pointsr[pR] (pR/pI - 1) - 1]]
Out[•]= True
```

Near pR = pI, aggregate consumption must slope down with pR because the switching term is zero.

```
In[•]:= TheoryGuru[{SignConditions[[4]], Θ[h[pR]] > 0,
  0 ≤ F[criticalfixedcostat[pR, pR]] ≤ 1,
  criticalfixedcostat[pR, pR] ≤ 0 ⇒ F[criticalfixedcostat[pR, pR]] == 0},

 Δ^(1,0,0,0)[pR, pR, pR, pR] == (1 - F[criticalfixedcostat[pR, pR]]) Θ[h[pR]] H'[pR] < 0]
Out[•]= True
```

with pR > pI ... if pR0 ≤ pI0, then cheaper Rx in the past not only increases total opioid consumption in the present, but must also increase present Im consumption.

```
In[•]:= IfOutsideFSupport[pR_, pI_] := criticalfixedcostat[pR, pI] < 0 ⇒
          F[criticalfixedcostat[pR, pI]] == F'[criticalfixedcostat[pR, pI]] == 0
```

```
In[•]:= TheoryGuru[{pR > pI > 0, 0 < pR0 < pI0, Assumeϴ[pR0][[{1, 3}]],
          H[pR0] > 0, H′[pR0] < 0, SignConditions[[{1, 7, 8}]],
          RoySign[pR0, pI0],
          DefineLR[pR0], DefineLR'[pR0], 0 < ΔI[pR, pI, pR0, pI0] < Δ[pR, pI, pR0, pI0],
          F'[f[pR, pI] × ϴ[h[pR0]]] ≥ 0, IfOutsideFSupport[pR0, pI0]},

         Δ^(0,0,1,0)[pR, pI, pR0, pI0] < 0 ∧ ΔI^(0,0,1,0)[pR, pI, pR0, pI0] < 0 /. rRoysId]
Out[•]= True
```

Proposition 6

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In[•]:=
```
crosslr = D[ΔI[pR, pI, pR, pI], pR] pR / ΔI[pR, pI, pR, pI];
```

In[•]:=
```
arclr = (1 - h[pR]/h[pI]) / (1 - pR/pI);
```

In[•]:=
```
TheoryGuru[{pR ≠ pI, SignConditions[[{1, 3, 4, 8}]], Rest@Assume0discrete[pR, pI],
  DefineLR[pI], DefineLR[pR], DefineLR'[pR], Last@Assume0[pR], PositivePrices},
 arclr < 0 ∧
  pointlr[pR] < 0]
```

Out[•]= True

PROPOSITION 6 (Sufficient statistics sign the long-run Rx-price effect). long-run cross-price elasticity of Im demand $CROSSLR \equiv \frac{d \ln \Delta_I(p_R,p_I,p_R,p_I)}{d \ln p_R}$, the elasticity $ARCLR \equiv \frac{1-\frac{h(p_R)}{h(p_I)}}{1-\frac{p_R}{p_I}} < ARCSR$, and the long-run point elasticity $POINTLR \equiv POINTSR$,

(a) The long-run effect of the Rx price on opioid consumption can be signed as:

$$Sign\left[\frac{d \ln \Delta(p_R,p_I,p_R,p_I)}{d \ln p_R}\right] = Sign\left[\frac{1-r}{r} CROSSLR \frac{ARCLR}{POINTLR}\left(\frac{p_R}{p_I}-1\right)-\right.$$

(b) The Rx price can reduce opioid consumption in the long run even when it i the short run.

```
In[•]:= TheoryGuru[{r == 1 - ΔI[pR, pI, pR, pI] / Δ[pR, pI, pR, pI], Δ[pR, pI, pR, pI] > 0, 0 < r < 1,
  SignConditions[{3, 4}], DefineLR[pI], DefineLR[pR],
  DefineLR'[pR], Last@Assume∂[pR], PositivePrices},
 SameSign[D[Δ[pR, pI, pR, pI], pR] pR / Δ[pR, pI, pR, pI],
  (1 - r) / r crosslr arclr / pointlr[pR] (pR / pI - 1) - 1]]
```

```
Out[•]= True
```

```
In[•]:= Not@TheoryEmpty[{r == 1 - ΔI[pR, pI, pR, pI] / Δ[pR, pI, pR, pI], Δ[pR, pI, pR, pI] > 0, 0 < r < 1,
    SignConditions, DefineLR[pI], DefineLR[pR], DefineLR'[pI], DefineLR'[pR],
    Assume∂[pR], CDFConditions, PositivePrices, Assume∂discrete[pR, pI],
    Δ^(1,0,0,0)[pR, pI, pR, pI] > 0(* increases in the short run *),
    D[Δ[pR, pI, pR, pI], pR] < 0(* decreases in long run *)} /. rRoysId]
```

```
Out[•]= True
```

Proof. (a) Evaluate the p_R derivative of (6). Eliminate $F()$, $F'()$, $H(p_R)$, $H'(p_R)$ derivative of v using Roy's identity and the definitions of r, CROSSLR, ARCLR, Factoring out the positive factor ($-r$ POINTLR) yields (9). Part (b) is proved by e p_R exceeds p_I enough for Rx prices to increase consumption in the short run but th consumption on tastes satisfies $\theta'(h(p_R)) = [H(p_R) - (v(p_I, y) - v(p_R, y))H'(p$ $(0, \theta(h(p_R))/h(p_R))$. Rx prices must reduce steady-state consumption in this CROSSLR is zero. QED

6(a) proof steps

(Solve eliminates variables from a system of equations)

```
In[•]:= D[Δ[pR, pI, pR, pI], pR] pR / Δ[pR, pI, pR, pI] == -point r ((1 - r)/r cross arc/point (pR/pI - 1) - 1) /.
  First@Solve[{r == 1 - ΔI[pR, pI, pR, pI]/Δ[pR, pI, pR, pI], cross == crosslr,
    DefineLR[pR], point == pointlr[pR], DefineLR[pI], DefineLR'[pR]},
   {F[criticalfixedcostat[pR, pI]], F'[criticalfixedcostat[pR, pI]], H[pR],
    H'[pR], H[pI], h'[pR]}] /. First@Solve[arc == arclr, h[pR]] // Simplify
```

```
Out[•]= True
```

6(b) proof steps

in this case [] CROSSLR is zero

```
In[•]:= TheoryGuru[{Θ'[h[pR]] == 1/(H[pR] - (v[pI, y] - v[pR, y]) × H'[pR]/H[pR]),
   DefineLR'[pR], DefineLR[pR], Most@AssumeΘ[pR], SignConditions[[3 ;; 4]]},
  crosslr == 0 ∧
   0 < Θ'[h[pR]] < Θ[h[pR]]/h[pR] /. rRoysId]
```

```
Out[•]= True
```

```
In[•]:= TheoryGuru[{r == 1 - ΔI[pR, pI, pR, pI]/Δ[pR, pI, pR, pI], Δ[pR, pI, pR, pI] > 0, 0 < r < 1,
   SignConditions[[{4, 7, 8}]],
   First@AssumeΘ[pR], CDFConditions, First@PositivePrices,
   Δ^(1,0,0,0)[pR, pI, pR, pI] > 0(* increases in the short run *),
   crosslr == 0} /. rRoysId,
  D[Δ[pR, pI, pR, pI], pR] < 0(* decreases in long run *) /. rRoysId]
```

```
Out[•]= True
```

Other

Near pR = pI, aggregate consumption must slope down with pR because the switching term is zero.

```
In[•]:= TheoryGuru[{r == 1 - ΔI[pR, pI, pR, pI] / Δ[pR, pI, pR, pI], Δ[pR, pI, pR, pI] > 0, 0 < r < 1,
    SignConditions[{4, 8}], DefineLR[pI], DefineLR[pR], DefineLR'[pR],
    Most@Assume⊖[pR], PositivePrices, Last@Assume⊖discrete[pR, pI],
    pR == pI} /. rRoysId,
  D[Δ[pR, pI, pR, pI], pR] < 0 (* decreases in long run *) /. rRoysId]
Out[•]= True
```

From the expression (6), Rx prices affect steady-state Im consumption only through switching.

```
In[•]:= D[ΔI[pR, pI, pR, pI], pR] == 0 /. {F'[___] :> 0 (* turn off switching *)}
Out[•]= True
```

Main Appendix

On-line Appendix: Short- and long-run demand with a wider range of price changes

Theory Backup for Mulligan "Prices and Policies in Opioid Markets" *JPE*

Methods note to readers and users: **TheoryGuru**, **TheoryOverlap**, and **Simplify** are not "trained" and do not have "memory." Every time they are evaluated, they solve each problem from the beginning and without approximation, using the principles of "elementary algebra and geometry" (in the Alfred Tarski sense of this phrase). The TheoryGuru algorithm is explained by Mulligan, Davenport, and England.

Output of **True** means that the assumptions are (i) together sufficient for the hypothesis and (ii) mutually compatible. **TheoryOverlap** output of "equivalent" means that, conditional on the assumptions, each of the two hypotheses is necessary and sufficient for the other.

Grey cells in this notebook contain definitions. As notebook initialization cells, they will (with user permission) be automatically evaluated before the user evaluates any other cell in this notebook.

Brown cells are quotes from Mulligan "Prices and Policies in Opioid Markets" *JPE*.

White cells can be evaluated or edited as the user wishes. The evaluation results are already shown.

Setup

Results

Lemma 2

Proposition 1

Proposition 2

Proposition 2 appendix proof

Proposition 3

Long-run definition

Proposition 4

Short-run definition

Proposition 5

Proposition 6

Main Appendix

See the Mathematica Notebook JPEProofsMulliganAppendix.nb

On-line Appendix: Short- and long-run demand with a wider range of price changes

In[•]:=
```
Imshss = F[Θ[h[pR0]] × f[pR0, pI0]];
```

In[•]:=
```
Δgen[pR_, pI_, pR0_, pI0_] := shImboth Θ[h[pI0]] × H[pI] + shRxboth Θ[h[pR0]] × H[pR] +
  (Imshss - shImboth) Θ[h[pI0]] × H[pR] (* Ims that switched to Rx *) +
  (1 - Imshss - shRxboth) Θ[h[pR0]] × H[pI] (* Rxs that switched to Im *)
```

In[•]:=
```
Farg00 = Θ[h[pR0]] × f[pR0, pI0];
FargI01 = Θ[h[pI0]] × f[pR, pI];
Imboth = {Farg00 ≤ FargI01 ⟺ shImboth == F[Farg00],
   shImboth == F[Farg00] ∨ shImboth == F[FargI01]};
(* Tarski way of saying shImboth == F[min{Farg00,FargI01}] *)
```

In[•]:=
```
FargR01 = Θ[h[pR0]] × f[pR, pI];
Rxboth = {FargR01 ≥ Farg00 ⟺ shRxboth == 1 - F[FargR01],
   shRxboth == 1 - F[FargR01] ∨ shRxboth == 1 - F[Farg00]};
(* Tarski way of saying shRxboth == F[max{Farg00,FargR01}] *)
```

f[pR,pI] ≥ f[pR0,pI0] is the case described in equation (7) in the main text.

In[•]:=
```
TheoryGuru[{f[pR, pI] ≥ f[pR0, pI0], RoySign[pR0, pI0], First@Imboth, First@Rxboth,
  Most@Assume@discrete[pR0, pI0], SameSign[pR0 - pI0, h[pI0] - h[pR0]]},

 Δgen[pR, pI, pR0, pI0] == Δ[pR, pI, pR0, pI0]]
```

Out[•]=
```
True
```

If f[pR,pI] < f[pR0,pI0] but Θ[h[pI0]] f[pR,pI] ≥ Θ[h[pR0]] f[pR0,pI0] then marginal changes in current prices induce no shifting and aggregate demand is:

```
In[•]:= TheoryGuru[{f[pR, pI] < f[pR0, pI0], FargI01 ≥ Farg00,
          First@Imboth, Rxboth, First@Assume⊖discrete[pR0, pI0]},

         ∆gen[pR, pI, pR0, pI0] ==
           F[Farg00] × ⊖[h[pI0]] × H[pI] + (1 - F[Farg00]) ⊖[h[pR0]] × H[pR] ∧
          D[F[Farg00], pR] == D[F[Farg00], pI] == 0 (* no switching due to current prices *)]
Out[•]= True
```

If pR0 > pI0 ... The case shown in the main text (equation (7)) is where the ratio equals or exceeds one and includes any Rx price increase or Im decrease beginning from a steady state.

```
In[•]:= TheoryGuru[{pR0 > pI0, f[pR, pI] ≥ f[pR0, pI0], RoySign[pR0, pI0]},
          f[pR, pI] / f[pR0, pI0] ≥ 1]
Out[•]= True
```

```
In[•]:= TheoryGuru[{(pR > pR0 ∧ pI == pI0) ∨ (pR == pR0 ∧ pI < pI0),
          RoySign[pI0, pI], RoySign[pR0, pR]},
         f[pR, pI] ≥ f[pR0, pI0]]
Out[•]= True
```

Intermediate values for the ratio, which include sufficiently small Rx price decreases or Im increases beginning from a steady state, changes the maximum term from its first argument to second.

```
In[•]:= TheoryGuru[{pR0 > pI0, Assume⊖discrete[pR0, pI0]⟦1 ;; 2⟧, RoySign[pR0, pI0],
          f[pR, pI] < f[pR0, pI0], ⊖[h[pI0]] × f[pR, pI] ≥ ⊖[h[pR0]] × f[pR0, pI0]},
         0 < ⊖[h[pR0]] / ⊖[h[pI0]] ≤ f[pR, pI] / f[pR0, pI0] < 1]
Out[•]= True
```