Online Appendix

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Likelihood Ratio Test

The likelihood ratio test (LRT) is widely regarded as one of the most commonly used methods for assessing the performance of nested models. In this study, we compare our proposed improved LM test to LRT. LRT can be done as follows. One would create a restricted model $\boldsymbol{M_1}$ in which all of the free parameters $\boldsymbol{\theta_1}$ to be tested are simultaneously set to zero, assuming these parameters do not contribute to explaining the variance in the data. The more general model $\boldsymbol{M_2}$ has added parameters $\boldsymbol{\theta_r}$, so all the parameters in the general model are $\boldsymbol{\theta_2 = \theta_1 + \theta_r}$, thus $\boldsymbol{\theta_1}$ is a subset of $\boldsymbol{\theta_2}$.

LRT compares the χ^2 test statistics of two nested models by evaluating the difference in χ^2 values obtained from the two runs, as well as by calculating the corresponding difference in degrees of freedom between the restricted model M_1 and the more parameterized model M_2. It assesses whether the χ^2 difference, also following a χ^2distribution under the null hypothesis, is statistically significant. This tests the hypothesis that the covariance matrices of the two models are equivalent. If this null hypothesis is rejected, it indicates that the additional parameters $\boldsymbol{\theta_r}$ in the more complex model M_2 significantly improve the model's fit to the data. With q_1 free parameters in $\boldsymbol{\theta_r}$, $\boldsymbol{\theta_2}$ thus has q_2 free parameters, where $q_2 = q_1 - r$. When fitting the same covariance matrix with *p* variables, the degrees of freedom associated with M_1 and M_2 are $df_1 = (p^* - q_1)$ and $df_2 = (p^* - q_2) = (p^* - q_1 - r)$, respectively. Let $\widehat{\boldsymbol{\theta}}_{\boldsymbol{1}}$ and $\widehat{\boldsymbol{\theta}}_{\boldsymbol{2}}$ represent the estimations of $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$. The LRT compares the χ^2 test statistics of the two models, denoted as $nF(\widehat{\boldsymbol{\theta}}_{\boldsymbol{1}})$ and $nF(\widehat{\boldsymbol{\theta}}_{\boldsymbol{2}})$ respectively. The degrees of freedom associated with the LRT statistic are $df_1 - df_2$. To simplify, $\boldsymbol{\theta_r}$ is assumed to be fixed at 0 in the M_1 model. Hence, to compare the difference between M_1 and M_2, we test if $\boldsymbol{\theta_r}$ differs from 0. Therefore, the LRT statistic can be formulated as:

$$LRT = n\left[F(\widehat{\boldsymbol{\theta}}_{\boldsymbol{1}}) - F(\widehat{\boldsymbol{\theta}}_{\boldsymbol{2}})\right] \sim \chi_r^2.$$

How do we fit likelihood ratio test?

First, we based on the univariate LM test to draw 12 suggested omitted parameters, so that we have the same set of testing parameters as the improved LM test. Based on these 12 suggested omitted parameters, we add one of them at a time, that way we create a set of nested models. The likelihood

ratio tests compare a pair of models at a time sequentially. If a model is better than the other one, then we will see that χ^2 statistic is statistically significant.

Table A1. Test Statistics by Different Sample Sizes (Full results)

N=100

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	92.098	0.000	67.646	0.000	**63.877**	**0.000**	**363.19**	**0.000**
2	**F3, x16**	24.355	0.000	23.663	0.000	**19.996**	**0.000**	**337.11**	**0.000**
3	**F2, x20**	20.773	0.000	22.001	0.005	**27.681**	**0.000**	**313.42**	**0.000**
4	**F1, x9**	17.747	0.000	18.401	0.007	**19.313**	**0.001**	**283.98**	**0.000**
5	x4, x8	12.349	0.000	10.732	0.058			283.32	0.416
6	F2, x2	11.601	0.001	10.763	0.029	6.206	0.094	**277.23**	**0.014**
7	x4, x12	11.393	0.001	10.854	0.022	5.722	0.099	**272.45**	**0.029**
8	x4, x7	10.763	0.001	10.934	0.021			**267.48**	**0.026**
9	x5, x8	10.058	0.002	9.124	0.057			264.64	0.092
10	x12, x15	9.999	0.002	10.155	0.034	5.020	0.133	**259.81**	**0.028**
11	x6, x4	9.644	0.002	8.254	0.035	7.359	0.081	**254.87**	**0.026**
12	F2, x3	8.573	0.003	7.374	0.063			254.62	0.620

N=150

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	168.551	0.000	137.789	0.000	**115.089**	**0.000**	**375.610**	**0.000**
2	**F1, x9**	74.895	0.000	69.773	0.000	**40.894**	**0.000**	**309.720**	**0.000**
3	**F3, x16**	30.653	0.000	28.847	0.002	**12.019**	**0.006**	**285.180**	**0.000**
4	**F2, x20**	22.704	0.000	24.366	0.001	**39.441**	**0.000**	**241.480**	**0.000**
5	F3, x9	22.238	0.000	19.485	0.015	1.056	0.474	241.480	0.931
6	F2, x2	20.780	0.000	17.325	0.003	1.066	0.483	240.800	0.412
7	F1, x16	19.006	0.000	18.947	0.005	4.279	0.176	237.060	0.053
8	x6, x4	16.978	0.000	13.988	0.015	6.939	0.106	**232.230**	**0.028**
9	x6, x12	16.882	0.000	15.180	0.009	1.586	0.405	231.400	0.363
10	F2, x5	16.355	0.000	13.907	0.012	1.031	0.495	230.880	0.469
11	F2, x8	16.268	0.000	12.154	0.031	3.972	0.230	228.680	0.138
12	f3, x6	15.809	0.000	19.202	0.040	1.786	0.343	227.400	0.258

N=250

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	243.499	0.000	227.021	0.000	**128.070**	**0.000**	**474.88**	**0.000**
2	**F1, x9**	101.591	0.000	92.615	0.000	**71.814**	**0.000**	**387.54**	**0.000**
3	F3, x6	92.413	0.000	86.370	0.000	6.920	0.066	384.99	0.111
4	**F2, x20**	57.578	0.000	57.658	0.000	**63.686**	**0.000**	**312.61**	**0.000**
5	**F3, x16**	50.632	0.000	51.008	0.000	**29.380**	**0.000**	**258.99**	**0.000**
6	F2, x8	25.454	0.000	23.029	0.001	1.606	0.385	258.12	0.351
7	x6, x20	24.994	0.000	24.250	0.001	1.386	0.434	257.88	0.622
8	F2, x1	21.83	0.000	20.145	0.001	1.450	0.411	256.61	0.260
9	F2, x3	21.321	0.000	19.433	0.001	0.983	0.495	256.61	0.999
10	F1, x13	19.161	0.000	18.067	0.002	4.793	0.149	253.16	0.063
11	x3, x12	18.871	0.000	19.316	0.002	6.919	0.075	**246.85**	**0.012**
12	F2, x5	18.293	0.000	16.324	0.005	0.969	0.503	246.78	0.795

N=300

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	267.745	0.000	246.558	0.000	**182.242**	**0.000**	**514.030**	**0.000**
2	**F1, x9**	155.903	0.000	152.754	0.000	**103.410**	**0.000**	**367.540**	**0.000**
3	**F2, x20**	68.739	0.000	70.415	0.000	**70.999**	**0.000**	**266.730**	**0.000**
4	F1, x20	32.597	0.000	30.873	0.007	1.319	0.457	266.610	0.736
5	**F3, x16**	30.947	0.000	30.117	0.000	**29.494**	**0.000**	**227.190**	**0.000**
6	f2, x5	27.228	0.000	25.242	0.000	0.753	0.547	226.690	0.480
7	x6, x20	26.383	0.000	28.026	0.001	1.563	0.412	226.410	0.593
8	F2, x8	24.648	0.000	23.483	0.001	1.039	0.477	226.410	0.986
9	F2, x1	21.296	0.000	20.915	0.002	1.941	0.337	225.410	0.317
10	F2, x4	21.083	0.000	20.786	0.003	1.253	0.449	225.250	0.692
11	x8, x3	20.953	0.000	20.128	0.004	3.119	0.267	223.340	0.167
12	x6, x12	20.022	0.000	20.205	0.006	1.624	0.430	223.240	0.756

N=350

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	332.671	0.000	305.722	0.000	**203.534**	**0.000**	**584.87**	**0.000**
2	**F1, x9**	182.737	0.000	168.950	0.000	**133.946**	**0.000**	**419.97**	**0.000**
3	**F2, x20**	87.01	0.000	87.739	0.000	**88.283**	**0.000**	**291.16**	**0.000**
4	F3, x6	65.913	0.000	62.475	0.000	3.770	0.176	**281.6**	**0.002**
5	x6, x20	60.066	0.000	57.062	0.000	3.544	0.207	279.11	0.115
6	**F3, x16**	47.652	0.000	48.129	0.000	**31.355**	**0.000**	**231.68**	**0.000**
7	F2, x4	31.121	0.000	27.909	0.000	1.589	0.383	231.6	0.781
8	x11, x15	29.194	0.000	29.707	0.000	**11.806**	**0.012**	**219.58**	**0.001**
9	F2, x3	27.776	0.000	24.910	0.001	2.146	0.306	218.73	0.358
10	F2, x5	26.409	0.000	22.512	0.001	1.797	0.367	217.98	0.387
11	F2, x8	26.066	0.000	23.136	0.001	1.161	0.483	217.68	0.581
12	F2, x2	25.335	0.000	22.741	0.000	2.280	0.287	215.92	0.185

N=400

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	288.109	0.000	282.443	0.000	**214.186**	**0.000**	**537.27**	**0.000**
2	**F1, x9**	206.289	0.000	207.344	0.000	**115.641**	**0.000**	**365.33**	**0.000**
3	**F2, x20**	73.277	0.000	73.805	0.000	**80.792**	**0.000**	**252.84**	**0.000**
4	**F3, x16**	41.481	0.000	41.395	0.000	**80.792**	**0.000**	**208.28**	**0.000**
5	F2, x5	28.641	0.000	29.372	0.000	2.363	0.290	205.69	0.108
6	F1, x20	26.982	0.000	27.045	0.005	3.365	0.233	203.6	0.148
7	F2, x4	25.02	0.000	27.045	0.005	2.504	0.284	201.82	0.182
8	x20, x11	24.793	0.000	25.034	0.000	2.073	0.360	199.95	0.171
9	x5, x10	23.098	0.000	23.713	0.001	6.893	0.098	**194.35**	**0.018**
10	x5, x8	21.474	0.000	21.412	0.001	2.779	0.279	192.38	0.160
11	F1, x11	21.401	0.000	22.131	0.002	2.092	0.338	190.94	0.229
12	F2, x8	21.241	0.000	21.832	0.002	1.052	0.490	190.93	0.925

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	538.391	0.000	483.535	0.000	**302.512**	**0.000**	**720.28**	**0.000**
2	**F1, x9**	328.768	0.000	306.362	0.000	**189.343**	**0.000**	**466.39**	**0.000**
3	**F2, x20**	92.473	0.000	97.219	0.000	**120.855**	**0.000**	**321.34**	**0.000**
4	F3, x6	86.143	0.000	90.821	0.000	1.113	0.486	321.02	0.570
5	**F3, x16**	69.436	0.000	67.891	0.000	**46.686**	**0.000**	**255.71**	**0.000**
6	F2, x5	53.813	0.000	46.616	0.000	2.420	0.275	254.17	0.215
7	F3, x9	51.487	0.000	45.364	0.010	3.392	0.238	251.4	0.096
8	F2, x8	48.234	0.000	40.207	0.000	0.898	0.500	251.34	0.813
9	F2, x1	45.938	0.000	40.969	0.000	1.162	0.449	251.25	0.763
10	F2, x3	39.532	0.000	33.452	0.000	1.476	0.414	250.68	0.451
11	x6, x20	38.695	0.000	39.874	0.000	1.840	0.378	249.74	0.332
12	F1, x10	35.594	0.000	33.082	0.000	5.693	0.106	**244.64**	**0.024**

N=700

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	795.318	0.000	766.975	0.000	**437.878**	**0.000**	**794.400**	**0.000**
2	**F1, x9**	215.067	0.000	214.862	0.000	**183.251**	**0.000**	**561.740**	**0.000**
3	**F2, x20**	203.848	0.000	201.634	0.000	**187.452**	**0.000**	**321.070**	**0.000**
4	F3, x6	128.206	0.000	135.198	0.000	1.177	0.484	319.520	0.214
5	F1, x20	108.18	0.000	105.039	0.000	1.102	0.470	319.460	0.800
6	F2, x8	82.385	0.000	79.944	0.000	3.063	0.240	**315.180**	**0.039**
7	**F3, x16**	77.868	0.000	78.092	0.000	**60.078**	**0.000**	**240.780**	**0.000**
8	x6, x20	67.495	0.000	68.539	0.000	2.224	0.326	239.580	0.273
9	F2, x4	59.623	0.000	58.419	0.000	1.181	0.454	239.500	0.787
10	F2, x3	58.6	0.000	56.355	0.000	1.630	0.416	238.640	0.353
11	F2, x5	57.406	0.000	55.430	0.000	1.055	0.480	238.400	0.624
12	x6, x8	43.807	0.000	42.612	0.000	1.829	0.401	237.680	0.397

N=1000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		Likelihood Ratio Test	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	1059.751	0.000	1013.785	0.000	**622.506**	**0.000**	**1231.510**	**0.000**
2	**F1, x9**	551.727	0.000	541.939	0.000	**312.393**	**0.000**	**752.360**	**0.000**
3	**F2, x20**	262.35	0.000	266.353	0.000	**265.379**	**0.000**	**407.060**	**0.000**
4	F3, x6	130.052	0.000	140.235	0.000	1.257	0.455	404.750	0.129
5	**F3, x16**	128.449	0.000	128.355	0.000	**97.400**	**0.000**	**266.690**	**0.000**
6	F2, x8	101.117	0.000	96.034	0.000	1.369	0.460	266.590	0.761
7	F1, x20	95.897	0.000	91.887	0.000	1.125	0.477	266.590	0.929
8	x6, x20	79.496	0.000	83.095	0.000	3.020	0.270	264.710	0.171
9	F2, x5	78.413	0.000	74.127	0.000	1.188	0.471	264.710	0.940
10	F2, x3	76.011	0.000	74.053	0.000	1.042	0.484	264.330	0.537
11	F2. x4	74.874	0.000	72.359	0.000	1.862	0.402	263.740	0.445
12	F2, x1	73.657	0.000	70.275	0.000	1.403	0.447	263.600	0.702

N=2000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	1962.459	0.000	1952.303	0.000	**1212.599**	**0.000**	**2087.21**	**0.000**
2	**F1, x9**	943.124	0.000	944.285	0.000	**598.461**	**0.000**	**1240.16**	**0.000**
3	**F2, x20**	587.028	0.000	586.605	0.000	**528.742**	**0.000**	**538.86**	**0.000**
4	F1, x20	283.966	0.000	277.914	0.000	1.541	0.427	538.5	0.552
5	**F3, x16**	281.457	0.000	284.288	0.000	**232.153**	**0.000**	**224.93**	**0.000**
6	F2, x8	207.045	0.000	207.408	0.000	1.190	0.460	224.84	0.755
7	F3, x6	187.422	0.000	198.224	0.000	1.196	0.479	224.75	0.766
8	x6, x20	153.37	0.000	156.145	0.000	1.003	0.495	224.65	0.751
9	F2, x3	144.77	0.000	145.718	0.000	1.028	0.487	224.05	0.438
10	F2, x5	140.543	0.000	141.952	0.000	1.995	0.374	224.02	0.887
11	F2, x1	116.233	0.000	116.989	0.000	0.946	0.510	222.33	0.192
12	F2, x4	114.175	0.000	114.271	0.000	9.140	0.059	**215.66**	**0.010**

Robustness of the Improved LM Test

Varying Factor Correlations

To further examine the robustness of the improved LM test in covariance structure, we examine these statistical properties by probing varying factor correlations, indicators per factor, and loadings. First, we begin the test with low factor correlations by setting the factor correlations as: (F1, F2) = 0.13, (F1, F3) = 0.1, and (F2, F3) = 0.18. The factor loadings and the number of factors per factor remain the

same as in the original model. We find that when factor correlations are low, the improved LM test becomes more efficient at detecting omitted parameters. The performances of the improved LM test and the likelihood ratio test become similar when the sample sizes exceed 100. However, when sample sizes are smaller than 100, the improved LM test still outperforms the LRT.

For high factor correlations, we set the factor correlations as: (F1, F2) = 0.65, (F1, F3) = 0.7, and (F2, F3) = 0.8. We find that high factor correlations tend to behave differently than low factor correlations. The improved LM test tends to accept false parameters, except when N=100. In other sample sizes in this study, the improved LM test falsely detects one additional omitted parameter. However, the improved LM test still outperforms the LRT, which tends to falsely detect three or four additional omitted parameters. Overall, when the factor correlations are high, it increases the potential relationships among factor loadings and residuals. Still, the improved LM test delivers outstanding performance.

Low Factor Correlations

$$\mathbf{\Phi} = \begin{bmatrix} 1 & & \\ 0.13 & 1 & \\ 0.10 & 0.18 & 1 \end{bmatrix}$$

High Factor Correlations

$$\mathbf{\Phi} = \begin{bmatrix} 1 & & \\ 0.65 & 1 & \\ 0.70 & 0.80 & 1 \end{bmatrix}$$

Table A2. Low Factor Correlations

N=100

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	60.932	0.000	53.067	0.000	50.141	0.000	**362.49**	**0.000**
2	**F2, x20**	37.404	0.000	37.194	0.000	22.686	0.000	**315.72**	**0.000**
3	F1, x20	29.672	0.000	28.973	0.000	0.948	0.497	315.2	0.469
4	**F3, x16**	25.418	0.000	26.322	0.000	8.953	0.034	**284.37**	**0.000**
5	**F1, x9**	15.863	0.000	16.266	0.025	19.323	0.000	**261.69**	**0.000**
6	x3, x14	10.051	0.002	10.686	0.020	6.581	0.075	**255.25**	**0.011**
7	x4, x11	8.984	0.003	9.437	0.038	3.460	0.223	251.43	0.051
8	F2, x4	8.875	0.003	8.211	0.044	1.273	0.454	251.01	0.515
9	x6, x11	8.064	0.005	8.824	0.044	2.754	0.272	249.31	0.192
10	F3, x1	7.305	0.007	7.725	0.067			**243.89**	**0.020**
11	F2, x24	6.827	0.009	7.492	0.048	4.271	0.140	240.16	0.054
12	x5, x11	6.704	0.010	7.052	0.059			238.44	0.190

N=150

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	93.549	0.000	96.666	0.000	**102.587**	**0.000**	**431.09**	**0.000**
2	**F1, x9**	74.239	0.000	74.825	0.000	**45.244**	**0.000**	**351.33**	**0.000**
3	**F2, x20**	46.276	0.000	44.563	0.000	**53.075**	**0.000**	**296.64**	**0.000**
4	**F3, x16**	26.57	0.000	27.294	0.000	**23.957**	**0.000**	**268.17**	**0.000**
5	x6, x20	25.647	0.000	22.951	0.003	1.253	0.471	268.13	0.845
6	F3, x6	17.288	0.000	18.813	0.038	1.126	0.465	267.95	0.671
7	F1, x12	15.881	0.000	16.930	0.004	2.049	0.379	266.67	0.258
8	F2, x3	13.088	0.000	13.248	0.005	2.476	0.295	264.46	0.137
9	x12, x3	8.544	0.003	10.035	0.039	3.412	0.231	261.58	0.090
10	F2, x5	7.851	0.005	9.083	0.034	1.498	0.415	261.11	0.495
11	F1, x14	7.55	0.006	8.292	0.063			259.16	0.163
12	x20, x1	7.167	0.007	8.374	0.051			256.83	0.126

N=200

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	129.148	0.000	132.049	0.000	**189.628**	**0.000**	**527.73**	**0.000**
2	**F1, x9**	96.736	0.000	96.592	0.000	**60.397**	**0.000**	**409.11**	**0.000**
3	**F2, x20**	95.234	0.000	92.628	0.000	**107.738**	**0.000**	**283.53**	**0.000**
4	x6, x20	60.975	0.000	58.134	0.000	5.720	0.122	281.63	0.169
5	**F3, x16**	56.573	0.000	56.833	0.000	**29.174**	**0.000**	**215.65**	**0.000**
6	F2, x5	12.703	0.000	14.346	0.009	2.192	0.322	214.51	0.284
7	F1, x14	9.528	0.002	10.839	0.025	5.400	0.158	211.14	0.067
8	x6, x9	9.174	0.002	9.715	0.024	1.025	0.483	211.14	0.969
9	F1, x20	9.087	0.003	12.098	0.030	1.787	0.403	209.59	0.213
10	F1, x13	8.388	0.004	8.950	0.035	2.504	0.285	208.29	0.254
11	F1, x12	8.354	0.004	9.662	0.036	1.721	0.401	207.58	0.399
12	x20, x3	8.052	0.005	9.536	0.041	5.016	0.157	203.99	0.058

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	485.053	0.000	421.198	0.000	**225.103**	**0.000**	**800.350**	**0.000**
2	**F1, x9**	265.282	0.000	242.423	0.000	**142.420**	**0.000**	**583.370**	**0.000**
3	**F2, x20**	191.987	0.000	193.447	0.000	**123.820**	**0.000**	**322.420**	**0.000**
4	F1, x20	97.899	0.000	86.324	0.000	0.954	0.507	322.420	0.966
5	**F3, x16**	84.702	0.000	83.921	0.000	**58.086**	**0.000**	**232.880**	**0.000**
6	x6, x20	58.97	0.000	66.617	0.000	1.606	0.413	232.370	0.478
7	F2, x8	49.468	0.000	40.816	0.000	1.173	0.491	231.900	0.493
8	F2, x4	46.935	0.000	38.490	0.000	0.781	0.549	231.900	0.973
9	F2, x3	42.394	0.000	34.504	0.000	0.731	0.551	231.840	0.804
10	F2, x5	40.422	0.000	32.449	0.000	1.027	0.492	231.830	0.940
11	F2, x1	34.25	0.000	28.318	0.000	0.763	0.536	230.700	0.288
12	F2, x2	29.197	0.000	23.038	0.003	1.724	0.426	230.280	0.516

N=1000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	708.561	0.000	710.403	0.000	**846.215**	**0.000**	**1544.45**	**0.000**
2	**F1, x9**	486.137	0.000	490.303	0.000	**315.185**	**0.000**	**982.67**	**0.000**
3	**F2, x20**	429.001	0.000	428.120	0.000	**422.994**	**0.000**	**403.84**	**0.000**
4	x6, x20	317.4	0.000	314.413	0.000	3.245	0.263	402.27	0.211
5	**F3, x16**	131.355	0.000	130.182	0.000	**100.472**	**0.000**	**250.34**	**0.000**
6	F3, x6	71.884	0.000	72.057	0.000	1.981	0.379	249.26	0.298
7	F2, x3	49.915	0.000	50.732	0.000	2.555	0.313	247.56	0.192
8	F2, x5	44.363	0.000	46.577	0.000	1.390	0.419	247	0.454
9	F2, x8	41.153	0.000	43.445	0.000	1.103	0.490	246.65	0.553
10	F2, x4	40.881	0.000	42.976	0.000	1.213	0.481	246.42	0.635
11	F1, x20	37.506	0.000	40.830	0.000	1.102	0.486	246.41	0.923
12	F1, x11	36.728	0.000	37.736	0.000	3.155	0.274	244.41	0.157

Table A3. High Factor Correlations

N=100

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	41.907	0.000	38.962	0.001	**23.512**	**0.006**	**347.060**	**0.000**
2	F3, x6	29.587	0.000	30.073	0.004	1.931	0.373	346.920	0.706
3	**F1, x9**	26.238	0.000	25.459	0.003	**28.725**	**0.001**	**319.520**	**0.000**
4	**F2, x20**	14.735	0.000	13.936	0.038	**17.435**	**0.008**	**301.050**	**0.000**
5	x20, x14	13.416	0.000	13.605	0.014	4.194	0.208	**295.340**	**0.017**
6	x7, x11	12.615	0.000	12.749	0.024	6.963	0.092	295.340	0.963
7	**F3, x16**	11.231	0.001	12.466	0.021	**13.769**	**0.015**	**279.050**	**0.000**
8	x7, x3	11.052	0.001	11.134	0.021	4.117	0.170	**273.460**	**0.018**
9	x14, x19	10.165	0.001	11.203	0.025	6.042	0.137	**268.900**	**0.033**
10	x15, x17	9.570	0.002	10.408	0.036	6.502	0.094	**262.840**	**0.014**
11	x3, x11	8.210	0.004	8.161	0.058			260.440	0.121
12	x3, x4	7.464	0.006	7.972	0.075			256.910	0.060

N=150

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	44.696	0.00	45.507	0.013	**33.619**	**0.000**	**344.100**	**0.000**
2	**F1, x9**	31.586	0.00	33.163	0.001	**33.781**	**0.000**	**299.870**	**0.000**
3	**F2, x20**	21.423	0.00	21.726	0.006	7.913	0.066	**289.980**	**0.002**
4	F1, x20	17.806	0.00	18.604	0.015	7.493	0.125	**281.700**	**0.004**
5	F1, x16	14.743	0.00	15.077	0.006	1.859	0.388	280.300	0.236
6	x1, x13	14.354	0.00	14.707	0.006	**8.309**	**0.044**	**270.010**	**0.001**
7	x16, x4	13.305	0.00	13.429	0.007	6.308	0.091	266.490	0.061
8	F3, x9	12.996	0.00	13.814	0.021	0.965	0.486	265.890	0.438
9	x6, x15	12.752	0.00	13.352	0.014	7.785	0.067	**255.990**	**0.002**
10	x16, x11	12.377	0.00	13.196	0.007	6.540	0.061	**251.440**	**0.033**
11	**F3, x16**	12.288	0.00	12.728	0.017	9.213	0.062	**244.100**	**0.007**
12	x4, x5	11.668	0.00	11.859	0.014	2.787	0.266	241.870	0.136

N=200

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	89.11	0.000	80.174	0.000	**58.207**	**0.000**	**359.910**	**0.000**
2	**F1, x9**	64.29	0.000	61.536	0.000	**43.849**	**0.000**	**294.690**	**0.000**
3	**F2, x20**	26.043	0.000	26.252	0.004	**18.502**	**0.005**	**277.370**	**0.000**
4	**F3, x16**	18.347	0.000	18.584	0.002	**15.146**	**0.003**	**256.890**	**0.000**
5	x9, x3	17.486	0.000	17.249	0.002	**8.612**	**0.030**	**248.100**	**0.003**
6	F1, x20	14.602	0.000	16.296	0.035	4.691	0.141	**242.570**	**0.019**
7	F3, x6	13.922	0.000	14.909	0.044	2.571	0.322	241.620	0.328
8	x5, x8	13.144	0.000	14.036	0.006	5.368	0.106	**237.240**	**0.036**
9	F2, x7	12.727	0.000	13.220	0.020	1.718	0.400	236.690	0.457
10	F1, x13	12.502	0.000	12.203	0.012	4.092	0.186	234.100	0.108
11	F3, x9	12.237	0.000	13.468	0.037	1.337	0.443	233.940	0.683
12	F1, x14	11.527	0.001	12.253	0.009	2.551	0.307	232.390	0.214

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	164.205	0	161.801	0.000	**106.944**	**0.000**	**446.700**	**0.000**
2	**F1, x9**	88.361	0	87.683	0.000	**98.705**	**0.000**	**344.040**	**0.000**
3	**F3, x16**	32.303	0	33.085	0.000	**19.113**	**0.003**	**319.680**	**0.000**
4	**F2, x20**	32.067	0	31.884	0.000	**31.672**	**0.000**	**278.750**	**0.000**
5	F3, x6	31.265	0	32.651	0.000	1.565	0.400	278.660	0.764
6	F3, x9	29.569	0	30.356	0.000	1.795	0.359	277.800	0.356
7	x20, x19	20.414	0	20.591	0.001	**11.887**	**0.017**	**267.320**	**0.001**
8	x8, x7	19.054	0	19.405	0.004	6.634	0.081	**260.960**	**0.012**
9	F2, x8	15.326	0	16.054	0.005	1.559	0.410	260.180	0.377
10	x5, x13	15.042	0	15.696	0.007	8.035	0.054	**252.600**	**0.006**
11	x8, x3	14.733	0	15.831	0.004	2.603	0.281	250.900	0.192
12	x4, x14	14.435	0	15.218	0.007	7.659	0.058	**243.930**	**0.008**

N=1000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	366.254	0.000	361.243	0.000	**180.103**	**0.000**	**596.550**	**0.000**
2	**F1, x9**	182.128	0.000	182.761	0.000	**150.891**	**0.000**	**375.770**	**0.000**
3	**F3, x6**	117.022	0.000	118.322	0.000	0.966	0.497	373.910	0.173
4	F3, x9	64.765	0.000	65.903	0.000	0.982	0.516	372.500	0.234
5	F2, x20	58.591	0.000	58.196	0.000	**71.717**	**0.000**	**298.200**	**0.000**
6	**F3, x16**	54.699	0.000	55.265	0.000	**33.139**	**0.000**	**257.260**	**0.000**
7	x4, x5	33.114	0.000	34.254	0.000	4.192	0.201	254.220	0.081
8	F2, x8	29.85	0.000	29.849	0.000	1.190	0.482	253.930	0.592
9	x5, x12	29.526	0.000	29.757	0.000	3.728	0.211	250.970	0.085
10	x10, x12	27.98	0.000	29.342	0.000	**8.361**	**0.044**	**243.270**	**0.006**
11	F1, x16	27.139	0.000	28.064	0.000	1.462	0.446	242.710	0.454
12	F2, x1	25.286	0.000	26.123	0.000	2.019	0.362	241.690	0.313

Varying Number of Indicators Per Factor

To explore the performance of the improved LM test with different numbers of indicators per factor, we choose to test 5 and 12 loadings per factor.

5-Indicators Per Factor

For the 5-indicator model, we have the following factor loadings and factor correlation matrix:

$$\boldsymbol{\Lambda}' = \begin{bmatrix} 0.5, 0.424, 0.581, 0.48, 0.55, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0.65, 0, 0, 0, 0.55, 0.38, 0.6, 0.85, 0.6, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.65, 0.58, 0.62, 0.49, 0.62, 0.64 \end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & & \\ 0.173 & 1 & \\ 0.408 & 0.262 & 1 \end{bmatrix}$$

12-Indicators Per Factor

To test how 12 indicators per factor affect the performance of the improved LM test, we use the following factor loading and factor correlation design:

$$\boldsymbol{\Lambda}' = \begin{bmatrix} 0.65, 0.65, 0.7, 0.7, 0.7, 0.7, 0.6, 0.5, 0.5, 0.5, 0.6, 0.55, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0.5, 0, 0, 0, 0, 0, 0, 0.7, 0.5, 0.5, 0.65, 0.5, 0.5, 0.6, 0.55, 0.6, 0.45, 0.5, 0.45, 0, 0, 0, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.45, 0, 0, 0, 0, 0, 0, 0, 0, 0.5, 0.5, 0.5, 0.6, 0.6, 0.6, 0.7, 0.70, 0.450, 0.5, 0.65, 0.55 \end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & & \\ 0.3 & 1 & \\ 0.4 & 0.5 & 1 \end{bmatrix}$$

To ensure the covariance matrix is positive definite, we modify the factor loadings and factor correlations for both the 5-indicator and 12-indicator models. In the 5-indicator model, high factor correlations can result in a covariance matrix that is not positive definite, causing convergence issues, particularly with small sample sizes. To address this, we lower the factor correlations in the 5-indicator model, enabling the model to run successfully. Additionally, we set the population model with two additional parameters: (F2, x2) and (F3, x10). For the 12-indicator model, we set the population model with four additional parameters: (F2, x28), (F1, x13), (F3, x16), and (F2, x6). These parameters are omitted in their respective analysis models. Therefore, we expect the improved LM test to detect these omitted parameters correspondingly.

The results show consistent patterns. As Table A4 shows, when the number of indicators is 5, the statistical power of the improved LM test weakens with smaller sample sizes, though it still performs better than the LRT. In contrast, Table A5 shows that with 12 indicators, both the improved LM tests

and the LRT demonstrate equivalent performance. However, when N=300, the improved LM test outperforms the LRT.

Table A4. 5-Indicators Per Factor

N=100

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F3, x10**	37.400	0.000	36.535	0.000	**16.151**	**0.011**	**128.951**	**0.000**
2	**F2, x2**	33.010	0.000	30.966	0.001	**24.859**	**0.000**	**91.892**	**0.000**
3	F1, x10	19.054	0.000	18.827	0.015	1.526	0.396	91.527	0.545
4	f3, x2	8.362	0.004	10.071	0.089			89.875	0.199
5	x7, x8	7.475	0.006	9.170	0.040	5.704	0.080	**84.427**	**0.020**
6	x6, x12	6.695	0.010	7.352	0.055			**77.054**	**0.007**
7	F3, x4	6.556	0.010	5.995	0.089			**70.236**	**0.009**
8	x10, x7	6.377	0.012	7.065	0.061			67.724	0.113
9	x10, x14	5.383	0.020	6.326	0.099			66.979	0.388
10	x1, x11	4.113	0.043	4.570	0.115			65.764	0.271
11	x1, x12	4.049	0.044	4.692	0.137			**61.438**	**0.038**
12	F2, x4	3.763	0.052	4.602	0.147			61.233	0.650

N=200

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F3, x10**	70.115	0.000	64.942	0.000	**38.960**	**0.000**	**127.321**	**0.000**
2	**F2, x2**	26.225	0.000	21.693	0.027	**20.182**	**0.002**	**92.602**	**0.000**
3	F3, x9	19.891	0.000	17.334	0.002	3.066	0.217	89.901	0.100
4	x2, x9	11.820	0.001	13.475	0.023	1.435	0.437	89.577	0.569
5	x10, x14	9.485	0.002	8.242	0.058			88.837	0.389
6	x10, x9	7.768	0.005	6.570	0.063			88.660	0.674
7	F1, x10	7.695	0.006	6.737	0.181			86.759	0.168
8	x9, x6	6.497	0.011	11.099	0.089			86.507	0.616
9	x10, x12	6.468	0.011	6.135	0.095			84.850	0.198
10	x2, x7	5.898	0.015	7.687	0.063			81.974	0.090
11	x3, x4	5.878	0.015	7.121	0.087			**76.803**	**0.023**
12	x7, x15	5.050	0.025	5.870	0.115			73.151	0.056

N=300

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F3, x10**	90.856	0.000	90.551	0.000	**40.332**	**0.000**	**148.301**	**0.000**
2	**F2, x2**	58.402	0.000	57.441	0.000	**42.434**	**0.000**	**87.091**	**0.000**
3	F3, x9	16.693	0.000	16.033	0.006	3.055	0.250	85.820	0.260
4	F3, x8	14.629	0.000	15.468	0.005	4.003	0.160	82.153	0.055
5	F3, x2	11.860	0.001	12.243	0.028	0.916	0.514	82.151	0.971
6	F2, x4	11.205	0.001	11.540	0.020	4.329	0.159	78.433	0.054
7	x10, x11	11.120	0.001	12.126	0.017	3.050	0.245	76.721	0.191
8	x10, x8	9.581	0.002	10.490	0.021	2.024	0.359	75.720	0.317
9	x7, x15	8.644	0.003	9.737	0.033	7.667	0.058	**68.972**	**0.009**
10	F3, x6	7.044	0.008	8.681	0.067			68.863	0.742
11	x10, x15	7.005	0.008	7.098	0.052			68.856	0.935
12	x2, x9	6.984	0.008	7.794	0.050			68.010	0.357

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F3, 10**	201.014	0.000	197.0602	0.000	**72.095**	**0.000**	**219.622**	**0.000**
2	**F2, x2**	136.377	0.000	131.367	0.000	**49.903**	**0.000**	**79.244**	**0.000**
3	F1, x10	38.392	0.000	33.852	0.019	1.035	0.484	78.724	0.471
4	F3, x9	33.370	0.000	32.237	0.000	1.121	0.481	78.698	0.871
5	F3, x6	30.226	0.000	29.281	0.000	1.309	0.458	78.698	0.991
6	F3, x8	20.525	0.000	19.937	0.003	1.778	0.396	78.695	0.958
7	x10, x6	20.465	0.000	20.237	0.002	1.036	0.484	77.856	0.360
8	x9, x6	16.426	0.000	21.456	0.023	1.156	0.468	77.728	0.720
9	x10, x11	15.756	0.000	15.514	0.004	3.013	0.235	75.76	0.161
10	x2, x9	15.731	0.000	17.283	0.004	1.845	0.381	75.672	0.767
11	x8, x14	14.215	0.000	14.970	0.008	8.407	0.057	**68.844**	**0.009**
12	x6, x8	13.067	0.000	17.412	0.032	2.118	0.375	68.044	0.371

Table A5. 12-Indicators Per Factor

N=100

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	78.191	0.000	83.895	0.000	**134.959**	**0.000**	**807.590**	**0.000**
2	**F1, x13**	70.741	0.000	71.819	0.000	**55.350**	**0.000**	**708.870**	**0.000**
3	F3, x6	47.369	0.000	38.845	0.011	2.436	0.310	708.770	0.756
4	**F2, x28**	29.439	0.000	24.026	0.005	**34.100**	**0.000**	**679.500**	**0.000**
5	**F3, x16**	25.592	0.000	27.217	0.016	**17.474**	**0.001**	**634.420**	**0.000**
6	F1, x16	17.067	0.000	14.581	0.025	1.220	0.470	634.350	0.791
7	F2, x1	11.358	0.001	10.604	0.025	1.399	0.429	632.520	0.176
8	F3, x1	10.528	0.001	10.203	0.022	2.013	0.315	630.970	0.213
9	x29, x36	9.237	0.002	9.609	0.021	**7.206**	**0.042**	**622.680**	**0.004**
10	F2, x35	9.051	0.003	9.002	0.023	3.241	0.238	620.600	0.150
11	x18, x20	8.905	0.003	9.792	0.037	6.953	0.062	**612.820**	**0.005**
12	x28, x29	8.849	0.003	7.990	0.046	5.432	0.113	**608.350**	**0.034**

N=200

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	228.34	0.00	190.742	0.000	**226.139**	**0.000**	**928.07**	**0.000**
2	**F2, x28**	132.093	0.00	133.615	0.000	**146.055**	**0.000**	**813.04**	**0.000**
3	**F1, x13**	131.812	0.00	119.635	0.000	**77.737**	**0.000**	**675.77**	**0.000**
4	**F3, x16**	97.05	0.00	86.211	0.000	**30.897**	**0.000**	**582.36**	**0.000**
5	F3, x6	77.574	0.00	72.984	0.004	1.082	0.490	581.73	0.428
6	F3, x13	46.315	0.00	35.594	0.007	1.444	0.413	581.38	0.556
7	F1, x16	41.899	0.00	33.670	0.002	1.830	0.382	581.33	0.808
8	x13, x16	33.522	0.00	26.670	0.000	1.490	0.447	581.28	0.830
9	x28, x16	24.574	0.00	22.794	0.001	3.353	0.197	578.19	0.079
10	x6, x1	19.803	0.00	17.102	0.001	2.354	0.322	576.98	0.272
11	x6, x28	19.463	0.00	19.543	0.009	2.749	0.295	575.61	0.242
12	F2, x32	19.428	0.00	19.573	0.003	3.784	0.217	573.02	0.107

N=300

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	221.084	0.000	222.643	0.000	**373.677**	**0.000**	**1252.080**	**0.000**
2	**F1, x13**	202.705	0.000	210.428	0.000	**129.587**	**0.000**	**1003.840**	**0.000**
3	**F2, x28**	183.313	0.000	167.474	0.000	**188.098**	**0.000**	**847.300**	**0.000**
4	**F3, x16**	130.012	0.000	129.396	0.000	**54.044**	**0.000**	**653.600**	**0.000**
5	F3, x6	122.427	0.000	108.683	0.000	2.288	0.308	652.000	0.205
6	F1, x16	46.094	0.000	50.496	0.000	0.882	0.523	651.930	0.783
7	x13, x16	43.626	0.000	47.956	0.000	2.716	0.307	650.480	0.230
8	F2, x5	25.399	0.000	26.701	0.000	2.305	0.324	649.580	0.343
9	F2, x32	25.394	0.000	23.674	0.002	1.740	0.378	648.930	0.421
10	F3, x13	25.221	0.000	44.820	0.028	1.168	0.462	648.770	0.688
11	x29, x30	23.209	0.000	23.647	0.000	**8.976**	**0.036**	**639.830**	**0.003**
12	F2, x4	22.214	0.000	22.793	0.001	1.959	0.373	639.110	0.398

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F1, x13**	461.937	0	404.435	0.000	**198.847**	**0.000**	**1410.39**	**0.000**
2	**F2, x6**	400.032	0	367.705	0.000	**596.082**	**0.000**	**1142.54**	**0.000**
3	**F3, x16**	292.493	0	264.810	0.000	**94.089**	**0.000**	**875.36**	**0.000**
4	F3, x13	245.116	0	170.708	0.000	5.119	0.110	874.63	0.394
5	**F2, x28**	241.162	0	268.713	0.000	**346.318**	**0.000**	**563.13**	**0.000**
6	F1, x16	187.052	0	151.074	0.000	0.964	0.509	563.12	0.956
7	x13, x16	117.424	0	91.630	0.000	3.345	0.253	560.75	0.123
8	F3, x6	94.49	0	120.618	0.001	4.224	0.138	556.96	0.052
9	F1, x28	77.131	0	58.237	0.017	1.014	0.485	556.95	0.933
10	F2, x3	54.215	0	47.388	0.000	2.013	0.336	555.02	0.165
11	x16, x3	45.495	0	36.470	0.000	2.315	0.311	553.61	0.235
12	F2, ~x5	33.462	0	27.694	0.001	1.151	0.484	553.61	0.951

N=1000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	780.100	0.000	820.228	0.000	**1045.898**	**0.000**	**2459.97**	**0.000**
2	**F1, x13**	711.316	0.000	736.515	0.000	**368.448**	**0.000**	**1540.19**	**0.000**
3	**F3, x16**	573.377	0.000	566.802	0.000	**194.870**	**0.000**	**1066.67**	**0.000**
4	F3, x6	522.391	0.000	485.847	0.000	1.822	0.380	1066.49	0.671
5	F1, x16	336.246	0.000	340.162	0.000	1.828	0.368	1064.19	0.129
6	**F2, x28**	332.581	0.000	322.924	0.000	**595.541**	**0.000**	**573.91**	**0.000**
7	F3, x13	249.036	0.000	290.799	0.000	2.412	0.331	572.66	0.265
8	x13, x16	161.386	0.000	163.551	0.000	0.974	0.505	572.33	0.562
9	F2, x5	58.493	0.000	62.417	0.000	4.157	0.196	569.75	0.109
10	x6, x16	57.389	0.000	59.037	0.000	1.132	0.485	569.61	0.708
11	F2, x1	50.178	0.000	53.127	0.000	1.700	0.384	569.04	0.447
12	F2, x4	49.058	0.000	53.468	0.000	1.252	0.464	568.84	0.658

Varying Magnitudes of Factor Loadings

To explore the extent to which factor loadings affect the performance of the improve LM test, we create two sets of loadings: Low and high. For the low factor loading model, we have the following factor loadings and factor correlation matrix:

Low Factor Loadings

$$\boldsymbol{\Lambda}' = \begin{bmatrix} 0.3, 0.3, 0.3, 0.3, 0.3, 0.4, 0.4, 0.35, 0.45, 0, 0, 0, 0, 0, 0,0, \ 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0.45, 0, 0, \ 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.35, 0, 0, 0, 0.55, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, \ 0, 0, 0, 0, 0, 0, 0, 0.45, \ 0.3, 0.3, 0.3, 0.3, 0.4, 0.4, 0.3, 0.3 \end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & & \\ 0.3 & 1 & \\ 0.4 & 0.5 & 1 \end{bmatrix}$$

High Factor Loadings

For the high factor loading model, we have the following factor loadings and factor correlation matrix:

$$\boldsymbol{\Lambda}' = \begin{bmatrix} 0.8, 0.8, 0.8, 0.8, 0.8, 0.9, 0.9, 0.85, 0.9, 0, 0, 0, 0, 0, 0,0, \ 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0.9, 0, 0, \ 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.85, 0, 0, 0, 0.9, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, \ 0, 0, 0, 0, 0, 0, 0, 0.9, \ 0.8, 0.8, 0.8, 0.8, 0.9, 0.9, 0.8, 0.8 \end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & & \\ 0.3 & 1 & \\ 0.4 & 0.5 & 1 \end{bmatrix}$$

We find that the magnitudes of factor loadings influence the performance of the improved LM test, and this effect is dependent on the sample size. When sample sizes are greater than 400, we find that the improved LM test delivers efficient and robust performance compared to the LRT. However, low factor loadings in smaller sample sizes tend to have stronger impacts on the detection of omitted variables and convergence. We find that when sample sizes are smaller than 400, the models encounter convergence issues, mainly because the covariance matrix of latent variables becomes not positive definite. In contrast, with high factor loadings, both the improved LM test and LRT perform well across all sample sizes in this study. However, the improved LM test consistently demonstrates a statistical edge in detecting correct parameters compared to the LRT.

Table A6. Low Factor Loadings

N=400

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	20.061	0.000	25.247	0.056	**16.576**	**0.010**	**333.08**	**0.000**
2	F3, x6	32.47	0.000	24.543	0.050	1.204	0.436	332.69	0.530
3	x7, x1	15.245	0.000	18.303	0.006	5.754	0.104	**327.92**	**0.029**
4	**F1, x9**	14.886	0.000	16.698	0.119	**7.199**	**0.038**	**311.32**	**0.000**
5	F3, x9	13.244	0.000	11.557	0.147	1.362	0.418	310.41	0.341
6	**F3, x16**	9.764	0.002	12.627	0.138	**9.927**	**0.022**	**300.03**	**0.001**
7	F3, x7	9.167	0.002	12.308	0.041	3.292	0.181	299.73	0.584
8	x6, x20	9.07	0.003	13.583	0.043	1.103	0.486	296.57	0.075
9	F2, x7	8.635	0.003	10.437	0.062	2.868	0.265	293.94	0.105
10	F1, x16	8.36	0.004	10.175	0.156	1.251	0.434	293.35	0.441
11	**F2, x20**	8.289	0.004	16.106	0.122	**14.331**	**0.016**	**263.93**	**0.000**
12	x18, x21	8.19	0.004	12.140	0.027	4.401	0.190	260.7	0.072

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	36.225	0.000	33.149	0.004	**10.458**	**0.025**	**307.870**	**0.000**
2	F3, x6	35.652	0.000	33.176	0.005	1.284	0.448	**307.850**	**0.000**
3	x6, x20	25.177	0.000	30.737	0.002	3.394	0.263	**293.750**	**0.000**
4	**F2, x20**	24.87	0.000	22.958	0.076	**10.593**	**0.026**	**306.330**	**0.000**
5	**F1, x9**	15.835	0.000	20.820	0.009	**9.432**	**0.012**	**241.850**	**0.000**
6	F1, x20	15.793	0.000	22.626	0.010	1.105	0.462	241.800	0.825
7	x9, x7	14.181	0.000	19.883	0.009	4.596	0.204	239.460	0.126
8	**F3, x16**	13.875	0.000	18.163	0.056	**16.633**	**0.003**	**220.670**	**0.000**
9	F3, x9	13.315	0.000	12.421	0.104	**7.630**	**0.048**	**212.890**	**0.005**
10	x16, x21	11.465	0.001	16.556	0.011	5.106	0.151	209.310	0.058
11	x20, x9	11.054	0.001	16.865	0.010	4.287	0.187	**204.950**	**0.037**
12	x20, x14	9.727	0.002	14.402	0.017	6.277	0.088	**200.100**	**0.028**

N=700

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	39.708	0.000	37.885	0.003	**22.035**	**0.001**	**370.86**	**0.000**
2	F3, x6	33.269	0.000	32.607	0.003	1.252	0.467	370.74	0.734
3	**F2, x20**	28.788	0.000	27.318	0.034	**22.331**	**0.001**	**329.76**	**0.000**
4	**F1, x9**	27.376	0.000	28.850	0.003	**15.658**	**0.001**	**278.03**	**0.000**
5	**F3, x16**	23.307	0.000	23.340	0.013	**19.770**	**0.001**	**246.33**	**0.000**
6	x9, x8	16.132	0.000	17.350	0.003	2.553	0.316	244.49	0.174
7	x16, x19	15.868	0.000	16.991	0.005	**10.105**	**0.035**	**234.74**	**0.002**
8	F1, x16	15.379	0.000	16.669	0.009	1.872	0.340	233.55	0.274
9	x1, x7	11.175	0.001	12.115	0.020	5.614	0.112	**228.61**	**0.026**
10	F3, x9	10.957	0.001	12.830	0.095	0.931	0.508	228.54	0.796
11	x6, x20	10.815	0.001	12.212	0.029	1.346	0.447	228.24	0.584
12	x14, x21	10.441	0.001	11.363	0.019	6.216	0.093	**222.72**	**0.019**

N=1000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F1, x9**	58.984	0.000	62.220	0.000	**30.689**	**0.000**	**398.99**	**0.000**
2	**F3, x16**	54.975	0.000	53.113	0.004	**38.293**	**0.000**	**380.21**	**0.000**
3	**F2, x6**	54.463	0.000	54.411	0.000	**22.254**	**0.001**	**316.53**	**0.000**
4	F3, x6	51.42	0.000	51.119	0.000	1.750	0.419	316.13	0.527
5	**F2, x20**	38.254	0.000	36.668	0.022	**23.690**	**0.001**	**237.82**	**0.000**
6	F1, x16	37.387	0.000	40.281	0.000	2.574	0.281	233.98	0.050
7	F3, x9	36.524	0.000	34.381	0.030	1.188	0.471	233.32	0.418
8	x16, x21	22.832	0.000	26.541	0.001	8.124	0.051	**225.63**	**0.006**
9	x6, x20	17.209	0.000	21.863	0.005	1.636	0.419	223.74	0.168
10	x3, x7	13.956	0.000	17.609	0.012	5.227	0.140	**219.73**	**0.045**
11	x16, x6	12.291	0.000	15.575	0.012	2.469	0.317	217.93	0.180
12	F3, x1	11.998	0.001	5.561	0.011	4.092	0.172	**214.05**	**0.049**

Table A7. High Factor Loadings

N=100

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F3, x16**	53.23	0.000	44.991	0.004	**25.018**	**0.000**	**388.74**	**0.000**
2	**F2, x6**	41.87	0.000	36.235	0.001	**28.377**	**0.000**	**351.97**	**0.000**
3	**F2, x20**	37.745	0.000	32.974	0.007	**27.885**	**0.001**	**315.78**	**0.000**
4	**F1, x9**	21.268	0.000	21.743	0.004	**16.710**	**0.002**	**279.31**	**0.000**
5	F3, x6	20.779	0.000	19.139	0.049	1.131	0.462	279.23	0.770
6	x9, x3	14.834	0.000	14.988	0.003	**8.663**	**0.027**	**270.35**	**0.003**
7	F2, x21	14.5	0.000	12.896	0.018	4.001	0.157	266.94	0.065
8	F1, x12	13.549	0.000	12.993	0.009	**6.901**	**0.043**	**257.88**	**0.003**
9	x16, x10	12.251	0.000	11.882	0.006	6.413	0.066	**252.49**	**0.020**
10	x20, x12	10.961	0.001	11.073	0.012	3.627	0.186	249.39	0.078
11	x16, x17	10.418	0.001	10.746	0.029	3.406	0.241	247.04	0.125
12	x21, x12	9.593	0.002	9.847	0.034	1.633	0.395	246.38	0.416

N=300

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	275.93	0.000	276.700	0.000	**562.319**	**0.000**	**817.25**	**0.000**
2	**F1, x9**	209.86	0.000	211.901	0.000	**196.257**	**0.000**	**515.14**	**0.000**
3	**F2, x20**	128.525	0.000	127.522	0.000	**152.197**	**0.000**	**354.73**	**0.000**
4	**F3, x16**	106.783	0.000	105.149	0.000	**111.193**	**0.000**	**205.65**	**0.000**
5	x6, x20	87.282	0.000	86.555	0.000	1.052	0.496	205.53	0.734
6	F3, x6	83.759	0.000	83.726	0.000	4.874	0.131	203.18	0.125
7	F1, x15	20.623	0.000	21.242	0.001	1.691	0.394	202.29	0.345
8	F2, x8	18.284	0.000	19.519	0.003	1.001	0.500	202.23	0.804
9	x6, x12	17.951	0.000	18.420	0.000	2.246	0.365	202.16	0.784
10	F1, x12	15.639	0.000	16.806	0.004	0.809	0.530	201.86	0.586
11	F3, x12	15.25	0.000	15.818	0.005	2.232	0.346	200.66	0.273
12	x6, x15	14.545	0.000	15.472	0.002	4.486	0.192	197.56	0.079

N=500

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	433.662	0.000	434.277	0.000	**664.337**	**0.000**	**1277.97**	**0.000**
2	**F1, x9**	347.478	0.000	347.727	0.000	**346.671**	**0.000**	**756.69**	**0.000**
3	**F2, x20**	221.892	0.000	220.501	0.000	**215.782**	**0.000**	**479.7**	**0.000**
4	F3, x6	168.598	0.000	169.013	0.000	5.442	0.106	**447.54**	**0.000**
5	**F3, x16**	154.575	0.000	153.615	0.000	**141.152**	**0.000**	**254.11**	**0.000**
6	x6, x20	145.585	0.000	145.388	0.000	1.416	0.434	254.08	0.848
7	F2, x8	43.358	0.000	44.265	0.000	1.911	0.350	252.69	0.238
8	x6, x12	34.384	0.000	35.055	0.000	1.897	0.362	250.94	0.186
9	x20, x12	33.352	0.000	34.028	0.000	1.562	0.410	250.58	0.550
10	F3, x12	29.333	0.000	29.731	0.000	3.666	0.214	247.83	0.097
11	x20, x11	25.946	0.000	27.273	0.000	5.513	0.119	**243.57**	**0.039**
12	x20, x8	24.982	0.000	25.731	0.000	2.960	0.260	241.47	0.147

N=1000

		Univariate LM Test		Bootstrap LM Test		Bootstrap W Test		LRT	
	Parameters	LM test	P-values	LM test	P-values	Chi-square	P-values	Chi-square	P-values
1	**F2, x6**	911.297	0.00	912.366	0.000	**1681.605**	**0.000**	**2427.85**	**0.000**
2	**F1, x9**	798.267	0.00	798.635	0.000	**693.119**	**0.000**	**1220.3**	**0.000**
3	**F2, x20**	490.452	0.00	490.115	0.000	**550.420**	**0.000**	**642.49**	**0.000**
4	**F3, x16**	284.323	0.00	282.955	0.000	**276.200**	**0.000**	**255.31**	**0.000**
5	x6, x20	264.367	0.00	264.714	0.000	8.478	0.057	**247.11**	**0.004**
6	F3, x6	254.426	0.00	255.271	0.000	1.143	0.476	246.97	0.705
7	F2, x8	89.066	0.00	91.158	0.000	3.188	0.254	244.94	0.154
8	x6, x12	53.928	0.00	54.597	0.000	1.479	0.419	244.56	0.538
9	x6, x11	50.576	0.00	50.717	0.000	1.433	0.429	244.17	0.531
10	x9, x8	47.468	0.00	48.918	0.000	3.124	0.264	242.05	0.145
11	F1, x11	42.107	0.00	42.928	0.000	1.502	0.432	241.69	0.553
12	F1, x12	40.789	0.00	41.050	0.000	1.522	0.422	241.18	0.475

Huddy and Khatib's (2007) survey questions on the 2002 student sample.

Q20. How similar do you feel to the average American?

1. Very similar
2. Somewhat similar
3. Not very similar
4. Not at all

Q22. When you hear a non-American criticizing Americans, to what extent do you feel you are being personally criticized?

1. A great deal
2. Somewhat
3. Very little

4. Not at all

Q23. How well does the term American describe you?

1. Very well
2. Somewhat well
3. Not very well
4. Not at all

Q24. When talking about Americans, how often would you say "we" rather than "they"

1. Most of the time
2. Some of the time
3. Occasionally
4. Never

Q26. How good does it make you feel when you see the American flag flying:?

1. Extremely good
2. Very good
3. Somewhat good
4. Or not very good

Q27. How angry does it make you feel, if at all, when you hear someone criticizing the United States:

1. Extremely angry
2. Somewhat angry
3. Not very angry
4. Not at all angry

Q28. How proud do you feel when you hear the national anthem?

1. Extremely proud
2. Very proud
3. Somewhat proud
4. Or not very proud

Q37. How strongly do you agree or disagree with the following statements? There is too much criticism of the US in the world, and we as its citizens should not criticize it.

1. Strongly agree
2. Somewhat agree
3. Somewhat disagree

4. Strongly disagree

Q39. For the most part, people who protest and demonstrate against US policy are good, upstanding, intelligent people.

1. Strongly agree
2. Somewhat agree
3. Somewhat disagree
4. Strongly disagree

Q40. If another country disagreed with an important United States policy that I knew little about, I would not necessarily support my country's position.

1. Strongly agree
2. Somewhat agree
3. Somewhat disagree
4. Strongly disagree