Additional notes for quantitative results

1 Calibration algorithm

Disutility from labour is given by $h(\ell) = \frac{\ell^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$, which is common to all workers. We reintroduce $\pi(\ell)$ function, which is a probability of high output realisation at performance pay jobs. We suppose it takes a power function form $\pi(\ell) = \bar{\pi}\ell^{\rho}$, $\rho \in (0, 1]$, with the inverse function $\pi^{-1}(x) = (\frac{x}{\bar{\pi}})^{\frac{1}{\rho}}$. Define auxiliary disutility from effort expressed as probability of high output $h_{pp}(x) = \bar{h}_{pp}\frac{x^{1+\frac{1}{\varepsilon_{pp}}}}{1+\frac{1}{\varepsilon_{pp}}}$, where $h_{pp}(x) = h(\pi^{-1}(x))$.[1] Denote the variance of log-earnings risk at performance-pay jobs $Var(\log z_{pp} \mid \theta) = \beta^2(1-\pi(l_{pp}))\pi(l_{pp})$ by σ^2_β.

We match the following moments and assumptions:

1. Ratio of variance of log-earnings at the two job types: $m_1 = \frac{\lambda_\theta^{-2}+\sigma^2_{\theta,pp}+\sigma^2_\beta}{\lambda_\theta^{-2}+\sigma^2_{\theta,fp}}$
2. Assumption: risky bonuses explain share $m_2 > 0$ of excess variance of log-earnings at pp jobs: $m_2 = \frac{\sigma^2_\beta}{\sigma^2_\beta+\sigma^2_{\theta,pp}-\sigma^2_{\theta,fp}} = m_2 \implies \sigma^2_{\theta,pp} = \frac{1-m_2}{m_2}\sigma^2_\beta + \sigma^2_{\theta,fp}$.
3. Ratio of mean earnings at the two job types: $m_3 = \frac{\mathbb{E}[\theta_{pp}]\pi}{\mathbb{E}[\theta_{fp}]\ell_{fp}}$.
4. Cross-sectional variance of log-earnings $m_4 = Var(\log z)$.

The calibration algorithm proceeds as follows:

1. Guess value of σ^2_{fp}.
2. Calculate $\sigma^2_\beta = m_2(m_1-1)(\lambda_\theta^{-2} + \sigma^2_{\theta,fp})$ (follows from moments 1 and 2)
3. Calculate $\beta = \sqrt{\frac{\sigma^2_\beta}{\pi(1-\pi)}}$ (from the definition of σ^2_β)
4. Calculate $\varepsilon_{pp} = \sigma^2_\beta\frac{b}{\underline{z}\beta} = \sigma^2_\beta\frac{e^\beta-1}{\beta}$ (from the effort FOC)
5. Calculate $\rho = \frac{1+\varepsilon}{\varepsilon}\frac{\varepsilon_{pp}}{1+\varepsilon_{pp}}$ (by inverting $\varepsilon_{pp} = \frac{\rho\varepsilon}{1+\varepsilon-\rho\varepsilon}$ from footnote above)
6. Calculate $\bar{h}_{pp} = \bar{\pi}^{-\frac{1+\varepsilon}{\rho\varepsilon}}\frac{1+\frac{1}{\varepsilon_{pp}}}{1+\frac{1}{\varepsilon}}$ (from the footnote above)
7. Given ρ and h_{pp}, calculate $\bar{\pi}$
8. Calculate $\ell_{fp} = (1-p)^{\frac{\varepsilon}{1+\varepsilon}}$.
9. Calculate $\mu_{pp} - \mu_{fp}$ by matching m_3

$$m_3 = \frac{\mathbb{E}[\theta_{pp}]}{\mathbb{E}[\theta_{fp}]}\frac{\pi}{\ell_{fp}} = e^{\mu_{pp}-\mu_{fp}+\frac{\sigma^2_{\theta,pp}-\sigma^2_{\theta,fp}}{2}}\frac{\pi}{\ell_{fp}} = e^{\mu_{pp}-\mu_{fp}+\frac{1-m_2}{m_2}\frac{\sigma^2_\beta}{2}}\frac{\pi}{\ell_{fp}} \tag{1}$$

$$\implies \mu_{pp} - \mu_{fp} = \log\left(m_3\frac{\ell_{fp}}{\pi}\right) - \frac{1-m_2}{m_2}\frac{\sigma^2_\beta}{2} \tag{2}$$

[1] It is easy to show that $\varepsilon_{pp} = \frac{\rho\varepsilon}{1+\varepsilon-\rho\varepsilon}$ and $\bar{h}_{pp} = \bar{\pi}^{-\frac{1+\varepsilon}{\rho\varepsilon}}\frac{1+\frac{1}{\varepsilon_{pp}}}{1+\frac{1}{\varepsilon}}$.

10. Calculate base pay of performance-pay worker with $\theta = 1$: $\underline{z}_{pp}(1) = \frac{\pi}{1-\pi+\pi e^{\beta}}$.
11. Calculate cross-sectional variance of log earnings

$$Var(\log z) = (m_1 s_{pp} + 1 - s_{pp})\left(\lambda_{\theta}^{-2} + \sigma_{\theta,fp}^2\right) + s_{pp}(1-s_{pp})\left(\log\left(e^{\mu_{pp}} \cdot \underline{z}_{pp}(1)\right) + \pi\beta - \log(e^{\mu_{fp}} \cdot \ell_{fp})\right)^2.$$

It is does not equal m_4, adjust the initial guess of σ_{fp}^2.

2 Analytical formula for $\mathcal{E}^*$ for computation of top tax rates

Denote the elasticity of labour effort of type θ with respect to the top tax rate by $\varepsilon_{\ell,1-\tau}(\theta) = \frac{\hat{\ell}(\theta)}{-\hat{\tau}} \frac{1-\tau}{\ell(\theta)}$. Otherwise, notation is as in the proof of Proposition 3 in the Online Appendix. The average elasticity among top earners, weighted by expected earnings, is equal

$$\mathcal{E} = \underbrace{\int_{\theta^*}^{\theta^{**}} \frac{\theta\ell}{\overline{Z}} \varepsilon_{\ell,1-\tau} \frac{dF_\theta(\theta)}{1-F_\theta(\theta^*)}}_{=e_1} + \underbrace{\int_{\theta^{**}}^{\infty} \frac{\theta\ell}{\overline{Z}} \varepsilon_{\ell,1-\tau} \frac{dF_\theta(\theta)}{1-F_\theta(\theta^*)}}_{=e_2}. \tag{3}$$

Calculating e_1. It follows from Lemma 9 in the Online Appendix that for $\theta \in (\theta^*, \theta^{**})$

$$\varepsilon_{\ell,1-\tau}(\theta) = X\left(\frac{1+\tilde{R}_0/\bar{z}}{1-\underline{z}/\bar{z}} - \frac{\tilde{R}_0/\bar{z}}{\ell + (1-\ell)\underline{z}/\bar{z} + \tilde{R}_0/\bar{z}} \frac{\bar{z}-z^*}{\bar{z}}\right) \underset{z^*\to\infty}{\to} \frac{X^*}{1-e^{-h'(\ell^*)}} \tag{4}$$

where $\tilde{R}_0 = \frac{R_0}{1-\tau}$ and $X(\theta) = \left(-\frac{\ell(\theta)^2}{\underline{z}} \frac{\partial^2 \Pi}{\partial \ell^2}\right)^{-1}$ with limit X^* as $z^* \to \infty$ (see Lemma 9 in the Online Appendix for the exact expression). Then, for some $t_1 \in (\theta^*, \theta^{**})$ and $t_2 > \theta^{**}$, we have

$$e_1 = \frac{\int_{\theta^*}^{\theta^{**}} \theta\ell\varepsilon_{\ell,1-\tau} dF_\theta(\theta)}{\int_{\theta^*}^{\infty} \theta\ell dF_\theta(\theta)} \overset{MVT}{=} \varepsilon_{\ell,1-\tau}(t_1) \frac{\ell(t_1)}{\ell(t_2)} \frac{\int_{\theta^*}^{\theta^{**}} \theta dF_\theta(\theta)}{\int_{\theta^*}^{\infty} \theta dF_\theta(\theta)} \tag{5}$$

$$= \varepsilon_{\ell,1-\tau}(t_1) \frac{\ell(t_1)}{\ell(t_2)} \left(1 - \left(\frac{\theta^*}{\theta^{**}}\right)^{\rho^*-1}\right) \underset{z^*\to\infty}{\to} \frac{X^*}{1-e^{-h'(\ell^*)}} \left(1 - e^{-(\rho^*-1)h'(\ell^*)}\right) \tag{6}$$

Calculating e_2. It follows from Lemma 9 in the Online Appendix that for $\theta > \theta^{**}$

$$\varepsilon_{\ell,1-\tau}(\theta) = X\left(1 - \frac{\ell\bar{z}/\underline{z} + 1 - \ell + \ell\tilde{R}_0/\bar{z}}{\ell\bar{z}/\bar{z} + 1 - \ell + \tilde{R}_0/\bar{z}} \cdot \frac{\underline{z}-z^*}{\underline{z}} - \frac{\tilde{R}_0/\bar{z}}{\ell + (1-\ell)\underline{z}/\bar{z} + \tilde{R}_0/\bar{z}} \frac{\bar{z}-z^*}{\bar{z}}\right). \tag{7}$$

Ignoring the terms which vanish as $z^* \to \infty$, we can write

$$\varepsilon_{\ell,1-\tau}(\theta) \sim X \frac{\ell\bar{z}/\underline{z} + 1 - \ell + \ell\tilde{R}_0/\bar{z}}{\ell\bar{z}/\bar{z} + 1 - \ell + \tilde{R}_0/\bar{z}} \cdot \frac{z^*}{\underline{z}} \equiv \tilde{\varepsilon}_{\ell,1-\tau}(\theta). \tag{8}$$

By a straightforward application of MVT, any terms in $\varepsilon_{\ell,1-\tau}(\theta)$ which vanish in the limit have no impact $\lim_{z^*\to\infty} e_2$. Thus, when calculating the limit of e_2, we can replace $\varepsilon_{\ell,1-\tau}(\theta)$ with $\tilde{\varepsilon}_{\ell,1-\tau}(\theta)$. Proceeding in this way, we obtain

$$\lim_{z^*\to\infty} e_2 = \lim_{z^*\to\infty} \frac{\int_{\theta^{**}}^{\infty} \theta\ell\tilde{\varepsilon}_{\ell,1-\tau} dF_\theta(\theta)}{\int_{\theta^*}^{\infty} \theta\ell dF_\theta(\theta)} \overset{MVT}{=} \lim_{z^*\to\infty} \frac{\ell(t_1)}{\ell(t_2)} \frac{t_1}{\theta^{**}} \tilde{\varepsilon}_{\ell,1-\tau}(t_1) \frac{\theta^{**}\int_{\theta^{**}}^{\infty} dF_\theta(\theta)}{\int_{\theta^*}^{\infty} \theta dF_\theta(\theta)}. \quad (9)$$

Plug in the expression for $\tilde{\varepsilon}_{\ell,1-\tau}(t_1)$ and use $\frac{t_1}{\theta^{**}} \frac{z^*}{\underline{z}(t_1)} \to 1$ as $z^* \to \infty$, as well as the properties of the Pareto distribution, to get

$$\lim_{z^*\to\infty} e_2 = \lim_{z^*\to\infty} X(t_1) \frac{\rho^* - 1}{\rho^*} \left(\frac{\theta^*}{\theta^{**}}\right)^{\rho^*-1} = X^* \frac{\rho^* - 1}{\rho^*} e^{-(\rho^*-1)h'(\ell^*)}. \quad (10)$$

Summing up, we have

$$\mathcal{E}^* = \lim_{z^*\to\infty} [e_1 + e_2] = \frac{X^*}{1 - e^{-h'(\ell^*)}} \left(1 - e^{-(\rho^*-1)h'(\ell^*)}\right) + X^* \frac{\rho^* - 1}{\rho^*} e^{-(\rho^*-1)h'(\ell^*)}. \quad (11)$$