Phase 1: PSI Construction & Aggregation Sensitivity

Power-Sharing Index Methodological Documentation

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Table of contents

1 Introduction **3**
1.1 Relationship to Existing Approaches . . . 3

2 Aggregation Methods: Theoretical Rationale **4**
2.1 Additive (Arithmetic Mean) . . . 4
2.2 Multiplicative (Geometric Mean) . . . 4
2.3 Hybrid (Multiplicative × Polyarchy) . . . 4

3 Comparing Aggregation Methods **5**
3.1 Visual Comparison . . . 5
3.2 Era-Level Summary Statistics . . . 6
3.3 Correlation Across Methods . . . 7

4 Why Hybrid? Theoretical Justification **8**
4.1 1. Multiplicative Penalty for Exclusion . . . 8
4.2 2. Procedural Democracy Cap . . . 8
4.3 Empirical Demonstration . . . 8

5 Sensitivity Analysis: Jackknife (Leave-One-Component-Out) **10**
5.1 Interpretation . . . 11

6 Alternative Specifications **12**
6.1 Specifications Tested . . . 13

7 Response to Historical Events **14**

8 Summary **15**
8.1 Key Findings . . . 15
8.2 Recommended Specification . . . 15
8.3 Comparison to Sigman & Lindberg (2019) . . . 15

9 Save Results **16**

10 Session Info **17**

1 Introduction

This document constructs the Power-Sharing Index (PSI) and presents sensitivity analyses demonstrating robustness to specification choices. The hybrid aggregation method is selected and substantive conclusions are shown to be invariant to methodological decisions.

Output: `data/processed/psi_phase1_results.rda` (loaded by Phase 2 and Appendix).

1.1 Relationship to Existing Approaches

This analysis follows the general framework established by Sigman and Lindberg (2019) for constructing composite democracy indices, with two key departures:

1. **Aggregation transparency**: Sigman and Lindberg do not present sensitivity analyses for their aggregation choices. This document explicitly compares three aggregation methods and demonstrates that results are robust across specifications.
2. **Component selection logic**: Sigman and Lindberg recommend avoiding V-Dem variables that appear in existing indices to prevent circularity. This concern does not apply here because PSI measures a conceptually distinct phenomenon—cross-group power transfer rather than procedural democracy quality. The discriminant validity analysis in Phase 2 confirms this empirically: PSI shows a *negative* correlation with Electoral Democracy during the Herrenvolk era (1789–1865), demonstrating that it captures something qualitatively different from existing indices.

2 Aggregation Methods: Theoretical Rationale

Three aggregation approaches are considered, each with distinct theoretical implications for how exclusion across dimensions should be weighted.

2.1 Additive (Arithmetic Mean)

$$PSI_{add} = \frac{1}{5} \sum_{i=1}^{5} X_{i,norm}$$

Theoretical logic: High scores in some dimensions can compensate for low scores in others. A society with strong gender equality but weak racial equality would score moderately.

Implication: Partial inclusion in some dimensions can "offset" exclusion in others.

2.2 Multiplicative (Geometric Mean)

$$PSI_{mult} = \left(\prod_{i=1}^{5} X_{i,norm} \right)^{1/5}$$

Theoretical logic: Low values in *any* dimension pull down the overall score. A society cannot "average its way out of exclusion."

Implication: Power-sharing requires inclusion across *all* dimensions simultaneously. Zero in any dimension produces zero overall.

2.3 Hybrid (Multiplicative $\times$ Polyarchy)

$$PSI_{hybrid} = Polyarchy_{norm} \times \left(\prod_{i=1}^{5} X_{i,norm} \right)^{1/5}$$

Theoretical logic: Combines the multiplicative penalty for exclusion with a procedural democracy cap. Even perfect inclusion cannot exceed the quality of democratic institutions.

Implication: Power-sharing is meaningful only within functioning democratic procedures. Authoritarian regimes with nominal inclusion still score low.

3 Comparing Aggregation Methods

3.1 Visual Comparison

```
usa_psi |>
  select(year, psi_additive, psi_multiplicative, psi_hybrid) |>
  pivot_longer(-year, names_to = "Method", values_to = "PSI") |>
  mutate(Method = case_when(
    Method == "psi_additive" ~ "Additive",
    Method == "psi_multiplicative" ~ "Multiplicative",
    Method == "psi_hybrid" ~ "Hybrid"
  ) |> factor(levels = c("Additive", "Multiplicative", "Hybrid"))) |>
  ggplot(aes(x = year, y = PSI, color = Method)) +
  geom_line(linewidth = 1) +
  geom_vline(xintercept = c(1865, 1920, 1965), linetype = "dashed", alpha = 0.4) +
  scale_color_manual(values = c("Additive" = "black",
                                "Multiplicative" = ELEC_COLOR,
                                "Hybrid" = PSI_COLOR)) +
  scale_y_continuous(limits = c(0, 1), labels = percent) +
  labs(title = "PSI by Aggregation Method",
       subtitle = "All three methods show same substantive pattern; differ in magnitude",
       x = NULL, y = "Power-Sharing Index") +
  theme_psi()
```

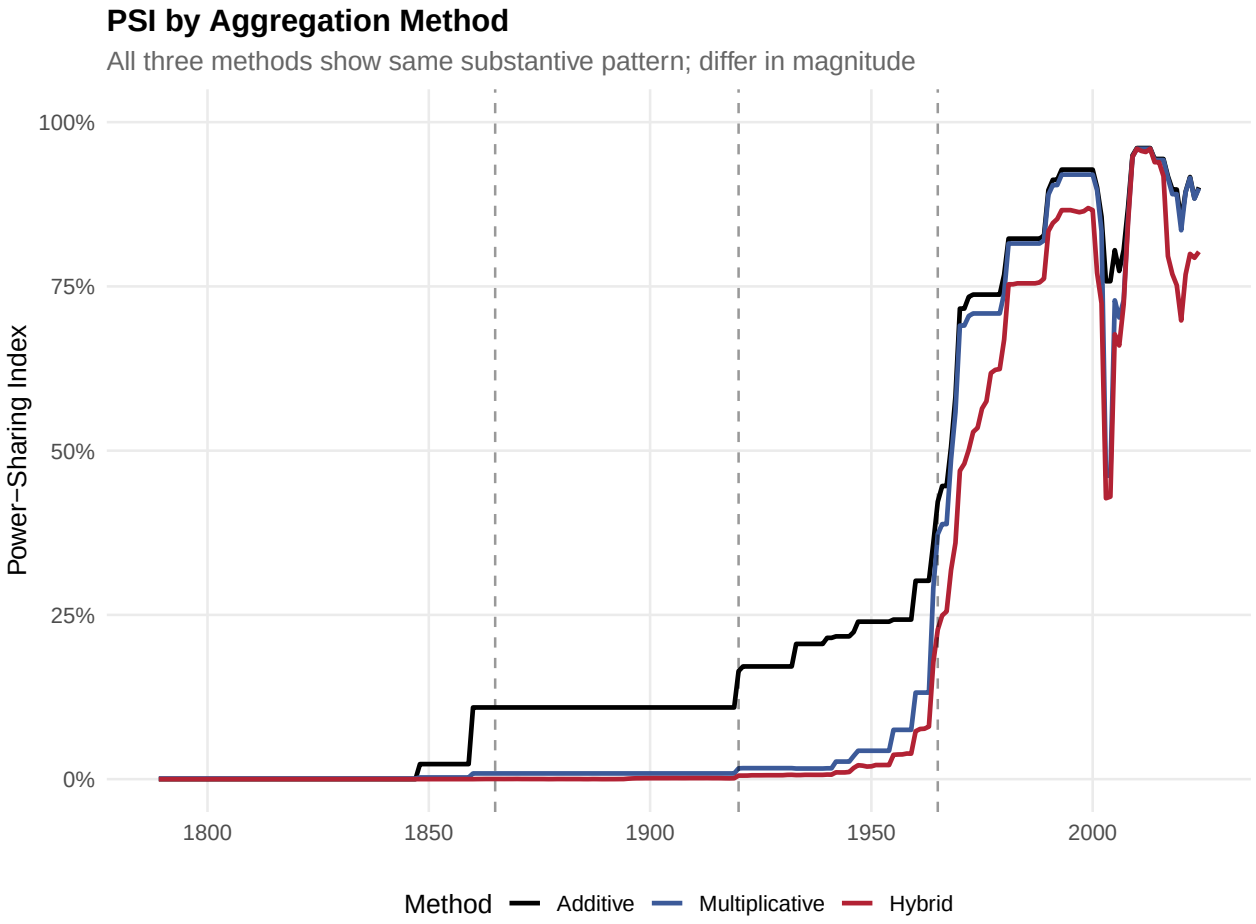

Figure 1: PSI by Aggregation Method

Note. Vertical dashed lines mark the end of the Civil War (1865), women's suffrage (1920), and the Voting Rights Act (1965). All three methods produce the same substantive pattern; they differ only in magnitude.

3.2 Era-Level Summary Statistics

```
usa_psi |>
  group_by(era) |>
  summarise(
    N = n(),
    `Additive Mean` = mean(psi_additive, na.rm = TRUE),
    `Multiplicative Mean` = mean(psi_multiplicative, na.rm = TRUE),
    `Hybrid Mean` = mean(psi_hybrid, na.rm = TRUE),
    .groups = "drop"
  ) |>
  gt() |>
  tab_header(
```

Table 1

PSI by Era and Aggregation Method

All methods show same pattern: near-zero →low →high

era	N	Additive Mean	Multiplicative Mean	Hybrid Mean
Herrenvolk Democracy (1789-1865)	76	0.01	0.00	0.00
Reconstruction (1865-1877)	12	0.11	0.01	0.00
Partial Inclusion (1877-1965)	88	0.17	0.03	0.01
Power-Sharing Dilemma (1965+)	60	0.82	0.80	0.71

```
    title = "PSI by Era and Aggregation Method",
    subtitle = "All methods show same pattern: near-zero → low → high"
  ) |>
  fmt_number(columns = -c(era, N), decimals = 2)
```

Note. Means rounded to 2 decimal places. All methods show the same ordering across eras.

3.3 Correlation Across Methods

```
tibble(
  Comparison = c("Additive vs. Multiplicative",
                 "Additive vs. Hybrid",
                 "Multiplicative vs. Hybrid"),
  Correlation = c(
    cor(usa_psi$psi_additive, usa_psi$psi_multiplicative, use = "complete.obs"),
    cor(usa_psi$psi_additive, usa_psi$psi_hybrid, use = "complete.obs"),
    cor(usa_psi$psi_multiplicative, usa_psi$psi_hybrid, use = "complete.obs")
  )
) |>
  gt() |>
  tab_header(
    title = "Correlation Across Aggregation Methods",
    subtitle = "High correlations indicate robust measurement"
  ) |>
  fmt_number(columns = Correlation, decimals = 3)
```

Note. Pearson correlations computed on complete cases across the full time series (1789–2024).

Table 2

Correlation Across Aggregation Methods

High correlations indicate robust measurement

Comparison	Correlation
Additive vs. Multiplicative	0.980
Additive vs. Hybrid	0.972
Multiplicative vs. Hybrid	0.994

4 Why Hybrid? Theoretical Justification

The hybrid method is preferred for two theoretical reasons:

4.1 1. Multiplicative Penalty for Exclusion

The multiplicative component ensures that exclusion in *any* dimension reduces the overall score. This reflects the theoretical claim that power-sharing requires inclusion across multiple dimensions simultaneously.

Consider a hypothetical society with perfect gender equality (1.0) but complete racial exclusion (0.0):

- **Additive**: $(1.0 + 0.0 + 0.5 + 0.5 + 0.5)/5 = 0.50$ (moderate score)
- **Multiplicative**: $(1.0 \times 0.001 \times 0.5 \times 0.5 \times 0.5)^{1/5} = 0.07$ (near-zero)

The multiplicative approach correctly identifies this as a society with minimal cross-group power-sharing.

4.2 2. Procedural Democracy Cap

The Polyarchy multiplier ensures that PSI cannot exceed the quality of democratic institutions. This addresses a potential concern: could an authoritarian regime with nominal inclusion score highly?

The hybrid formulation prevents this. A regime with Polyarchy = 0.2 cannot exceed PSI = 0.2, regardless of inclusion scores. Power-sharing is meaningful only within functioning democratic procedures.

4.3 Empirical Demonstration

```r
usa_psi |>
  ggplot(aes(x = year)) +
  geom_line(aes(y = psi_multiplicative, color = "Multiplicative (uncapped)"), linewidth = 0.8) +
  geom_line(aes(y = psi_hybrid, color = "Hybrid (Polyarchy-capped)"), linewidth = 0.8) +
  geom_line(aes(y = polyarchy_norm, color = "Polyarchy (cap)"), linewidth = 0.6, linetype = "da
  scale_color_manual(values = c("Multiplicative (uncapped)" = ELEC_COLOR,
                                "Hybrid (Polyarchy-capped)" = PSI_COLOR,
                                "Polyarchy (cap)" = "gray50")) +
  scale_y_continuous(limits = c(0, 1), labels = percent) +
  labs(title = "How the Hybrid Method Works",
       subtitle = "Multiplicative score is capped by procedural democracy quality",
       x = NULL, y = "Index Value", color = NULL) +
  theme_psi()
```

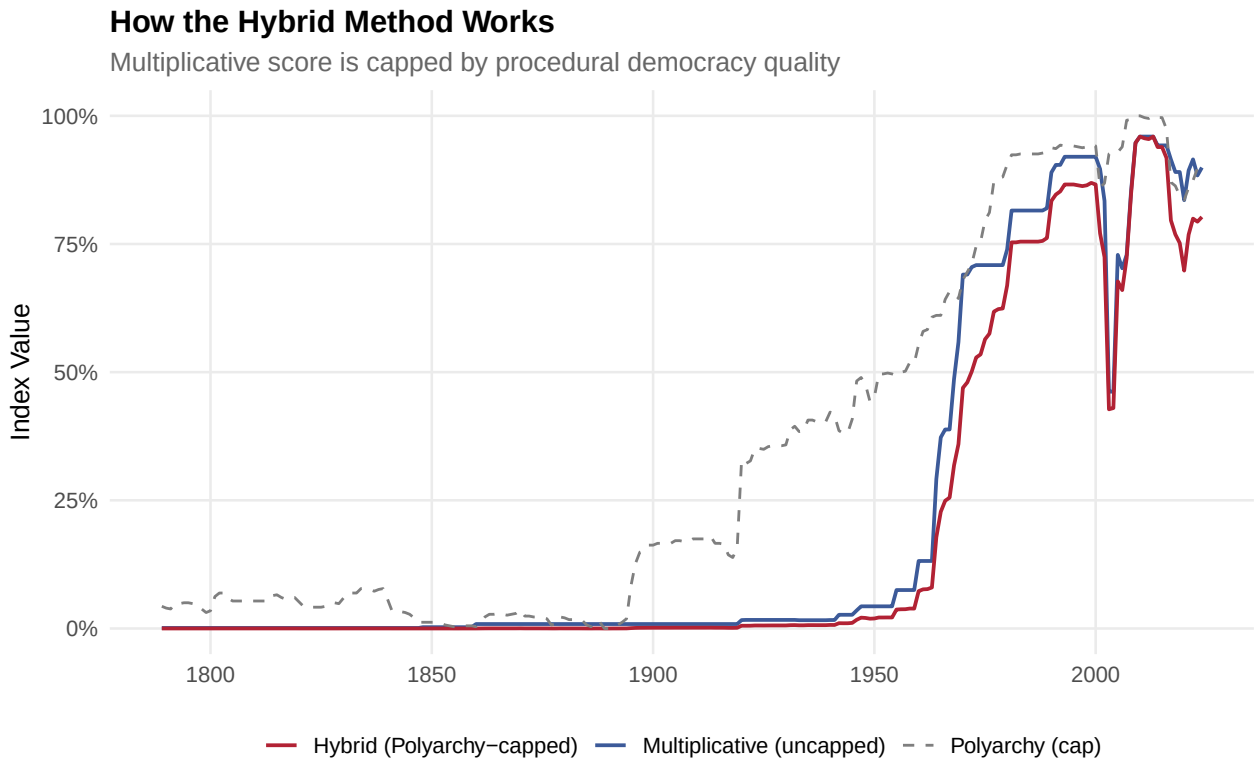

Figure 2: How the Hybrid Method Works

Note. The hybrid PSI (red) tracks the multiplicative score (blue) but is bounded above by the normalized Polyarchy score (gray dashed). This ensures power-sharing is measured within democratic procedures.

5 Sensitivity Analysis: Jackknife (Leave-One-Component-Out)

To assess whether any single component is driving results, each component is dropped in turn and the index is recalculated.

```
usa_psi |>
  mutate(
    Full = psi_additive,
    `Drop: Social Group` = (v2pepwrgen_norm + v2clsocgrp_norm + v2cltort_norm + v2clkill_norm)
    `Drop: Gender` = (v2pepwrsoc_norm + v2clsocgrp_norm + v2cltort_norm + v2clkill_norm) / 4,
    `Drop: Civil Liberties` = (v2pepwrsoc_norm + v2pepwrgen_norm + v2cltort_norm + v2clkill_nor
    `Drop: Torture` = (v2pepwrsoc_norm + v2pepwrgen_norm + v2clsocgrp_norm + v2clkill_norm) / 4
    `Drop: Killings` = (v2pepwrsoc_norm + v2pepwrgen_norm + v2clsocgrp_norm + v2cltort_norm) /
  ) |>
  select(year, Full, starts_with("Drop")) |>
  pivot_longer(-year, names_to = "Version", values_to = "PSI") |>
  mutate(Version = factor(Version, levels = c("Full", "Drop: Social Group", "Drop: Gender",
                                              "Drop: Civil Liberties", "Drop: Torture", "Drop
  ggplot(aes(x = year, y = PSI, color = Version, linewidth = Version)) +
  geom_line() +
  scale_color_manual(values = c("Full" = "black", "Drop: Social Group" = "#1b9e77", "Drop: Gend
                                "Drop: Civil Liberties" = "#7570b3", "Drop: Torture" = "#e7298a
  scale_linewidth_manual(values = c(1.5, rep(0.6, 5))) +
  scale_y_continuous(limits = c(0, 1), labels = percent) +
  labs(title = "Jackknife Sensitivity Analysis",
       subtitle = "Dropping any single component does not change substantive pattern",
       x = NULL, y = "PSI (Additive)") +
  theme_psi() +
  guides(linewidth = "none")
```

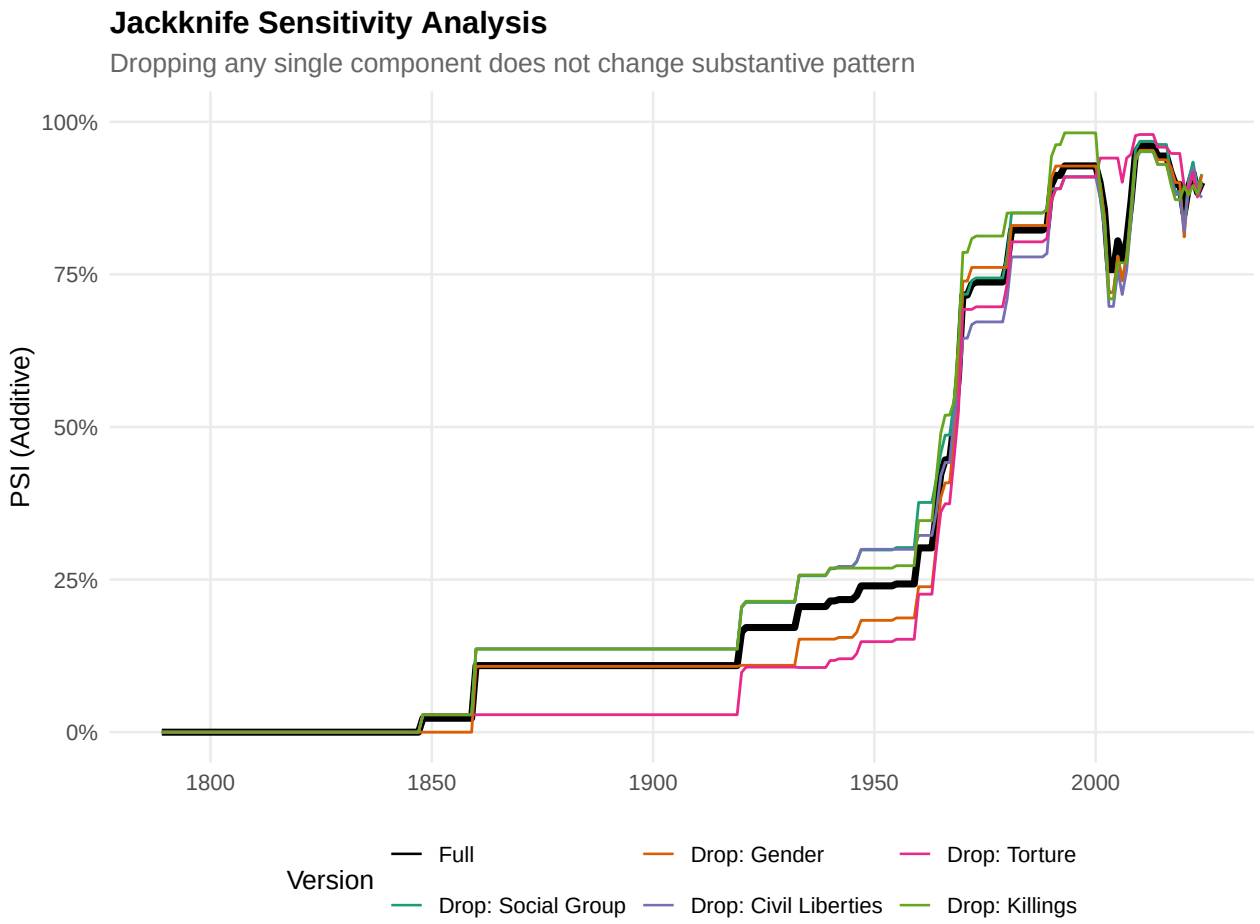

Figure 3: Jackknife Sensitivity Analysis

Note. Black line shows the full five-component index. Colored lines show the index with each component dropped in turn. When a drop line falls above the full index, that component was pulling the score down.

5.1 Interpretation

Key findings:

- **Gender and Civil Liberties** pull scores down most in early periods (as expected—women's exclusion and racial exclusion)
- **Torture** pulls scores down in middle periods
- No single component drives the overall pattern
- Substantive conclusions (four eras, post-1965 shift) are invariant to component exclusion

6 Alternative Specifications

Beyond the three primary methods, additional specifications are tested to ensure robustness.

```
usa_psi |>
  mutate(
    Additive = psi_additive,
    Geometric = psi_multiplicative,
    Hybrid = psi_hybrid,
    Minimum = pmin(v2pepwrsoc_norm, v2pepwrgen_norm, v2clsocgrp_norm, v2cltort_norm, v2clkill_
    `Weighted (2x Power)` = (2*v2pepwrsoc_norm + 2*v2pepwrgen_norm + v2clsocgrp_norm + v2cltor
  ) |>
  select(year, Additive, Geometric, Hybrid, Minimum, `Weighted (2x Power)`) |>
  pivot_longer(-year, names_to = "Specification", values_to = "PSI") |>
  ggplot(aes(x = year, y = PSI, color = Specification)) +
  geom_line(linewidth = 0.7) +
  scale_y_continuous(limits = c(0, 1), labels = percent) +
  labs(title = "Alternative Specifications",
       subtitle = "All specifications show same substantive pattern",
       x = NULL, y = "PSI") +
  theme_psi()
```

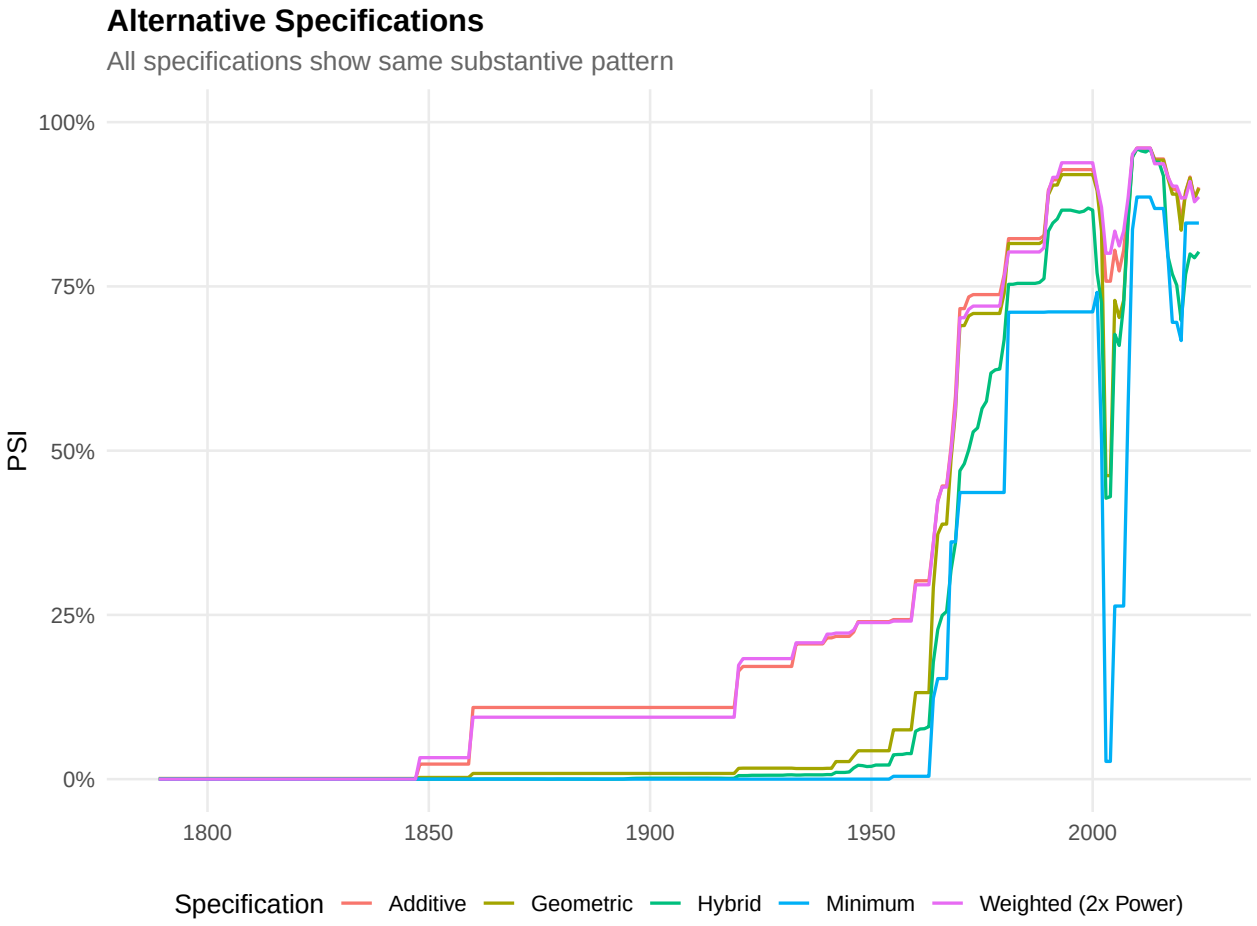

Figure 4: Alternative Specifications

Note. All five specifications produce the same substantive periodization, differing only in magnitude.

6.1 Specifications Tested

Specification	Formula	Rationale
Additive	Arithmetic mean of 5 components	Standard approach
Geometric	Geometric mean of 5 components	Penalizes any-dimension exclusion
Hybrid	Geometric × Polyarchy	Adds procedural cap
Minimum	Lowest component score	Maximum penalty for exclusion
Weighted (2x Power)	Double-weights power distribution	Tests sensitivity to weighting

7 Response to Historical Events

A properly constructed index should respond to known historical events that affected cross-group power distribution.

```
events <- tribble(
  ~Event, ~Year_Before, ~Year_After, ~Expected,
  "19th Amendment (1920)", 1919, 1921, "Increase",
  "Voting Rights Act (1965)", 1964, 1966, "Major Increase",
  "Shelby County v. Holder (2013)", 2012, 2014, "Decline"
)

get_psi <- function(yr, method = "additive") {
  val <- if (method == "additive") {
    usa_psi$psi_additive[usa_psi$year == yr]
  } else {
    usa_psi$psi_hybrid[usa_psi$year == yr]
  }
  if (length(val) == 0) NA_real_ else val
}

events |>
  rowwise() |>
  mutate(
    PSI_Before = get_psi(Year_Before),
    PSI_After = get_psi(Year_After),
    Change = PSI_After - PSI_Before,
    Direction = if_else(Change > 0, "\u2191", "\u2193")
  ) |>
  ungroup() |>
  select(Event, Expected, PSI_Before, PSI_After, Change, Direction) |>
  gt() |>
  tab_header(
    title = "PSI Response to Historical Events",
    subtitle = "Index moves in expected direction for all major events"
  ) |>
  fmt_number(columns = c(PSI_Before, PSI_After, Change), decimals = 3)
```

Note. PSI values from the additive specification. All three events produce changes in the expected direction, providing construct validity evidence.

Table 4

PSI Response to Historical Events

Index moves in expected direction for all major events

Event	Expected	PSI_Before	PSI_After	Change	Direction
19th Amendment (1920)	Increase	0.109	0.172	0.062	↑
Voting Rights Act (1965)	Major Increase	0.359	0.446	0.087	↑
Shelby County v. Holder (2013)	Decline	0.961	0.944	-0.017	↓

8 Summary

8.1 Key Findings

1. **Robustness**: All aggregation methods produce the same substantive pattern. Conclusions do not depend on methodological choices.
2. **Jackknife stability**: No single component drives results. The index is not an artifact of any particular variable.
3. **Construct validity**: The index responds appropriately to known historical events (19th Amendment, VRA, Shelby County).

8.2 Recommended Specification

Hybrid (Multiplicative × Polyarchy) is recommended because:

- Multiplicative aggregation correctly penalizes exclusion in any dimension
- Polyarchy cap ensures power-sharing is measured within democratic procedures
- Results are robust to alternative specifications

8.3 Comparison to Sigman & Lindberg (2019)

Aspect	Sigman & Lindberg	This Analysis
Sensitivity analysis	Not presented	Full jackknife and alternative specifications
Aggregation choice	Single method	Three methods compared, robustness demonstrated
Component selection	Avoid existing indices	Selection based on conceptual fit; distinctiveness shown empirically

9 Save Results

Output saved to `data/processed/psi_phase1_results.rda`. Render Phase 2 next.

10 Session Info

```
R version 4.5.2 (2025-10-31)
Platform: aarch64-apple-darwin20
Running under: macOS Tahoe 26.3

Matrix products: default
BLAS:   /System/Library/Frameworks/Accelerate.framework/Versions/A/Frameworks/vecLib.framework/
LAPACK: /Library/Frameworks/R.framework/Versions/4.5-arm64/Resources/lib/libRlapack.dylib;  LA

locale:
[1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8

time zone: America/Indiana/Indianapolis
tzcode source: internal

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] kableExtra_1.4.0   knitr_1.50         ggcorrplot_0.1.4.1 psych_2.5.3
 [5] scales_1.4.0       gt_1.1.0           lubridate_1.9.4    forcats_1.0.0
 [9] stringr_1.5.1      dplyr_1.1.4        purrr_1.0.4        readr_2.1.5
[13] tidyr_1.3.1        tibble_3.3.0       ggplot2_4.0.2      tidyverse_2.0.0
[17] pacman_0.5.1       here_1.0.1

loaded via a namespace (and not attached):
 [1] generics_0.1.4     xml2_1.3.8         stringi_1.8.7      lattice_0.22-7
 [5] hms_1.1.3          digest_0.6.37      magrittr_2.0.3     evaluate_1.0.3
 [9] grid_4.5.2         timechange_0.3.0   RColorBrewer_1.1-3 fastmap_1.2.0
[13] rprojroot_2.0.4    jsonlite_2.0.0     tinytex_0.57       viridisLite_0.4.3
[17] textshaping_1.0.1  mnormt_2.1.1       cli_3.6.5          rlang_1.1.7
[21] withr_3.0.2        yaml_2.3.10        tools_4.5.2        parallel_4.5.2
[25] tzdb_0.5.0         vctrs_0.7.1        R6_2.6.1           lifecycle_1.0.5
[29] fs_1.6.6           pkgconfig_2.0.3    pillar_1.10.2      gtable_0.3.6
[33] glue_1.8.0         systemfonts_1.2.3  xfun_0.54          tidyselect_1.2.1
[37] rstudioapi_0.17.1  farver_2.1.2       htmltools_0.5.8.1  nlme_3.1-168
[41] labeling_0.4.3     svglite_2.2.1      rmarkdown_2.29     compiler_4.5.2
[45] S7_0.2.1
```