Extension 1
Different weight on wages and employment in the utility function
monopolist equilibrium output (see paper)

$q = \frac{(a - w)}{2 - n}$

$$q = \frac{a - w}{2 - n} \tag{1}$$

Union utility function

$V = (w - e \cdot k)^{\theta} \cdot q^{(1-\theta)}$

$$V = (-e\,k + w)^{\theta}\, q^{1-\theta} \tag{2}$$

Insert Eq. (1) into Eq.(2)

$subs\left(q = \frac{a - w}{2 - n},\ V = (w - e \cdot k)^{\theta} \cdot q^{(1-\theta)}\right)$

$$V = (-e\,k + w)^{\theta} \left(\frac{a - w}{2 - n}\right)^{1-\theta} \tag{3}$$

$\xrightarrow{\text{differentiate w.r.t. w}}$

$$0 = \frac{(-e\,k + w)^{\theta}\,\theta \left(\frac{a - w}{2 - n}\right)^{1-\theta}}{-e\,k + w} - \frac{(-e\,k + w)^{\theta} \left(\frac{a - w}{2 - n}\right)^{1-\theta} (1 - \theta)}{a - w} \tag{4}$$

$\xrightarrow{\text{solve for w}}$ optimal wage set by the union

$$[[w = -e\,k\,\theta + a\,\theta + e\,k]] \tag{5}$$

Optimal wage, linear combination of market size and environmental concern/reservation wage

$w = -e\,k\,\theta + a\,\theta + e\,k$

$$w = -e\,k\,\theta + a\,\theta + e\,k \tag{6}$$

$\xrightarrow{\text{evaluate at point}}$ check refrence model $(\theta = \frac{1}{2})$

$$w = \frac{1}{2}\,e\,k + \frac{1}{2}\,a \tag{7}$$

Substitute Eq. (7) into Eq. (1) to get the optimal output

$subs\left(w = -e\,k\,\theta + a\,\theta + e\,k,\ q = \frac{a - w}{2 - n}\right)$

$$q = \frac{e\,k\,\theta - a\,\theta - e\,k + a}{2 - n} \tag{8}$$

$\overset{\text{simplify}}{=}$

$$q = \frac{(\theta - 1)\,(-e\,k + a)}{-2 + n} \tag{9}$$

Profits in equilibrium
Substitutions of the optimal wage and output into the profit function (see paper)

$subs\left(q = \frac{(\theta - 1)\,(-e\,k + a)}{-2 + n},\ w = -e\,k\,\theta + a\,\theta + e\,k,\ \pi = (a - q + n \cdot q) \cdot q - w \cdot q - z \cdot (1 - k)^2\right)$

$$\pi = \frac{\left(\frac{n\,(\theta - 1)\,(-e\,k + a)}{-2 + n} + a - \frac{(\theta - 1)\,(-e\,k + a)}{-2 + n}\right)(\theta - 1)\,(-e\,k + a)}{-2 + n} \tag{10}$$

$$-\frac{(-ek\theta+a\theta+ek)(\theta-1)(-ek+a)}{-2+n}-z(1-k)^2$$

simplify

$$\pi=\frac{1}{(-2+n)^2}\left(\left(-(-2+n)^2z+e^2(\theta-1)^2\right)k^2+\left(2(-2+n)^2z-2ae(\theta-1)^2\right)k\right. \tag{11}$$
$$\left.-(-2+n)^2z+a^2(\theta-1)^2\right)$$

differentiate w.r.t. k

$$0=\frac{2\left(-(-2+n)^2z+e^2(\theta-1)^2\right)k+2(-2+n)^2z-2ae(\theta-1)^2}{(-2+n)^2} \tag{12}$$

solve for k → optimal level of emission

$$\left[\left[k=\frac{ae\theta^2-2ae\theta-n^2z+ae+4nz-4z}{e^2\theta^2-2e^2\theta-n^2z+e^2+4nz-4z}\right]\right] \tag{13}$$

$$k=\frac{ae\theta^2-2ae\theta-n^2z+ae+4nz-4z}{e^2\theta^2-2e^2\theta-n^2z+e^2+4nz-4z}$$

$$k=\frac{ae\theta^2-2ae\theta-n^2z+ae+4nz-4z}{e^2\theta^2-2e^2\theta-n^2z+e^2+4nz-4z} \tag{14}$$

simplify

$$k=\frac{ae(\theta-1)^2-(-2+n)^2z}{-(-2+n)^2z+e^2(\theta-1)^2} \tag{15}$$

evaluate at point → check basic model

$$k=\frac{\frac{1}{4}ae-(-2+n)^2z}{-(-2+n)^2z+\frac{1}{4}e^2} \tag{16}$$

simplify

$$k=\frac{-4(-2+n)^2z+ae}{-4(-2+n)^2z+e^2} \tag{17}$$

Substitute Eq. (15) into Eq. (9)

$$subs\left(k=\frac{ae(\theta-1)^2-(-2+n)^2z}{-(-2+n)^2z+e^2(\theta-1)^2},\ q=\frac{(\theta-1)(-ek+a)}{-2+n}\right)$$

$$q=\frac{(\theta-1)\left(-\frac{e\left(ae(\theta-1)^2-(-2+n)^2z\right)}{-(-2+n)^2z+e^2(\theta-1)^2}+a\right)}{-2+n} \tag{18}$$

simplify → optimal output as function of the network externalities, environmental damage and union wage sensitivity

$$q=-\frac{(\theta-1)z(-2+n)(a-e)}{-(-2+n)^2z+e^2(\theta-1)^2} \tag{19}$$

$$-(-2+n)^2 z + e^2 (\theta-1)^2 = 0$$

$$-(-2+n)^2 z + e^2 (\theta-1)^2 = 0 \tag{20}$$

$\xrightarrow{\text{solve for e}}$ non-negativity condition on output production $e < \frac{\sqrt{z}\,(2-n)}{1-\theta}$

$$\left[\left[e = \frac{\sqrt{z}\,(-2+n)}{\theta-1}\right], \left[e = -\frac{\sqrt{z}\,(-2+n)}{\theta-1}\right]\right] \tag{21}$$

the more the union is wage oriented, the more the environmental concern can be higher

$$k = \frac{a\,e\,(\theta-1)^2 - (-2+n)^2 z}{-(-2+n)^2 z + e^2 (\theta-1)^2}$$

$$\frac{a\,e\,(\theta-1)^2 - (-2+n)^2 z}{-(-2+n)^2 z + e^2 (\theta-1)^2} = 0$$

$$\frac{a\,e\,(\theta-1)^2 - (-2+n)^2 z}{-(-2+n)^2 z + e^2 (\theta-1)^2} = 0 \tag{22}$$

$\xrightarrow{\text{solve for a}}$

$$\left[\left[a = \frac{(-2+n)^2 z}{e\,(\theta-1)^2}\right]\right] \tag{23}$$

Comparative statics

$$k = \frac{a\,e\,(\theta-1)^2 - (-2+n)^2 z}{-(-2+n)^2 z + e^2 (\theta-1)^2} \xrightarrow{\text{differentiate w.r.t. n}}$$

$$0 = -\frac{2\,(-2+n)\,z}{-(-2+n)^2 z + e^2 (\theta-1)^2} + \frac{2\left(a\,e\,(\theta-1)^2 - (-2+n)^2 z\right)(-2+n)\,z}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^2} \overset{\text{simplify}}{=} 0$$

$$= \frac{2\,(-2+n)\,z\,e\,(\theta-1)^2\,(a-e)}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^2} \xrightarrow{\text{differentiate w.r.t. theta}} 0 = \frac{4\,(-2+n)\,z\,e\,(\theta-1)\,(a-e)}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^2}$$

$$-\frac{8\,(-2+n)\,z\,e^3\,(\theta-1)^3\,(a-e)}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^3} \overset{\text{simplify}}{=} 0 =$$

$$-\frac{4\left(e^2 (\theta-1)^2 + (-2+n)^2 z\right) z\,(-2+n)\,e\,(\theta-1)\,(a-e)}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^3}$$

$$q = -\frac{(\theta-1)\,z\,(-2+n)\,(a-e)}{-(-2+n)^2 z + e^2 (\theta-1)^2} \xrightarrow{\text{differentiate w.r.t. n}}$$

$$0 = -\frac{(\theta-1)\,z\,(a-e)}{-(-2+n)^2 z + e^2 (\theta-1)^2} - \frac{2\,(\theta-1)\,z^2\,(-2+n)^2\,(a-e)}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^2} \overset{\text{simplify}}{=} 0 =$$

$$-\frac{\left(e^2 (\theta-1)^2 + (-2+n)^2 z\right) z\,(\theta-1)\,(a-e)}{\left(-(-2+n)^2 z + e^2 (\theta-1)^2\right)^2} \xrightarrow{\text{differentiate w.r.t. theta}} 0 =$$

$$-\frac{2e^2(\theta-1)^2 z(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2}-\frac{\left(e^2(\theta-1)^2+(-2+n)^2 z\right) z(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2}$$
$$+\frac{4\left(e^2(\theta-1)^2+(-2+n)^2 z\right) z(\theta-1)^2(a-e)e^2}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^3}\overset{\text{simplify}}{=}0$$
$$=\frac{\left((\theta-1)^4 e^4+6z(\theta-1)^2(-2+n)^2 e^2+z^2(-2+n)^4\right) z(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^3}$$

optimal wage as function of the network externalities, environmental damage and union wage sensitivity

Substitute Eq. (15) in Eq. (6)

$$subs\left(k=\frac{a e(\theta-1)^2-(-2+n)^2 z}{-(-2+n)^2 z+e^2(\theta-1)^2},\ w=-e k\theta+a\theta+e k\right)$$

$$w=-\frac{e\left(a e(\theta-1)^2-(-2+n)^2 z\right)\theta}{-(-2+n)^2 z+e^2(\theta-1)^2}+\theta a+\frac{e\left(a e(\theta-1)^2-(-2+n)^2 z\right)}{-(-2+n)^2 z+e^2(\theta-1)^2} \quad (24)$$

$\overset{\text{simplify}}{=}$

$$w=\frac{a(\theta-1)^2 e^2+z(-2+n)^2(\theta-1)e-a z\theta(-2+n)^2}{-(-2+n)^2 z+e^2(\theta-1)^2} \quad (25)$$

Total pollution

$P=k\cdot q$

$$P=k q \quad (26)$$

$$subs\left(k=\frac{a e(\theta-1)^2-(-2+n)^2 z}{-(-2+n)^2 z+e^2(\theta-1)^2},\ q=-\frac{(\theta-1) z(-2+n)(a-e)}{-(-2+n)^2 z+e^2(\theta-1)^2},\ P=k q\right)$$

$$P=-\frac{\left(a e(\theta-1)^2-(-2+n)^2 z\right)(\theta-1) z(-2+n)(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} \quad (27)$$

$\xrightarrow{\text{evaluate at point}}$ check basic model

$$P=\frac{1}{2}\frac{\left(\frac{1}{4}a e-(-2+n)^2 z\right) z(-2+n)(a-e)}{\left(-(-2+n)^2 z+\frac{1}{4}e^2\right)^2} \quad (28)$$

$\overset{\text{simplify}}{=}$

$$P=\frac{2z(a-e)\left(-4(-2+n)^2 z+a e\right)(-2+n)}{\left(-4(-2+n)^2 z+e^2\right)^2} \quad (29)$$

Effect of wage orientation on investment in technology, $\frac{\partial k}{\partial\theta}>0$

$$k = \frac{a\,e\,(\theta-1)^2-(-2+n)^2 z}{-(-2+n)^2 z+e^2(\theta-1)^2} \xrightarrow{\text{differentiate w.r.t. theta}}$$

$$0 = \frac{2\,a\,e\,(\theta-1)}{-(-2+n)^2 z+e^2(\theta-1)^2} - \frac{2\left(a\,e\,(\theta-1)^2-(-2+n)^2 z\right)e^2(\theta-1)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} \overset{\text{simplify}}{=} 0 =$$

$$-\frac{2\,e\,(\theta-1)\,z\,(-2+n)^2(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} \xrightarrow{\text{differentiate w.r.t. theta}} 0 =$$

$$-\frac{2\,e\,z\,(-2+n)^2(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} + \frac{8\,e^3(\theta-1)^2 z\,(-2+n)^2(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^3} \overset{\text{simplify}}{=} 0$$

$$= \frac{6\,(a-e)\left(e^2(\theta-1)^2+\frac{1}{3}(-2+n)^2 z\right)e\,z\,(-2+n)^2}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^3}$$

$$2\,e\,(1-\theta)\,z\,(-2+n)^2(a-e)=0$$

$$2\,e\,(1-\theta)\,z\,(-2+n)^2(a-e)=0 \qquad \textbf{(30)}$$

$\xrightarrow{\text{solve for theta}}$

$$[[\theta=1]] \qquad \textbf{(31)}$$

$$q = -\frac{(\theta-1)\,z\,(-2+n)\,(a-e)}{-(-2+n)^2 z+e^2(\theta-1)^2} \xrightarrow{\text{differentiate w.r.t. theta}} \frac{\partial q}{\partial\theta} < 0$$

$$0 = -\frac{z\,(-2+n)\,(a-e)}{-(-2+n)^2 z+e^2(\theta-1)^2} + \frac{2(\theta-1)^2 z\,(-2+n)\,(a-e)\,e^2}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} \overset{\text{simplify}}{=} 0$$

$$= \frac{(a-e)\left(e^2(\theta-1)^2+(-2+n)^2 z\right)z\,(-2+n)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2}$$

Market size at which the pollution-reducing effect occurs

$$P = -\frac{\left(a\,e\,(\theta-1)^2-(-2+n)^2 z\right)(\theta-1)\,z\,(-2+n)\,(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} \xrightarrow{\text{differentiate w.r.t. n}}$$

$$0 = \frac{2\,(-2+n)^2 z^2(\theta-1)\,(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} - \frac{\left(a\,e\,(\theta-1)^2-(-2+n)^2 z\right)(\theta-1)\,z\,(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2}$$

$$- \frac{4\left(a\,e\,(\theta-1)^2-(-2+n)^2 z\right)(\theta-1)\,z^2(-2+n)^2(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^3}$$

$\xrightarrow{\text{solve for a}}$

$$[[a=e],[a=(z(3e^2n^2\theta^2-6e^2n^2\theta-12e^2n\theta^2+n^4z+3e^2n^2+24e^2n\theta+12e^2\theta^2-8n^3z-12e^2n-24e^2\theta+24n^2z+12e^2-32nz+16z))/(e(e^2\theta^4-4e^2\theta^3+3n^2\theta^2z+6e^2\theta^2-6n^2\theta z-12n\theta^2z-4e^2\theta+3n^2z+24n\theta z+12\theta^2z+e^2-12nz-24\theta z+12z))]]$$

$$a = \left(z\left(3\,e^2\,n^2\,\theta^2 - 6\,e^2\,n^2\,\theta - 12\,e^2\,n\,\theta^2 + n^4\,z + 3\,e^2\,n^2 + 24\,e^2\,n\,\theta + 12\,e^2\,\theta^2 - 8\,n^3\,z - 12\,e^2\,n - 24\,e^2\,\theta + 24\,n^2\,z + 12\,e^2 - 32\,n\,z + 16\,z\right)\right) \Big/ \left(e\left(e^2\,\theta^4 - 4\,e^2\,\theta^3 + 3\,n^2\,\theta^2\,z + 6\,e^2\,\theta^2 - 6\,n^2\,\theta\,z - 12\,n\,\theta^2\,z - 4\,e^2\,\theta + 3\,n^2\,z + 24\,n\,\theta\,z + 12\,\theta^2\,z + e^2 - 12\,n\,z - 24\,\theta\,z + 12\,z\right)\right)$$

$$\overset{\text{simplify}}{=} a = \frac{3\left(e^2\,\theta^2 - 2\,e^2\,\theta + \frac{1}{3}\,n^2\,z + e^2 - \frac{4}{3}\,n\,z + \frac{4}{3}\,z\right) z\,(-2+n)^2}{e\,(\theta-1)^2\left(e^2\,\theta^2 - 2\,e^2\,\theta + 3\,n^2\,z + e^2 - 12\,n\,z + 12\,z\right)}$$

Numerical examples to construct Figure 3
Left side

$$P = -\frac{\left(a\,e\,(\theta-1)^2 - (-2+n)^2\,z\right)(\theta-1)\,z\,(-2+n)\,(a-e)}{\left(-(-2+n)^2\,z + e^2\,(\theta-1)^2\right)^2} \xrightarrow{\text{evaluate at point}}$$

$$P = \frac{1}{2}\,\frac{\left(\frac{1}{4}\,a\,e - (-2+n)^2\,z\right) z\,(-2+n)\,(a-e)}{\left(-(-2+n)^2\,z + \frac{1}{4}\,e^2\right)^2} \xrightarrow{\text{evaluate at point}} P$$

$$= \frac{2.475\,(0.60750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.050625\right)^2} \xrightarrow{\text{evaluate at point}} P$$

$$= \frac{3.025\,(0.90750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.075625\right)^2}$$

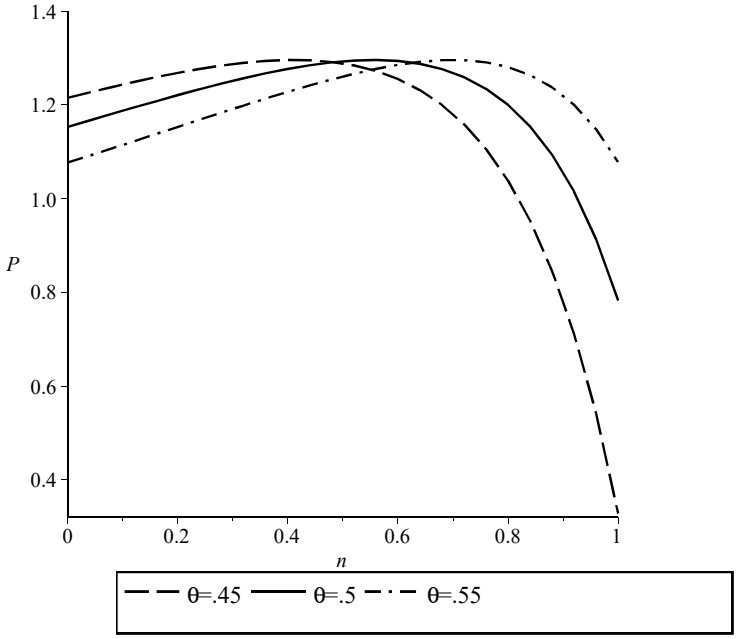
$$P = \frac{3.025\,(0.90750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.075625\right)^2}, P = \frac{2.75\,(0.750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.0625\right)^2}, P$$

$$= \frac{2.475\,(0.60750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.050625\right)^2}$$

$$P = \frac{3.025\,(0.90750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.075625\right)^2}, P = \frac{2.75\,(0.750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.0625\right)^2}, P \quad \textbf{(32)}$$

$$= \frac{2.475\,(0.60750 - (-2+n)^2)\,(-2+n)}{\left(-(-2+n)^2 + 0.050625\right)^2}$$

$\rightarrow$



Right side

$$P = -\frac{\left(a\,e\,(\theta-1)^2-(-2+n)^2 z\right)(\theta-1)\,z\,(-2+n)\,(a-e)}{\left(-(-2+n)^2 z+e^2(\theta-1)^2\right)^2} \xrightarrow{\text{evaluate at point}}$$

$$P = \frac{0.990\,\left(0.48600-(-2+n)^2\right)(-2+n)}{\left(-(-2+n)^2+0.129600\right)^2} \xrightarrow{\text{evaluate at point}} P$$

$$= \frac{1.10\,\left(0.600-(-2+n)^2\right)(-2+n)}{\left(-(-2+n)^2+0.1600\right)^2} \xrightarrow{\text{evaluate at point}} P$$

$$= \frac{1.210\,\left(0.72600-(-2+n)^2\right)(-2+n)}{\left(-(-2+n)^2+0.193600\right)^2}$$

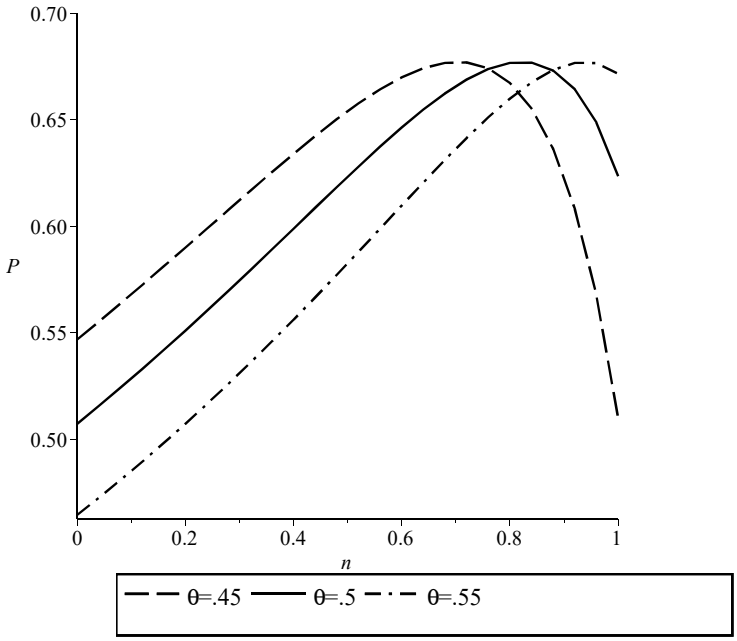
$$P = \frac{1.210\,\left(0.72600-(-2+n)^2\right)(-2+n)}{\left(-(-2+n)^2+0.193600\right)^2}, P = \frac{1.10\,\left(0.600-(-2+n)^2\right)(-2+n)}{\left(-(-2+n)^2+0.1600\right)^2}, P$$

$$= \frac{0.990\,(0.48600 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.129600)^2}$$

$$P = \frac{1.210\,(0.72600 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.193600)^2}, P = \frac{1.10\,(0.600 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.1600)^2}, P \quad \textbf{(33)}$$

$$= \frac{0.990\,(0.48600 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.129600)^2}$$

→



Extension 2
Government decision of the abatement technology
Output as a function of the wage

$$subs\left(w = \frac{1}{2}\,e\,k + \frac{1}{2}\,a\,,\ q = \frac{(a-w)}{2-n}\right)$$

$$q = \frac{\frac{1}{2}\,a - \frac{1}{2}\,e\,k}{2-n} \quad \textbf{(34)}$$

simplify
=

$$q = \frac{e\,k - a}{-4 + 2\,n} \quad \textbf{(35)}$$

$$q = \frac{(a - w)}{2 - n}$$

Substitute the optimal wage (see paper) into the basic union utility function

$$subs\left(q = \frac{e\,k - a}{-4 + 2\,n},\ w = \frac{1}{2}\,e\,k + \frac{1}{2}\,a,\ V = (w - e \cdot k) \cdot q\right)$$

$$V = \frac{\left(\frac{1}{2}\,a - \frac{1}{2}\,e\,k\right)\,(e\,k - a)}{-4 + 2\,n}$$

$$\overset{\text{simplify}}{=}$$

$$V = -\frac{(-e\,k + a)^2}{-8 + 4\,n} \tag{37}$$

Substitute optimal output and wage into the profit fuction

$$subs\left(q = \frac{e\,k - a}{-4 + 2\,n},\ w = \frac{1}{2}\,e\,k + \frac{1}{2}\,a,\ \pi = (a - q + n \cdot q) \cdot q - w \cdot q\right)$$

$$\pi = \frac{\left(\frac{n\,(e\,k - a)}{-4 + 2\,n} + a - \frac{e\,k - a}{-4 + 2\,n}\right)\,(e\,k - a)}{-4 + 2\,n} - \frac{\left(\frac{1}{2}\,e\,k + \frac{1}{2}\,a\right)\,(e\,k - a)}{-4 + 2\,n}$$

$$\overset{\text{simplify}}{=}$$

$$\pi = \frac{1}{4}\,\frac{(-e\,k + a)^2}{(-2 + n)^2}$$

$$\pi = \frac{1}{4}\,\frac{(-e\,k + a)^2}{(-2 + n)^2} - z \cdot (1 - k)^2$$

$$\pi = \frac{1}{4}\,\frac{(-e\,k + a)^2}{(-2 + n)^2} - z\,(1 - k)^2 \tag{40}$$

Consumer's surplus formula

$$CS = \frac{(1 - n) \cdot q^2}{2}$$

$$CS = \frac{1}{2}\,(1 - n)\,q^2 \tag{41}$$

$$subs\left(q = \frac{e\,k - a}{-4 + 2\,n},\ CS = \frac{1}{2}\,(1 - n)\,q^2\right)$$

$$CS = \frac{1}{2}\,\frac{(1 - n)\,(e\,k - a)^2}{(-4 + 2\,n)^2} \tag{42}$$

$$\xrightarrow{\text{simplify symbolic}}$$

$$CS = -\frac{1}{8}\,\frac{(-1 + n)\,(-e\,k + a)^2}{(-2 + n)^2} \tag{43}$$

No additional environmental damage

The government chooses the socially optimal level of emissions

$SW = V + \pi + CS$

$$SW = V + \pi + CS \tag{44}$$

$$subs\left(V = -\frac{(-e\,k + a)^2}{-8 + 4\,n},\ \pi = \frac{1}{4}\,\frac{(-e\,k + a)^2}{(-2 + n)^2} - z\,(1 - k)^2,\ CS = \frac{1}{2}\,\frac{(1 - n)\,(e\,k - a)^2}{(-4 + 2\,n)^2},\ SW = V + \pi + CS\right)$$

$$SW = -\frac{(-e\,k + a)^2}{-8 + 4\,n} + \frac{1}{4}\,\frac{(-e\,k + a)^2}{(-2 + n)^2} - z\,(1 - k)^2 + \frac{1}{2}\,\frac{(1 - n)\,(e\,k - a)^2}{(-4 + 2\,n)^2} \tag{45}$$

differentiate w.r.t. k →

$$0 = \frac{2\,(-e\,k + a)\,e}{-8 + 4\,n} - \frac{1}{2}\,\frac{(-e\,k + a)\,e}{(-2 + n)^2} + 2\,z\,(1 - k) + \frac{(1 - n)\,(e\,k - a)\,e}{(-4 + 2\,n)^2} \tag{46}$$

solve for k →

$$\left[\left[k = \frac{3\,a\,e\,n + 8\,n^2\,z - 7\,a\,e - 32\,n\,z + 32\,z}{3\,e^2\,n + 8\,n^2\,z - 7\,e^2 - 32\,n\,z + 32\,z}\right]\right] \tag{47}$$

$$k = \frac{3\,a\,e\,n + 8\,n^2\,z - 7\,a\,e - 32\,n\,z + 32\,z}{3\,e^2\,n + 8\,n^2\,z - 7\,e^2 - 32\,n\,z + 32\,z}$$

$$k = \frac{3\,a\,e\,n + 8\,n^2\,z - 7\,a\,e - 32\,n\,z + 32\,z}{3\,e^2\,n + 8\,n^2\,z - 7\,e^2 - 32\,n\,z + 32\,z} \tag{48}$$

simplify =

$$k = \frac{8\,n^2\,z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z}{8\,n^2\,z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z} \tag{49}$$

Positivity conditions Eq. (49) (Eq. (22) in the main text)

$3\,a\,e\,n + 8\,n^2\,z - 7\,a\,e - 32\,n\,z + 32\,z = 0$

$$3\,a\,e\,n + 8\,n^2\,z - 7\,a\,e - 32\,n\,z + 32\,z = 0 \tag{50}$$

solve for a →

$$\left[\left[a = -\frac{8\,z\,(n^2 - 4\,n + 4)}{e\,(3\,n - 7)}\right]\right] \tag{51}$$

$0 = 8\,n^2\,z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z$

$$0 = 8\,n^2\,z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z \tag{52}$$

solve for e →

$$\left[\left[e = \frac{2\sqrt{-(6\,n - 14)\,z}\,(-2 + n)}{3\,n - 7}\right], \left[e = -\frac{2\sqrt{-(6\,n - 14)\,z}\,(-2 + n)}{3\,n - 7}\right]\right] \tag{53}$$

Optimal output

$$subs\left(k = \frac{8\,n^2\,z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z}{8\,n^2\,z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z},\ q = \frac{e\,k - a}{-4 + 2\,n}\right)$$

$$q = \frac{\frac{e\,(8\,n^2 z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z)}{8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z} - a}{-4 + 2\,n} \tag{54}$$

simplify

$$= q = -\frac{4\,z\,(-2+n)\,(a-e)}{3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z} \tag{55}$$

$3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z$

$$3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z \tag{56}$$

simplify

$$= 8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z \tag{57}$$

Emission levels

$$subs\left(k = \frac{8\,n^2 z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z}{8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z},\ q = -\frac{4\,z\,(-2+n)\,(a-e)}{3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z},\ P = k \cdot q\right)$$

$$P = -\frac{4\,(8\,n^2 z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z)\,z\,(-2+n)\,(a-e)}{(8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z)\,(3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z)} \tag{58}$$

$\xrightarrow{\text{differentiate w.r.t. n}}$ pollution reducing effect of network externalities, market size

$$0 = -\frac{4\,(3\,a\,e + 16\,n\,z - 32\,z)\,z\,(-2+n)\,(a-e)}{(8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z)\,(3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z)} \tag{59}$$
$$- \frac{4\,(8\,n^2 z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z)\,z\,(a-e)}{(8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z)\,(3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z)}$$
$$+ (4\,(8\,n^2 z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z)\,z\,(-2+n)\,(a-e)\,(3\,e^2 + 16\,n\,z - 32\,z))\Big/((8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z)^2\,(3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z))$$
$$+ (4\,(8\,n^2 z + (3\,a\,e - 32\,z)\,n - 7\,a\,e + 32\,z)\,z\,(-2+n)\,(a-e)\,(3\,e^2 + 16\,n\,z - 32\,z))\Big/((8\,n^2 z + (3\,e^2 - 32\,z)\,n - 7\,e^2 + 32\,z)\,(3\,e^2 n + 8\,n^2 z - 7\,e^2 - 32\,n\,z + 32\,z)^2)$$

$\xrightarrow{\text{solve for a}}$

$$\left[[a = e],\ \left[a = (8\,z\,(3\,e^2 n^3 - 8\,n^4 z - 21\,e^2 n^2 + 64\,n^3 z + 48\,e^2 n - 192\,n^2 z - 36\,e^2 + 256\,n\,z - 128\,z))\Big/(e\,(48\,n^3 z + 3\,e^2 n - 312\,n^2 z - 7\,e^2 + 672\,n\,z - 480\,z))\right]\right] \tag{60}$$

$$a = \frac{8\,z\,(3\,e^2 n^3 - 8\,n^4 z - 21\,e^2 n^2 + 64\,n^3 z + 48\,e^2 n - 192\,n^2 z - 36\,e^2 + 256\,n\,z - 128\,z)}{e\,(48\,n^3 z + 3\,e^2 n - 312\,n^2 z - 7\,e^2 + 672\,n\,z - 480\,z)}$$

$$a = \frac{8\,z\,(3\,e^2\,n^3 - 8\,n^4\,z - 21\,e^2\,n^2 + 64\,n^3\,z + 48\,e^2\,n - 192\,n^2\,z - 36\,e^2 + 256\,n\,z - 128\,z)}{e\,(48\,n^3\,z + 3\,e^2\,n - 312\,n^2\,z - 7\,e^2 + 672\,n\,z - 480\,z)}$$

$$\overset{\text{simplify}}{=} a = \frac{8\,z\,(-2+n)^2\left(-\frac{8}{3}\,n^2\,z + \left(\frac{32}{3}\,z + e^2\right)n - \frac{32}{3}\,z - 3\,e^2\right)}{e\left(16\,n^3\,z - 104\,n^2\,z + (e^2 + 224\,z)\,n - 160\,z - \frac{7}{3}\,e^2\right)} \xrightarrow{\text{simplify symbolic}} a$$

$$= \frac{8\,z\,(-2+n)^2\,(3\,e^2\,n - 8\,n^2\,z - 9\,e^2 + 32\,n\,z - 32\,z)}{e\,(48\,n^3\,z + 3\,e^2\,n - 312\,n^2\,z - 7\,e^2 + 672\,n\,z - 480\,z)}$$

Numerical example to construct Figure 4

$$P = \frac{1}{2}\,\frac{\left(\frac{1}{4}\,a\,e - (-2+n)^2\,z\right)z\,(-2+n)\,(a-e)}{\left(-(-2+n)^2\,z + \frac{1}{4}\,e^2\right)^2} \xrightarrow{\text{evaluate at point}}$$

$$P = \frac{2.850000000\,(0.4500000000 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.02250000000)^2} \xrightarrow{\text{evaluate at point}} P$$

$$= \frac{2.950000000\,(0.1500000000 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.002500000000)^2} \xrightarrow{\text{evaluate at point}} P$$

$$= \frac{2.900000000\,(0.3000000000 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.01000000000)^2}$$

$$P = -\frac{4\,(3\,a\,e\,n + 8\,n^2\,z - 7\,a\,e - 32\,n\,z + 32\,z)\,z\,(-2+n)\,(a-e)}{(3\,e^2\,n + 8\,n^2\,z - 7\,e^2 - 32\,n\,z + 32\,z)^2} \xrightarrow{\text{evaluate at point}}$$

$$P = -\frac{22.8\,(8\,n^2 - 26.6\,n + 19.4)\,(-2+n)}{(8\,n^2 - 31.73\,n + 31.37)^2} \xrightarrow{\text{evaluate at point}} P =$$

$$-\frac{23.6\,(8\,n^2 - 30.2\,n + 27.8)\,(-2+n)}{(8\,n^2 - 31.97\,n + 31.93)^2} \xrightarrow{\text{evaluate at point}} P =$$

$$-\frac{23.2\,(8\,n^2 - 28.4\,n + 23.6)\,(-2+n)}{(8\,n^2 - 31.88\,n + 31.72)^2}$$

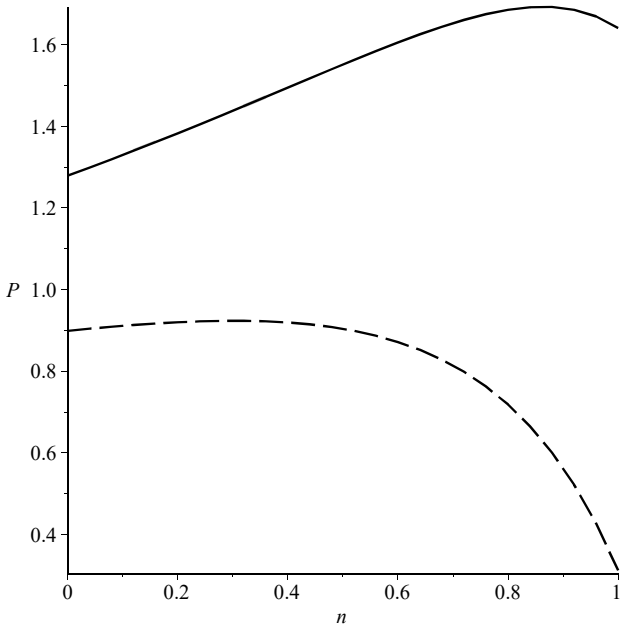
$$P = \frac{2.850000000\,(0.4500000000 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.02250000000)^2}, P =$$

$$-\frac{22.8\,(8\,n^2 - 26.6\,n + 19.4)\,(-2+n)}{(8\,n^2 - 31.73\,n + 31.37)^2}$$

$$P = \frac{2.850000000\,(0.4500000000 - (-2+n)^2)\,(-2+n)}{(-(-2+n)^2 + 0.02250000000)^2}, P = \qquad \textbf{(61)}$$

$$-\frac{22.8\,(8\,n^2 - 26.6\,n + 19.4)\,(-2+n)}{(8\,n^2 - 31.73\,n + 31.37)^2}$$

$\rightarrow$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1.4
1.2
P 1.0
0.8
0.6
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0
0.2
0.4
0.6
0.8
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