Table 1: Marginal Effects and Variances for Various Multiplicative Interaction Models (Double and Triple Interaction Terms)

Case	Equation	Marginal Effect	Variance
1a	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_3) + 2Z cov(\hat{\beta}_1 \hat{\beta}_3)$
1b	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$	$\frac{\partial Y}{\partial Z} = \beta_2 + \beta_3 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_2) + X^2 var(\hat{\beta}_3) + 2X cov(\hat{\beta}_2 \hat{\beta}_3)$
2	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_6 ZW$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_4 Z$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_4) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4)$
3	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_5 XW + \beta_6 ZW$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_4 Z + \beta_5 W$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_4) + W^2 var(\hat{\beta}_5) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4) + 2W cov(\hat{\beta}_1 \hat{\beta}_5) + 2ZW cov(\hat{\beta}_4 \hat{\beta}_5)$
4	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_5 XW + \beta_6 ZW + \beta_7 XZW$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_4 Z + \beta_5 W + \beta_7 ZW$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_4) + W^2 var(\hat{\beta}_5) + Z^2 W^2 var(\hat{\beta}_7) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4) + 2W cov(\hat{\beta}_1 \hat{\beta}_5) + 2ZW cov(\hat{\beta}_1 \hat{\beta}_7) + 2ZW cov(\hat{\beta}_4 \hat{\beta}_5) + 2W Z^2 cov(\hat{\beta}_4 \hat{\beta}_7) + 2ZW^2 cov(\hat{\beta}_5 \hat{\beta}_7)$

Table 2: Marginal Effects and Variances for Various Multiplicative Interaction Models (Quadratic Terms)

Case	Equation	Marginal Effect	Variance
1	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + 4X cov(\hat{\beta}_1 \hat{\beta}_2)$
2	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + 4X cov(\hat{\beta}_1 \hat{\beta}_2)$
3a	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X + \beta_4 Z$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + Z^2 var(\hat{\beta}_4)$ $+4X cov(\hat{\beta}_1 \hat{\beta}_2) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4)$ $+4XZ cov(\hat{\beta}_2 \hat{\beta}_4)$
3b	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ$	$\frac{\partial Y}{\partial Z} = \beta_3 + \beta_4 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial Z}} = var(\hat{\beta}_3) + X^2 var(\hat{\beta}_4) + 2X cov(\hat{\beta}_3 \hat{\beta}_4)$
4a	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ$ $+\beta_5 X^2 Z$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X + \beta_4 Z$ $+2\beta_5 XZ$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + Z^2 var(\hat{\beta}_4)$ $+4X^2 Z^2 var(\hat{\beta}_5)$ $+4X cov(\hat{\beta}_1 \hat{\beta}_2) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4)$ $+4XZ cov(\hat{\beta}_2 \hat{\beta}_4) + 4XZ cov(\hat{\beta}_1 \hat{\beta}_5)$ $+8X^2 Z cov(\hat{\beta}_2 \hat{\beta}_5) + 4XZ^2 cov(\hat{\beta}_4 \hat{\beta}_5)$
4b	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ$ $+\beta_5 X^2 Z$	$\frac{\partial Y}{\partial Z} = \beta_3 + \beta_4 X + \beta_5 X^2$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial Z}} = var(\hat{\beta}_3) + X^2 var(\hat{\beta}_4) + X^4 var(\hat{\beta}_5)$ $+2X cov(\hat{\beta}_3 \hat{\beta}_4) + 2X^2 cov(\hat{\beta}_3 \hat{\beta}_5)$ $+2X^3 cov(\hat{\beta}_4 \hat{\beta}_5)$