Multiplicative Interaction Models

Thomas Brambor, William Roberts Clark, Matt Golder

Multiplicative interaction models are common in the quantitative political science literature. This is so for good reason. Institutional arguments frequently imply that the relationship between political inputs and outcomes varies depending on the institutional context. Moreover, models of strategic interaction typically produce conditional, rather than unconditional, hypotheses. Indeed, it could be argued that any causal claim implies a set of conditions that need to be satisfied before a purported cause is sufficient to bring about its effect. It is little wonder then that conditional hypotheses such as 'an increase in X is associated with an increase in Y when condition Z is met, but not otherwise' are ubiquitous in all fields of political science. It has been well established that the intuition behind conditional hypotheses is captured quite well by multiplicative interaction models. Given the ubiquity and centrality of conditional hypotheses in political science, it is somewhat surprising that the execution of multiplicative interaction models is often flawed and inferential errors are common. In "Understanding Interaction Models: Improving Empirical Analyses" (*Political Analysis* 14: 63-82), we present several simple suggestions that are easy to implement and that we believe can dramatically improve empirical analyses employing these types of models.

- We strongly encourage analysts to use multiplicative interaction models whenever the hypothesis they want to test is conditional in nature.
- We recommend that scholars include all constitutive terms in their interaction model specifications.
- We remind scholars not to interpret constitutive terms as if they are unconditional marginal effects.
- We urge analysts not to forget to calculate substantively meaningful marginal effects and standard errors.

While these suggestions may seem obvious to some, an examination of three leading political science journals from 1998 to 2002 indicate that our commonsense recommendations are rarely reflected in common practice. In fact, of the 156 articles that we found to employ interaction models, only 16 (10%) actually included all constitutive terms, did not make mistakes interpreting these terms, and calculated substantively meaningful marginal effects and standard errors.

If you find any of the information on this page useful for your research, we would be grateful if you could cite the paper listed above.

Note: We would like to acknowledge a typo in Eq. 12 of our article. Currently, this equation reads "Y = b + b1X + b2Z + b2XZ". This should, of course, read "Y = b + b1X + b2Z + b3XZ". Thanks to Mike Ward for spotting this.

Standard Errors	Computer Code	Literature Survey	Replications	Articles

Standard Errors

Below are two tables illustrating a variety of multiplicative interaction models, marginal effects, and variances. The tables are based on those in Aiken and West (1991). If you would like to download these tables as a pdf document, click here.

Table 1: Marginal Effects and Variances for Various Interaction Models (One Modifying Variable and Two Modifying Variables)

Case	Equation	Marginal Effect	Variance
1a	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_3 Z$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_3) + 2Z cov(\hat{\beta}_1 \hat{\beta}_3)$
1b	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$	$\frac{\partial Y}{\partial Z} = \beta_2 + \beta_3 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_2) + X^2 var(\hat{\beta}_3) + 2X cov(\hat{\beta}_2 \hat{\beta}_3)$
2	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_6 ZW$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_4 Z$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_4) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4)$
3	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_5 XW + \beta_6 ZW$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_4 Z + \beta_5 W$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_4) + W^2 var(\hat{\beta}_5) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4) + 2W cov(\hat{\beta}_1 \hat{\beta}_5) + 2ZW cov(\hat{\beta}_4 \hat{\beta}_5)$
4	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_5 XW + \beta_6 ZW + \beta_7 XZW$	$\frac{\partial Y}{\partial X} = \beta_1 + \beta_4 Z + \beta_5 W + \beta_7 ZW$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + Z^2 var(\hat{\beta}_4) + W^2 var(\hat{\beta}_5) + Z^2 W^2 var(\hat{\beta}_7) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4) + 2W cov(\hat{\beta}_1 \hat{\beta}_5) + 2ZW cov(\hat{\beta}_1 \hat{\beta}_7) + 2ZW cov(\hat{\beta}_4 \hat{\beta}_5) + 2WZ^2 cov(\hat{\beta}_4 \hat{\beta}_7) + 2ZW^2 cov(\hat{\beta}_5 \hat{\beta}_7)$

Table 2: Marginal Effects and Variances for Various Interaction Models (Quadratic Terms)

Case	Equation	Marginal Effect	Variance
1	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + 4X cov(\hat{\beta}_1 \hat{\beta}_2)$
2	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + 4X cov(\hat{\beta}_1 \hat{\beta}_2)$
3a	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X + \beta_4 Z$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + Z^2 var(\hat{\beta}_4) + 4X cov(\hat{\beta}_1 \hat{\beta}_2) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4) + 4XZ cov(\hat{\beta}_2 \hat{\beta}_4)$
3b	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ$	$\frac{\partial Y}{\partial Z} = \beta_3 + \beta_4 X$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial Z}} = var(\hat{\beta}_3) + X^2 var(\hat{\beta}_4) + 2X cov(\hat{\beta}_3 \hat{\beta}_4)$
4a	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ + \beta_5 X^2 Z$	$\frac{\partial Y}{\partial X} = \beta_1 + 2\beta_2 X + \beta_4 Z + 2\beta_5 XZ$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial X}} = var(\hat{\beta}_1) + 4X^2 var(\hat{\beta}_2) + Z^2 var(\hat{\beta}_4) + 4X^2 Z^2 var(\hat{\beta}_5) + 4X cov(\hat{\beta}_1 \hat{\beta}_2) + 2Z cov(\hat{\beta}_1 \hat{\beta}_4) + 4XZ cov(\hat{\beta}_2 \hat{\beta}_4) + 4XZ cov(\hat{\beta}_1 \hat{\beta}_5) + 8X^2 Z cov(\hat{\beta}_2 \hat{\beta}_5) + 4XZ^2 cov(\hat{\beta}_4 \hat{\beta}_5)$
4b	$\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 Z + \beta_4 XZ + \beta_5 X^2 Z$	$\frac{\partial Y}{\partial Z} = \beta_3 + \beta_4 X + \beta_5 X^2$	$\hat{\sigma}^2_{\frac{\partial Y}{\partial Z}} = var(\hat{\beta}_3) + X^2 var(\hat{\beta}_4) + X^4 var(\hat{\beta}_5) + 2X cov(\hat{\beta}_3 \hat{\beta}_4) + 2X^2 cov(\hat{\beta}_3 \hat{\beta}_5) + 2X^3 cov(\hat{\beta}_4 \hat{\beta}_5)$

Standard Errors | Computer Code | Literature Survey | Replications | Articles

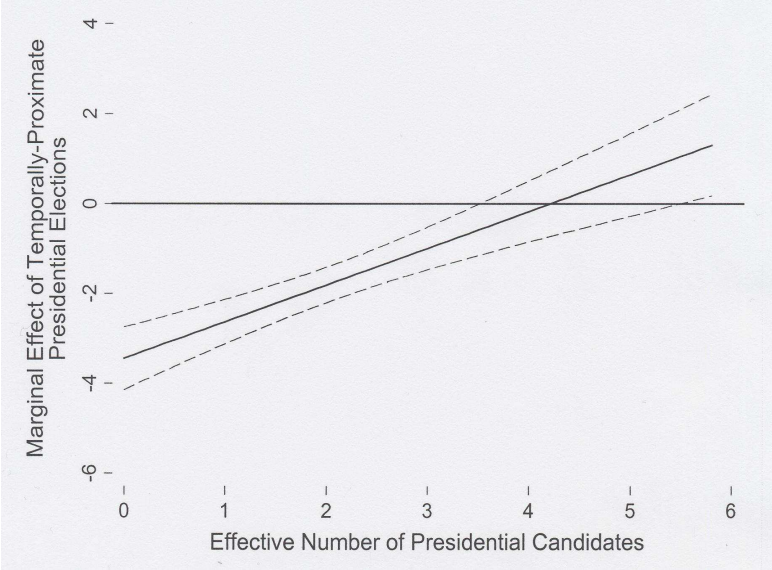
Computer Code

As we suggest in our paper, it is often necessary for the analyst to go beyond the typical results table in order to convey quantities of interest such as the marginal effect of X on Y. If the modifying variable is dichotomous, this simply requires the analyst to present four numbers - the marginal effect of X when the modifying variable is 0 and when the modifying variable is one, along with the two corresponding standard errors. If the conditioning variable is continuous the analyst must work a little harder. In our paper, we suggest that a simple figure can succinctly illustrate the marginal effect of X and the corresponding standard errors across the full range of the modifying variable. Below, we provide code in STATA to do this for three types of multiplicative interaction models. It should be easy to adapt this code to deal with other interaction models.

Continous Dependent Variable with Single Modifying Variable
(Equivalent to Case 1a in Table 1 above)

[Code][Detailed Explanation of Code]

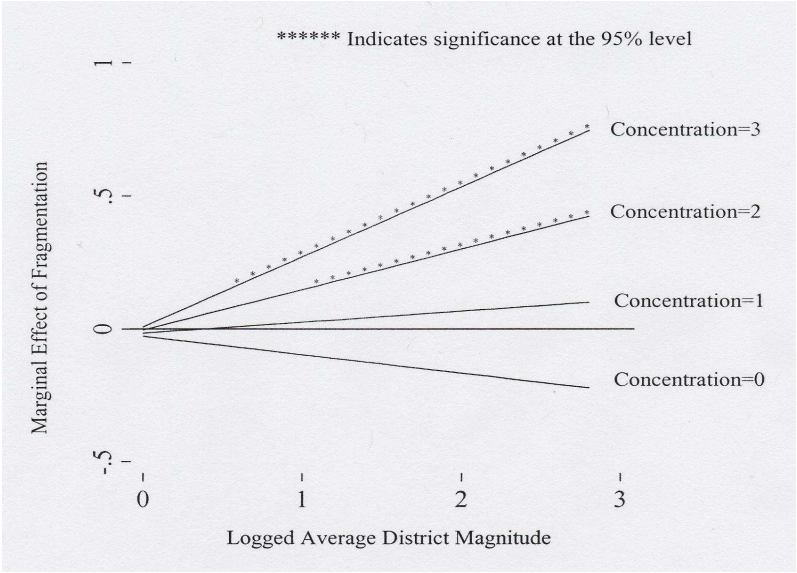
The example below shows how the marginal effect of temporally-proximate presidential elections on the effective number of electoral parties changes with the number of presidential candidates. The solid sloping line indicates how the marginal effect of temporally proximate presidential elections changes with the number of presidential candidates. 95% confidence intervals around the line allow us to determine the conditions under which presidential elections have a statistically significant effect on the number of electoral parties - they have a statistically significant effect whenever the upper and lower bounds of the confidence interval are both above (or below) the zero line. The figure includes a zero line specifically to help the reader determine when the marginal effect is significant. The marginal effect and the variance necessary to compute confidence intervals are shown in Case 1a in Table 1 above.



Continous dependent variable with Two Modifying Variables
(Equivalent to Case 4 in Table 1 above)

[Code][Detailed Explanation of Code]

The example below shows how the marginal effect of ethnic fragmentation on the effective number of legislative parties changes with logged average district magnitude and ethnic group concentration. The figure plots the marginal effect of ethnic fragmentation (the solid sloping lines) as the permissiveness of the electoral system changes and at substantively meaningful levels of group concentration. In this case, we indicate when the marginal effect of ethnic fragmentation is significant at the 95% level by placing a star at these points. We do this because the use of confidence intervals for each sloping line would make the figure hard to read. Of course, those analysts who dislike the deification of the rather arbitrary 95% confidence level might prefer to continue using confidence intervals and provide four separate figures -- one for each level of group concentration. The marginal effect and variance necessary to compute confidence intervals are shown in Case 4 in Table 1 above.



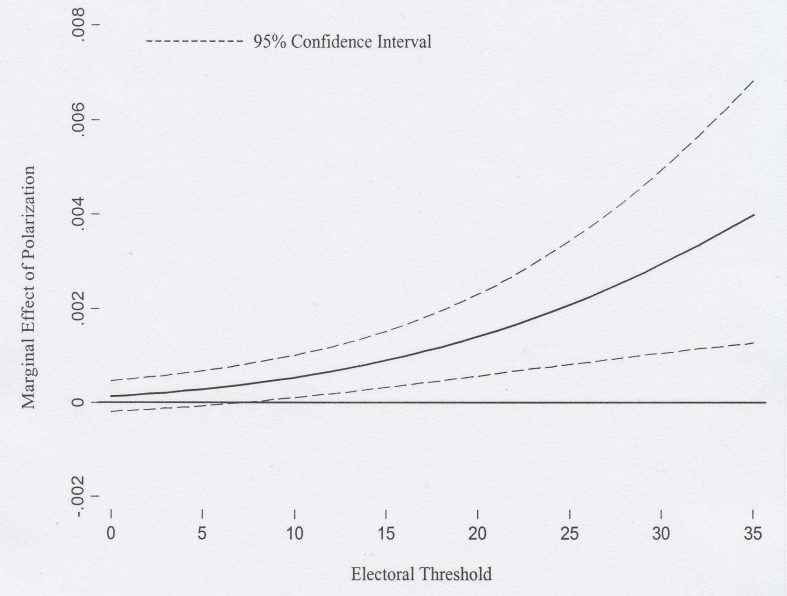
Limited dependent variable with Single Modifying Variable
(A Probit Example)

[Code][Detailed Explanation of Code]

It is almost as easy to use a figure to convey the quantities of interest when the dependent variable is limited instead of continuous. The code presented below can easily be adapted to a variety of limited dependent variables -- dichotomous dependent variables (logit, probit etc.), multichotomous dependent variables (multinomial logit, conditional logit, multinomial probit etc.), ordered dependent variables (ordered probit etc.), duration models (exponential, weibull, generalized gamma etc.). The intuition behind the figures for limited dependent variable models with interaction terms is quite straightforward. Consider an interaction model with a single modifying variable. Typically, the analyst is interested in how some X affects the predicted probability or expected duration of some Y. Let the modifying variable be Z for this example. The code essentially has the following sequence.

1. Estimate your model.
2. Obtain 1,000 or 10,000 values of the model parameters through simulation and STATA's 'drawnorm' command.
3. Calculate the predicted probability or expected duration of Y when X is at some baseline value and the modifying variable Z is at 0 (or its lowest value). You should set any other independent variables at some level of interest - perhaps the mean for continuous variables or the median for dichotomous variables. The specific equation for predicted probabilities or expected durations are available for most models in a several econometric textbooks.
4. Calculate the predicted probability or expected duration of Y when X is increased by some value, keeping the modifying variable Z at 0 and all of the other independent variables at the same level as before. The analyst has to decide what a meaningful increase in X would be - this might be a one unit change, some percentage change, or a standard deviation change etc..
5. Create a first difference by subtracting the predicted probability calculated in (3) from the predicted probability calculated in (4).
6. Repeat steps (3), (4), and (5) for each level of Z.
7. Graph the first difference (the effect of an increase in X on Y) across the observed range of Z. Confidence intervals can be created using the 1,000 or 10,000 simulated values of the first differences.

The solid line in the figure below shows how a one unit increase in party system polarization (from its mean) affects the probability of pre-electoral coalition formation across the observed range of electoral threshold. We can see that party system polarization only increases the probability of pre-electoral coalition formation significantly when electoral thresholds are greater than seven.



Standard Errors | Computer Code | Literature Survey | Replications | Articles

Survey of the Literature

We conducted a systematic examination of three leading, non-specialized political science journals (*American Political Science Review*, *Journal of Politics*, *American Journal of Political Science*) from 1998 to 2002. During the five year period from 1998 to 2002 we found 149 articles that employed interaction models of one variety or another. We coded each article for whether they implemented the recommendations that we proposed at the top of this page. A summary of our results are shown in the table below.

Recommendation	Yes	No	Total

Include all constitutive terms	107 (69%)	49 (31%)	156
Interpret constitutive terms correctly*	38 (38%)	63 (62%)	101
Provide range for marginal effect	86 (55%)	70 (45%)	156
Provide measure of uncertainty	34 (22%)	122 (78%)	156
* Only 101 articles interpreted constitutive terms			

'**Include all constitutive terms**' is self-explanatory. '**Interpret constitutive terms correctly**' means not interpreting the coefficients on constitutive terms as unconditional marginal effects. '**Provide range for marginal effect**' and '**Provide measure of uncertainty**' require the analyst to calculate the marginal effect for some independent variable for at least one value of the modifying variable other than zero and provide some measure of uncertainty such as a standard error or confidence interval. We were very liberal on these last two criteria and coded articles that reported predicted probabilities under two or more different scenarios as having met our recommendations even though these are not marginal effects or first differences (or the quantities of interest). If we had not done this, considerably fewer articles would have been coded as having implemented our recommendations.

A complete list of these articles, along with additional details, can be found by clicking here (Sample). We do not mean to suggest that the conclusions reached in any of these articles are necessarily wrong. After all, we have not conducted detailed reanalyses of all of these studies. However, we do believe that there is a potential for some conclusions in these articles to be incorrect. This is why we encourage people to conduct replications of these studies.

Although we have gone to great pains to avoid any errors in our survey, we are only human and errors may remain. In some cases, it was hard to code particular articles because it was not always clear how certain variables were constructed or what model specification was actually used. If your article is included in our sample and you believe that it has been erroneously classified, we would be happy to hear from you. Our goal in conducting this survey is not to cause offence but to improve future empirical research. As a result, we will immediately correct any errors. Please email Thomas Brambor, William Roberts Clark, or Matt Golder.

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Replications

In our survey of the literature, we listed several articles that estimated multiplicative interaction models that omit at least one constitutive term, interpret the coeffcient on constitutive terms as unconditional marginal effects, or fail to calculate marginal effects and standard errors across a substantively meaningful range of the modifying variable(s). We believe that these types of mistakes are so common in the literature that analysts should critically re-evaluate, and where necessary re-specify, models employing interaction terms before using their results as the basis for future research. Substantively different conclusions from those in the original analyses often arise when this is done. For a discussion of the benefits of replication studies see Gary King's "Replication, Replication" and "The Future of Replication".

Below we present several replications of analyses using multiplicative interaction models that we have conducted in the course of our own research. We hope that others will also conduct replications of other analyses using such models and will add them to the list below. This might be a useful assignment for a first or second semester class in quantitative methods. If anyone would like to do this, they should send the replication to Matt Golder at mgolder@fsu.edu.

- Golder, Matt. 2003. "Electoral Institutions, Unemployment and Extreme Right Parties: A Correction." *British Journal of Political Science*. 33:3, pp. 525-534.
  - This is a reanalysis of Jackman, Robert & Karin Volpert. 1996. "Conditions Favoring Parties of the Extreme Right in Western Europe." *British Journal of Political Science* 25: 501-521.

- Brambor, Thomas, William Roberts Clark & Matt Golder. 2004. "Gubernatorial and Presidential Coattails in Brazil." Unpublished replication, New York University.
  - This is a reanalysis of Samuels, David. 2000. "The Gubernatorial Coattails Effect: Federalism and Congressional Elections in Brazil." *Journal of Politics* 62: 240-253. [Replication materials]

- Brambor, Thomas, William Roberts Clark & Matt Golder. 2004. "Electoral System Choice." Unpublished replication, New York University.
  - This is a reanalysis of Boix, Carles. 1999. "Setting the Rules of the Game: The Choice of Electoral Systems in Advanced Democracies." *American Political Science Review* 93: 609-624. [Replication materials]

- Brambor, Thomas, William Roberts Clark & Matt Golder. 2005. "Are African Party Systems Different?" Under review, Florida State University.
  - This is a reanalysis of Mozaffar, Shaheen, James Scarritt and Glen Galaich. 2003. "Electoral Institutions, Ethnopolitical Cleavages, and Party Systems in Africa's Emerging Democracies." *American Political Science Review* 97: 379-390.
  - For a longer and more detailed replication, see "Are African Party Systems Different?" (2004). Unpublished manuscript, New York University.

- Clark, William Roberts & Matt Golder. 2004. "Rehabilitating Duverger's Theory: Testing the Mechanical and Strategic Modifying Effects of Electoral Laws." Unpublished manuscript, New York University.
  - This includes a reanalysis of Amorim Neto, Octavio & Gary Cox. 1997. "Electoral Institutions: Cleavage Structures and the Number of Parties." *American Journal of Political Science* 41: 149-174.

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Articles

Aiken, Leona & Stephen West. 1991. *Multiple Regression: Testing and Interpreting Interactions*. London: Sage Publications.

Allison, Paul D. 1977. "Testing for Interaction in Multiple Regression." *American Journal of Political Science* 83:144-153.

Bernhardt, Irwin & Bong S. Jung. 1979. "The Interpretation of Least Squares Regression with Interaction or Polynomial Terms." *The Review of Economics and Statistics* 61: 481-483.

Braumoeller, Bear. forthcoming. "Hypothesis Testing and Multiplicative Interaction Terms." *International Organization*.

Busemeyer, Jerome & Lawrence Jones. 1983. "Analysis of Multiplicative Combination Rules When the Causal Variables are Measured with Error." *Psychological Bulletin* 93: 549-562.

Cleary, Paul & Ronald Kessler. 1982. "The Estimation and Interpretation of Modifier Effects." *Journal of Health and Social Behavior* 23: 159-169.

Cox, D.R. 1984. "Interaction." *International Statistical Review* 52: 1-31.

Friedrich, Robert. 1982. "In Defense of Multiplicative Terms in Multiple Regression Equations." *American Journal of Political Science* 26: 797-833.

Gill, Jeff. 2001. "Interpreting Interactions and Interaction Hierarchies in Generalized Linear Models: Issues and Applications." Presented at the Annual Meeting of the American Political Science Association, San Francisco.

Griepentrog, Gary, J. Michael Ryan & L. Douglas Smith. 1982. "Linear Transformations of Polynomial Regression Models." *The American Statistician* 36: 171-174.

Jaccard, James & Choi Wan. 1995. "Measurement Error in the Analysis of Interaction Effects Between Continuous Predictors Using Multiple Regression: Multiple Indicator and Structural Equation Approaches." *Pyschological Bulletin* 117: 348-357.

Jaccard, James, Choi Wan & Robert Turrisi. 1990. "The Detection and Interpretation of Interaction Effects Between Continuous Variables in Multiple Regression." *Multivariate Behavioral Research* 25: 467-478.

Kam, Cindy & Robert Franzese. 2003. "Modeling and Interpreting Interactive Hypotheses in Regression Analysis: A Brief Refresher and Some Practical Advice." Unpublished manuscript, University of Michigan.

Nagler, Jonathan. 1991. "Scobit: An Alternative to Logit and Probit." *American Journal of Political Science* 38: 230-255.

Rosnow, Ralph & Robert Rosenthal. 1989. "Definition and Interpretation of Interaction Effects." *Psychological Bulletin* 105: 143-146.

Rosnow, Ralph & Robert Rosenthal. 1991. "If You're Looking at the Cell Means, You're Not Looking at *Only* the Interaction (Unless All Main Effects Are Zero)." *Psychological Bulletin* 110: 574-576.

Wright, Gerald. 1976. "Linear Models for Evaluating Conditional Relationships." *American Journal of Political Science* 2: 349-373.

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