Distributed Bridge for Silicon Sensor, 0–300°C

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Abstract

This paper proposes a distributed bridge measurement circuit for silicon integrated mechanical sensors operating in the 0–300°C range, addressing limitations in existing designs for high-precision transducers of physical quantities. It reviews conventional methods, including heteroepitaxial structures on sapphire, dielectric-insulated mesa bridges, "shear" piezoresistive elements (X-ducer analogs), and resistor full-bridge circuits (RMT), highlighting issues like temperature restrictions (typically $\leq 200°C$), mechanical stresses from lattice mismatches, and poor reproducibility of elastic element dimensions. The proposed method dopes the silicon elastic element to increase majority carrier concentration, selecting average resistivity ($\rho_{ee} \approx 0.5$–$10\,\Omega \cdot \text{cm}$) and channel depth to ensure 2–10 times higher conductivity in piezoresistive channels without isolating p-n junctions. Modeling uses differential equations for charge carrier redistribution, solved via Bessel functions, yielding expressions for strain-sensitivity coefficient $K(T, \varepsilon_y)$ and input resistance $R_{in}(T)$. Experimental validation on beam and membrane transducers confirms extended ranges (e.g., -60 to $325°C$ for $\rho_{ee} = 0.5\,\Omega \cdot \text{cm}$) with intrinsic errors $\leq 1\%$ and complementary temperature errors $< 0.05\%/°C$.

1 Field of study

The data presented relate to the field of semiconductor devices and can be used in the manufacture of high-precision semiconductor transducers of physical quantities for operation in a wide temperature range.

To convert the strain of an elastic element into an electrical signal, we use DC bridge measuring circuit of four piezoresistors [1].

The conversion characteristic of such a circuit is determined by the transfer function k, whose dependence on temperature and strain changes resistance of piezoresistors, according to [2] is represented in the form:

$$\mathrm{k} = K_0(T) + K\left(\varepsilon_y; T\right) \cdot \varepsilon_y, \tag{1}$$

where $K_0(T)$ is the initial transfer coefficient of the measuring circuit in the absence of the input mechanical quantity; $K\left(\varepsilon_y; T\right)$ - strain-sensitivity coefficient of the transducer.

Reducing the error of silicon integrated resistor transducers by bringing the value and temperature dependence of the initial transfer coefficient close to zero is possible if the requirements to the manufacturing technology and design are fulfilled, among which are:

- minimizing the deviation of resistivity of resistors from the average values and identical behavior of the characteristics of resistors in a certain temperature range;
- minimization of the values and scatters of the initial deformations in the elastic element in the areas of strain gauge placement plus identical behavior of the characteristics of these values in a certain temperature range;
- the equality of all thermal resistances in the channels of the heat sinks "piezoresistor - elastic element - sensor body";
- minimizing the temperature gradient in the plane of piezoresistors on the elastic element.

These requirements are satisfied in the two known topological solutions for the symmetric four-branch bridge circuit:

- the square contact areas, centered at the vertices of the square, are interconnected by four single-band piezoresistive channels of longitudinal and transverse piezoresistors;
- four contact areas are located in the area of the piezoresistive sensing element similar to the Hall transducer.

2 Overall information about silicon integrated transducers for temperature range $0 \div 300^{\circ}C$

There is a known method of manufacturing converter with symmetric bridge circuit [3]. On an insulating substrate by chemical etching methods create a silicon mesa structure of the bridge circuit, which contains in the vertices of the square contact pads connected with each other by single-band piezoresistive channels. Representatives of the known method are technical solutions:

- sensor with a heteroepitaxial silicon bridge circuit on a monocrystalline sapphire elastic element;
- sensor with a monocrystalline (or polycristalline) silicon bridge circuit that is recessed in a layer of glass or other dielectric formed on a silicon elastic element.

The general disadvantage of the above methods is the difficulty to reproduce accurately the small thickness of the elastic element in the production of miniature

sensors: - in the first case, due to the need for high-temperature gas profiling with $\alpha - Al_2O_3$ in the second case - the nonreproducibility of the geometric dimensions of the dielectric layer in the process of sintering. The disadvantage of the method for the second solution is also that the low values of the modulus of elasticity of materials, used as dielectrics, significantly limit the operating range of conversion of the mechanical value.

The next common disadvantage is the occurrence of mechanical stresses in silicon strain gauges, due to the mismatch between the structure of the substrate or insulating layer of the structure of single-crystal silicon. In the first case, silicon with a cubic lattice structure is grown on an α - modification of sapphire with a hexagonal lattice, in the second - on an amorphous glass structure. These stresses, due to the mismatch of thermal expansion coefficients, are temperature-dependent and lead to:

- a basic static error due to strain of the elastic element (under normal conditions) and an complementary temperature error;
- an additional restriction on the correlation values of the thicknesses of the elastic element, insulation and mesa-resistors.

The articles [4][5] describe a method of manufacturing a silicon integral transducer of mechanical quantities with a "shear" piezoresistive sensing element, which is a piezo analogue of the Hall transducer (X-ducer). In an n-type silicon wafer with crystallographic orientation in the plane (100), a p-type sensing element region is formed by diffusion doping. Two pairs of contacts are created inside the area on the orthogonal axes.

When measuring a mechanical quantity, a direct current, passing through the longitudinal axis of the element, generates a strain-dependent potential difference on a pair of contacts located on the transverse axis.The advantage of the known method, consisting in reducing the value of the initial transfer coefficient and its temperature dependence, does not lead to a significant decrease in the additive component of the complementary temperature error, since the strain sensitivity of "shear" strain gauge elements is 2-3 times less than the strain sensitivity of a single-band strain gauge channel in the full bridge circuit.[6]

The second disadvantage of the method is that the operating temperature range for such converters is limited to $+100^oC$. The boundary temperature is determined by the diffusion component of the reverse current, which grows strongly at these temperatures and is proportional to the surface area of the p-n junction that insulates the sensitive elements from the elastic element.

The disadvantage is that the small input resistance of the "shear" strain gauge sensing element does not allow using a significant voltage for its power supply, and the use of low resistance material sharply worsens the temperature characteristics of the sensor.

A method of manufacturing resistance silicon mechanical transducers is also known[7]. We will call it RMT. Resistor full-bridge circuit contains four contact pads located on the vertices of the square and connected by single-band piezoresistive channels. Piezoresistive channels and contact pads are formed in

the body of the silicon wafer by doping with an impurity that increases the concentration of minority carriers to form an insulating p-n junction. Selective chemical etching on the reverse side of the wafer, relative to the circuit, forms the resilient element and rigid base regions.

The design drawing of the sensor (see Fig. 4 in [7]) does not indicate the type of isolation of the bridge circuit from the elastic element. Therefore, we can assume that the following variants are possible for the RMT: isolating $p-n$-junction and non-isolating junction, when the piezoresistors and the elastic element have the same type of conductivity.

Characteristics of the converter are determined by the design parameters in the conditions of electrical isolation of the bridge circuit relative to the elastic element and rigid base. For this purpose the surface resistance of doped layers, etch depth and resistivity of the silicon wafer must provide at least 10 times higher conductivity in the layers of the bridge circuit than in the body of the elastic element.

The limitation imposed on the design parameters is as follows:

$$\frac{1}{10 \cdot R_{bc}} \geq \frac{1}{R_{ee}} \tag{2}$$

where R_{bc} is the input resistance of the bridge circuit, and R_{ee} is the resistance of the elastic element between the contacts at the input of the bridge circuit. Conditions (2) must be met over the entire operating temperature range of the transmitter.

Let us estimate the values of silicon resistivity for the piezoresistor of the bridge circuit $\tilde{\rho}_{bc}$ and the elastic element $\tilde{\rho}_{ee}$ in order that the relation (2) is fulfilled.

In the evaluation we will consider the elastic element as a flat plate with thickness H_{ee}. The value of H_{ee} usually varies from $3 \cdot 10^{-3}$ cm to $3 \cdot 10^{-2}$ cm.

Let's calculate the input resistance R_{in} for a parallel connected bridge circuit 52 with a square part of elastic element 54 by means of common contact pads 55 and 57 [7].

To determine the electrical resistance of the square section between the contact pads (taking into account their geometrical dimensions) can only be determined by numerical calculation. Let's choose their relative sizes for the calculation: square 1×1; contacts $0,2 \times 0,2$.

The resistance of the square section of the elastic element between the contact pads 55-57(see [7]) is:

$$R_{ee} = 1.383 \cdot \frac{\rho_{ee}}{H_{ee}} \tag{3}$$

where ρ_{ee} is the resistivity of the silicon elastic element; H_{ee} is the thickness of the "plate elastic element".

Take the number of squares of any resistor channel of the bridge circuit between the boundaries of contacts to be 10. Then the relative width of the resistor is 0.06.

The resistance of the isolated bridge circuit is equal:

$$R_{bc} = 10 \cdot \frac{\tilde{\rho}_{bc}}{X_j} \tag{4}$$

where $\tilde{\rho}_{bc}$ is the average volumetric resistivity of the piezoresistor channel; X_j is the thickness of the piezoresistor channel at which $\tilde{\rho}_{bc}$ was averaged.

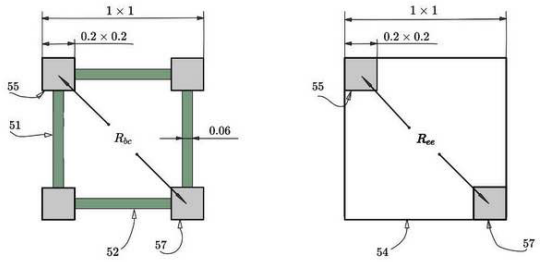

Figure 1: Configurations of the bridge circuit layers and elastic element in Fig.4 in [7]

If we take the thickness of the ion-alloy piezoresistor to be $X_j = \left(0.33 \cdot 10^{-4} \div 0.36 \cdot 10^{-4}\right)\ cm$, which is the characteristic value, and $H_{ee} =$

$\left(3 \cdot 10^{-3} \div 3 \cdot 10^{-2}\right)\ cm$, then the restrictions take on the form:

$$\rho_{ee} \approx (0.63 \div 6.3) \cdot 10^3 \cdot \tilde{\rho}_{bc} \tag{5}$$

The smallest value of $\tilde{\rho}_{bc}$, in ion-alloyed silicon piezoresistors, appears to be $\tilde{\rho}_{bc} = 2.8 \cdot 10^{-3}\ \Omega \cdot \text{cm}$.

The input resistance of the bridge circuit configured in [7] with the piezoresistors 51 isolated from the elastic element in this case will be: $R_{bc} = 800\ \Omega$. Then to be able to vary the thickness of the elastic element in the range of $3 \cdot 10^{-3}$ cm $\leq$ $\mathrm{H_{ee}}$ $\leq$ $3 \cdot 10^{-2}$ cm, the resistivity of silicon ρ_{ee} must be: $\rho \geq 18\ \Omega \cdot \text{cm}$.

Next, we estimate the critical temperature T_{int}, at which the impurity semiconductor of the elastic element will turn into its intrinsic one. At such transformation, the condition $\frac{1}{10 \cdot R_{bc}} \geq \frac{1}{R_{ee}}$ will certainly (and with excess) be violated due to a sharp increase in the electrical conductivity of the high-resistance elastic element.

We will use the expression in [8] on page 41 for this estimation:

$$\mathrm{T_{int}}(\rho) = 273 \left(\frac{10}{4,5 + \log \rho} - 1 \right); \tag{6}$$

$$\mathrm{T_{int}}(\rho = 18\ \Omega \cdot \text{cm}) = 200^{\circ}\mathrm{C} \tag{7}$$

Thus, the upper limit of the operating temperature range of this transducer, if the piezoresistors of the bridge circuit are not isolated by a p-n junction, cannot be higher than $+200^{\circ}$C.

In the case when the piezoresistors are isolated from the elastic element by a p-n junction, the upper limit of the temperature range of the transducer is limited (in RMT) by the leakage current through the p-n junction. Its value (in the RMT) is determined by the ratio $I_{\text{rev}} \leq 1 \cdot 10^{-1} \cdot \frac{V_{\text{in}}}{R_{\text{in}}}$. According to our estimate, for example, at $V_{in} = 5\ V$ the leakage current should be $6.26 \cdot 10^{-4}\ A$. At this value of current the change in the sensitivity of the transducer, in fact, the error, will reach 10%.

The value of current at which the change in sensitivity will not exceed the norm in [9] is determined by another relationship and is equal:

$$\text{I}_{\text{revN}} = 4 \cdot 10^{-4} \cdot \frac{V_{\text{in}}}{R_{\text{in}}} = 4 \cdot 10^{-4} \cdot \frac{5}{800} = 2.5 \cdot 10^{-6}\ A \tag{8}$$

If the current is further increased, the shunt effect of the resistive part of the resilient element material on one of the arms of the bridge circuit can change the value of the transfer ratio by more than 1% relative to its nominal value.

To estimate the RMT, we will take the following dimensions of the bridge circuit in Figure 1: piezoresistor width $3 \cdot 10^{-3}\ cm$; piezoresistor length $3 \cdot 10^{-2}\ cm$; contact size $(1 \cdot 10^{-2} \times 1 \cdot 10^{-2})\ cm^2$. Then p-n junction area equals $4.36 \cdot 10^{-4}\ cm^2$. To determine the upper limit of temperature range of transducer operation we will use characteristic values of leakage current, given in [10]. At p-n junction area $4.36 \cdot 10^{-4}\ cm^2$ and room temperature $+\mathbf{20^{\circ}C}$ $(273\ K)$ the current $I_{rev0} = 8.2 \cdot 10^{-13}\ A$.

Let's use the formula $I_{revN} = I_{rev0} \cdot e^{38.13 \cdot \left(\frac{T_{per}-293}{T_{per}}\right)}$ to calculate T_{per} [10]. The resulting value $T_{per} = +208\ ^0C$ is the temperature that limits the scope of the RMT transducer with an insulating p-n junction.

Because the area of bridge piezoresistor circuits is usually larger than the value we took to estimate, the real value of this temperature will not exceed approximately $+180\ ^0C$.

A common reason in both cases for limiting the operating temperature range is the sharp decrease in resistance of the elastic element, connected in parallel to the input and output of the bridge circuit, at the temperature of the beginning of intrinsic conductivity. There will be a decrease in its sensitivity to the input value ([2]on page 51) according to the law

$$\text{V}_{\text{out}} = V_{oc} \frac{R_{ee}}{R_{ee} + R_{bc}} \tag{9}$$

where V_{out} - output signal of the bridge circuit; V_{oc} - open-circuit voltage of the bridge circuit; R_{ee} is the resistance of the elastic element connected in parallel to the output of the bridge circuit. The resulting sharp drop in the temperature dependence defines the upper limit of the transducer's operating temperature range [11].

A common disadvantage for all the considered methods is the additional temperature error of the bridge measuring circuit when supplied with stabilized voltage, which is largely determined by the value of the temperature coefficient of sensitivity of the piezoresistor. Depending on the impurity concentration, its

value varies in the range $(0.13 \div 0.3)$ $\%/K$.
This error requires additional time-consuming individual adjustment of the transducer.

3 Detailed description of the silicon transducer measuring bridge circuit.

The objective is to extend the temperature range and reduce the temperature error by effectively redistributing the current of the charge carriers in the channels and in the elastic element.

The problem is solved by the fact that in the known method of manufacturing silicon resistance integral mechanical transducer, doping is carried out with impurities that increase the concentration of majority carriers in the elastic element and select the average resistivity of the elastic element, the depth of doping, geometric dimensions of the channel in accordance with the ratio:

$$\Gamma_{kt} = \sqrt{\frac{1}{n-1}\sum_{i=0}^{n}\frac{1}{(T_n-T_0)^2}\left\{\frac{\left[2\frac{X_j\cdot b}{a}+\frac{\tilde{\rho}_{bc0}}{\rho_{ee0}}\Lambda\right]\left[K_0+K_1\frac{T_0}{T_n}+K_2\left(\frac{T_0}{T_n}\right)^2\right]}{\left[2\frac{X_j\cdot b}{a}+\frac{\tilde{\rho}_{bc0}\left[1+\alpha_{bc}(T_n-T_0)+\beta_{bc}(T_n-T_0)^2\right]}{\rho_{ee0}\left[1+\alpha_{ee}(T_n-T_0)+\beta_{ee}(T_n-T_0)^2\right]}\cdot\Lambda\right][K_0+K_1+K_2]}-1\right\}^2};$$

$$\mathrm{T}_n = T_2\frac{n-i}{n}+T_1\frac{i}{n} \tag{10}$$

where Γ_{kt} is the limit value of the permissible temperature error, K^{-1}; T_1, T_2, are the limit values of the working range temperature,K; T_n are temperature values in intervals n selected for summation; T_0 is the value of the initial temperature within the working range, K; $\tilde{\rho}_{bc0}$ - average value of the channel resistivity at T_0, $\Omega\cdot cm$; X_j - channel doping depth, equal to the depth of p-junction at the opposite type of plate conductivity, cm; a - channel length ,cm; b - channel width, cm; $\alpha_{bc}, \beta_{bc}, \alpha_{ee}, \beta_{ee}$ - approximation coefficients of temperature dependences of channel and plate resistivity by polynomial of the second degree of the form $\rho(T) = \rho_0\left[1+\alpha(T-T_0)+\beta(T-T_0)^2\right]$, K^{-1}, K^{-2}; K_0, K_1, K_2 - approximation coefficients of the temperature dependence of the channel strain-sensitivity coefficient by a second-degree polynomial of the form $K(T) = K_0 + K_1\frac{T_0}{T} + K_2\left(\frac{T_0}{T}\right)^2$; n is the number of temperature intervals selected for summation,$i = 0, 1, \ldots, n$ are positive numbers;
Λ - parameter of redistribution of current of charge carriers in channels and elastic element, cm, determined by measuring the values of input resistance of the bridge circuit, resistance of electrically isolated channel from the ratio:

$$\frac{\rho_{eeT1}\left(\frac{1}{R_{bcT1}}-\frac{1}{R_{T1}}\right)+\Lambda\cdot R_{T1}\left(\frac{1}{2R_{bcT1}}-\frac{1}{R_{T1}}\right)}{\rho_{eeT2}\left(\frac{1}{R_{bcT2}}-\frac{1}{R_{T2}}\right)+\Lambda\cdot R_{T2}\left(\frac{1}{2R_{bcT2}}-\frac{1}{R_{T2}}\right)} = \frac{1+\frac{\Lambda\cdot R_{T1}}{2\rho_{eeT1}}}{1+\frac{\Lambda\cdot R_{T2}}{2\rho_{eeT2}}} \tag{11}$$

where R_{bcT1}, R_{bcT2} are values of input resistances of the circuit at temperatures T_1 and T_2, respectively ; R_{T1},R_{T2}-values of channel resistances at temperatures, respectively T_1 and T_2 ; ρ_{eeT1},ρ_{eeT2} are the values of the resistivities of the silicon wafer (i.e., the elastic element) at temperatures, respectively T_1 and T_2.

A significant difference in the design of the transducer is the electrical connection of the elastic element to the bridge measuring circuit using a $p^+ - p$ junction instead of $p - n$.

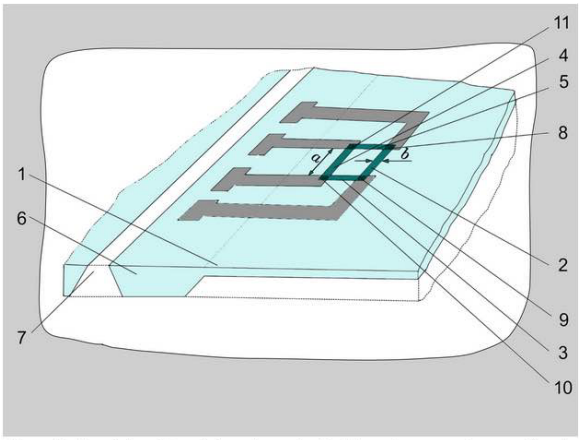

Figure 2: A resistor silicon integral mechanical transducer made according to the proposed method.

The essence of the construction is explained by the drawings: Figure 2 shows a piezoresistor silicon integral transducer of mechanical quantities, made by the proposed method; Figure 3 shows the structure of the selected for modeling characteristics of the deformation transducer, made by the proposed method; Figure 4 - the same, second projection A-A section in Figure 3; Figure 5 - an equivalent diagram of the discrete analog model of the transducer in Figure 3 and Figure 4.

The physical meaning of the fabrication method is as follows:

- finding a ratio between the electrical characteristics of the elastic element 1 and the piezoresistive channels 2,3,4,5 (sensing elements) that allows extending the temperature range beyond the critical temperature of the analog; in this case, the temperature dependence of sensitivity should not have sharp drops;
- the use of a parallel connected section of silicon elastic element 1 with a positive temperature coefficient of resistance as an element of the compensation circuit of the temperature dependence of the bridge sensitivity when a stabilized voltage is applied.

In the method proposed by the authors, the elastic element 1 is not only a link for converting mechanical quantity into deformation, but also an element for

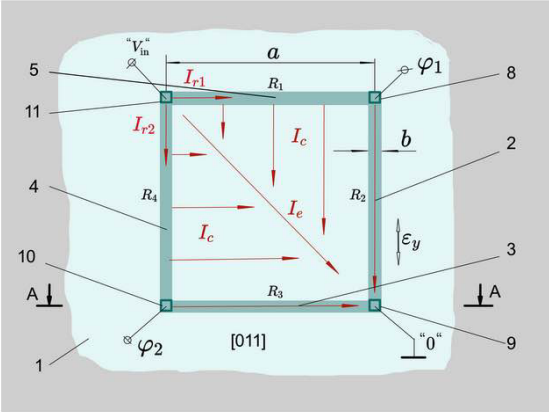

Figure 3: The structure of the selected for modeling characteristics of the deformation transducer.

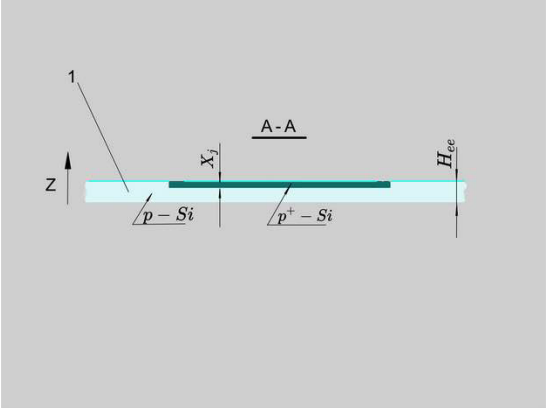

Figure 4: The A-A projection of the section in Figure 3.

adjusting the temperature dependence of the output signal. In this case, it has absolutely accurate information about the ambient temperature change, because it combines the functions of heat transfer to piezoresistive channels 2,3,4,5. Such manufacturing increases the reliability of transducer 6 in the area of high (more than 200°C) temperatures, because it does not require any additional element for adjusting the characteristics (or temperature measurement). Parametric failure of the elastic element section 1 is possible only simultaneously with its mechanical destruction.

The method of determining the parameters of the piezoresistive channels 2,3,4,5 and the elastic element 1 consists of measurements and calculations. The expressions for the calculations by the authors are obtained as follows.

For modeling we chose the shown in Figure 3 and Figure 4 deformation transducer design consisting of a silicon elastic element 1 of p-type with orientation

in the plane (100) and thickness H_{ee}, in which equal square contact areas 8,9,10, 11 with sides b with centers in vertices of square with sides $(a+b)$, are interconnected by four single-band piezoresistive channels of p+-type 2,3,4,5 oriented along crystallographic directions [011]with width equal to b and thickness - X .

The following conditions and assumptions are chosen:

- $a \gg H_{ee}$;
- the strain component ε_y is directed along the crystallographic trajectory [011];
- the bridge circuit is fed from the stabilized voltage source V_{in} through the diagonal contacts 9,11;
- current I_r is evenly distributed with respect to the plane of cross section of channels 2,3,4,5; its distribution is similar in neighboring channels 2,3 and 4,5 with common contacts 11,9, respectively;
- the current in the elastic element 1 is evenly distributed in the cross section in the direction Z along the normal to the flat surface of the elastic element 1; currents are absent outside the circuit area bounded by channels 2,3,4,5; inside the area are characterized by components I_c and I_e; the direction of components I_c is normal relative to the sides of neighboring channels 4,5 with a common power contact 11; current component I_e is directed along the diagonal connecting power contacts 9,11;
- distribution of concentration of majority carriers in the channels 2,3,4,5 is uniform; the level of doping is characterized by the value of resistivity equal to the mean value of resistivity at real non-uniform distribution of impurity;$R_{Sc} = \frac{\tilde{\rho}_{bc}}{X_j}$ where R_{Sc} - values of surface resistance of channels 2,3,4,5 , $\Omega/\square$; X_j - doping depth of channels 2,3,4,5,cm, equal to depth of p-n junction, if elastic element 1 of silicon n-type;
- the level of doping of elastic element 1 is characterized by the value of resistivity of the initial material ρ_{ee}.

As a discrete analogue of the transducer model, we use the equivalent circuit of a long line with parameters distributed along the X-axis, shown in Figure 5, for which the differential equations linking currents and potentials have the form:

$$\begin{gathered} \left\{ \begin{array}{l} I_{r1}(x) = -\frac{d\varphi_1(x)}{dx} \cdot \frac{b}{R_{Sc1}}; \\ I_{r2}(x) = -\frac{d\varphi_2(x)}{dx} \cdot \frac{b}{R_{Sc4}}; \end{array} \right. \left\{ \begin{array}{l} I_c(x) = -\frac{dI_{r1}(x)}{dx}; \\ I_c(x) = -\frac{dI_{r2}(x)}{dx}; \end{array} \right. \\ I_e(x) = -\frac{dV_{in}(x)}{dx} \cdot \frac{b \cdot H_{ee}}{\sqrt{2} \cdot \rho_{ee}}; \quad \frac{dI_e(x)}{dx} = 2I_c(x). \end{gathered} \tag{12}$$

After transformations (12), the differential equations look like

$$\left\{ \begin{array}{l} \frac{d^2\Upsilon_1(x)}{dx^2} = x^{-1}C_1 \cdot \Upsilon_1(x); \\ \frac{d^2\Upsilon_2(x)}{dx^2} = x^{-1}C_2 \cdot \Upsilon_2(x), \end{array} \right. \tag{13}$$

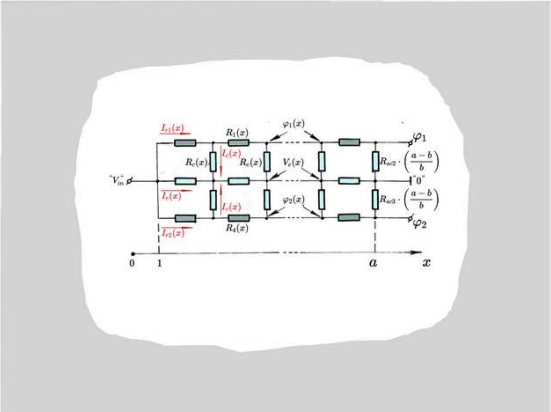

Figure 5: An equivalent diagram of the discrete analog model of the transducer in Figure 3 and Figure 4.

where

$$\Upsilon_1(x) = \varphi_1(x) - V_{in}(x);$$
$$\Upsilon_2(x) = \varphi_2(x) - V_{in}(x);$$
$$C_1 = \left(2\sqrt{2} + \frac{R_{Sc1}}{\rho_{ee}} H_{ee}\right);$$
$$C_2 = \left(2\sqrt{2} + \frac{R_{Sc4}}{\rho_{ee}} H_{ee}\right).$$

The solution of equations (13) has the form [12]:

$$\Upsilon(x) = \sqrt{x} Z_1(i \cdot 2\sqrt{Cx}) \tag{14}$$

where $Z_1(i \cdot 2\sqrt{Cx})$ is a Bessel function with an imaginary unit.
The following boundary conditions were used:
at $x = 1 : \Upsilon_1(1) = \Upsilon_2(1) = 0;\ \varphi_1(1) = \varphi_2(1) = V_{in};$
$V_{in}(1) = V_{in}$;
at $x = a :\ \Upsilon_1(a) = \varphi_1;\ \Upsilon_2(a) = \varphi_2;\ V_{in}(a) = 0;$
$\varphi_1(a) = \varphi_1;\ \varphi_2(a) = \varphi_2;$
$I_{r_1}(a) = \varphi_1 \frac{b}{R_{Sc2} \cdot a};\ I_{r_2}(a) = \varphi_2 \frac{b}{R_{Sc3} \cdot a};$
$I_{in} = I_e(a) + 2I_c(a);\ I_e(a) = I_{in} - \varphi_1 \frac{b}{R_{Sc2} \cdot a} - \varphi_2 \frac{b}{R_{Sc3} \cdot a};$
with which, the solution of equations (13) determined the following relations for circuit parameters:

$$V_{out} = \varphi_1 - \varphi_2 = V_{in} \cdot \left(\frac{R_2}{R_1 + R_2 + \frac{R_1 \cdot R_2 \cdot b}{\rho_{ee} \cdot a} H_{ee} \cdot \Phi} - \frac{R_3}{R_4 + R_3 + \frac{R_3 \cdot R_4 \cdot b}{\rho_{ee} \cdot a} H_{ee} \cdot \Phi} \right); \tag{15}$$

$$R_{in} = \left[\frac{1 + 2\frac{R_2 \cdot b}{\rho_{ee} \cdot a} H_{ee} \cdot \Phi}{R_1 + R_2 + \frac{R_1 \cdot R_2 \cdot b}{\rho_{ee} \cdot a} H_{ee} \cdot \Phi} + \frac{1}{R_3 + R_4 + \frac{R_3 \cdot R_4 \cdot b}{\rho_{ee} \cdot a} H_{ee} \cdot \Phi} + \frac{b \cdot H_{ee}}{\rho_{ee} \cdot a\sqrt{2}} \right]^{-1}; \tag{16}$$

where V_{out} is the value of the output of the bridge circuit, V; R_{in} - value of input resistance of the bridge circuit, Ω; R_1, R_2, R_3, R_4 - resistance values of piezoresistive channels 5,4,3,2 electrically isolated from each other in the four shoulders of the bridge circuit, Ω; Φ is a dimensionless coefficient for the strain transducer model, which characterizes the redistribution of the charge carrier current in the channels and the elastic element.

We assume that

$$\begin{gathered} R_1 = R_2 = R_3 = R_4 = R_{bc} = \tilde{\rho}_{bc} \cdot \frac{a}{b \cdot X_j}; \\ \frac{dR_2}{d\varepsilon_x} = -\frac{dR_1}{d\varepsilon_x} = \frac{dR_4}{d\varepsilon_x} = -\frac{dR_3}{d\varepsilon_x} = R_{bc} \cdot K_{bc}; \end{gathered} \tag{17}$$

where K_{bc} is the strain-sensitivity coefficient of electrically isolated piezoresistive channels 2,3,4,5. Then the strain-sensitivity coefficient of the transducer and the input resistance of the bridge circuit are equal:

$$K\left(T, \varepsilon_y\right) = \frac{1}{V_{in}} \frac{dV_{out}}{d\varepsilon_y} = \frac{2K_{bc}(T)}{2 + \frac{\tilde{\rho}_{bc}(T)}{\rho_{ee}(T)} \cdot \frac{H_{ee} \cdot \Phi}{X_j}}; \tag{18}$$

$$R_{in}(T) = 2R_{bc}(T) \left[\frac{1 + \frac{R_{bc}(T)}{\rho_{ee}(T)} \cdot \frac{H_{ee} \cdot \Phi \cdot b}{a}}{2 + \frac{R_{bc}(T)}{\rho_{ee}(T)} \cdot \frac{H_{ee} \cdot \Phi \cdot b}{a}} + 0.354 \frac{H_{ee} \cdot b}{\rho_{ee}(T) \cdot a} \right]^{-1} \tag{19}$$

where $K_{bc}(T) = K_0 + K_1 \frac{T_0}{T} + K_2 \left(\frac{T_0}{T}\right)^2$ is the strain-sensitivity factor of electrically isolated piezoresistive channels 2,3,4,5 with temperature dependence determined according to [13] by second degree polynomial in case the temperature parameter t, $t = \frac{T_0}{T}$. This is true for silicon piezoresistors [14]; $\alpha_{bc}[1/K], \beta_{bc}\left[1/K^2\right]$ are approximation coefficients; $\tilde{\rho}_{bc0}$ - is the resistivity at $T = T_0$; $R_{bc}(T) = \rho_{bc}(T) \cdot \frac{a}{b \cdot X_j} = R_{bc0}\left[1 + \alpha_{bc}\left(T - T_0\right) + \beta_{bc}\left(T - T_0\right)^2\right]$ - resistance value of electrically isolated piezoresistive channels 2,3,4,5; R_{bc0} - - is the resistance at $T = T_0$; $\rho_{ee}(T) = \rho_{ee0}\left[1 + \alpha_{ee}\left(T - T_0\right) + \beta_{ee}\left(T - T_0\right)^2\right]$ - is the resistivity of the initial silicon wafer 7 with the temperature dependence defined by the polynomial of the second degree; $\alpha_{ee}[1/K], \beta_{ee}\left[1/K^2\right]$ are approximation coefficients.

For any transducer design, the condition $a \leq H_{ee}$ is possible. Then the assumptions about the uniformity of current distribution in the elastic element along the direction Z will not be adequate to the nature of the distribution in the real object. Therefore, instead of H_{ee} its effective value H_{ef} is used. The redistribution parameter of the current of charge carriers in the channels and the elastic element in this case is equal to:

$$\Lambda = \frac{H_{ef} \cdot \Phi \cdot b}{a}. \tag{20}$$

The numerical value of the parameter Λ is determined by joint measurements [15]. With known values of temperatures inside the selected operating range T_1

and T_2, the values R_{inT1} and R_{inT2} are measured. For the same temperature values, the values R_{bcT1} , R_{bcT2} are calculated. If α_{bc} and β_{bc} are unknown a priori, R_{bcT1} and R_{bcT2} are measured on specially designed test samples . The same is true for ρ_{eeT1} and ρ_{eeT2}. Using the system of 2 equations derived from (19), we obtain the ratio for calculating Λ (11).

The dependence of the parameter Λ on the resistivity ρ_{ee} was determined by measuring test samples with a thickness of $H_{EE} = 1 \cdot 10^{-2} cm$. The change in Λ values for $\rho_{ee} = (0.5 \div 10)\ \Omega \cdot cm$ was within measurement error. For the test samples where piezoresistive channels were formed by boron ion implantation with a doping dose of $2 \cdot 10^{15} cm^{-2}$,$\Lambda = (9.1 \pm 0.5) \cdot 10^{-3} cm$.

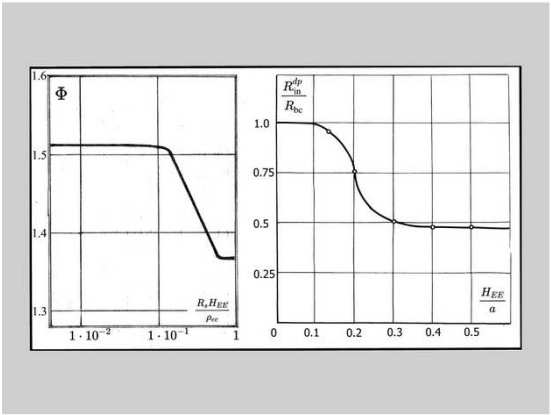

Figure 6: Experimentally obtained dependences of variation of coefficient Φ and input resistance on design parameters of the transducer with distributed parameters.

Unification of the sensor design according to the input mechanical quantities will be related to changing the slope of the conversion characteristic by selecting the thickness of the elastic element. An important condition is the independence of the circuit parameters from the thickness of H_{ee}. From the graph in Figure 6, which shows the experimental dependence of the input resistance change R_{in} on the calculated parameter $\frac{H_{ee}}{a}$, it can be seen that this condition will take place at $\frac{H_{ee}}{a} \geqslant 0.3$.

The limit of the admissible value of the complementary temperature error of the transducer Γ_{kt} is determined by geometric summation of the errors due to the change in the strain-sensitivity coefficient of the transducer relative to the initial temperature T_0 as the temperature changes within the operating range, i.e.

$$\Gamma_{kt} = \frac{1}{K_0} \sqrt{\frac{1}{n-1} \sum_{i=0}^{n} \left[\frac{K - K_0}{\left(T_2 \frac{n-i}{n} + T_1 \frac{i}{n} - T_0 \right)} \right]^2} \tag{21}$$

Finally, taking into account relation (18), the temperature dependence of the parameters in (18), and the value of the parameter Λ, determined by equality (20), we obtain expression (10) for Γ_{kt} .

4 Manufacturing samples of small-size silicon mechanical sensors.

It is necessary to make a silicon resistor integral pressure differential transmitter with the following specifications:

- pressure range of $0 \div 0.06$ MPa;
- margin of admissible basic error $+1\%$;
- operating temperature range $+50°\text{C} - +250°\text{C}$;
- power supply voltage 6 V;
  For sensors of this accuracy class, according to [16], the limit of admissible additional temperature error should be $\Gamma_{kt} = 0.06\ \%/C$.

The previously developed sensor design for the indicated pressures with a resistor silicon integral transducer with piezoresistors, the channels of which are isolated from the elastic element by p-n junction, was chosen. The transducer design consists of: a silicon membrane elastic element of thickness $H_{ee} = 1 \cdot 10^{-2}$cm; square contact areas 6,9,10,11 with side dimensions $b = 2 \cdot 10^{-3}$ cm, connected to each other by four ion-doped single-band piezoresistive channels of 2,3,4,5 p-type; channels of width $b = 2 \cdot 10^{-3}$ cm are oriented along the crystallographic direction [011] . For the selected sensor design, the dependences of the coefficients α_{bc}, β_{bc} on $\tilde{\rho}_{bc}$ in the range $(5 \cdot 10^{-3} \div 5 \cdot 10^{-2})\quad \Omega \cdot cm$ in the temperature range $-60°\text{C} - +170°\text{C}$.

The initial silicon wafers doped with boron type KDB-0.5, KDB-1.0, KDB-10 ($\rho_{ee}\,(25°\text{C}) = 0.5; 1; 10\quad \Omega \cdot cm$) were chosen.

The temperature dependence of resistivity for silicon wafers has no sharp decreases up to the intrinsic conductivity temperature, and according to [17] in the $50°\text{C} \div 250°\text{C}\quad (323\ \text{K} \div 523\ \text{K})$ determined

$$\rho_{ee}(T) = 2.2\left[1 + 5.3 \cdot 10^{-3}(T - 423) + 8.2 \cdot 10^{-6}(T - 423)^2\right]\ \ [\Omega \cdot cm]$$

The dependences of the parameter Λ on the average resistivity $\tilde{\rho}_{bc}$ and the doping depth X_j during the formation of piezoresistive boron ion implantation channels were measured experimentally. It was found that during the doping process with the parameters: energy 80 keV; doping dose $2 \cdot 10^{15}\ cm^{-2}$; average resistivity $\tilde{\rho}_{bc} = 7.3 \cdot 10^{-3}\Omega \cdot$ cm; alloying depth $X_j = 0.86 \cdot 10^{-4}\ cm$, the input resistance values of the transducer at temperatures $T_1 = 323\ K$ and $T_1 = 598\ K$ are $R_{in1} = 338\ \Omega$ and $R_{in2} = 598\ \Omega$ respectively. The temperature dependence of the strain-sensitivity coefficient $K_{bc}(T)$ and resistivity $\tilde{\rho}_{bc}(T)$ in the temperature range $50°\text{C} \div 250°\text{C}$ are determined by the relations:

$$K_{bc}(T) = 10 + 60\frac{423}{T} - 14\left(\frac{423}{T}\right)^2;$$

$$\tilde{\rho}_{bc}(T) = 8 \cdot 10^{-3}\left[1 + 1.1 \cdot 10^{-3}(T - 423) + 1.5 \cdot 10^{-6}(T - 423)^2\right]\quad [\Omega \cdot cm]$$

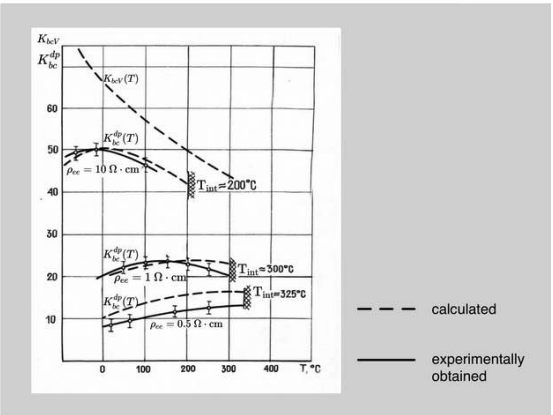

Figure 7: Calculated and experimentally obtained temperature changes of strain-sensitivity coefficients for transducers with distributed parameters and electrically isolated from the elastic element.

Parameter value $\Lambda = 2.67 \cdot 10^{-3}\ cm$, complementary temperature error of the sensor $\Gamma_{kt} = 0.0386\ \%/C$.

Figure 7 shows the calculated and experimentally obtained temperature changes of strain-sensitivity coefficients for transducers with distributed parameters ($K_{bc}^{dp}(T)$) and electrically isolated from the elastic element ($K_{bcV}(T)$). For the design of small-sized sensors selected design with distributed parameters with $\rho_{ee} = 1\ \Omega \cdot \text{cm}$, for which in the range of $(0 \div +300)^{\circ}C$ experimentally determined the value of the parameter $\gamma_{mc0} = 0.04\%/\ ^{0}C$.

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