SI 2.2 Michelson interferometer (The University of Sydney)

Experiment 24. The Michelson Interferometer

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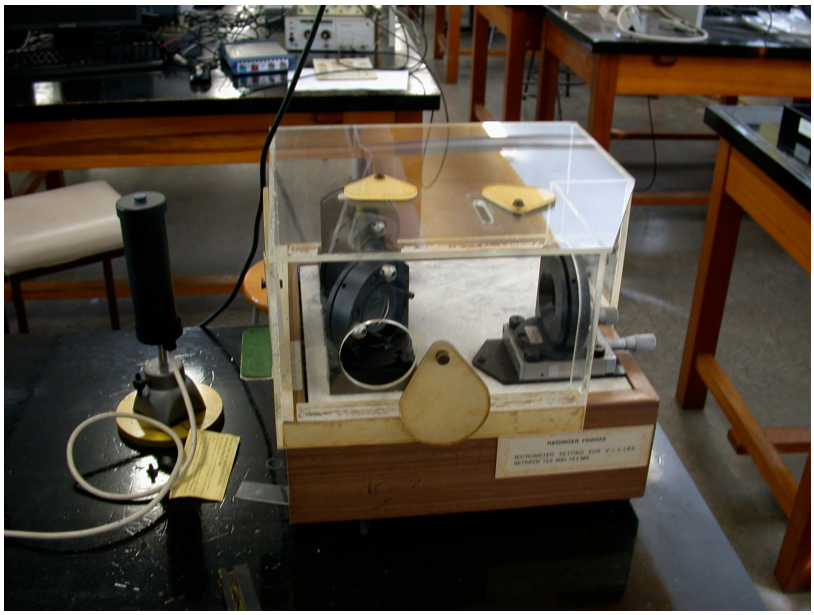

Updated WJT May 15, 2014

General References
Hecht, E., *Optics*, 4th ed., Addison-Wesley, 2001

1 Safety

1. The light sources used in this experiment (Na lamp, Hg lamp and incandescent lamp) can become quite hot. Take care not to touch the actual lamp housings when the lamps are on.
2. To avoid back and neck strain, when looking into the interferometer choose a low chair or stool so your eyes are approximately level with the exit aperture of the interferometer.

2 Objectives

In this experiment you will use the Michelson interferometer to investigate some aspects of the spatial and temporal coherence of light. In particular you will

- Familiarise yourself with the Michelson interferometer
- Investigate the phenomenon of fringe localisation
- Use the interferometer to study atomic spectra
- Observe the channeled spectrum of a white light source
- Find the "white light fringe" for a white light source
- Use the interferometer to measure the refractive index of a transparent material

The **prework** for this experiment will be found in section 6.1.

3 Introduction

The Michelson interferometer is historically important for its use by Michelson and Morley in 1887 to provide experimental evidence against the theory of the luminiferous aether. Michelson subsequently used the interferometer to measure the length of the standard metre in terms of the wavelength of an atomic spectral line. In 1907 he became the first U.S. citizen[1] to receive the Nobel Prize in Physics "for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid."

Modified versions of the Michelson interferometer are still widely used both in research and technology. Applications include Fourier transform spectroscopy (see Experiment 28), the testing of precision optical components and optical fibre communications.

Broadly speaking there are two classes of interferometers:

- Wavefront-splitting interferometers like the Young's slit interferometer use two or more apertures to select different parts of an extended wavefront. The radiation from these apertures is then combined to form an interference pattern. One example of this class is the Michelson *stellar* interferometer which was used by Michelson and Pease in 1921 to make the first direct measurement of the angular diameter of a star (α Orionis). Modern optical stellar interferometers and radio synthesis telescopes are examples of this kind of interferometer.
- Amplitude-splitting interferometers use a partially reflecting mirror ("beamsplitter") to divide an incoming beam of light into two beams. These beams can either be recombined using the same beamsplitter (Michelson interferometer) or a separate beam combiner can be used (Mach-Zehnder interferometer).

Figure 24-1 shows the layout of the Michelson interferometer used in this experiment. The theory of fringe formation in the Michelson interferometer is discussed in the next section.

[1] Michelson was born in Poland but his parents emigrated to California when he was two years old.

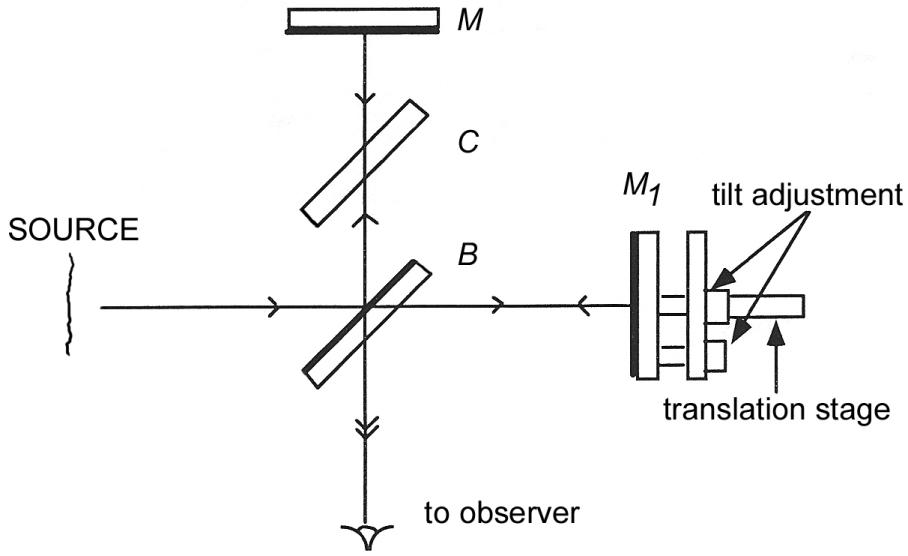

Fig. 24-1 : The layout of the Michelson interferometer used in this experiment. See the text for details.

4 Theory

4.1 Fringe localisation

4.1.1 Real, non-localised fringes

If a Michelson interferometer is illuminated with *collimated* monochromatic light interference will occur everywhere in the output beam. If a screen such as a white card is placed in the output beam one will see interference fringes on the card. Thus the fringes are real and, because they occur everywhere in the output beam, they are said to be non-localised.[2]

If the basic Michelson interferometer is illuminated with light from an extended, uncollimated light source most of the rays will not propagate parallel to the optical axis and consequently will not produce interference fringes in the output beam. Some rays will still propagate parallel to the axis and these will produce non-localised real fringes. They will be difficult to see because of the bright "incoherent" background due to the other rays.

4.1.2 Virtual fringes at infinity

Figure 24-2 shows a simplified diagram of a Michelson interferometer (for clarity the compensator plate is omitted and the beamsplitter is assumed to have zero thickness). An extended source S is used to illuminate the interferometer. M_1' is the virtual image of M_1 and is located at a distance d from M. The *optical path difference* (OPD) between the two arms of the interferometer is $x = 2d$.

We assume that the interferometer is adjusted so both mirrors are perpendicular to the optical axis of the interferometer. In this case M and M_1' are parallel. A ray traced from a point on the extended source is also shown. The ray reflected from M (shown in green) and the ray reflected from M_1'

[2]This arrangement, using collimated light, is known as a Twyman-Green interferometer.

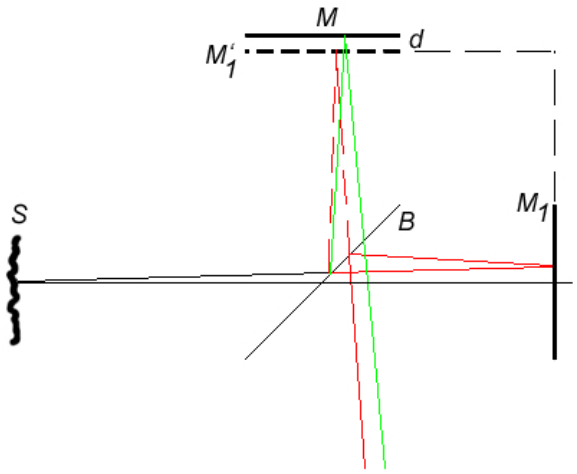

Fig. 24-2 : Fringe localisation: when the two mirrors are parallel the fringes are localised "at infinity."

(shown in red) are parallel and thus appear to come from infinity. The OPD between the two rays will be $2d \sec \alpha$. All the rays emitted at an angle α to the optical axis will emerge parallel to the surface of a cone of half angle α.

If a telescope focused on infinity is used to view the output of the interferometer the light from the cone of rays will form a ring in the image plane having an angular diameter α. Thus the pattern one sees is a set of circular bright and dark fringes.

Because the rays appear to come from infinity the fringes are said to be localised at infinity. They are sometimes called Haidinger fringes. As d becomes smaller the angular scale of the fringe pattern increases and when $d = 0$ M and M_1' are coincident and the field of view is uniformly illuminated: the mirrors are said to be in *optical contact*. As we shall see later this position also coincides with the "white light fringe" (WLF) position and the fringes localised at infinity can be used to help locate the WLF position.

4.1.3 Localised tilt fringes

In Figure 24-3 the separation between M and M_1' is small and M_1' is deliberately tilted at a small angle θ to the optical axis. In this case the rays that reach the observer are no longer parallel, but appear to come from the wedge formed by M and M_1'. To view the resulting fringe pattern a lens must be focussed on this wedge (if viewed with the naked eye your eye will naturally focus on this point). The fringes will be equidistant straight bands parallel to the apex of the wedge and their separation will depend on the angle θ.

As d increases the variation in the angle of incidence across the field of view becomes significant; the fringes become curved and their visibility decreases.

In an ideal Michelson interferometer these tilt fringes are perfectly straight when $d = 0$. Any imperfections in the beamsplitter or mirrors will introduce phase errors that distort the fringes and

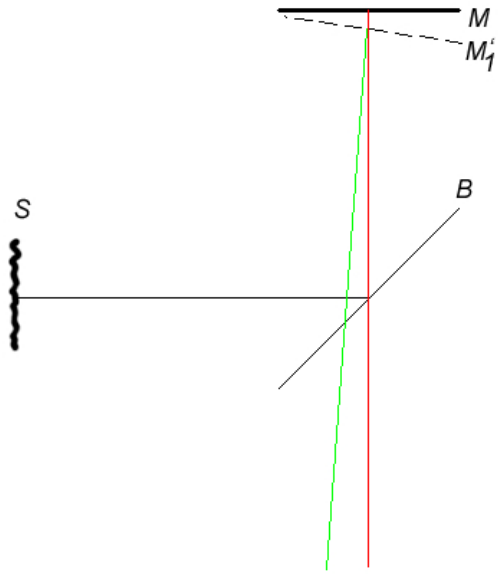

Fig. 24-3 : Fringe localisation: when the two mirrors are tilted the fringes are localised in the virtual wedge formed by M and M_1'. The angle in this diagram is greatly exaggerated.

irregularities as small as $\lambda/20$ can be detected. This makes the Michelson interferometer suitable for testing the optical quality of surfaces and it is widely used in the optical fabrication industry for this purpose.

4.2 Temporal coherence

If the interferometer in Figure 24-1 is carefully adjusted so the two mirrors are perpendicular to the optical axis of the interferometer the light that reaches the observer will be uniformly illuminated. As the position of the mirror M_2 is changed the field of view will vary from bright to dark due to interference. Let $I_{\max}$ and $I_{\min}$ be the maximum and minimum intensities in this "fringe pattern." The Michelson fringe visibility is defined to be

$$V(x) = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{1}$$

It is easy to show that $0 \leq V \leq 1$ and in general the fringe visibility will depend on the optical path difference (OPD) x between the two arms of the interferometer. If the source is monochromatic $V = 1$ and will be independent of x. For real light sources $V \leq 1$ and becomes smaller as $|x|$ increases. The visibility will equal 1 when $x = 0$. The full width at half maximum (FWHM) of $V(x)$ is called the *coherence length* of the light source. Lasers, for example, can have a coherence length of many metres whilst an ordinary incandescent light has a coherence length of $\sim 1\mu$m.

In this experiment you will normally be working with tilt fringes. If α is the angle between the mirror M and the virtual mirror M_1' (see above) the OPD x will vary across the field of view and in this case the visibility refers to the visibility of the tilt fringes.

Figure 24-4 is a plot of the output intensity of a Michelson interferometer as a function of the OPD x. There are several features of this plot:

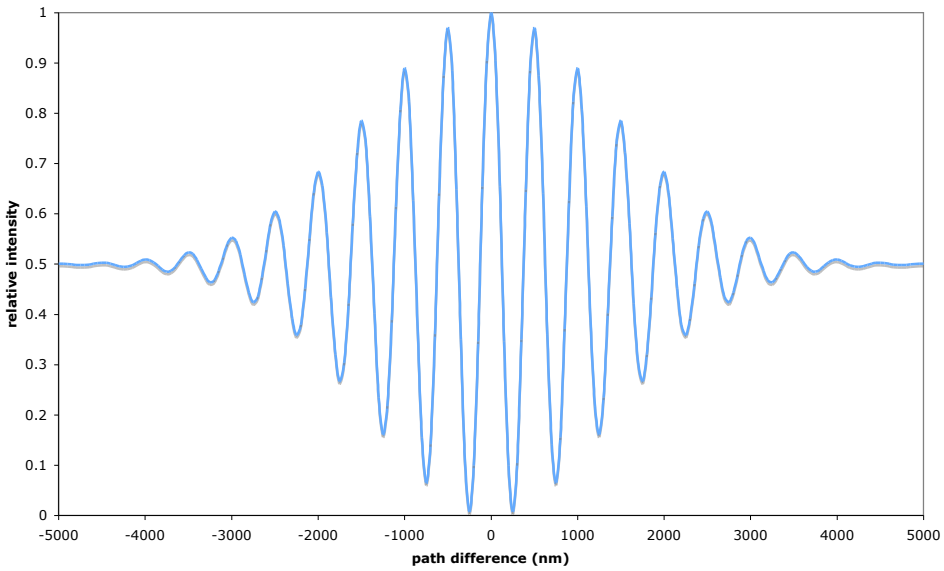

Fig. 24-4 : The fringe pattern observed with a non-monochromatic source. The fringe visibility given by Michelson's formula $V = (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ varies with optical path length. It is a maximum when the OPD is zero.

- The fringes are equally spaced. The *period* of the fringes, expressed in terms of the optical path difference, will be denoted by L.
- The fringe visibility $V(x)$is a maximum at $x = 0$.
- The fringe visibility is the *envelope function* of the fringe pattern and is symmetric around $x = 0$.

The mathematical form of the intensity of this fringe pattern can therefore be expressed as

$$I(x) = I_0[1 + V(x)\cos\{2\pi x/L\}]$$

It is usually more convenient to write this in complex form (we follow engineering practice and use $j = \sqrt{-1}$):

$$I(x) = I_0[1 + V(x)e^{-2\pi jx/L}]$$

It is always understood that the actual fringe pattern is the real part of $I(x)$. We are only interested in the oscillatory term, which we can call the fringe signal:

$$S(x) = V(x)e^{-2\pi jx/L}$$

We next investigate the relationship between $V(x)$, L and the properties of the light source.

4.2.1 The visibility as a Fourier transform

If the light is strictly monochromatic the fringe signal will be

$$I(x) = I_0[1 + e^{-2\pi jx/\lambda}]$$

where λ is the wavelength of the light. The fact that λ appears as λ^{-1} complicates things and we use instead the *spectroscopic wavenumber*[3] $\sigma = 1/\lambda$:

$$I(x) = I(\sigma)[1 + e^{-2\pi j\sigma x}]$$

where we have explicitly indicated that the intensity can vary as a function of σ.

Real light sources are not monochromatic. If the light source is *incoherent* the phase between two different wavelengths λ and $\lambda + \delta\lambda$ fluctuates randomly and extremely rapidly. The result is that each wavelength generates its own interference pattern independent of the other wavelengths and to find the total fringe signal we simply integrate over all wavelengths

$$I(x) = \int_{-\infty}^{\infty} I(\sigma)d\sigma + \int_{-\infty}^{\infty} I(\sigma)e^{-2\pi j\sigma x}d\sigma$$

It is convenient to introduce the "fringe signal" $S(x)$:

$$I(x) = \left(\int_{-\infty}^{\infty} I(\sigma)d\sigma\right)[1 + S(x)] = I_0[1 + S(x)]$$

where

$$S(x) = I_0^{-1}\int_{-\infty}^{\infty} I(\sigma)e^{-2\pi j\sigma x}d\sigma \tag{2}$$

This will be recognised as a (normalised) Fourier transform and we have the important result that *the signal $S(x)$ is the Fourier transform of the spectral intensity distribution $I(\sigma)$*. In particular, the Michelson fringe visibility $V(x) = |S(x)|$ (the function $S(x)$ is in general complex).

4.2.2 The coherence length

We have defined the coherence length l to be the FWHM of $V(x)$ Using the properties of the Fourier transform one can show that

$$l \approx \delta\sigma^{-1} \approx \frac{\lambda_0^2}{\delta\lambda} \tag{3}$$

where $\delta\sigma$ is the FWHM of the light source, centred on the wavenumber σ_0, and $\delta\lambda$ is the bandwidth centred on the wavelength $\lambda_0 = \sigma_0^{-1}$.

[3]The spectroscopic wavenumber should not be confused with the circular wavenumber $k = 2\pi/\lambda$ which is frequently encountered in the theory of wave propagation. Various symbols are used for the spectroscopic wavenumber; the US National Institute for Standards and Technology recommends σ and we follow that here.

5 Apparatus

5.1 The Michelson interferometer

Figure 24-1 shows the layout of the Michelson interferometer used in this experiment. The mirrors, beamsplitter and compensator plate are mounted on a marble slab for stability. The apparatus is housed in a clear plastic box to exclude drafts and to keep the optics clean. Light from the source is split at the dielectric beamsplitter B. Approximately half the light is transmitted to the mirror M_1 while the other half is reflected to the mirror M. A second glass plate called the compensator (C) is placed between B and M to ensure that the amount of glass in each path is the same.[4] The two mirrors reflect the light back to B where the beams are recombined. The combined beams emerge from the interferometer and can be viewed either directly by eye or with another optical instrument.

The mirror M_1 has three adjustments:

- Two micrometers fixed to the mirror mount allow M_1 to be tilted horizontally and vertically. *Always use "baby steps" when adjusting these mirrors!*
- A third micrometer moves the mirror along the optical axis; i. e., either towards or away from the beamsplitter. Because of the way the translation stage has been mounted, a clockwise rotation of the micrometer *increases* the optical path difference. There is also a digital gauge attached to the translation stage; this is much easier to read than the micrometer and should be used for all measurements (except when calibrating the gauge).

5.2 Light sources, filters and screens

There are three light sources used in this experiment: a sodium vapour lamp, a mercury vapour lamp and an ordinary incandescent lamp. Please note that all three lamps become quite hot when they are operating.

A plastic diffusing screen should be placed between the lamps and the interferometer to reduce glare and provide more uniform illumination. There is a hole in the plastic cover over the interferometer and the screen is designed to fit in the hole.

Interference filters are provided to isolate the different lines of the mercury spectrum. To use a filter slide it into one of the spring-loaded support blocks and place the block on the small shelf in front of the input to the interferometer. The reflecting side of the filter should face towards the light source.

5.3 Viewing apparatus

In this experiment the most important viewing apparatus is your eye! In parts of the experiment, however, you will use viewing aids. These include:

- A telescope that has a fixed focus at infinity.

[4] Unequal glass paths will introduce *dispersion* in the interferometer. As discussed in Appendix B dispersion will cause a shift in the position of the white light fringe position. Another effect is to reduce the visibility of the fringes. It is therefore important to ensure that the glass paths in the two arms of the interferometer are equal.

- A small spectroscope.
- An adjustable slit.

5.4 Other equipment

For the final part of this experiment you will measure the refractive index of a glass plate. A micrometer is provided to measure the thickness of the plate.

6 Procedure

6.1 Prework

Note: These questions should be answered before coming to the lab. Answer the questions on a sheet of paper and glue or staple the sheet in your logbook when you commence the experimental work.

1. A certain light source emits radiation with a mean wavelength of 600 nm. When it is viewed with a Michelson interferometer it is found that the fringe visibility periodically drops to zero as the optical path length is changed. The optical path distance between successive minima is 3.0 mm. What can you conclude about this light source? [Hint: See Appendix A.2.]

2. The uncertainty in the energy of an excited state is given by the Heisenberg uncertainty principle:

$$\Delta E \Delta t \gtrsim \frac{\hbar}{2} \tag{4}$$

   When the state decays it emits radiation. What is the uncertainty in the frequency of this radiation? The uncertainty in wavelength, $\Delta\lambda$, is known as the *natural linewidth* of the radiation. If the wavelength of the radiation is λ_0, what is $\Delta\lambda$?

6.2 Initial setup

The digital gauge displays *relative*, not absolute, displacement. When the gauge control unit is turned on it will display 0.000. Toggling the RESET switch will also reset the display to zero. The following procedure should be used to calibrate the gauge:

1. Turn the control unit on.
2. Rotate the micrometer screw in a counterclockwise direction until the micrometer scale reading is 25.00 mm. Note that one turn of the micrometer screw corresponds to a displacement of 0.50 mm.
3. Press the reset switch on the control unit. The display will now read 0.000 mm. The gauge is now calibrated.

As noted previously a clockwise rotation of the micrometer increases the OPD x. The gauge readout is

$$d = x/2 + d_{\rm cal}$$

where $d_{\rm cal}$ is the gauge offset. As long as the gauge is not reset this offset can be ignored. However, if the gauge is accidentally reset or the control unit has been turned off you will need to repeat the calibration procedure. Note also that

$$d = 25.00 - d_{\rm mic}$$

where d is the gauge reading and $d_{\rm mic}$ is the micrometer reading.

Turn on the sodium lamp. There are several different designs of lamp that are used; check if there any special instructions before switching on the lamp. It will take a few minutes for the lamp to stabilise.

The approximate location of the $d = 0$ or "optical contact" position is marked on the interferometer in terms of the micrometer setting. Adjust the micrometer to put the mirror slightly higher than the indicated range. For example, if the range marked on the instrument is 12.5–13.5 mm, set the micrometer to $\sim$14 mm. The corresponding gauge reading will be $\sim$11 mm.

Place the translucent screen with a cross drawn on it in the entrance to the interferometer and illuminate it with the Na lamp. View through the exit port of the interferometer and adjust the tilt of M_1 to superimpose the two images of the cross. You should see dark fringes crossing the field of view. *Using baby steps* tilt M_1 to reduce the number of fringes. Ideally you should obtain a uniformly illuminated field.

6.3 Fringe localisation

Place the telescope in the output beam and use it to observe the fringe pattern. This telescope is focussed at infinity. Describe what you see and make a sketch in your notebook. Gradually turn the micrometer in a clockwise direction, stopping frequently to observe the fringe pattern. What do you observe? What happens when you reach the $d = 0$ position? Explain your observations in terms of the discussion in section (4.1.2).

Estimate the $d = 0$ position according to the digital gauge, using the fringe pattern localised at infinity. Repeat your measurement several times, always approaching the $d = 0$ position by turning the micrometer clockwise (Why?). Use your results to estimate the best value for $d = 0$ and its uncertainty using the standard error of the mean (s.e.m.).

With M_1 set at the $d = 0$ position remove the telescope and observe the fringe pattern directly by eye. Tilt M_1 until there are approximately five fringes across the field of view. These fringes are the localised tilt fringes. Where are they located? Are they real or virtual?

Change the separation (d) by $1 \sim 2$ mm and describe how the pattern changes.

If necessary, adjust d until there are high contrast tilt fringes and place one of the translucent screens in the exit aperture of the interferometer and look at the screen from an oblique angle (do not look directly into the interferometer). You should see very faint fringes *on* the translucent screen. You may need to use black cloth to block the room light. Also it helps to see the fringes if you translate M_1 through a small distance. This will cause the fringes to move and you may see them more easily.

Remove the translucent screen from the *input* side of the interferometer and replace it with a narrow slit. This should improve the visibility of the non-localised fringes.

Explain your observations in terms of the discussion in sections (4.1.1) & (4.1.3). Why does the slit improve the visibility of the non-localised fringes?

When finished, remove the slit and replace the translucent screen in the input beam.

C1 ▷

6.4 Temporal coherence

6.4.1 The sodium spectrum

Observe the tilt fringes from the Na lamp directly by eye. The Na spectrum consists of two closely spaced lines (the "sodium doublet") centred on the mean wavelength 589.3 nm. Starting from the $d = 0$ position slowly turn the micrometer and observe how the visibility of the fringes decreases. You should find a position where the visibility is zero; i.e., the fringes disappear. Record the gauge readout at this position (call it d_1).

Return to the $d = 0$ position and turn the micrometer in the opposite direction. Again you should find a position where the visibility becomes zero. Record this position (call it d_2).

The optical path difference between these two points will be $X = 2(d_2 - d_1)$. Why is there a factor of two?

Repeat your measurements several times in order to estimate the best value of X and its s.e.m. Use Eq. (6) in the Appendix to calculate the wavelength difference (with uncertainties!) between the two lines in the Na doublet.

The fringe visibility is symmetrical around the $d = 0$ position. Use your values of d_1 and d_2 to estimate the $d = 0$ position and its uncertainty. Compare this value with the one you found using the fringes localised at infinity. Which method is more accurate?

6.4.2 The mercury spectrum

Replace the Na lamp with the Hg lamp. Observe the Hg spectrum with the small hand spectroscope provided. The most prominent lines in the spectrum are at 435.8 nm (blue), 546.1 nm (green) and 578.0 nm (yellow). You should be able to see the green and yellow mercury lines. Adjust the slit width of the spectroscope to make this as narrow as possible (but still let light through!). You may also need to adjust the focus. Do you notice any difference between the green and yellow lines?

Remove the spectroscope, reset M_1 to the $d = 0$ position and place the yellow interference filter on the shelf in front of the input aperture of the interferometer. Change the OPD and observe what happens. *Qualitatively* what can you say about the yellow Hg line?

Return to the $d = 0$ position and repeat your observations using the green filter. Again, what can you say *qualitatively* about the green line.

Change the OPD and estimate where the fringe visibility is reduced by 50%. Record the micrometer

reading. As with the Na lamp, do this on both sides of the $d = 0$ position. Repeat your measurements several times in order to estimate the s.e.m. and use your results to calculate the coherence length of the green mercury line and its linewidth (see section 4.2.2).

C2 ▷

6.5 White light

6.5.1 Channel fringes and white light fringes

Set the interferometer to the $d = 0$ position. Using the green Hg line observe the tilt fringes. Adjust the tilt until the fringes are as broad as you can make them. Do not make any adjustments to the tilt until you have found the white light fringes (see below)!

Use the hand spectroscope to view the light from the interferometer. If necessary, adjust the slit width and focus to produce sharply defined spectral lines.

Remove the green filter and the mercury lamp. Illuminate the interferometer using the incandescent lamp. You should now see the continuous spectrum of the incandescent lamp in the spectroscope. Scan through the range of d indicated on the front of the interferometer *using baby steps* until you find fringes in the spectrum. You may need to do this several times until you find the fringes.

Once you have obtained the fringes observe how the spacing changes with OPD. These fringes are called channel fringes and the spectrum is often called the channelled spectrum. Explain how channel fringes are formed (HINT: what is the phase difference between the two light beams in the interferometer?). Carefully adjust the interferometer until there are only a few fringes visible.

If you remove the spectroscope you should see coloured fringes. Tilt M_1 until there are $5 \sim 10$ fringes across the field of view. Describe what you see. The central fringe is called the "white light fringe" and it exists exactly at the point $d = 0$. Centre the white light fringe in the pattern and record the micrometer reading. As before, take several readings of the digital gauge (moving the micrometer away from the WLF position each time and resetting onto the WLF position) to estimate the position of optical contact and its s.e.m. Compare this with the other two methods you used.

C3 ▷

6.5.2 Determination of the refractive index of glass

One use of the Michelson interferometer is the measurement of the refractive indexes of transparent substances. It is most commonly used to measure the refractive indexes of gases or liquids that are placed in a cell in one arm of the interferometer.[5] In this part of the experiment you will use the interferometer to measure the refractive index of a thin glass plate.

If we insert a plate having the *group* refractive index n_g into one arm of the interferometer (see Appendix B) the OPD will be increased by

$$2(n_g - 1)t$$

[5] This variation on the Michelson interferometer is known as a Rayleigh interferometer.

where t is the physical thickness of the plate (the plate *adds* group OPD equal to $2n_g t$ but *removes* an amount of air having the OPD $2 \times 1.000 \times t$).

The group index for common optical glass is $n_g \approx 1.55$. Measure the thickness of the glass plate using the micrometer provided and estimate by how much the OPD will change.

Using the white light source set up the interferometer for white light fringes as in the previous section. Insert the glass plate into one arm of the interferometer using the slot in the cover. The fringes will disappear. Using your estimate move the micrometer to where you expect to find fringes (make sure you move the mirror in the correct direction!).

If you cannot find the fringes, temporarily replace the lamp with the Hg lamp. Using the green filter follow the procedure in the previous section to spread the fringes out. Using the white light source and the spectroscope find the channel fringes and then locate the WLF position. Record the gauge reading for the new WLF position and determine its s.e.m.

If the measured shift in the WLF position is Δ then the change in OPD will be $x = 2\Delta$, hence $x = 2(n_g - 1)t$ or $\Delta = (n_g - 1)t$. Use your measured shift and uncertainty to determine the group refractive index of the glass and its uncertainty.

C4 ▷

Appendices

A Some examples of Fourier transforms

We use Eq. (2) to calculate the interference pattern expected for some simple situations.

A.1 Monochromatic light

In the case of monochromatic radiation of wavelength λ_0 the mean wavenumber will be $\sigma_0 = \lambda_0$ and

$$\tilde{I}(s) = \delta(s)$$

where $\delta(s)$ is the Dirac delta-function. It follows that $V(d) = 1$ and the interference signal will be

$$S(x) = e^{-2\pi j x/\lambda_0}$$

The real signal will be proportional to $\cos(2\pi x/\lambda_0)$.

A.2 Two wavelengths of equal intensity

We assume that the light consists of two "lines" of equal intensity. The real fringe signal will be simply

$$S(x) = [\cos(2\pi\sigma_1 x) + \cos(2\pi\sigma_2 x]/2 \tag{5}$$

This is completely analogous to the formula for acoustic beats, with $\sigma_{1,2}$ corresponding to the frequencies of the two sound waves and x corresponding to t (see Figure 24-5). We can use the

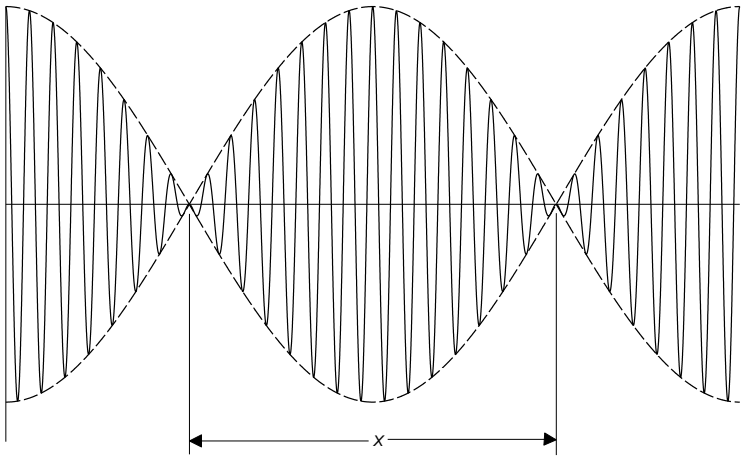

Fig. 24-5 : The fringe pattern seen when the light source consists of two wavelengths.

standard trigonometric sum-to-product formula to write this as

$$S(x) = \cos\left(2\pi\frac{\sigma_1 - \sigma_2}{2}x\right)\cos\left(2\pi\frac{\sigma_1 + \sigma_2}{2}x\right)$$

The fringe frequency is $\overline{\sigma} = (\sigma_1 + \sigma_2)/2$ and is modulated by a slowly varying cosine function that has a frequency equal to $(\sigma_1 - \sigma_2)/2$. The modulation period is $[(\sigma_1 - \sigma_2)/2]^{-1}$.

Let X be the distance between two successive minima (or maxima) of the visibility. As can be seen from Figure 24-5 the period of the modulation is $2 \times X$ so

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2X}$$

We can write this in terms of wavelengths as follows:

$$\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \approx \frac{\lambda_2 - \lambda_1}{\bar{\lambda}^2} = \frac{1}{X} \tag{6}$$

where $\bar{\lambda}$ is the mean wavelength.

B White light fringes and dispersion

The shift in the white light fringe position can be used to determine the refractive index of materials (see section 6.5.2). However, the effects of dispersion (the variation of the refractive index with wavelength or wavenumber) cannot be ignored. If a material with refractive index n and a thickness

t is placed in one arm of the interferometer and there is a difference d in the position of the two mirrors (to be precise, d is the air-path difference between the two arms of the interferometer), the total OPD will be

$$x(\sigma) = 2d + 2[n(\sigma) - 1]t$$

where σ is the wavenumber. Let σ_0 be the central (or average) wavenumber of the light being used. Intuitively we expect that the white light fringe will occur when $x(\sigma_0) = 0$ or when

$$d = -[n(\sigma_0) - 1]t \tag{7}$$

This is wrong! The phase for this setup will be

$$\phi(\sigma) = 2\pi\sigma x(\sigma) = 4\pi\sigma[n(\sigma) - n(\sigma_0]t$$

The OPD will vary with wavenumber: $x = 2[n(\sigma) - n(\sigma_0]t$. This is shown in Fig. (24-6) for the case of BK7 glass with a thickness of 0.5 mm. The phase and OPD at σ_0 are indeed zero but there is an approximately linear variation in phase across the band, corresponding to an OPD variation of more than 0.3 mm. If we were to view the output of the interferometer by eye no white light fringes would be seen; if we were to use a spectroscope the spectrum would be crossed by a very large number of closely spaced channel fringes.

We know that the phase varies linearly with σ if the OPD is not zero: $\phi(\sigma) = 2\pi\sigma x$. We can choose x to make this linear variation approximately cancel the variation due to dispersion. When this condition is met white light fringes will be seen.[6]

To find the location of the WLF we expand the phase in a Taylor's series around σ_0:

$$\phi(\sigma) = \phi(\sigma_0) + 4\pi(\sigma - \sigma_0)\left[d - (n(\sigma_0) - 1)t + \sigma_0 t\frac{dn}{d\sigma}\right] + \cdots \tag{8}$$

where only the first order term has been included.

White light fringes will be seen when the phase is constant (but not necessarily zero) across the bandwidth. To first order the phase will be constant when the second term on the right hand side of Eq. (8) is zero and this occurs when

$$d = \left[n(\sigma_0) + \sigma_0\frac{dn}{d\sigma} - 1\right]t$$

or

$$d = (n_g - 1)t \tag{9}$$

where

$$n_g(\sigma_0) = n(\sigma_0) + \sigma_0\frac{dn}{d\sigma} = n(\lambda_0) - \lambda_0\frac{dn}{d\lambda}$$

is the *group* refractive index. For standard optical materials in the visible part of the spectrum the group index is always larger than the normal, or *phase* index. One commonly used glass, BK7, has a (phase) refractive index of 1.52 but a group index of 1.55.

Fig. (24-7) shows the optical path variation when d is calculated using Eq. (9). The variation is now less than 0.008 mm and white light fringes will be seen at this point.[7]

[6]The discussion here ignores the question of fringe visibility. As the amount of dispersion increases the fringe visibility will decrease and the fringes will been spread out over a relatively large range of OPD. If the glass path difference is small these effects are relatively unimportant.

[7]The calculations are based on data given in Tango (1990), Applied Optics **29**, 516. The approximately quadratic variation in the OPD is due to the higher order terms that were omitted in Eq. (8).

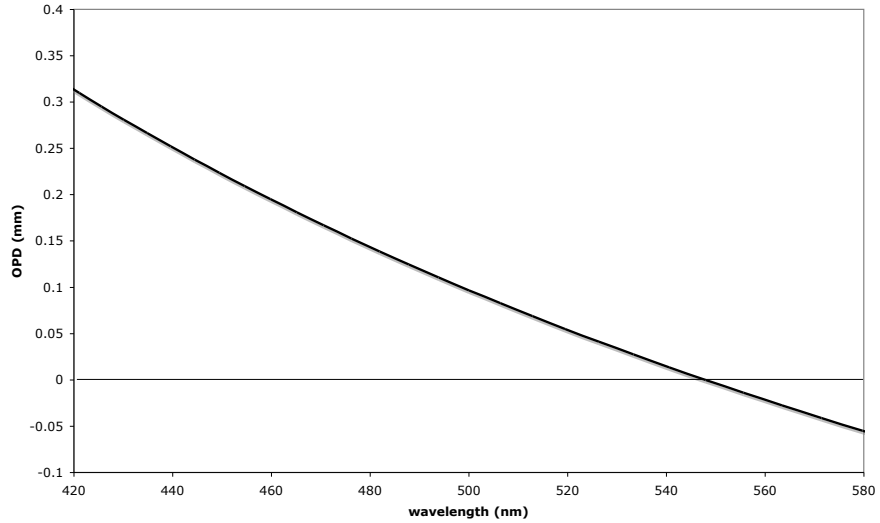

Fig. 24-6 : The variation in optical path difference plate of BK7 glass having thickness $t = 0.5$ mm in one arm when the phase OPD is equal in both arms of the interferometer.

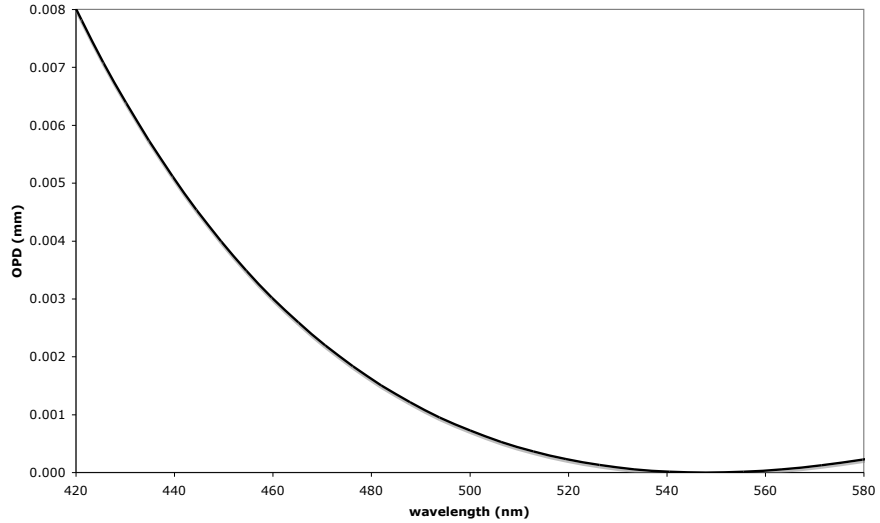

Fig. 24-7 : The variation in optical path difference plate of BK7 glass having thickness $t = .5$ mm in one arm when the group OPD is equal in both arms of the interferometer.

The group velocity v_g is related to the group index by $v_g = cn_g^{-1}$. When pulses travel along a transmission line (or optical fibre) they propagate at the group velocity, so it may be surprising to see the group index/velocity appear in the context of an interferometer illuminated by continuous radiation. However, we know from Fourier analysis that a short pulse consists of a wide range of frequencies. Whether we have a narrow pulse propagating along an optical fibre or an interferometer illuminated with white light, in both cases the radiation consists of a wide range of frequencies and it is the group velocity (or group refractive index) that must be used.

A variation on the Rayleigh interferometer is often used to measure the index of refraction of gases. An evacuated cell is placed in one arm of the interferometer (for symmetry an identical cell can be placed in the other arm) and the interferometer is illuminated with monochromatic light. The gas is slowly added to the cell. As the OPD changes the fringes will move across the detector. By counting the fringes as the gas is added one can calculate the OPD due to the gas and hence its (phase) refractive index.