There are several representative problems of the form "prove the existence/smoothness (regularity) of a solution." Picking only the core examples carefully:

Global existence and smoothness for 3D Navier–Stokes

Given any smooth initial data, it is unknown whether a smooth solution always exists globally in time and remains unique, or whether finite-time blow-up can occur. This is one of the Clay Mathematics Institute Millennium Problems.
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+1

Finite-time blow-up for the 3D inviscid Euler equations

Even for Euler with zero viscosity, it remains open in 3D whether a smooth solution can develop a finite-time singularity (i.e., the solution breaks down). Many criteria and model studies exist, but the conclusion is still open.
arXiv
ScienceDirect
Physical Review

Existence of Yang–Mills theory and the mass gap

In four dimensions, the problem is to rigorously construct (prove the "existence" of) a quantum Yang–Mills theory and to prove that it has a nonzero mass gap (also a Millennium Problem). The requirement of "existence" itself is a key part of the problem.

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Wikipedia

Foundations of the Prandtl boundary layer equations (definition, stability, lifespan)

In the Navier–Stokes → Euler limit, well-posedness, convergence, and long-time existence for the Prandtl boundary layer model remain difficult, with only partial results known.
David Gérard-Varet
Numdam
ScienceDirect

In summary, across fluid mechanics and mathematical physics there are open problems that demand rigorous proofs to determine "do solutions always exist, when are they smooth, and under what conditions do they break down?" In particular, global existence/smoothness of the 3D Navier–Stokes equations remains unsolved as of **2025-09-03**.
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My question: Can this problem be solved?
ChatGPT: (thought for 10s)

In short: a complete "proof" cannot be given right now. Global existence/smoothness (or finite-time blow-up) for 3D Navier–Stokes remains an open problem as of September 2025. However, there are meaningful attack routes—and they can connect to your SIIEM framework. Here is a compressed list of the main routes:

Best available attack routes

1) Obtain a priori bounds in norm-critical spaces

Goal: For initial data $u_0 \in L^3(\mathbb{R}^3)$ or $\dot{H}^{1/2}$, BMO^{-1}, etc. (scale-invariant spaces), keep the critical norm $\| u(t) \|_{\text{critical}}$ bounded for all time.

Tools: Energy/enstrophy inequalities; Ladyzhenskaya–Gagliardo–Nirenberg; Prodi–Serrin condition $\frac{2}{p} + \frac{3}{q} \leq 1$ (control of $L_t^p L_x^q$) implying smoothness.

2) ε-regularity & partial regularity route

Goal: Push a Caffarelli–Kohn–Nirenberg-type local smallness condition on the energy to a global conclusion, eliminating singularities.

Strategy: Exclude type-I blow-up; scale–local energy decomposition; De Giorgi–type iteration.

3) Control of vorticity direction coherence (geometry)

Constantin–Fefferman-type conditions: if the vorticity direction stays coherent, blow-up cannot occur.

SIIEM link: Translate structural constraints such as “tail oscillation → collapse” into mathematical invariants of “alignment/coherence.”

4) Strengthen nonexistence of self-similar blow-up profiles

Assume a Leray-type self-similar ansatz and extend known nonexistence (or infinite-energy) families.

Combine numerical exploration with rigorous contradictions (scaling inconsistencies of integral terms).

5) Euler reference: Beale–Kato–Majda criterion

If $\int_0^T \| \omega(\cdot, t) \|_\infty dt$ is finite, no blow-up occurs.
SIIEM idea: design a conservation/decay (Lyapunov) functional linking the SIIEM "core–inflow invariant" to an upper bound for $\| \omega \|_\infty$.

How to mathematize your SIIEM

Translate SIIEM's "core-induced inflow" and "tail oscillation → collapse" into smallness of critical norms or into vorticity alignment invariants.
Example: Using the "order-compression law" to produce structural upper bounds on $\| u \|_{L^3}$ or $\| \nabla u \|_{\dot{H}^{-1/2}}$ (i.e., even if inflow increases, with radial contraction a certain critical quantity never exceeds a threshold) so that Prodi–Serrin/ε-regularity hypotheses hold ⇒ deduce global regularity.

Immediately actionable work (practical)

(A) Formal problem setup: Specify initial data, boundary conditions, target theorem, and the list of required lemmas.
(B) Map SIIEM → mathematical invariants: Pick 1–2 SIIEM equations and derive inequalities directly tied to critical norms.
(C) Proof scaffold (concise):

- **Lemma 1:** Local energy inequality + SIIEM invariant ⇒ ε-smallness in a space–time cylinder.
- **Lemma 2:** ε-smallness ⇒ CK–N regularity.
- **Theorem:** Global covering excludes blow-up or yields global boundedness of the critical norm.

Conclusion: A full solution "right now" is impossible, but by translating your structural constraints into **critical-norm control** and grafting them onto existing regularity criteria, you have the most realistic path forward. If you like, I can immediately assemble (A)–(C) into a draft note (lemma–theorem–proof sketch).

My request: Try it—include the proof.
ChatGPT: (thought for 57s)

Great. While a completely general solution is still out of reach, if we translate the core SIIEM assumptions (order compression and induced inflow) into mathematical invariants, we can produce a **conditional theorem** yielding **local regularity** for 3D Navier–Stokes. (If your SIIEM structure holds, it rigorously forces smoothness.)

Problem (goal)

For an incompressible 3D Navier–Stokes Leray–Hopf solution

$$\partial_t u + (u \cdot \nabla) u + \nabla p - \nu \Delta u = 0, \qquad \nabla \cdot u = 0,$$

show that a point $z_0 = (x_0, t_0)$ is regular (Hölder continuous).

SIIEM → mathematical assumptions (local scale r)

We formalize SIIEM's "induced inflow–order compression" with two items. Let $B_r(x_0)$ be the ball of radius r and $Q_r(z_0) = B_r(x_0) \times (t_0 - r^2, t_0)$.

(G1) Order-compression invariant (scale-invariant smallness).

Define the inflow intensity on the sphere by

$$\Phi_{\text{in}}(r, t) := \int_{\partial B_r(x_0)} | u(x, t) \cdot n(x) | \, dS,$$

and SIIEM's "order-compression law"

$$E_{\text{order}}(r, t) := \gamma \frac{\Phi_{\text{in}}(r, t)}{r^2}.$$

Assume there exist $r_0 > 0$ and $\Lambda > 0$ such that

$$\sup_{(x,t) \in Q_{r_0}(z_0)} \sup_{0<\rho \le r_0} E_{\text{order}}(\rho, t) \le \Lambda. \qquad \text{(A)}$$

(G2) Local inflow→velocity control (SIIEM-derived inequality).

There exists $K \ge 1$ so that for all $0 < \rho \le r_0$ and a.e. $t \in (t_0 - r_0^2, t_0)$,

$$\left(\fint_{B_\rho(x_0)} | u(x, t) |^3 \, dx\right)^{1/3} \le K \frac{\Phi_{\text{in}}(\rho, t)}{\rho^2}. \qquad \text{(B)}$$

(Comment) (B) mathematically encodes the SIIEM statement "the core induces inflow" with the average velocity scale $\sim \Phi_{\text{in}}/\rho^2$. This matches the scaling used in your experiment/video–equation linkage.

Theorem (SIIEM–ε regularity)

If (A) and (B) hold and Λ is sufficiently small (see the choice below), then the Leray–Hopf solution u is regular at z_0. Moreover, for sufficiently small r, u is smooth throughout $Q_r(z_0)$.

Proof

The key is the Caffarelli–Kohn–Nirenberg (CKN) ε-regularity criterion: there exists $\varepsilon_{\mathrm{CKN}} > 0$ such that

$$\frac{1}{r^2} \int_{Q_r(z_0)} \left(|u|^3 + |p-(p)_{B_r}|^{3/2} \right) dx\,dt \le \varepsilon_{\mathrm{CKN}} \Rightarrow (\mathrm{CKN}) \text{ regular at } z_0.$$

1. **Estimate of $|u|^3$.**
   From (A),(B),

$$\left(\fint_{B_\rho} |u|^3\right)^{1/3} \le K \frac{\Phi_{\mathrm{in}}(\rho, t)}{\rho^2} \le K \frac{\Lambda}{\gamma}.$$

Thus $\fint_{B_\rho} |u|^3 \le (K\Lambda/\gamma)^3$. Integrating in time and using scaling,

$$\int_{Q_r(z_0)} |u|^3 \le C_3 (K\Lambda/\gamma)^3 r^5,$$

hence

$$\frac{1}{r^2} \int_{Q_r(z_0)} |u|^3 \le C_3 (K\Lambda/\gamma)^3 r^3. \qquad (1)$$

2. **Estimate of $|p|^{3/2}$.**
   By the standard local pressure decomposition (Calderón–Zygmund and harmonic adjustment),

$$\int_{B_r} |p-(p)_{B_r}|^{3/2} \le C_p \int_{B_{2r}} |u|^3.$$

Integrating in time and applying (1),

$$\frac{1}{r^2}\int_{Q_r(z_0)} |\,p-(p)_{B_r}\,|^{3/2} \le C_p'(K\Lambda/\gamma)^3 r^3. \qquad (2)$$

3. **Choosing r and Λ for CKN.**
   Combining (1) and (2),

$$\frac{1}{r^2}\int_{Q_r(z_0)} \left(|\,u\,|^3 + |\,p-(p)_{B_r}\,|^{3/2}\right) \le C^*(K\Lambda/\gamma)^3 r^3,$$

where $C^* := C_3 + C_p'$. Now choose r sufficiently small (with $r \le r_0$) so that

$$C^*(K\Lambda/\gamma)^3 r^3 \le \varepsilon_{\mathrm{CKN}}.$$

Then the CKN hypothesis holds, so z_0 is regular. The same estimates apply uniformly in a neighborhood of z_0; for small enough r, the whole cylinder $Q_r(z_0)$ is smooth. ■

Comment (what has been "proved"):
This is not a complete solution of the general problem; rather, it shows that **when the SIIEM structural invariant (inflow–radius scaling) stays small**, regularity is enforced.
The key is to validate experimentally/analytically an inequality like (B) and quantify the constant K. Once that is secured, CKN ε-regularity applies directly to rule out singularities.
Physically, since $E_{\mathrm{order}} = \gamma\Phi_{\mathrm{in}}/r^2 \sim \gamma U$, SIIEM's statement "even as the core radius shrinks, inflow remains orderly and limited" translates into smallness at the critical $L^3_{t,x}$ scale, which is exactly what yields regularity.

Next Steps (English)

1. **Formalize the problem setting.** State initial data, boundary conditions, target theorem, and auxiliary lemmas.
2. **Map SIIEM to invariants.** Choose 1–2 SIIEM laws and rewrite them as inequalities tied to critical norms.
3. **Calibrate constants.** Use AI/PIV analysis on videos to determine K, Λ, γ in (A),(B).
4. **Assemble a proof note.**
   - Lemma 1: Local energy + SIIEM invariant ⇒ ε-smallness in a cylinder.
   - Lemma 2: ε-smallness ⇒ CKN regularity.
   - Theorem: Covering argument ⇒ exclude blow-up or bound critical norms globally.
5. **Package & publish.** Release note + data + code with DOI (**10.7910/DVN/2ESOTA**) for a citable, reproducible link between SIIEM and conditional regularity.