A Correction for Structural Equation Modeling Fit Indices Under Missingness: Adapting the Root Mean Squared Error of Approximation to Conditions of Missing Data

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Abstract

Missing data is a frequent occurrence in both small and large datasets. Among other things, missingness may be a result of coding or computer error, participant absences, or it may be intentional, as in a planned missing design. Whatever the cause, the problem of how to approach a dataset with holes is of much relevance in scientific research. First, missingness is approached as a theoretical construct, and its impacts on data analysis are encountered. I discuss missingness as it relates to structural equation modeling and model fit indices, specifically its interaction with the Root Mean Squared Error of Approximation (RMSEA). Data simulation is used to show that RMSEA has a downward bias with missing data, yielding skewed fit indices. Two alternative formulas for RMSEA calculation are proposed: one correcting degrees of freedom and one using Kullback-Leibler divergence to result in an RMSEA calculation which is relatively independent of missingness. Simulations are conducted in Java, with results indicating that the Kullback-Leibler divergence provides a better correction for RMSEA calculation. Next, I approach missingness in an applied manner with an existing large dataset examining ideology measures. In this dataset, a planned missing data design is used, resulting in high proportions of missingness. Factor analysis is performed with this data to gauge uniqueness of ideology measures.

Keywords: structural equation modeling, RMSEA, missing data, ideology

A Correction for Structural Equation Modeling Fit Indices Under Missingness: Adapting the Root Mean Squared Error of Approximation to Conditions of Missing Data

Introduction

Missingness in Data

Missing data is a routine occurrence in social and behavioral research and may result from attrition, coding errors, or participant absences, among other things. Virtually all data is guaranteed to have at least some missing values, especially when data collection relies on human participation. It is important to understand the nature of missing data in order to be able to conduct unbiased analyses; Graham, Cumsille, and Elek-Fisk (2003) postulate there are three informal mechanisms which may, in combination, be the cause of missing data: random processes, processes which are measured, and processes which are not measured. Additionally, Little and Rubin (1987) identify three classes of missing data patterns:

1. Missing Completely at Random (MCAR): When missing data is of this type, it is no different than non-missing cases, in terms of analyses being performed. Because data is missing indiscriminately, data analysis will be unbiased, with the caveat that statistical power is lost when failing to account for missing data. MCAR is the most desirable missing data pattern because the cause of missingness is completely known, and analysis is unaffected by missingness. A common circumstance resulting in data MCAR is a planned missing design.

2. Missing at Random (MAR): Here, missing data depends on values of variables that are present in the dataset, allowing missingness to be fully described by observed variables. Missingness is thus related to a particular variable, but it is not related to the value of the variable that has missing data. Accounting for values which "cause" the missing data will produce unbiased results in an analysis (Wayman, 2003). For example, suppose a survey contains, among other items, two questions about spiders. The first question asks the

participant to rate the extent to which he considers himself to be arachnophobic, and the second question asks how frequently he encounters spiders in the workplace. If the participant answers that he is extremely afraid of spiders, it may be more likely that he will not answer the second question given the fearful subject matter. Thus, the missing value in the second variable can be accounted for by the value of the first variable. Both MCAR and MAR are considered *ignorable* missingness conditions, given the ability to perform unbiased data analyses.

3. Missing Not at Random (MNAR): Missing data of this type is *nonignorable* because the missing data depends on values which the researcher has not measured. That is, the value of the variable missing is related to the reason why it is missing. For example, a very wealthy participant may choose not to answer a question regarding income *due to* his level of income.

Structural Equation Modeling

Structural Equation Modeling (SEM) is a general statistical technique used for testing and estimating relationships among variables. Specifically, SEM aims for parsimony in applying models to datasets of interest. One of the many reasons for the widespread use of SEM as an analytic technique is its handling of missing data. SEM manages missing data via full-information maximum likelihood (FIML) estimation, making it a useful tool when missing data is present or pervasive. FIML has been shown to produce unbiased parameter estimates under conditions of ignorable missingness, in addition to outperforming other methods for managing missing data, such as listwise deletion and multiple imputation (Enders & Bandalos, 2001).

A critical feature of SEM is the use of fit indices, such as the root mean squared error of approximation (RMSEA), to gauge the extent to which a model-implied covariance matrix resembles a sample covariance matrix. As an alternative to traditional hypothesis testing measures which evaluate specific aspects of data structures, fit statistics encompass

the entirety of a model. Fit values indicate only the average fit of a model, thus collapsing numerous inconsistencies into a single measure (Kline, 2011; Steiger, 2007). Though SEM fit statistics are widely used, Davey (2005) and Hoyle (2012) report that under conditions of missingness, fit statistics become flawed, with fit bias increasing as the percentage of missing data grows. Despite this being a documented problem, to date, no solutions have been proposed.

RMSEA is one index which suffers bias in the presence of missing data. RMSEA is a parsimony-adjusted fit index, meaning that its calculation accounts for model complexity via the inclusion of the degrees of freedom (df) value. Given two models with similar fit to identical data, parsimony-adjusted fit indices favor the simpler model (Kline, 2011). In contrast to other fit indices which give an indication of goodness-of-fit, RMSEA measures badness-of-fit, where a value of zero indicates best fit, a value of .05 or lower indicates good fit, and higher values indicate poor fit (Browne & Cudeck, 1993; Hu & Bentler, 1999). RMSEA is defined as

$$RMSEA = \sqrt{\text{Max}\left(0, \frac{\chi^2 - df}{(N-1)df}\right)} \tag{1}$$

Since RMSEA theoretically follows a noncentral chi-square distribution, it is calculated as measuring the amount of noncentrality ($\chi^2 - df$) per the product, ($(N-1)df$), of degrees of freedom df and adjusted sample size ($N-1$). With no missing data, the product of degrees of freedom and adjusted sample size give a certain measure of the amount of information within the relevant dataset. An increase in missing data rates yields a decreasing χ^2 value, although sample size and degrees of freedom remain constant. With all other values unaffected by the introduction of missing data, the decreased χ^2 value in the numerator will result in a lower RMSEA value, indicating better model fit than when data were complete.

Summary of this article

In the present article, we will discuss the problem that missing data introduces into the realm of model fit estimates (particularly RMSEA) in structural equation modeling. Two possible solutions will be introduced, along with data simulations and plots used to illustrate the problem and demonstrate in what ways the proposed solutions are better than the naive RMSEA calculation. An application for ideological constructs is included to demonstrate how this correction may be applied in substantive contexts. Out of a large planned missing designed study, five individual difference questionnaires measuring various aspects of ideology are examined. The aim is twofold: assessing the extent to which proposed ideological constructs are unique and evaluating the parsimony of existing models of ideology.

Methods

Mathematical Basis of Correction

For full data, RMSEA is defined as

$$RMSEA_{naive} = \sqrt{\text{Max}\left(0, \frac{\chi^2 - df}{(N-1)df}\right)} \tag{2}$$

where df is the degrees of freedom for the model, N the sample size, and χ^2 the difference in minus two log likelihood between the model and saturated model for a specific dataset. When there is model misspecification, χ^2 will be distributed as chi-squared with df degrees of freedom, resulting in an expected value of zero and decreasing variance as N grows larger. For a model with no misspecification, however, χ^2 will grow in the order of N, and the fraction will converge to a non-zero expected value. When missing data percentages increase, the minus two log likelihood of the model and of the saturated model decrease, resulting in a decreasing χ^2 value and a successive decrease in RMSEA values as a whole. Effectively, the naive RMSEA calculation yields values unaffected by missingness in cases of no misspecification but produces lower, and thus more desirable, values under conditions

of model misspecification with missingness. In order to use RMSEA as a reliable indicator of model misfit, its values must remain constant as missingness percentage increases. To achieve this, two possible corrections are proposed.

Degrees of freedom correction. First, let p be the ratio of non-missing data points, i.e. $p = 1$ for complete data, $p = .5$ for cases where half of the data is missing, and $p = 0$ for completely missing data. With a missingness of p, χ^2 is distributed as a non-central chi-square distribution with df degrees of freedom times p (Fan & Sivo, 2005). The distribution's mean becomes $N \cdot df \cdot p$ rather than simply $N \cdot df$, as it is when no data is missing. As in the naive RMSEA calculation, we subtract the mean from χ^2 in the numerator and divide by the mean in the denominator, yielding our first proposed correction for RMSEA under missingness:

$$RMSEA_{df\ corrected} = \sqrt{\text{Max}\left(0, \frac{\chi^2 - pdf}{pNdf}\right)} \tag{3}$$

Kullback-Leibler divergence correction. The second suggestion offers a solution correcting the naive RMSEA distribution rather than the mean. Instead of using χ^2, the difference between the minus two log likelihoods for the model and saturated model, we propose using a $2N$ multiple of the Kullback-Leibler divergence between the two distributions

$$KL = \text{Trace}\left(\Sigma^{-1}S\right) + (\mu - m)^T \Sigma^{-1} (\mu - m) - k - \ln\left(\frac{|S|}{|\Sigma|}\right) \tag{4}$$

where Σ and μ represent the model distribution, S and m represent the saturated model distribution, and k is the number of variables, also known as the dimensionality of the distribution. KL divergence coincides with the χ^2 value for complete datasets but does not change with missingness in expectation if $S \neq \Sigma$ in the population. In order to obtain a term within the square root sign which remains constant as N increases, we must adapt N to pN, given our choice of using $2N$ times KL rather than the χ^2 distribution:

$$RMSEA_{KL} = \sqrt{\text{Max}\left(0, \frac{p \cdot N \cdot KL - df}{pNdf}\right)} \tag{5}$$

Due to the maximum function in the RMSEA equation, there are conditions in which $RMSEA_{KL}$ may still increase with missingness. For models with very low misspecification or datasets with high missing percentages, the second value within the square root sign may become negative. The argument of the max remains constant in expectation, but for high variance, the negative instances are cut off by the max function, resulting in a non-zero expectation of the RMSEA itself.

Design

The simulation study utilized two 2-variable structural equation models created in Onyx (von Oertzen, Brandmaier, & Tsang, 2014). In the first, used for data generation, two observed variables were created, each with variance one and covariance between them set at intervals ranging from 0.0 to .90 (Figure 1). The second model (Figure 2) is similar to the first, with two observed variables, each with freely estimated variance and a covariance value of 0.0. The second model is used to fit the simulated data.

In addition to generating raw data, simulation setup included creating a number of variables in Java and establishing a structured environment in which missing values could be simulated. All data were simulated with $N = 1000$ and 1000 trials, and missing value ratios ranged from 0.0 to .75, increasing in increments of .05, such that simulated missing value ratios were computed as $\{0.0,\ .05,\ .10,\ \cdots,\ .75\}$.

Following data generation, two types of missing data were simulated: data missing completely at random (MCAR) and data missing at random (MAR). MCAR data were simulated by randomly deleting variables by way of introducing a random number generator into the model. A uniformly distributed random real number between 0 and 1 was generated, and if that number was smaller than the missingness ratio specified in that specific trial block (where the missingness ratio ranged in value from 0 to .75), a missing value occurred. The program looped through each participant and each variable within each participant, such that any variable for any participant could be set as missing.

For data MAR, the simulation was more complicated. Because data MAR depend on values of non-missing data, missingness was introduced among variable two only. This restricted missingness ratio values to half of what they were for data MCAR, i.e. values ranged from 0.0 to .40, because singly one, rather than two, variables could have designated missing values. The way in which we modeled data MAR involved the introduction of a sigmoid function

$$f(x) = \frac{1}{(1 + e^{-x})}$$

into the simulation. For each trial, the sigmoid curve was rescaled such that it intersected the y-axis at the point of specified missingness and the y-values ranged from 0 to 1 in accordance with the possible missingness ratios. Three cases were created, depending on the missingness ratio values ($miss$) and x, the value of the first variable generated by the model for a given participant and trial number. In cases 2 and 3, another variable, $missProb$, representing the probability of a missing value in the second variable, was created.

Case 1: $miss = 0.0$, no restrictions placed on x: All data in the first variable was retained such that no missing values were present in the data.

Case 2: $miss > 0$, $x < 0$: For cases of missingness values greater than zero and negative x-values, a rescaling of the sigmoid curve was conducted such that the curve intercepted the y-axis at the $miss$ value, rather than the default y-intercept of .50, which occurs without rescaling. The curve was rescaled by multiplying $f(x)$ by the ratio between the $miss$ value and .50, as shown below

$$missProb = \frac{1}{(1 + e^{-x})} \frac{miss}{.50} \tag{6}$$

and the value of $missProb$ was set to equal the value given by the function for a particular x-value.

Case 3: $miss > 0$, $x \geq 0$: For cases of missingness values greater than zero and positive x-values, a rescaling of the sigmoid curve was once again conducted such that the

curve intercepted the y-axis at the $miss$ value, and $missProb$ values ranged from $miss$ to 1.0 for positive x-values. A complete equation for how the rescaling was performed is given below.

$$missProb = \frac{1}{(1 + e^{-x}) - \frac{.50 - miss}{1 - miss}} \frac{1 - miss}{.50} \tag{7}$$

As in the case with data MCAR, simulation values for data MAR were deleted at random according to whether a random number between 0 and 1 was less than $missProb$ for a specific participant and trial. If the number was smaller than $missProb$, then the value for data in the second variable was set to missing.

Results

Plot Descriptions

Data simulations in Java revealed consistent patterns for RMSEA calculations. RMSEA values were less related to missingness patterns (MCAR vs. MAR) and more related to covariance values between variables. For a covariance value of 0.125, MAR and MCAR data act in a similar manner, in terms of general data patterns. Similarly, MAR and MCAR data with model covariance values of 0.0 appear to be alike.

MAR, cov = .125. For data MAR with a covariance of 0.125 between variables (Figure 3), the naive RMSEA values scale linearly with missingness. For the data simulation conducted here, a missingness percentage of zero results in an RMSEA value of approximately 0.07, which indicates that the model has mediocre fit. With only 20% missingness, however, the RMSEA value drops to 0.05, indicating good fit. With an additional decline of 20% in percent of data reported, 40% missingness total, we see the RMSEA value decreasing by more than one one-hundredth, confirming the linearly declining pattern observed. The df corrected RMSEA calculation also shows a slight decline with increasing missingness, although not as drastically as naive values. A linear regression was run and the slope of the naive values is nearly 2.5 times steeper than df corrected values ($m_{naive} = -.0034$, $m_{df} = -.0014$). KL corrected RMSEA values remain

constant across all levels of missingness, with only a slight increase near 35% missingness.

MCAR, cov = .125. For data MCAR also with a covariance value of 0.125 between variables (Figure 4), we see a similar pattern as was observed with data MAR above. In calculations for MCAR data, however, the trends are even more dramatic. Note the difference in scale between data MCAR and MAR; as previously described, data MAR can have missing percentages only half of what data MCAR missing percentages can be. Naive RMSEA values continue to decline linearly as missingness percentage increases. The naive RMSEA values form a line which is over 3.5 times more steep than the df corrected value line ($m_{naive} = -.0037$, $m_{df} = -.001$). With increasing percentages of missing data, naive values reach the .05 cutoff rather quickly and continue to decline far below the original RMSEA value of approximately .07. Values for df corrected RMSEA decline and nearly reach the .05 cutoff value but do not cross the threshold into the range of RMSEA values indicating "good" fit, i.e. values $\leq .05$

As the covariance values increase, thus increasing model misspecification, the result patterns observed for covariance equal to .125 continue and become more pronounced. The trend in the data is maintained, with naive values scaling downward as percent missingness increases, df corrected values being less linearly dependent on missing percentages, and KL corrected values being almost completely independent of percent missingness. Figures 5 and 6 display results for data simulation when model covariance $cov = .90$.

MAR, MCAR, cov = 0.0. For models with no covariance between variables and thus no model misspecification, we observe no decline in RMSEA values for any of the three calculations (Figures 7 and 8). Instead, naive RMSEA values remain constant at a level of almost zero, even as missingness increass. The proposed calculation values, for both df corrected and KL corrected, increase as missingness increases. When missing percentage values reach 35-40%, both MAR and MCAR KL corrected values show an incrase in terms of RMSEA value.

Summary

Among models with some misspecification, we see a a downward bias with the Naive RMSEA calculation while the KL corrected and df corrected RMSEA calculations remain relatively constant. This trend appears among both MCAR and MAR data simulations among models with both low (cov = .125) and high (cov = .90) model misspecification.

For the models having no misspecification, an upward trend for KL and df corrected values is observed when missingness values are high, but not when they are low. This is due to a floor effect in which df and KL corrected values are found using a maximum function. This results in only positive RMSEA values being reported, skewing the mean in a positive direction. When the expression beneath the square root sign is evaluated and is thus no longer required to be positive, the floor effect becomes less pronounced, and the differences in RMSEA values between the naive, df, and KL calculations are much smaller in proportion.

Application: Ideology

Introduction to Ideology

Ideology can be understood as "an interrelated set of moral and political attitudes that possess cognitive, affective, and motivational components" (Jost, 2006; Tedin 1987). This rather non-specific definition encapsulates a wide variety of conceptions and measures or components of ideology. For example, Right-Wing Authoritarianism is an ideology variable measuring an individual's willingness to submit to authority and adhere to societal conventions and norms. Right-Wing Authoritarianism is assessed with measures such as shortened 10-item or 15-item scales which include only a subset of the original 30 items (Zakrisson, 2005; McFarland, 2005). Likewise, measures such as Social Dominance Orientation, Protestant Work Ethic, Belief in a Just World, and Need to Evaluate each assess a set of attitudes, beliefs, values or expectations that prior research suggests are distinct but related (Pratto, Sidanius, Stallworth, & Malle, 1994; Mirels & Garrett, 1971;

Dalbert, Lipkus, Sallay, & Goch 2001; Bizer et al., 2004). One organizing theme for ideology is a simple left-right political dimension (Bonanno & Jost, 2006; Jost, Nosek, & Gosling, 2008). Here, left refers to endorsing social change in order to achieve greater political, economic, or social equality, and right means supporting traditional hierarchical social order in opposition of change toward equality (Lipset, Lazarsfeld, Barton, & Linz, 1954/1962).

Across the variety of measures of ideology, there are hundreds of items each possibly assessing a unique component or quality. However, it is unknown whether each item or survey measures uniquely one thing, whether that one thing is the construct that the survey intended to measure, or whether the variety of different surveys measure the same aspects of ideology in slightly different ways. Scale developers created items intended to measure their construct of interest – Right-Wing Authoritarianism or Belief in Just World, for example. These scales and constructs were developed independently, and there is no logical reason to expect that the independent efforts of researchers resulted in the ideal selection and combination of constructs to characterize ideology. Moreover, there is no logical reason to assume that the individual items are more similar to the other items in that scale than to items in other scales.

The purpose of this investigation is to test whether ideology scales can be refactored into a more efficient or effective set of constructs. Can we collapse across questionnaires to conclude there are fewer core ideological facets than the number of measures would suggest? How parsimonious is the structure of ideology? Structural equation modeling is used to begin to answer these questions, and RMSEA values help evaluate the effectiveness of competing models. Due to computing resource limitations, this project takes only the first step in this refactoring effort by comparing three models of ideology within a specified subset of data. Five individual difference questionnaires are selected for analysis, and one-factor, two-factor, and five-factor solutions for this data are compared to assess ideological parsimony and examine model fit in the presence of missing data.

Methods

Ideology 1.0 is an extremely large, multivariate, planned missing data design. In total, there are over 100 ideology measures, including ∼40 implicit items and ∼30 explicit self-report items matching topics measured implicitly, 25 individual difference questionnaires, and dozens of additional individual self-report items. For any given session, a participant was assigned a random selection of ideology measures, typically requiring 15 minutes to complete. Participants were allowed to complete as many sessions as they wished, with no guarantee that they would not encounter measures already seen in a previous session. By requiring participants to complete only a small subset of all ideology measures, we decrease the likelihood of participant attrition, due to the shorter study time. In fact, it would not be plausible to expect participants to complete all ideology measures, given the vast number of items assessed in the study.

Planned Missingness. The planned missing design of Ideology 1.0 allows for assessment of significantly more measures than would be possible using a traditional study design in which all participants complete all measures. Graham, Hofer, and MacKinnon (1996) illustrate that planned missing designs permit the collection of more survey items with fewer participants, thus reducing both participant burden and cost. Planned missing designs are useful in a variety of contexts, including longitudinal studies, studies where there are considerable numbers of measures, such as here, or studies that involve expensive data collection methods (Enders, 2010). Because planned missing designs use random assignment to conditions, the missing data pattern is MCAR, allowing for unbiased analyses of results.

Individual difference questionnaires. In order to begin to answer the question of the parsimony of ideology in a manageable fashion, I began the investigation by examining five individual difference questionnaires (IDQs) that I hypothesized might be related in some way. The questionnaires and a brief description of each are listed below.

Social dominance orientation. Social Dominance Orientation (SDO) is a construct rooted in social dominance theory, which accounts for psychological and social forces contributing to the formation and maintenance of social hierarchies (Sidanius, Pratto, Martin, & Stallworth, 1991). SDO is related to the nature of group relations, specifically, group-based inequality and prejudice and measures the extent to which an individual desires his in-group to dominate and be superior to out-groups. The SDO scale has 16 items, and confirmatory factor analysis suggests that all items represent one latent construct (Pratto et al., 1994). Sample item: *If certain groups stayed in their place, we would have fewer problems.*

Protestant work ethic. Protestant Work Ethic (PWE) finds its origins in religion, emphasizing hard work, frugality, and diligence as a display of salvation in the Christian faith. Mirels and Garrett (1971) view PWE as an ascetic dispositional variable and find PWE scores correlate with certain occupational interests. Typically, occupations with interest patterns positively correlated with PWE scores place emphasis on adherence to traditional role-appropriate behavior and make few demands on an individual's emotional sensitivity or capacity for imagination. In fact, occupations which value and encourage innovation and creativity are negatively correlated with PWE score. Katz and Hass (1988) found significant positive correlations between anti-black sentiments and PWE scores, suggesting that there may be a correlation between PWE and SDO scales. The PWE scale has 11 items, and unrotated principal component analysis confirms the notion that a single, general component underlies PWE (Mirels & Garrett, 1971). Sample item: *A distaste for hard work usually reflects a weakness of character.*

Belief in a just world. Belief in a Just World (BJW) is the concept that good things happen to good people and bad things happen to bad people. Individuals are motivated to believe they they live in a world that is just and where individuals get what they deserve. This notion is so important that individuals will take actions to correct injustice or psychologically restore justice, such as engaging in victim-blaming (Dalbert et

al., 2001). Lerner and Miller (1978) postulate that this belief serves an adaptive role in enabling individuals to confront their world as if it were secure and orderly and that without this belief, it would be difficult to justify the commitment to long-range goals. BJW and religiosity are significantly correlated, suggesting people may seek justice in the form of believing in an omniscient and almighty god (Dalbert et al., 2001). The BJW scale has six items and factor analysis supports a two-factor model of belief in a just versus unjust world, suggesting BJW can be represented by a single construct. Sample item: *Justice always prevails over injustice.*

Need to evaluate. Need to Evaluate (NE) measures the extent to which an individual spontaneously engages in evaluative responding, such as rating objects or experiences as good or bad (Jarvis & Petty, 1996). People high on NE frequently have many evaluative thoughts and hold attitudes toward a wide variety of objects, whereas those low on NE take part in evaluation to a lesser extent and thus do not hold as many attitudes (Bizer et al., 2004). Differences in NE may be related to core religious or moral beliefs which dictate the extent to which an individual assesses or judges his circumstances and forms corresponding opinions. Whether an individual has a strictly defined moral compass may be reflected in how frequently he engages in evaluative thought, suggesting NE may be related to PWE. The NE scale has three items but was combined with the BJW scale for analysis, resulting in a BJWEval composite scale having nine items. Sample NE item: *Compared to the average person, do you have fewer opinions about whether things are good or bad, about the same number of opinions, or more opinions?*

Right-wing authoritarianism. Right-Wing Authoritarianism (RWA) was developed as a refinement of the authoritarian personality theory which emphasizes an absolute deference to authority, frequently through oppression of one's subordinates (Altemeyer, 1981). RWA was created through Altemeyer's observation that three aspects of authoritarianism strongly covaried – conventionalism, authoritarian aggression, and authoritarian submission – thus forming a distinct, new social attitude dimension. RWA

and SDO are consistently related, both predicting various forms of prejudice, ethnocentrism, and homophobia (Pratto et al., 1994). The difference between RWA and SDO, however, lay in the fact that RWA focuses on in-group deference to authority, whereas SDO focuses on dominance over out-groups. In this way, RWA can be understood to be an intra-group phenomenon and SDO an inter-group phenomenon (Zakrisson, 2005). RWA and PWE have also been shown to correlate, both emphasizing submission and obedience (Mirels & Garrett, 1971). Original RWA scales have 30 items, but two reduced-item scales have been used as reliable measures of RWA. Zakrisson (2005) created a 15-item scale (RWAZ) which included shortened and less extreme versions of original RWA questions. Sample item: *Our country needs a powerful leader, in order to destroy the radical and immoral currents prevailing in society today.* A second shortened scale (RWAM), created and used by McFarland (2005), has 10 items. Sample item: *Our country will be destroyed someday if we do not smash the perversions eating away at our moral fiber and traditional beliefs.* The RWAZ and RWAM scales were not combined and were included in the analysis as separate IDQs.

Missing percentages. Ideology 1.0 has 321,701 participants total and 107,883 participants who completed any item of one or more of the five individual difference questionnaires of interest. Data cleaning involved isolating cases of missingness and removing participants who had missing values for all variables of interest. For the relevant dataset, that is, the data set for which only variables corresponding to the five IDQs were retained, the cumulative rate of missingness across all variables was 91.4%. The individual datasets for each IDQ had lower percentages of missingness (SDO: 75.1%, BJWEval: 55.8%, PWE: 63.7%, RWAZ: 73.6%, RWAM: 60.2%).

KL correction implementation. A beta version of the KL corrected RMSEA calculation is currently implemented as an estimate option in the developer mode of Onyx, an SEM graphical user interface program. Onyx is written in Java and relies on Java's computational abilities to run user-specified analyses and presents estimates in an

easy-to-read format following model convergence (von Oertzen et al., 2014). Unfortunately, due to the large size of the ideology dataset, factor models of interest as specified in Onyx, namely the proposed one, two, and five-factor solutions for the subset of ideological data, were unable to converge in Java. Thus, $RMSEA_{KL}$ values are unavailable. This is a problem that is receiving further attention and is likely to be remedied soon.

Results

Before combining IDQs and performing confirmatory factor analyses (CFA) for the entire dataset, a one-factor CFA for each IDQ was performed using the *lavaan* package in R (Rosseel, 2012). The individual CFAs were used as verification that each IDQ represented a single construct. For each IDQ scale, the model was specified such that all questionnaire items load onto one latent factor representing the IDQ as a whole. Model fit was assessed via the use of two indices: Comparative Fit Index (CFI) and RMSEA. Hu and Bentler (1990) provide guidelines for CFI cutoff scores: values above .95 indicate great model fit, values of .90-.95 are considered adequate, and values below .90 suggest poor fit.

One-factor analysis for SDO revealed fair fit (CFI = .80, $RSMEA_{naive}$ = .039) with moderate factor loadings, ranging from .412 to .734 in absolute value and having an average absolute mean of .599. One-factor analysis for BJWEval revealed poor fit (CFI = .78, $RSMEA_{naive}$ = .057) with low factor loadings, ranging from .020 to .666 in absolute value and having an average absolute mean of .384. One-factor analysis for PWE revealed excellent fit (CFI = .98, $RSMEA_{naive}$ = .041) with high factor loadings, ranging from .535 to .994 in absolute value and having an average absolute mean of .849. One-factor analysis for RWAZ revealed adequate fit (CFI = .91, $RSMEA_{naive}$ = .048) with moderate factor loadings, ranging from .371 to .993 in absolute value and having an average absolute mean of .711. One-factor analysis for RWAM revealed poor fit (CFI = .79, $RSMEA_{naive}$ = .070) with low factor loadings, ranging from .396 to .787 in absolute value and having an average absolute mean of .592. Refer to tables 1 through 5 for individual factor loadings.

All KL corrected RMSEA values for individual one-factor CFAs were not obtainable as a result of time and computating power constraints. By chance, one model in this set of analyses appeared to converge, although the estimates given by the model indicate that a computation error likely occurred. Onyx estimated $RMSEA_{KL}$ to be .55 in the one-factor solution for BJWEval. This is almost a ten-fold increase in the RMSEA estimate when compared to the naive estimate, $RMSEA_{naive} = .057$. An RMSEA value of .55 is extremely high, especially considering the fact that previous studies have suggested that BJW and NE correlate and have confirmed that one factor models fit both scales with good model fit (Dalbert et al., 2001; Bizer et al., 2004). Based on findings in the current study and past literature, it is reasonable to assume that the $RMSEA_{KL}$ result given by Onyx is inaccurate.

Following the individual CFAs, three additional factor analyses were performed to gauge the extent to which the five chosen IDQs were related. The additional CFAs also assessed whether ideology, as described by these IDQs, could be represented in a more parsimonious manner by collapsing constructs across multiple measures. Because the dataset is so large and 61 variables were included in each model, a small subset of the data was used for these inclusive models. By random selection, 5000 participants were chosen from the clean dataset of interest, in which all participants had data for at least one of the five chosen IDQs.

In the one-factor analysis, variables representing items from all IDQs were modeled as loading onto a single factor named 'IDEO'. Results for this analysis are conflicting, with CFI indicating extremely poor fit and RMSEA indicating excellent fit (CFI = .748, $RSMEA_{naive} = .011$). Factor loadings are low, ranging from .047 to .751 in absolute value and having an average absolute mean of .507. For the five-factor analysis, each of the five IDQs was represented as a single factor with all corresponding item variables loading onto that factor. Results for this analysis are also conflicting, with CFI indicating poor fit and RMSEA indicating excellent fit (CFI = .874, $RSMEA_{naive} = .008$). Average absolute

factor loadings are moderately low, ranging from .415 to .598 (SDO: .598, BJWEval: .415, PWE: .536, RWAZ .552, RWAM: .571). In addition to assessing model fit via one and five factor solutions, a two factor solution was proposed. In this model, one factor, called 'BEL' for belief, represented PWE and BJWEval items and the other factor, named 'AUTH' for authority, represented SDO, RWAZ, and RWAM items. Results for this analysis are conflicting too, with CFI indicating poor fit and RMSEA indicating excellent fit (CFI = .820, $RSMEA_{naive}$ = .009). Average absolute factor loadings are moderately low (BEL: .476, AUTH: .556). Tables 6, 7, and 8 list individual factor loadings for the combined CFAs.

Discussion

In accordance with initial simulation results, factor analyses for the five IDQs of interest suggest uncorrected RMSEA estimates conclude good model fit, but in the presence of missing data, these values are artificially low. This is illustrated best by the disparity between CFI and $RMSEA_{naive}$ estimates for the one, two, and five-factor models that include variables from all five IDQs. The high missingness ratio present within the dataset at large (91.4%) is likely the cause of this inconsistency across fit indices. Although KL corrected RMSEA values are unavailable for these models, CFI results provide a sensible standard against which naive RMSEA values can be compared. Additionally, given the exceptionally high $RMSEA_{KL}$ value yielded by Onyx in the one-factor analysis of BJWEval, it is unlikely that useful insight could have been gleaned from further imprecise $RMSEA_{KL}$ values, even if the factor models in Onyx had converged.

Although this analysis is nowhere near comprehensive, it is not unreasonable to suggest that ideology may be more parsimonious than previously thought. $RMSEA_{naive}$ and CFI results show that a two-factor solution is comparable to a five-factor solution in its ability to describe the ideological data. As mentioned earlier, structural equation modeling seeks parsimony in describing data; therefore, a more parsimonious solution is preferred when differing models have similar fit estimates for the same dataset. Favoring a

two-factor solution for the structure of ideology in the context of this project is evidence that begins to support the notion that fewer factors may underlie ideology than the substantial number of measures indicate.

In addition to the absence of reliable RMSEA estimates, one of the largest limitations for these conclusions is the fact that only 5000 participants' data were used for the modeling of the one, two, and five factor solutions. The complete dataset is composed of over 100,000 participants, and therefore it is likely that this small sample, although randomly chosen, is not representative of the data at large. With less stringent time and computational resource limitations, a more comprehensive analysis can be performed. Additional limitations include restricting analysis to five IDQs and conducting a limited number of CFAs without extensive exploratory factor analysis (EFA) first. Conducting EFAs would have allowed for better informed models and guided additional solutions.

In the future it would be valuable to extend analysis to remaining IDQs and other ideology measures, such as implicit items and various explicit questions. This would allow for a more complete examination of ideology and provide a more robust answer to the question of ideological parsimony. Additionally, further investigation into the theoretical similarities of ideology measures is a necessary step in identifying which scales or items may be related and thus may yield ideology amenable to a collapsing of constructs. Identifying groups of related measures by means of investigation into the literature roots possible solutions in empirical findings.

General Discussion

Reason for correction

Even though RMSEA values are unaffected by missingness in cases of perfect model specification, no misspecification is not the condition for which RMSEA was constructed. The rationale behind RMSEA as a badness-of-fit index rests in the assumption that all models have some level of misspecification, a necessary condition since simplification is an

essential aspect of modeling; RMSEA allows for the evaluation of the degree of misspecification present within a model. With model misspecification and in the presence of missing data, the model shifts from having a normal distribution to a mixture of normal distributions defined by missingness patterns. Under these conditions, RMSEA is not guaranteed to be constant and data simulations confirm the linear dependence of RMSEA values on missingness percentages. Consequently, RMSEA must be corrected in cases of missing data.

Limitations and future directions

Although the data simulations strongly suggest that KL corrected RMSEA values are unaffected by missingness and provide a better indicator of model fit in the presence of missing data than do naive RMSEA or df corrected RMSEA values, a definite conclusion cannot be reached in the absence of further investigation. The models used to simulate and fit data in this paper were simple, having only two observed variables. In most cases, these models are vastly oversimplified and do not adequately represent more complicated datasets. Increasing the complexity of data simulation models and reassessing RMSEA trends will allow for broader conclusions to be made about the effectiveness of KL corrected RMSEA as a modified fit index. Additionally, all RMSEA corrections cause problems for the null, where the model is perfectly specified. Despite the fact that, as described above, RMSEA was not created for scenarios in which no misspecification is present, it is possible to argue that the proposed corrections are not suitable to use in all possible cases of model specification.

A possible objection to implementing the KL correction for RMSEA lay in the realization that, in presence of missingness, corrected RMSEA values indicate worse model fit than naive RMSEA values. Though the rationale for an RMSEA correction is rooted in theory and practice, it may be hard to accept that naive RMSEA values imply artificially good fit for models with missingness. Accepting the KL correction will force researchers to

reassess model fit, and there may be resistance if KL corrected RMSEA values point to a different conclusion than naive RMSEA values had indicated previously. Changing the standard en masse for RMSEA calculations may lessen resistance to such a change by creating an environment in which higher RMSEA values are acceptable and expected. RMSEA values may then become comparable across different levels of missingness, and preference will no longer be given to models where missing data rates are high.

Informing researchers about the benefits of an RMSEA correction and its ability to handle missing data may encourage the use of planned missing data designs, which are useful in a variety of contexts. In order to do this, it is important to provide the scientific community easy access to the correction. A feasible solution is writing a package for R which includes wrapper tools that will extract data from existing functions and calculate RMSEA corrected values. Although likely more difficult, integrating the KL correction into existing data software tools, such as SPSS and SAS, or popular R packages, such as *lavaan* and *psych*, would be useful for encouraging and supporting widespread use of corrected RMSEA estimates.

Conclusion

Missing data is pervasive in all areas of research, and given the frequency with which it occurs, it is necessary to understand how missingness operates in all stages of the research process. In this paper, both simulated and experimental data show the effect that missing data has on research outcomes, particularly in the way that RMSEA estimates change as missingness percentages increase. The Kullback-Leiber divergence is used to propose a correction to the problem of RMSEA's downward bias with missing data, and simulations confirmed that the proposed $RMSEA_{KL}$ correction remains constant across varying levels of misspecification and missing ratios. Although computational constraints and high missingness percentages limited the conclusions that could be reached for the applied dataset, support was found to suggest that ideology may be amenable to a

refactoring of constructs. More research must be done to verify these findings, but the fact that missing data plays a substantial role in research cannot be denied.

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Item	SDO
sdo1	1.00
sdo2	.412
sdo3	.628
sdo4	.402
sdo5	.569
sdo6	.700
sdo7	.610
sdo8	.552
sdo9	-.707
sdo10	-.697
sdo11	-.589
sdo12	-.720
sdo13	-.734
sdo14	-.650
sdo15	-.513
sdo16	-.499

Table 1

Standardized loadings for SDO one-factor CFA

Item	BJWEval
bjw1	1.00
bjw2	.479
bjw3	.459
bjw4	.666
bjw5	.623
bjw6	.611
eval1	-.113
eval2	-.106
eval3	.020

Table 2

Standardized loadings for BJWEval one-factor CFA

Item	PWE
pwe1	1.00
pwe2	.980
pwe3	.754
pwe4	.986
pwe5	.993
pwe6	.890
pwe7	.684
pwe8	.823
pwe9	.994
pwe10	.535
pwe11	.855

Table 3

Standardized loadings for PWE one-factor CFA

Item	RWAZ
rwaz1	1.00
rwaz2	.962
rwaz3	.700
rwaz4	-.616
rwaz5	.987
rwaz6	-.525
rwaz7	.694
rwaz8	-.371
rwaz9	.653
rwaz10	-.659
rwaz11	.717
rwaz12	-.613
rwaz13	.993
rwaz14	-.477
rwaz15	.986

Table 4

Standardized loadings for RWAZ one-factor CFA

Item	RWAM
rwam1	1.00
rwam2	-.396
rwam3	.787
rwam4	-.517
rwam5	-.474
rwam6	.783
rwam7	-.512
rwam8	.710
rwam9	-.502
rwam10	.648

Table 5

Standardized loadings for RWAM one-factor CFA

Item	IDEO	Item	IDEO	Item	IDEO	Item	IDEO	Item	IDEO
sdo1	1.00	bjw1	.625	pwe1	.425	rwaz1	.534	rwam1	0.616
sdo2	.334	bjw2	.508	pwe2	.432	rwaz2	-.539	rwam2	-0.294
sdo3	.556	bjw3	.468	pwe3	.431	rwaz3	.611	rwam3	0.732
sdo4	.358	bjw4	.570	pwe4	.677	rwaz4	-.449	rwam4	-0.525
sdo5	.606	bjw5	.578	pwe5	.544	rwaz5	.641	rwam5	-0.327
sdo6	.705	bjw6	.562	pwe6	.683	rwaz6	-.299	rwam6	0.751
sdo7	.655	eval1	-.146	pwe7	.319	rwaz7	.516	rwam7	-0.432
sdo8	.494	eval2	-.154	pwe8	.426	rwaz8	-.222	rwam8	0.724
sdo9	-.584	eval3	.047	pwe9	.550	rwaz9	.552	rwam9	-0.441
sdo10	-.569			pwe10	.334	rwaz10	-.472	rwam10	.700
sdo11	-.463			pwe11	.577	rwaz11	.648		
sdo12	-.711					rwaz12	-.523		
sdo13	-.657					rwaz13	.750		
sdo14	-.635					rwaz14	-.480		
sdo15	-.451					rwaz15	.560		
sdo16	-.496								

Table 6

Standardized loadings for one-factor CFA, where the single ideology factor is named IDEO

Item	SDO	Item	BJWEval	Item	PWE	Item	RWAZ	Item	RWAM
sdo1	1.00	bjw1	1.00	pwe1	1.00	rwaz1	1.00	rwam1	1.00
sdo2	.382	bjw2	.572	pwe2	.431	rwaz2	-.518	rwam2	-.333
sdo3	.551	bjw3	.522	pwe3	.438	rwaz3	.676	rwam3	.779
sdo4	.353	bjw4	.651	pwe4	.752	rwaz4	-.465	rwam4	-.550
sdo5	.566	bjw5	.645	pwe5	.624	rwaz5	.754	rwam5	-.339
sdo6	.765	bjw6	.636	pwe6	.768	rwaz6	-.304	rwam6	.765
sdo7	.622	eval1	-.118	pwe7	.312	rwaz7	.605	rwam7	-.465
sdo8	.463	eval2	-.145	pwe8	.468	rwaz8	-.253	rwam8	.709
sdo9	-.696	eval3	.032	pwe9	.625	rwaz9	.522	rwam9	-.493
sdo10	-.715			pwe10	.335	rwaz10	-.538	rwam10	.709
sdo11	-.514			pwe11	.606	rwaz11	.660		
sdo12	-.815					rwaz12	-.564		
sdo13	-.729					rwaz13	.774		
sdo14	-.700					rwaz14	-.493		
sdo15	-.537					rwaz15	.598		
sdo16	-.562								

Table 7

Standardized loadings for five-factor CFA, where the factors are named according to scale name: SDO, BJWEval, PWE, RWAZ, and RWAM

Item	BEL	Item	BEL	Item	AUTH	Item	AUTH	Item	AUTH
bjw1	1.00	pwe1	.424	sdo1	1.00	rwaz1	.546	rwam1	.641
bjw2	.575	pwe2	.426	sdo2	.378	rwaz2	-.540	rwam2	-.316
bjw3	.519	pwe3	.437	sdo3	.571	rwaz3	.635	rwam3	.747
bjw4	.635	pwe4	.738	sdo4	.360	rwaz4	.464	rwam4	-.541
bjw5	.649	pwe5	.619	sdo5	.632	rwaz5	.683	rwam5	-.332
bjw6	.639	pwe6	.756	sdo6	.724	rwaz6	-.316	rwam6	.759
eval1	-.116	pwe7	.307	sdo7	.649	rwaz7	.533	rwam7	-.463
eval2	-.148	pwe8	.465	sdo8	.494	rwaz8	-.249	rwam8	.710
eval3.	.042	pwe9	.620	sdo9	-.609	rwaz9	.531	rwam9	-.461
		pwe10	.332	sdo10	-.605	rwaz10	-.501	rwam10	.698
		pwe11	.606	sdo11	-.456	rwaz11	.642		
				sdo12	-.722	rwaz12	-.557		
				sdo13	-.676	rwaz13	.753		
				sdo14	-.653	rwaz14	-.516		
				sdo15	-.465	rwaz15	.560		
				sdo16	-.532				

Table 8

Standardized loadings for two-factor CFA, where the factors are named BEL for belief (PWE and BJWEval) and AUTH for authority (SDO, RWAZ, RWAM)

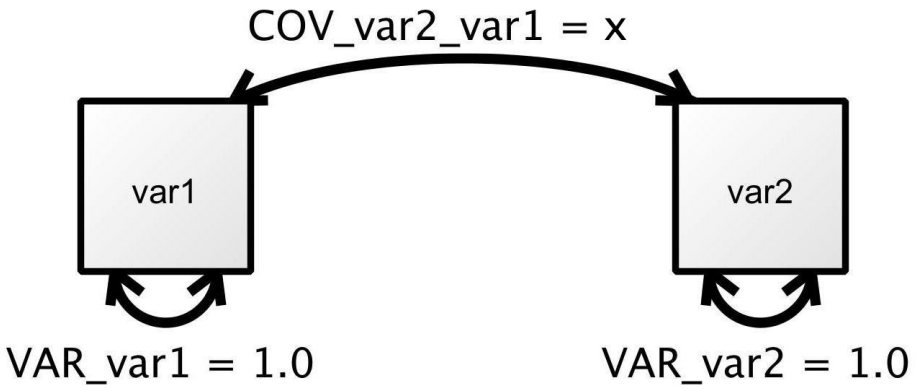


Figure 1. Model used for data simulation; $x = \{0.0,\ .125, \cdots,\ .90\}$

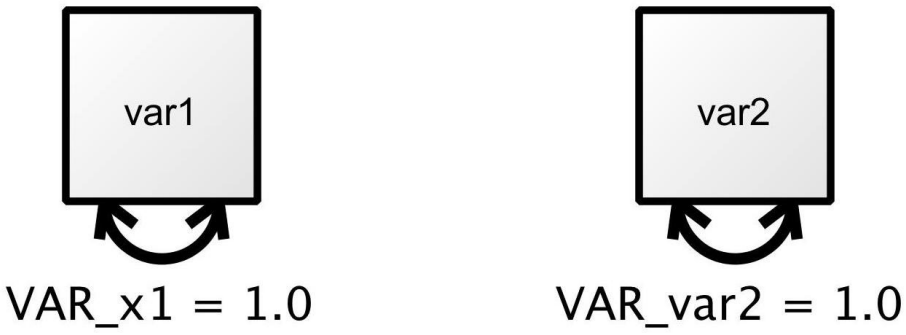


Figure 2. Model used to fit data

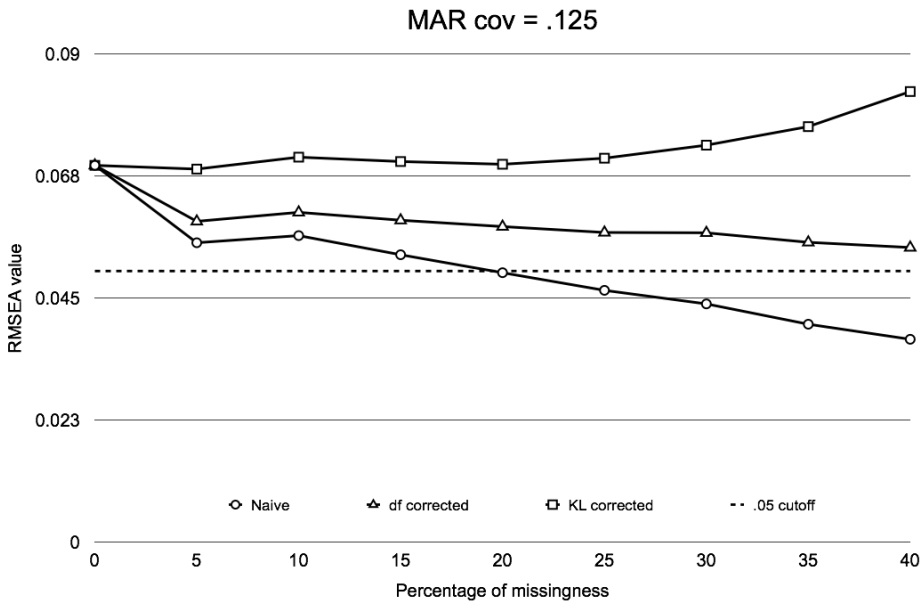

MAR cov = .125
0.09
0.068
0.045
0.023
0
RMSEA value
Naive
df corrected
KL corrected
.05 cutoff
0
5
10
15
20
25
30
35
40
Percentage of missingness

Figure 3

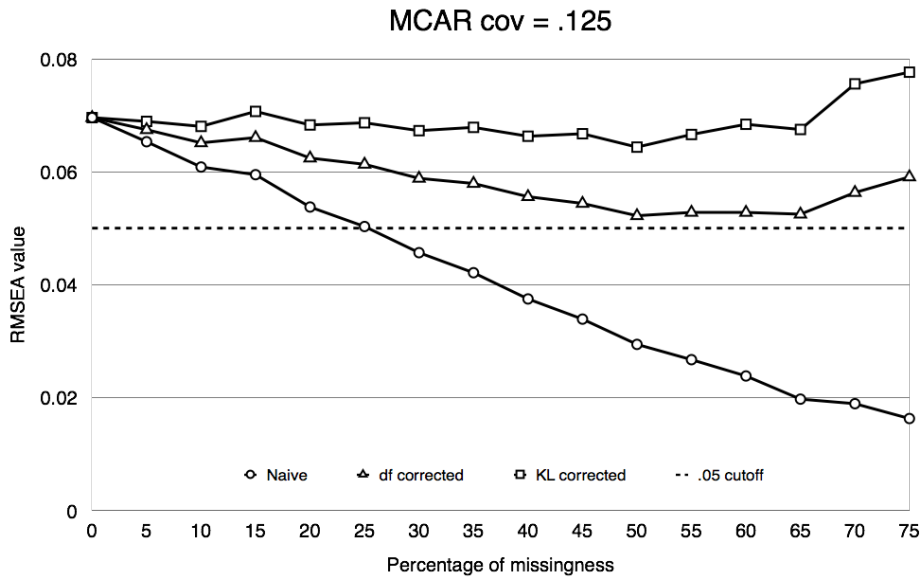

MCAR cov = .125
RMSEA value
0.08
0.06
0.04
0.02
0
Naive
df corrected
KL corrected
.05 cutoff
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
Percentage of missingness

Figure 4

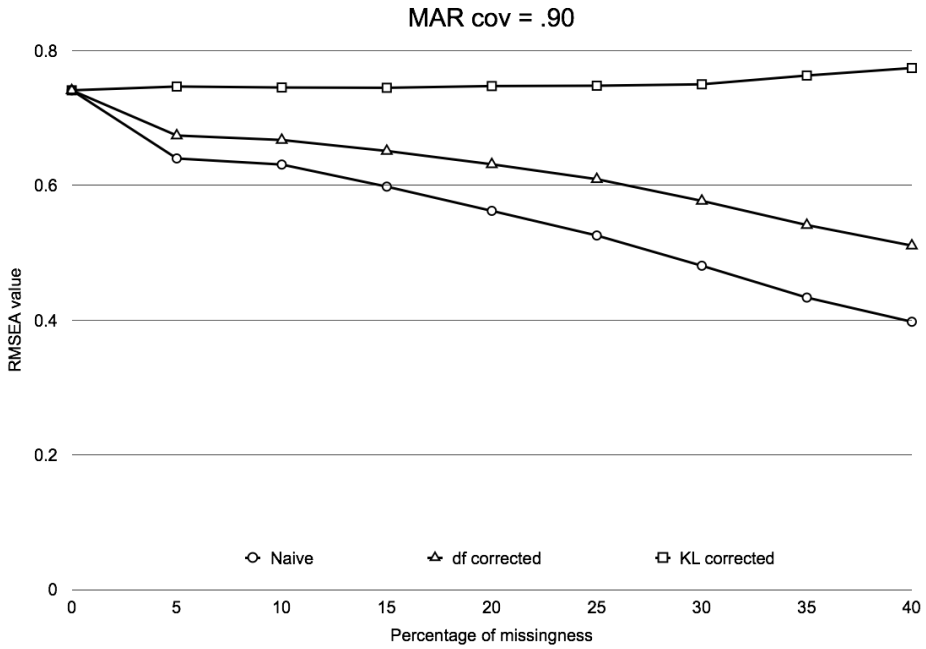

MAR cov = .90
0.8
0.6
0.4
0.2
0
RMSEA value
Naive
df corrected
KL corrected
0
5
10
15
20
25
30
35
40
Percentage of missingness

Figure 5

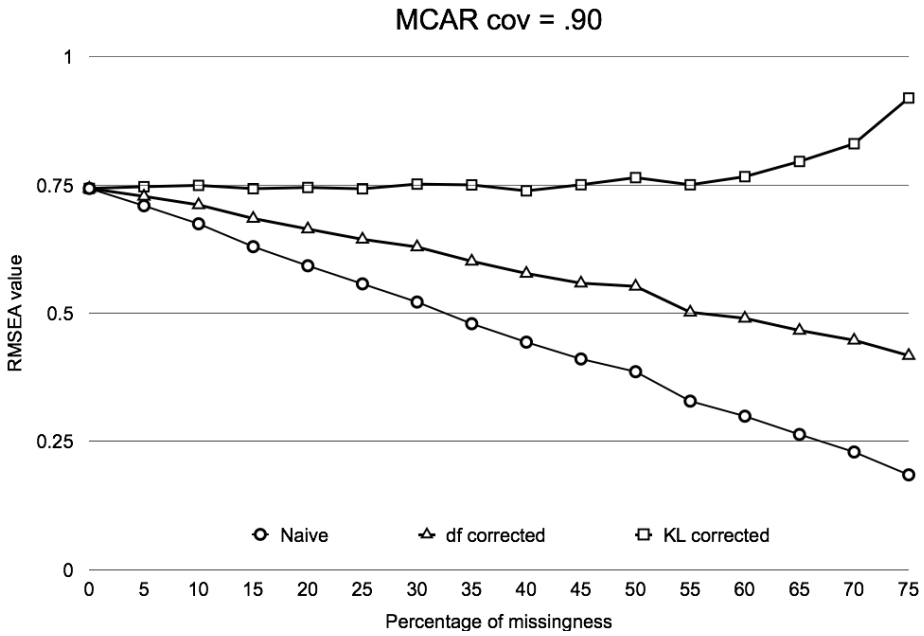
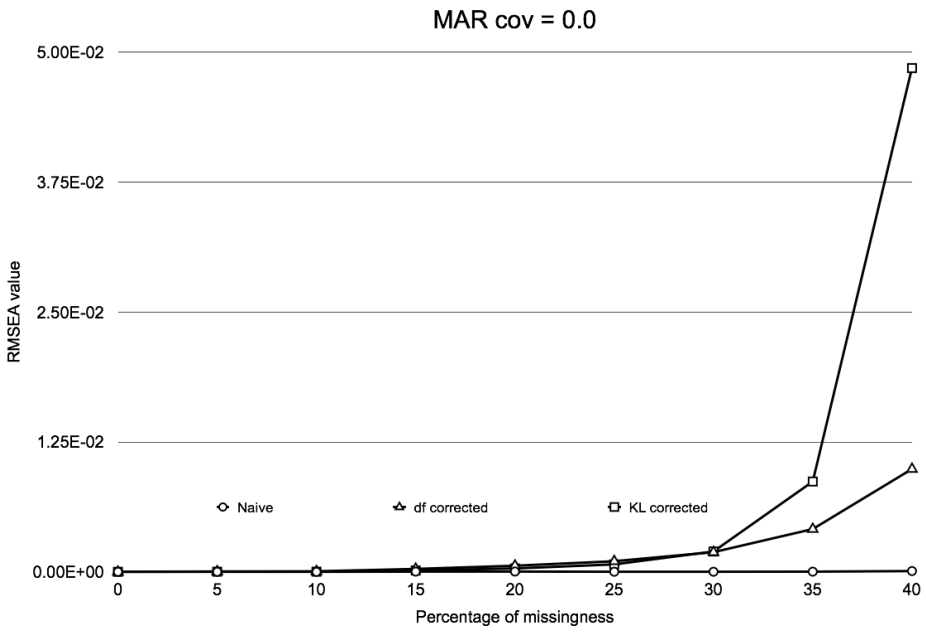

MCAR cov = .90
1
0.75
0.5
0.25
0
RMSEA value
Naive
df corrected
KL corrected
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
Percentage of missingness

Figure 6


MAR cov = 0.0
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3.75E-02
2.50E-02
1.25E-02
0.00E+00
RMSEA value
Naive
df corrected
KL corrected
0
5
10
15
20
25
30
35
40
Percentage of missingness

Figure 7

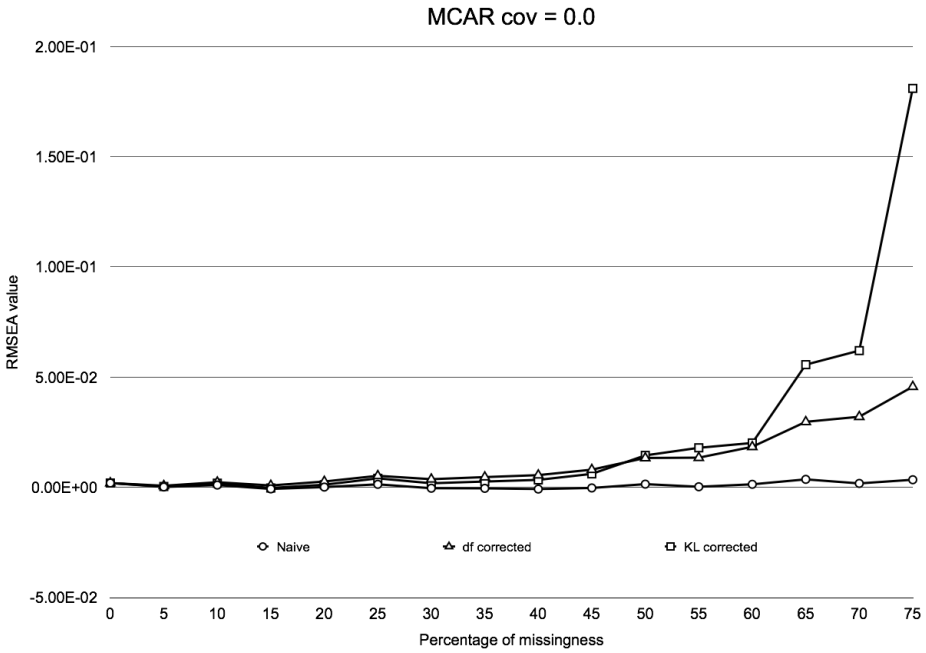
MCAR cov = 0.0
2.00E-01
1.50E-01
1.00E-01
5.00E-02
0.00E+00
-5.00E-02
RMSEA value
Naive
df corrected
KL corrected
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
Percentage of missingness

Figure 8