Trade Flows, Multilateral Resistance, and Firm Heterogeneity

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A Technical appendix

A.1 Theory

A.1.1 Derivation of gravity equation

HMR's set-up yield imports by country i from country j of:

$$M_{ij} = \left(\frac{c_j t_{ij}}{\alpha P_i}\right)^{1-\sigma} N_j Y_i V_{ij}, \tag{1}$$

(cf. HMR equation (6)). Assuming trade balance yields $Y_j = \sum_{i \in I_j} M_{ij}$, such that summing both sides of HMR's expression gives:

$$Y_j = \left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j \sum_{i \in I_j} \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij} \Rightarrow \left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j = \frac{Y_j}{\sum_{i \in I_j} \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij}} \tag{2}$$

Using this expression to substitute factor costs from equation (1) then yields bilateral imports of:

$$M_{ij} = Y_i Y_j \frac{1}{\sum_{i \in I_j} \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij}} \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} V_{ij}.$$

Dividing and multiplying by Y^{I_j}, the combined incomes of all importers from country j, $Y^{I_j} \equiv \sum_{i \in I_j} Y_i$, then gives:

$$M_{ij} = \frac{Y_i Y_j}{Y^{I_j}} \left(\frac{t_{ij}}{P_i \widehat{P}_j}\right)^{1-\sigma} V_{ij}, \tag{3}$$

where

$$\widehat{P}_j^{1-\sigma} \equiv \sum_{i \in I_j} \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} s_i^{I_j} V_{ij} \tag{4}$$

is outward multilateral resistance, in which we define $s_i^{I_j} \equiv Y_i / Y^{I_j}$. The expression for P_i, or inward multilateral resistance, follows from the definition of the CES price index for consumers in country i. This is given by:

$$P_i^{1-\sigma} = \int_{\gamma \in \Gamma_i} p_i(\gamma)^{1-\sigma} d\gamma$$

(cf. HMR equation 2) where $p_i(\gamma)$ is the price faced by consumers in i of product variety γ in the set of products consumed in i, Γ_i. By monopolistic competition, firms set prices such that $p_i(a) = t_{ij} c_j a / \alpha$, where a is the (inverse) productivity draw of firm of type a, and $\alpha \equiv 1 - (1/\sigma)$ is a transformation of the elasticity of substitution, capturing the mark-up. Using this in the price index for all import origins yields:

$$P_i^{1-\sigma} = \sum_{j \in J_i} \left(\frac{t_{ij} c_j}{\alpha}\right)^{1-\sigma} N_j V_{ij}$$

since each exporter j has measure N_j firms and where $V_{ij} \equiv \int_{a_L}^{a_{ij}} a^{1-\sigma} dG(a)$, in which a_L is the lower bound on the support of (inverse) productivity draws, which has c.d.f. $G(a)$. Using equation (2) then gives $P_i^{1-\sigma} = \sum_{j \in J_i} \left(t_{ij} / \widehat{P}_j\right)^{1-\sigma} s_j^{J_i} r_{ij} V_{ij}$, where $s_j^{J_i} \equiv Y_j / Y^{J_i}$ and where $r_{ij} \equiv Y^{J_i} / Y^{I_j}$. The latter is

a scaling factor that increases as the collective size of exporters to i grows larger relative to the size of importers from j. Recall that a higher P_i encourages more trade between i and j, as it reflects higher inward multilateral resistance. When r_{ij} is high on average, P_i tends to be low, discouraging exports from j to i. Since $r_{ij} \equiv \left(\sum_{j\in J_i} Y_j\right) / \left(\sum_{i\in I_j} Y_i\right)$ is a constant for pair $i-j$ when the sets of active traders $\{I_j, J_i\}$ are constant, and we hold these sets constant in our comparative statics (justified by the empirical result that little of the expansion of trade is due to country entry – see HMR and Table 5 of the text), we can discard it for comparative statics purposes, and use:

$$P_i^{1-\sigma} = \sum_{j\in J_i} \left(\frac{t_{ij}}{\widehat{P}_j}\right)^{1-\sigma} s_j^{J_i} V_{ij}, \tag{5}$$

Equations (3), (5) and (4) correspond to equations (1), (2) and (3) in the text.

The gravity equation for the extensive margin (equation 4 in the text) is derived as follows. As in HMR, a firm in country j producing variety γ faces demand of $x_j(\gamma) = \left(Y_j/P_j^{1-\sigma}\right) p_j(\gamma)^{-\sigma}$. A firm with (inverse) productivity a in country j serving consumers in i has a cost function $C_{ij}(a) = at_{ij}c_j x_i(\gamma) + c_j f_{ij}$. Using the firm's profit-maximising resulting price $p_i(a) = t_{ij}c_j a/\alpha$, the profit for a firm with productivity a exporting from j to i is:

$$\pi_{ij}(a) = (1-\alpha)\left(\frac{t_{ij}c_j a}{\alpha P_i}\right)^{1-\sigma} Y_i - c_j f_{ij}.$$

Now define a cut-off level of (inverse) productivity a_{ij} such that a firm with this productivity makes zero profit. The cut-off is given by:

$$a_{ij} = \left[\frac{(1-\alpha)\, Y_i}{c_j f_{ij}}\right]^{\frac{1}{\sigma-1}} \frac{\alpha P_i}{t_{ij}c_j},$$

(see HMR equation 4). Next, use (2) above to write

$$c_j = \alpha\left(\frac{Y^{I_j} N_j \widehat{P}_j^{1-\sigma}}{Y_j}\right)^{\frac{1}{\sigma-1}},$$

such that the cut-off can be written as:

$$a_{ij}^{\sigma-1} = \frac{(1-\alpha)}{N_j}\frac{Y_i Y_j}{Y^{I_j}}\frac{1}{f_{ij}}\left(\frac{t_{ij}}{P_i\widehat{P}_j}\right)^{1-\sigma},$$

which is the gravity equation for the extensive margin given by equation 4 in the text.

A.1.2 Proof of Lemma 1: Effect of country size on extensive margin elasticity

Proof. For $h = i, j$,

$$\begin{aligned}
\left.\frac{\partial\varphi_{ij}}{\partial y_h}\right|_{P_i,\hat{P}_j} &= \left.\frac{\partial}{\partial y_h}\frac{\delta e^{\delta z_{ij}}}{e^{\delta z_{ij}}-1}\right|_{P_i,\hat{P}_j} \\
&= \frac{\delta e^{\delta z_{ij}}}{e^{\delta z_{ij}}-1}\left(\delta - \frac{\delta e^{\delta z_{ij}}}{e^{\delta z_{ij}}-1}\right)\left.\frac{\partial z_{ij}}{\partial y_h}\right|_{P_i,\hat{P}_j} \\
&= -\frac{\delta^2 e^{\delta z_{ij}}}{\left(e^{\delta z_{ij}}-1\right)^2} < 0,
\end{aligned}$$

since $\partial z_{ij}/\partial y_h|_{P_i,\hat{P}_j} = 1$ for $h = i, j$. For fixed world output and given active traders, the elasticity of φ_{ij} with respect to (log) GDP will have the same sign as the elasticity of φ_{ij} with respect to GDP shares,

since shares $s_h^{X_h} \equiv Y_h/Y^{X_h}$, $h = i, j$, $X = J, I$ are trivially increasing in output levels Y_h. Therefore:

$$\frac{\partial \varphi_{ij}}{\partial s_h} < 0, \quad h = i, j.$$

■

A.1.3 Taylor approximation

Following Baier and Bergstrand, we take a first-order Taylor approximation of the MR indices. Our model contains an additional effect due to the impact of MR on the extensive margin: the price indices contain extensive margin terms, which themselves contain a system of price indices. In order to extract the exogenous components of the system, we make the further assumption that the extensive margin terms that appear in the price index equations (but not the main gravity equation) can be decomposed according to $V_{ij} \simeq \widetilde{Z}_{ij}^{\delta} \left(P_i \widehat{P}_j\right)^{\delta(\sigma-1)}$, or $w_{ij} \simeq \delta \widetilde{z}_{ij} + \ln \left(P_i \widehat{P}_j\right)^{\delta(\sigma-1)}$. This approximated extensive margin value approaches that computed under the assumption of a Pareto distribution for δz_{ij} sufficiently large, since $w_{ij} = \ln\left\{e^{\delta z_{ij}} - 1\right\} \simeq \delta z_{ij} = \delta \widetilde{z}_{ij} + \ln \left(P_i \widehat{P}_j\right)^{\delta(\sigma-1)}$ when this is the case. Using this, and defining $W_{ij} \equiv \widetilde{Z}_{ij}^{\delta}$ yields price indices of

$$\widehat{P}_j^{-k} = \sum_{i \in I_j} t_{ij}^{1-\sigma} s_i^{I_j} P_i^k W_{ij} \tag{6}$$

$$P_i^{-k} = \sum_{j \in J_i} t_{ij}^{1-\sigma} s_j^{J_i} \widehat{P}_j^k W_{ij}, \tag{7}$$

where we have used that $(\delta+1)(\sigma-1) = (\frac{k-\sigma+1}{\sigma-1}+1)(\sigma-1) = k$. The terms (6) and (7) are those which we Taylor approximate. We do this around a symmetric centre in which trade frictions are positive but symmetric and sufficiently low for all countries to trade. Then $t_{ij} = t > 1$ for all i, j. Similarly, $W_{ij} = W$, $P_i = P$, $I_j = J_i$ and $\widehat{P}_j = \widehat{P}$. This yields the system

$$\widehat{P}^{-k} = t^{1-\sigma} P^k W \sum_i s_i = t^{1-\sigma} P^k W,$$

$$P^{-k} = t^{1-\sigma} \widehat{P}^k W \sum_j s_j = t^{1-\sigma} \widehat{P}^k W,$$

which has a solution at $P = \widehat{P}$. Any observed variations in t_{ij} are pertubations from this centre which can be approximated using Taylor's method. These can be asymmetric in the sense that $t_{ij} \neq t_{ji}$. To do this, we use

$$e^{-k \ln \widehat{P}_j} = \sum_{i \in I_j} e^{\ln s_i^{I_j}} e^{(1-\sigma) \ln t_{ij}} e^{k \ln P_i} e^{\ln W_{ij}}, \tag{8}$$

$$e^{-k \ln P_i} = \sum_{j \in J_i} e^{\ln s_j^{J_i}} e^{(1-\sigma) \ln t_{ij}} e^{k \ln \widehat{P}_j} e^{\ln W_{ij}}, \tag{9}$$

Then expanding the LHS of (9) around $\ln P$ and the RHS around $\ln t$, $\ln W$, and $\ln \widehat{P}$ gives

$$\begin{aligned} P^{-k} - kP^{-k}[\ln P_i - \ln P] = &\sum_{j \in J_i} s_j^{J_i} t^{1-\sigma} W \widehat{P}^k + \sum_{j \in J_i} s_j^{J_i} (1-\sigma) t^{1-\sigma} W \widehat{P}^k [\ln t_{ij} - \ln t] \\ &+ \sum_{j \in J_i} s_j^{J_i} t^{1-\sigma} W \widehat{P}^k [\ln W_{ij} - \ln W] + \sum_{j \in J_i} s_j^{J_i} k t^{1-\sigma} W \widehat{P}^k [\ln \widehat{P}_j - \ln \widehat{P}]. \end{aligned}$$

Divide through by $P^{-k} = t^{1-\sigma} W \widehat{P}^k$ and use that $\sum_{j \in J_i} s_j^{J_i} = 1$ to give

$$\begin{aligned} -k[\ln P_i - \ln P] = & \sum_{j \in J_i} s_j^{J_i} (1-\sigma)[\ln t_{ij} - \ln t] \\ & + \sum_{j \in J_i} s_j^{J_i} [\ln W_{ij} - \ln W] + \sum_{j \in J_i} s_j^{J_i} k[\ln \widehat{P}_j - \ln \widehat{P}] \\ & = \sum_{j \in J_i} s_j^{J_i} (1-\sigma) \ln t_{ij} - (1-\sigma) \ln t \\ & \quad + \sum_{j \in J_i} s_j^{J_i} \ln W_{ij} - \ln W + \sum_{j \in J_i} s_j^{J_i} k \ln \widehat{P}_j - k \ln \widehat{P}. \end{aligned}$$

Use $-k \ln P = (1-\sigma) \ln t + \ln W + k \ln \widehat{P}$ to give

$$\ln P_i = \frac{1}{k} \sum_{j \in J_i} s_j^{J_i} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right] - \sum_{j \in J_i} s_j^{J_i} \ln \widehat{P}_j.$$

We can obtain the following expression for exporter resistance analogously:

$$\ln \widehat{P}_j = \frac{1}{k} \sum_{i \in I_j} s_i^{I_j} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right] - \sum_{i \in I_j} s_i^{I_j} \ln P_i.$$

Substituting the first of these into the second yields:

$$\begin{aligned} \ln \widehat{P}_j = & \frac{1}{k} \sum_{i \in I_j} s_i^{I_j} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right] \\ & - \sum_{i \in I_j} s_i^{I_j} \left[\frac{1}{k} \sum_{j \in J_i} s_j^{J_i} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right] - \sum_{j \in J_i} s_j^{J_i} \ln \widehat{P}_j \right], \end{aligned}$$

such that adding $\ln P_i + \ln \widehat{P}_j$ gives:

$$\begin{aligned} \ln P_i \widehat{P}_j = & -\frac{1}{k} \sum_{i \in I_j} s_i^{I_j} \sum_{j \in J_i} s_j^{J_i} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right] \\ & + \frac{1}{k} \sum_{j \in J_i} s_j^{J_i} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right] + \frac{1}{k} \sum_{i \in I_j} s_i^{I_j} \left[(\sigma - 1) \ln t_{ij} - \ln W_{ij} \right]. \end{aligned}$$

Writing the approximated price indices in terms using subscript h as the generic exporter and l as the generic importer, we have

$$\begin{aligned} \ln P_i \widehat{P}_j = & -\frac{1}{k} \sum_{l \in I_j} s_l^{I_j} \sum_{h \in J_i} s_h^{J_i} \left[(\sigma - 1) \ln t_{lh} - \ln W_{lh} \right] \\ & + \frac{1}{k} \sum_{h \in J_i} s_h^{J_i} \left[(\sigma - 1) \ln t_{ih} - \ln W_{ih} \right] + \frac{1}{k} \sum_{l \in I_j} s_l^{I_j} \left[(\sigma - 1) \ln t_{lj} - \ln W_{lj} \right]. \end{aligned}$$

Using $(\sigma - 1) \ln t_{ij} = \gamma d_{ij}$ and $\ln W_{ij} = \delta \widetilde{z}_{ij}$, our MR approximation is then given by

$$\ln P_i \widehat{P}_j = -\frac{1}{k} \sum_{l \in I_j} s_l^{I_j} \sum_{h \in J_i} s_h^{J_i} \left[\gamma d_{lh} - \delta \widetilde{z}_{lh} \right] + \frac{1}{k} \sum_{h \in J_i} s_h^{J_i} \left[\gamma d_{ih} - \delta \widetilde{z}_{ih} \right] + \frac{1}{k} \sum_{l \in I_j} s_l^{I_j} \left[\gamma d_{lj} - \delta \widetilde{z}_{lj} \right].$$

Our comparative statics require the term $\frac{\partial \ln(P_i P_j)^{\sigma-1}}{\partial d} = (\sigma-1)\frac{\partial \ln(P_i P_j)}{\partial d}$, so we write

$$(\sigma-1)\ln P_i\widehat{P}_j = \frac{(\sigma-1)}{k}\left\{-\sum_{l\in I_j} s_l^{I_j}\sum_{h\in J_i} s_h^{J_i}\left[\gamma d_{lh}-\delta\widetilde{z}_{lh}\right]+\sum_{h\in J_i} s_h^{J_i}\left[\gamma d_{ih}-\delta\widetilde{z}_{ih}\right]+\sum_{l\in I_j} s_l^{I_j}\left[\gamma d_{lj}-\delta\widetilde{z}_{lj}\right]\right\},$$

and use $\delta = \frac{k-\sigma+1}{\sigma-1}$, such that $(\delta+1)(\sigma-1)=k$, finally yielding the approximation:

$$(\sigma-1)\ln P_i\widehat{P}_j = \frac{1}{\delta+1}\left\{-\sum_{l\in I_j} s_l^{I_j}\sum_{h\in J_i} s_h^{J_i}\left[\gamma d_{lh}-\delta\widetilde{z}_{lh}\right]+\sum_{h\in J_i} s_h^{J_i}\left[\gamma d_{ih}-\delta\widetilde{z}_{ih}\right]+\sum_{l\in I_j} s_l^{I_j}\left[\gamma d_{lj}-\delta\widetilde{z}_{lj}\right]\right\}.$$

A.1.4 Proof of Implication 1: Bilateral changes

Proof. The trade elasticity for a bilateral change is given by

$$\begin{aligned}
\xi_{ij}^B &= (\sigma-1)+\varphi_{ij}(\sigma-1)-(1+\varphi_{ij})\frac{\partial \ln\left(P_i\widehat{P}_j\right)^{\sigma-1}}{\partial d_{ij,ji}},\\
\frac{\partial \ln\left(P_i\widehat{P}_j\right)^{\sigma-1}}{\partial d_{ij,ji}} &= (\sigma-1)\left(-s_i^{I_j}s_j^{J_i}-s_j^{I_j}s_i^{J_i}+s_j^{J_i}+s_i^{I_j}\right), \qquad (10)\\
&\Rightarrow \xi_{ij}^B = (\sigma-1)(1+\varphi_{ij})\left(1+s_i^{I_j}s_j^{J_i}+s_j^{I_j}s_i^{J_i}-s_j^{J_i}-s_i^{I_j}\right).
\end{aligned}$$

or $\xi_{ij}^B = \gamma(1+\varphi_{ij})\left(1+s_i^{I_j}s_j^{J_i}+s_j^{I_j}s_i^{J_i}-s_j^{J_i}-s_i^{I_j}\right)$ if $\gamma=\sigma-1$. For a change in the size of importer i such that $\partial s_i^{J_i} = \partial s_i^{I_j} = \partial s_i$,

$$\frac{\partial \xi_{ij}^B}{\partial s_i} = \frac{\partial \xi_{ij}^{gross}}{\partial s_i}\left(1+s_i^{I_j}s_j^{J_i}+s_j^{I_j}s_i^{J_i}-s_j^{J_i}-s_i^{I_j}\right)+\xi_{ij}^{gross}\left(s_j^{J_i}+s_j^{I_j}-1\right).$$

Use Lemma 1 to find

$$\frac{\partial \xi_{ij}^B}{\partial s_i} < 0 \text{ if } s_j^{J_i}+s_j^{I_j}<1.$$

For a change in the size of exporter j such that $\partial s_j^{J_i} = \partial s_j^{I_j} = \partial s_j$,

$$\frac{\partial \xi_{ij}^B}{\partial s_j} = \frac{\partial \xi_{ij}^{gross}}{\partial s_j}\left(1+s_i^{I_j}s_j^{J_i}+s_j^{I_j}s_i^{J_i}-s_j^{J_i}-s_i^{I_j}\right)+\xi_{ij}^{gross}\left(s_i^{I_j}+s_i^{J_i}-1\right),$$

Use Lemma 1 to find

$$\frac{\partial \xi_{ij}^B}{\partial s_j} < 0 \text{ if } s_i^{I_j}+s_i^{J_i}<1.$$

■

A.1.5 Proof of Implication 2: Multilateral Changes

Proof. The trade elasticity for a multilateral change is given by

$$\xi_{ij}^{M} = (\sigma-1) + \varphi_{ij}(\sigma-1) - (1+\varphi_{ij})\frac{\partial \ln\left(P_i\widehat{P}_j\right)^{\sigma-1}}{\partial d},$$

$$\frac{\partial \ln\left(P_i\widehat{P}_j\right)^{\sigma-1}}{\partial d} = (\sigma-1)\left[-\sum_{l\in I_j} s_l^{I_j}\left(1-s_l^{J_i}\right) + \left(1-s_i^{J_i}\right) + \left(1-s_j^{I_j}\right)\right] \tag{11}$$

$$= (\sigma-1)\left(1+\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} - s_i^{J_i} - s_j^{I_j}\right), \tag{12}$$

$$\Rightarrow \xi_{ij}^{M} = (\sigma-1) + \varphi_{ij}(\sigma-1) - (1+\varphi_{ij})(\sigma-1)\left(1+\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} - s_i^{J_i} - s_j^{I_j}\right). \tag{13}$$

The second line follows from the fact that, for multilateral changes, $\partial d_{ij} = \partial d$, $i \neq j$, $\partial d_{ii} = 0$, such that the component of MR due to world trade resistance $-\sum_{l\in I_j} s_l^{I_j}\sum_{h\in J_i} s_h^{J_i}\left[\gamma d_{lh} - \delta\widetilde{z}_{lh}\right]$ changes according to

$$-\gamma(1+\delta)\sum_{l\in I_j} s_l^{I_j}\left[s_1^{J_i}\partial d_{l1} + s_2^{J_i}\partial d_{l2} + ... + \left(s_l^{J_i}\times 0\right) + ... + s_n^{J_i}\partial d_{ln}\right]$$
$$= -\gamma(1+\delta)\,\partial d\sum_{l\in I_j} s_l^{I_j}\left(1-s_l^{J_i}\right),$$

generating the term $\sum_{l\in I_j} s_l^{I_j}\left(1-s_l^{J_i}\right) = 1-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i}$ in (11) and (12) above. Then (13) can be simplified to yield:

$$\xi_{ij}^{M} = (\sigma-1)(1+\varphi_{ij})\left(-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} + s_i^{J_i} + s_j^{I_j}\right),$$

or $\xi_{ij}^{M} = \gamma(1+\varphi_{ij})\left(-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} + s_i^{J_i} + s_j^{I_j}\right)$ if $\gamma = \sigma - 1$. Then:

(a) Ignoring MR gives the gross elasticity as $\gamma(1+\varphi_{ij}) > 0$ but including the MR term $\left(-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} + s_i^{J_i} + s_j^{I_j}\right)$ implies $\xi_{ij}^{M} \gtrless 0$ since $\left(-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} + s_i^{J_i} + s_j^{I_j}\right) \gtrless 0$.

(b) (i) If the extensive margin does not operate, the elasticity is $\xi_{ij}^{M}\big|_{\varphi_{ij}=0} = \gamma\left(-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} + s_i^{J_i} + s_j^{I_j}\right)$. Then as importer size changes $\partial s_i = \partial s_i^{J_i} = \partial s_i^{I_j}$, $\frac{\partial\,\xi_{ij}^{M}\big|_{\varphi_{ij}=0}}{\partial s_i} = \gamma\left(1-s_i^{I_j}-s_i^{J_i}\right)$, which is positive if $s_i^{I_j} + s_i^{J_i} < 1$, and as exporter size changes $\partial s_j = \partial s_j^{I_j} = \partial s_j^{J_i}$, $\frac{\partial\,\xi_{ij}^{M}\big|_{\varphi_{ij}=0}}{\partial s_j} = \gamma\left(1-s_j^{I_j}-s_j^{J_i}\right)$, which is positive if $s_j^{I_j} + s_j^{J_i} < 1$.

(b) (ii) If the extensive margin operates, the sign of $\frac{\partial \xi_{ij}^{M}}{\partial s_i}$ is subject to two competing effects. For changes in importer size:

$$\frac{\partial \xi_{ij}^{M}}{\partial s_i} = \underbrace{\frac{\partial \xi_{ij}^{gross}}{\partial s_i}}_{-\text{ve}}\underbrace{\left(-\sum_{l\in I_j} s_l^{I_j}s_l^{J_i} + s_i^{J_i} + s_j^{I_j}\right)}_{+\text{ve or }-\text{ve}} + \underbrace{\xi_{ij}^{gross}\left(1-s_i^{I_j}-s_i^{J_i}\right)}_{+\text{ve}}.$$

The MR effect from (b) (i) increases the elasticity for larger countries, but larger countries experience smaller direct effects of trade costs on the extensive margin as shown in Lemma 1. The response of the overall elasticity to changes in importer size is therefore ambiguous so that $\frac{\partial \xi_{ij}^{M}}{\partial s_i} \lesseqgtr 0$. An analogous result

can be shown for exporter size, using b(i) above. ■

A.2 Empirical implementation

A.2.1 Description of variables

The dependent variable is the log of exports in 1986 measured in constant (2000) US dollars. The main variables used are distance: the log of distance in km between the importer and exporter; border: a binary variable indicating whether the country pair shares a common physical boundary; island: a binary variable indicating whether at least one country is an island (in HMR, this is described as an indicator of whether both countries are an island); landlocked: a binary variable indicating whether at least one country is landlocked (in HMR, this is described as an indicator of whether both countries are landlocked); legal: a binary variable indicating whether or not the country pair share the same legal origin; language: a binary variable indicating whether the country pair share a common language; colonial ties: a binary variable indicating whether one country every colonised the other; FTA: a binary variable indicating whether or not the country pair formed a regional trade agreement; religion: a variable constructed by HMR indicating how similar the religious composition is in the country pair (% Protestants in j multiplied by % Protestants in i + % Catholics in j multiplied by % Catholics in i + % Muslims in j multiplied by % Muslims in i);[1]exporter and importer dummies to capture fixed effects.

To construct GDP Shares, GDP data are sourced from the World Bank's World Development Indicators. Where necessary, WDI data were combined to meet the country definitions in the trade data (for example combining Belgium and Luxembourg). Where possible, missing observations were supplemented with data from the United Nations Common Database (UNCDB). Otherwise, a GDP of 0.1 was inputted manually. The denominator is constructed based on the sum of the individual countries' GDPs. Shares are based on the subset of countries the importer imports from or the exporter exports to. Except in the example of a small exporter with few trading partners exporting to a large exporter, we have achieved similar results at the bilateral level simply by using the share of world GDP, where world GDP is the value provided by the WDI.

A.2.2 Description of empirical methodology

Here, we describe the methodology used for estimation and the comparative statics exercises.

Estimation

1. Estimate a probit model for the probability of positive exports from j to i. The first stage includes the bilateral variables listed in the data appendix together with importer and exporter dummies. The set of bilateral variables must include a variable which will be omitted from the second stage. Theoretically, such a variable should affect the fixed costs of exporting, but not variable costs. For the full sample, HMR use religion for this purpose.[2]

2. Generate predicted probabilities ($\hat{\rho}$) of positive exports. Many of these are practically indistinguishable from unity such that many values of the Inverse Mills Ratios are not distinguished from each other. The approach in HMR converts all $\hat{\rho} > 0.9999999$ to values of $\hat{\rho} = 0.9999999$.

[1] We use the version employed by HMR in their paper. An updated version has subsequently become available on Elhanan Helpman's website but does not affect any of our results.

[2] Some countries export to everyone else or import from everyone else. The fixed effects are thus perfect predictors and cannot be included in the probit. HMR omit such variables from the study. However, an alternative option might be to generate predicted probabilities equal to 0.9999999, as done in step 2 for a number of countries, and continue from there.

3. Predict the Inverse Mills Ratio $\widehat{\eta}^*_{ij} = \frac{\phi(\widehat{z}^*_{ij})}{\Phi(\widehat{z}^*_{ij})}$, where $\phi(.)$ is the the standard normal density function and $\Phi(.)$ is the standard normal distribution function. Generate $\widehat{z}^*_{ij}$ as a linear function of the probit variables and $\widehat{\widetilde{z}}^*_{ij} = \widehat{z}^*_{ij} + \ \widehat{\eta}^*_{ij}$.

4. Estimate the second stage including the predicted controls for firm selection $\log(e^{\delta\widehat{\widetilde{z}}^*_{ij}} - 1)$ and country-selection $\widehat{\eta}^*_{ij}$. HMR07 use maximum likelihood while HMR08 use non-linear least squares (NLS). We use NLS. Because of the large number of dummies (over 300), we follow Carayol (2006) and use the function evaluator program version of the nl command in *Stata*. Our results, while extremely close, do not precisely match HMR and are in fact closer to those in HMR07. Estimates are presented in the accompanying table for convenience.

5. Generate predicted values for trade $\hat{m}$ including a predicted value $w = \log(e^{\hat{\delta}\widehat{\widetilde{z}}^*} - 1)$.

Simulations Generate an alternative measure of the variable of interest. In the case of distance, the new measure is 10% lower than the original.

1. Generate a new latent variable using estimated coefficients together with the new distance measure.

2. Generate the new predicted probabilities $\hat{\rho}'$, setting values above 0.9999999 equal to 0.9999999 as before.

3. Keep the originally estimated Inverse Mills Ratio $\widehat{\eta}^*_{ij}$. The original estimate of unobserved trade frictions, conditional on the same countries trading, is still the prediction based on the original values.

4. Generate new predictions $\widehat{\widetilde{z}}^{*\prime}$, using the same value for $\widehat{\eta}^*_{ij}$ but the new predicted probabilities in $\Phi^{-1}(\hat{\rho})$. Use the estimate $\hat{\delta}$ and $\widehat{\widetilde{z}}^{*\prime}$ to generate an alternative prediction of the non-linear term $w' = \log(e^{\hat{\delta}\widehat{\widetilde{z}}^{*\prime}} - 1)$.

5. Generate the alternative predicted values for trade $\hat{m}'$ based on the new distance values and w'.

6. Calculate the gross elasticity $\xi = \frac{\hat{m}' - \hat{m}}{|\log 0.9|}$. For country-pairs with $\hat{\rho}' > 0.9999999$, assign the elasticity generated by a value of $\hat{\rho}' = 0.9999999$. By doing this, HMR are taking a probability of 0.9999999 to imply a certain proportion of firms exporting.

7. Calculate the bilateral net elasticities applying the appropriate country shares adjusting for trading partners using either $\xi^B_{ij} = \gamma + \varphi_{ij}\gamma_p - (1+\varphi_{ij})\frac{\gamma+\delta\gamma_p}{1+\delta}\left(-s_i^{I_j}s_j^{J_i} - s_j^{I_j}s_i^{J_i} + s_j^{J_i} + s_i^{I_j}\right)$ or $\xi^M_{ij} = \gamma + \varphi_{ij}\gamma_p - (1+\varphi_{ij})\frac{\gamma+\delta\gamma_p}{1+\delta}\left(1 + \sum_{l\in I_j} s_l^{I_j}s_l^{J_i} - s_i^{J_i} - s_j^{I_j}\right)$. By ignoring the potential difference in estimates of γ and γ_p, one could simply attain the desired elasticity by multiplying the gross elasticity by the share terms.

A.3 Regression results

Table A.1 contains the regression results. The first column gives the results from standard OLS estimates ignoring firm heterogeneity. Column 2 gives first-stage probit estimates and column 3 gives the second-stage regression, which includes controls for country selection (the Inverse Mills Ratio $\widehat{\overline{\eta}}^*_{ij}$) and the proportion of firms exporting (represented by the non-linear parameter δ). In all cases, multilateral resistance is controlled for in estimation by the use of exporter and importer fixed effects. For completeness, column 4 includes results based on the semi-paramteric polynomial approximation for the proportion of firms exporting.

References

[1] Carayol, T. (2006), '"Expression too long" error with nl command', Statalist

Table A.1: Estimated gravity equations. Dependent variable is bilateral trade flows.

	(1) OLS	(2) Probit	(3) NL	(4) Polynomial
Distance $(-\gamma)$	-1.176***	-0.660***	-0.799***	-0.862***
	(0.0269)	(0.0206)	(0.0352)	(0.038)
Border	0.458***	-0.382***	0.832***	0.786***
	(0.118)	(0.0941)	(0.124)	0.149
Island	-0.391***	-0.345***	-0.17	-0.2
	(0.107)	(0.0751)	(0.097)	0.12
Landlocked	-0.561***	-0.181	-0.447***	-0.482
	(0.164)	(0.105)	(0.128)	0.135
Legal	0.486***	0.0964**	0.388***	0.385***
	(0.043)	(0.0297)	(0.039)	0.044
Language	0.176**	0.284***	0.023	0.045
	(0.0556)	(0.0381)	(0.059)	0.066
Colonial ties	1.299***	0.325	1.003***	1.038***
	(0.132)	(0.292)	(0.091)	0.092
Currency Union	1.364***	0.492***	1.024***	1.106***
	(0.216)	(0.132)	(0.269)	0.249
FTA	0.759***	1.985***	0.378**	0.457***
	(0.162)	(0.314)	(0.144)	0.108
Religion	0.102	0.261***		
	(0.09)	(0.061)		
$\widehat{\overline{\eta}}^*$			0.392***	1.131***
			(0.049)	0.137
$\widehat{\delta}$			0.716***	
			(0.049)	
$\widehat{z}^*$				3.602***
				0.348
$\widehat{z}^{*2}$				-0.782
				0.112
$\widehat{z}^{*3}$				0.064***
				0.011
Constant	3.168***	3.813***	517.2	4.197***
	(0.788)	(0.371)	(764.8)	0.517
Observations [R^2]	11,146 [0.709]	24,649	11,146 [0.718]	11,146 [0.721]

Notes: Significance at 0.001 & 0.01 levels denoted with '***' and '**' respectively. Standard errors in parentheses (based on 50 bootstrap replications in columns 3 and 4).