Strategic Party Placement with a Dynamic Electorate

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Online Appendix: Modeling with Directional Theory

In the directional model of voter utility, voters vote for candidates who are on the same side of policy space as themselves before they will vote for any candidate on the opposite side of the space (Rabinowitz and Macdonald, 1989). Additionally, the more a politician emphasizes the symbolism of one side, the higher he or she will be rated by voters on that side and the lower he or she will be rated by voters on the other side. The unidimensional result with two parties is convergence to the outside bounds of the policy space, though candidates stop short of taking a position for which they could be labeled an extremist.[1]

Equation 15 formalizes the way voters directionally evaluate parties:

$$\begin{aligned} U_{m_t}(A) &= m_t \times \theta_A + V_t \text{ where } [V_1 = V > 0, V_2 = 0] \\ U_{m_t}(D) &= m_t \times \theta_D \end{aligned} \tag{15}$$

Just as in Equation 1 in the paper, U is the voter's utility from a party, m_t is the median voter's ideal point at time $t \in \{1, 2\}$, θ is the party's position, and V is a constant non-ideological utility that the first median voter (m_1) adds to its evaluation of party A in the first election. The parameters are the same as with the proximity model. The main difference is that the voter's utility is a function of the product of his or her ideal

[1]Several researchers have tried to combine the proximity and directional theories of electoral competition, typically by adding a new consideration to the proximity model. Namely, Adams, Merrill & Grofman add a discounting factor (2005), and Kedar adds a strategic compromise consideration (2005a,b).

point with the party's stated position, rather than the negative squared distance between the ideal point and party position.[2] For simplicity, party positions are restricted to the policy space ($\theta \in \Theta = [-1, 1]$), and I assume that no position in this space would be considered extreme by voters. To keep the game interesting, the game assumes that the present median voter and the future median voter are on different sides of the issue. Without loss of generality, the present median voter is conservative ($m_1 > 0$) and the future median voter is liberal ($m_2 < 0$).

Much like the deterministic proximity-based game in the paper, there is only an equilibrium in a special case. Interestingly, the condition of the special case depends not on movement of the median, but whether the valence advantage is large relative to the present median voter's ideological extremity. If the advantaged party has a non-ideological advantage more than twice the ideal point of the present median voter, then party A will win the present election even if it takes a very liberal position that appeals to the future median voter. Therefore, both parties will take the most liberal possible positions when the valence advantage is large relative to the present median voter's ideal point, so they will tie in the future election. When valence is not big enough relative to the present median voter's ideal point, there is no pure strategy equilibrium because for any set of positions that the parties might take, one party would change its behavior in order to win or tie an additional election. Propositions 4 & 5 formally demonstrate these ideas, then the next section considers a stochastic version of the directional game.

Proposition 4 *Under directional utilities, when $V > 2m_1$, it is a Nash equilibrium if the parties converge to the most liberal point in the policy space.*

Proof If both parties play $\theta = -1$, then A earns $1 + \frac{\delta}{2}$ for winning the first election and

[2]We can use the simple product to capture ideological utility if we assume that zero is ideologically neutral, negative numbers represent liberalism, and positive numbers represent conservatism. This product gives the voter positive utility for any party on the same side, negative utility for any party on the opposing side, and zero utility if either the voter or party is neutral on the issue. As either the voter or the party move further to an extreme, the absolute size of the voter's utility increases.

tying the second. D earns $\frac{\delta}{2}$ for tying the second. If either unilaterally defects, then it will lose the second election outright, diminishing utility by $\frac{\delta}{2}$, without gaining anything. This is because A will win the first election for $\theta_A = -1$ against any position D can take in the policy range $\theta_D \in [-1, 1]$. Since neither party will defect, $\theta_A = \theta_D = -1$ is a Nash equilibrium. □

Proposition 5 *Under directional voter utilities, when $V \leq 2m_1$, there is no pure strategy Nash equilibrium.*

Proof Under these circumstances, D will win the first election if:

$$\begin{aligned} m_1\theta_A + V &< m_1\theta_D \\ \theta_A + \frac{V}{m_1} &< \theta_D \end{aligned} \tag{16}$$

Otherwise, A will win. Therefore, D's best response function is:

$$\begin{aligned} B_D(\theta_A): \quad -1 \leq \theta_D < \theta_A \quad & \text{if } \theta_A + \frac{V}{m_1} \geq 1 \\ \theta_A + \tfrac{V}{m_1} < \theta_D \leq 1 \quad & \text{if } \theta_A + \frac{V}{m_1} < 1 \end{aligned} \tag{17}$$

This is because whenever $\theta_A + \frac{V}{m_1} \geq 1$, party A has taken a sufficiently conservative position that the valence advantage will exceed any ideological advantage with voter m_1 that party D can edge out. In this case, D should play for the voter in the second election (m_2). Whenever $\theta_A + \frac{V}{m_1} < 1$, though, party D can win the more valuable first election by taking a position conservative enough to stimulate an ideological advantage with the first voter that exceeds the valence advantage.

On the other hand, party A can win both elections, so its best response function is:

$$B_A(\theta_D): \quad \begin{array}{ll} \theta_D - \frac{V}{m_1} < \theta_A < \theta_D & \text{if } \theta_D - \dfrac{V}{m_1} \geq -1 \\ -1 \leq \theta_A < \theta_D & \text{if } \theta_D - \dfrac{V}{m_1} < -1 \end{array} \tag{18}$$

In order to win the second election, party A has to play something lower than θ_D. In order to simultaneously win the first election, though, A needs to be close enough to D that party D's ideological advantage with the first voter is smaller than A's valence advantage.

Given these best response functions, there is no Nash equilibrium for the game when valence does not exceed the ideological difference. First, in the case when $\theta_A + \frac{V}{m_1} \geq 1$, for an equilibrium to exist the parties would have to play in such a way that $\theta_D < \theta_A$ and $\theta_D > \theta_A$, which is impossible. Second, for an equilibrium to exist in the case when $\theta_A + \frac{V}{m_1} < 1$, the parties would have to play such that $\theta_A + \frac{V}{m_1} < \theta_D$ (i.e., $\theta_D - \frac{V}{m_1} > \theta_A$) and $\theta_D - \frac{V}{m_1} < \theta_A$, which is also impossible. Hence, there is no pure strategy equilibrium whenever $V \leq 2m_1$. $\square$

The Directional Model with Probabilistic Voting

Similar to the stochastic model with proximity utilities, this subsection presents a version of the game in which voters evaluate parties' issue positions directionally, but vote probabilistically. This is again created by adding a shock to voter utilities in the form of a random draw from a probability distribution:

$$\begin{aligned} U_{m_t}(A) &= m_t \times \theta_A + V_t + \epsilon_{At} \text{ where } [V_1 = V > 0, V_2 = 0] \\ U_{m_t}(D) &= m_t \times \theta_D + \epsilon_{Dt} \end{aligned} \tag{19}$$

Equation 19 resembles Equation 15, with ϵ as a stochastic utility term. This game does not yield a closed-form solution, so I again ran simulations to determine the equilibria for fixed parameters.[3] In all treatments, both parties converge to the same extreme of the issue space, be it liberal or conservative. If the present median voter is more extreme in its position than the future median voter, then the parties always take the most extreme position on the side of the present median voter. If the future median voter is more extreme than the present median voter, then the results vary. For low values of the future election, the parties will take the most extreme position on the side of the present median voter. When the value of the future election reaches a certain level, the parties switch to taking the most extreme position on the side of the future median voter. Where the parties switch strategies depends on their level of uncertainty: as the variance of the voters' stochastic utility term decreases, the value of the future election that draws parties to the other side of the issue decreases. Hence, the effect that the value of the future has on party strategies is conditional on uncertainty and relative extremity of the present and future electorates.

[3]The ϵ terms still have a Gumbel distribution, which means parties' expected utilities are a sum in which each term includes a logistic distribution. This creates the same insolubility problem from the stochastic proximity game.