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THE DEVELOPMENT OF A STAND MODEL FOR DOUGLAS FIR

by

ROBERT MONTAGUE
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A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in the Department
of
FORESTRY

We accept this thesis as conforming to the
required standard

Charlotte Fraser .....
Philip H. Haddock .....
J. H. G. Smith .....

THE UNIVERSITY OF BRITISH COLUMBIA

October, 1964

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ABSTRACT

Supervisor: Professor J. H. G. Smith

A mathematical model has been developed to describe the growth of trees in stands of Douglas fir (Pseudotsuga menziesii (Mirb.) Franco) from age ten to age 100 years. An initial square pattern of spacing was assumed. At age ten years the trees were assumed to be open-grown, that is, growing in diameter at breast height at a maximum rate. A regression of d. b. h. on age was obtained from eighteen open-grown, Douglas fir trees measured on the Saanich Peninsula, Vancouver Island. The relationship derived from these data agreed with further data collected elsewhere in the coastal regions of British Columbia and Washington and in the interior of British Columbia. The d. b. h. growth of individual trees was predicted by five-year periods. Relationships between crown width and d. b. h. were calculated from data on 426 open-grown, Douglas fir trees. There was a close correlation between crown width and root spread for open-grown trees. A multiple regression equation was obtained for height of 869 trees on d. b. h. and basal area per acre. All regression equations calculated for use in the model were highly significant statistically.

The model is initiated with a matrix of 15 x 15 trees (or tree "locations"). The initial d. b. h. of each tree is specified and, from the crown width/d. b. h. regressions, the crown width of each tree is calculated. As long

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as the tree remains free of competition, this calculated crown width is reduced by 40 per cent by the reduction factor "REDFAC", to give the "competitive" crown width. This was because it was found that, in young Douglas fir plantations, there could be considerable overlapping of the crowns before d. b. h. growth was reduced. As soon as competition sets in the original 40 per cent reduction is systematically reduced. The proportion of the circumference of each tree that is occupied by the crowns of surrounding competitors is then calculated. This proportion indicates the amount of competition to which the tree is being subjected and varies between zero, if the tree is open-grown, and one or more, if the tree is completely enclosed by the surrounding competitors. The five-year d. b. h. growth of each tree is then determined from the d. b. h./age regression described above. D. b. h. increment is reduced in value by the proportion of the crown occupied by competitors. If the reduction is sufficiently great, continued survival of the tree is considered unlikely, and the tree is assumed to have died. The periodic d. b. h. growth of the surviving trees is calculated at five-year intervals to age 100 years.

All calculations are performed using an I. B. M. 7090 electronic computer. A summary of the structure of the stand can be printed out at the end of each five-year period if required. Height growth can be described by modifying the stand model by including an appropriate regression equation. Similarly, volume growth can be estimated by

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modifying the basic stand model.

The mathematical model developed here satisfactorily describes the growth of Douglas fir stands on an individual tree basis, over a wide range of site conditions, stand densities, amounts and distributions of mortality and thinning regimes. Field data cannot be secured to evaluate the accuracy of all the tests made. However there are no gross errors in absolute values and results are accurate proportionately.

The model described here can aid the forester in managing Douglas fir stands in the Pacific Northwest. By simulating the growth of his stands from age ten to age 100 years in a few minutes he can study questions that would otherwise require several human generations to evaluate.

J. H. G. Smith

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INTRODUCTION

The present study was initiated in 1962 to develop a stand model to describe the growth of Douglas fir (Pseudotsuga menziesii (Mirb.) Franco) stands under the conditions normally encountered in the coastal region of British Columbia. A stand model should describe mathematically the growth of a stand by individual trees rather than by average height, average diameter at breast height, total basal area and volume (either total or merchantable) as in published yield tables. Yield tables generally describe the "normal" or "fully stocked" stand although in Europe (e. g. Hummel and Christie, 1953) and, more recently, in North America (e. g. Clutter, 1963) attempts have been made to forecast growth under various thinning regimes. New Zealand tables for Douglas fir (Duff, 1956) take the initial spacing in the plantations into account. These tables have the defect that once away from the normal or prescribed it is not possible to forecast yield on any sound basis.

In the past ten years there has been a marked change in thinning and planting practice which has outdated many of the yield tables. In the perfect stand model the effect of the removal of any group of trees by thinning or natural mortality can be described by the model in terms of increased or decreased growth and, what is more important, the distribution of that growth on the remaining trees can

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also be estimated. If such a model could be obtained it would be of great value to the forest manager in determining optimum initial spacing in plantations and the most economical method of thinning. From present day knowledge of the growth of stands and of individual trees it should be possible to obtain a satisfactory approximation to the 'perfect' model.

For the purposes of the present study a satisfactory model was defined as one which gave estimates of mean diameter growth, basal area growth and number of trees per acre which fell within the bounds of the data given in several yield tables for Douglas fir (Barnes, (U. B. C. Forest Club, 1959), McArdle et al., 1949, and Duff, 1956) and which showed no abnormalities in diameter distribution. In the model no attempt has been made to describe height growth of individual trees in detail, due to the impracticality of measuring the large number of tree heights required, or volume growth, although the latter is usually considered to be closely correlated with basal area growth. Height-diameter relationships can be calculated for individual stands from empirical data. It should also be noted that, although the calculations have been made on individual trees, the model should not be used to describe the growth of any particular tree but rather the stand as a whole. It is anticipated that the greatest value of the model will be for comparing different methods of stand management.

Most of the information on the growth of Douglas fir used in this study is based on data collected at the

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University Research Forest, Haney, British Columbia since 1949. Much of this information has been published by Smith et al. (1961) and by Griffith (1960). Further data were collected in the summer of 1963 in the interior of British Columbia (Paul Lake, near Kamloops), in the drier parts of the coastal region of British Columbia (Saanich Peninsula, Vancouver Island), and on the higher rainfall, western slopes of the Cascade Mountains in Washington State (Wind River).

The growth of a forest stand is a very complex subject and one which has probably been studied more intensively than any other topic in forestry. The principles of the mensurational aspects of growth - height and diameter growth, stocking, density, yield, thinning and competition - are generally understood but attempts to fit mathematical models to the basic theory have either been complicated and limited in their use (Clutter, 1962 and 1963; Crane, 1962; Czarnowski, 1961, and Meyer, 1930) or else did not give the satisfactory results that had been desired (Staebler, 1951). Due to the complexity of the growth patterns which involve extensive computations, researchers in the past have also been physically limited in the amount of work they could undertake. With the advent of modern, high-speed, electronic computers and their widespread use in forestry, this problem has been reduced and the forest researcher's horizon has been greatly widened (see Csizmazia, 1963). The advantages of these advances have been summarized by Jeffers (1962):

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> For the first time in human history, not only is it possible for us to undertake large scale forest enumerations showing the state of the forest at a particular time, it is possible to project these enumerations forward with a fair degree of reliability, and to build from this information, mathematical models of the forest and its production. The effect of management decisions on this model may then be tested, so that the forester can have a fair idea of the consequence of his actions, before treatments are ever applied.

This paragraph emphasizes the importance of stand models in forest management and also the need to establish a model, or models, which will satisfactorily predict growth under a wide range of stand conditions.

The growth of an individual tree is dependent on a number of factors. Baker (1950) outlined these as:

1. The site factors, which are virtually fixed and can be but slightly modified by the forester's art.
2. The inherent capacity of the leaves to carry on photosynthesis (tolerance is involved here).
3. The input of light energy, water and nutrients.
4. The photosynthetic area.

Until recently, the forester could control, or influence, only the third and fourth factors. By fertilizing, irrigating or draining, the forester can now improve the quality of the site and, in tree-breeding, it is possible to select those trees having the greatest growth capacity. The factors influencing site are multitudinous and, because of this, no two forest sites are ever exactly alike. In addition to the soil characteristics (depth, permeability, organic material content and nutrient content), site is affected by climate (precipitation, temperature, hours of sunshine, frost and winds), topography (type of slope, steepness, aspect

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and elevation), and the artificial effects of man and domestic animals. In natural stands growth may vary from year to year due to climatic changes so that growth studies should always be averaged over a number of years to eliminate such variation as far as is possible.

Not only does growth vary from site to site but growth within a stand on a particular site is also variable. If we consider a stand where all the trees are the same age and species, and have sufficient room that they are not in competition with each other, there will be still considerable variation in growth (Baker, 1923). These differences will be due to two sources: first, the inherited genetical differences between trees, and second, local differences in site due to changes in microclimate, microtopography, the presence of local pockets of fertility, the presence of root systems of a former stand and many other causes. This partially explains the lack of success in predicting the growth of individual trees (Staebler, 1951). In predicting growth, even if it is on an individual tree basis, the overall effect must be considered and not the individual effect.

Smith (1964) described progress to date and the problems involved in the preparation of a model of stand development from stem analysis. In order to apply his data secured by stem analysis in the testing of various alternatives in forest management, it was necessary to develop a comprehensive, mathematical model of the growth of Douglas fir stands which can be manipulated quickly and easily. The

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writer has been concerned for two years with the preparation and testing of the required mathematical model.

In this thesis preparation of the model will be described. The developed model will be used to test the effects of various amounts and distributions of mortality following planting on basal area growth. Four different spacings ranging from 3.3 x 3.3 ft. to 13.2 x 13.2 ft. will be tested on three different site qualities - poor (site index 120), medium (site index 140) and good (site index 160). Two wider spacings, 16.5 x 16.5 ft. and 19.8 x 19.8 ft., will be tested on the medium site quality. Different types and intensities of thinning will be tested in an attempt to determine the best thinning schedule for the management of Douglas fir plantations in the coastal region of British Columbia.

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PART I

SUMMARY OF PAST WORK

As stated previously, very little attention has been paid to the development of comprehensive stand models, due to the enormity of the mathematical computations involved. Staebler (1951) suggested a number of hypotheses about the growth of Douglas fir. His first hypothesis was that a tree's growth varied inversely to the competition which it received from neighbouring trees. It was assumed that competition was directly proportional to some function of the competing trees (e.g. diameter at breast height or crown class) and inversely proportional to their distance apart. All the competing trees taken together explain the growth of the study tree. Staebler's "area of overlap" hypothesis is probably of most interest in relation to the present study. According to this hypothesis, trees require a circular area, which varies with the d. b.h. of the tree, in which to grow. Two trees which are growing so close together that their circles overlap are considered to be competing with each other. The growth of any tree is inversely proportional to the amount of this overlap, d' (Fig. 1a). Staebler suggested that competition might be better measured by the area of overlap rather than the lineal measurement. In the present study an angular measure, θ, has been used to measure tree competition (Fig. 1b). Staebler's third hypothesis was to

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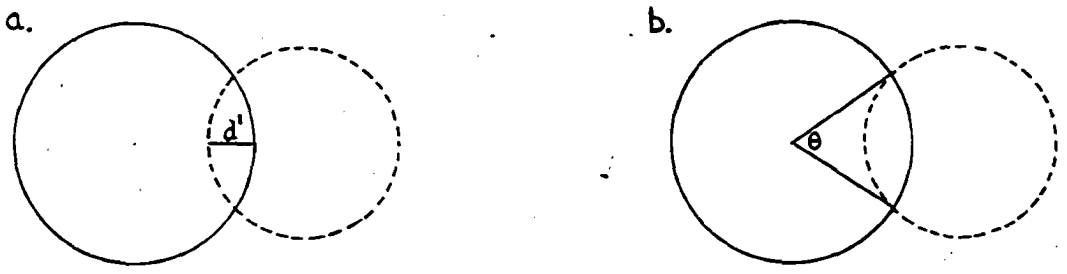

Fig. 1.(a) Staebler's method of measuring tree competition by the overlap method (from Staebler, 1951: Fig. 4).

(b) The method of measuring tree competition in the present study.

define competition as a fraction of "full growth". "Full growth" was defined as two standard deviations above the basic diameter growth/d. b. h. curve for the stand. A curve of "full growth" could then be used as uniform reference point for any stand. Unfortunately, the regression equations that Staebler developed to describe the diameter growth of individual trees did not remove much of the variation in growth between trees, possibly due to the small size of the sample he was able to use (40 trees).

Staebler's research outlined above is probably the only attempt that has been made to develop a mathematical model to describe the growth of a forest stand on an individual tree basis. Other methods have used stand averages; these are described below together with other factors which affect the growth of forest stands.

The Spatial Pattern of Forest Stands

"Spatial pattern" is the arrangement of tree stems within the stand. This arrangement is usually clearly definable

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in the early life of a plantation as the trees will normally have been planted in some geometrical arrangement at roughly equal distances. A square pattern is usually adopted as this gives a balanced, equal space to each tree and is easy to apply in practice. Although there is always a certain amount of mortality, and sometimes ingrowth of naturally regenerated trees, the planting pattern can usually be seen for some time. Planting the trees closer within the rows than the distance between the rows is a fairly common practice in poplar cultivation and may become more important in the establishment of Douglas fir plantations in the Pacific Northwest. Van Slyke (1964 a,b) has adopted the systematic designs of Nelder (1962) for testing the effects of spacing and rectangularity in forest stands.

The trees in a natural stand are never equally spaced and their spatial pattern is often indescribable by mathematical formulae. Where regeneration is dense and competition is intense, or when the stand becomes older, the spatial pattern does become more regular.

Development of pattern can be illustrated simply. If a number of pennies are placed in a flat tray so that they do not overlap and the tray then tilted so that the coins slide to one edge it will be seen that, except for the edge pennies, each penny is surrounded by six other pennies. If the coins could now be compressed so that the spaces between them were eliminated, the coins would change their shape to six-sided polygons whose centres would be located at the

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corners of equilateral triangles. Such a phenomenon is common in nature when the units, or cells, are occupying all the available space as, for example, in a honeycomb. Dice (1952) adopted triangular spacing in describing the spatial pattern of plants. For these reasons it may be argued that the spatial pattern of natural stands should be triangular since the stand will tend to utilize the site completely. However, few stands have 100 per cent crown closure and by adopting an arbitrary square spacing, allowance can be made for this. Hummel (1954) suggested that square spacing was at least as justifiable as triangular spacing because the distribution of trees in a stand was never quite regular. It is also simpler in practice and foresters are more accustomed to think in terms of it. Bright (1914), Crane (1962), Lemmon and Schumacher (1963), Smith (1958 and 1963) and Wiley (1959) have also adopted the square spacing concept to allow for the spaces between crowns in the canopy.

The argument that a stand is only fully utilizing a site when the crown closure is more or less complete is fallacious in some instances. In South Africa it was found that root competition set in long before the crowns of plantation-grown, exotic pines came into competition (Hiley, 1948 and 1954). When water is scarce, growth is more influenced by root restriction than by crown restriction (Hiley, 1959). Part of the root system of trees normally extends beyond the perimeters of the crowns and may interlock with the roots of the surrounding trees (Spurr, 1952). This

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has been found specifically to be so in Douglas fir (Hengst, 1938; and McMinn, 1955 and 1963). McMinn (1963), however, found that the greatest concentration of feeding roots was confined to an area considerably smaller than the area occupied by the crown and that the spread of the root systems was restricted by the presence of other root systems in the soil. Bright (1914), who studied yellow pine (Pinus ponderosa Laws.), claimed that the roots rarely extended beyond the crown. It appears probable that he did not measure the full extent of the root systems as he confined his measurements to the roots of wind-blown trees.

The various methods of describing pattern in plant ecology have been described in detail by Greig-Smith (1957). Foster and Johnson (1963c) have described the pattern and frequency distribution of forest disease in Douglas fir plantations on Vancouver Island. Although the initial pattern of spacing must have been regular, or nearly so, and the plantations had only been established thirteen years, the distribution of the living trees was described as regular in only one of the five areas sampled. In the two areas with the heaviest mortality the distribution was aggregated whilst in the remaining two areas it was irregular. Root-rot affected trees were also aggregated whereas trees affected by frost-lesion and sunscald conformed to an irregular pattern. The negative binomial distribution was found to adequately describe the distribution of root-rot affected trees.

The actual spatial pattern of a forest canopy can best be determined from crown canopy photographs taken from

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below or from stand maps. Stand maps have been drawn by some European foresters but these have generally been of uneven-aged stands (Miegroet, 1950) or else are not extensive enough to draw definite conclusions (Kostler, 1953).

The Effect of Initial Spacing on Growth

The "initial spacing" in plantations is the planting distance or, in naturally regenerated stands, it is the average distance between 'established' seedlings. What is meant by 'established' will depend to a large extent on the site and species. Dick (1963) defined an established ponderosa pine seedling as "one at least 1 foot in height or growing in height at a rate of at least 1/10-foot per year". These figures should probably be doubled for Douglas fir in the coastal region of British Columbia because of increased competition from more vigorous ground vegetation.

Many spacing trials have been carried out to test the effects on growth of different planting distances. In the Pacific Northwest a series of Douglas fir plantations was established at Wind River, Washington, in 1925 at spacings varying from 4 x 4 to 12 x 12 ft. The establishment and the results of this experiment have been described in detail in various unpublished reports of the Pacific Northwest Forest and Range Experiment Station of the United States Forest Service (Isaac, 1926; Isaac and Meagher, 1936; Isaac and Petersen, 1940; and Reukema, 1961). Less detailed reports have been published by Eversole (1955) and by Reukema (1959).

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Morse (1962) has carried out an economic analysis using data from the Wind River experiment to determine the optimum initial spacing in forest plantations. The 1960 measurements from this experiment (Reukema, 1961) showed that basal area decreased with increase in spacing while cubic volume (trees 1.5 in. d. b. h. and larger) increased with wider spacing. This increase in volume was more pronounced when only trees 6.5 in. in d. b. h. and greater were considered. The d. b. h. of the average tree in the 12 x 12 ft. spacing was more than twice that in the 4 x 4 ft. spacing. Merchantable cubic volume in the 12 x 12 ft. spacing was three times that in the 4 x 4 ft. spacing. Net periodic annual growth (1957-1960) by all measures was progressively better with wider spacing. Mortality was confined to trees six inches d. b. h. and under. The largest trees per acre contained 57 per cent of the total stand cubic volume in the 12 x 12 ft. spacing but only nineteen per cent in the 4 x 4 ft. spacing. Total height of the dominant and codominant trees appeared to be greatest in the widest spacings (Reukema, 1959).

In Great Britain, Mackenzie (1951) found that initial spacing (4 x 4, 6 x 6 or 8 x 8 ft.) had no effect on the height growth of Douglas fir or the other conifers tested. He also found that there was considerable interlacing of the branches in the closer plantings. The dominant height growth of southern pines, established at spacings from 4 x 4 to 16 x 16 ft., was not affected by spacing during the first fourteen growing seasons (Ware and Stahelin, 1948). Similar results were

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obtained with Norway spruce (Picea abies Karst.) in Bavaria (Guillebaud, 1951). It seems reasonable to suppose that, providing the average height of the dominant trees or of the 100 largest trees per acre, is used as the parameter, spacing does not affect the first 30 to 40 years' height growth, except at the extremes of open spacing or very close spacing. On the other hand, diameter growth is greatly improved with wider spacing because of the extra space available for crown development.

Until recently, especially in Europe, the practice has been to establish plantations at relatively close spacings (4 x 4 to 6 x 6 ft.) and then to thin early in the rotation. This results in high initial costs of establishment and, where early thinnings are unsaleable, further financial loss. If thinning is ignored the crop's rotation may have to be lengthened. Smith (1958) and Smith et al. (1961) have suggested establishing stands at wide spacings which will permit the rapid growth rate characteristic of open-growth trees during the early part of the rotation but which, by the time the trees have reached harvestable size, would be normally stocked. The initial spacings could be as much as twelve feet or greater. Such an idea is not entirely new. Stafford (1931) advocated "skeleton planting", that is, planting the ultimate stand, on the Swann Forest in Massachusetts. In South Africa plantations have been established at spacings of 9 x 9 ft. or greater for a number of years (Hiley, 1959). Cromer and Pawsey (1957) found for radiata pine (Pinus radiata D. Don.) that the optimum spacing for maximum merchantable

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volume growth at age fifteen years was between 9 x 9 and 10 x 10 ft. Cook (1963) observed that the traditional 6 x 6 ft. spacing in the northeastern United States was costly to establish, produced much unusable wood and required cultural operations that were difficult to perform with modern equipment and labour. He advocated a 6 x 10 ft. planting program which would produce adequate stocking and maximum merchantable volume on well formed stems.

It is almost certain that wide initial spacing will be more extensively used in the establishment of Douglas fir in the Pacific Northwest in the future. To test the effect of spacings varying from 3 x 3 ft. to 15 x 15 ft., experimental plantations of Douglas fir were established in 1957 on the University Research Forest at Haney (U. B. C. Faculty of Forestry, 1959). Four of these plantations are shown as of October, 1963, in Plates Ia-d. The poorer growth in the plantation established at 12 x 12 ft. is probably due to soil compaction as a result of logging the previous stand.

The Distribution of Trees in a Forest Stand

The distribution or "manner of occurrence" of trees in a forest stand gives information on the regularity, or irregularity, of the scatter of the trees. It is important that the distribution is known as, for instance, the volume growth behavior of tightly clustered groups of trees of all sizes will be different from the behavior of evenly spaced trees of more or less uniform size growing under similar site

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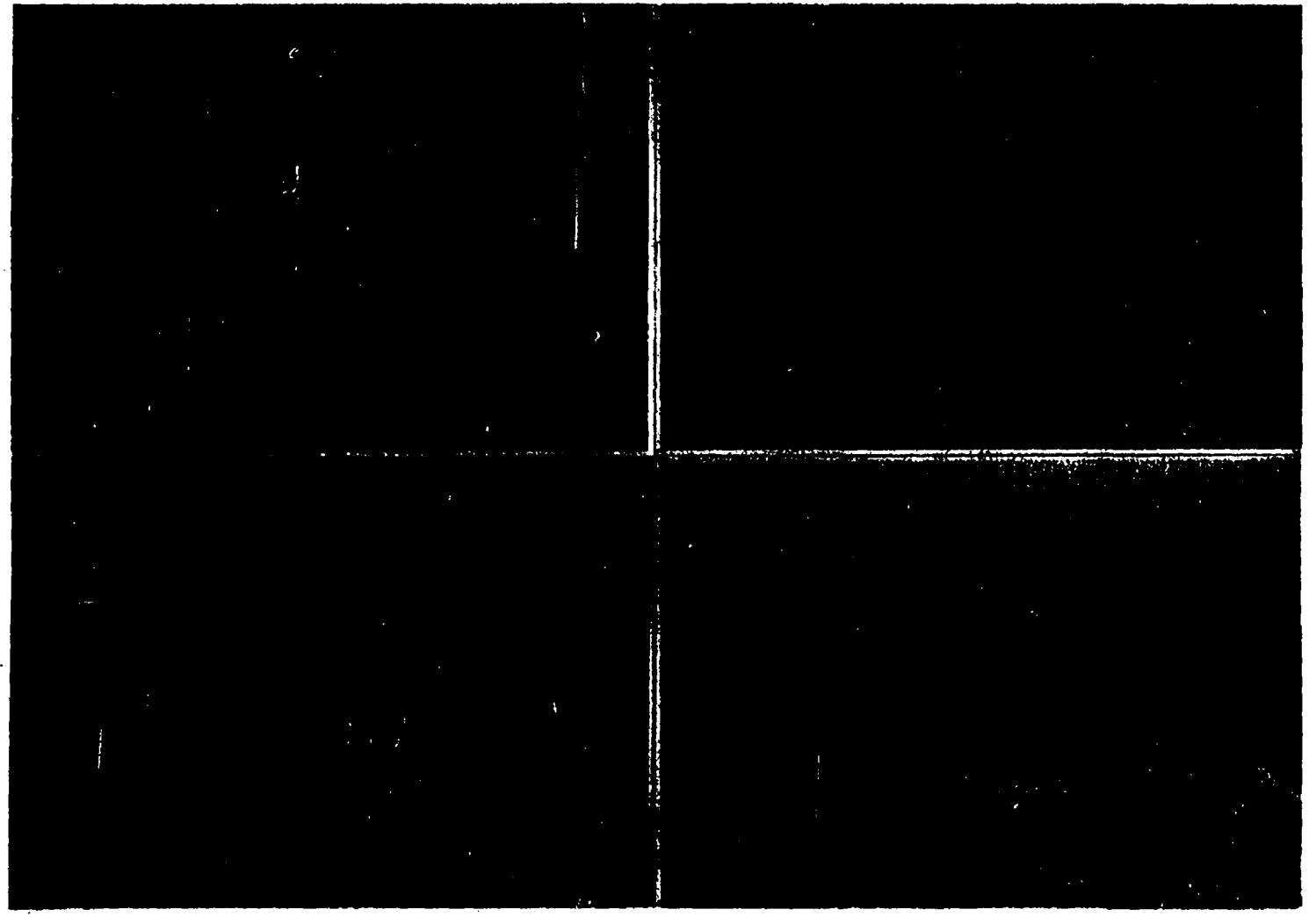

PLATE I: Douglas fir plantations on the University Research Forest, Haney, B. C., planted as two-year-old seedlings in the fall of 1957 at different spacings: (a) 3 x 3 ft. (b) 6 x 6 ft. (c) 9 x 9 ft. (d) 12 x 12 ft. Photographed in September, 1963.

conditions (Grosenbaugh, 1948). The highest yields should be obtained when the trees are spaced in a regular pattern (Foster and Johnson, 1963b).

A summary of the frequency distributions that have been found to have applications in forestry is given in Appendix I. A description of some of the distributions found in sampling forest stands has been given by Smith and Ker (1958).

The distribution of the trees is obtained by taking a number of sample plots and recording the number of trees in each plot or, alternatively, dividing the plot into a number of sub-plots and counting the number of stocked sub-plots, a sub-plot being classified as stocked if it contains an established seedling of the desired species. The parameters of most distributions are estimated from the mean and/or the variance of the sample (see Appendix I). The distribution will vary according to the size of quadrat used (Smith and Ker, 1957).

Growth

Diameter at Breast Height and Basal Area

Breast height is taken at 4 ft. 6 in. in North America and in New Zealand, 4 ft. 3 in. in Great Britain and 1.3 metres (4.265 ft.) on the continent of Europe. The differences in diameter growth in these three systems can be considered to be negligible. The age of a plantation in Great

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Britain and the rest of Europe and in New Zealand, is usually reckoned from the date of the plantation's establishment and not from the date of germination of the seed as in North America. These facts should be remembered when consulting yield tables.

The methods of predicting diameter growth that are commonly used in forestry have been described by Spurr (1952) and by Husch (1963). For Douglas fir in Washington and Oregon Spurr (1952) gave the following formulae for basal area growth:

$G_B = 85.31 - 2.995B$; or $G_B = 87.30 - 2.039B - 0.396A$;

where G_B is the basal area growth in square feet per acre for a period of fifteen years, B is the basal area at the beginning of the period, and A is the age of the stand. Spurr later studied the growth of Douglas fir in New Zealand and found that, regardless of the type of thinning carried out, the relationship between mean basal area of the 100 largest trees per acre and age was linear (Spurr, 1963). For predicting the mean diameter in inches of the 100 largest trees per acre ($\overline{D}$) Spurr gave the equation:

$$\overline{D} = \sqrt{13.5A - 160}$$

where A is the age of the plantation in years from the date of establishment.

Warrack (1959a) studied the diameter growth following thinning of an eighteen-year-old stand of Douglas fir and found that initial diameter was the best single criterion for estimating diameter increment. The inclusion of other

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variables, such as crown width or crown index (crown width x crown length), did not significantly improve the results. Rouse (1962) has published estimates of radial growth of Douglas fir for five- and ten-year periods for different initial diameters. His estimates were based on the British yield tables for Douglas fir (Hummel and Christie, 1953). Ker (1953) and Smith et al. (1961) described the growth of individual Douglas fir trees. A detailed study of the radial growth of Douglas fir and its relation to climate and soil has been made by Griffith (1960) on the University Research Forest over a period of five years. Crane (1962) has given formulae for predicting the average basal area increment per tree for radiata pine depending on whether the stand is open-grown, dense, or in the transition period between open-grown and dense.

Not only is it necessary to predict diameter and basal area growth of both stands and individual trees, it is also necessary to be able to describe the manner in which the distribution of diameter classes varies with age. One of the earliest, and probably the most extensive, studies in this field was carried out by Meyer (1930). Although diameter distributions may approximate the normal distribution, forcing them to do so often leads to serious errors in stand distribution tables. For this reason graphical methods are considered more acceptable than mathematical methods in fitting frequency distribution curves (Spurr, 1952). To be analysed mathematically the frequency distributions have to

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be defined by parameters which take into account departures from the symmetrical, normal distribution. These parameters are the coefficient of asymmetry (a measure of skewness) and the coefficient of excess (which represents approximately the extent to which the actual distribution differs in height from the corresponding normal curve). The Charlier Type A curve deviates around the normal frequency curve and is derived from:

$$F(x) = \frac{\mu_0}{\delta}\left[\phi_0(x) + \beta_3\phi_3(x) + \beta_4\phi_4(x)\right]$$

where:

$\phi_0(x) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(x-b)^2}{2\delta^2}} = \delta\phi(x)$, $\phi(x)$ being the normal probability function

$\phi_3(x) = \delta^4\phi^{III}(x)$, where $\phi^{III}(x)$ is the third derivative of $\phi(x)$

$\phi_4(x) = \delta^5\phi^{IV}(x)$, where $\phi^{IV}(x)$ is the fourth derivative of $\phi(x)$

μ_0 = number of trees,

β_3 = coefficient of asymmetry,

β_4 = coefficient of excess,

δ = standard deviation,

b = average diameter of the stand.

The Charlier Type B curve progresses from extreme negative symmetry to the normal curve and is derived from:

$$F(xw-c) = B_0\psi(x) + B_1\Delta\psi(x) + B_2\Delta^2\psi(x) + B_3\Delta^3\psi(x) + B_4\Delta^4\psi(x)$$

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where:

$$\psi(x) = \frac{e^{-2}\sin \pi x}{\pi}\left[\frac{1}{x} - \frac{\lambda}{x} - \frac{\lambda}{1(x-1)} - \frac{\lambda^2}{2(x-2)} - \frac{\lambda^3}{3(x-3)} \cdots\cdots\right]$$

$$\Delta\psi(x) = \left|\frac{1}{x} - \frac{\lambda}{1(x-1)}\right|; \quad \Delta^2\psi(x) = \left|\frac{\lambda}{1(x-1)} - \frac{\lambda^2}{2(x-2)}\right|; \text{ etc., etc.}$$

w, c, λ and the Bs are parameters which are defined by the conditions of the solution. Meyer (1930) fitted Type A curves to 113 distributions of Douglas fir. He found that the coefficient of asymmetry, β_3, decreased rapidly at first with increase in age but then more or less levelled off. The coefficient of excess, β_4, was positive (i. e. central class frequencies raised) but decreased with age and became negative. Type B curve fittings were superior to Type A when the average diameter was small (less than seven inches). Prodan (1953) also has fitted Charlier Type A curves to diameter frequency distributions of even-aged stands. Besides using the coefficients of asymmetry and excess in fitting diameter distributions of loblolly pine (Pinus taeda L.), Nelson (1964) also used the gamma distribution (see Appendix I) and obtained satisfactory results with the same distribution. Anderson (1937) has described the application of Fourier's series in forest mensuration.

A simple method of obtaining the range of a diameter distribution, given the mean, has been suggested by Smith and Ker (1960). They found that the minimum diameter is seldom less than half the mean and the maximum usually never more than twice the mean. Later, Smith et al. (1961) noted that in

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plantation-grown Douglas fir in New Zealand the upper limit was only 1.6 times the mean. Vezina (1963) found that in dense, natural stands of balsam fir (Abies balsamea (L.) Mill.), the most vigorous trees had diameters of about twice the average but also observed that the ratio of largest-to-average d. b. h. tended to decrease with age. This would indicate that the diameter distribution was probably becoming more normal.

Height Growth

The relationship of height to age is the best guide to site quality (Spurr, 1952). In spite of this important fact height growth and distribution have been less extensively studied than diameter growth and distribution. This is partly due to the fact that the height of standing trees is more difficult to measure and takes longer than the measurement of diameter at breast height. Spurr (1952) has described the form of typical height-over-age curves. For a short period in the early life of the tree height growth is exponential. There is then a long period when the growth curve is linear. The curve then gradually becomes horizontal as it reaches the maximum height for the site. Smith et al. (1960) have discussed the relative merits of natural and conventional height/age curves and Smith (1962) has given factors for converting height at any age between 10 and 100 years to height at age 50 years (site index).

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Mathematical expressions for describing height growth have been produced by Meyer (1940) and Coile (Schumacher, 1962). Meyer's formula is:

$$Y_c = H(1-e^{-ax})$$

where Y_c = height, x = age, and H = maximum height. The constant, a, varied between 0.04 and 0.12. Coile's equation is:

$$\log H = a - 6.528(1/A)$$

where H is the average height of 40 trees per acre in the dominant and codominant classes and A is the age in years.

The best practice is to predict mean height growth for the 100 largest diameter trees per acre rather than for the average height of dominants and codominants or the average stand height as has been done in the past. Mean height of the 100 largest trees is not so greatly affected by the density of stocking as is average height and therefore gives more consistent growth trends. Because age or thinning may reduce the number of stems per acre to below 100, Spurr (1963) tentatively suggested that the mean height of the 40 largest diameter stems per acre would be a better measure of stand height.

Stoate and Crossin (1959) claimed that the relation between height and girth (or diameter) at breast height can be used as an index of site where age is unknown or indeterminable. Height-over-girth curves were drawn for codominant trees on different sites and it was found that the girth at which the curve levelled out increased with improvement in

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site quality.

Volume Growth

The true, or cubic foot, volume curve follows the same sigmoid pattern as the height and diameter curves except that the period of exponential growth at the beginning of the life of the trees is more prolonged (Spurr, 1952). An equation has been developed to describe the growth curve of loblolly pine by Clutter (1962 and 1963). This is of the form:

$$\log_e V = a + b_1 S + b_2 \log_e B + b_3 A^{-1}$$

where V is the volume inside bark in cubic feet per acre, A is the stand age in years, S is the site index in feet, and B is the basal area in square feet. Clutter's equation for volume increment is obtained by differentiating the above equation with respect to age:

$$\frac{dV}{dA} = b_2 V B^{-1}(dB/dA) - b_3 V A^{-2}$$

where $\frac{dB}{dA} = - B(\log_e B)A^{-1} + C_o A^{-1} B + C_1 BSA^{-1}$. Thus:

$$\frac{dV}{dA} = b_2 V(\log_e B)A^{-1} + b_2 C_o V A^{-1} - b_2 C_1 VSA^{-1} - b_3 VA^{-2}.$$

The actual equations obtained for loblolly pine were:

$$\log_e V = 2.8076 + 0.015108S + 0.94931(\log_e B) - 21.863A^{-1}$$

and: $\frac{dV}{dA} = V(5.7907A^{-1} - 0.78166(\log_e B) + 3.6562 + 0.01741S)A^{-1}$.

Similar equations have been developed by Buckman (1962) for red pine (Pinus resonosa Ait.).

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Yield tables for Douglas fir have been produced by Barnes and by Alexander (both in U. B. C. Forest Club, 1959) and by McArdle et al. (1949) for the coastal region of the Pacific Northwest. Yield tables for Douglas fir in California have been developed by Schumacher (1930). Hummel and Christie (1953) published yield tables for plantation-grown Douglas fir in Great Britain. These plantations have been thinned according to standard Forestry Commission practice and the yield tables give the yield from thinnings as well as the main crop. Barnes (1956) discussed the application of the British tables in the Pacific Northwest. Grandjean and Van Soest (1953) published similar tables for the Netherlands and Pardé (1956) adapted the British tables for French plantations. Yield tables have been published for unthinned Douglas fir in New Zealand (Duff, 1956).

Stand Density

The three important factors describing stand density are trees per acre, mean height and mean diameter (Hummel, 1954). The product of these three variables is proportional to the "bole area" (Lexen, 1943; and Hummel, 1954) or the "cambial area" (Anucin, 1960). The bole area remains more or less constant from the time of canopy closure until the limit of height growth is reached. Although it is cumbersome to apply, it has the advantage that it is directly proportional to the actual growing surface of the stem. The product of number of trees and mean diameter is related to basal area.

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According to Hummel (1954) a more practical index of density, especially for thinning, is Hart's stand density index which is the average distance between trees expressed as a percentage of the mean top height of the 100 largest trees per hectare. Hummel suggested that it would be better to work with the largest 250 trees per hectare (or, approximately, the largest 100 trees per acre). An index of twenty per cent should be found after thinning to the British C/D grade.

Czarnowski (1961) put forward the hypothesis:

> In pure, even-aged stands of a given species growing on land of identical site quality and under conditions of comparable competition for growing space, the number of trees per unit of land area is inversely proportional to the square of the mean height of the stand.

For a measure of density Czarnowski defined the "crowding factor" which was the ratio of the actual number of trees per unit of land area to the normal number. The average d. b. h. of the stand is proportional to this crowding factor. Another measure of density used by Czarnowski was the "compactness factor", the ratio between the actual volume per acre and the maximum volume attainable for the site.

The use of "number of trees per acre" has the disadvantage that this number may vary widely without affecting density (Spurr, 1952). Basal area is not so affected and is simple, objective and easy to use but has the disadvantage that it gives equal weight to non-functioning heartwood and functioning sapwood and also that cambial (or bole) area is the first exponent of diameter and not the second (Nelson and Brender, 1963). Reineke's stand density index (Spurr,

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1952) gives the number of trees per acre, N, when the tree of average basal area has a d. b. h. of ten inches. This index can be obtained from the reference curve:

$$\log_{10}N = -1.6051\log_{10}D + K$$

where K is a constant varying with the species.

The crown width/d. b. h. ratios of Smith et al. (1961) can also be used as an index of density. Stands will then vary in density from a CW/D ratio of 0.7 for dense, to about one for well-stocked or "normal" stands, to two or more for nearly open-grown trees. Briegleb (1952) obtained regressions for crown width and crown length on tree height and d. b. h. for Douglas fir. Having done this he was able to calculate the "crown projection" and the crown surface area and suggested that, as these indices remained more or less constant regardless of stocking, they could be used as measures of density.

Other methods of measuring density have been discussed by Vezina (1962).

Thinning

Once the forest stand has been established there is probably no way in which the forester can alter its development so much as by thinning. In Europe and other countries where forest management is intensive, thinning is carried out at regular intervals from the time of canopy closure to the time of final harvesting. The heaviest thinnings are probably undertaken in South Africa where the trees are practically in

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a state of free growth for a large part of the rotation. Based on South African experience with exotic pines, Hiley (1948) drew up a schedule for thinning Douglas fir. Thinning in Britain is not heavy but the thinning intervals are short, varying between three years for young stands to five or six years for more mature stands (Hummel and Christie, 1953). In New Zealand, Douglas fir plantations are thinned on a ten-year cycle from age 30 years, removing one-third of the basal area at each thinning (Spurr, 1963). In the Pacific Northwest thinning has not been carried much farther than the experimental stage.

It is generally accepted that the increased growing space caused by thinning results in the improved diameter growth of the remaining trees, with the result that the merchantable volume is often increased (Mulloy, 1946). The immediate response may at first be a reduction in diameter growth of the remaining trees compared with unthinned trees due to 'shock' (Staebler, 1956a).

Growth reduction may be due to one of three reasons. Firstly, if the stand remains unthinned until the crowns of the trees become very small, then the trees will often not respond to thinning because their release increases total respiration more than total photosynthesis (Kramer and Kozlowski, 1960). Large amounts of carbohydrate are removed in the respiration of cambial tissue and the exposure of the stems to the direct heating effect of the sun results in greatly increased respiration in the stem tissues. Trees in

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overcrowded stands may possess insufficient food supplies to quickly develop enlarged crowns capable of increased photosynthesis. It is therefore important to thin early while the crowns are sufficiently large to shift the carbohydrate balance in favour of photosynthesis over respiration. The second possible reason is that on release the stem puts on increased growth at the base at the expense of growth higher up (Larson, 1963). Finally, reduction in diameter growth may be due to damage of the residual trees during the felling and extraction of the thinned trees. 'Shock' following thinning is probably confined to dense stands that are heavily thinned. Stephens and Spurr (1948) detected an immediate response to thinning in a twenty-year-old stand of red pine, radial growth being increased by 41 per cent within 24 hours. In this particular stand root competition was probably the limiting factor rather than crown competition as the soil was sandy. Part of this increase may have been due to swelling and not actual growth.

Height growth of trees is less influenced by thinning than is diameter growth (Hall, 1954). Kittredge (1927) and Mulloy (1946) found that thinning young stands of red pine did not affect height growth. In South Africa, thinning was found to affect the mean height growth but did not affect top height (Hiley, 1959). Staebler (1956) observed a reduction in height growth in the first season following thinning of Douglas fir in an experiment at Wind River, Washington. At Snow Creek, however, Worthington (1961) found that the mean

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annual height growth was increased from 2.2 to 2.4 ft. in the six years following a crown thinning in similar stands. Adams (1936) found that increased height growth occurred only in exceptional circumstances and that growth could be retarded for a few years following thinning. Spurr (1952) stated that at least some open-grown conifers do not attain the height of forest-grown trees and that a very high density will adversely affect height growth through stagnation. Generally, top height growth, or the height growth of dominant and codominant trees, is not affected by thinning. Any differences in height growth noted following thinning are probably due to the fact that mean height growth was measured.

The effect of thinning on volume growth varies. Unless the thinning removes such a large proportion of the trees that the site is not fully occupied for many years, the overall result is a gain in gross volume (including volume from thinnings) as a result of thinning. Hanzlik (1924), in describing a Norway spruce thinning experiment in Sweden, showed that the volume of an unthinned stand increased by 91 per cent in fifteen years as opposed to figures of 72, 63 and 27 per cent for stands treated with various thinning grades. Total volume production was far greater in the heavily thinned plots. Kittredge (1927) found that the periodic volume increment of red pine could be increased by 15 to 23 per cent by thinning. Li (1923) found this increase to be as high as 23 to 34 per cent for white pine (Pinus strobus L.). Of this increase, 43 to 57 per cent was attributed to utilization of

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material lost by mortality in the unthinned stand and the remainder due to accelerated production in the thinned stand. Moller (1947) refuted the hypothesis that thinning increases gross volume increment. German experiments indicated that the degree of thinning, even within very wide limits, had no influence on gross increment over an extended period of time. The increased increment caused by heavy thinning lasted only ten to twenty years. Generally, one would expect thinning, providing that it is not excessively heavy, to increase the merchantable volume of the stand and the gross volume yield but to have little effect on the total cubic-foot volume of the stand.

Various methods of thinning and thinning schedules have been derived to suit different conditions. Of interest to foresters in the Pacific Northwest is the work of Heiberg and Haddock (1955), Warrack (1959b) and Staebler (1960). Heiberg and Haddock suggested that site index 150 Douglas fir should have thirteen thinnings between the 28th and the 85th years to reduce the number of stems from 735 to 47 per acre. Under such a schedule volume production is 39 and value 54 per cent greater than in unmanaged stands. This number of thinnings is considerably greater than that usually advocated in this region and, apart from the delay in their commencement, resembles European practice. Unless the stands were relatively free growing to their 28th year it would probably be more economical if a "cleaning" had been done much earlier in the rotation so that the trees were at the spacing desired

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at age 28 or, if planting was carried out, to plant the trees at that spacing. Warrack (1959b) prescribed thinning to the d/D ratio, where d is the average d. b. h. of the thinnings and D the average d. b. h. of the crop before thinning. A ratio of 0.65 indicates a cleaning, 0.65 to 0.75 is a low thinning, 0.75 to 0.90 is a severe low, or a light crown, thinning; and a ratio greater than 1.00 is a selection thinning. Thinning schedules were given for a natural stand and for a plantation showing the number of trees, average d. b. h. and height, basal area and volume before and after thinning.

Staebler (1960) based his thinning schedules on the assumption that the total production of cubic volume by a stand of a given composition on a given site was constant and optimum for a wide range of stocking densities. The length of his thinning cycle corresponded to a ten-ft. increase in the total height of dominants and codominants. For site index 170 it was assumed that the average d. b. h. at age twenty years was eight in. The schedule was drawn up so that the diameter growth in the 21st year was one-half inch and the growth rate decreased thereafter at a rate of 0.0035 in. each year. The main disadvantage with Staebler's model is that he assumed all the trees in the stand, including those removed in the thinning, were the same size.

These three papers show an advance in stand management in the Pacific Northwest over the past twelve years for, in 1952, Warrack was advocating not thinning until there were

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sufficient trees (without removing all the largest) to make it profitable. This would have been at an age of 35 years and might in practice have led to exploitation fellings rather than thinnings. Alternatively, a thinning could have been carried out before the crop was twenty years old, removing all but the dominants and best codominants.

In other regions, Hummel (1954) has defined thinning treatments by means of Hart's stand density index and Johnston and Waters (1961) have suggested controlling thinnings by means of a basal area/top height curve. Spurr (1948) advocated row thinnings as they are cheaper and practically as efficient as selective thinnings. They are good for improving root and soil conditions but lead to uneven crown development. Row thinnings may be useful in young plantations in the first and second thinnings or until the thinned produce becomes merchantable. Little and Mohr (1963) recommended the removal of every third row when thinning loblolly pine.

The effect of thinning on the growth of individual Douglas fir trees has been studied by Staebler (1956b) and Krueger (1959). In a 41-year-old natural stand, Staebler found that, after three years, the effect of release produced a greater d. b. h. growth in dominant trees than in codominant or intermediate trees. There was a progressive increase in diameter growth as from zero to three competitors were cut from around each tree. This increase was most marked between zero and one and least marked between two and three competitors cut. Dominant trees which were thought to be growing at

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the maximum rate possible for the site responded well to thinning. The amount of release did not affect stem form (Yerkes, 1960). Krueger (1959) reported on a similar experiment carried out in a 30-year-old Douglas fir plantation. The results were similar to those obtained in the natural stand except that there were no significant differences in diameter growth between the different amounts of release.

Guillebaud and Hummel (1949) made observations on the movement of tree classes in Douglas fir (and other coniferous) plantations in Great Britain subjected to different grades of thinning. In nearly every case, dominants showed a net loss and subdominants (intermediates) a net gain. It was possible for codominants to move up to the dominant crown class.

Thinning may also affect the eventual yield of a stand in indirect ways. Rishbeth (1951) has reported that thinning increased the risk of attack by butt-rot (Fomes annosus Fr.) in Douglas fir plantations in East Anglia. Conversely, Weir and Hubert (1919) claimed that the thinning of western hemlock (Tsuga heterophylla (Raf.) Sarg.) and grand fir (Abies grandis Lindl.) made conditions less favourable for fungal attack. Mulloy (1946) found that thinning reduced storm damage in red pine. It has been suggested that thinning may reduce the volume of standing timber destroyed by fire by as much as 50 per cent (Staebler, 1955a). All these factors, although to a large extent unpredictable, affect the growth of the forest stand.

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Natural Mortality

Natural mortality can be caused by the suppression of the weaker trees by surrounding, more vigorous trees, by insects, by fungi, by extremes of climate, by fire or by a combination of any of these effects. Often the cause of death cannot be attributed to any particular one of these. Usually one agent causes a decline in a tree's vigor, a second carries it further and, occasionally, a third agent completes the killing process. The loss to the forest can take two forms, complete loss of individual trees due to death or a weakening of the trees causing a decline in the growth processes with subsequent loss in volume increment. Spurr (1962) found that radiata pine trees growing on pumice in New Zealand began to exhibit a marked decrease in growth from at least six to eleven years before death. Trees showing a $(\text{d. b. h.})^2$ increment of less than four $(\text{inches})^2$ had a life expectancy of less than eight years.

Mortality due to Fungal Attack

The most important fungus causing death and decay in Douglas fir stands in the Pacific Northwest is the laminated root-rot, Poria weirii Murr. It is present in most Douglas fir stands but is not always serious (Childs, 1955). Trees of all ages and sizes are attacked but more especially those trees from about 40 to 125 years old (Childs, 1960). The disease occurs in patches or centres of infection (Plate II)

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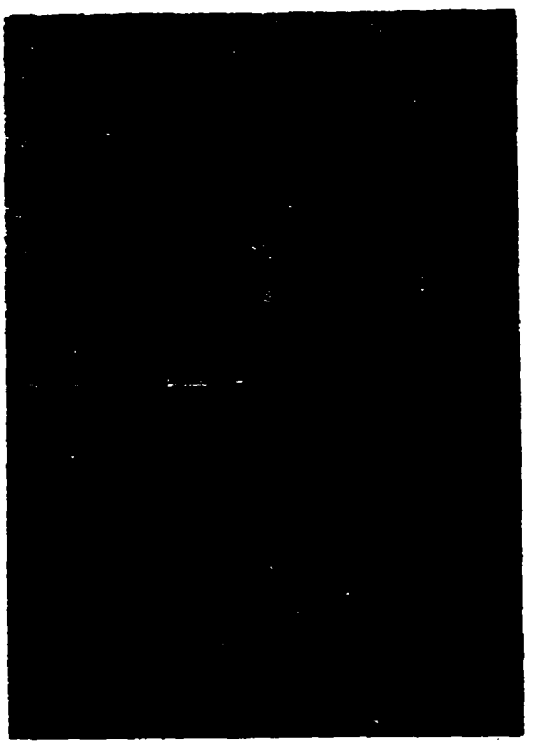

PLATE II: Group-dying of Douglas fir caused by *Poria weirii* Murr., Wind River, Washington. Photographed in August, 1963.

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from a few hundred square feet to an acre or more in extent. The distribution of the disease has been described as "erratic" (Anon., 1955). Within a typical centre there will be several trees, standing or down, that have been dead for differing lengths of time. A few of the living trees in the centre may be showing signs of attack - leaning, thin crowns or poor colour. In a 112-year-old stand of Douglas fir the percentage of living trees having infection visible on the stump was found to vary inversely with the distance to the nearest tree killed by the disease (Anon., 1955). Within ten feet, 88 per cent of the trees were affected but at a distance greater than 50 feet only four per cent were affected. Infection was usually on the portion of the stump nearest the dead tree but the extent of decay within the tree showed no correlation with the distance from the killed tree. The rate of damage usually doubles every ten to twenty years (Childs, 1955). The fungus spreads when spores infect wounds at or near the base of living trees (Childs, 1960). The fungus can live in dead roots for 50 years or more. In young stands, damage increases in geometric proportion to age as the infection centres enlarge. Infected trees seldom respond well to release from competition and are often wind-thrown within a few years if not killed by the fungus (Childs, 1955). The wind-thrown trees often provide breeding material for bark beetles (Wright and Lauterbach, 1958).

Foster and Johnson (1959a, 1959b and 1960) have carried out a series of disease-sampling studies in young

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(thirteen to seventeen years) Douglas fir plantations on Vancouver Island. In the first thirteen years after planting the Douglas fir had suffered a 57 per cent mortality. There was a high incidence of root rot. The shoe-string fungus, Armillaria mellea (Vahl. ex. Fr.) Quel., was most prevalent but Poria weirii and Fomes annosus (recorded for the first time in a plantation in British Columbia) were also present. The root disorders were aggregated or contagious and followed the negative binomial distribution when the stands were sampled with plots varying in size from 4/400 to 36/400 acres. Terminal leader dieback, frost-lesions and sunscald were also observed. These injuries were usually randomly distributed and followed the Poisson model. In the two later papers (Foster and Johnson, 1959b and 1960) the red heart-rot, Stereum sanguinolentum A. & S., was found to be present and it was thought that this fungus would lead to mortality in the future. Most of the openings caused by root-rot were less than 9.9 ft. in diameter and of insufficient size to support a tree more than 22 years old. Only 8.2 per cent of the 574 infection centres were greater than 9.9 ft. in diameter and 1.6 per cent were greater than 15.2 ft. (Foster and Johnson, 1963a). Further details of the assessment of pattern, frequency distribution and sampling of forest disease in Douglas fir plantations have been given in Foster and Johnson (1963c).

Fomes annosus is very common in Douglas fir plantations in Great Britain. Rishbeth (1951) found that trees

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were killed near infected stumps soon after planting. Stumps caused by thinning are colonized by the fungus and act as centres of infection for the surrounding trees. It may be advisable to delay thinning until the trees are over 25 years, at which age they are more resistant to attack.

Other fungi that may cause damage in Douglas fir stands in the Pacific Northwest have been described by Harvey (1962).

Mortality due to Insect Attack

The only insect causing mortality and volume loss of economic importance in the Pacific Northwest is the Douglas fir bark beetle, Dendroctonus pseudotsugae Hopk. According to Evenden and Wright (1955) it is present at all times, killing scattered trees and small groups (probably in association with root-rot). When it throws off the controlling effects of its natural enemies it becomes epidemic, killing much of the Douglas fir over large areas in a few years. It can apparently kill healthy trees but prefers windfalls, damaged or defoliated trees and logging slash. Mathers (1951) found that in a 700-acre stand near Quesnel, British Columbia, that the beetles spread in a northeasterly direction. There were fourteen pockets of attack ranging in size from two to twenty trees. There was no correlation between the trees killed and crown class, shape or size of crown or d. b. h. but there was with tree vigour, those trees with a slower growth (possibly due to fungal infection) being most liable to attack. Thomas

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and Craig (1958) found that winter injury due to frost-weakened trees, particularly dominants and codominants, had made them more susceptible to attack by Dendroctonus. In the Millicoma Forest, Oregon, which has 150,000 acres of predominantly Douglas fir, Dendroctonus was responsible for 59 per cent of the average annual mortality. Walters (1954) classified trees as to susceptibility to attack by age and vigour. Older, slower-growing trees were most liable to infection. A typical group of beetle-killed trees is shown in Plates IIIa and b.

Mortality caused by insects, unless it is associated with fungal infection, is generally scattered or randomly distributed in the forest stand. Little harm is caused by small "kills" as the surrounding trees benefit from the release (Hoffman and Anderson, 1945). Foster and Johnson (1963a) suggested that such mortality may be beneficial in overstocked stands although it is detrimental in understocked stands.

It can be seen that the usual pattern of mortality in Douglas fir stands follows the pattern of, first, infection by root-rot, which makes the tree more liable to windthrow which in turn provides suitable breeding material for bark beetles (Mathers, 1951). When conditions are suitable, the bark beetle population rises and an epidemic may occur.

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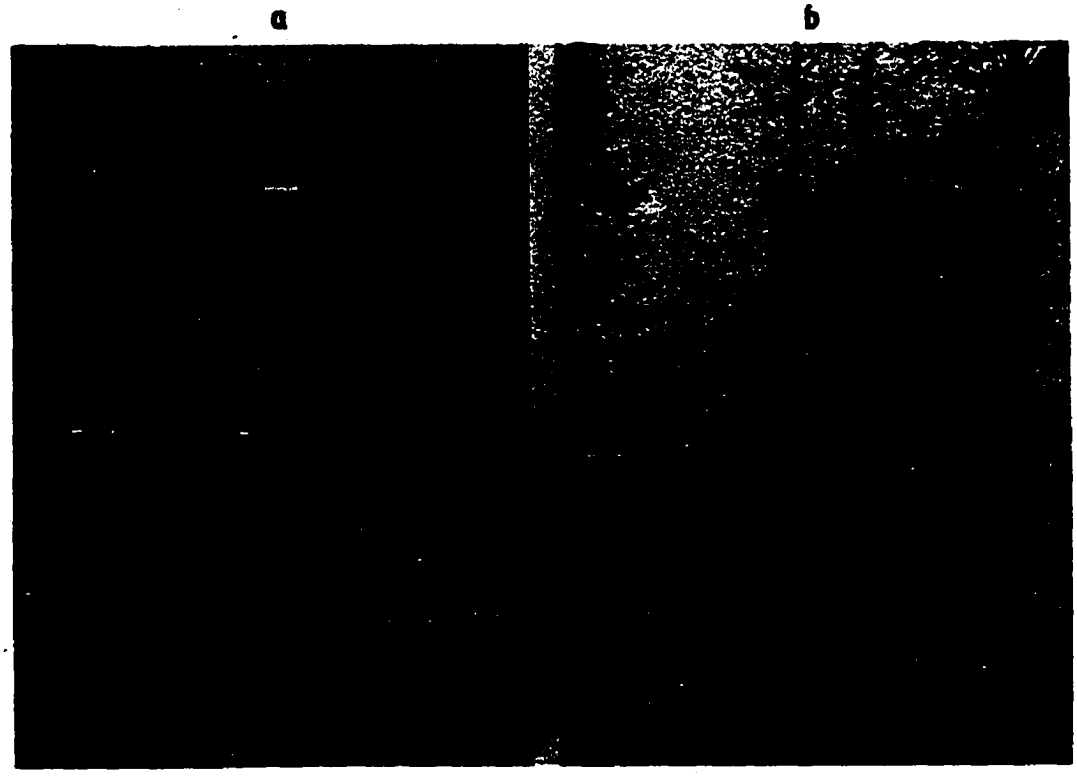

PLATE III(a,b): Group-dying of Douglas fir caused by Dendroctonus pseudotsugae Hopk., Paul Lake, Kamloops. Photographed in July, 1963.

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Natural Mortality through Suppression

Whereas the number of trees killed by insects, fungi or climatic extremes and their distribution cannot be accurately forecast, an attempt can be made to forecast natural mortality through suppression. It is known that, providing external forces do not come into play, the smallest, or suppressed trees in a stand will be the first to die as the stand becomes older. From permanent sample plots it is possible to tell, for any site quality, how many trees per acre there will be in a normally stocked stand at any given age. From this it is possible to estimate the number of trees that should die in a given period. Because the largest trees and the smallest trees are never evenly spaced over the area, the mortality will not remove entirely the smallest diameter classes but will be generally confined to them. Staebler (1953) has estimated the mortality in fully-stocked stands of young-growth Douglas fir (aged 26 to 93 years) on 36 permanent sample plots in Washington and Oregon. Plots where "irregular" mortality had occurred were rejected. He derived two equations for determining the percentage of trees that will die in a ten-year period. These are:

for dominants and codominants,

$$\% \text{ mortality} = 4.96 + 0.08(\text{age}) - 0.41(\text{d.b.h.})$$

$$(R = 0.266)$$

for intermediates and suppressed,

$$\% \text{ mortality} = 13.01 + 0.54(\text{S.I.}) + 0.61(\text{age}) - 783(\text{d.b.h.})$$

$$(R = 0.715)$$

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Gross yield and mortality tables were given in a later paper (Staebler, 1955b). Further data on mortality in Douglas fir have been given by Eversole (1955) and Griffith (1960).

Conclusions

From this review of the literature it can be seen that much is known qualitatively but little is known quantitatively about the growth of Douglas fir. Our quantitative knowledge is confined to the published yield tables based on "normal" or fully-stocked stands and to more or less localized research projects which, although yielding much valuable information, cannot be applied on a general basis in the Douglas fir region with any degree of reliability. In the development of a mathematical stand model assumptions will therefore have to be made on the basis of the available information.

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PART II

DEVELOPING A STAND MODEL FOR DOUGLAS FIR

The object of the present study is to develop a mathematical model that will describe the growth of a Douglas fir stand from an age of ten years, by which time it is assumed that all the trees will have reached breast height (4.5 ft.), to an age when the stand might normally be expected to be harvested. This is assumed to be at, or before, 100 years.

A sound mathematical model should have as its basis sound biological theory. Unfortunately our knowledge of the growth of forest stands is not by any means complete, particularly in the period between the onset of competition between trees in the stand and the time when mortality, through suppression of the weaker trees, occurs and the stand becomes normally stocked (cf. the open-to-normal concept of Smith et al., 1961). Table 111 of Smith et al. (1961) indicated that the length of time taken to grow Douglas fir stands to an average d. b. h. of twelve inches can be reduced by 30 to 40 per cent if the stands are established at such an open-spacing that they become "normal" when the average d. b. h. is twelve in. Because of the gaps in our knowledge of tree growth, certain assumptions have been made in developing the model which it is hoped will be justified when the model is compared with field conditions. All assumptions made will be discussed fully.

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Stand Model I

This model has been described in detail in an earlier report (Newnham, 1963).

In this, as in later models, initial square spacing was assumed, that is, trees could be located only at the intersections of a square lattice. The pattern could be modified by omitting trees from certain locations. The reasons for adopting square spacing have been discussed in Part I of this thesis. Its main advantage in model development is that it facilitates computations. Stocking was assumed to be 1,000 trees per acre at age ten years, decreasing to 250 trees at age 50 years. A basic matrix of 100 trees was used with each tree being given a rank number depending on the magnitude of its d. b. h., rank No. 1 being the largest tree and rank No. 100 the smallest.

Competition was evaluated by comparing the rank number of each tree with those of the surrounding trees. If a "competitor" had a lower rank number than the tree being studied, the rank number of the tree was increased by an amount inversely proportional to the distance of the "competitor" from the tree. At the end of each five-year period those trees having the greatest increase in rank number were considered to have "died" until the desired level of stocking was obtained. The process was repeated at five-year intervals to age 50 years. Diameter growth was predicted by assuming a constant rate of basal area growth for open-grown or free-growing, trees (Spurr, 1952). The five-year d. b. h. growth

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of each tree was then reduced by an amount proportional to the rank position of the tree and also its increase in rank during the five-year period.

Although the method used in this model was based on arbitrary assumptions, the results (see Newnham, 1963) conformed to a pattern that might well be assumed to occur in nature. It had the advantage that the calculations were straightforward and could be rapidly carried out. The main disadvantage was the difficulty of adapting it for different initial spacings and distributions of trees as it was designed to consider only those competitors within 13.2 feet of the tree being studied. For this reason work on this model was discontinued.

Stand Model II

The early work on the development of this model has been previously described (Newnham, 1963 and 1964). Most of this work consisted of varying the values of the parameters used in the model in order to make it give results which compared favourably with the published yield table data.

The Basic Principles and Assumptions

Data were collected in the interior of British Columbia (Paul Lake, near Kamloops), in the coastal region of British Columbia (Saanich Peninsula, Vancouver Island and the University Research Forest, Haney) and on the western slopes of the Cascade Mountains in Washington State (Wind River).

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The relationship between crown width (the sum of the measurements of the longest branch on two sides of the tree) and diameter at breast height outside bark was calculated from these. This relationship could best be described by two straight-line regressions, one for trees less than three in. d. b. h., based on data collected from a seven-year-old plantation established at Haney at a spacing of 9 x 9 ft. in which the crowns were not overlapping (Plate Ic), and one for trees three in. in d. b. h. or greater, based on the data collected from the remaining sites (Fig. 2). These regressions are:

trees < 3 in. d. b. h. $CW = 2.270 + 2.399D$ $r = .820$,
$N = 274$, $S = \pm 0.765$ ft.

trees ≥ 3 in. d. b. h. $CW = 5.031 + 1.423D$ $r = .917$
$N = 152$, $S = \pm 4.517$ ft.

These results are nearly identical to those published by Smith et al. (1961) and the more recent results obtained by Smith and Jakoy (1963, unpublished data) from measurements obtained on the University Research Forest at Haney.

The relationship between diameter at breast height and age was also studied for these open-grown trees by taking increment borings. Age was determined by counting the number of annual branch-whorls below breast height and adding this number to the number of rings on the increment boring. The relationship was found to be persistently linear for the Kamloops data (Fig. 3) and also for the poorer sites on the

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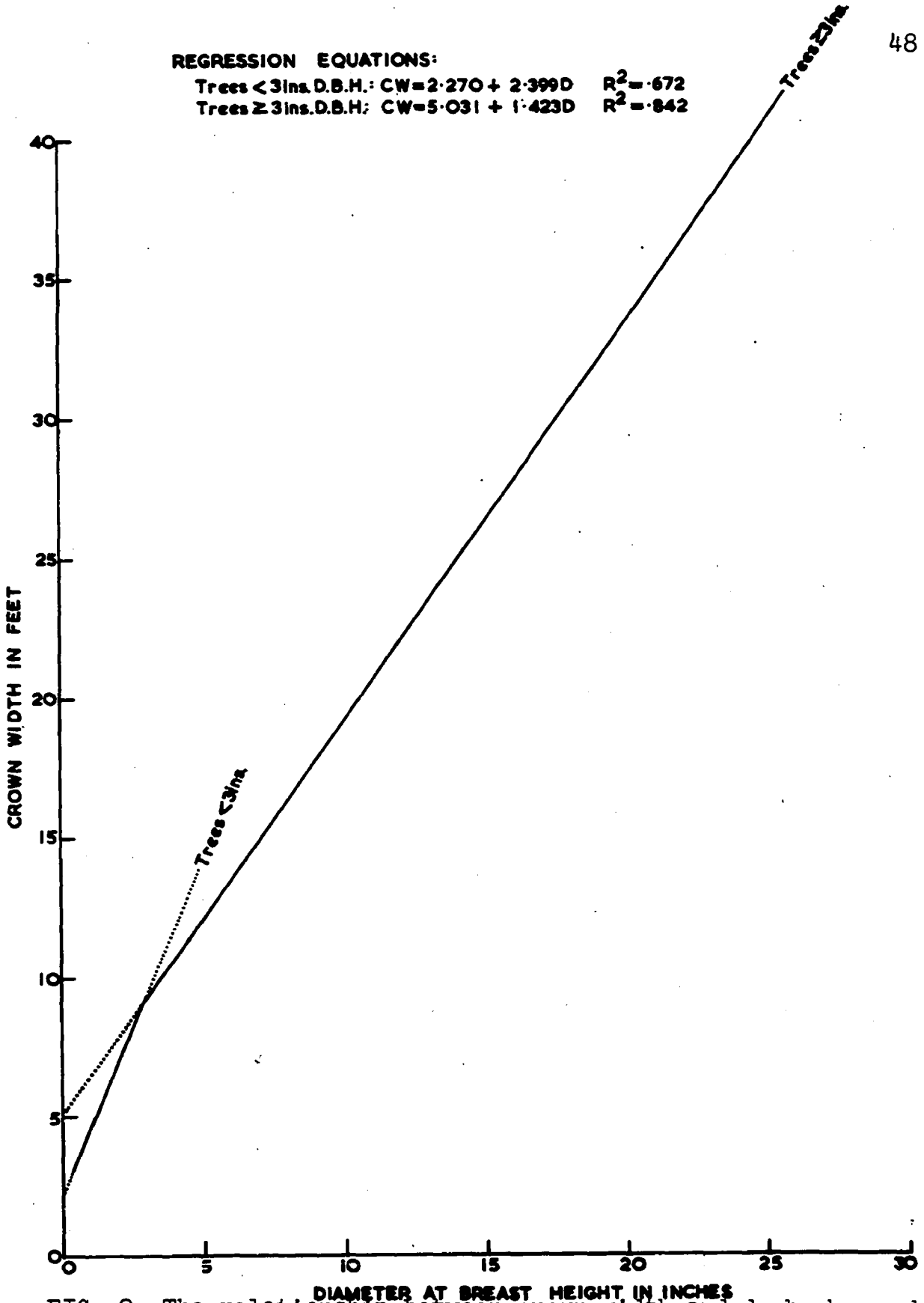

FIG. 2: The relationship between crown width and d. b. h. o. b. of open-grown Douglas fir.

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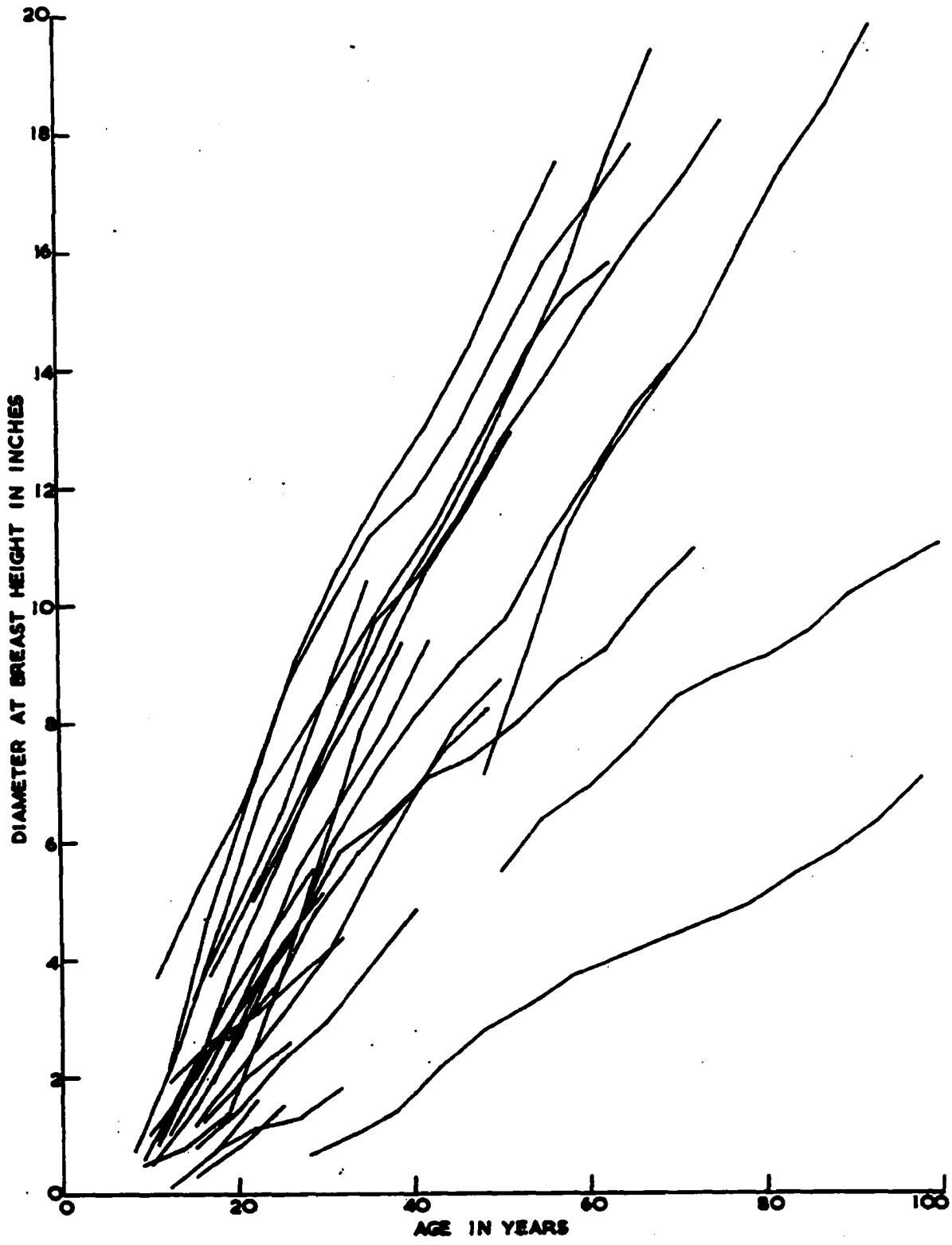

FIG. 3: The relationship between d. b. h. i. b. and age. Open-grown Douglas fir, Paul Lake.

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Saanich Peninsula (Fig. 4) and at Wind River (Fig. 5). On the better sites, the rate of diameter growth tended to decrease with increase in age. For the Saanich data a regression was calculated for five-year radial increment ($_{5yr}R_g$) on age, diameter at age 25 years (D_{25} - interpolated where necessary from the d. b. h./age curves of the individual trees) and the initial diameter at the beginning of each five-year period (D_i). Trees were rejected if there were any apparent errors in age estimation. The regression was:

$$_{5yr}R_g = 0.9920 - 0.07283D_i + 0.15388D_{25} + 0.01969Age - 0.00005029Age^2$$

$N = 141$(from 18 trees), $R = 0.784$, $s = \pm 0.270$ in.

To facilitate programming, the regression of D_{25} on D_{10}, the diameter at age ten years, was later calculated ($D_{25} = 1.706 + 2.754D_{10}$, $r = 0.950$, $s = 1.128$ in.) and the final regression became:

$$_{5yr}R_g = 0.06338 - 0.07223D_i + 0.4237D_{10} + 0.01969Age - 0.00005029Age^2$$

In the model this last regression has been applied to outside bark measurements whereas the regression is based on inside bark measurements. The error involved was not great enough to cause any serious defect in the model because it could only underestimate growth in d. b. h. by eight to twelve per cent.

The Saanich data were selected as they were representative of site conditions in drier parts of the coastal

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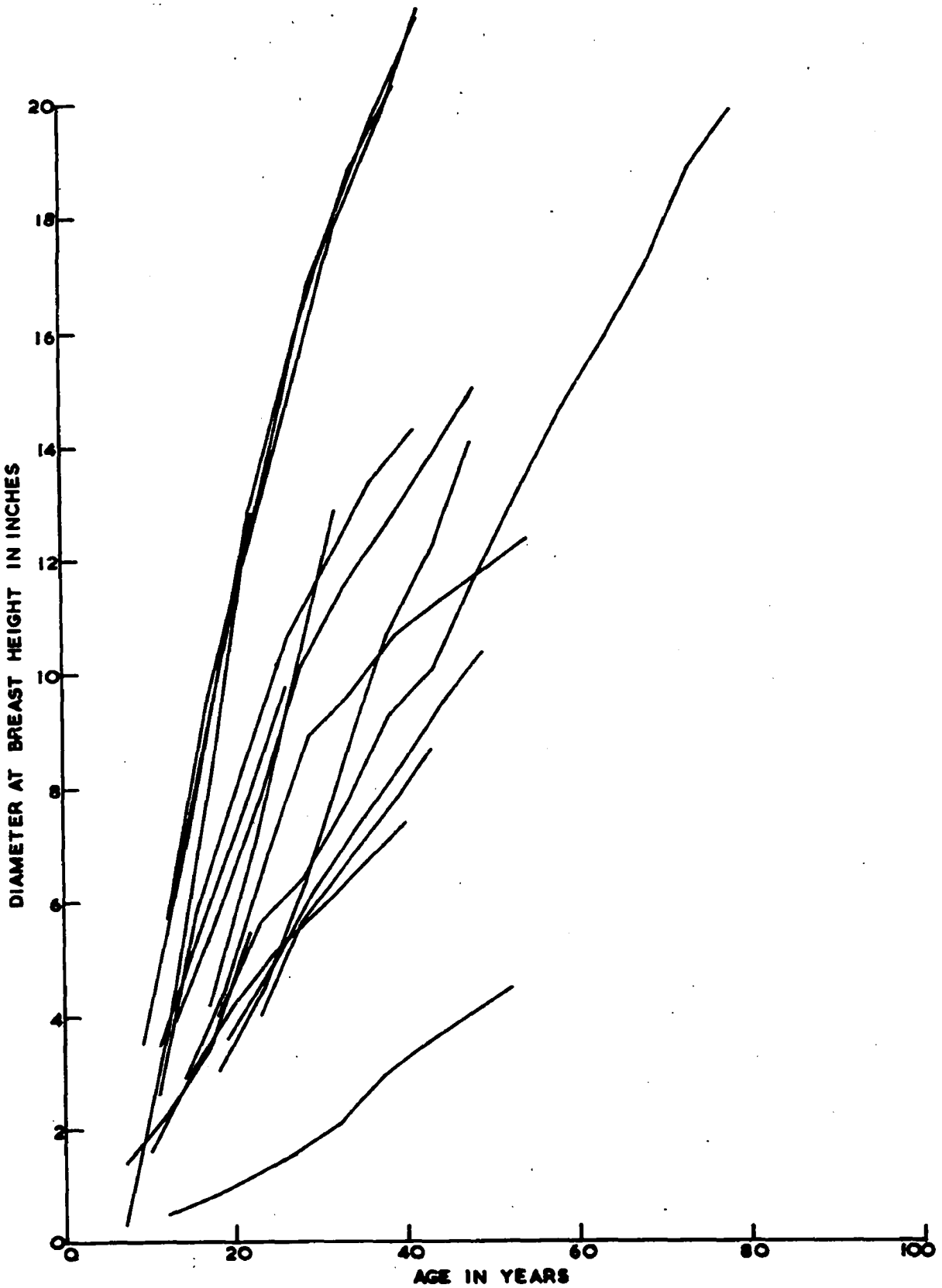

FIG. 4: The relationship between d. b. h. i. b. and age. Open-grown Douglas fir, Saanich.

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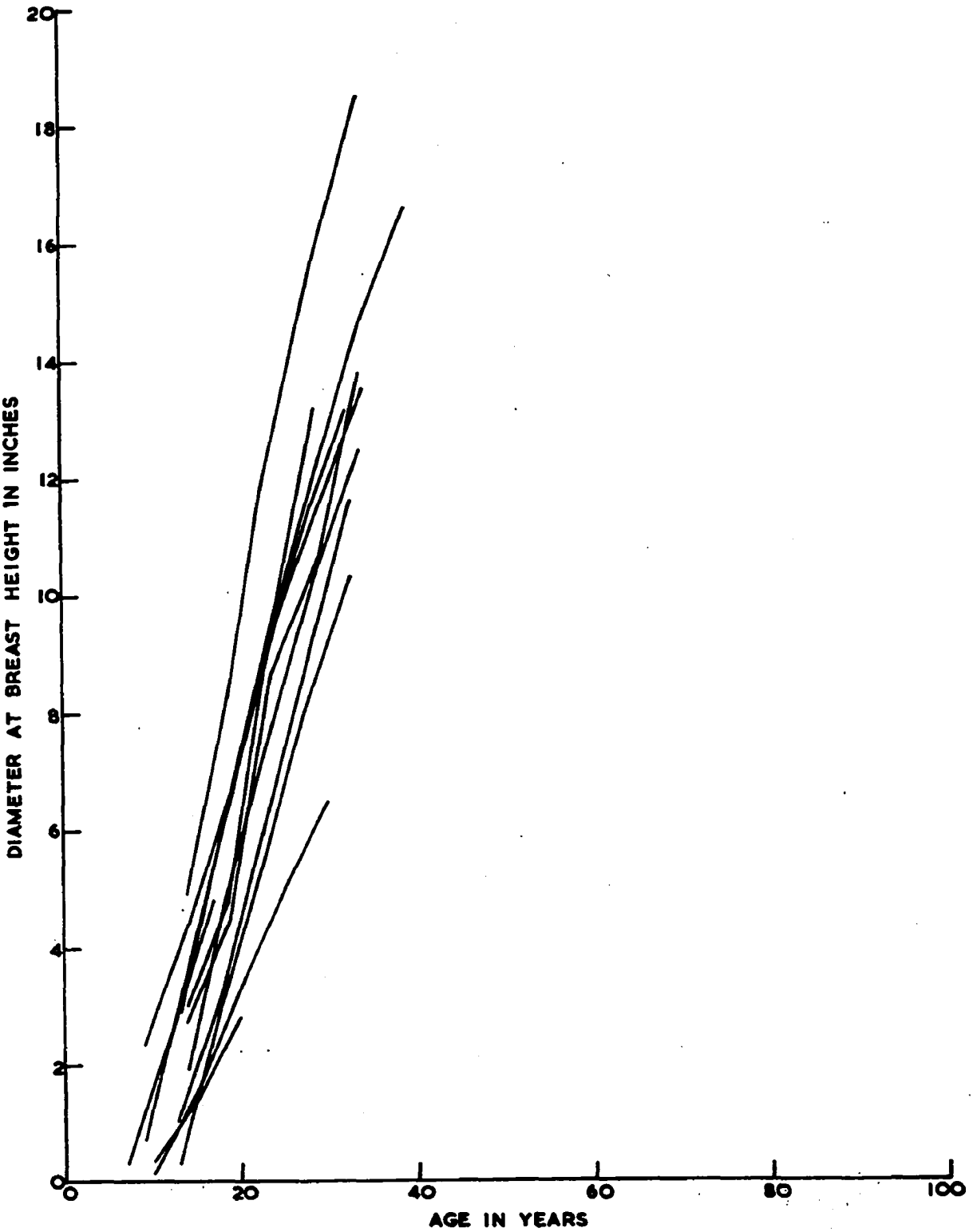

FIG. 5: The relationship between d. b. h. i. b. and age. Open-grown Douglas fir, Wind River.

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region. The Kamloops data were rejected on the grounds that their inclusion would have given too much weight to the poorer sites not often encountered in the coastal region; the Wind River data were not included as the trees were not sufficiently old. Diameter/age curves were constructed using the above regression and are shown in Fig. 6. It can be seen that the calculated diameter/age relationship is more or less linear on the poorer sites but shows some curvilinearity on the better ones.

For the model it was necessary to estimate the approximate age at which trees, initially established as open-grown, came into competition with one another. As the determination of root spread is difficult in practice, crown spread has often been used as an indicator of root spread. This relationship was studied in detail on the open-grown Douglas fir trees at Paul Lake, Kamlopps. The annual precipitation in this region is low (fifteen to twenty inches) and the soils are therefore dry. The general mass of roots was found not to spread much beyond the extent of the crown (Fig. 7) although occasional roots spread to greater distances. When studied in the other areas sampled this relationship also appeared to hold. Crown spread was therefore assumed to be a good indicator of root spread when considering the competitive status of trees.

In the seven-year-old plantation established at Haney at an initial spacing of 3 x 3 ft. it was found that, although the branches of the trees overlapped by as much as

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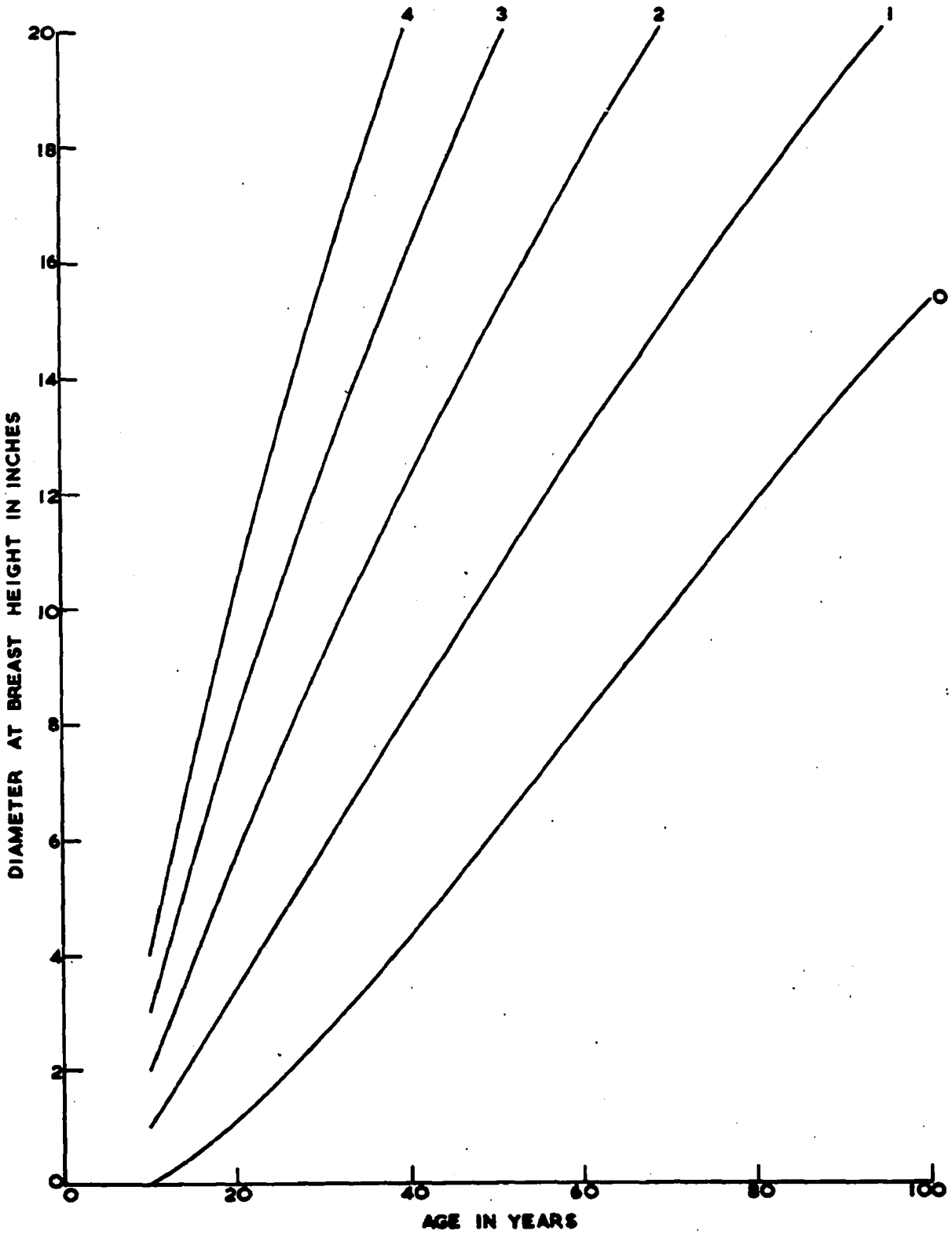

FIG. 6: Calculated d. b. h. o. b./age curves (by inch d. b. h. o. b. classes at age 10 years).

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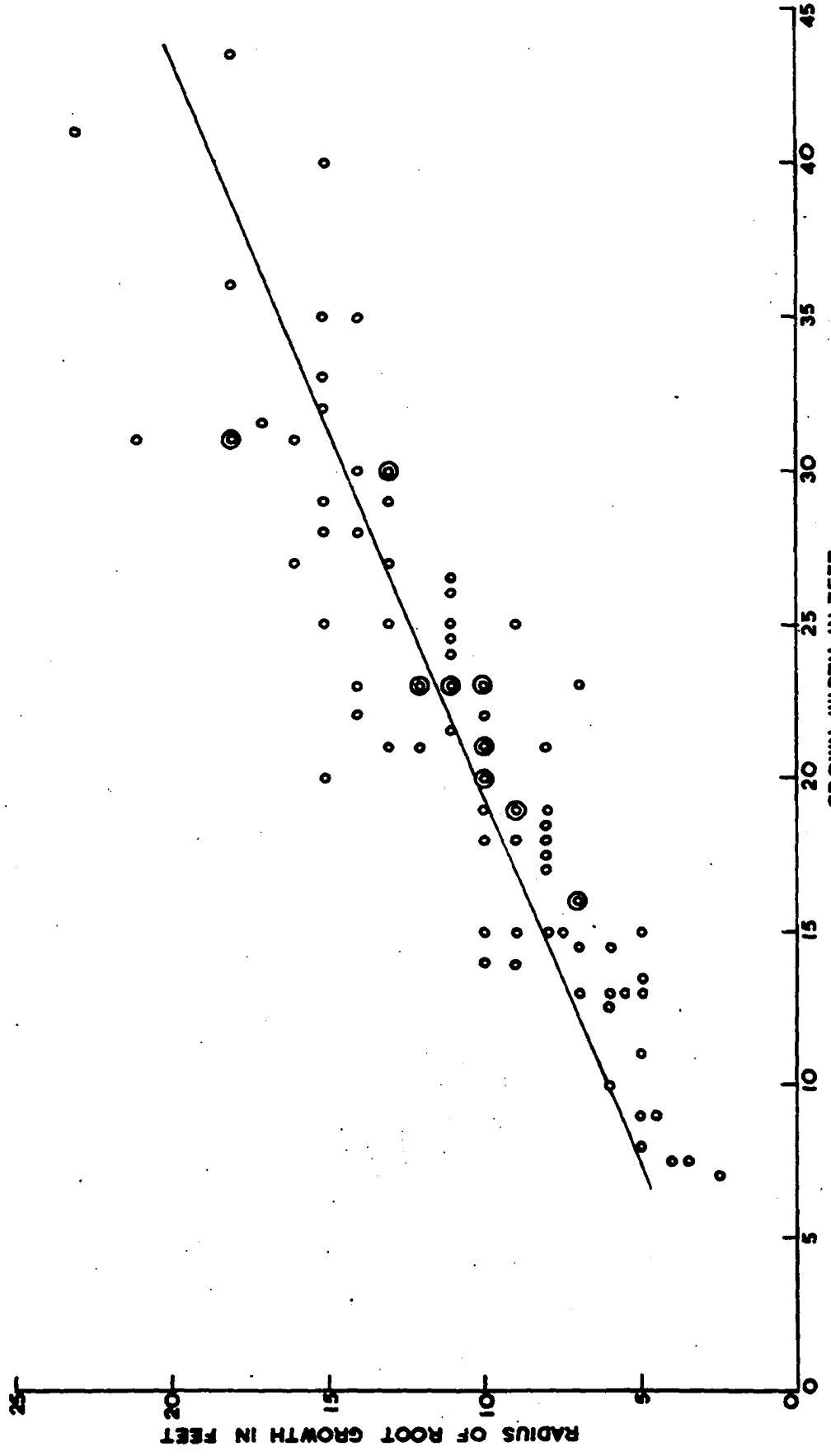

FIG. 7: The relationship between root growth and crown width of open-grown Douglas fir. Paul Lake.

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50 per cent, there was no apparent reduction in diameter growth compared with the plantations established at wider spacings which were still open-grown (see Fig. 8 and Plates Ia-d). To determine the competitive status of each tree in the early stages of the model, the crown widths (determined from the d. b. h. of each tree using the crown width/d. b. h. regression) were therefore multiplied by a reduction factor (the variable "REDFAC" in the FORTRAN program) which was chosen after testing several different values using the basic matrix of diameters. The new measurement thus obtained was called the competitive crown width.

For a starting point in the model, the diameters of a matrix of 15 x 15 trees were measured in the 6 x 6 ft. plantation (aged seven years) at Haney. These trees were growing on a good site (probably site index 170 feet at 100 years). To adapt these measurements to a poorer site more commonly encountered in the coastal region of British Columbia, it was assumed that the same diameter distribution would be found on site index 140 but at an age of ten years.

Description of the Stand Model

As stated, the model started with a matrix of 15 x 15 trees at age ten years. The number of trees per acre therefore varied with the initial spacing ranging from 4000 trees per acre at a 3.3 x 3.3 ft. spacing to 250 trees per acre at a 13.2 x 13.2 ft. spacing. To avoid "edge effects", it was assumed that this matrix was repeated every fifteen

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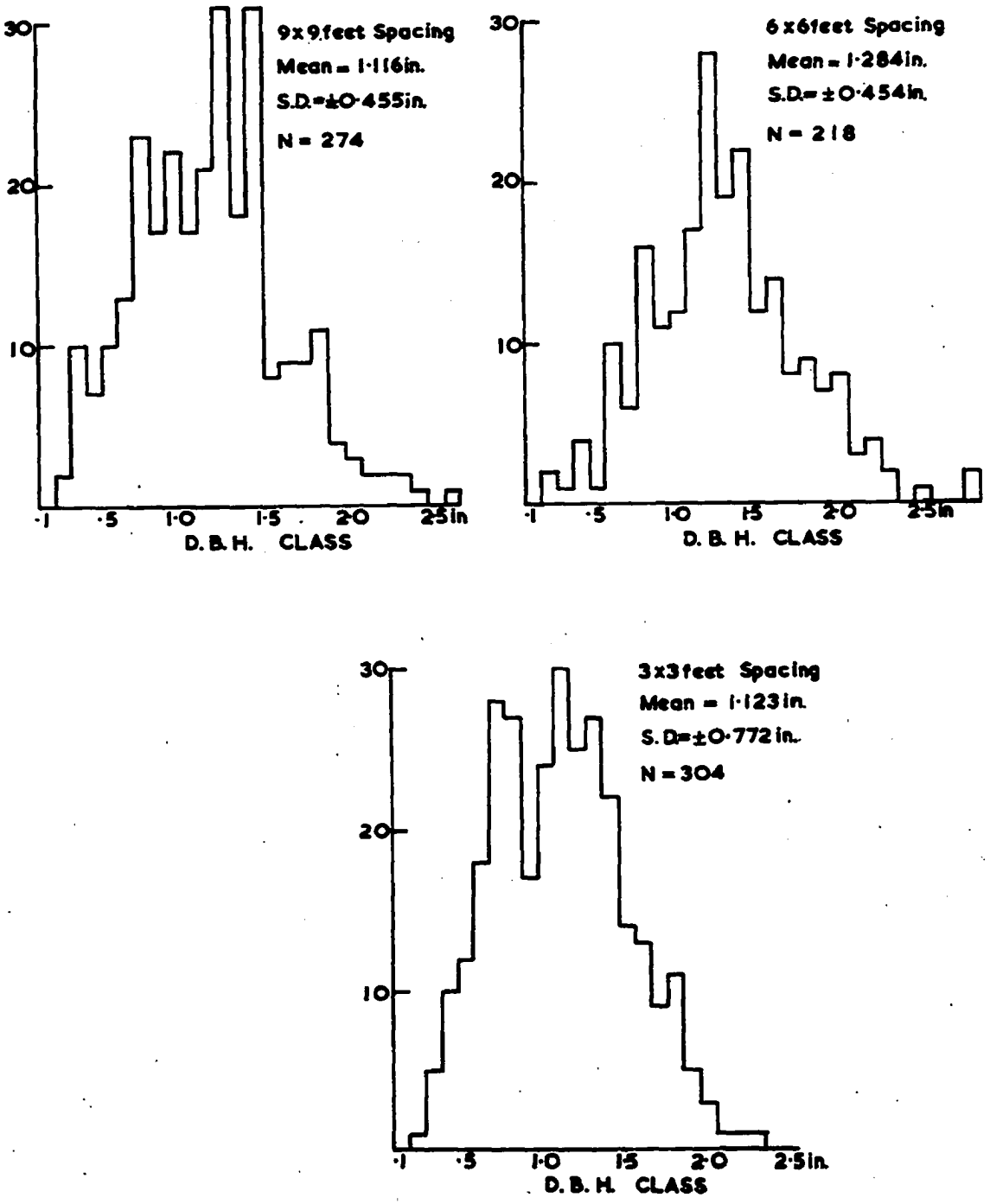

FIG. 8: Diameter frequency distributions of seven-year-old Douglas fir plantations established at different initial spacings. University Research Forest, Haney.

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rows and every fifteen columns. Thus the trees in row No. 15 were next to the trees in row No. 14 on one side and row No. 1 on the other. The diameters at breast height of these trees were known (Table 1) and from these the crown widths of the trees could be calculated (see Fig. 1) and the competitive crown widths obtained using the reduction factor, "REDFAC".

Taking one tree in the matrix at a time the model was tested to see if any of the surrounding trees were competing by determining whether the competitive crowns overlapped. If overlap occurred, the angle subtended at the centre of the crown by the two points of intersection of the competitive crown perimeters for each competitor was measured (in radians: 2π radians = 360°. See Fig. 1b). This measurement was weighted in each case by the ratio of the crown width of the competitor to the crown width of the tree being studied, thus recognizing that the trees with the larger crowns usually had the added advantage of being taller. The method of calculating this angle is given in the description of the FORTRAN program in Appendix II (see SUBROUTINE CROWN). For each tree the sum of these angles was divided by 2π to give the proportion of the circumference of the "competitive" crown of the tree occupied by the crowns of its competitors (the FORTRAN variable "SOC"). Thus a value, which varied between zero and one (or more if the circumference of the crown was occupied by several overlapping crowns), was obtained for the competitive status of each tree.

The sum of the angles subtended at the centre of a tree by the intersections of the crowns of the competing

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TABLE 1: The initial d. b. h. matrix used in the development of the model. Data from the University Research Forest, Haney, B. C.

Row No.	Diameter at breast height (inches)														
	Column No.														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.4	1.4	1.5	1.0	1.5	2.0	1.6	1.3	1.2	1.0	0.7	1.1	1.4	1.8	1.7
2	1.5	1.1	0.2	1.7	0.9	0.9	1.4	1.5	1.5	0.8	1.3	1.2	1.4	1.6	0.8
3	1.3	0.9	1.0	0.8	0.9	1.0	1.9	1.7	1.7	0.7	1.5	1.4	1.3	1.6	1.2
4	0.9	1.2	1.7	0.5	1.9	1.9	1.1	2.0	1.8	1.3	1.0	1.7	0.8	1.0	0.6
5	1.2	1.1	0.6	0.8	1.4	1.3	1.8	1.5	2.0	1.2	1.1	1.4	0.6	1.1	1.4
6	0.8	0.4	1.0	0.9	0.2	1.6	1.6	1.4	0.8	1.3	1.5	0.6	0.9	0.9	0.2
7	0.7	0.2	1.6	0.9	1.3	0.9	2.0	1.5	1.2	1.9	1.6	1.5	1.2	0.6	1.1
8	0.8	0.8	-	1.4	1.0	-	2.1	1.4	0.6	1.3	1.6	-	0.6	0.9	1.5
9	0.9	1.4	1.3	1.2	1.2	1.5	1.1	1.6	1.1	1.5	1.1	0.9	1.3	0.9	1.1
10	1.7	1.4	1.6	2.2	1.5	1.3	0.7	0.7	1.0	1.0	1.3	1.3	0.5	1.2	0.9
11	1.4	1.9	1.8	1.5	1.0	0.8	-	1.3	1.4	1.7	1.5	1.9	1.4	0.7	0.7
12	1.0	2.0	1.8	1.5	2.1	1.2	-	0.9	1.3	1.2	1.6	1.0	1.7	0.9	1.5
13	1.2	1.0	1.9	1.8	2.0	1.3	1.4	0.7	1.7	1.1	1.7	0.6	1.5	2.1	1.3
14	1.5	0.6	1.2	1.0	2.0	1.3	1.2	2.1	1.4	1.1	-	0.9	-	1.3	1.1
15	1.4	1.5	1.5	2.4	1.4	1.5	1.6	0.9	-	-	1.7	0.8	1.1	1.4	1.7

trees, as an index of competition, is related to the "number of sides free" index of competition used by Tinney and Malmberg (1948) and by Ker (1953). Ker found that for 65-year-old Douglas fir trees, radial growth at breast height increased with initial d. b. h., crown class (the weighting factor described above would have a similar effect in the model) and number of sides free. The variable "SOC" used in the present model differed only in that it measured "sides occupied" rather than "sides free". Staebler (1951) used the amount of crown overlap as a measure of competition (see Fig. 1a).

Using the regression described above for five-year radial growth on age, d. b. h. at age ten years, d. b. h. at the beginning of the five-year period and age, the model next calculated the five-year diameter increment for each tree, assuming that the tree was open-grown regardless of whether it was or was not so. This five-year increment was then reduced by an amount which depended on the competitive status of each tree (see lines 370-371 of the FORTRAN program in Appendix II). The amount of reduction would vary from zero for trees that were free-growing (i.e. "SOC" = 0) to 100 per cent for trees whose crowns were completely overlapped by surrounding competitors ("SOC" = 1). If this increment was not greater than a certain percentage ("DINC") of the d. b.h. at the beginning of the five-year period, the tree was considered to have died. If it was greater, the new diameter of the tree ("DAP5") was calculated. These new diameters were then used as a basis for

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calculating the next five years' growth of the stand and the process repeated to age 100 years.

The values of "DINC" used to define mortality were chosen arbitrarily. Attempts to obtain realistic values by sampling dead trees in the field were of little use as the variation in the last five years' d. b. h. growth of Douglas fir was very great. Some trees were able to survive and grow at the rate of 100 rings per inch for 25 years or more while others, apparently killed by insects or fungi, died with very little decrease in growth rate during the last five years of their lives. The values of "DINC" chosen range from five per cent at age ten years to 0.1 per cent at age 45 years or above. The inclusion of "DINC" values in the model was found to be necessary for, if it was assumed that death occurred only when the periodic diameter increment became zero, the onset of mortality was delayed and, when it did occur, was very heavy causing widespread depletion of the stand. This was because, in nature, mortality is a continuous process occurring every year, whereas in the model, mortality was a discrete process occurring only at the end of each five-year period. Thus trees that would have died at the beginning of the period remained "alive" to the end and thus reduced the growth of neighbouring trees, which would normally have been released, to a level where death might occur too.

As soon as mortality had started in the model the value of "REDFAC" was increased each five-year period by an

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amount "REDINC", again chosen arbitrarily. Competition in the model was based on the crown-dimensions of open-grown trees. Thus with the reduction in diameter growth as competition set in, the calculated competitive crown widths were less than they would have been had the trees remained competition-free. Increasing the value of "REDFAC" compensated for this. It should be noted that, in closed stands of Douglas fir after the initial period of intense competition, there may be considerable gaps in the crown canopy (see Plates IVa & b) whereas the roots may occupy these gaps and overlap to a considerable extent (McMinn, 1963).

Much of the early work in the development of the model involved selecting a combination of values of "REDFAC", "REDINC" and "DINC" which, when the model was run, would give results that fitted published yield table data (Figs. 9-11) satisfactorily. It was found (Newnham, 1964) that "REDFAC" controlled the age at which mortality first occurred and "REDINC" the amount of mortality thereafter. Reducing "REDFAC" delayed the onset of mortality; reducing "REDINC" decreased the amount of mortality.

When searching for possible competitors at each five-year re-appraisal the program was designed to consider all trees or, more correctly, all possible tree locations, within a distance of eight times the initial spacing of the tree being studied. The area around each tree was divided into octants (Fig. 12) and the closest tree in each octant was tested to see whether it was a competitor. If a competitor

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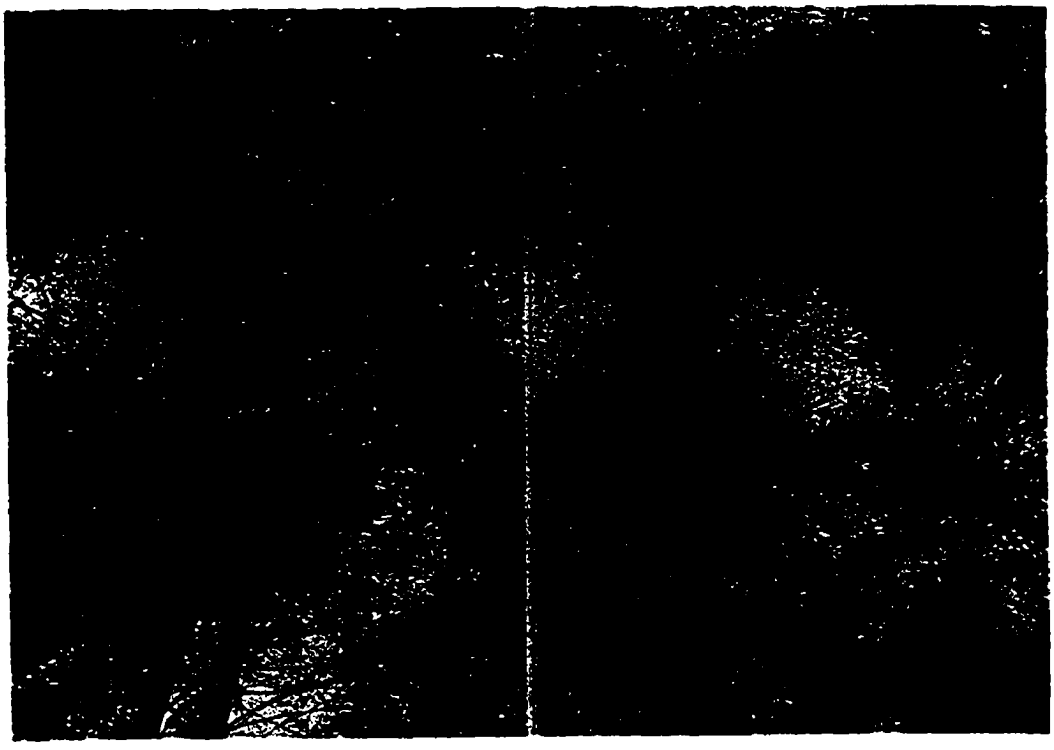

PLATE IV: Crown canopy photographs of 39-year-old Douglas fir plantations established at (a) 4 x 4 ft. (b) 10 x 10 ft., Wind River, Washington. Photographed in August, 1963.

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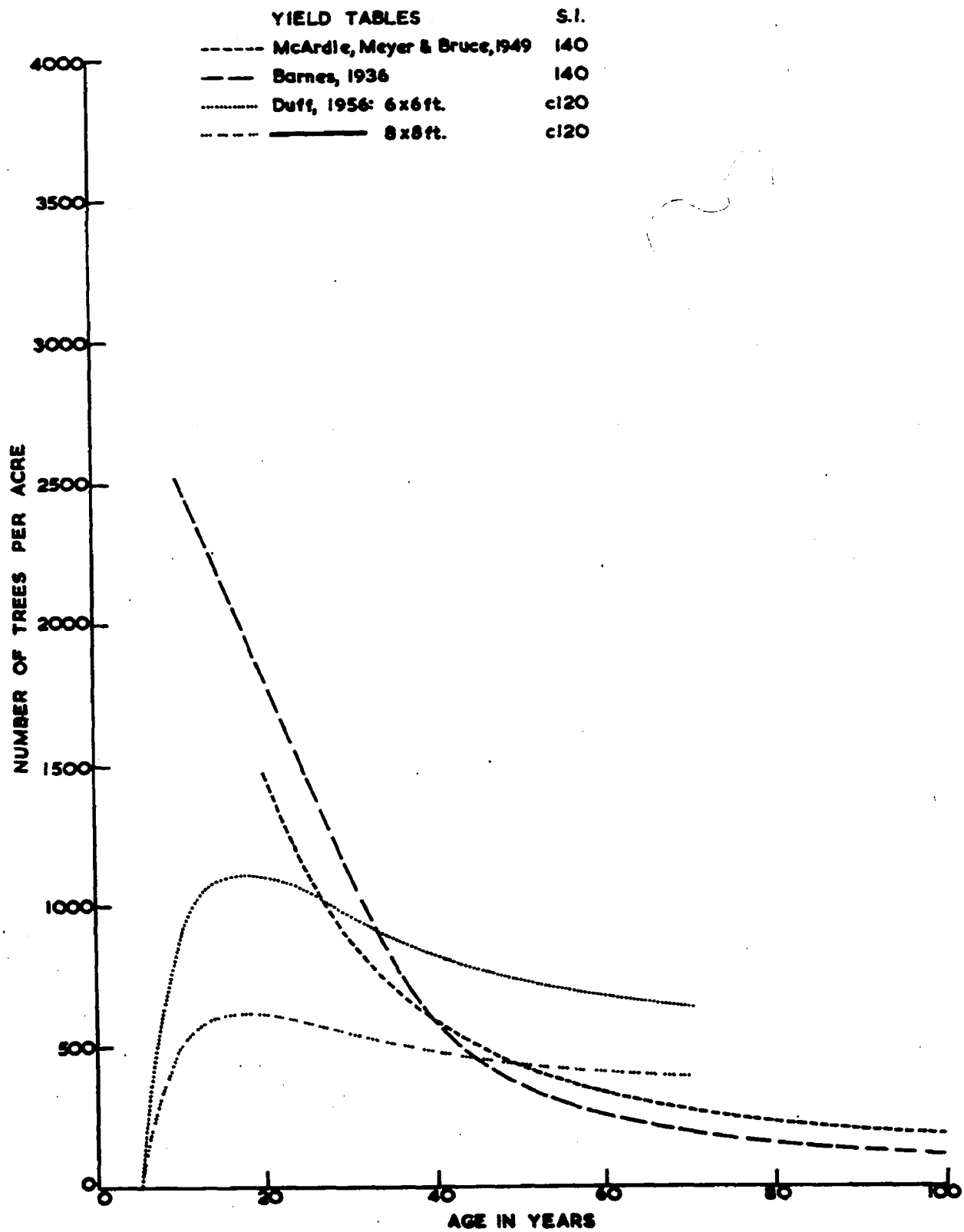

FIG. 9: The relationship between number of trees per acre and total age or years from planting (Duff, 1956).

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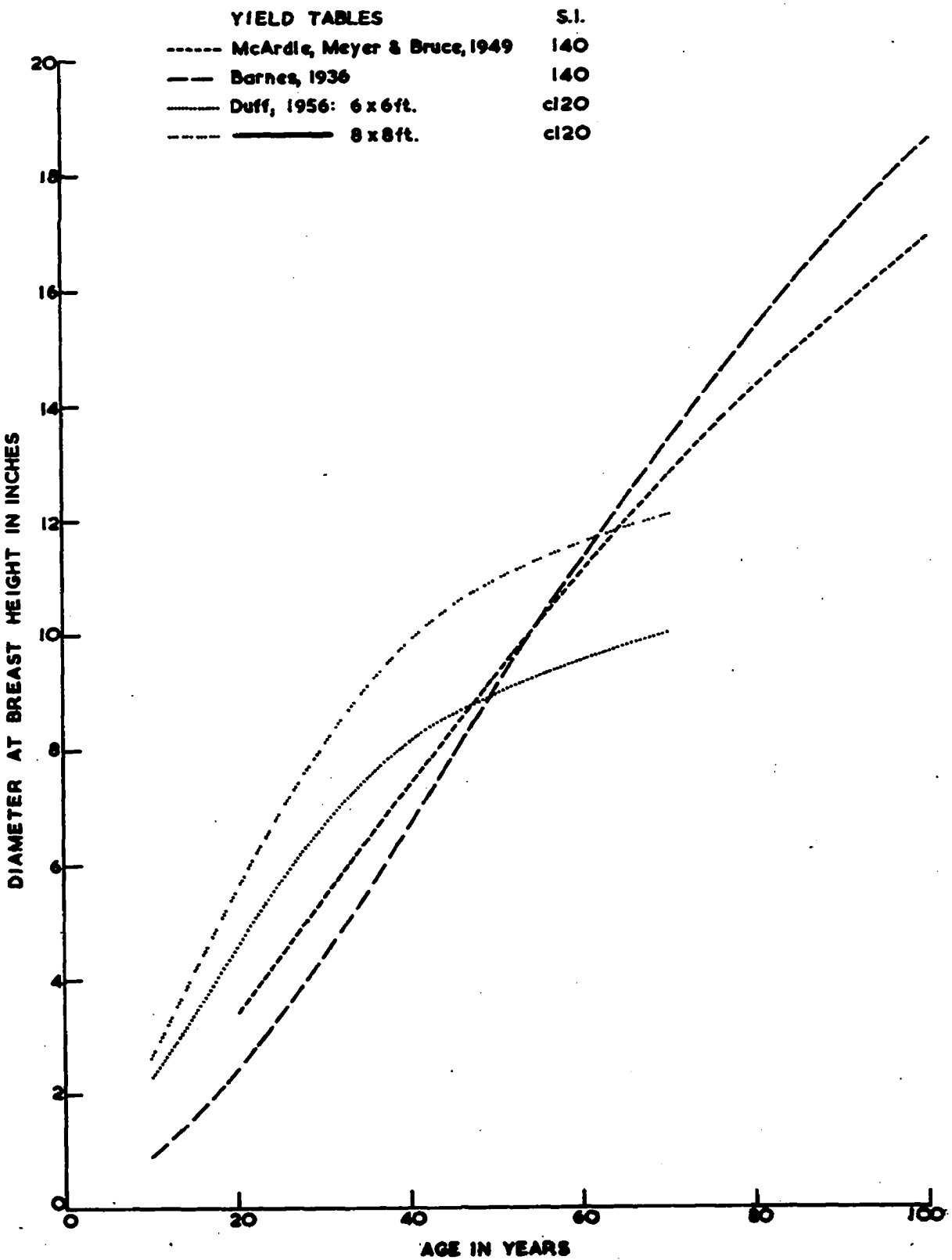

FIG. 10: The relationship between mean d. b. h. o. b. and total age or years from planting (Duff, 1956).

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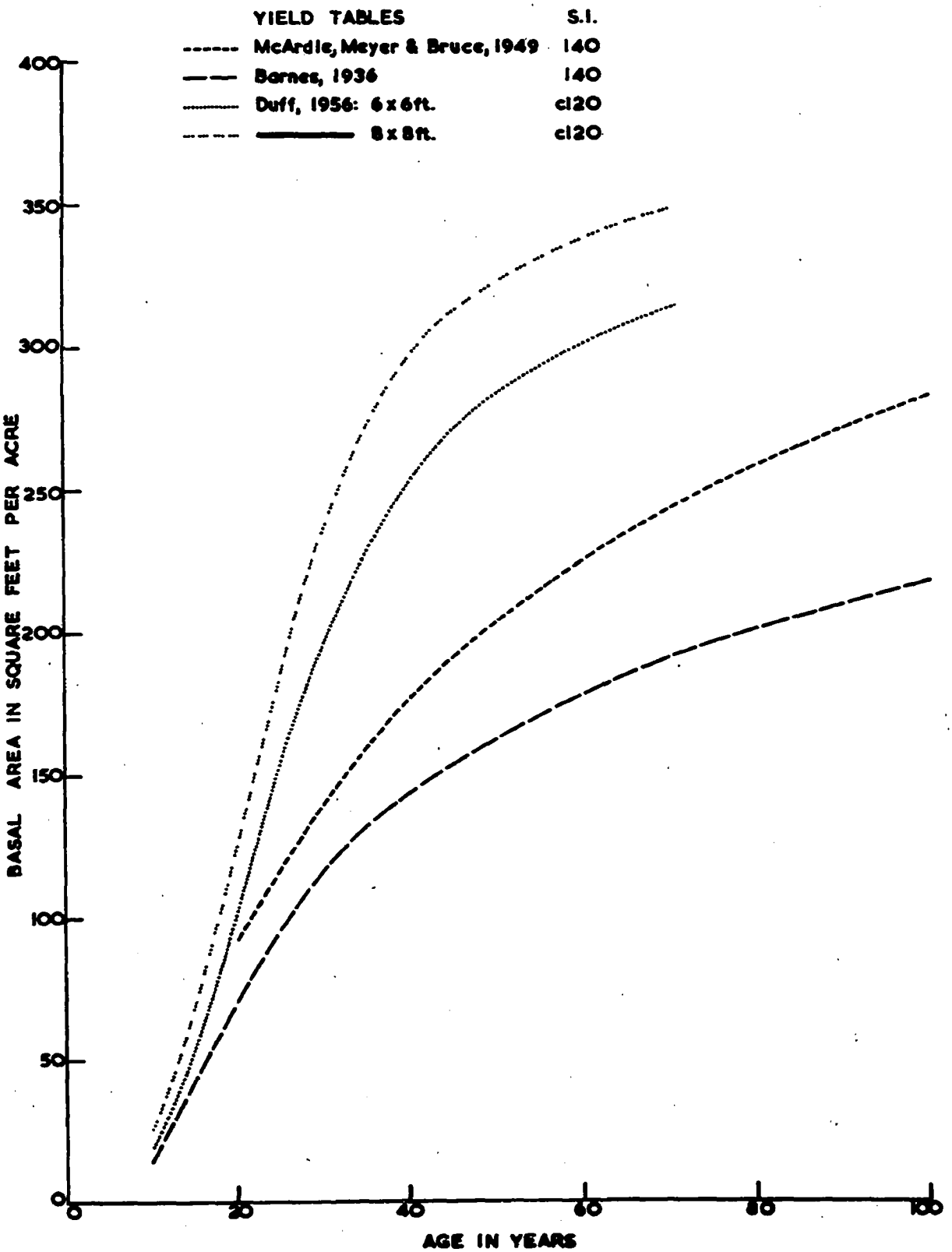

FIG. 11: The relationship between basal area per acre and total age or years from planting (Duff, 1956).

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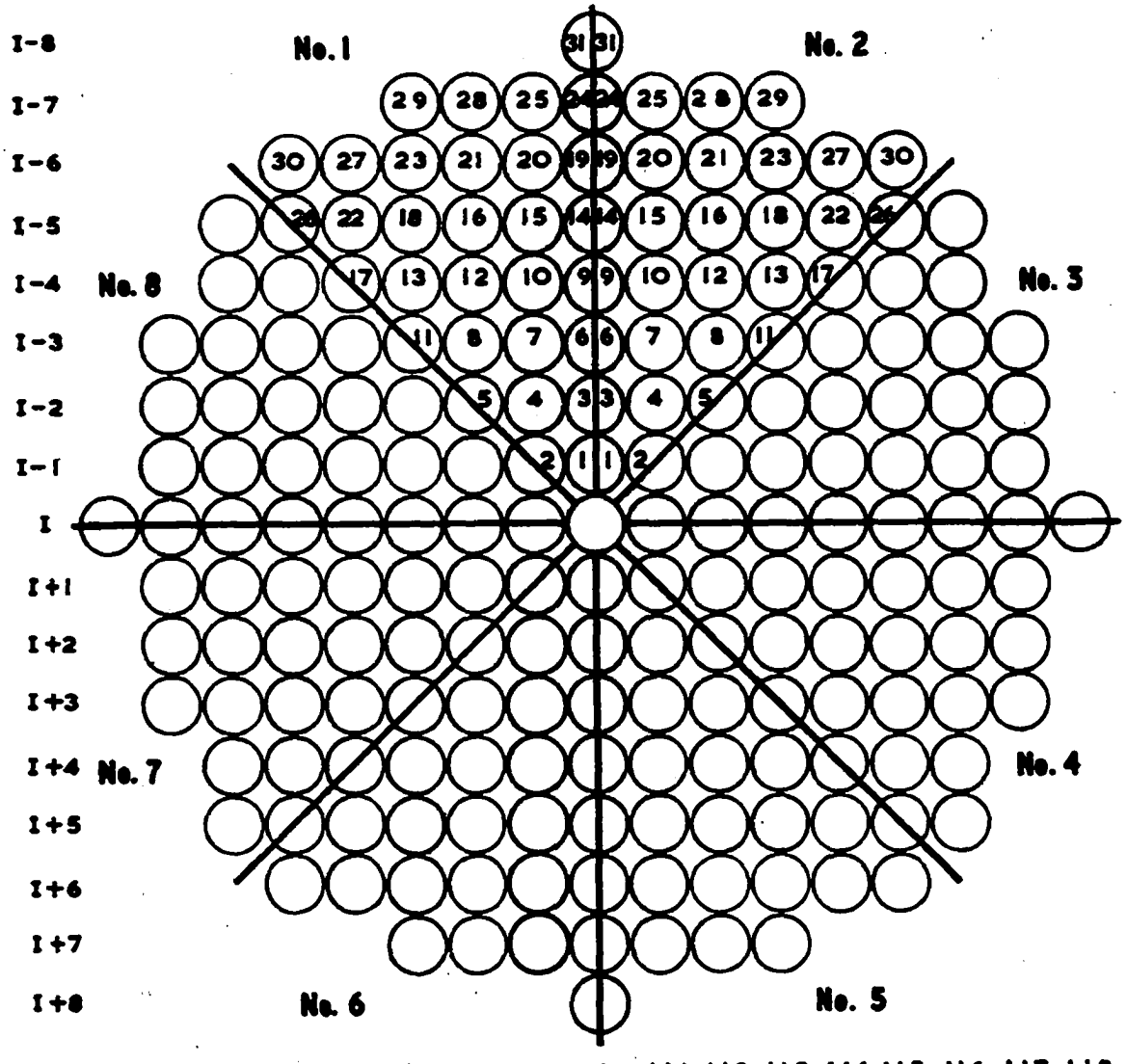

FIG. 12: The tree locations tested for possible competitors of the tree being studied (I, J) in the model. Octants are numbered 1 to 8, locations within each octant 1 to 31 in the order of increasing distance from tree (I, J).

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was only half in an octant, the competitive status of the tree was halved. A large section of the FORTRAN program (Lines 53-369) is required to calculate the distances to each location in the octant and to make the necessary tests involved.

The model assumed that if a tree was released from competition at any stage, its pattern of diameter growth until competition again set in would be that of an open-grown tree of the same d. b. h. and age. This ignored the possible shock effect (Staebler, 1956) discussed in Part I of this thesis. It was thought that the errors caused by making this assumption would be small.

Results

As stated earlier, the model was developed using a basic matrix of diameters from a 6 x 6 ft. plantation of Douglas fir on the University Research Forest at Haney. Ten (4.4 per cent) of the 225 trees in this matrix were dead or missing. There was no evidence to show that this mortality was not randomly distributed. Site index was approximately 140 according to Barnes (U. B. C. Forest Club, 1959). Four initial spacings (planting distances) were tested: 3.3 x 3.3, 6.6 x 6.6, 9.9 x 9.9 and 13.2 x 13.2 ft. Later the two wider spacings, 16.5 x 16.5 ft. and 19.8 x 19.8 ft., were tested. They are included in Fig. 13-15 in order that the effect of initial spacing on stand growth can be studied over very wide range of spacings. The program was run on the I. B. M. 7090 electronic computer at the University of Toronto which

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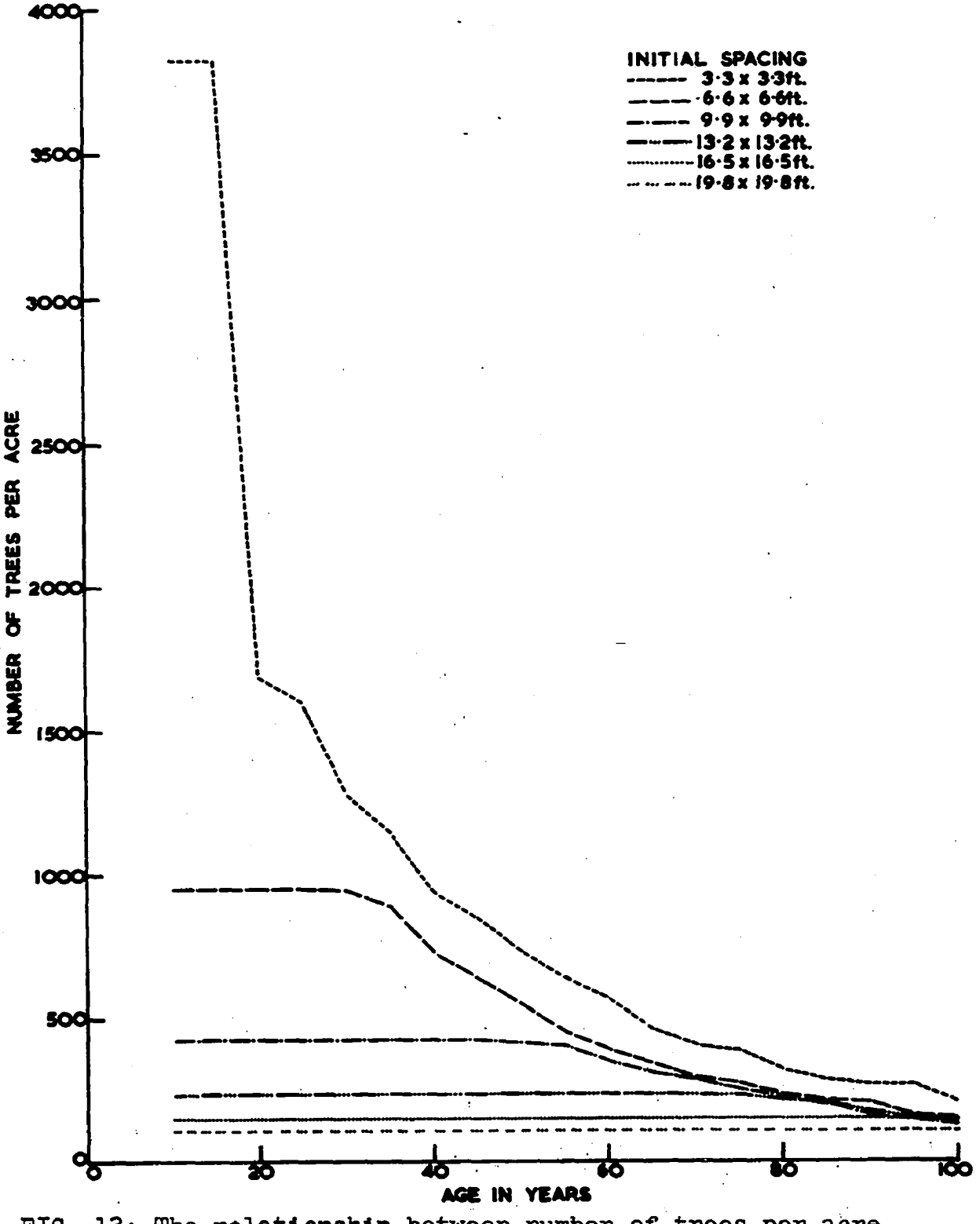

FIG. 13: The relationship between number of trees per acre and age. Run II-1.

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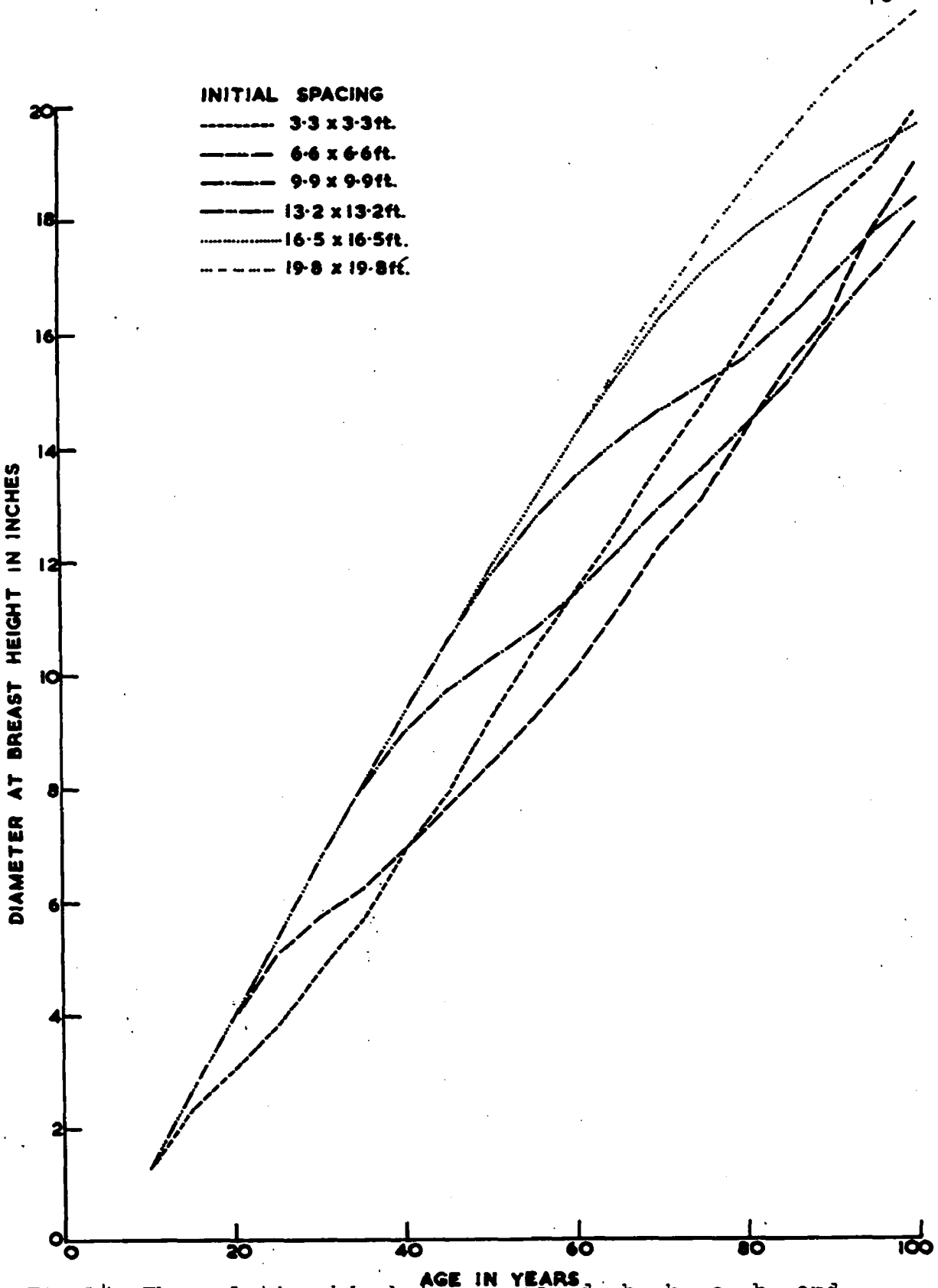

FIG. 14: The relationship between mean d. b. h. o. b. and age. Run II-1.

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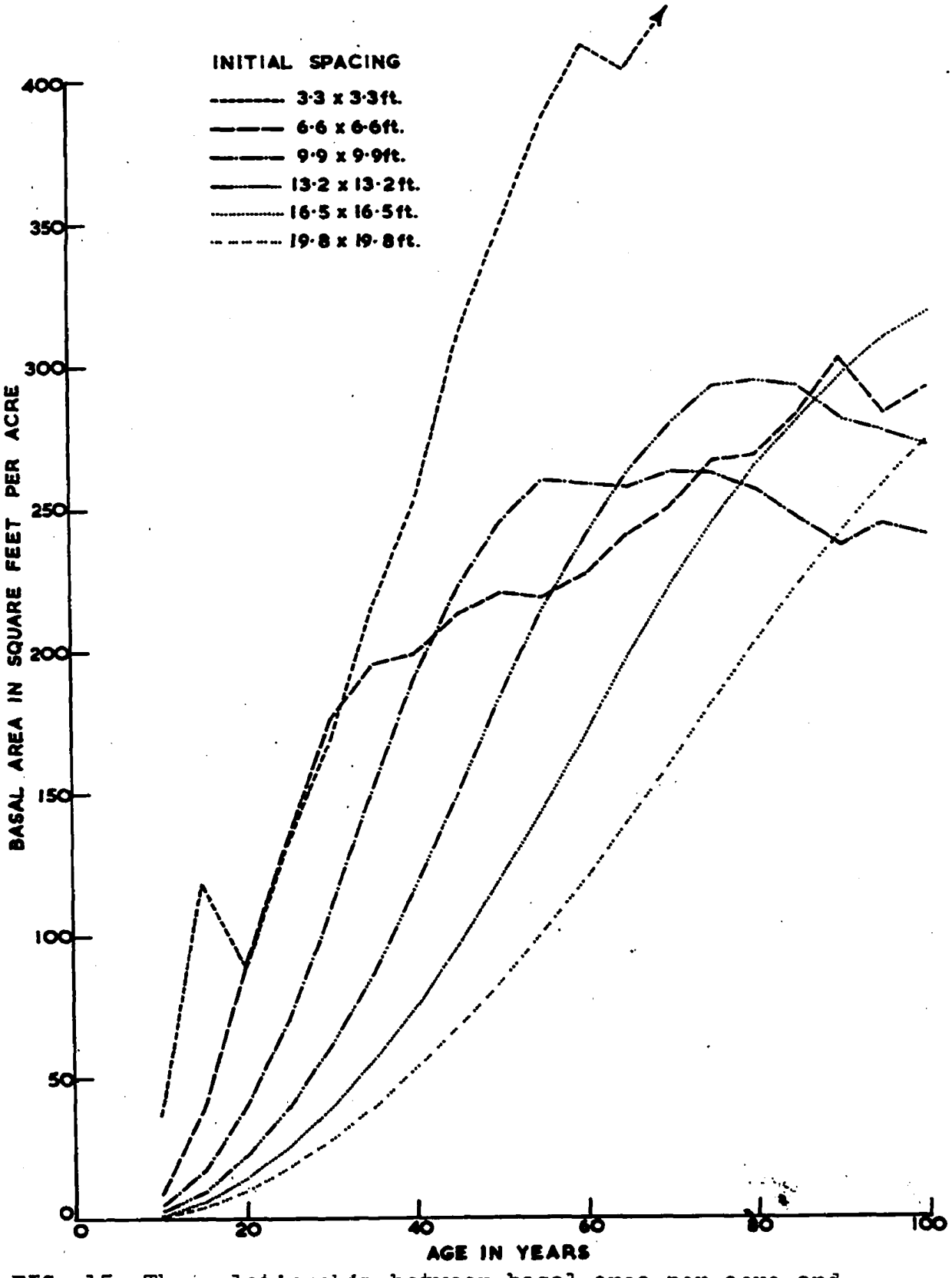

FIG. 15: The relationship between basal area per acre and age. Run II-1.

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is 300 times faster than an I. B. M. 1620. The time taken for program compilation, input, processing and output (but not listing the results) was about five minutes. The number of trees per acre, mean d. b. h. (with its variance and standard deviation), basal area per acre, mean and periodic basal area increments, diameter of the tree of mean basal area, gross basal area yield per acre and mortality were obtained at the end of each five-year period. In addition, the diameter frequency distribution table and the diameter matrix could be printed out as required. An example of the program output is given in Appendix II (Fig. 70).

Number of Trees per Acre (Fig. 13)

The number of trees per acre remained constant until mortality from competition set in at age twenty years (3.3 x 3.3 ft.), 30 years (6.6 x 6.6 ft.), 50 years (9.9 x 9.9 ft.), 70 years (13.2 x 13.2 ft.) or 95 years (16.5 x 16.5 ft.). Mortality, as defined here, did not occur at all in the 19.8 x 19.8 ft. spacing during the first 100 years of the life of the stand. Mortality did not usually occur until the number of trees at any age was greater than that given in the yield tables of McArdle et al. (1949). In the closest spacing (3.3 x 3.3 ft.) the stocking was considerably higher than the yield table data. Part of this difference was probably attributable to the fact that this model only takes account of natural mortality through suppression of the weaker trees by the more vigorous trees and not by

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agents that cause irregular mortality such as fungi, insects or wind. By age 80 years the differences in stocking due to the initial spacing have greatly diminished except for the 3.3 x 3.3 ft. spacing.

Average Diameter at Breast Height

The average diameter at breast height outside bark given in Fig. 14 is the simple arithmetic mean of all trees. Yield tables are based on those trees above a certain minimum d. b. h. (usually 1.5 inches). The New Zealand tables (Duff, 1956) differ from the yield tables of the Pacific Northwest in that the average d. b. h. given is the d. b. h. of the tree of average basal area. Age is calculated from the date of establishment of the plantation and not from the date of germination of the seed. These differences in nomenclature should be remembered when comparing the different yield tables and the stand model.

From Fig. 14 it can be seen that, in the early stages of the model, the average d. b. h. was greater than that of the yield tables of the Pacific Northwest due to the open-grown nature of the stands compared with the normal stands given in the yield tables. As competition set in the average d. b. h. growth falls off. After the first trees were removed as mortality, d. b. h. growth picked up again and remained more or less linear. The general trend of the d. b. h. growth curves followed those given by McArdle et al. (1949) and Barnes (U. B. C. Forest Club, 1959), except as

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outlined above. The average diameter of the 3.3 x 3.3 ft. spacing appeared to be higher than expected above age 80 years. This was because the model failed to determine the total amount of competition for each tree.

Basal Area per Acre

The relationship between basal area and age (Fig. 15) proved to be the most difficult to harmonize with the yield table relationships (Newnham, 1964) and the final model could probably still be improved. The basal area of the 3.3 x 3.3 ft. spacing was excessively high above 40 years, due to a combination of a high stocking of trees (Fig. 13) and a high average d. b. h. (Fig. 14). It appears that the model will be of limited use in performing tests on this spacing above 40 years, or when the number of trees in the matrix is reduced to less than 25. With the other three spacings, basal area growth was most rapid in the 6.6 x 6.6 ft. spacing, and least at the 19.8 x 19.8 ft. spacing. Basal area growth appears to level off at between 250 and 300 sq. ft. per acre. Basal area yield appeared closer to the yield tables of Barnes (U. B. C. Forest Club, 1959) than to those of McArdle et al. (1949).

"REDFAC"

The variation of the value of the reduction factor, "REDFAC", with age is shown in Fig. 16. It can be seen that its value remained constant at each spacing until mortality

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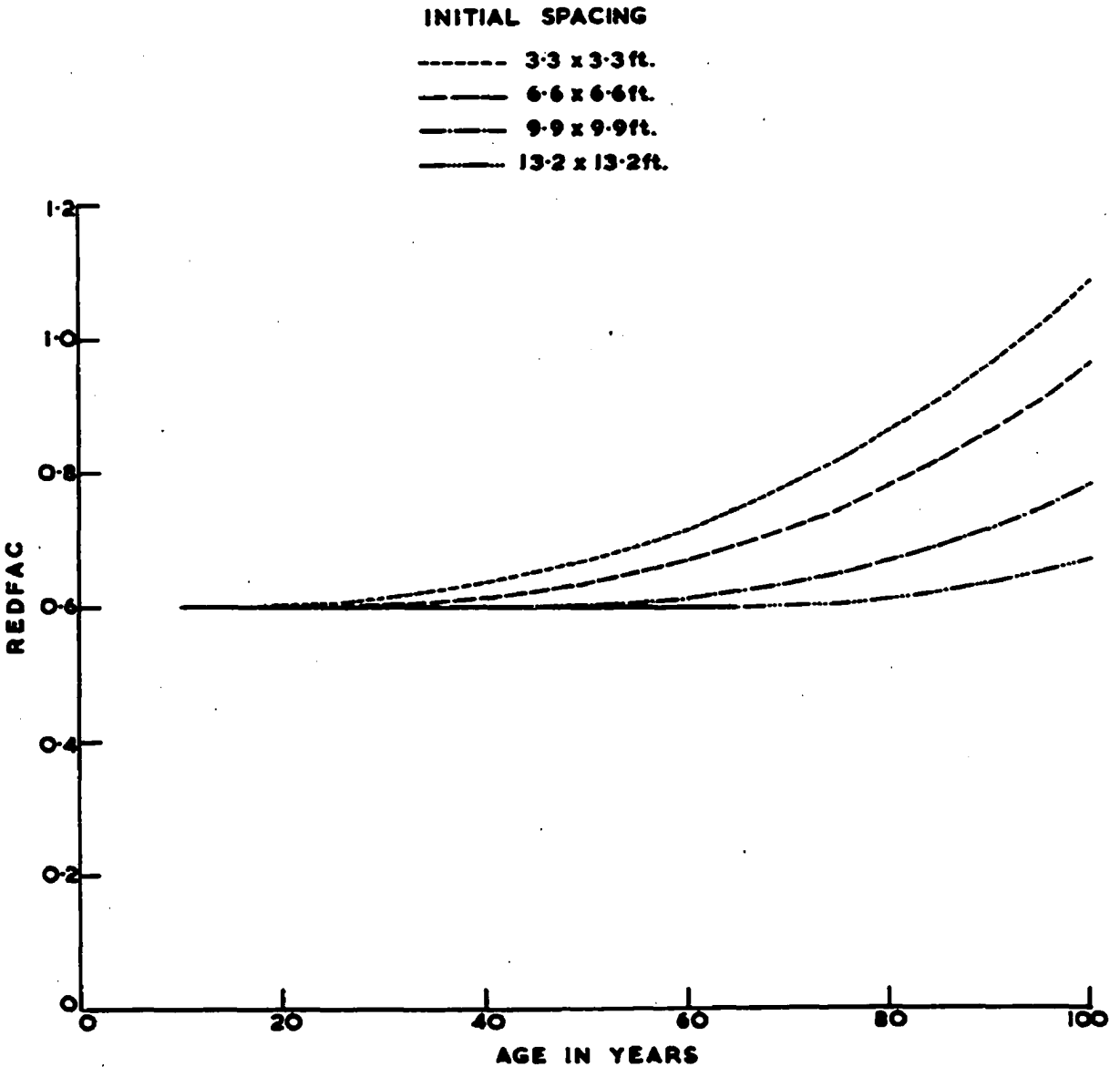

FIG. 16: The relationship between "REDFAC" and age. Run II-1.

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occurred. From then on it increased, the increase being more rapid with increase in age. Above a value of 1.0, "REDFAC" is no longer a "reduction" factor. The reasons for modifying the value of "REDFAC" have been explained above.

Diameter Frequency Distributions

Frequency polygons have been drawn at ten-year intervals on a trees-per-acre basis (Fig. 17-20). They show no significant departure from that which might be expected from small samples taken from plantations. The irregularities in the distributions of the last three or four decades of the closer spacings were probably due to the fact that, although the number of trees per acre in each age class was relatively large, the numbers in the matrix on which the distribution was based were relatively small. For example, one tree present in the matrix was equivalent to 17.8 trees per acre at 3.3 x 3.3 ft. spacing, but only 1.1 trees per acre at 13.2 x 13.2 ft. spacing. The small range in diameters probably was also due to the small number of trees.

The cumulative frequency distributions (Fig. 21 and 22) have the characteristic sigmoid shape of natural distributions. The range of diameters for any given mean increases with increase in initial spacing although this is probably due, in part, to the greater number of trees present in the matrix at the wider spacings.

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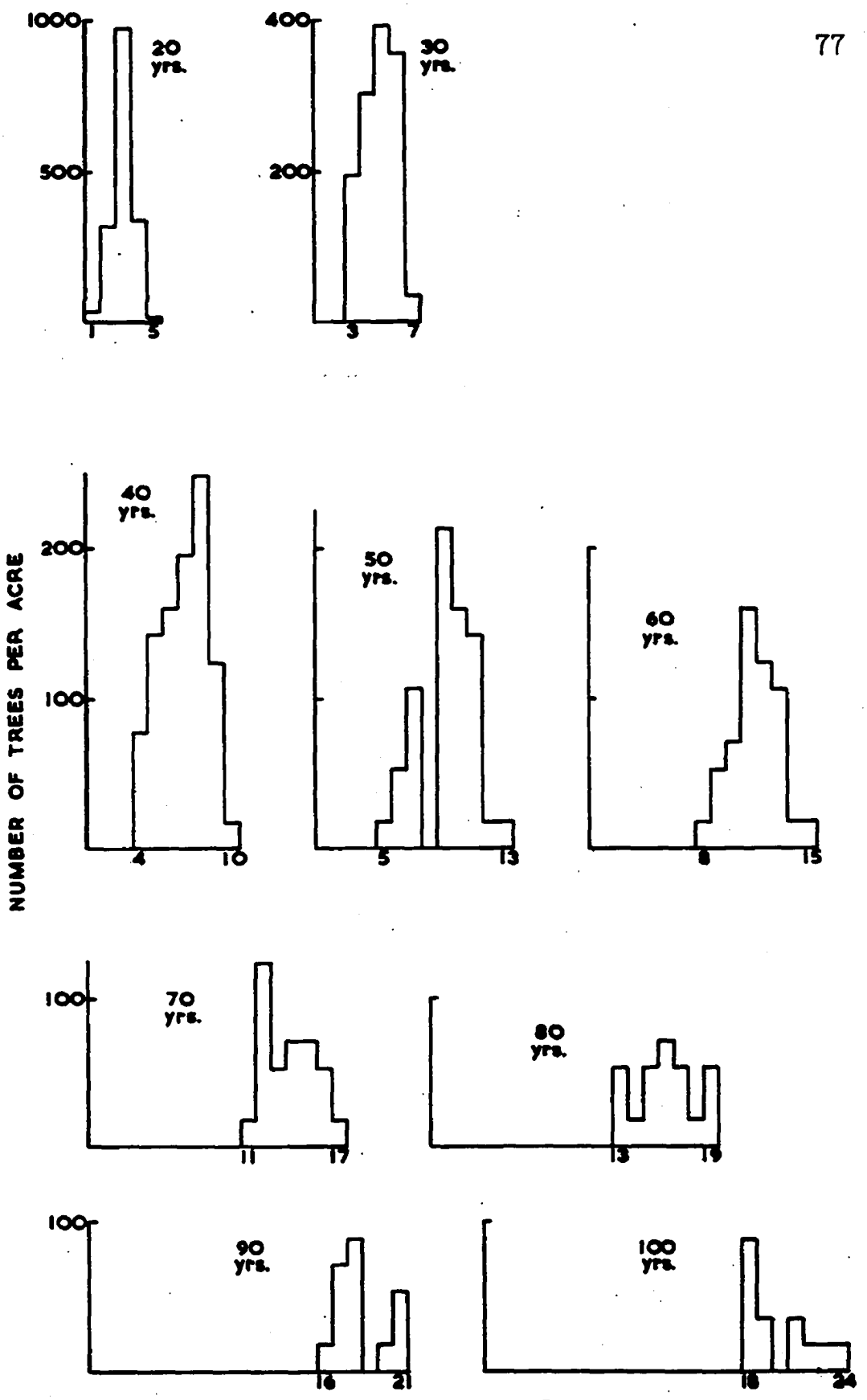

FIG. 17: Diameter frequency distributions. Spacing: 3.3 x 3.3 ft. Run II-1.

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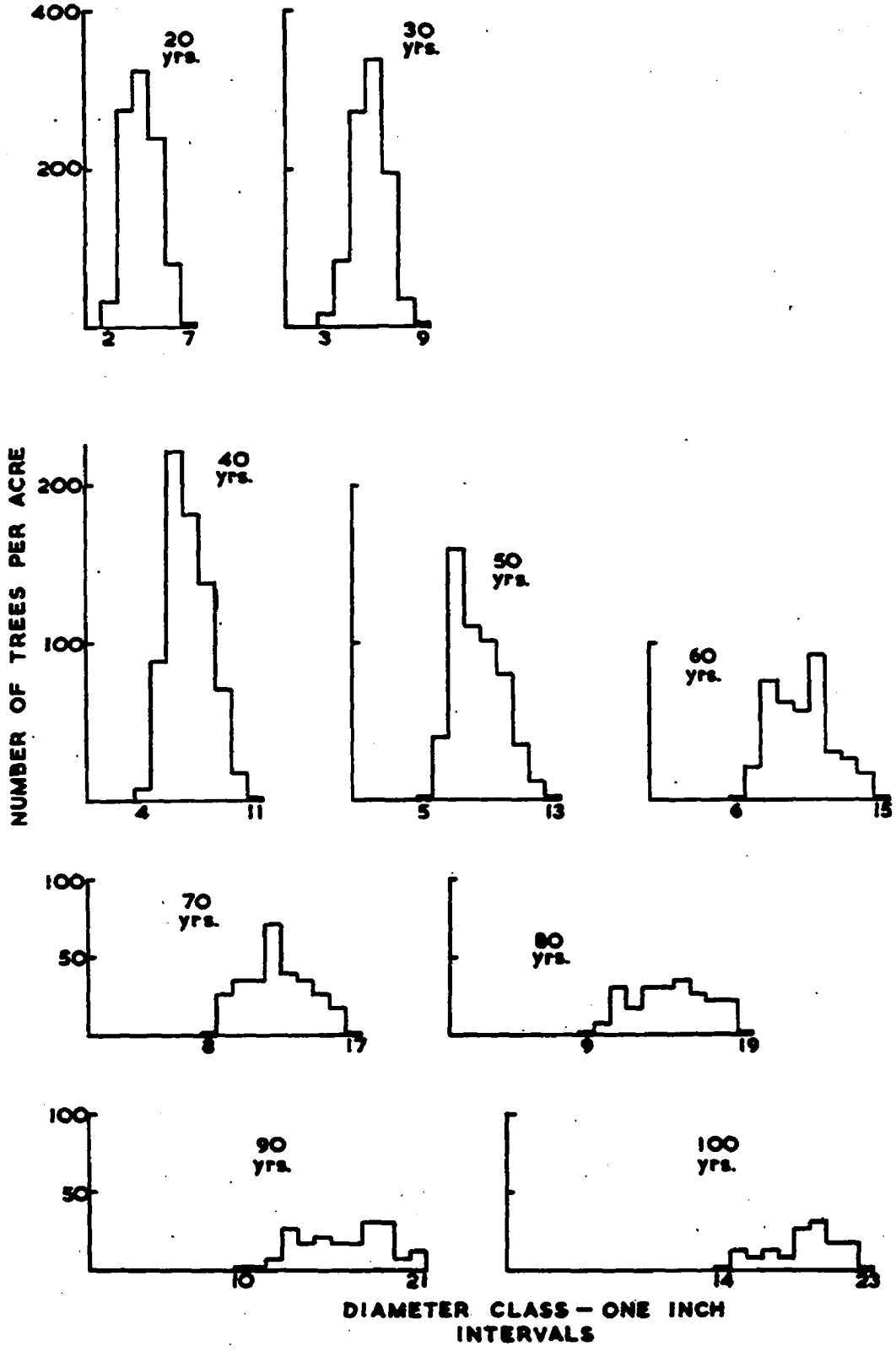

FIG. 18: Diameter frequency distributions. Spacing: 6.6 x 6.6 ft. Run II-1.

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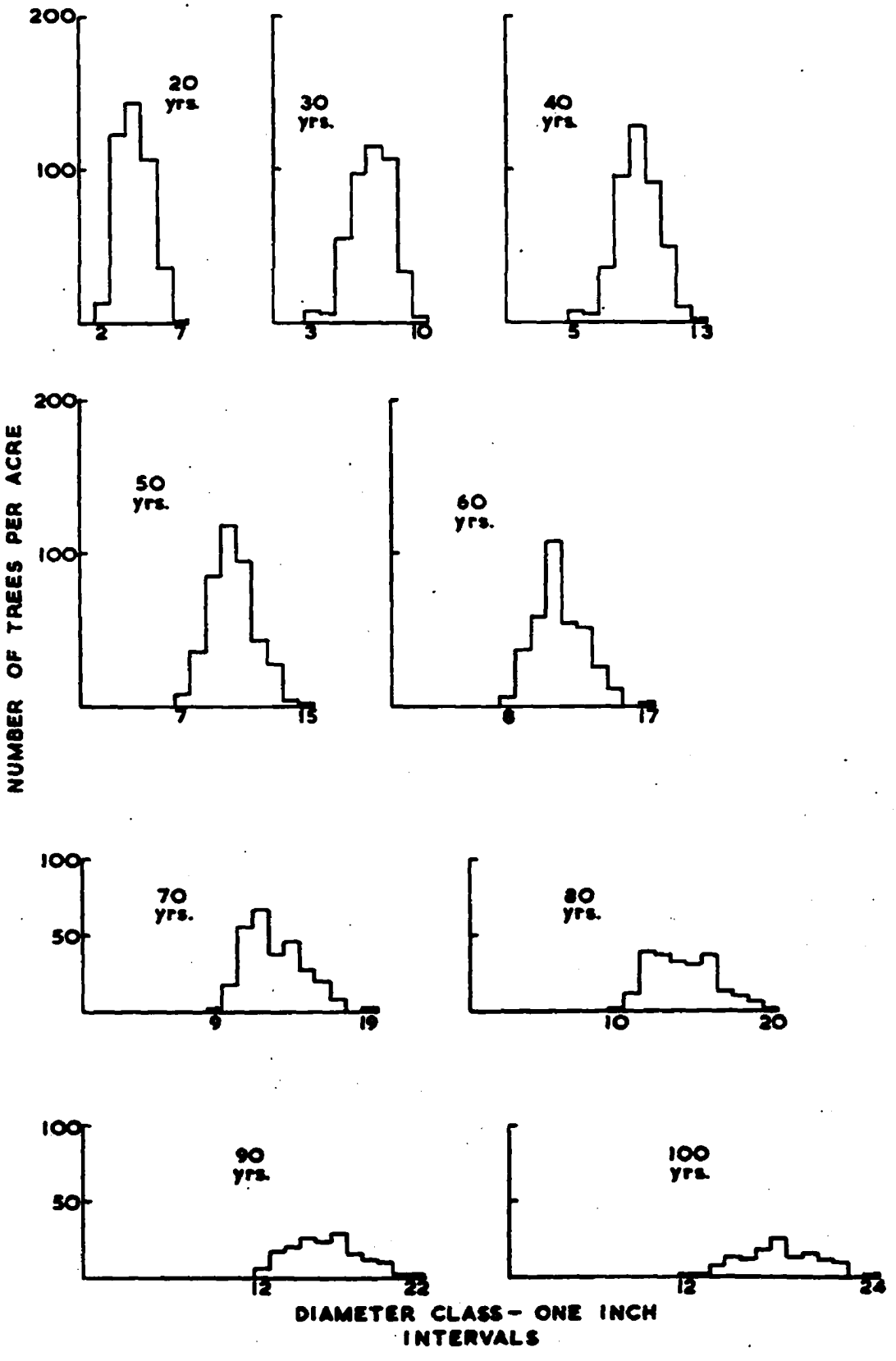

FIG. 19: Diameter frequency distributions. Spacing: 9.9 x 9.9 ft. Run II-1.

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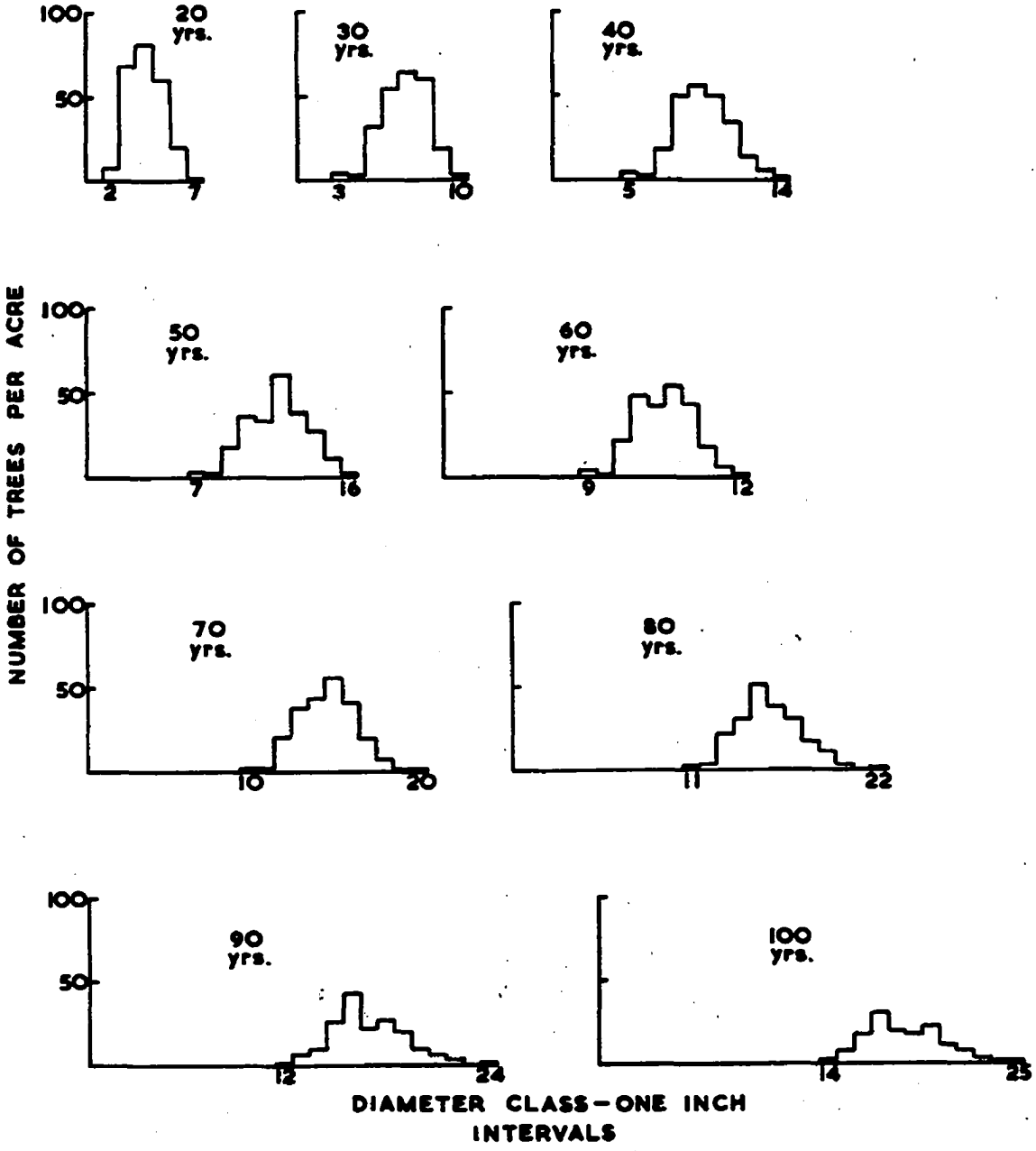

FIG. 20: Diameter frequency distributions. Spacing: 13.2 x 13.2 ft. Run II-1.

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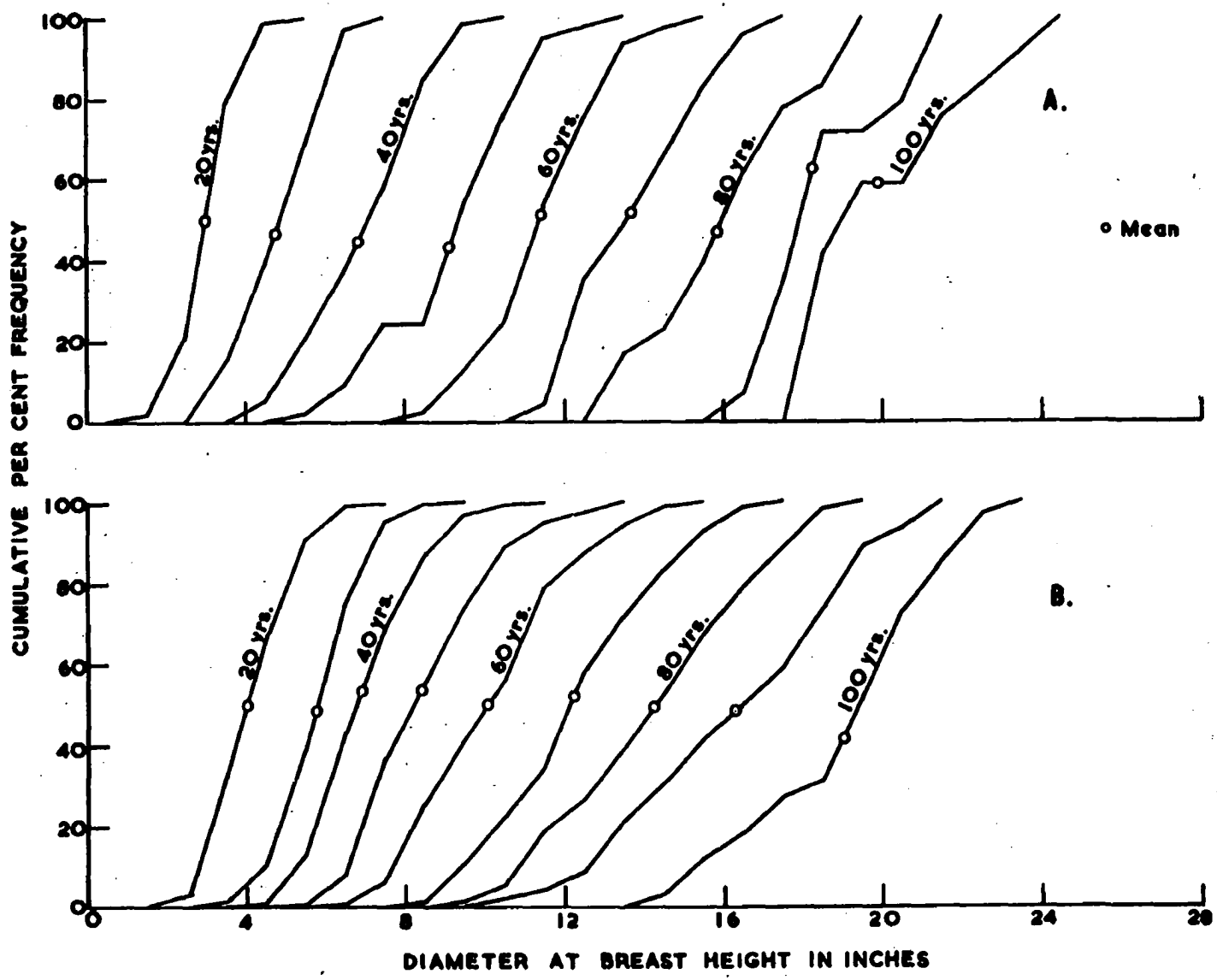

FIG. 21: Cumulative d. b. h. o. b. frequency distributions.
Spacing: (A) 3.3 x 3.3 ft.; (B) 6.6 x 6.6 ft. Run II-1.

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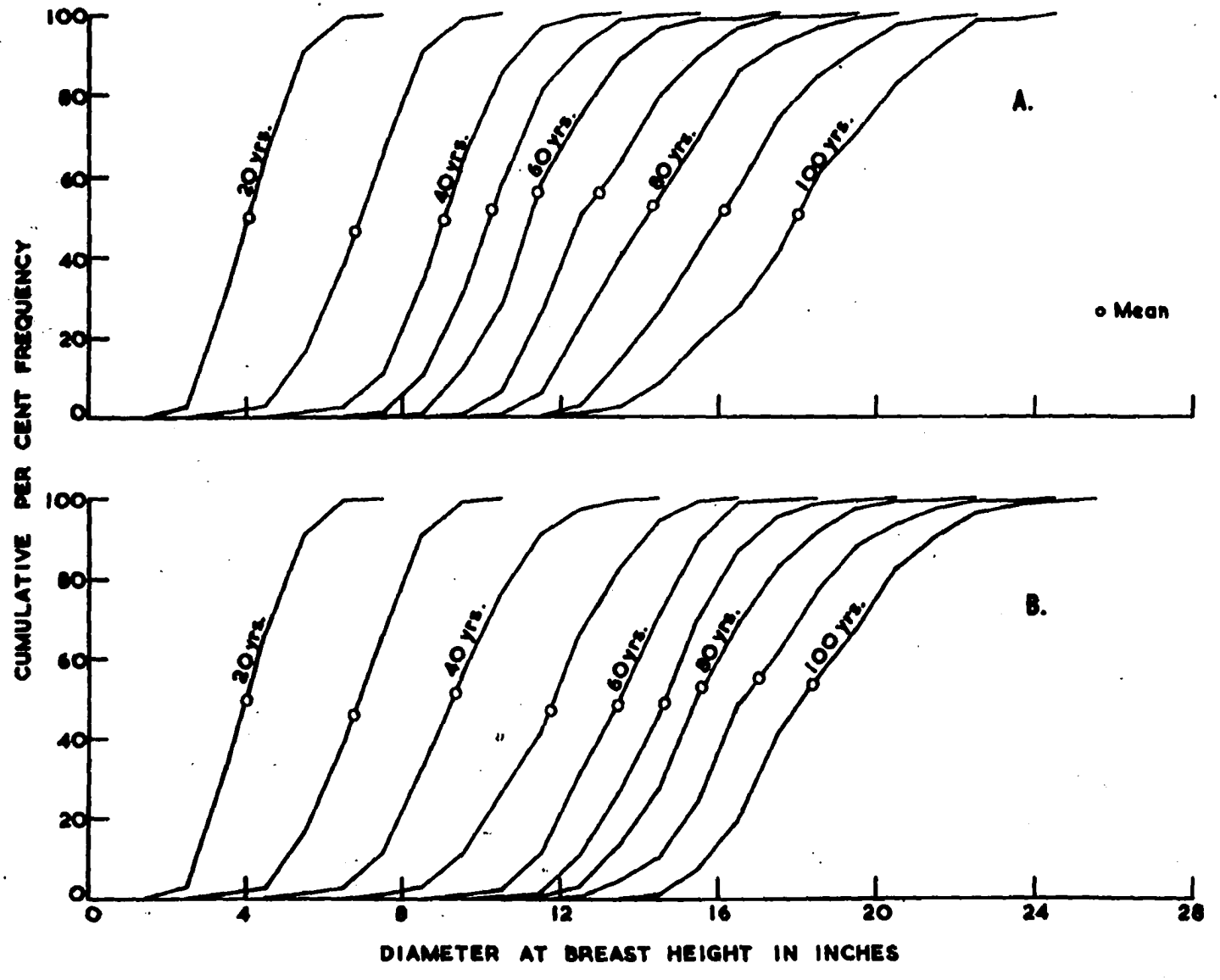

FIG. 22: Cumulative d. b. h. o. b. frequency distributions.
Spacing: (A) 9.9 x 9.9 ft.; (B) 13.2 x 13.2 ft. Run II-1.

Distributions of Trees

The stand structure of the basic matrix is shown in Fig. 23 and the development of the stand for the 6.6 x 6.6 ft. spacing is shown in Fig. 24. The final stand structure for each spacing is shown in Fig. 25. The stand development between age ten and age 100 years for the spacings not shown is similar to that of the 6.6 x 6.6 ft. spacing, the only major difference being the ages at which the various stages shown in Fig. 24 occur. For the 3.3 x 3.3 ft. spacings the stages are reached earlier and for the wider spacings they are reached later.

In order that the structures of the stands may be understood, the trees have been divided into four classes by d. b. h. (see Fig. 23). After competition has set in, these classes probably correspond to the four crown classes: dominant, codominant, intermediate and suppressed. However, where there is a concentration of trees in the upper d. b. h. class, it is unlikely that they would all be dominant trees, as would be indicated by this method. Conversely, a group of trees in the smallest d. b. h. class would contain trees in the higher crown classes. Using this diagrammatic representation of the stand, it is possible to trace movement of trees between crown classes and to see how the various distributions of trees, to be tested later in this thesis, are modified as the stand develops.

Although the plantation, on which the basic matrix of diameters is based, appeared remarkedly uniform in growth,

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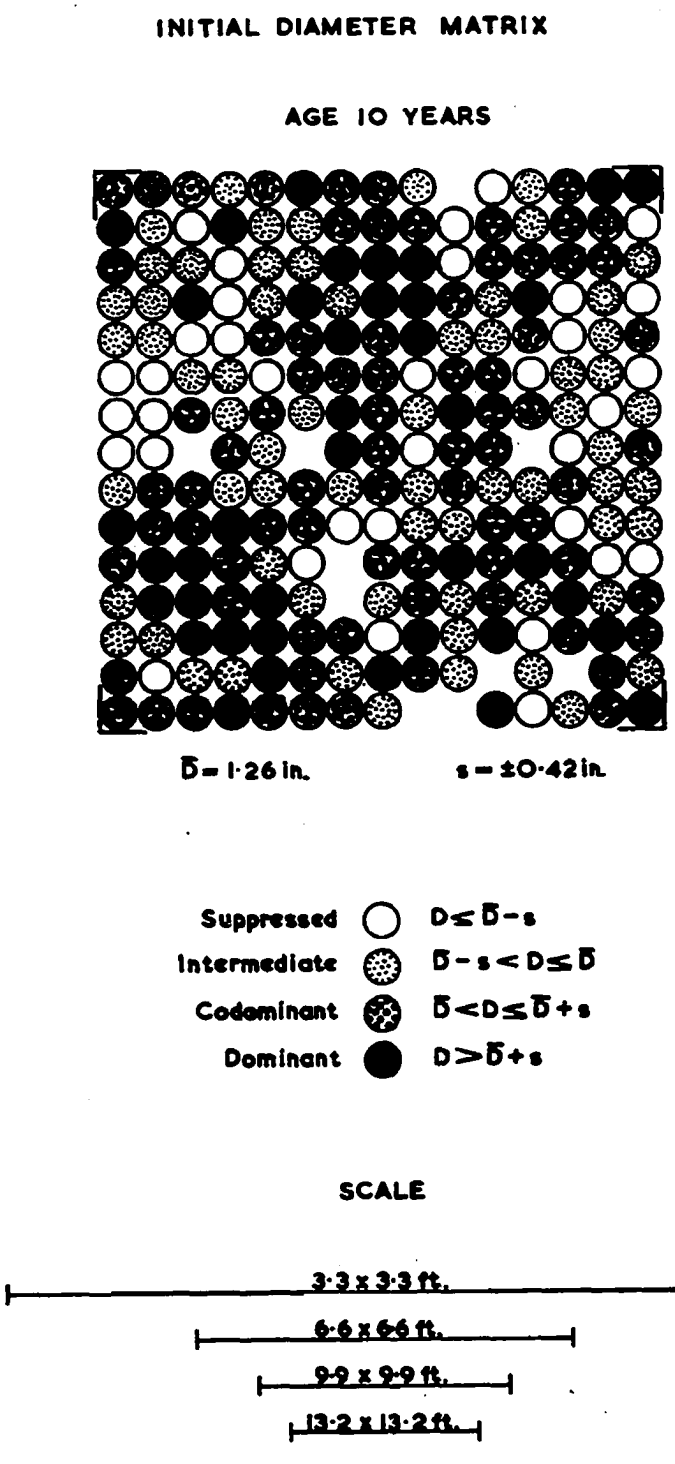

FIG. 23: Initial diameter matrix used for developing the model. Data collected from a Douglas fir plantation at Haney. (The circles representing trees are not drawn to scale.) Run II-1. The scales represent 66 ft. at each spacing.

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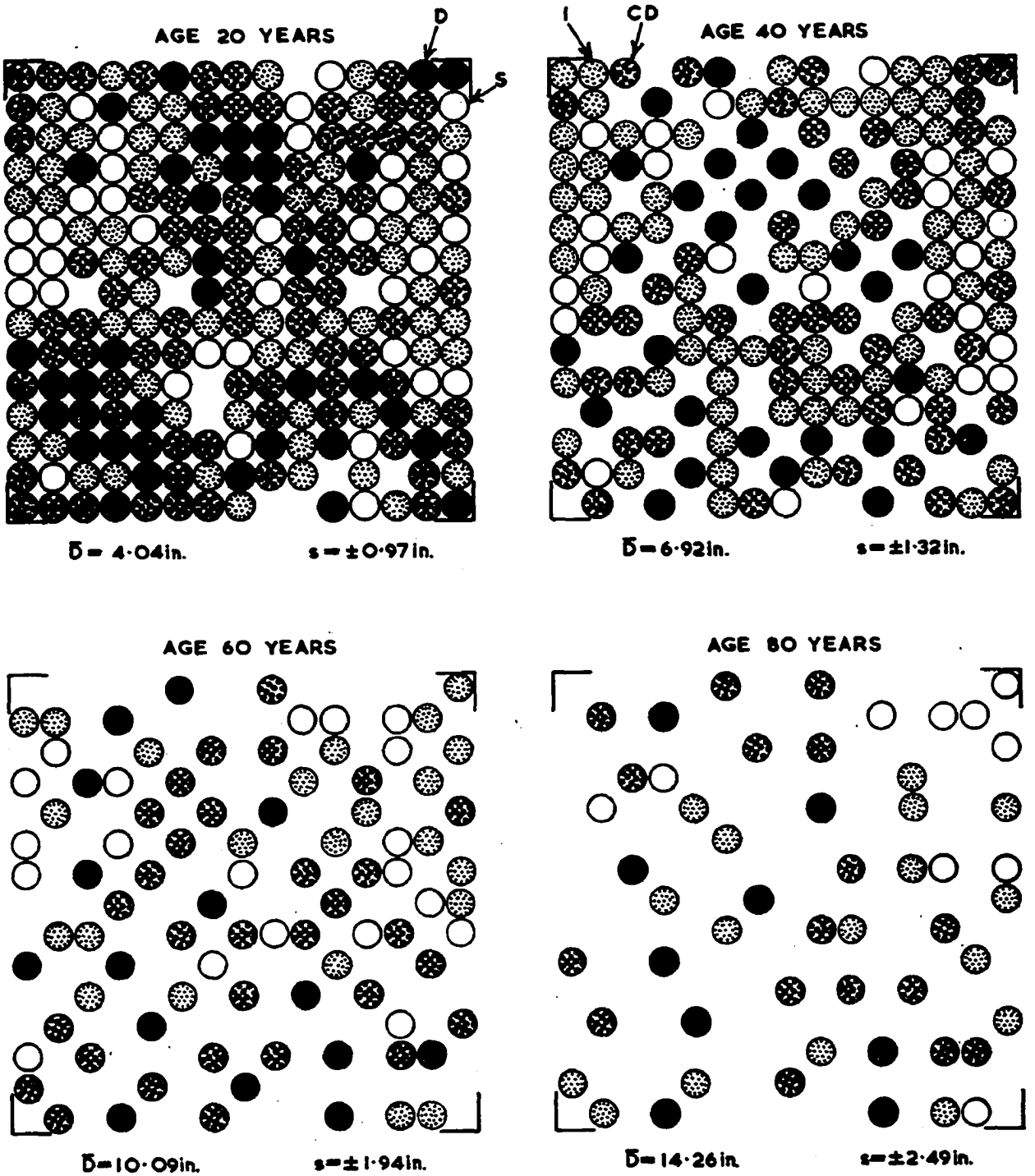

FIG. 24: Stand structure of the basic model. Spacing: 6.6 x 6.6 ft. Run II-1.

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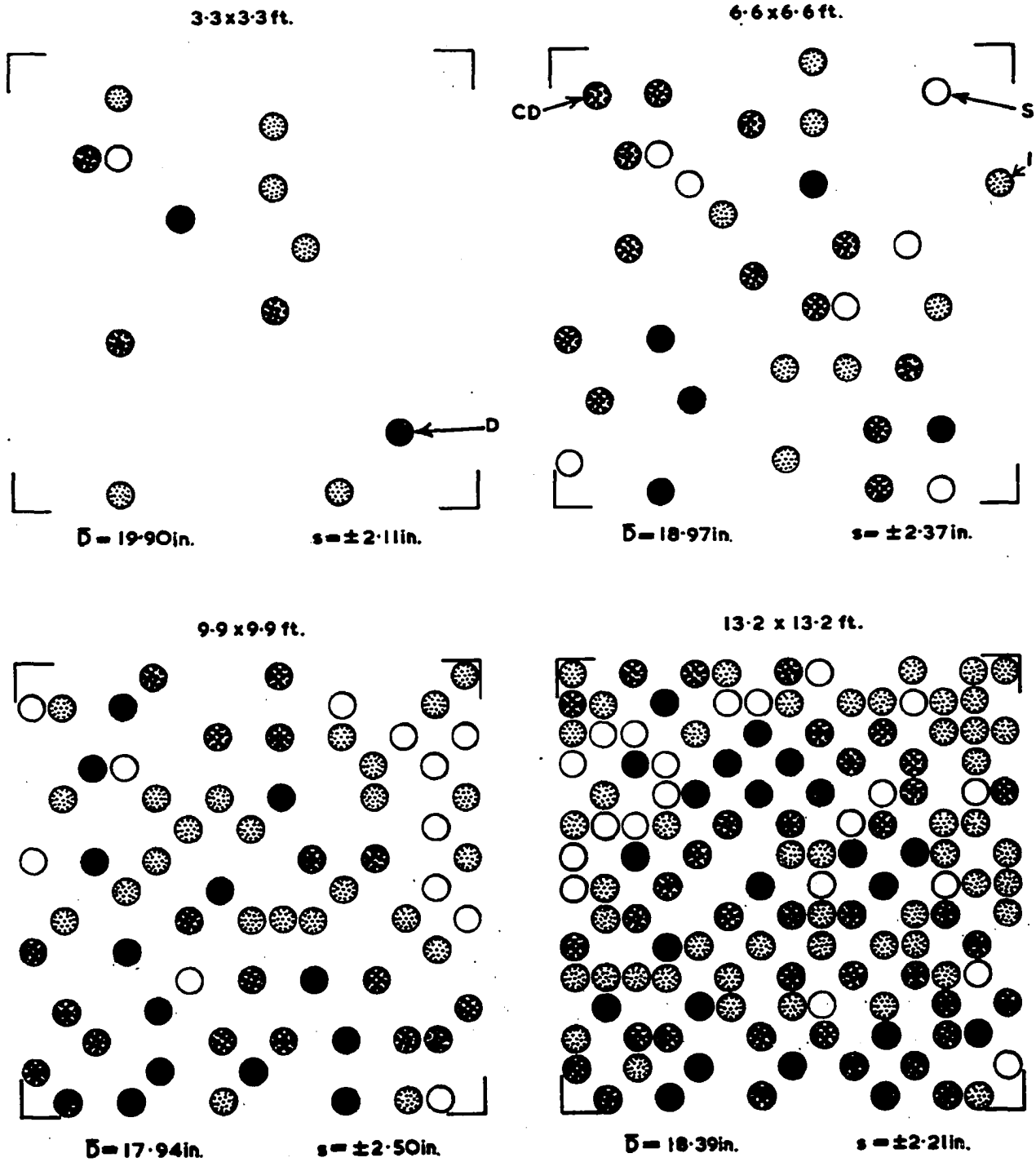

FIG. 25: Stand structure of the basic model at the different spacings at age 100 years. Run II-1.

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it can be seen that there is some site variation within the stand at age ten years. Mortality occurs first in the local patches of highest site quality (Fig. 24, age 40 years) as the competition is more intense. By age 60 years, mortality has occurred among the lower site groups and the trees have become more uniformly distributed. Before the number of trees is reduced to the level shown for the 3.3 x 3.3 ft. spacing at age 100 years (Fig. 25), the model ceases to function properly. The model only searches to a distance of eight times the initial spacing (26.4 ft. in this instance) for competitors. Therefore, at close spacings, some of the trees are considered to be free of competition in one or more octants, diameter growth is not reduced sufficiently and basal area consequently rises to the high level previously described (see Fig. 15).

Stand Model IIA

As it failed to give satisfactory results at the 3.3 x 3.3 ft. spacing above age 40 years, stand Model II was modified by prescribing a fixed amount of mortality each five-year period. This consisted of 0.5 per cent random mortality, which was applied to the stand each five-year period from the start of the model. A "competition" mortality was applied at the end of each five-year period as soon as five per cent of the stand had shown no increase in diameter growth during a five-year period. The values of this mortality were, in the first instance, interpolated from the

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yield table for Douglas fir of Barnes (U. B. C. Forest Club, 1959) but were later modified to suit the requirements of the model (Table 2). This competition mortality was not allocated

TABLE 2: Mortality by five-year periods for Douglas fir. Adapted from Barnes (U. B. C. Forest Club, 1959). Run IIA-1.

Age (yr.)	Previous 5 years' mortality (% of total no. of trees)
15	25
20	23
25	21
30	19
35	17
40	14
45	12
50	10
55	8
60	7
65	7
70	6
75	6
80	5
85	5
90	4
95	4
100	3

uniformly to all diameter classes but with decreased probability in classes greater than the mean. Within each class trees were selected at random for mortality. A detailed description of this method of applying mortality was given in an earlier report (Newnham, 1964).

This modified model was run using the same matrix of diameters as was used in stand Model II described above.

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Results (Fig. 26-28) show a reasonable correspondence with the published yield table data except that the basal area was again too high in the 3.3 x 3.3 ft. spacing. Diameter distributions usually covered a greater range of values than in stand model II. As there was no improvement over stand model II, the development of this model was not pursued further. It was also thought more desirable that the final model should generate its own competition mortality rather than to have it predetermined.

Conclusions

Of the three stand models tested, stand model I can be disregarded for further use owing to its severe limitations in testing different spacings. Stand model II meets the requirements of this thesis in that mortality is self-prescribed and it can be used to test a wide range of spacings. Stand model IIA could probably also be developed into a satisfactory model with further modification. It has the advantage, which could also be built into stand model II if required, that a small amount of random mortality is allocated at the end of each five-year period as well as competition mortality. Both stand models II and IIA are of only limited use with spacings as close as 3.3 x 3.3 ft.

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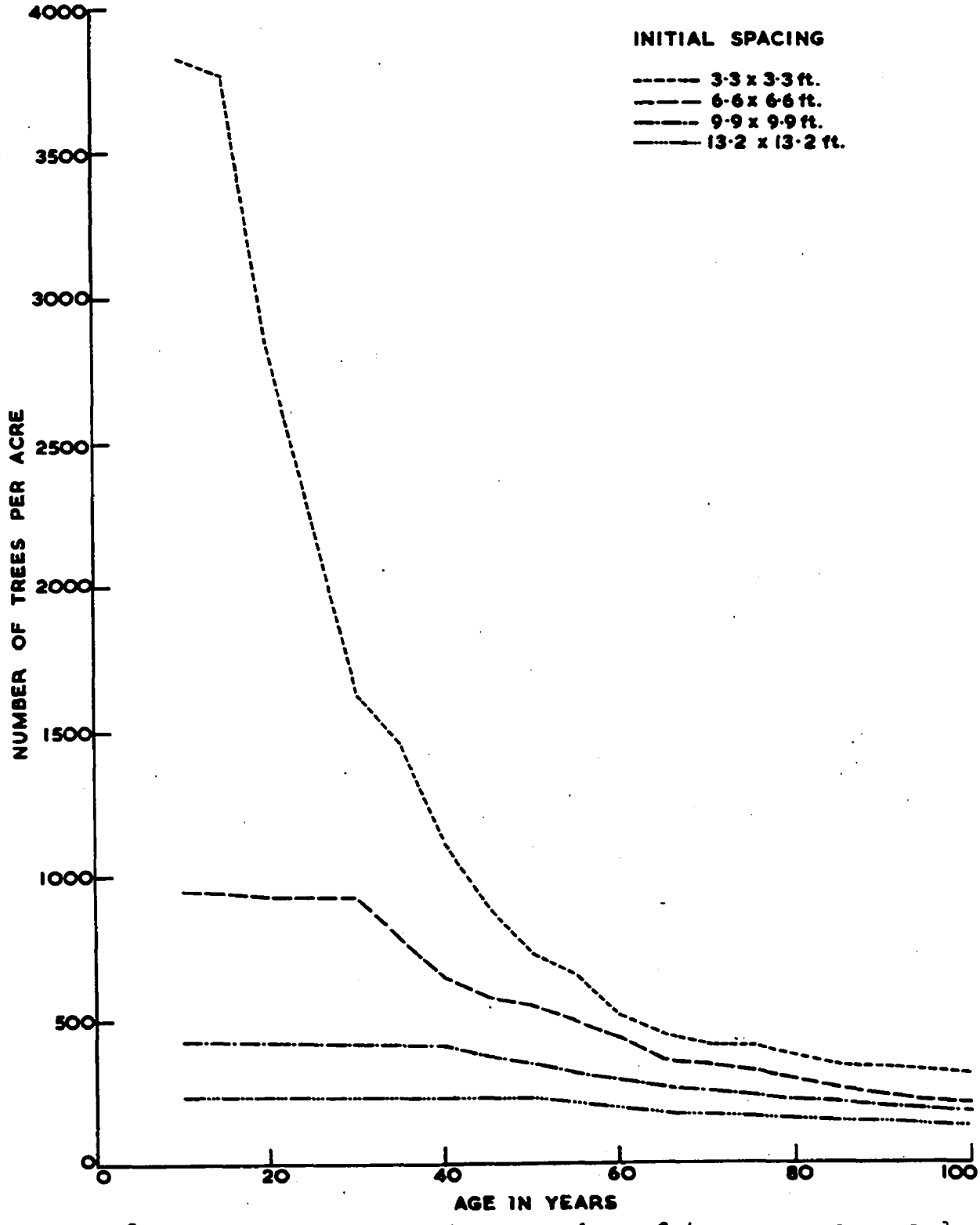

FIG. 26: The relationship between number of trees per acre and age. Run IIA-1.

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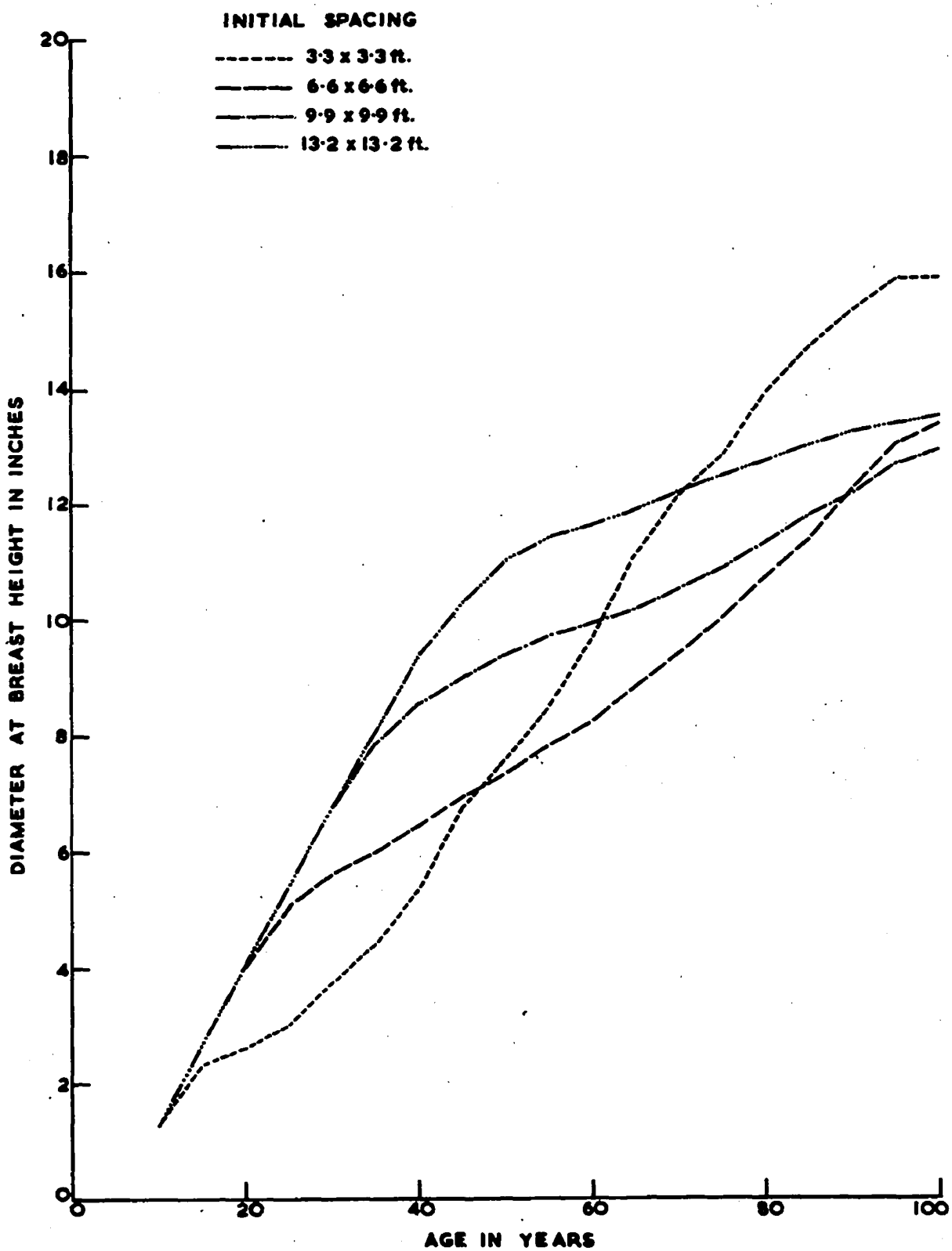

FIG. 27: The relationship between mean d. b. h. o. b. and age. Run IIA-1.

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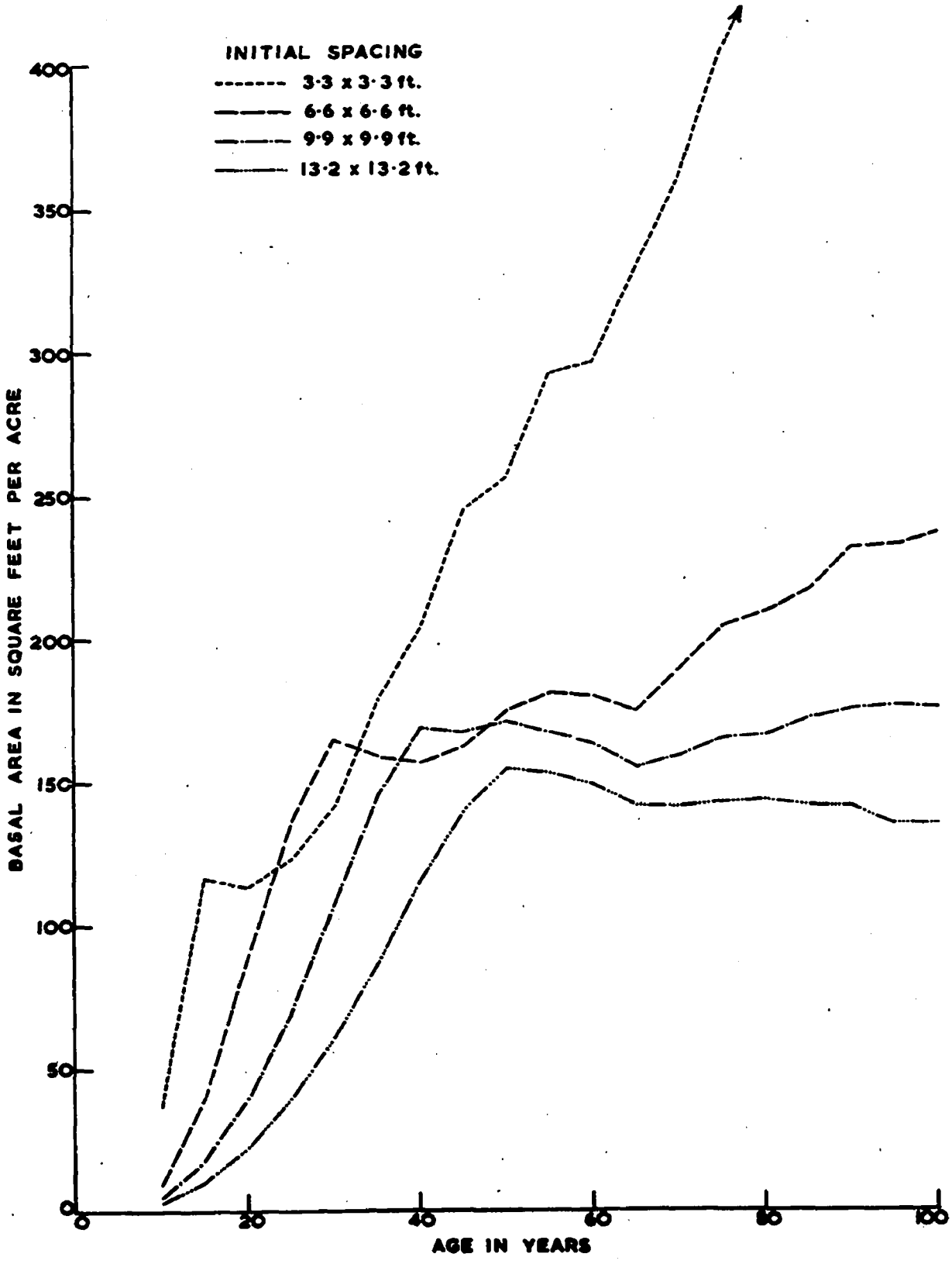

FIG. 28: The relationship between basal area per acre and age. Run IIA-1.

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PART III

TESTING STAND MODEL II

Having developed a satisfactory model for a normal or fully-stocked stand of site index 140 feet at 100 years, it was next necessary to test this model for stands having varying amounts and distributions of mortality following planting, for various sites and various thinning schedules. For all tests, except those used in testing site differences, the same diameter distribution as that used in developing the model, i.e. $N(\mu = 1.26, \sigma^2 = 0.177)$, was assumed. In the test runs, however, the theoretical normal distribution was used as opposed to the empirical distribution, obtained from the Douglas fir plantation at Haney, which was used in developing the model. For testing site differences each diameter in the basic matrix was multiplied by a constant, which varied with the site index being tested. The distribution of locations occupied by trees therefore remained constant over the range of sites tested while the distribution of diameters varied. An advantage of these methods of testing over field experiments is that all the site factors, except the factor being tested, are held constant.

Mortality Following Planting

Mortality following planting describes the number of trees missing, or dead, in the initial diameter matrix at age ten years. In the plantation which was used in developing the model this was 4.4 per cent which, in practice,

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even at spacings as wide as 13.2 x 13.2 ft., would be ignored. The purpose of these tests was to determine the amount of mortality that may occur at each spacing before the final yield would be affected. The distribution of the dead trees is important. If the dead trees occur in clumps, as opposed to a random distribution, the stand will take longer to return to full stocking (see Smith et al., 1961, Table 16).

Method of Generating Distributions

The distributions tested were: binomial (10, 30 and 50 per cent mortality), uniform or rectangular, (50 per cent mortality) and an artificial distribution consisting of two random infection centres (fourteen per cent mortality). To allocate each distribution of mortality, the basic matrix of 15 x 15 tree locations was divided into 9- location square plots. The number of plots with 0, 1, 2, ..., 9 trees was calculated from the density function for the appropriate distribution (see Appendix I); each plot was allocated one of these numbers at random. The locations of the trees within each plot were chosen at random and, finally, a diameter was allocated at random to each tree from a normal distribution ($\mu = 1.26$, $\sigma^2 = 0.177$).

The two random infection centres were chosen to represent the clumped mortality associated with, for example, Poria weirii (see Plate II). Two locations in the matrix were chosen at random as centres of infection. The infection was then assumed to have spread outwards from each centre,

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killing trees with probability inversely proportional to the distance from the centre location. To allocate this mortality to the initial matrix, it was assumed that all eight trees in the first square "shell" surrounding the centre location, four of the sixteen trees in the second "shell", two of the 24 trees in the third "shell" and one of the 32 trees in the fourth "shell", were dead. The allocation of the dead trees in the second, third and fourth shells was carried out at random. Apart from the two random infection centres no other mortality was introduced. It was assumed that the living trees surrounding the two centres were not "infected" and therefore showed no decline in vigour. This is usually not the case in practice but the problem of determining the amount of reduction in growth due to infection was outside the scope of the present project.

The distributions of mortality tested therefore cover random mortality, described by the binomial distributions, and clumped mortality, described by the uniform distribution. The random infection centres describe an extreme of aggregation. The binomial distribution was used to give a random distribution of mortality, instead of the Poisson distribution, which is more usually associated with randomness, as the greatest number of trees that could occur in a plot was limited to nine. The Poisson distribution requires that there be no upper bound.

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Results

For comparative purposes it has been assumed in these tests that the stand with only ten per cent mortality, binomially distributed, is fully stocked for each spacing at age ten years. A mortality of less than ten per cent following planting is unusual in practice. The Douglas fir plantation at Haney, which was used in the development of the model, was specially chosen as representative of the maximum stocking that could be expected in practice.

The results of the tests are summarised, graphically, in Fig. 29-40. Cumulative frequency distributions are given in Fig. 41-43 and the development of the stand structure under the different types of mortality is given for the 6.6 x 6.6 ft. spacing in Fig. 44-53.

As would be expected, the initial planting distance plays an important part in determining the length of time required by the stand to again reach full stocking, by basal area. At the closest spacing (Fig. 31), differences in basal area have disappeared by age twenty, regardless of the amount or distribution of mortality at age ten years. Doubling the planting distance increases the age at which recovery takes place to between 60 and 70 years, depending on the amount and distribution of mortality (Fig. 34). At a spacing of 9.9 x 9.9 ft., the stands with 30 and 50 per cent binomial mortalities have recovered by age 85 to 95 years but the clumped, uniform (or rectangular) 50 per cent mortality has not fully recovered at age 100 years (Fig. 37). At the widest

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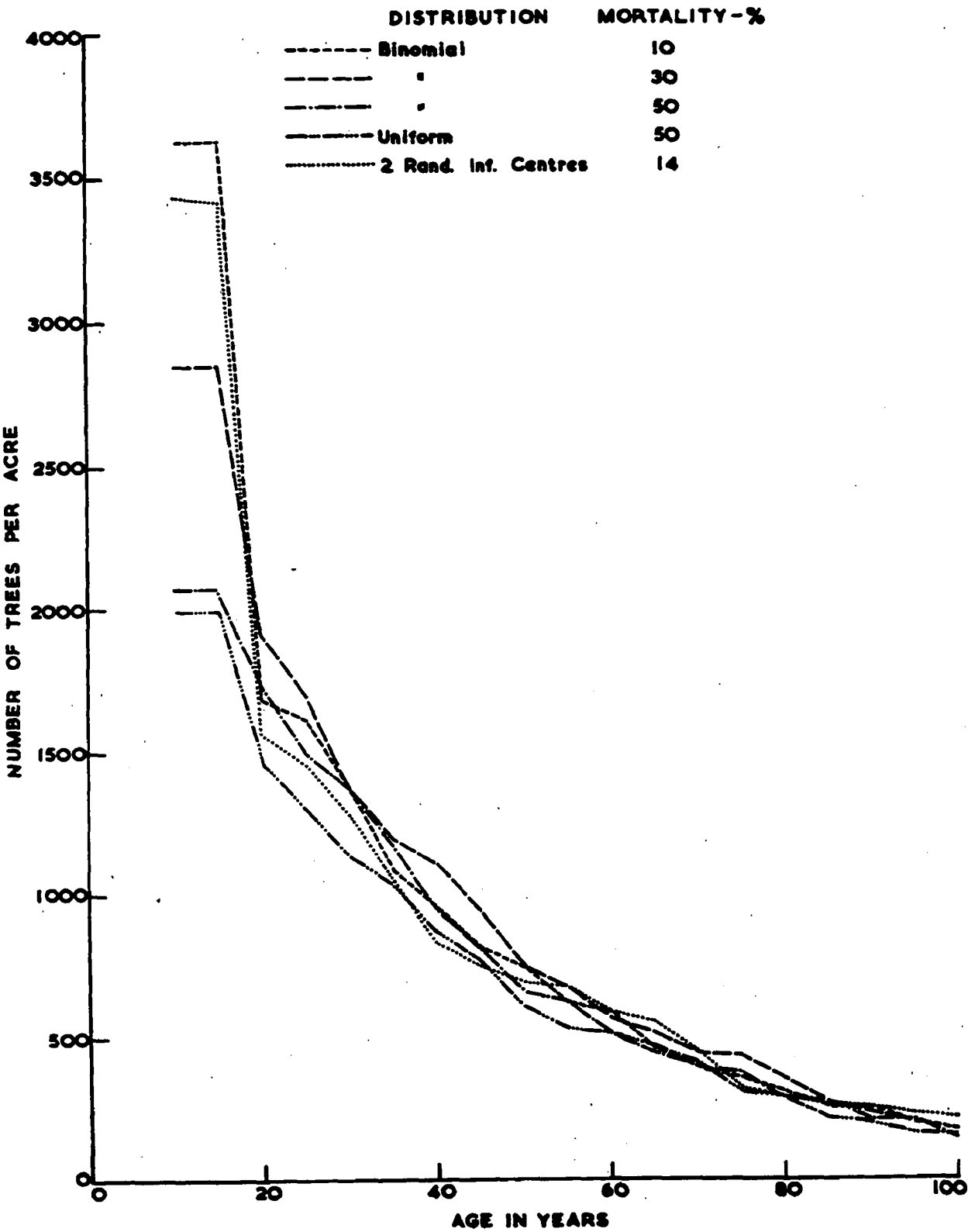

FIG. 29: The effect of amount and distribution of mortality following planting on number of trees per acre. Spacing: 3.3 x 3.3 ft. Runs II-2 to II-6.

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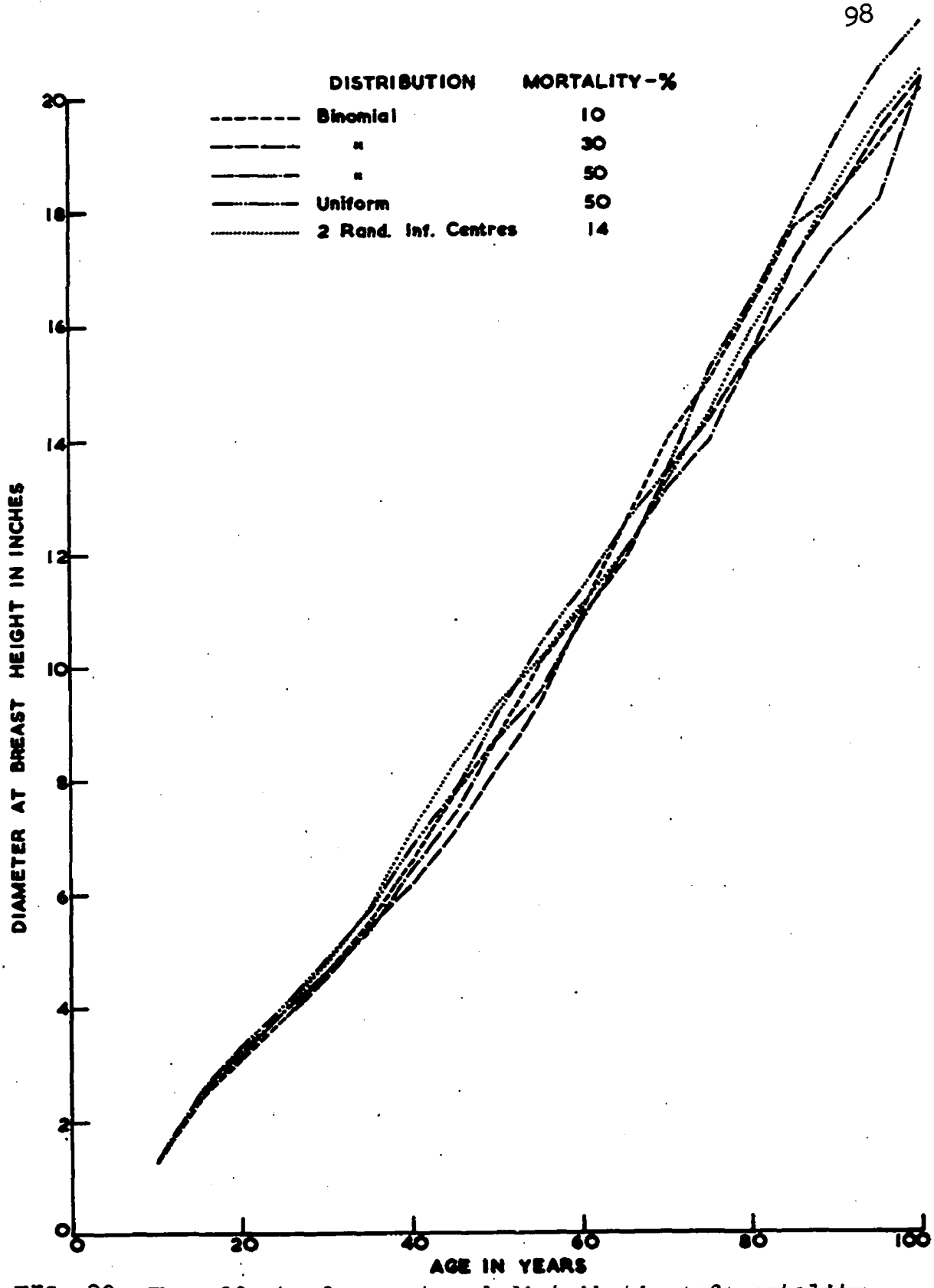

FIG. 30: The effect of amount and distribution of mortality following planting on mean d. b. h. o. b. Spacing: 3.3 x 3.3 ft. Runs II-2 to II-6.

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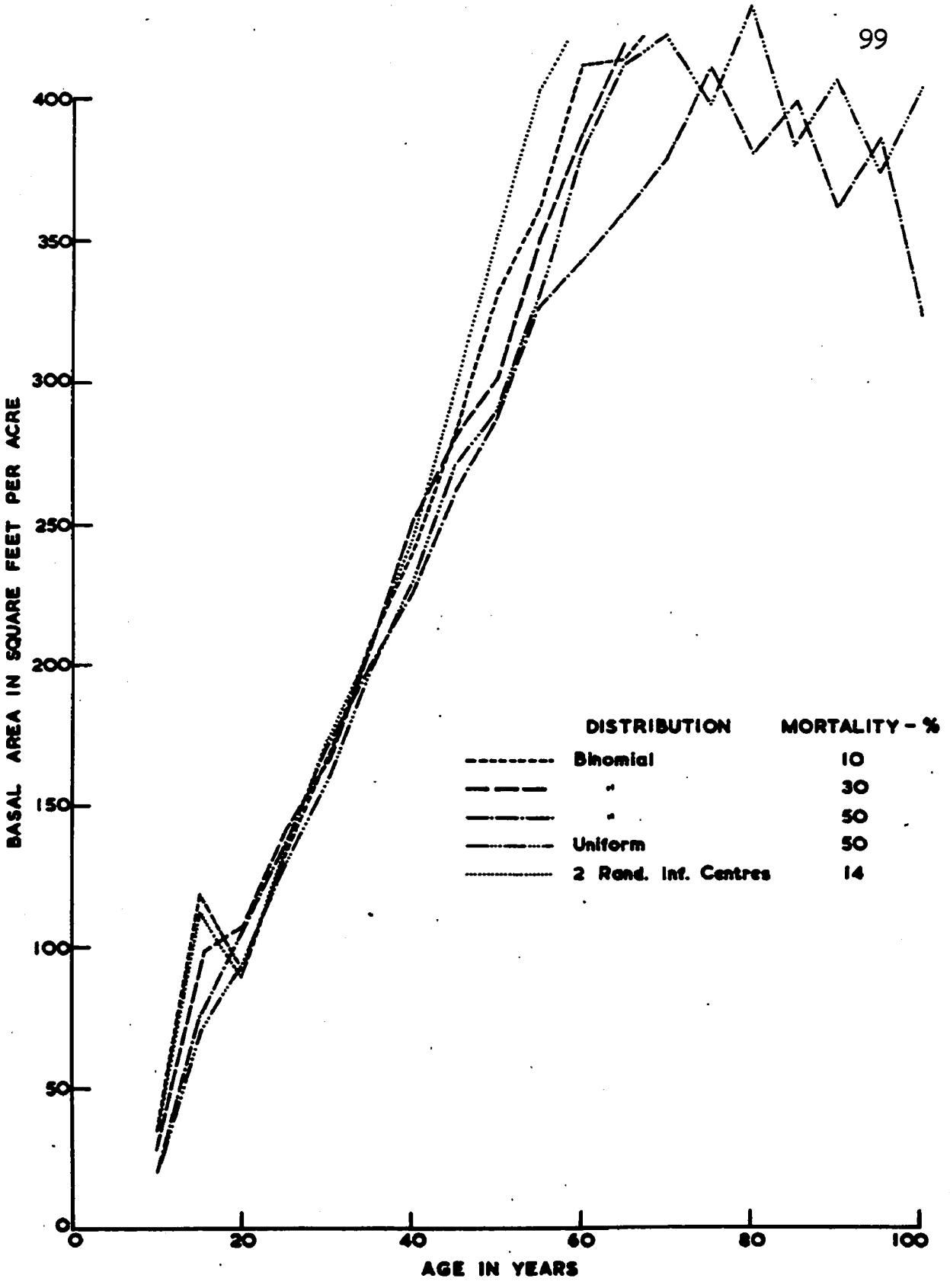

FIG. 31: The effect of amount and distribution of mortality following planting on basal area per acre. Spacing: 3.3 x 3.3 ft. Runs II-2 to II-6.

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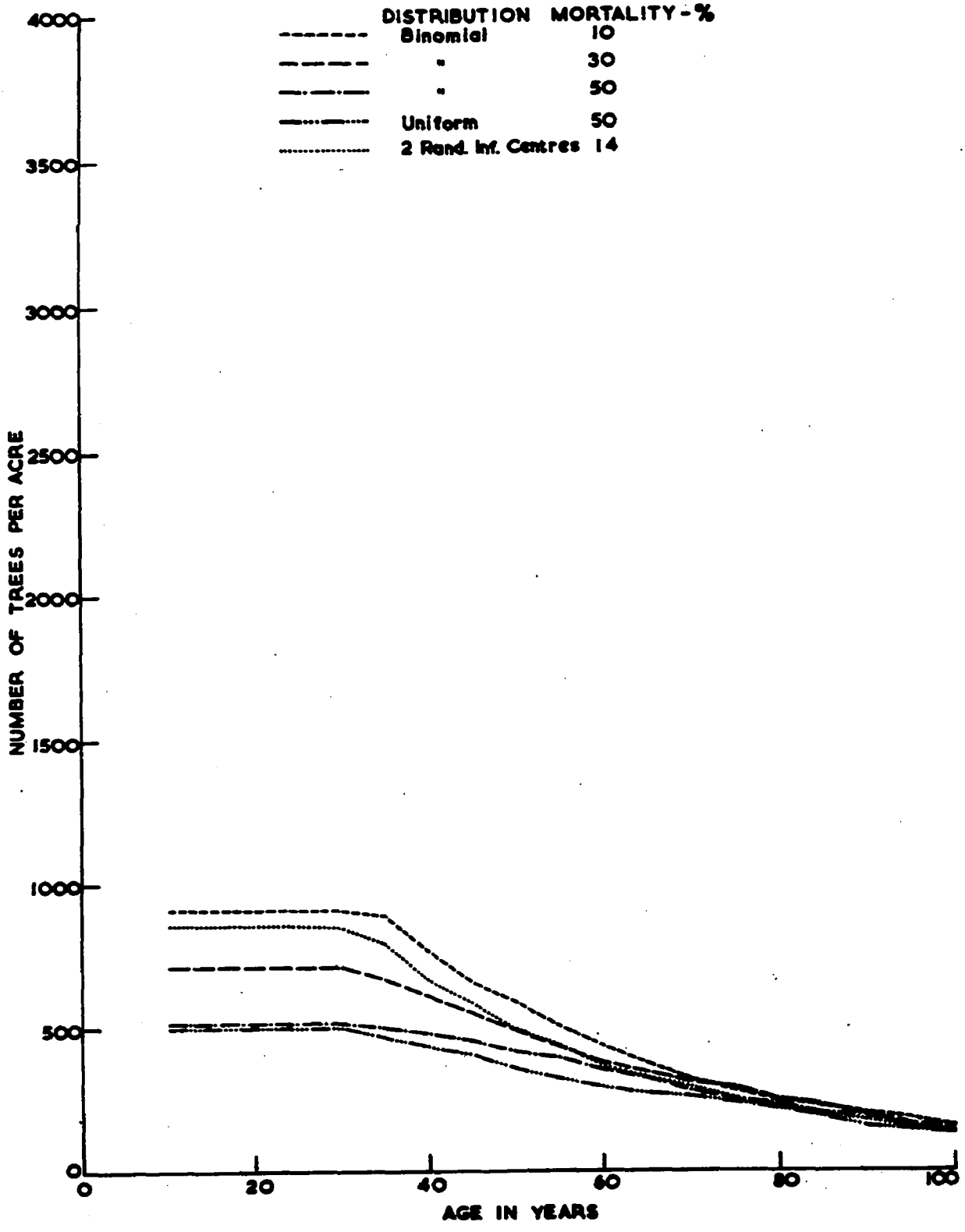

FIG. 32: The effect of amount and distribution of mortality following planting on number of trees per acre. Spacing: 6.6 x 6.6 ft. Runs II-2 to II-6.

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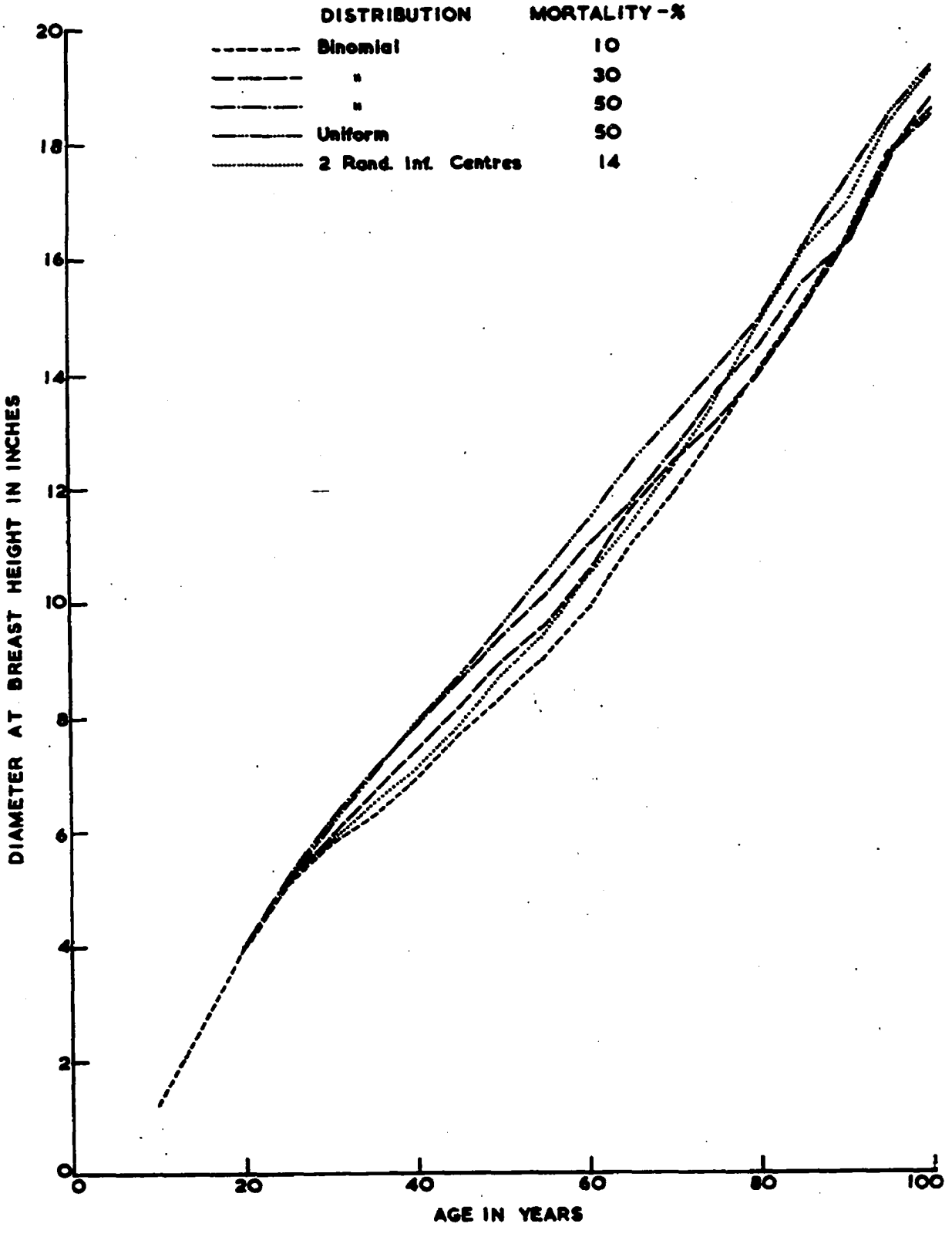

FIG. 33: The effect of amount and distribution of mortality following planting on mean d. b. h. o. b. Spacing: 6.6 x 6.6 ft. Runs II-2 to II-6.

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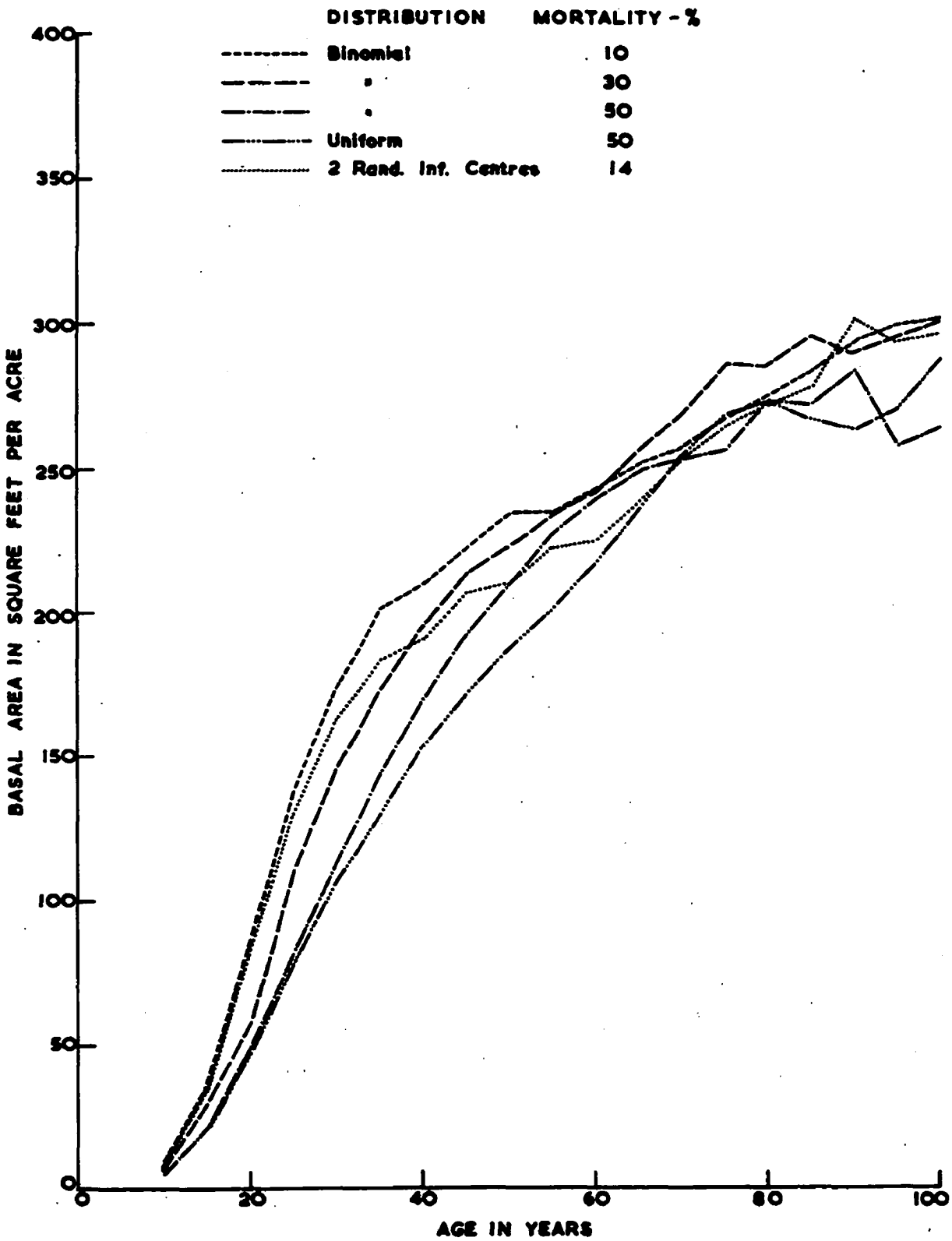

FIG. 34: The effect of amount and distribution of mortality following planting on basal area per acre. Spacing: 6.6 x 6.6 ft. Runs II-2 to II-6.

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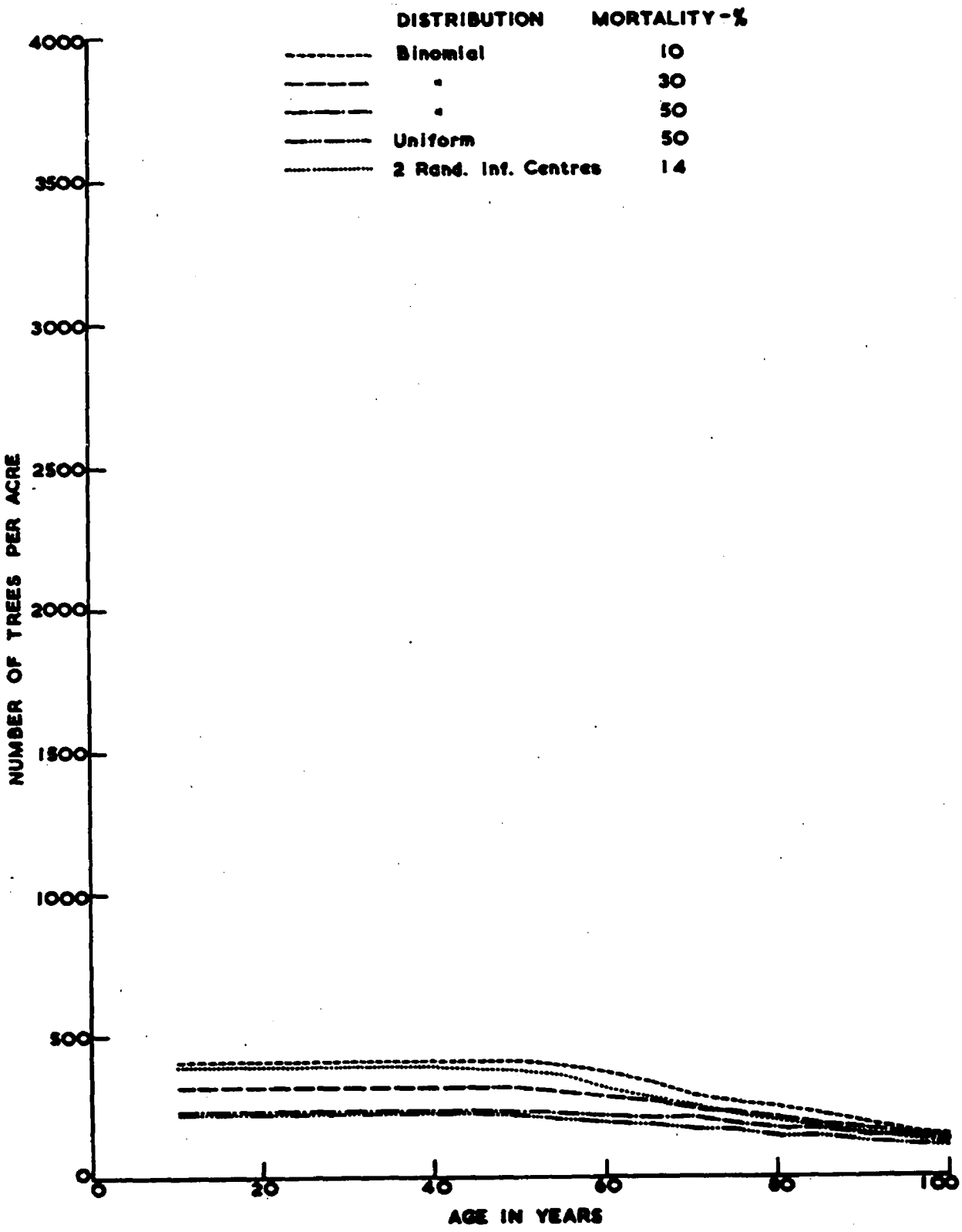

FIG. 35: The effect of amount and distribution of mortality following planting on number of trees per acre. Spacing: 9.9 x 9.9 ft. Runs II-2 to II-6.

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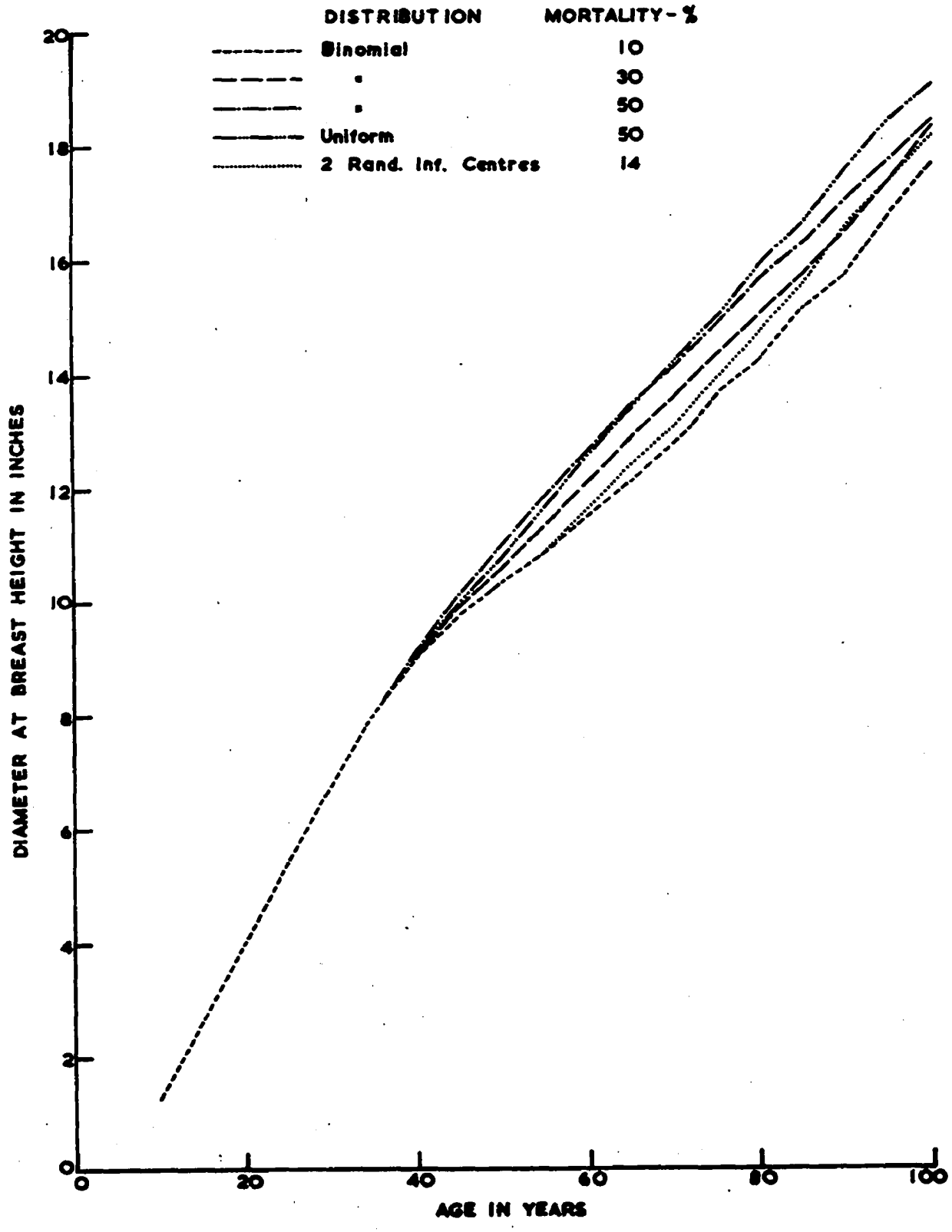

FIG. 36: The effect of amount and distribution of mortality following planting on mean d. b. h. o. b. Spacing: 9.9 x 9.9 ft. Runs II-2 to II-6.

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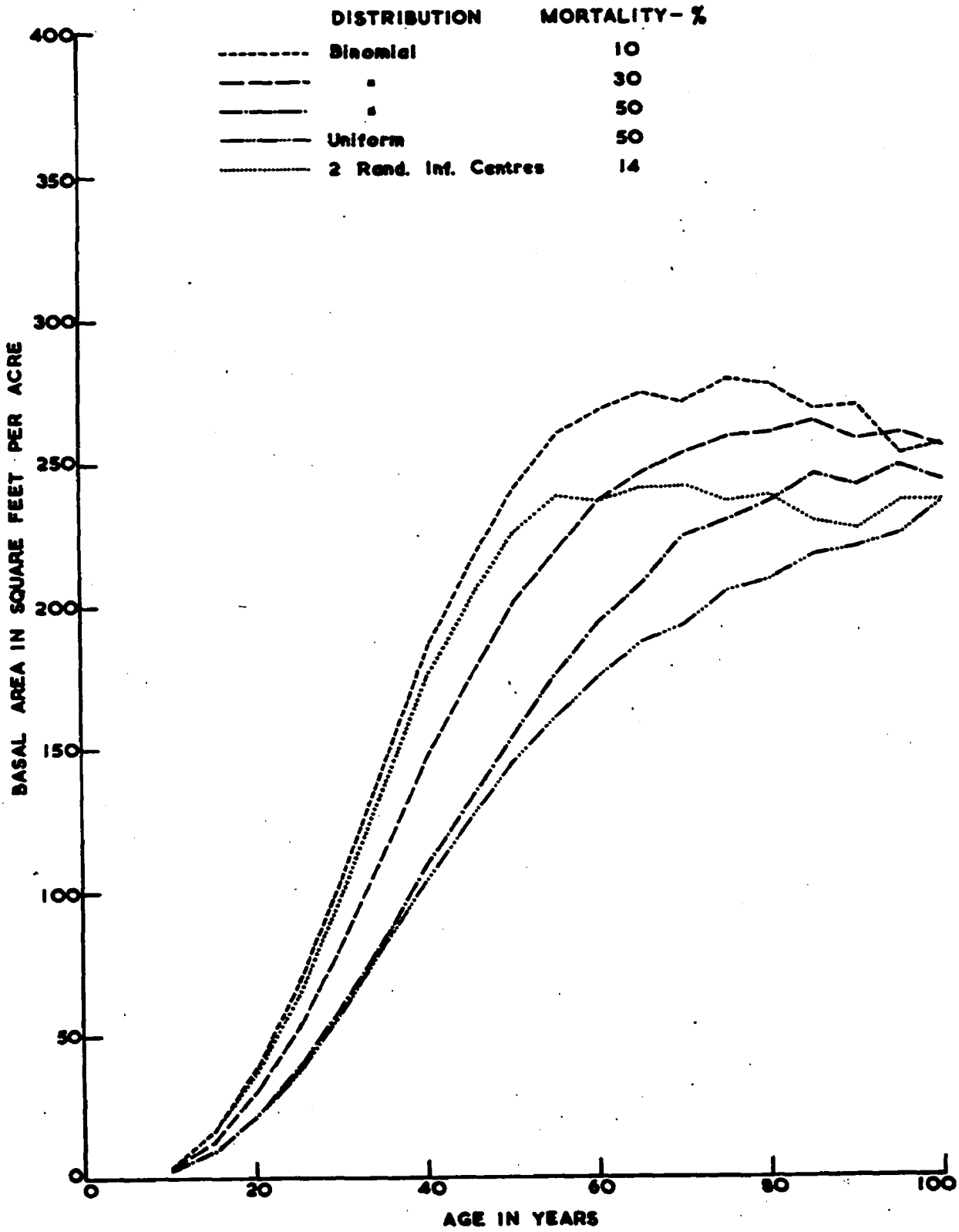

FIG. 37: The effect of amount and distribution of mortality following planting on basal area per acre. Spacing: 9.9 x 9.9 ft. Runs II-2 to II-6.

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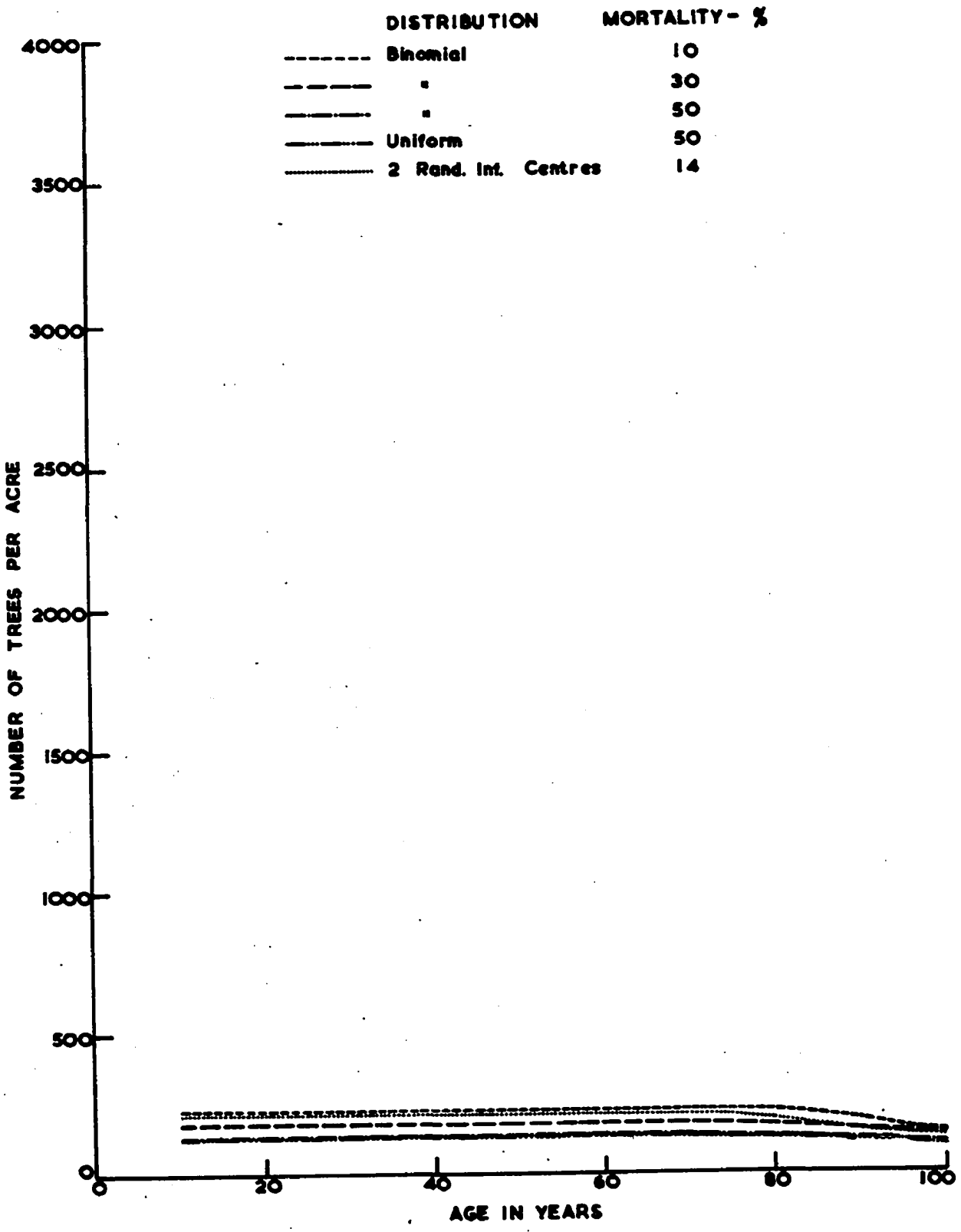

FIG. 38: The effect of amount and distribution of mortality following planting on number of trees per acre. Spacing: 13.2 x 13.2 ft. Runs II-2 to II-6.

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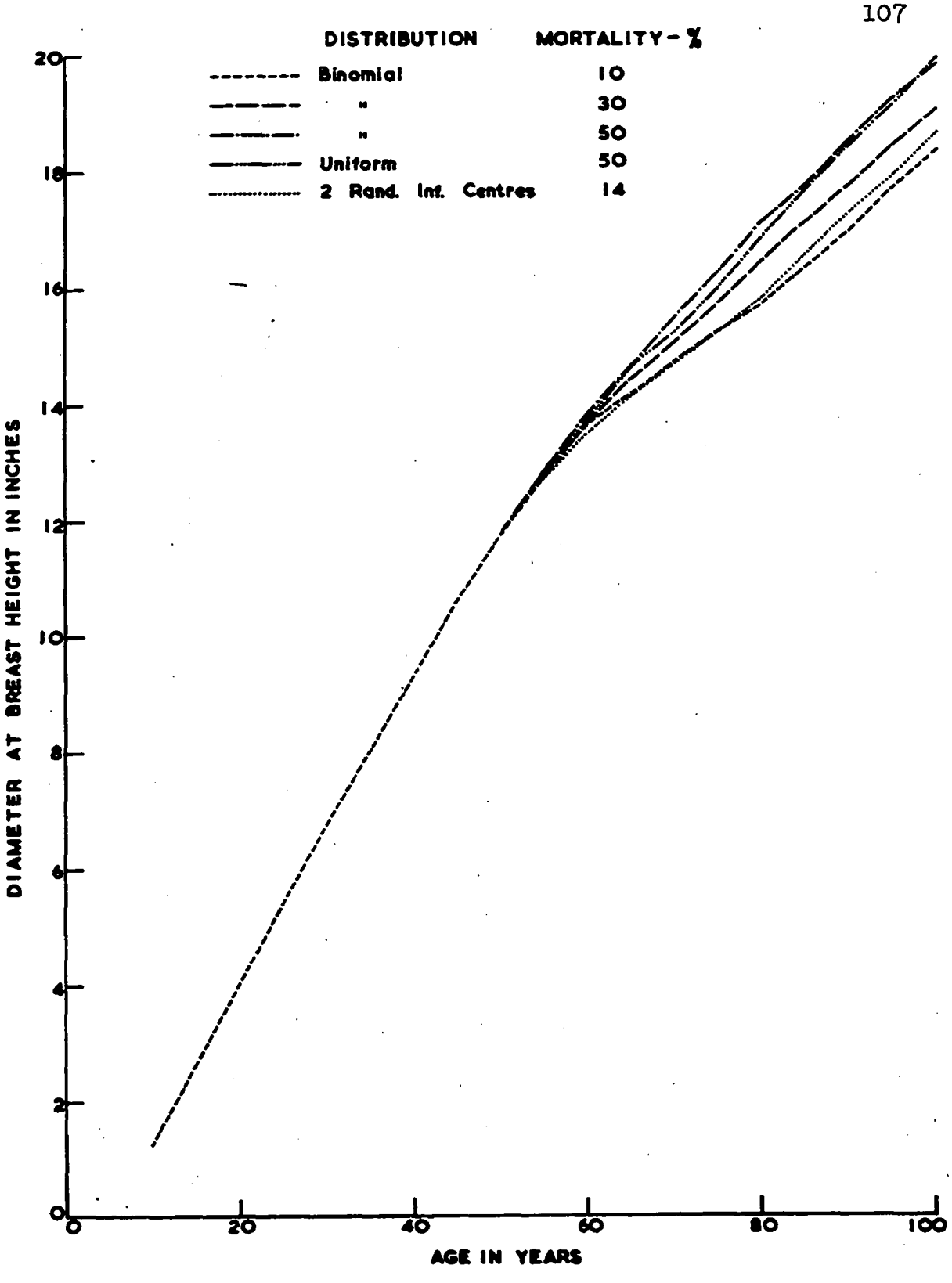

FIG. 39: The effect of amount and distribution of mortality following planting on mean d. b. h. o. b. Spacing: 13.2 x 13.2 ft. Runs II-2 to II-6.

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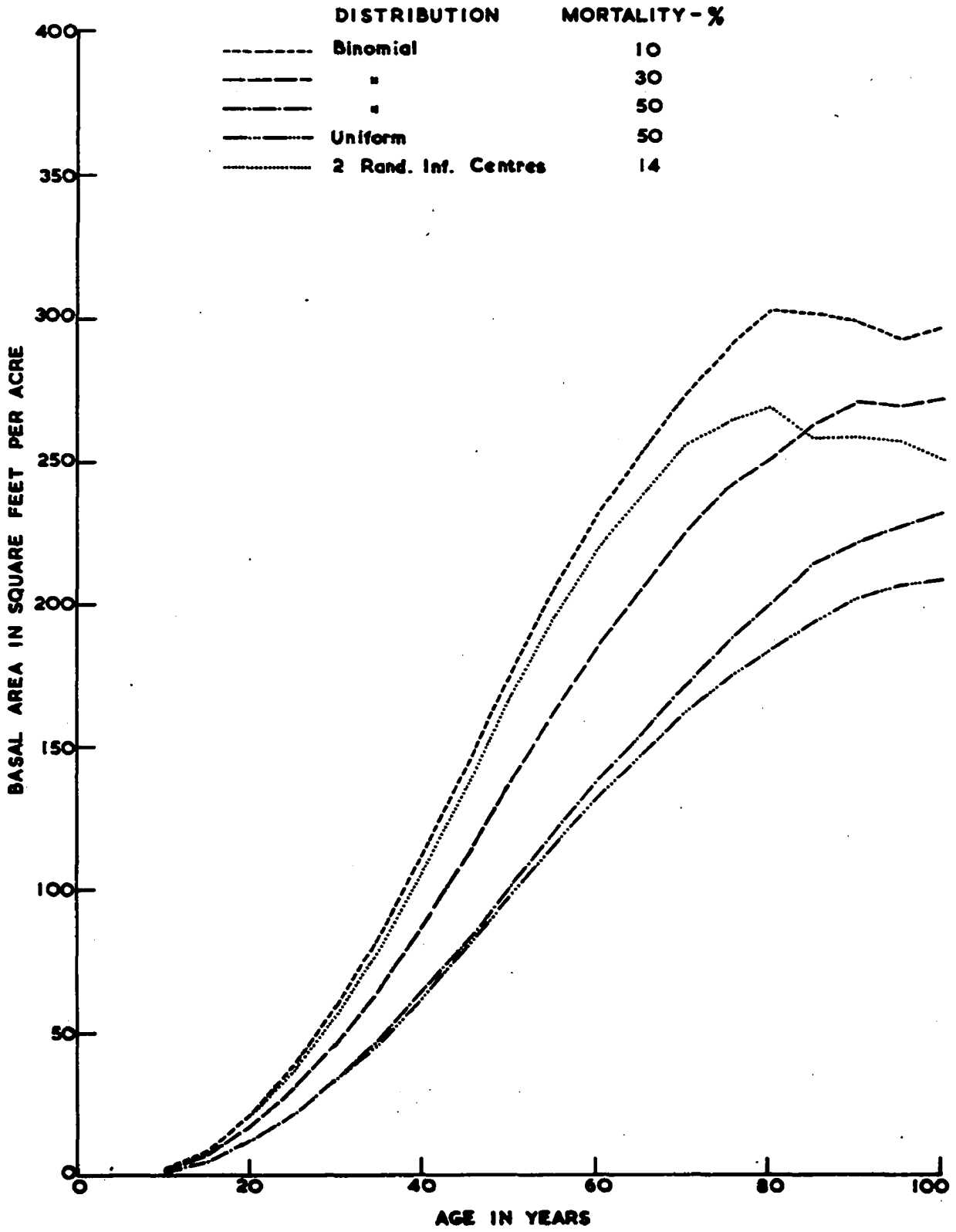

FIG. 40: The effect of amount and distribution of mortality following planting on basal area per acre. Spacing: 13.2 x 13.2 ft. Runs II-2 to II-6.

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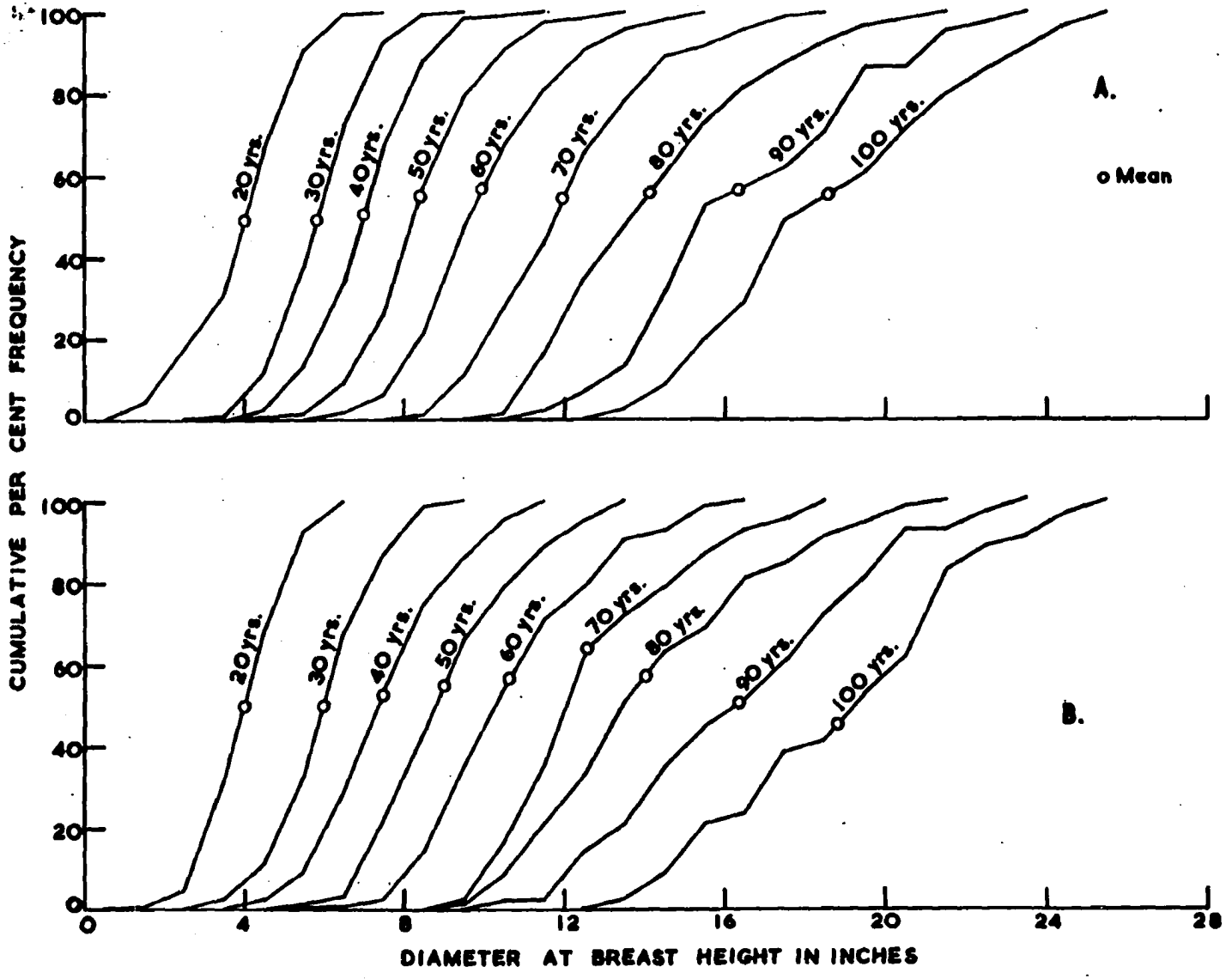

FIG. 41: Cumulative d. b. h. o. b. frequency distributions for (A) 10 and (B) 30 per cent binomial distributions of mortality following planting. Spacing: 6.6 x 6.6 ft. Runs II-2,3.

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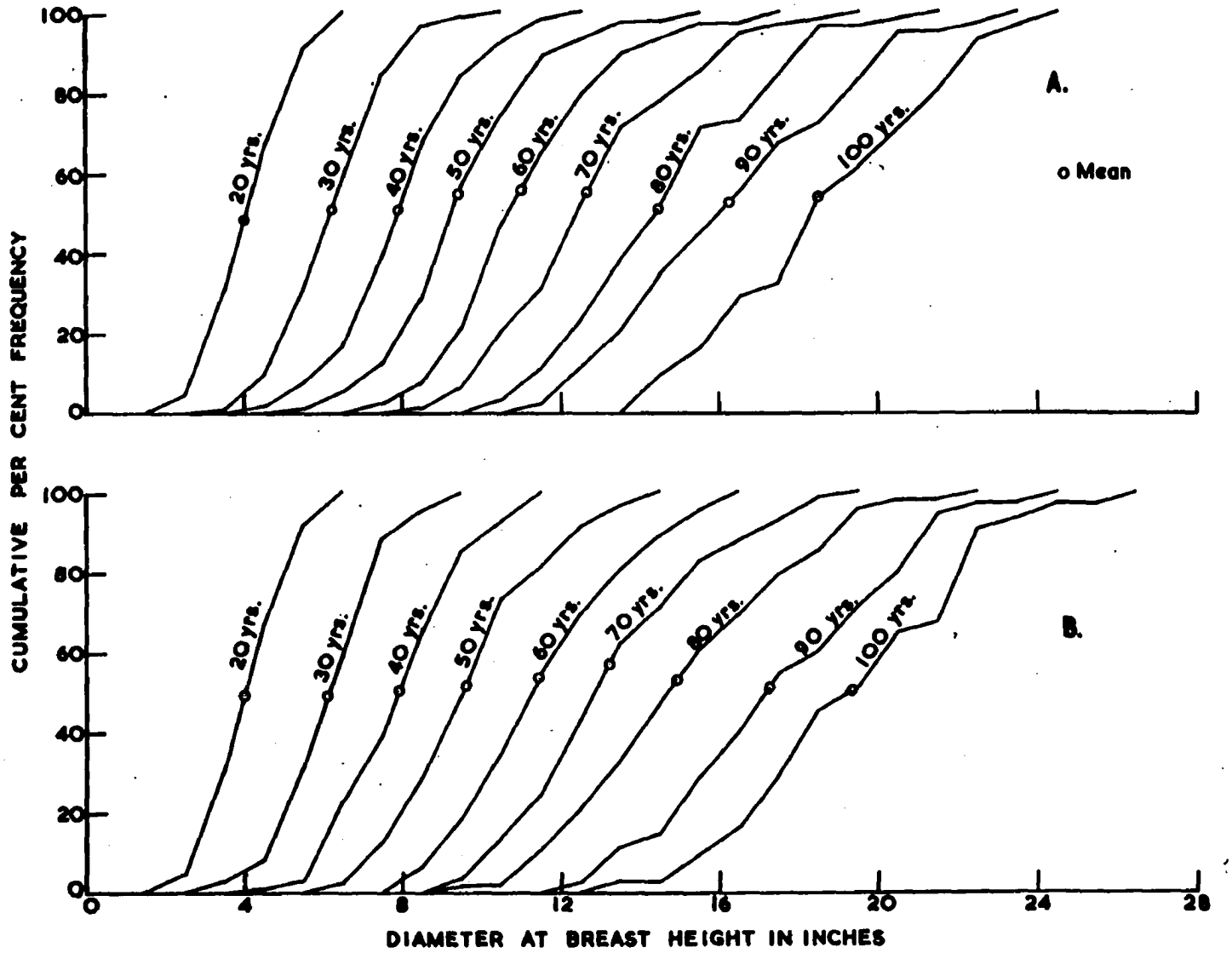

FIG. 42: Cumulative d. b. h. o. b. frequency distributions for (A) 50 per cent binomial and (B) 50 per cent uniform (rectangular) distributions of mortality following planting. Spacing: 6.6 x 6.6 ft. Runs II-4,5.

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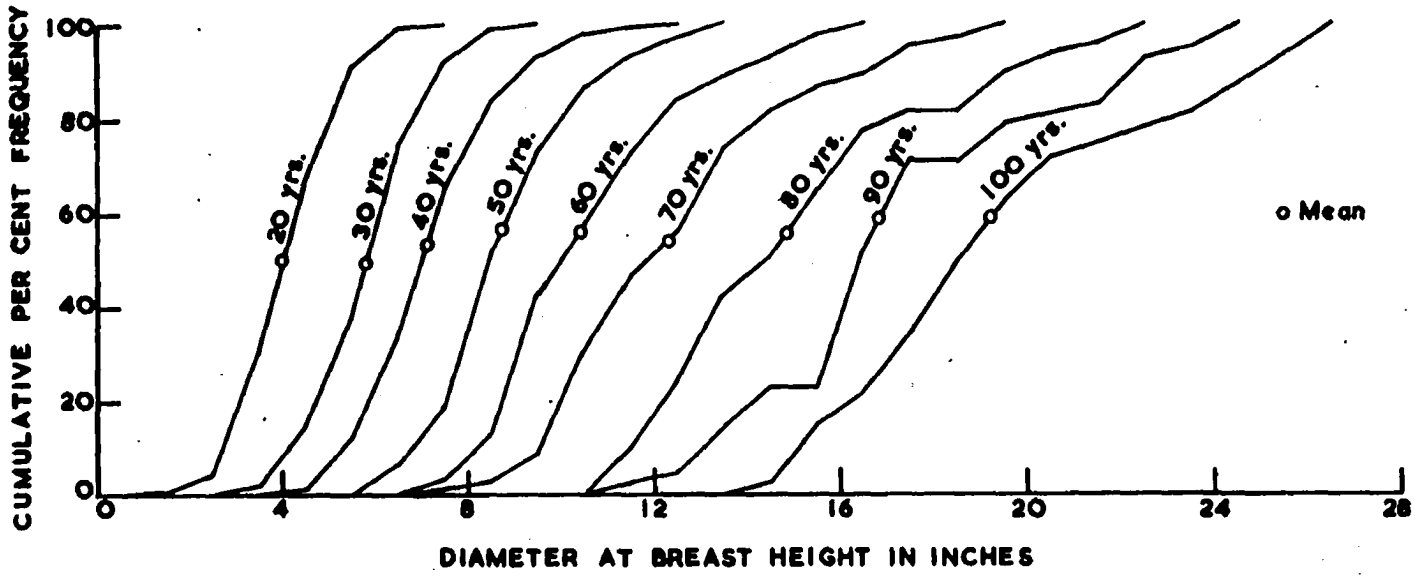

FIG. 43: Cumulative d. b. h. o. b. frequency distributions for two random infection centres (14 per cent mortality following planting). Spacing: 6.6 x 6.6 ft. Run II-6.

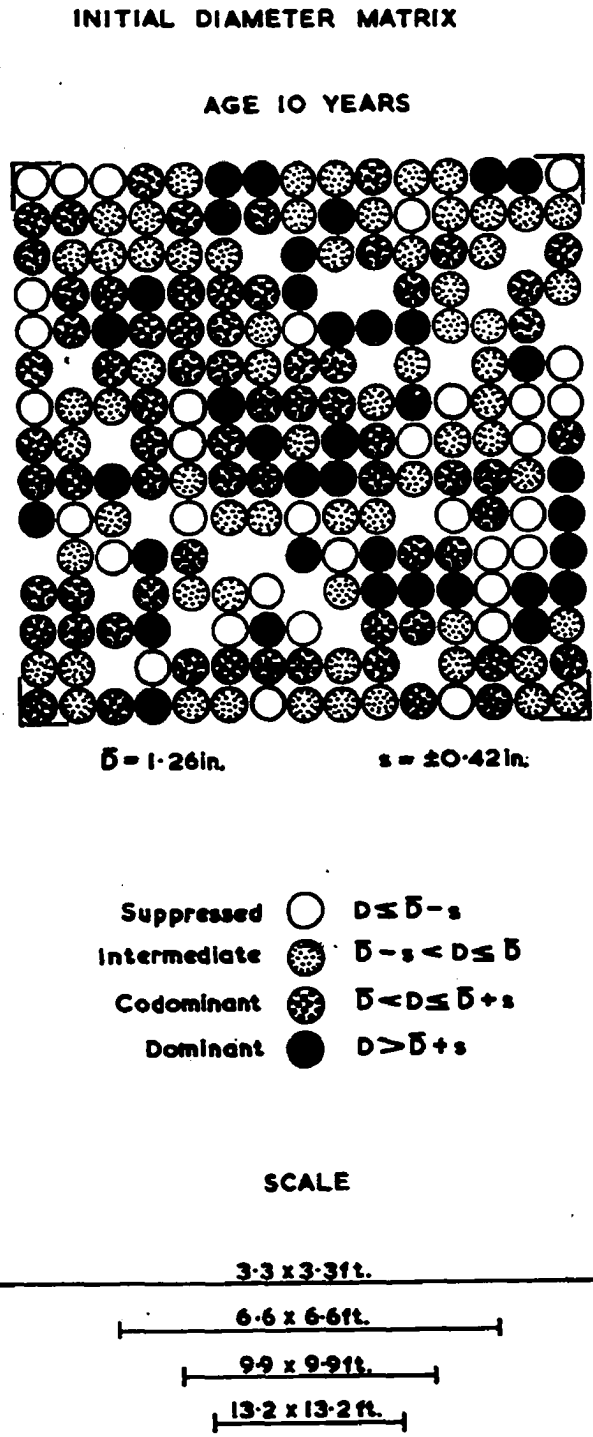

FIG. 44: Initial diameter matrix with 10 per cent binomial distribution of mortality following planting. Run II-2. The scales represent 66 ft. at each spacing.

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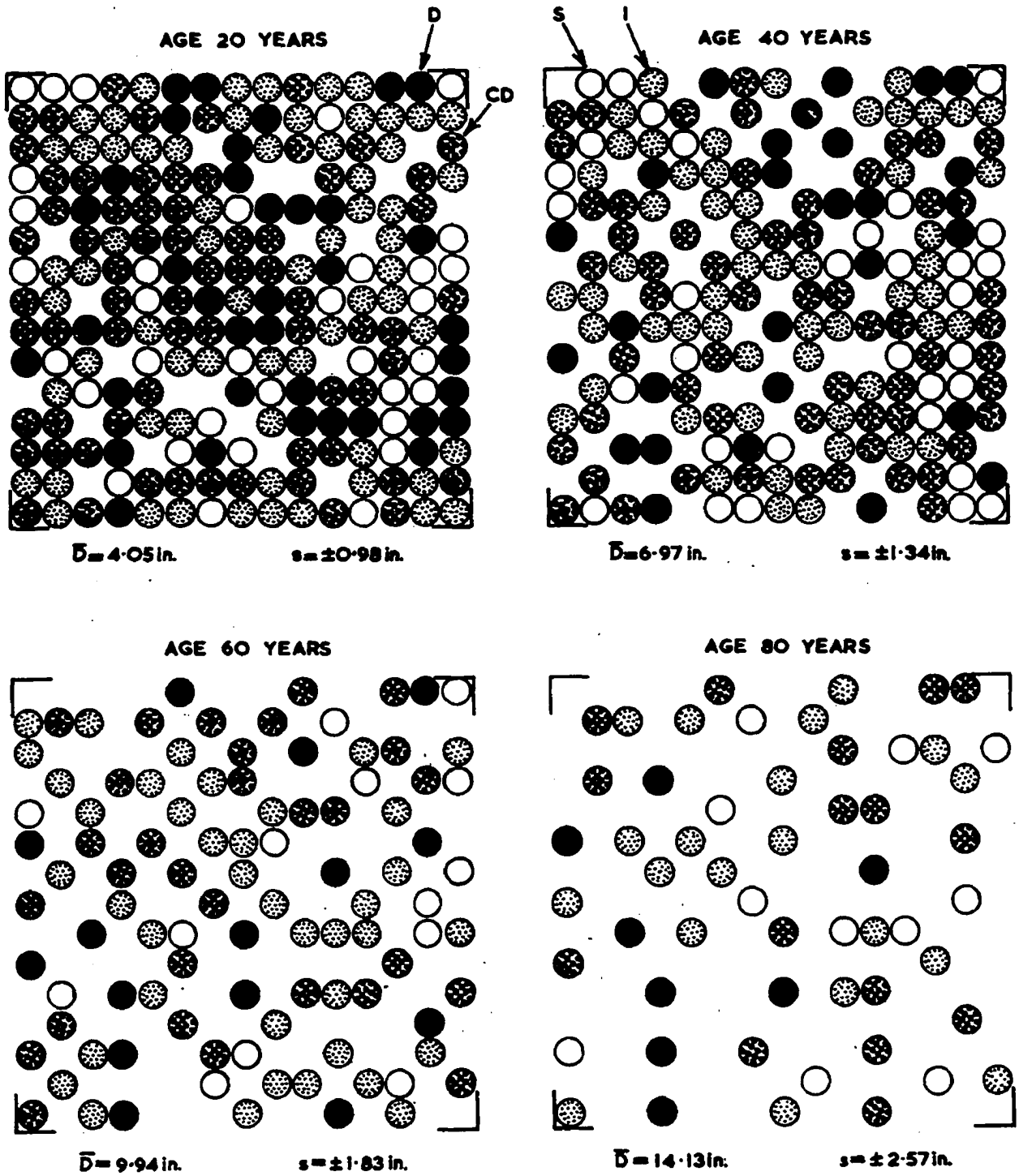

FIG. 45: The effect of amount and distribution of mortality following planting on stand structure. Binomial distribution (10 per cent mortality). Spacing: 6.6 x 6.6 ft. Run II-2.

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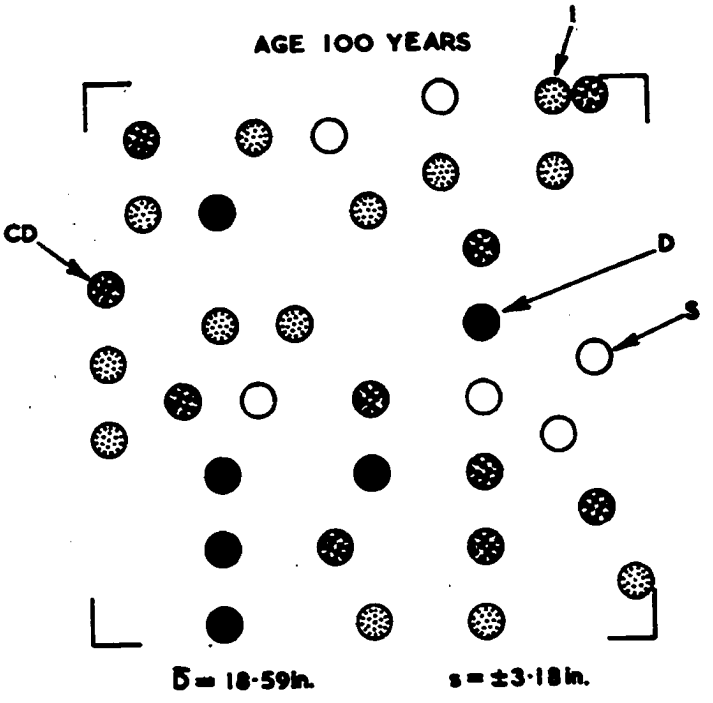

FIG. 45: Continued.

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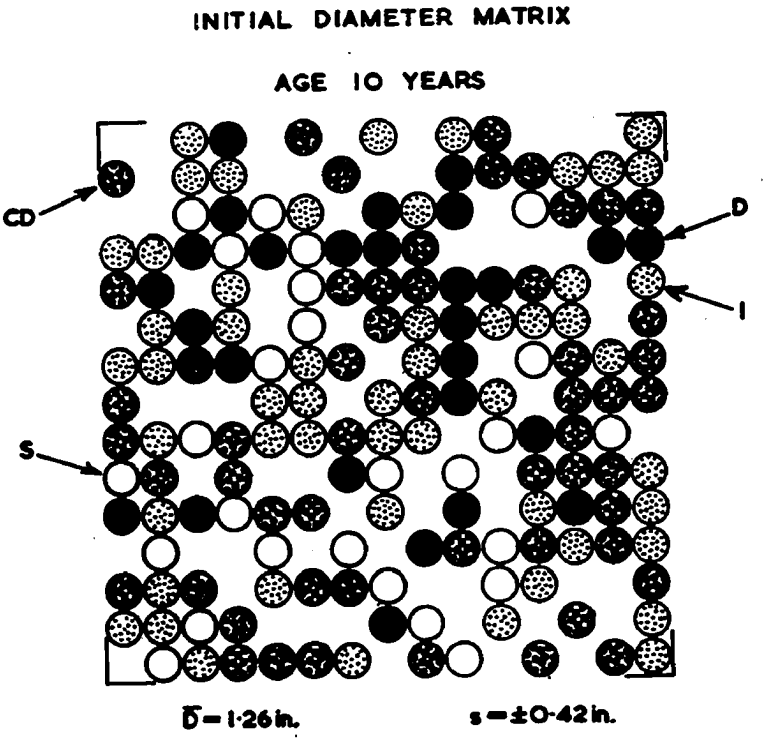

FIG. 46: Initial diameter matrix with 30 per cent binomial distribution of mortality following planting. Run II-3.

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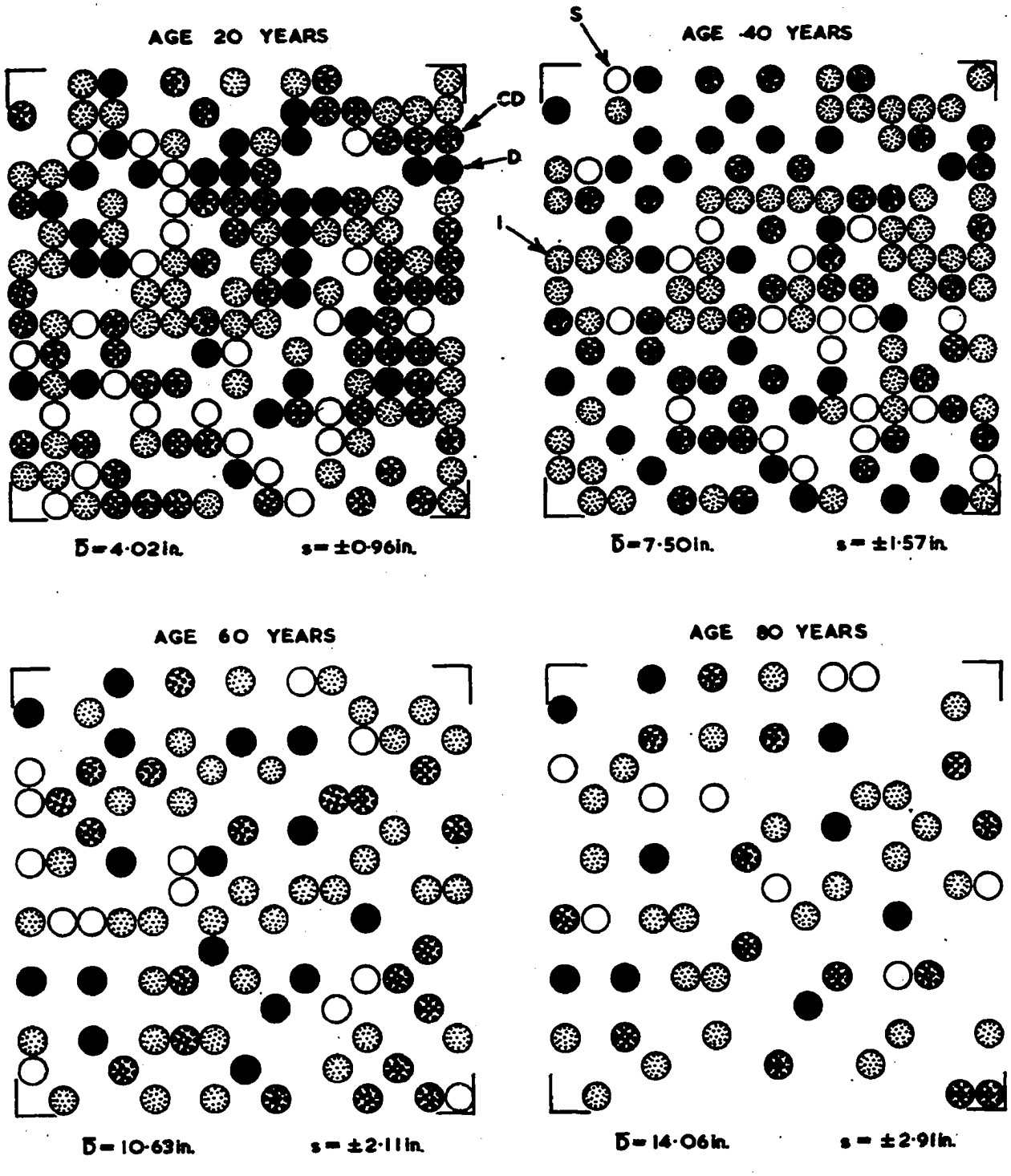

FIG. 47: The effect of amount and distribution of mortality following planting on stand structure. Binomial distribution (30 per cent mortality). Spacing: 6.6 x 6.6 ft. Run II-3.

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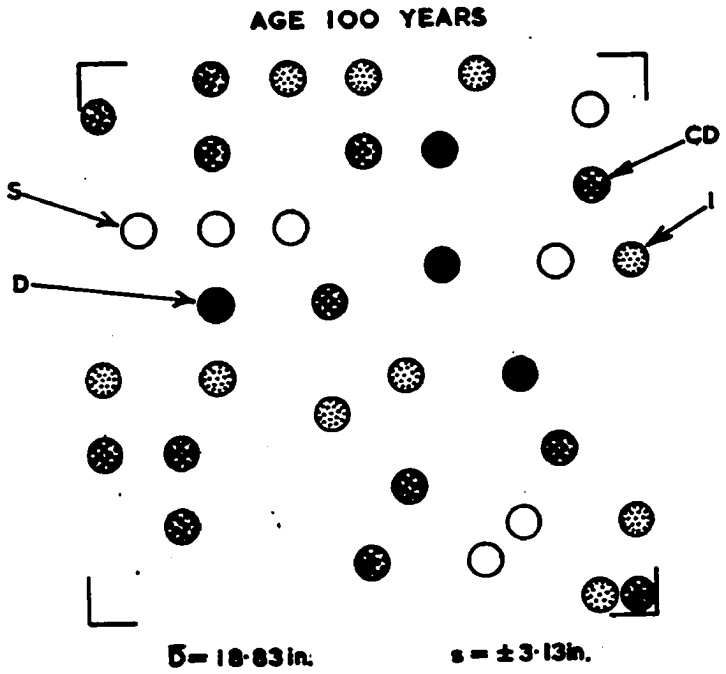

FIG. 47: Continued.

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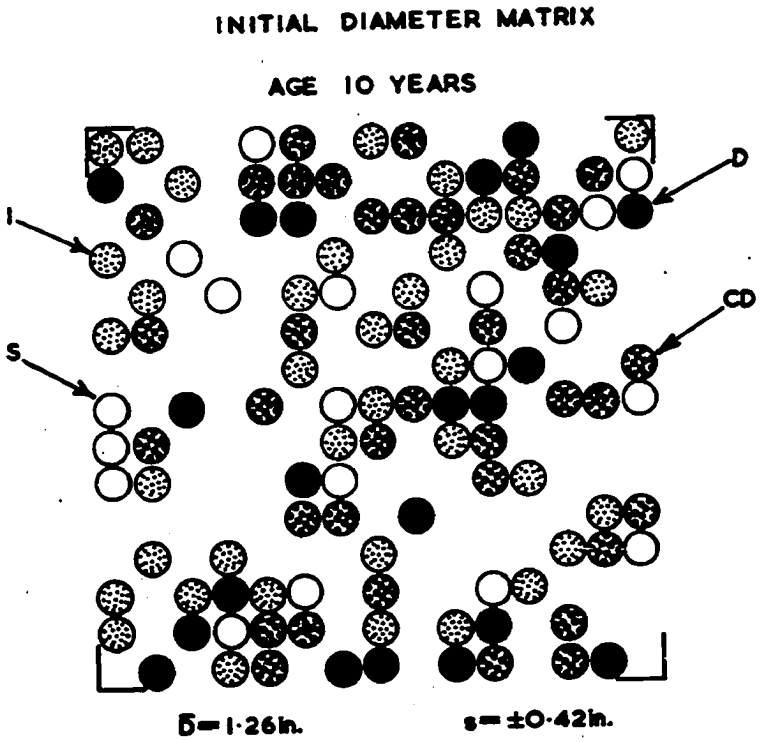

FIG. 48: Initial diameter matrix with 50 per cent binomial distribution of mortality following planting. Run II-4.

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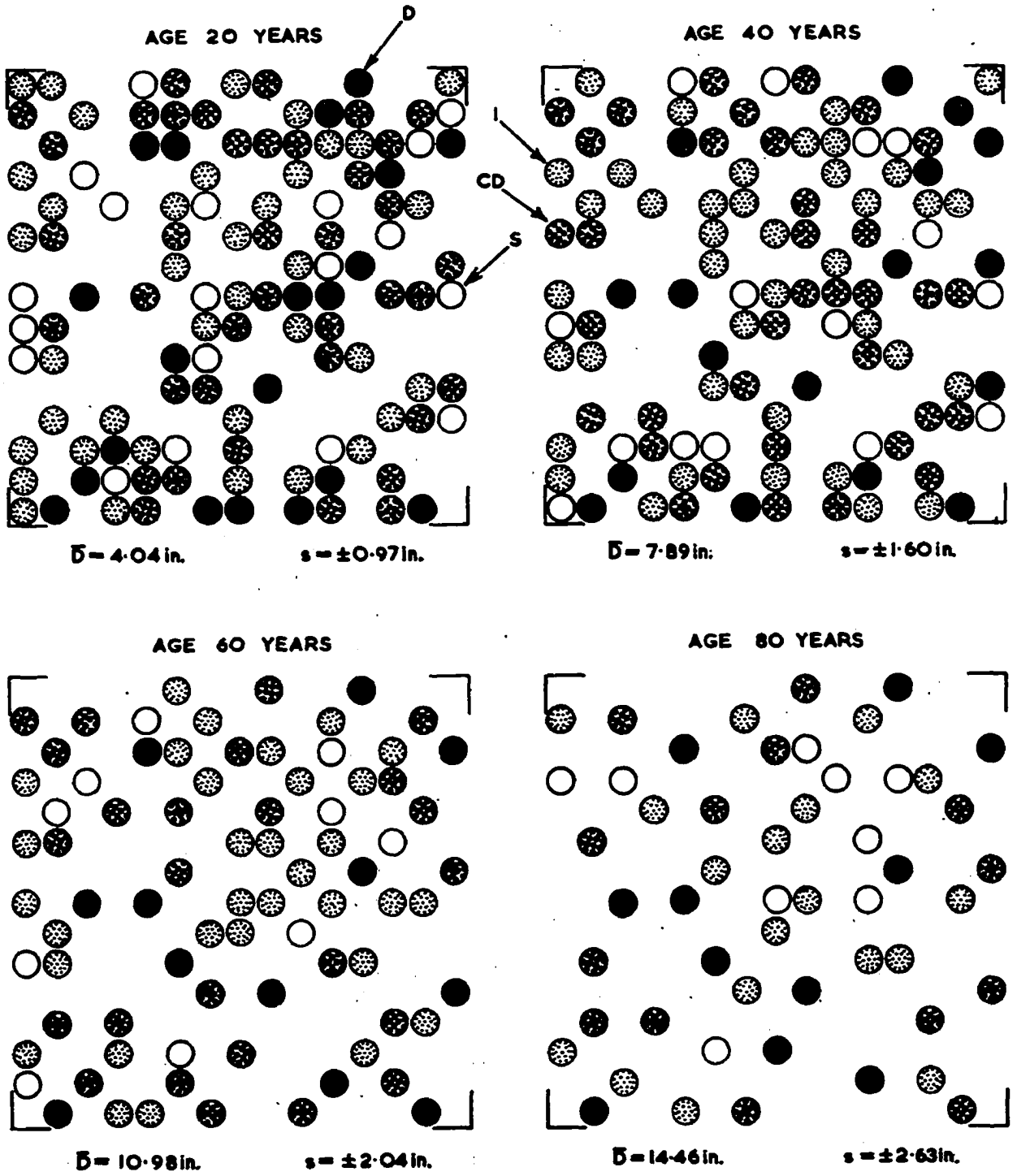

FIG. 49: The effect of amount and distribution of mortality following planting on stand structure. Binomial distribution (50 per cent mortality). Spacing: 6.6 x 6.6 ft. Run II-4.

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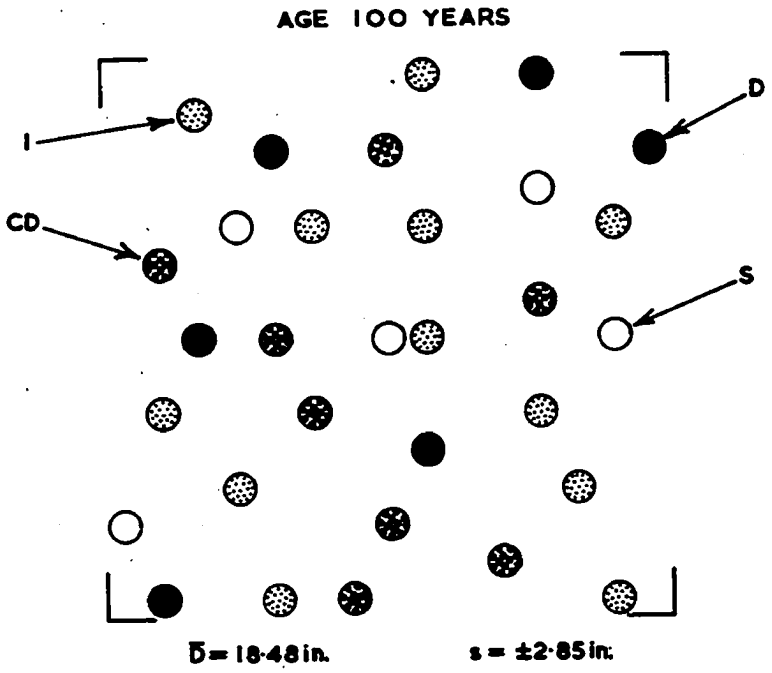

FIG. 49: Continued.

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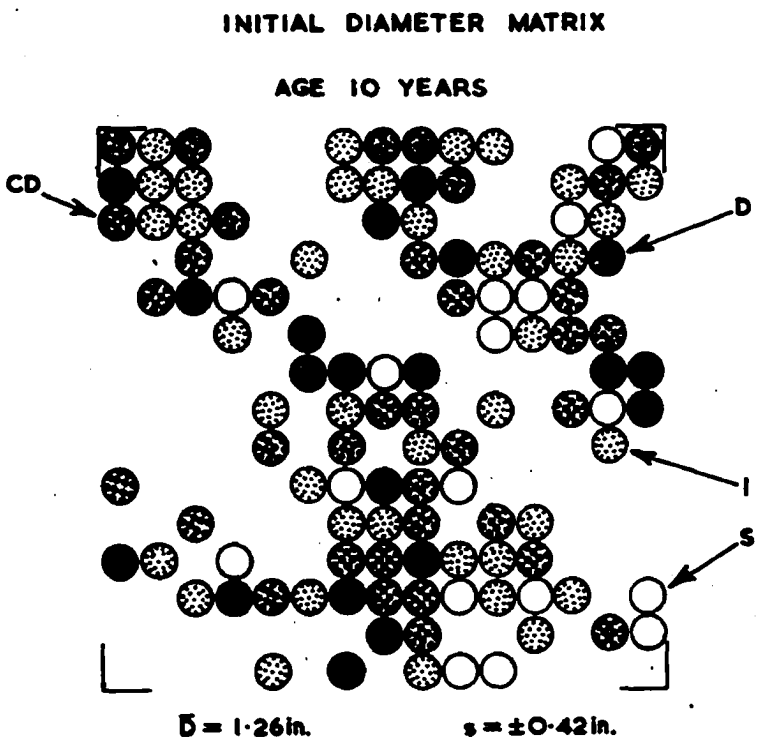

FIG. 50: Initial diameter matrix with 50 per cent uniform (rectangular) distribution of mortality following planting. Run II-5.

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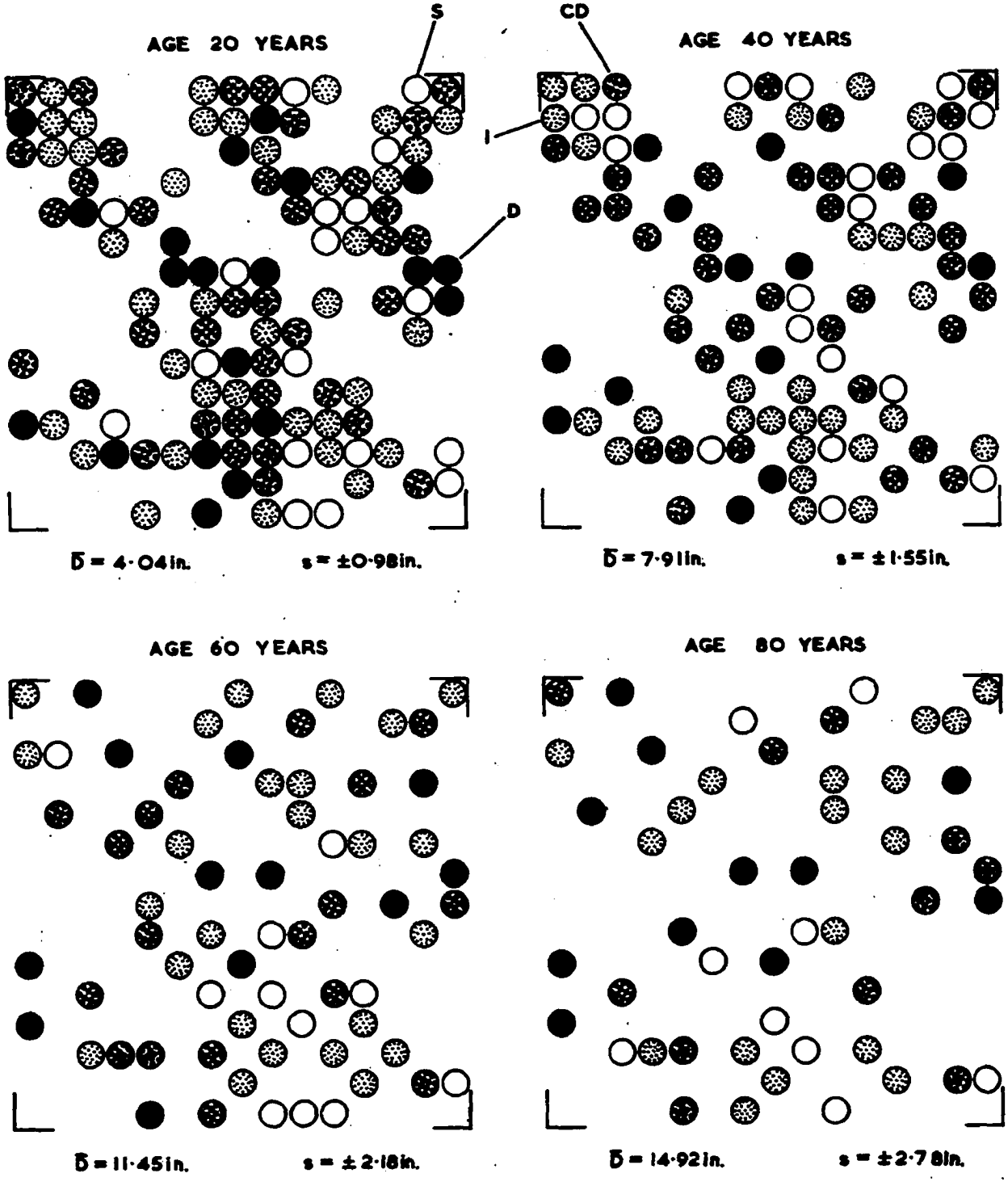

FIG. 51: The effect of amount and distribution of mortality following planting on stand structure. Uniform (rectangular) distribution (50 per cent mortality). Spacing: 6.6 x 6.6 ft. Run II-5.

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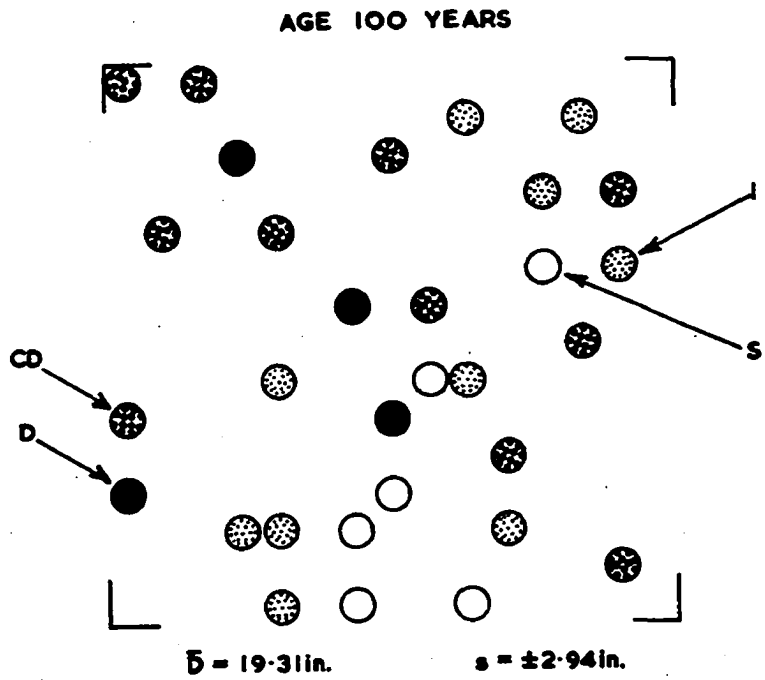

FIG. 51: Continued.

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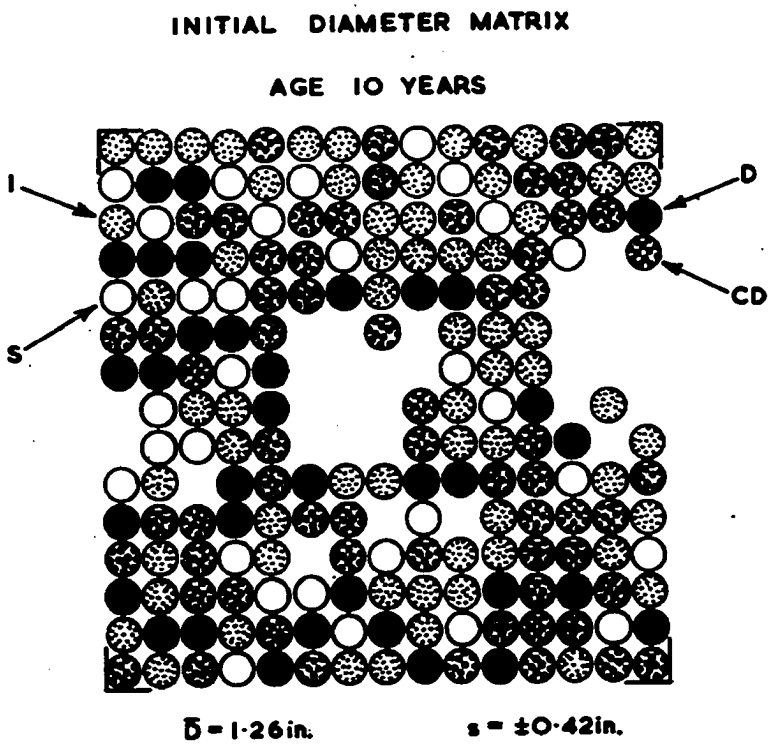

FIG. 52: Initial diameter matrix with two randomly located infection centres (14 per cent mortality). Run II-6.

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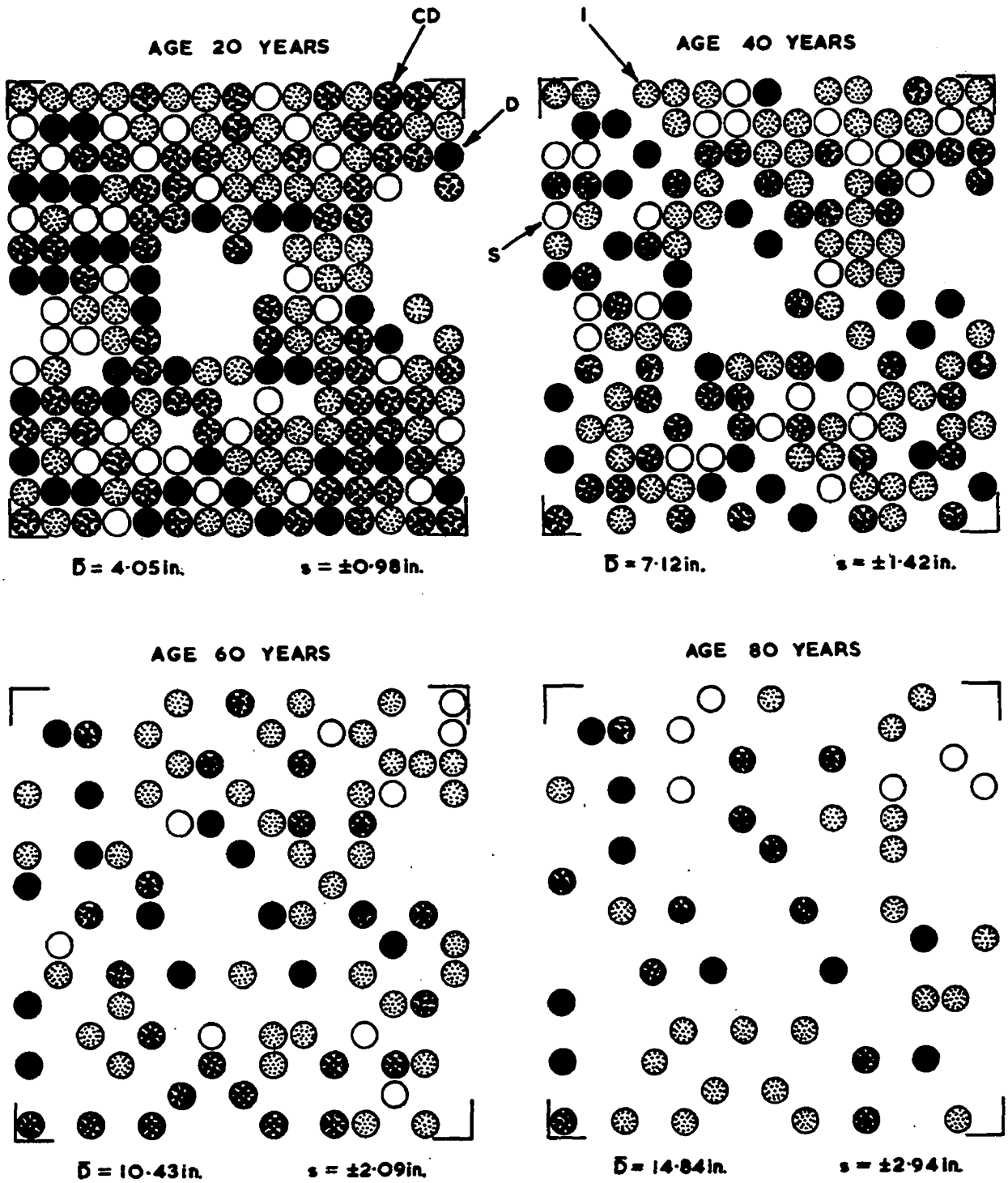

FIG. 53: The effect of amount and distribution of mortality following planting on stand structure. Two randomly located infection centres (14 per cent mortality). Spacing: 6.6 x 6.6 ft. Run. II-6.

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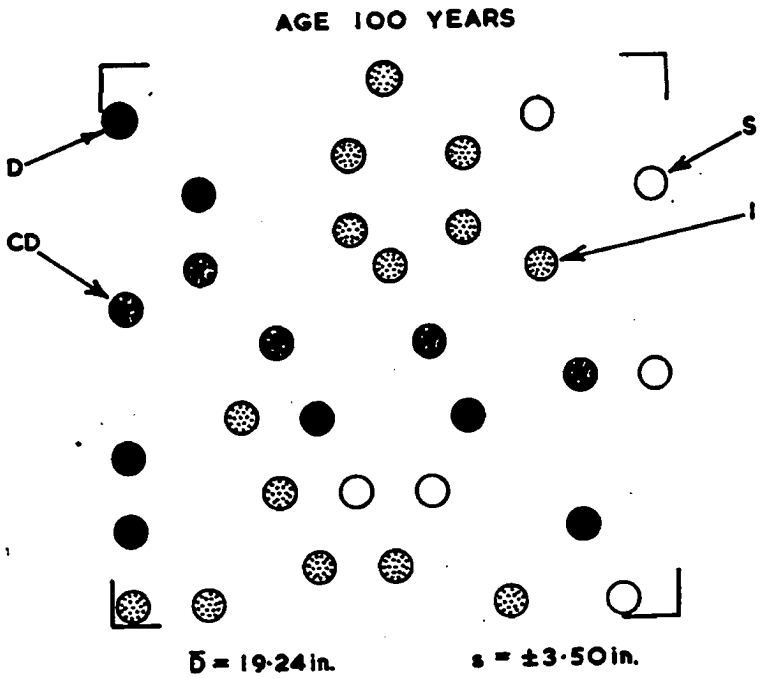

FIG. 53: Continued.

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spacing tested, 13.2 x 13.2 ft., only the 30 per cent binomial distribution of mortality and the two random infection centres are approaching full basal area stocking at age 100 years. Generally, it can be seen that the two clumped distributions take longer to recover than the random (binomial) distributions. There is also evidence to suggest that, at the two widest spacings, the maximum basal area for the two random infection centres will not be as great as that reached by the stands with binomial distributions of mortality. The reason for this is that each infection centre covers such a large area (36 and 64 milacres at 9.9 x 9.9 and 13.2 x 13.2 ft., respectively) that the surrounding stand is not capable of fully occupying the area before age 100 years.

The length of time before the differences in the number of trees per acre between the different types of mortality disappears, is longer than that for basal area (Fig. 29, 32, 35 and 38). This is because the surviving trees have less competition and are therefore able to accumulate rapid, individual, basal area growth. In the two clumped distributions tested, mortality is more rapid than with the random distributions in the first two or three periods after competition sets in. This is due to mortality occurring among the dense clumps of the surviving trees.

The diameter frequency distributions (Fig. 41-43) are not greatly affected by the distribution of the mortality and retain the characteristic sigmoid shape of normal distributions. A small secondary peak occurs towards the upper

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limit of the range after age 60 years in the stand with the two random infection centres (Fig. 43). This may be due to the "edge effect" as a result of which large trees are produced around each infection centre.

The development of the structure of the stands under the effects of the different types of planting mortality can be followed in Fig. 44-53. The development of the stands established at other than the 6.6 x 6.6 ft. spacing, which differs only in the age at which each stage is reached, is not reproduced here. If it is assumed that the four diameter groupings shown are equivalent to the dominant, codominant, intermediate and suppressed crown classes, it can be seen that there is a general movement down, although between two and five per cent of the trees move up. Mortality is usually confined to the "suppressed" and "intermediate" classes. Where the distribution of the trees is clumped, mortality occurs first within the clumps. These results do not differ from those that might occur in actual plantations (Guillebaud and Hummel, 1949; Warrack, 1952).

Site Quality

To test differences in site quality, each tree in the basic diameter matrix used to develop the model was multiplied by a constant to reduce the mean d. b. h. to 0.80 in. (approximately S. I. 120) or to 1.92 in. (approximately S. I. 160). The distribution of the trees in the matrix is therefore the same as that for the basic model.

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Results

Site indices 120 and 160 and site index 140 (the basic model), are compared in Fig. 54-58 for the 6.6 x 6.6 ft. spacing. Fig. 54 shows that the number of trees per acre is greatest for the poorest site and least for the best site as expected. The pattern of mean d. b. h. growth is similar for all three sites, the only difference being in the value of the mean (Fig. 55). The differences in basal area per acre (Fig. 56) are not as marked as those for d. b. h. but are in proportion to the yield table values (Barnes (U. B. C. Forest Club, 1959), McArdle et al., 1949). The cumulative diameter frequency distributions are shown in Fig. 57 and the stand structures at age 100 years in Fig. 58. The latter indicate that the stand development of the two sites has not been exactly parallel. Some of the locations occupied by trees in the site index 160 stand are not occupied in the site index 120 stand where the mortality was less.

Thinning

Thinning is the most important silvicultural operation that can improve the quality and condition of the stand once it has been established. It must not be confused with exploitation felling, which is carried out with the sole purpose of obtaining a monetary return, regardless of the condition of the residual stand. Thinning practice on the Continent of Europe has, until recently at least, been classical: "low" thinning where the dead, dying and

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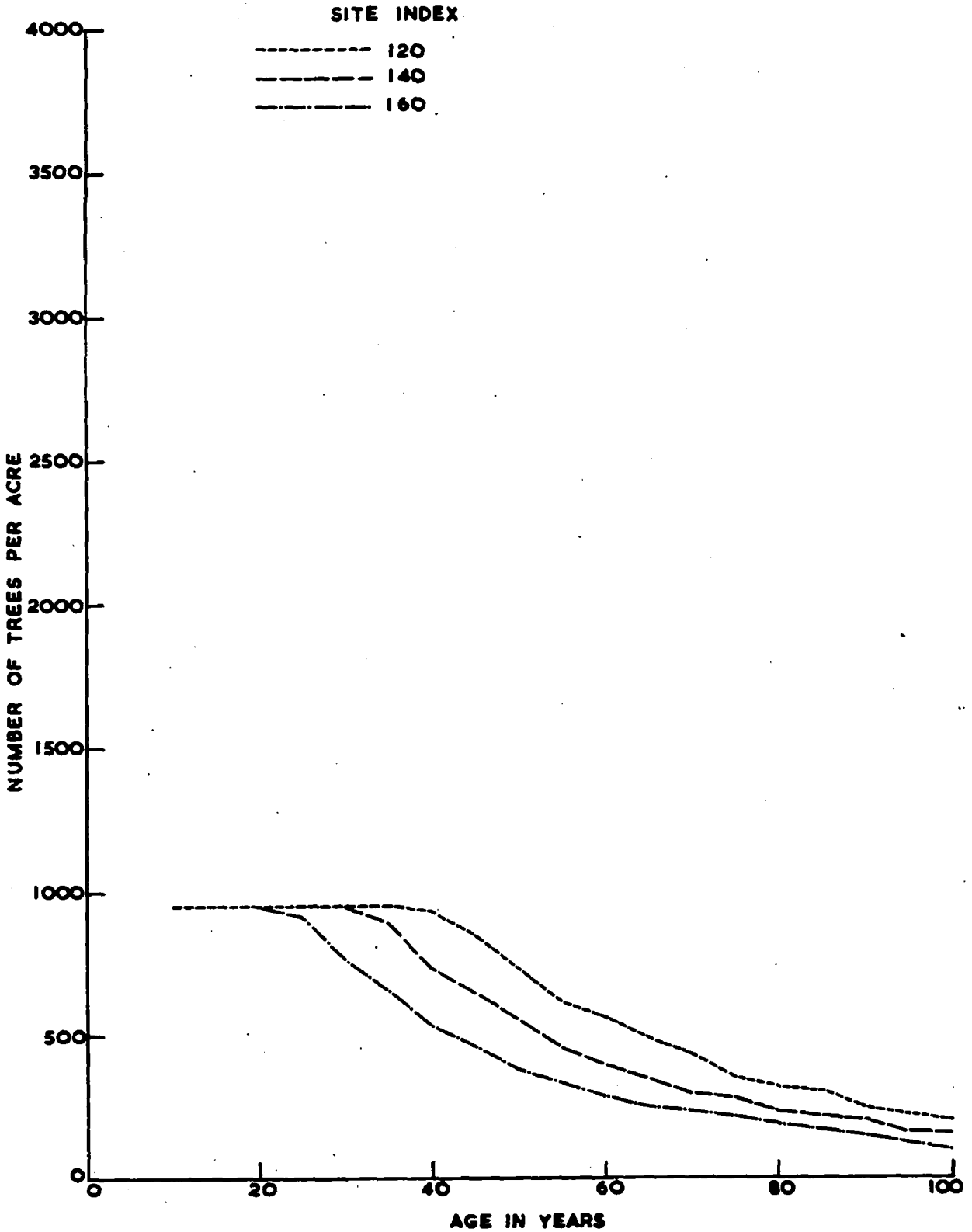

FIG. 54: The effect of site on number of trees per acre. Spacing: 6.6 x 6.6 ft. Runs II-2,7,8.

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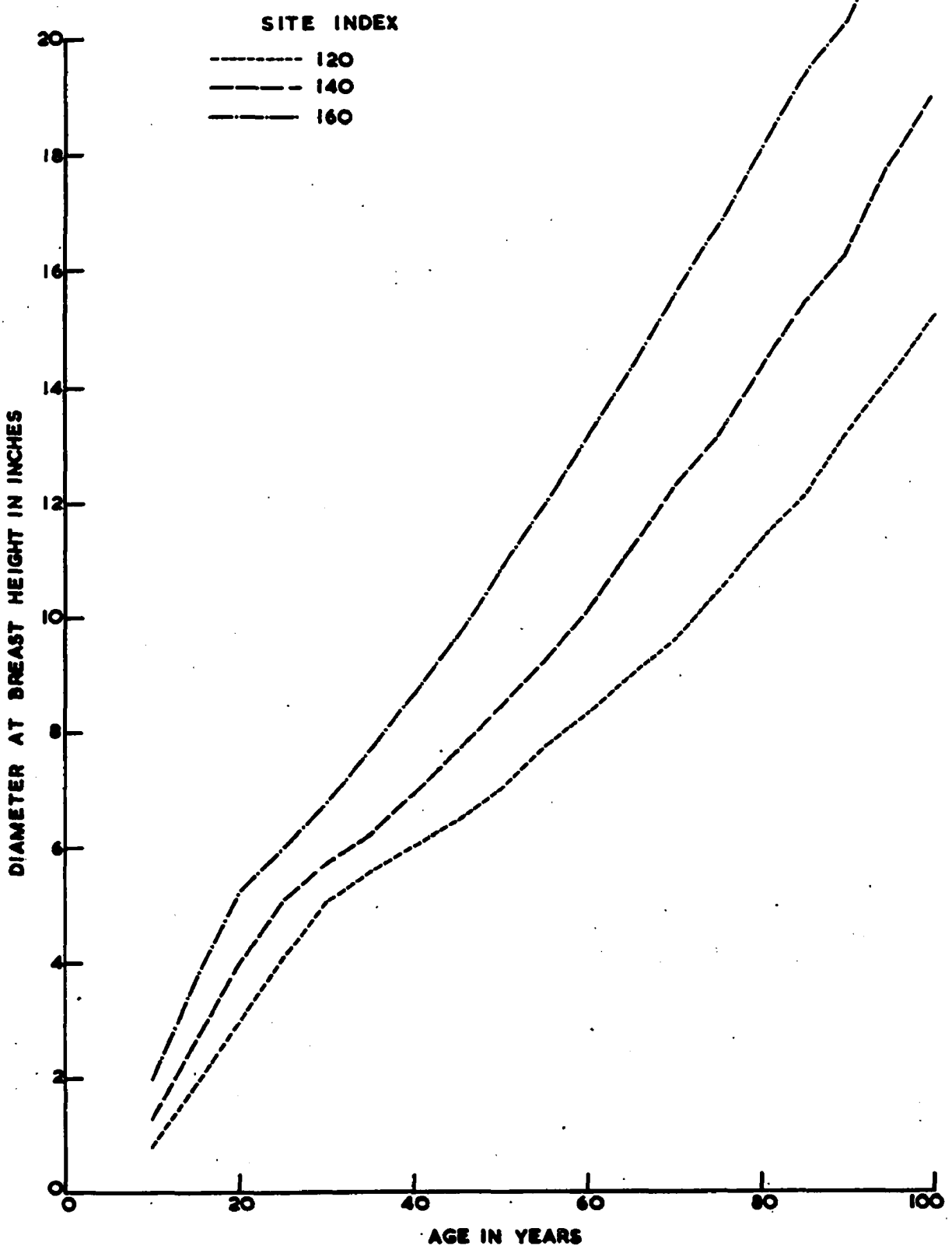

FIG. 55: The effect of site on mean d. b. h. o. b. Spacing: 6.6 x 6.6 ft. Runs II-2,7,8.

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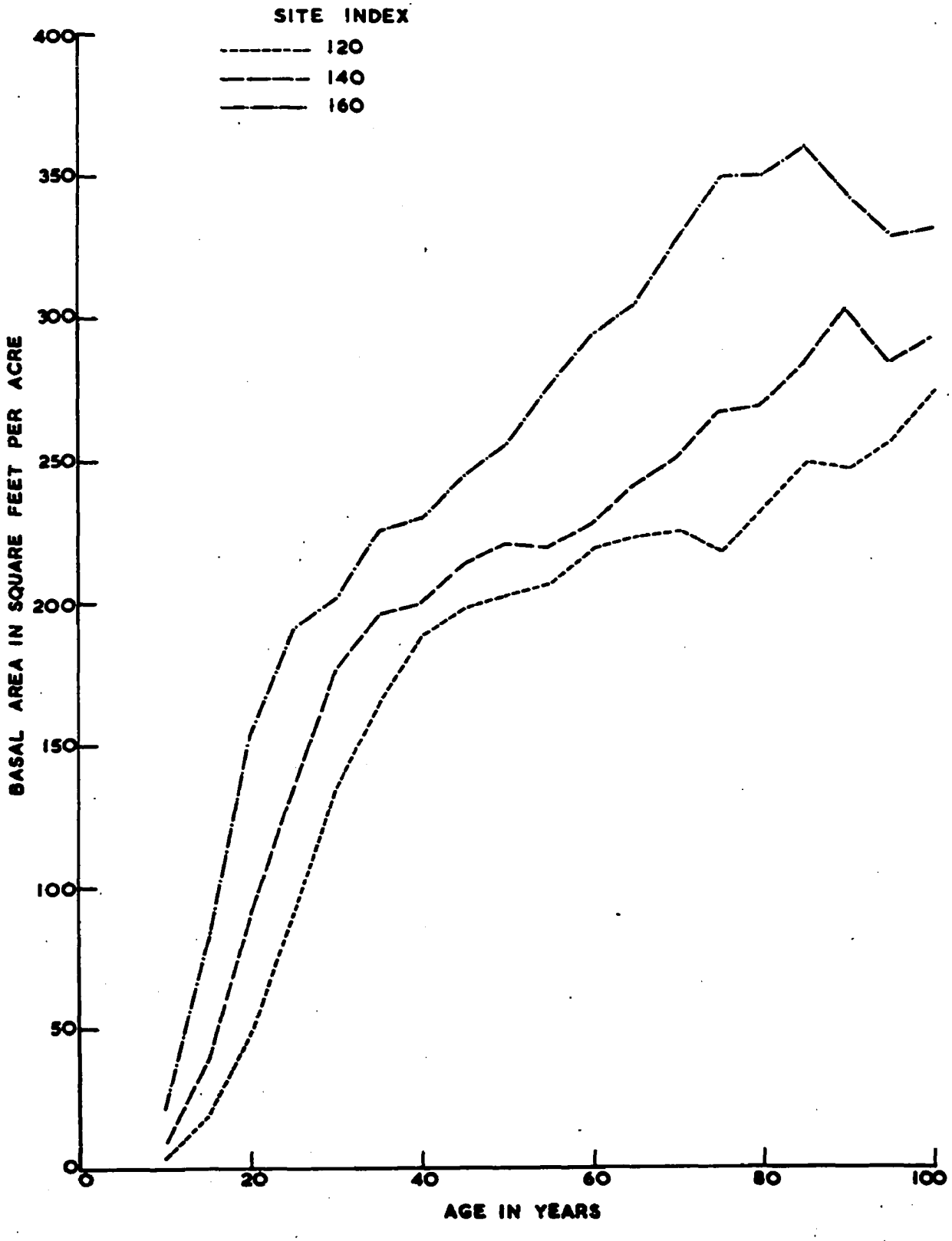

FIG. 56: The effect of site on basal area per acre. Spacing: 6.6 x 6.6 ft. Runs II-2,7,8.

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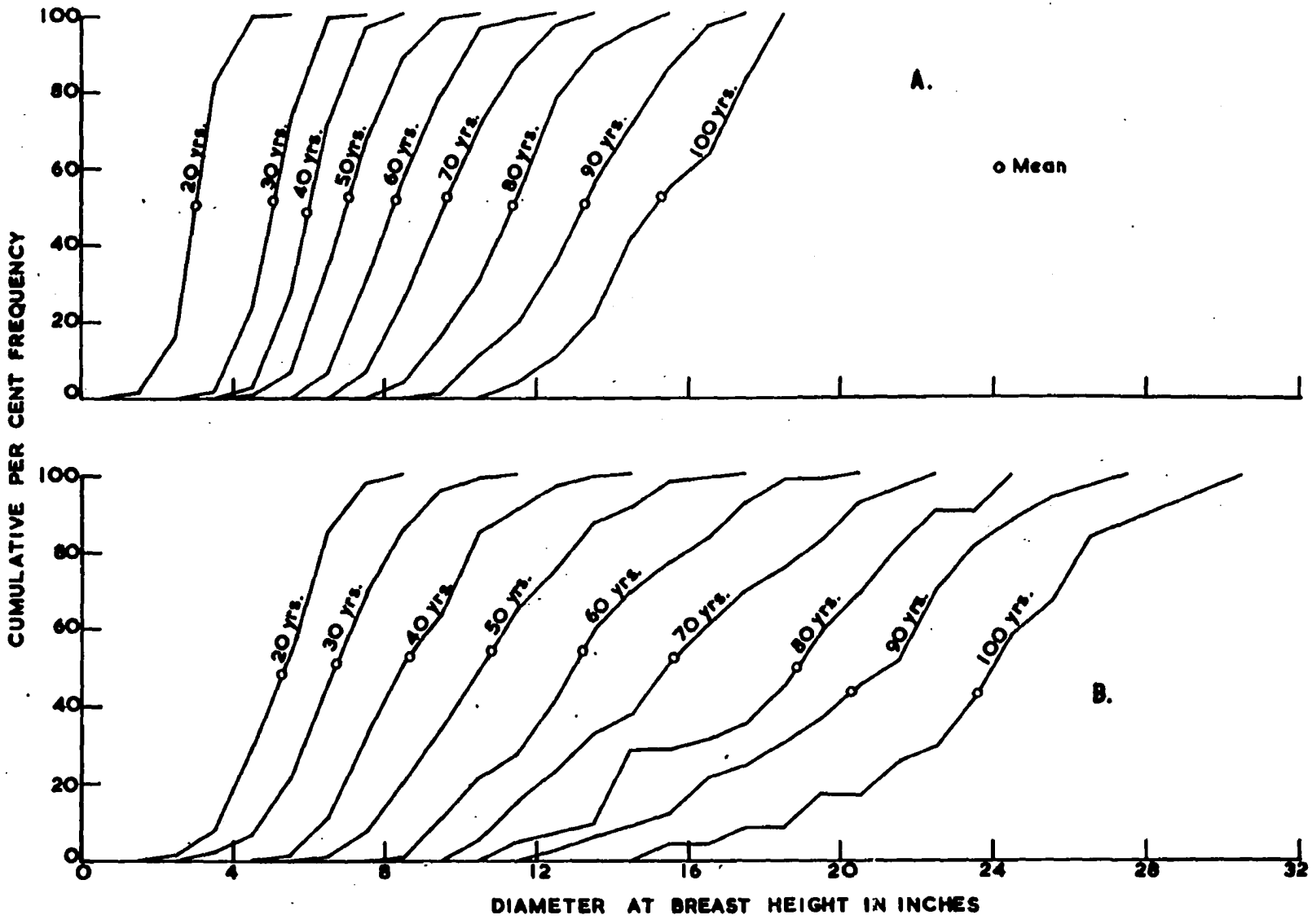

FIG. 57: Cumulative d. b. h. o. b. frequency distributions for (A) site index 120 and (B) site index 160. Spacing: 6.6 x 6.6 ft. Runs II-7,8. (See also Fig. 21b.)

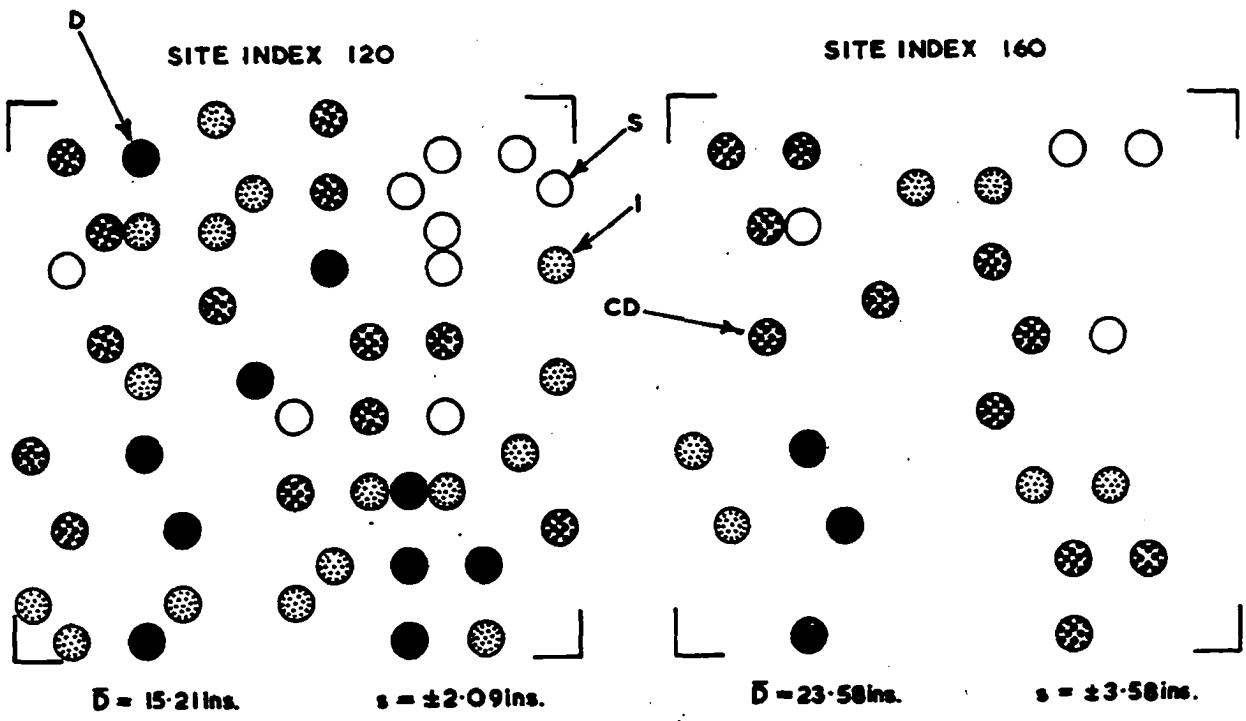

FIG. 58: The effect of site on stand structure at age 100 years. Spacing: 6.6 x 6.6 ft. Runs II-7,8. (See also Fig. 25.)

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diseased trees and the "whips" are the first to be removed, followed by the removal of suppressed and some intermediate trees to give more growing space for the dominant and co-dominant trees. In addition, "wolf" trees are often removed at the time of the first low thinning. Most of the trees removed in such thinnings, particularly those in the early life of the stand, were unusable and therefore there was little financial return. To make thinning more attractive economically, "crown" thinning was introduced. A crown thinning isolates a limited number of the best dominants in the stand by removing the larger competitors surrounding them. Most of the unusable suppressed and intermediate trees are left standing.

In both the low and crown thinnings described above the trees are subjectively chosen. The marking of trees for thinning is therefore a slow process, requiring considerable skill. Because of this, and the unsaleability of the smaller-sized trees, thinning has not gained the same prominence in the Pacific Northwest as it has in Europe.

To reduce the subjectivity in thinning, thinning prescriptions have been prescribed, describing the number of trees or the amount of basal area to be left after each thinning. The simplest of these prescriptions is row thinning in plantations (Spurr, 1948; Little and Mohr, 1963) where entire rows of trees are cut, regardless of the size or quality of the trees, or the size or quality of the trees in the residual stand. Such a thinning is mechanical,

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requiring little skill, and is usually practiced only in young plantations.

In South Africa, numerical thinnings have been carried out since about 1930 (Hiley, 1959). Before marking a plantation for thinning, the marker measures out a one-tenth acre plot and marks the trees on this plot in such a way that the prescribed number of trees are left. He continues to mark according to this density but with frequent checks to ensure he is maintaining his accuracy of marking. It has been found that this can be done quickly and accurately, by relatively unskilled labour. There is still a certain subjectiveness in selecting the trees to be removed. Although the numerical schedules are strictly adhered to, the thinning practices are usually silviculturally, as well as economically, acceptable.

It is therefore difficult to use the stand model that has been developed for this thesis to test the effect of various thinning regimes. However, there are two possible methods that can be used. First, knowing (from the matrix printed for each five-year period) the d. b. h. of each tree and its location in the stand at the end of any five-year period, it is possible to select those trees which, in the opinion of the reader, should be "thinned". The model is then run for a further five-or ten-year period and the process repeated. This is a subjective method. An objective method, and the one which is used here, is to prescribe the removal of all trees within certain diameter limits at the

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end of each thinning period of ten or twenty years. The limits used depend on the mean d. b. h. and its standard deviation. In the present instance, they have been selected to represent a moderate and a severe low thinning, and a crown thinning or selection felling. The growth of an unthinned (control) stand is summarized in Table 3.

Moderate Low Thinning

This removes all trees that are less than the mean d. b. h. minus one standard deviation ($\overline{D}$ - s) at the end of each thinning period (after the five-year mortality has been removed). Thinning commences at the end of the period in which competition mortality first occurs. According to the method which has been used in the model to allocate crown classes to each tree (see Fig. 23), this thinning removes all the trees in the suppressed crown class and no trees from any other crown class. It was thought that such a prescription would be equivalent to a light-to-moderate low thinning but, according to the d/D ratio (Warrack, 1959b), the thinning was more severe than expected.

The results are shown in Fig. 59 and 60 and in Table 4, for a ten-year thinning period. Although there is considerable reduction in the number of trees per acre compared with the unthinned stand (Fig. 59), the reduction in basal area is not so marked (Fig. 60). The increase in net basal area yield over the unthinned stand is at once apparent. Gross yield (stand + thinnings + mortality) was slightly

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TABLE 3: The growth of an unthinned stand. Site index: 140. Spacing: 6.6 x 6.6 ft. Run II-1.

Age (yr.)	Number of trees per acre		Mean d.b.h.o.b. (in.)	Basal area per acre (sq.ft.)		
	Stand	5-yr. mortality		Stand	5-yr. mortality	Gross cumulative
10	956		1.3	9		9
15	956		2.7	40		40
20	956		4.0	90		90
25	956		5.1	142		142
30	947	9	5.8	176	1	177
35	889	58	6.2	196	8	204
40	733	156	6.9	199	22	230
45	640	93	7.7	213	14	258
50	551	89	8.4	220	18	283
55	458	93	9.2	219	25	307
60	396	62	10.1	228	19	334
65	342	53	11.2	240	16	364
70	298	44	12.2	250	19	392
75	276	22	13.1	266	13	422
80	236	40	14.3	269	26	450
85	213	22	15.4	284	16	480
90	204	9	16.2	303	9	508
95	160	44	17.9	284	41	531
100	147	13	19.0	292	14	553

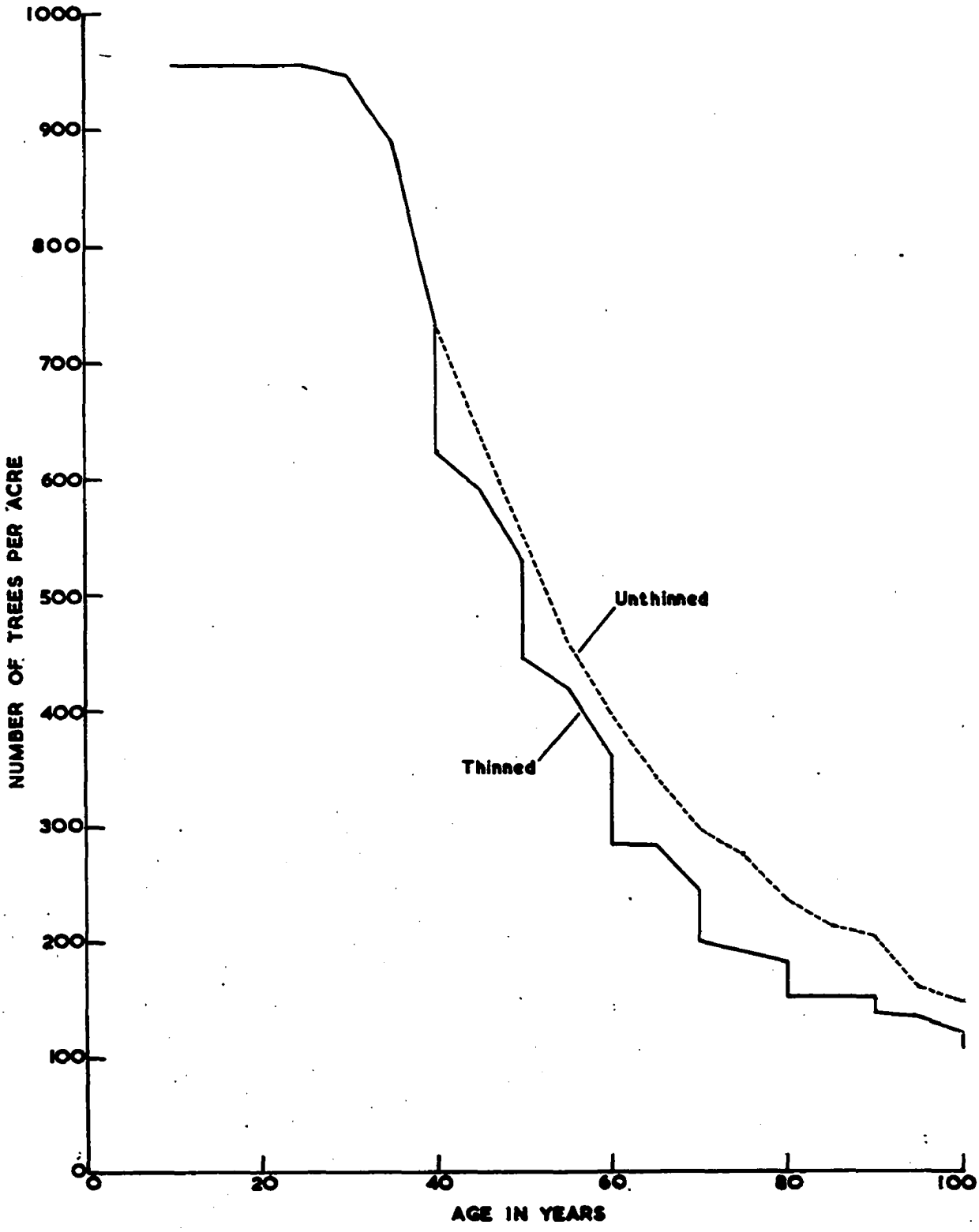

FIG. 59: The effect of thinning on number of trees per acre. All trees less than $(\bar{D} - s)$ removed at 10-year intervals. Spacing: 6.6 x 6.6 ft. Run II-9.

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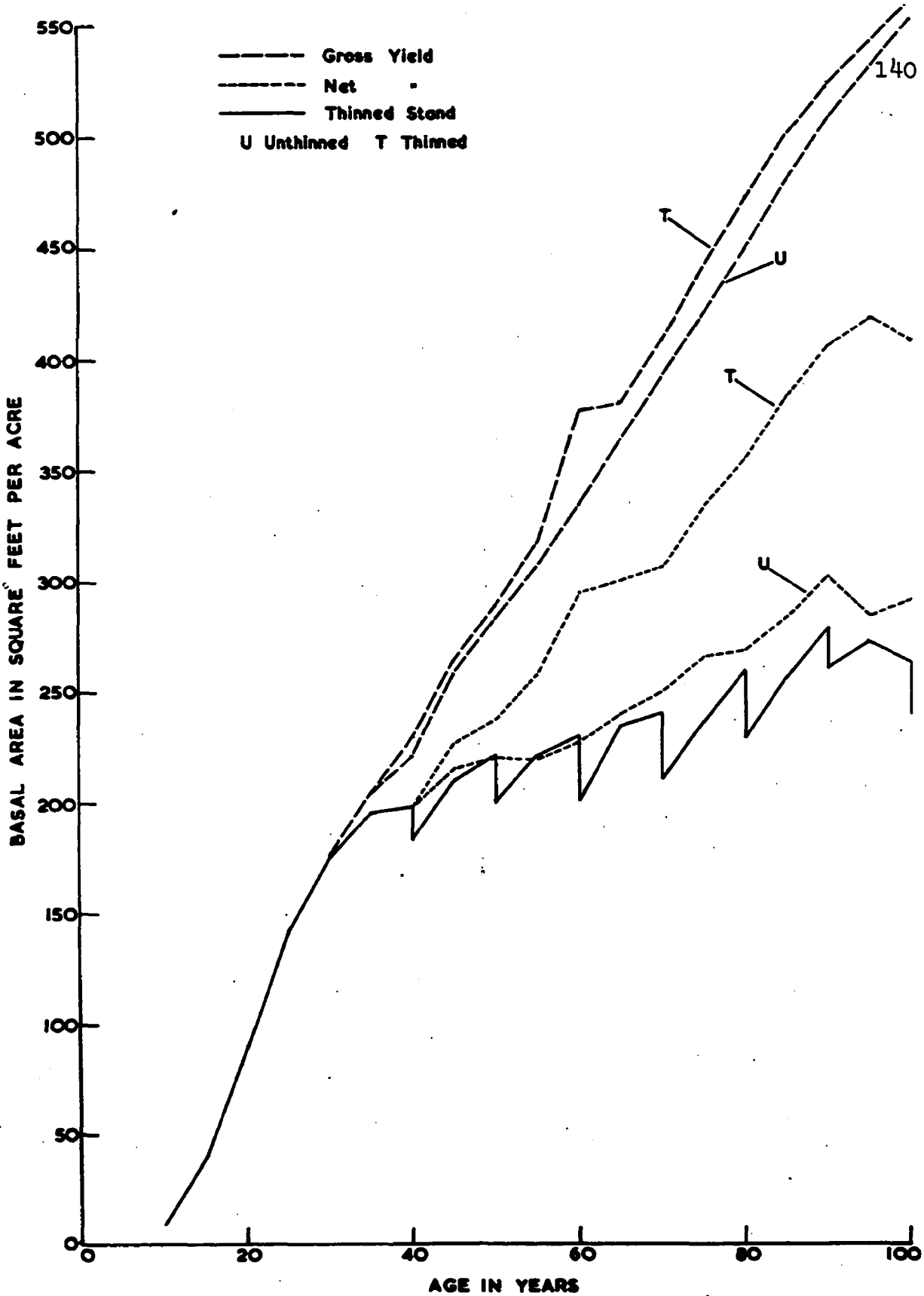

FIG. 60: The effect of thinning on gross, net (stand + thinnings) and stand basal area yield. All trees less than $(\overline{D} - s)$ removed at 10-year intervals. Spacing: 6.6 x 6.6 ft. Run II-9.

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TABLE 4: The effect of thinning on growth and yield. All trees less than ($\bar{D}$ - s) removed at intervals of 10 years. Spacing: 6.6 x 6.6 ft. Run II-9.

Age (yr.)	Number of trees per acre			Mean d.b.h.o.b.(in.)		d/D ratio	Basal area per acre (sq. ft.)				
	Stand (before thinning)	Removed as thinnings	5-yr. mortality	Stand (before thinning) D	Thinnings d		Stand (before thinning)	Removed as thinnings	5-yr. mortality	Net Yield (Stand + thinnings)	Cumulative gross yield (net yield + mortality)
10	956			1.3			9			9	9
15	956			2.7			40			40	40
20	956			4.0			90			90	90
25	956			5.1			142			142	142
30	947		9	5.8			176		1	176	177
35	889		58	6.2			196		8	196	204
40	733	111	156	6.9	5.0	0.72	199	15	22	199	230
45	591		31	8.0			211		7	226	264
50	529	84	62	8.6	6.7	0.78	222	21	14	237	289
55	418		27	9.7			222		8	258	318
60	360	76	58	10.7	8.4	0.79	230	30	20	296	376
65	284			12.2			235			301	381
70	244	44	40	13.4	11.2	0.84	241	30	22	307	409
75	191		9	15.0			236		8	332	442
80	182	31	9	16.1	13.5	0.84	260	31	8	356	473
85	151			17.6			257			384	501
90	151	13		18.4	16.1	0.88	280	19		406	524
95	133		4	19.3			273		7	418	543
100	120	13	13	20.0	17.7	0.88	263	23	26	408	559
Total		372	476					168	150		

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increased by thinning. Increasing the thinning period to twenty years (Fig. 61 and 62, Table 5) causes little reduction in the basal area of the stand but, compared with results from the ten-year thinning period, the net yield is reduced.

Severe Low Thinning

In this thinning none of the suppressed, codominant or dominant trees were removed and only those trees in the intermediate crown class between ($\overline{D}$ - s) and ($\overline{D}$ - 0.5s) were thinned. Such a thinning could be justified, silviculturally, on the grounds that the suppressed trees left are unlikely to seriously hamper the growth of the dominant trees expected to form the final crop and, economically, on the grounds that the intermediate trees removed are more valuable than the suppressed trees removed in the thinning previously described.

This thinning was tested with a ten-year thinning period (Fig. 63 and 64; Table 6) and a twenty-year period (Fig. 65 and 66, Table 7). On a ten-year cycle, net and gross basal area yields were the highest of the thinnings tested.

Crown Thinning or Selection Felling

This thinning removed a small number of codominant trees between ($\overline{D}$ + 0.75s) and ($\overline{D}$ + s) using a ten-year thinning period. Although the number of trees removed (Fig. 67, Table 8) was the least of all the thinnings tested (at ages 60 and 90 years there were no trees within the prescribed limits), the reduction in the basal area of the stand was

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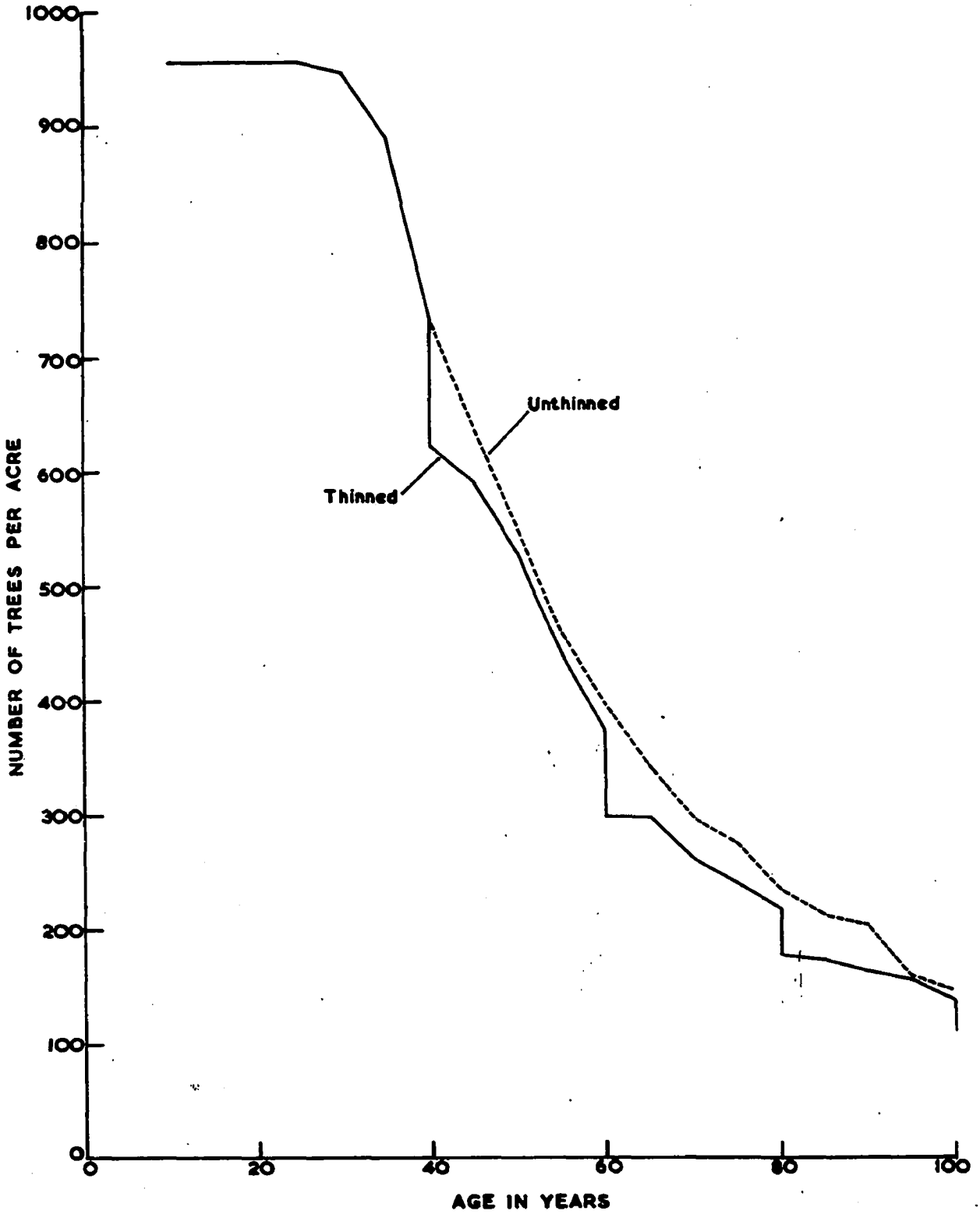

FIG. 61: The effect of thinning on number of trees per acre. All trees less than ($\bar{D}$ - s) removed at 20-year intervals. Spacing: 6.6 x 6.6 ft. Run II-10.

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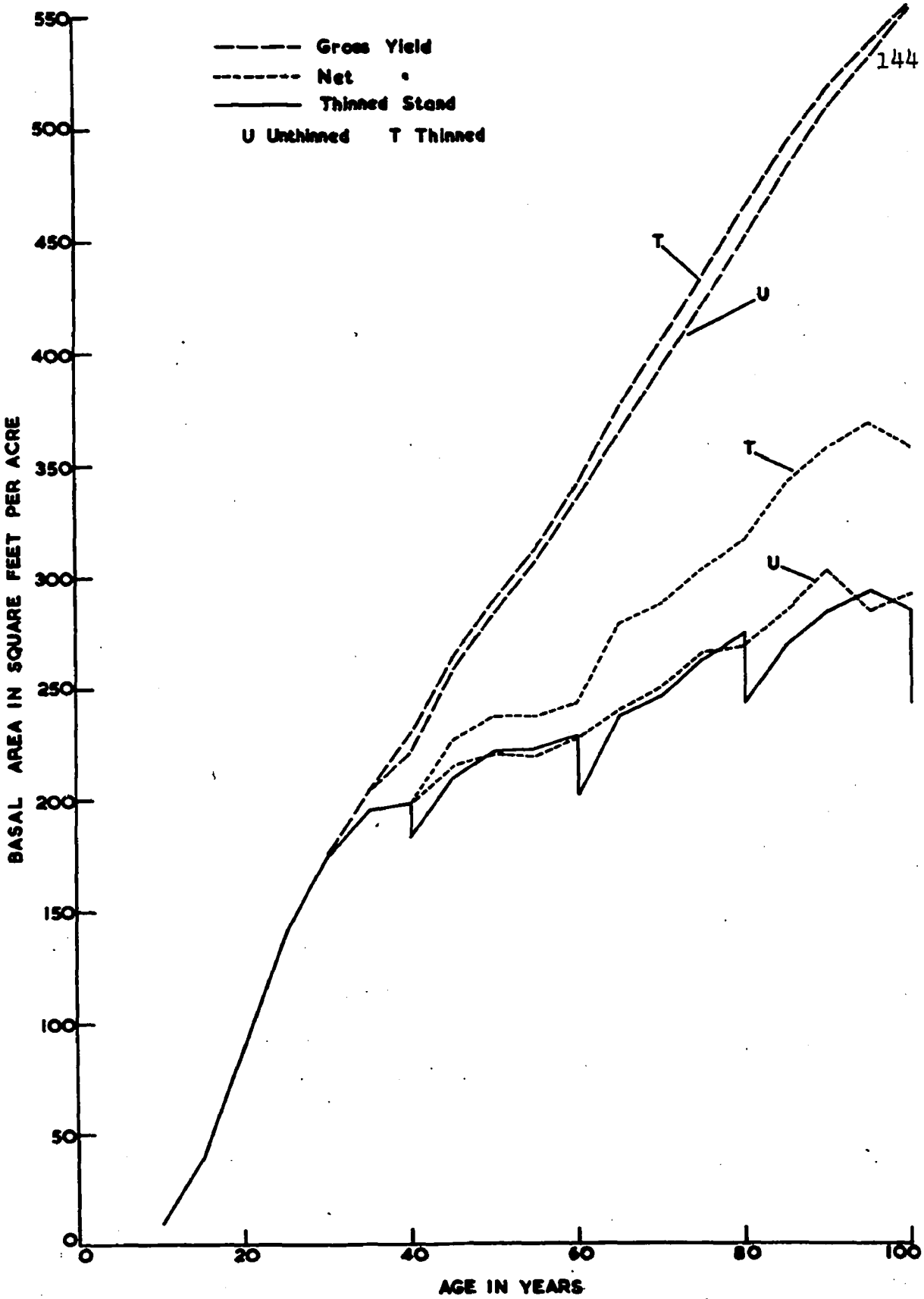

FIG. 62: The effect of thinning on gross, net (stand + thinnings) and stand basal area yield. All trees less than $(\bar{D} - s)$ removed at 20-year intervals. Spacing: 6.6 x 6.6 ft. Run II-10.

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TABLE 5: The effect of thinning on growth and yield. All trees less than ($\bar{D}$ - s) removed at intervals of 20 years. Spacing: 6.6 x 6.6 ft. Run II-10.

Age (yr.)	Number of trees per acre			Mean d.b.h.o.b.(in.)		d/D ratio	Basal area per acre (sq. ft.)				
	Stand (before thinning)	Removed as thinnings	5-yr. mortality	Stand (before thinning) D	Thinnings d		Stand (before thinning)	Removed as thinnings	5-yr. mortality	Net Yield (Stand + thinnings)	Cumulative gross yield (net yield + mortality)
10	956			1.3			9			9	9
15	956			2.7			40			40	40
20	956			4.0			90			90	90
25	956			5.1			142			142	142
30	947		9	5.8			176		1	176	177
35	889		58	6.2			196		8	196	204
40	733	111	156	6.9	5.0	0.72	199	15	22	199	230
45	591		31	8.0			211		7	226	264
50	529		62	8.6			222		14	237	289
55	440		89	9.5			222		24	237	313
60	374	76	67	10.4	7.9	0.76	228	26	22	243	341
65	298			12.0			237			278	376
70	262		36	13.0			247		18	288	404
75	240		22	14.0			262		13	304	433
80	218	40	22	15.1	12.0	0.80	275	32	17	316	463
85	173		4	16.8			269		4	341	492
90	164		9	17.7			284		10	357	517
95	156		9	18.5			293		11	366	538
100	138	27	18	19.4	17.0	0.88	285	42	26	357	555
Total		254	592					114	198		

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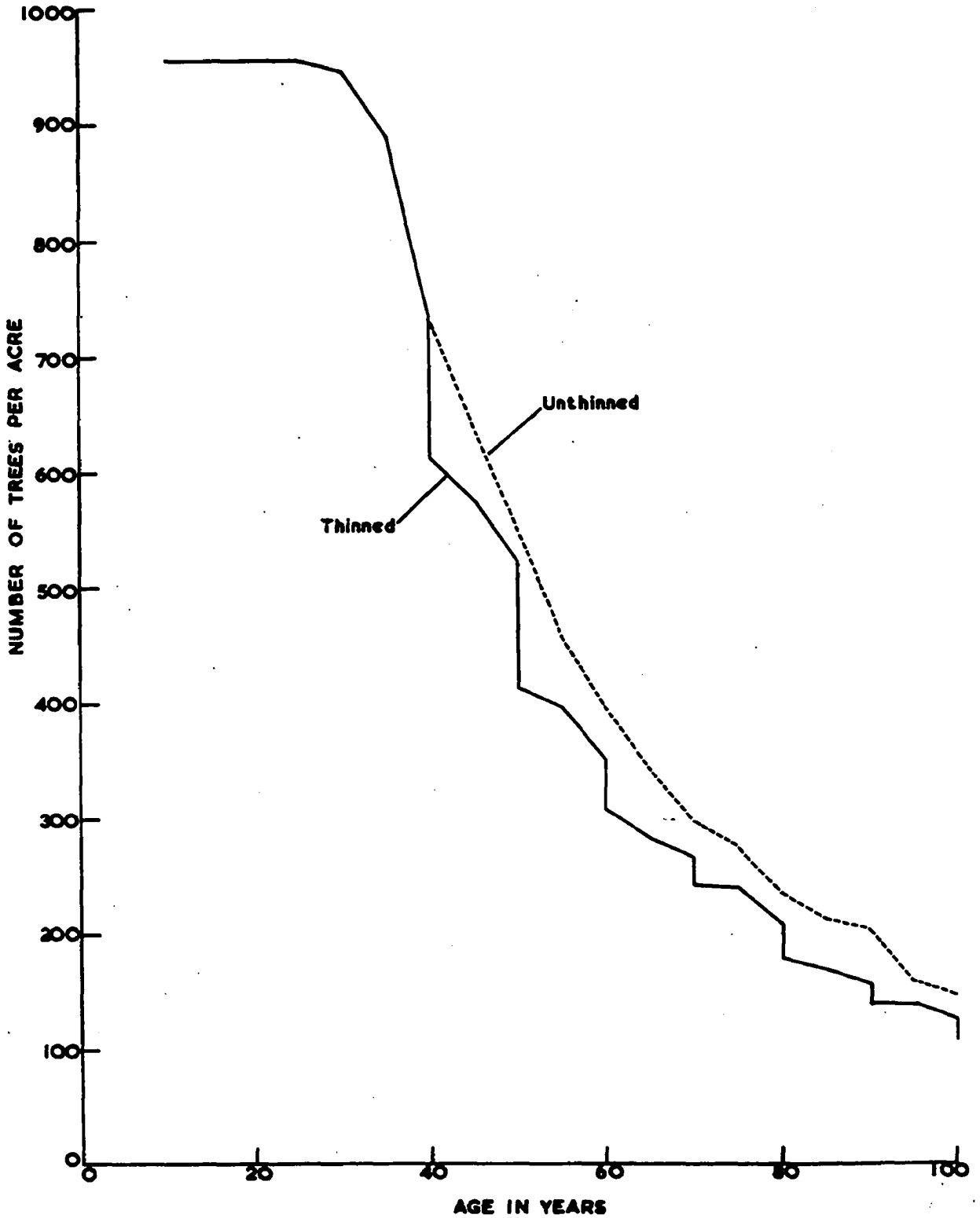

FIG. 63: The effect of thinning on number of trees per acre. All trees between $(\overline{D} - s)$ and $(\overline{D} - 0.5s)$ removed at 10-year intervals. Spacing: 6.6 x 6.6 ft. Run II-11.

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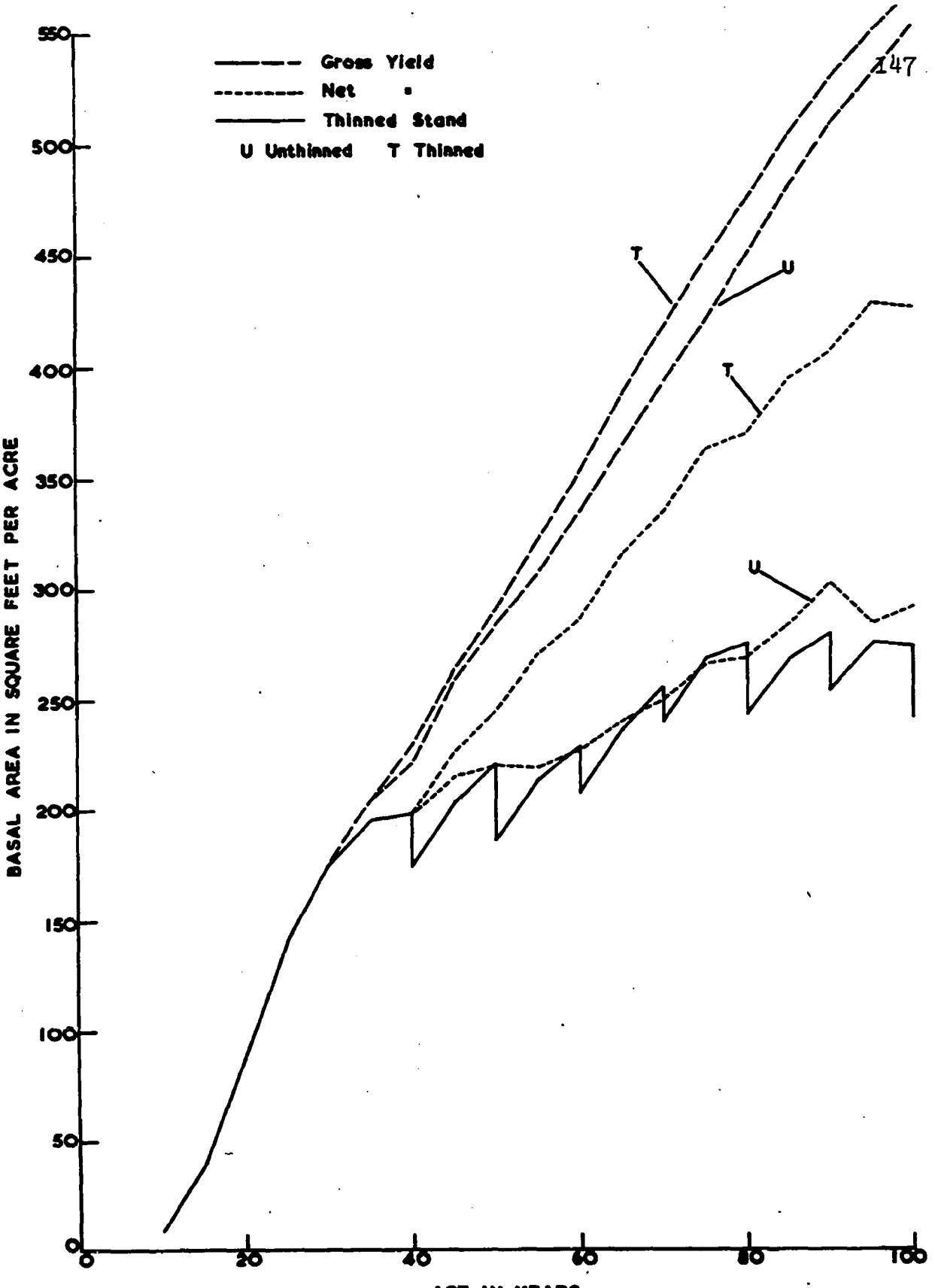

FIG. 64: The effect of thinning on gross, net (stand + thinnings) and stand basal area yield. All trees between $(\bar{D} - s)$ and $(\bar{D} - 0.5s)$ removed at 10-year intervals. Spacing: 6.6 x 6.6 ft. Run II-11.

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TABLE 6: The effect of thinning on growth and yield. All trees between ($\overline{D}$ - s) and ($\overline{D}$ - 0.5s) removed at intervals of 10 years. Spacing: 6.6 x 6.6 ft. Run II-11.

Age (yr.)	Number of trees per acre			Mean d.b.h.o.b. (in.)		d/D ratio	Basal area per acre (sq. ft.)				
	Stand (before thinning)	Removed as thinnings	5-yr. mortality	Stand (before thinning) D	Thinnings d		Stand (before thinning)	Removed as thinnings	5-yr. mortality	Net Yield (Stand + thinnings)	Cumulative gross yield (net yield + mortality)
10	956			1.3			9			9	9
15	956			2.7			40			40	40
20	956			4.0			90			90	90
25	956			5.1			142			142	142
30	947		9	5.8			176		1	176	177
35	889		58	6.2			196		8	196	204
40	733	120	156	6.9	6.0	0.87	199	24	22	199	230
45	573		40	7.9			203		6	227	263
50	524	111	49	8.6	7.5	0.86	221	34	9	245	290
55	396		18	9.8			213		6	271	322
60	351	44	44	10.8	9.4	0.88	229	22	14	287	352
65	284		22	12.2			236		7	315	387
70	266	22	18	13.1	11.4	0.87	255	16	10	335	417
75	240		4	14.1			268		3	363	448
80	209	31	31	15.4	13.6	0.89	274	32	21	369	475
85	169		9	16.9			267		4	394	504
90	156	18	13	18.0	16.4	0.91	280	26	12	406	528
95	138			19.0			275			428	550
100	125	18	13	20.0	18.5	0.92	274	33	19	427	568
Total		364	484					186	141		

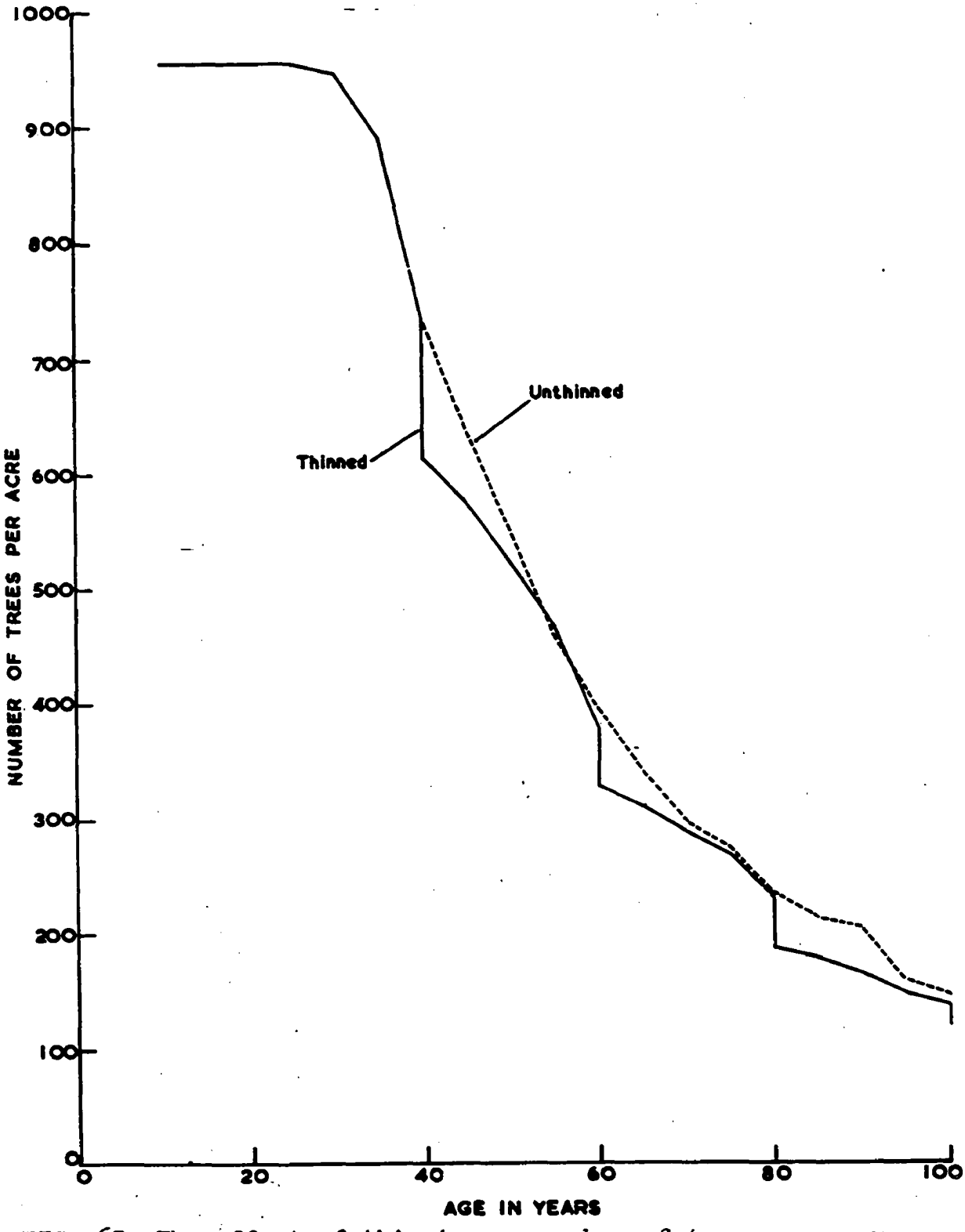

FIG. 65: The effect of thinning on number of trees per acre. All trees between ($\overline{D}$ - s) and ($\overline{D}$ - 0.5s) removed at 20-year intervals. Spacing: 6.6 x 6.6 ft. Run II-12.

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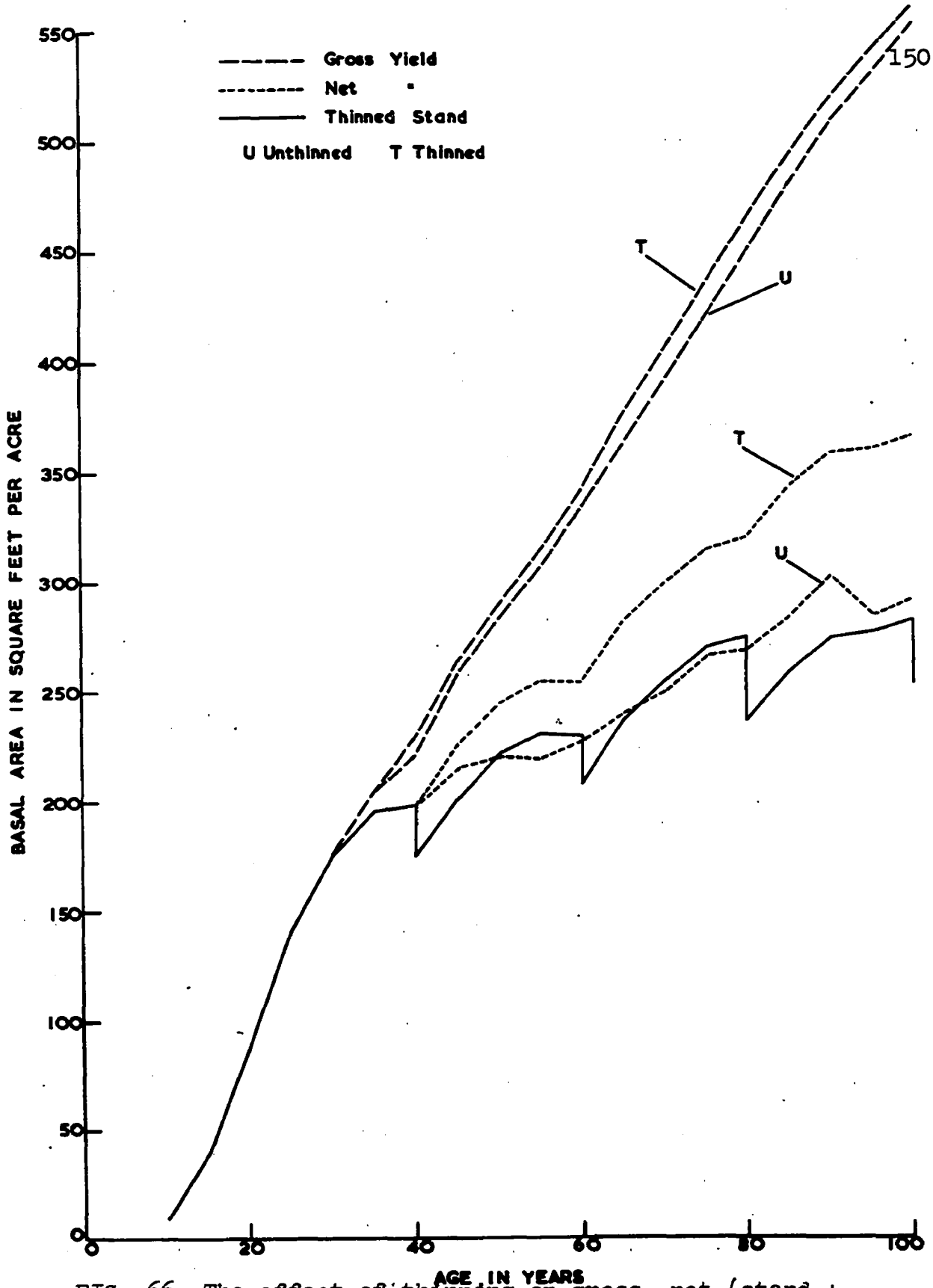

FIG. 66: The effect of thinning on gross, net (stand + thinnings) and stand basal area yield. All trees between $(\overline{D} - s)$ and $(\overline{D} - 0.5s)$ removed at 20-year intervals. Spacing: 6.6 x 6.6 ft. Run II-12.

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TABLE 7: The effect of thinning on growth and yield. All trees between ($\bar{D}$ - s) and ($\bar{D}$ - 0.5s) removed at intervals of 20 years. Spacing: 6.6 x 6.6 ft. Run II-12.

Age (yr.)	Number of trees per acre			Mean d.b.h.o.b. (in.)		d/D ratio	Basal area per acre (sq. ft.)				
	Stand (before thinning)	Removed as thinnings	5-yr. mortality	Stand (before thinning) D	Thinnings d		Stand (before thinning)	Removed as thinnings	5-yr. mortality	Net Yield (Stand + thinnings)	Cumulative gross yield (net yield + mortality)
10	956			1.3			9			9	9
15	956			2.7			40			40	40
20	956			4.0			90			90	90
25	956			5.1			142			142	142
30	947		9	5.8			176		1	176	177
35	889		58	6.2			196		8	196	204
40	733	120	156	6.9	6.0	0.87	199	24	22	199	230
45	573		40	7.9			203		6	227	263
50	524		49	8.6			221		9	245	290
55	467		58	9.4			231		15	255	315
60	378	49	89	10.4	8.9	0.86	230	21	27	253	341
65	311		18	11.7			237		6	282	376
70	289		22	12.5			255		13	300	406
75	267		22	13.4			270		15	315	436
80	231	44	36	14.6	12.7	0.87	275	39	23	320	464
85	178		9	16.1			259		6	343	493
90	164		13	17.3			274		10	358	519
95	147		18	18.5			276		20	360	540
100	138	18	9	19.2	17.4	0.90	283	29	14	367	560
Total		231	606					113	194		

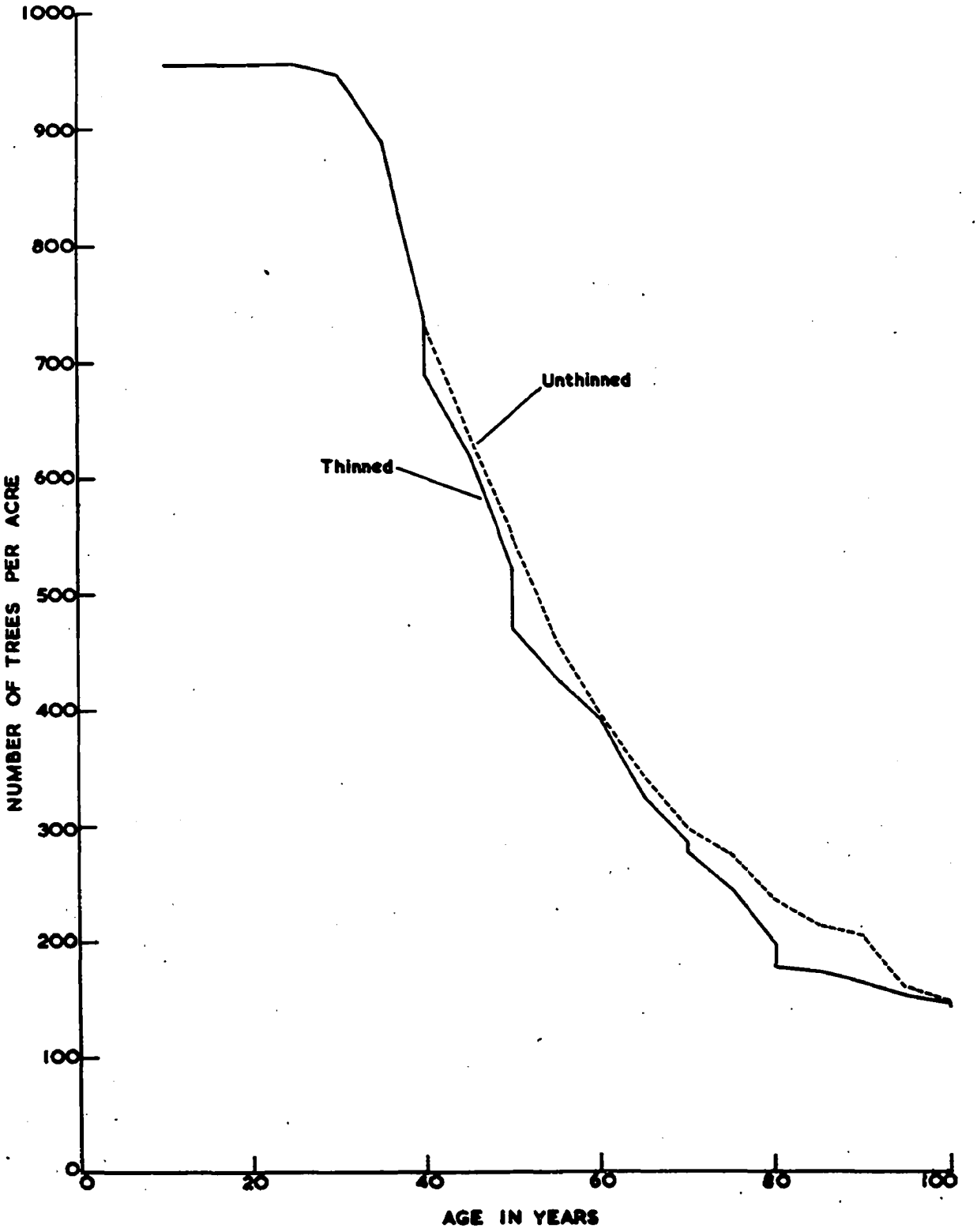

FIG. 67: The effect of thinning on number of trees per acre. All trees between $(\overline{D} + 0.75s)$ and $(\overline{D} + s)$ removed at 10-year intervals. Spacing: 6.6 x 6.6 ft. Run II-13.

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TABLE 8: The effect of thinning on growth and yield. All trees between ($\overline{D}$ + 0.75s) and ($\overline{D}$ + s) removed at intervals of 10 years. Spacing: 6.6 x 6.6 ft. Run II-13.

Age (yr.)	Number of trees per acre			Mean d.b.h.o.b.(in.)		d/D ratio	Basal area per acre (sq. ft.)				
	Stand (before thinning)	Removed as thinnings	5-yr. mortality	Stand (before thinning) D	Thinnings d		Stand (before thinning)	Removed as thinnings	5-yr. mortality	Net Yield (Stand + thinnings)	Cumulative gross yield (net yield + mortality)
10	956			1.3			9			9	9
15	956			2.7			40			40	40
20	956			4.0			90			90	90
25	956			5.1			142			142	142
30	947		9	5.8			176		1	176	177
35	889		58	6.2			196		8	196	204
40	733	44	156	6.9	8.1	1.17	199	16	22	199	230
45	618		71	7.6			202		11	218	260
50	524	53	93	8.4	9.9	1.18	209	28	20	225	285
55	427		44	9.0			198		11	243	314
60	391		36	9.8			215		10	260	341
65	324		67	10.8			219		21	264	367
70	285	9	40	11.9	14.5	1.22	231	10	16	275	395
75	244		31	13.0			236		12	291	423
80	196	18	49	14.3	17.4	1.22	230	30	32	284	448
85	173		4	15.0			225		3	309	476
90	164		9	16.0			242		9	326	502
95	151		13	17.3			258		9	342	527
100	146	4	4	18.2	21.8	1.20	277	12	3	361	549
Total		128	684					96	188		

the most marked (Fig. 68) due to the high mortality among the smaller trees in the residual stand, as well as the loss due to thinning. Net yield was also reduced and, above age 75 years, the gross yield of the thinned stand was a little less than that of the unthinned stand.

The results of these tests, although only covering a small proportion of many different types of thinning, indicate that the stand model can be used for testing the effects of thinning on the growth and yield of the stand. Any objective method of thinning, such as those used above, can be tested, provided the FORTRAN program for the model is modified to suit the individual's requirements.

Height Growth

In all the tests so far carried out using the model, no attempt has been made to predict height growth. The reasons for this omission have been stated earlier in this thesis. It is generally accepted that, within a stand, tree height is closely correlated with d. b. h., the relationship usually being curvilinear. The relationship will vary among stands due to differences in stocking, density of trees, age and site. To study this relationship, data for 869 Douglas fir sample trees were obtained from permanent sample plots located on the University Research Forest, Haney, the University Campus Forest, Vancouver, and from the experimental plots of the Research Division of the B. C. Forest Service at Cowichan Lake, Vancouver Island. The latter series of plots was located in plantations of Douglas fir which had

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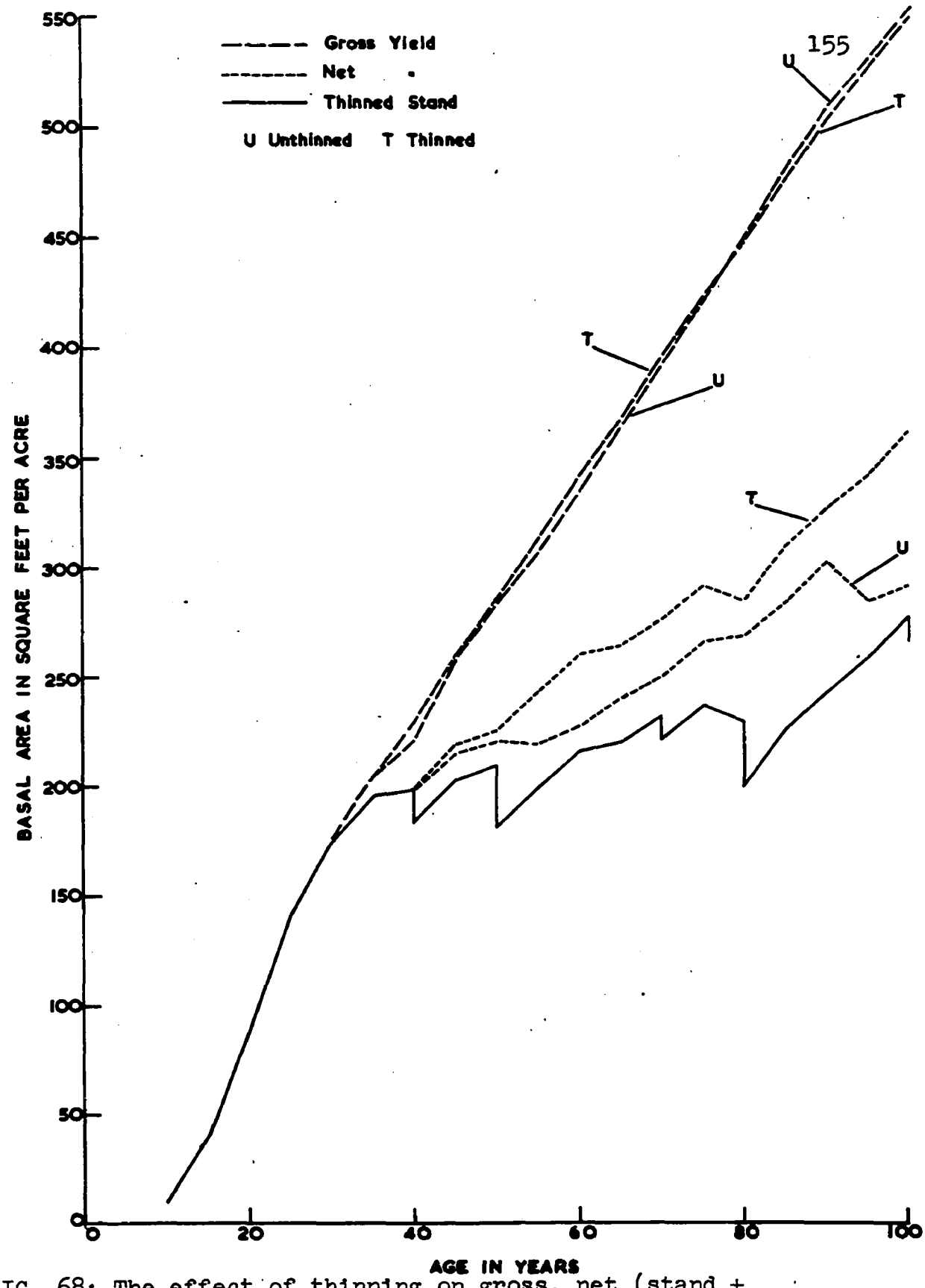

FIG. 68: The effect of thinning on gross, net (stand + thinnings) and stand basal area yield. All trees between $(\bar{D} + 0.75s)$ and $(\bar{D} + s)$ removed at 10-year intervals. Spacing: 6.6 x 6.6 ft. Run II-13.

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been thinned, leaving the residual stands at densities which varied between "normal" and "open-grown".

Using these data, a regression calculation was carried out of tree height on d. b. h., number of trees per acre, basal area per acre, site index and age. The results of this regression calculation are summarised in Table 9. A series of regression equations is given, each equation having one less independent variable than the previous equation. The variable eliminated each time was the one making the least absolute contribution to R^2. It appears that the regression equation $H = -11.083 + 8.27095D + 0.160482B - 0.154019D^2$, where H is the total height of the tree in feet, D is the d. b. h. o. b. in inches and B is the basal area of the stand in square feet per acre, is the most suitable for practical purposes. Inclusion of further variables does not greatly decrease the residual variance, s^2, whereas eliminating D^2 almost doubles the residual variance. Examples of height/d. b. h. curves, calculated from the above equation, are shown in Fig. 69 for stand basal areas between 60 and 240 sq. ft. per acre.

Conclusions

The model has proved to be satisfactory for the various tests performed in this part of the thesis. It is not possible at present, due to the unavailability of field data, to confirm that the absolute values for number of trees, mean d. b. h. and basal area are accurate. However, there are no significant departures, except when carrying out

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TABLE 9: Regression of tree height on d. b. h., number of trees per acre, basal area per acre, site index and age. The variable making the least absolute contribution to R^2 is eliminated each time. Number of trees = 869. Source of data: B. C. Forest Service, University Research Forest and Campus Forest. Mean height = 53.7 ft. Standard deviation = ± 34.74 ft.

Constant	Regression of height on:								R^2	s^2
	D.b.h. (in.)	No. of trees/ac.	B.A./ac. (sq.ft.)	Site Index	Age (yr.)	$(D.b.h.)^2$	$(D.b.h.)^3$	Log_e (d.b.h.)		
				Regression coefficients						
-15.415	8.3478	-0.005859	0.141596	0.060795	0.210877	-0.195037	0.001067	-2.1966	0.946	65.88
- 4.633	8.5384	-0.007905	0.179185	-	0.055343	-0.203969	0.001293	-2.6728	0.945	66.85
- 3.073	8.3799	-0.008939	0.189941	-	-	-0.191386	0.001041	-2.5069	0.945	66.92
-11.025	10.9741	-	0.157393	-	-	-0.273890	0.001818	-6.8723	0.939	74.26
-10.132	9.7477	-	0.158176	-	-	-0.192917	-	-4.8099	0.939	74.27
-11.084	8.2710	-	0.160482	-	-	-0.154019	-	-	0.938	74.94
- 6.099	11.0728	-	-	-	-	-0.222466	-	-	0.890	133.00
9.005	6.5691	-	-	-	-	-	-	-	0.844	187.48
Mean	6.81	727	120	139	35	69.96	954.0	1.6651		
S.D.	±4.86	±396	±65	±26	17	±103.84	±2240.6	±0.7475		
r for Ht.	0.919	-0.407	0.806	0.107	0.608	0.803	0.661	0.893		

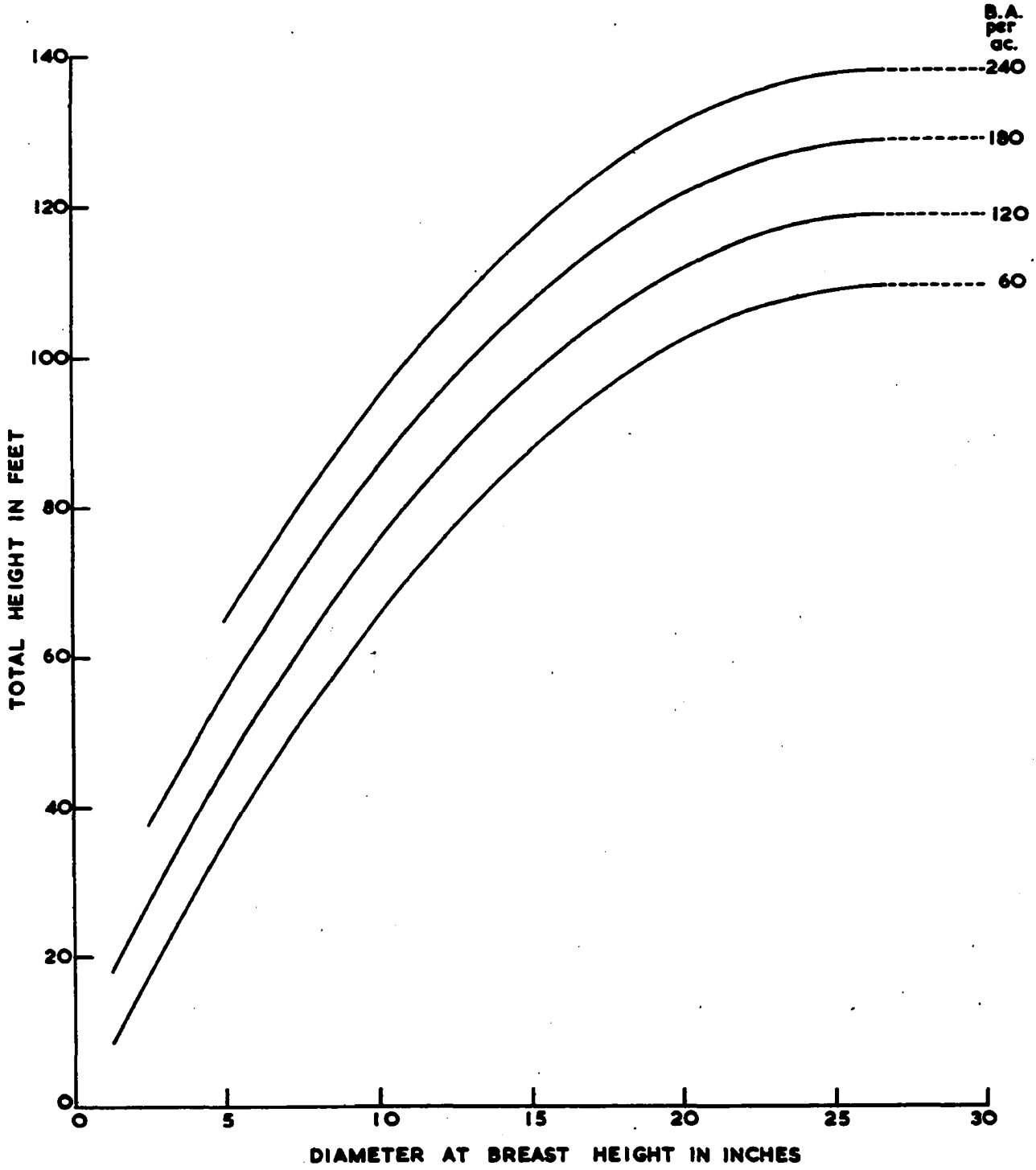

FIG. 69: Calculated tree height/d. b. h. o. b. curves by stand basal area in sq. ft. per ac.

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tests on some of the stands with an initial spacing of 3.3 x 3.3 ft., from that which might be expected to occur in nature. Reasons for the limitations of the model at close spacings have been given in Part II of this thesis.

Of particular interest, as it has seldom been described in natural stands or plantations, is the development of the structure of the stand after various amounts and distributions of mortality have been applied to the stand after planting. This will help the forest manager to decide whether to replant a young plantation which has suffered early mortality. The site indices tested cover the middle of the range of the site qualities encountered in the coastal region of British Columbia. To test site indices outside the range given, it may be necessary to modify the model to start at age five years for better sites, so that the trees have not already begun to compete with each other, or at age fifteen years for poorer sites, by which time the majority of the trees should have reached breast height.

The thinning tests that have been described are examples of how the model might be used rather than an exhaustive survey of such tests. The model can be modified by rewriting the FORTRAN program to meet the requirements of each individual case.

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SUMMARY AND SUGGESTIONS FOR FUTURE DEVELOPMENT

The object of this thesis was to develop a mathematical model to describe the growth of Douglas fir stands under the conditions commonly encountered in the coastal region of British Columbia. This has been done by describing the growth in diameter of each tree in a matrix of 225 trees, assuming the trees were located at the intersections of a square lattice. It was assumed that, at age ten years, these trees were open-grown. The five-year periodic growth in d. b. h. was calculated for each tree using a regression of d. b. h. on d. b. h. at the beginning of the period, d. b. h. at age ten years and total age, obtained from a sample of eighteen trees on the Saanich Peninsula, Vancouver Island. These growth data were in general agreement with those from younger trees collected at several other locations. The crown width of each tree was also calculated from the d. b. h., using regressions of crown width on d. b. h. obtained for 426 trees sampled in the interior and in the coastal regions of British Columbia and Washington.

Trees were assumed to continue to grow at the open-growth rate until the crowns overlapped. The five-year periodic diameter growth of each tree was then reduced by an amount depending on the proportion of the circumference of the crown occupied by the crowns of surrounding trees. This was repeated at five-year intervals to age 100 years using an I. B. M. 7090 electronic computer. Mortality was obtained by assuming that

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a tree was dead if the five-year periodic d. b. h. was reduced below a certain percentage of the d. b. h. of the tree. This minimum percentage growth expected for survival varied from five per cent at age ten years to 0.1 per cent at age 45 years or above.

The model worked satisfactorily over the range of conditions tested until the number of trees was reduced to below 25. This occurred only above age 40 years at the 3.3 x 3.3 ft. spacing so that the model is therefore of limited use when carrying out tests at older ages at this spacing. The basic model was developed using a matrix of diameters at breast height obtained from a plantation of Douglas fir on the University Research Forest at Haney, B. C. These diameters were modified to suit the requirements of spacing and age in the model. These data have been used as a "control" in testing the model under other conditions of growth.

The model does not attempt to describe height growth. Instead, the relationship between height and d. b. h. has been studied over a wide range of site quality, densities of stocking and age, and a multiple regression derived. This may be used to determine the height of any tree, given its d. b. h. and the basal area of the stand. Volume growth has not been estimated either, although this can be assumed to be correlated with basal area. Alternatively, knowing the height and d. b. h. of each tree, an estimate of tree volume can be obtained using the methcd of Newnham (1958) or of Smith and Breadon (1964).

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The assumptions that have been made in the model must be biologically justified. The first of these is that competition can be measured by the proportion of the crown of each tree occupied by the crowns of surrounding trees. In the model, this has been modified to allow a 40 per cent crown overlap before the amount of competition is sufficiently great to reduce diameter growth. Measurements taken in seven-year-old Douglas fir plantations on the University Research Forest support this assumption. It has been noted in Part II that root spread is closely related to crown spread. For growth, the tree requires light for photosynthesis and nutrients and moisture from the soil. If the crown of a tree is overlapped by the crowns of surrounding trees, shading of the lower branches occurs, and photosynthesis and the production of carbohydrates is reduced with the resultant reduction in tree growth. If the root systems of trees overlap, the competition for the available soil nutrients and moisture is increased until a point is reached where the root system is not able to obtain the maximum amount of nutrients and moisture that it is capable of transporting to the aerial part of the tree. Logically, competition must therefore be related to the proportion of the circumference of the tree occupied by the crowns of surrounding trees.

The assumption that the optimum rate of diameter growth is reduced by the proportion of the circumference of the crown (after reduction to allow for overlap) occupied by the crowns of competitors, is difficult to justify

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quantitatively, due to lack of information. The diameter growth of open-grown trees is known and it is known that, as competition sets in, diameter growth is reduced to the point where the tree dies. The relationship between the amount of competition and reduction in diameter growth used in this thesis was chosen because it was simple to use.

It has been assumed in the model that, if a tree was released from all competition, it would resume the rate of growth of an open-grown tree. This ignores any "shock" effect. Reasons for this "shock" effect have been discussed in Part I. Shock is most likely to occur when a stand, which has been allowed to become very dense with poorly developed crowns, is suddenly opened up. Nowhere are these conditions found in the model so that it is safe to ignore "shock". It is probable, however, that there is a time-lag before the tree resumes fully the optimum rate of growth. This time-lag is not taken into account by the model. The errors involved are not thought to be serious, although Reukema (1964) has shown that short-term growth of crown was not related to release. The model used here should apply generally.

Once a satisfactory model had been obtained it was next possible to test the effects of different distributions and amounts of mortality following planting (Table 10), different site qualities (Table 11), and different thinning regimes (Table 12) on the growth of the stand. The effects of different initial spacings (Table 13) had already been tested while developing the model. In the summary tables the 6.6 x

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TABLE 10: The effect of amount and distribution of mortality following planting on stand growth. Site index: 140. Mean d.b.h.o.b. at age 10 years: 1.26 in. Spacing: 6.6x6.6 ft.

Run	Distribution of mortality following planting	Amount of mortality (%)	Number of trees per acre at age 10 years	Age at which mean d.b.h. o.b. is 12 in.	Number of trees per acre when the mean d.b.h. o.b. is 12 in.	Basal area per acre when mean d.b.h. o.b. is 12 in. (sq.ft.)	Age 100 years		
							Number of trees per acre	Mean d.b.h. o.b. (in.)	Basal area per acre (sq.ft.)
II-1	Negligible	-	956	69	302	248	147	19.0	292
II-2	Binomial	10	907	70	319	257	156	18.6	302
II-3	Binomial	30	711	67	325	261	151	18.8	300
II-4	Binomial	50	516	66	317	250	138	18.5	263
II-5	Uniform (rectangular)	50	498	63	298	228	138	19.3	286
II.6	2 Rand. Inf. Centres	14	858	68	305	247	142	19.2	296

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TABLE 11: The effect of site quality on stand growth. Amount of mortality following planting: negligible. Spacing: 6.6 x 6.6 ft.

Run	Site Index	Age 10 years		Age at which mean d.b.h. o.b. is 12 in.	Number of trees per acre when mean d.b.h.o.b. is 12 in.	Basal area per acre when mean d.b.h. o.b. is 12 in. (sq.ft.)	Age 100 years		
		Number of trees per acre	Mean d.b.h.o.b. (in.)				Number of trees per acre	Mean d.b.h. o.b. (in.)	Basal area per acre (sq.ft.)
II-7	120	956	0.80	84	280	245	213	15.2	274
II-1	140	956	1.26	69	302	248	147	19.0	292
II-8	160	956	1.92	55	338	276	107	23.6	331

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TABLE 12: The effect of thinning on stand growth. Site index: 140. Mean d. b. h. o. b. at age 10 years: 1.26 in. Amount of mortality following planting: negligible. Spacing 6.6 x 6.6 ft.

Run	Trees removed between:		Thinning Period (yr.)	Number of thinnings before age 100 years	Age at which mean d.b.h. o.b.is 12 in.	Number of trees per acre when mean d.b.h. o.b.is 12 in.	Basal area per acre when mean d.b.h. o.b.is 12 in. (sq. ft.)	Thinnings		Total basal area yield (sq. ft.)	Age 100 years					
	lower limit	upper limit						Number of trees per acre	Basal area per acre (sq. ft.)		Number of trees per acre	Mean d.b.h. o.b. (in.)	Basal area per acre (sq. ft.)	Thinnings: Number of trees per acre	Thinnings: Basal area per acre (sq. ft.)	Total basal area yield (sq. ft.)
II-1	-	-	-	-	69	302	248	-	-	248	147	19.0	292	-	-	292
II-9	0	$\bar{D}$-s	10	6	64	284	228	271	66	294	120	20.0	263	359	146	409
II-10	0	$\bar{D}$-s	20	3	65	298	237	187	41	278	138	19.4	285	227	73	357
II-11	$\bar{D}$-s	$\bar{D}$-0.5s	10	6	64	289	230	275	79	309	125	20.0	274	346	153	427
II-12	$\bar{D}$-s	$\bar{D}$-0.5s	20	3	67	303	244	169	45	299	138	19.2	283	213	84	366
II-13	$\bar{D}$+0.75s	$\bar{D}$+s	10	4	71	270	224	106	54	278	146	18.2	277	124	84	361

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TABLE 13: The effect of initial spacing (planting distance) on stand growth. Site index: 140. Mean d.b.h.o.b. at age 10 years: 1.26 in. Distribution of mortality following planting: negligible. Run II-1

Initial spacing (ft.)	Number of trees per acre at age 10 yr.	Age at which mean d.h.b.o.b. is 12 in.	Number of trees per acre when mean d.b.h.o.b. is 12 in.	Basal area per acre when mean d.h.b.o.b. is 12 in. (sq. ft.)	Age 100 years		
					Number of trees per acre	Mean d.b.h.o.b. (in.)	Basal area per acre (sq. ft.)
3.3	3822	63	510	406	213	19.9	466
6.6	956	69	302	248	147	19.0	292
9.9	425	64	319	258	134	17.9	240
13.2	239	52	239	198	146	18.4	272
16.5	153	51	153	125	149	19.6	318
19.8	106	51	106	87	106	21.6	273

6.6 ft. spacing of the basic matrix has been taken as a "control", with which the results of the various tests may be compared. The age at which the stand reaches a mean d. b. h. of twelve inches is an indication of the minimum rotation (Smith et al., 1962).

In terms of absolute values it is not possible to confirm that these results are accurate. However, the basic model gives satisfactory fits to the yield table data of Barnes (U. B. C. Forest Club, 1959) and McArdle et al. (1949). It should be realised also that the results of these tests are based on one "run" in each case. Although it is possible to control site differences more readily in the stand model by using identical diameter distributions, tree distributions or site qualities as may be required, this is equivalent to a field experiment in which there is only one replicate of each treatment. The "runs" have been replicated in "degree" but not in "kind" due to the amount of computer time required. It is probable that, if not completely accurate in absolute values, the results give accurate, relative estimates of growth under the range of conditions tested. To confirm the accuracy of the results, further field data, largely not available at present, would have to be secured. Several recently established spacing and thinning trials may provide more suitable data in the long run.

This thesis has shown how a stand model can be developed for Douglas fir and has shown how the model may be used to study the growth of stands under various conditions.

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So far the model does not give estimates of volume growth although these may be obtained from individual diameter and height data. This could be built into the model by modifying the FORTRAN program.

The model also assumes that, after the initial mortality following planting, all mortality that occurs in the stand is due to suppression. In natural stands, besides suppression mortality, there is usually a small amount of random mortality due to insects, disease, or extremes of climate. In the stand model, such mortality could be obtained by a method similar to that described for stand model IIA (Newnham, 1964). At the end of each five-year period there is a three-digit random number associated with each tree. If this random number is less than the random mortality that has been prescribed, the tree "dies". If it is desired to prescribe a "clumped" mortality at a particular age, this can be accomplished by setting the random numbers of the trees to be "killed" equal to zero at that age.

The model can be used to test any prescribed thinning by modifying the FORTRAN program to meet the requirements of each case. In conjunction with the computer program of Kozak and Munro (1963), it can be used to find the theoretical distribution giving the best fit to the actual distribution of the trees in the stand at the end of any five-year period. From this it would be possible to find the best size of quadrat to use as a basis of sampling at the various initial spacings and stand ages. For these tests, a larger basic

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diameter matrix (at least 30 x 30 trees) could be required to give sufficient quadrats for satisfactory distribution fitting.

A basic square spacing has been assumed in all the tests performed with the model. The pattern of the spacing can be varied to a certain extent, as has been done in testing the different distributions of mortality following planting, by omitting trees from some of the locations on the square lattice. It is also most convenient to use a square pattern when programming the computer. Although the model can be modified to use a basic triangular, or polygonal, pattern the resulting program would be more complicated than that for square spacing.

It is not known whether it will be possible to adapt the model readily for use with other tree species. In theory it should be possible, providing the necessary crown width/ d. b. h. and diameter growth regressions are known for the species. Some difficulty may be encountered in obtaining satisfactory crown width/d. b. h. regressions for species, such as some of the spruces, which have a columnar crown habit.

In its final form, it should be possible to use the model to estimate, given sufficient basic information, the volume (even, perhaps, by grade) and values of sawn products, pulpwood, plywood, and waste for any Douglas fir stand, grown under various conditions. All that is required is a wealth of information on utilization, a large computer, the patience to write the necessary program, and time to test and refine the biological assumptions involved.

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APPENDIX I

THE DISTRIBUTIONS ENCOUNTERED IN FOREST RESEARCH

Introduction

Most researchers in forestry have at some time encountered the more common distributions described in forestry literature (Smith and Ker, 1957). It is the purpose of this section to present some of these distributions in a uniform manner based on the principles of mathematical statistics.

It is first necessary to give some definitions.

Definition 1: If X is a random variable, then the distribution function of X, F_X, is a function defined by

$$F_X = P\left[X \leq x\right]$$

for every real number x (Tucker, 1962). The term on the right-hand side of the above equation means "the probability of the event that the random variable, X, will take on a value less than, or equal to, x, a real number". Because it is a probability, $F_X(x)$ must take on a value between zero and one.

Definition 2: The discrete density function, $f_X(x)$, is defined by

$$f_X(x) = \begin{cases} P\left[X \leq x_n\right] & \text{if } x = x_n \\ 0 & \text{if } x \neq x_n \text{ for all } n. \end{cases}$$

Then $\sum_n f_X(x_n) = 1$, and $\sum_{\{n \mid x_n \leq x\}} f_X(x_n) = F_X(x)$ (Tucker, 1962)

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It should be noted that in discrete distributions the values of x can only be integers and also that the density function, $f_X(x)$, takes the value zero outside the range of the values of x on which the distribution is defined.

Definition 3: A random variable, X, is said to have an absolutely continuous distribution if there exists a function, $f_X(x)$, such that

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$$

for every real number, x. The function $f_X(x)$ is the density function of the random variable X (Tucker, 1962).

Definition 4: If $X_1,\ldots,X_n$ are n independent observations on a random variable, X, then

$$\text{the mean, } \bar{X}_n = (X_1 + \ldots + X_n)/n$$

$$\text{and tne variance, } s_n^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2/n\right).$$

For information about the properties of distribution functions, the various methods of obtaining estimates of the parameters describing a distribution from samples, expectations and consistent and unbiased estimators, reference should be made to any standard text on mathematical statistics (e. g. Kendall and Stuart, 1958, 1961; Tucker, 1962).

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Distributions encountered in forestry research

A. Discrete distributions

1. The binomial distribution

This distribution occurs where the experiment consists of a sequence of n (Bernoulli) trials which satisfy the following requirements:

(i) the outcome of each trial can only be one of two possible incompatible events, "success" (S) or "failure" (F),

(ii) the outcome of each trial is independent of the other trials, and

(iii) the probability, p, of S occurring does not vary from trial to trial.

If these conditions are met then the binomial density function is

$$f_X(x) = P[X = x] = \binom{n}{x} p^x(1 - p)^{n - x}$$

$$= \frac{n!}{(n - x)!\, x!}\, p^x(1 - p)^{n - x},\ x = 0,1,2,\ldots,n$$

where $0 < p < 1$ and x = number of "successes" in n trials.

Properties: $\overline{X}_n = np$; $s_n^2 = np(1 - p)$.

Estimator: $\hat{p} = \overline{X}_n/n$.

Tables of the binomial distribution function are given in Burington and May (1953).

2. The Poisson distribution

The binomial distribution approaches the Poisson

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distribution as a limit when $n \longrightarrow \infty$ and $p \longrightarrow 0$ in such a way that $np = m$ is a constant (Burington and May, 1953). The density function of the Poisson distribution is

$$f_X(x) = P[X = x] = \frac{m^x e^{-m}}{x!}, \quad x = 0,1,2,\ldots, \quad m>0$$

Because of the above limiting property, the Poisson distribution may be used to approximate the binomial distribution and vice versa.

Properties: $\bar{X}_n = m$; $s_n^2 = m$

Estimator: $\hat{m} = \bar{X}_n$

Tables of the Poisson distribution are given in Burington and May (1953).

3. The negative binomial distribution

The density function of the negative binomial distribution is

$$f_X(x) = P[X = x] = \begin{cases} (1 + \frac{m}{k})^{-k} & \text{if } x = 0 \\ \binom{k + x - 1}{x} (1 + \frac{m}{k})^{-k} (\frac{m}{m + k})^x & \text{if } x = 1,2,3,\ldots, \end{cases}$$

where $m>0$, $k>0$.

The Poisson distribution is obtained as a limit as $k \longrightarrow \infty$.

Properties: $\bar{X}_n = m$; $s_n^2 = m + \frac{m^2}{k}$.

Estimators: $\hat{m} = \bar{X}_n$; $\hat{k} = \frac{\bar{X}_n}{s_n^2 - \bar{X}_n}$ (Anscombe, 1949)

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Tables of the negative binomial distribution have been published by Grimm (1962). The writer has developed a FORTRAN program for the I. B. M. 1620 electronic computer which calculates tables of the distribution over the range of values of m and k normally encountered in practice.

In deciding which of the above three distributions should be used to fit empirical data, the relationship between $\overline{X}_n$ and s_n^2 should be studied. If $\overline{X}_n > s_n^2$, the binomial distribution is indicated; if $\overline{X}_n \cong s_n^2$, the Poisson distribution is indicated; and if $\overline{X}_n < s_n^2$, the negative binomial distribution is indicated. The Poisson distribution occurs when the individuals in the population are randomly distributed (see Pielou, 1959, for a discussion on randomness). If the variance is less than the mean, the individuals are more uniformly distributed. The negative binomial distribution indicates that the individuals occur in clumps or aggregates. The uniform (rectangular) distribution indicates extreme clumpiness.

4. The uniform (rectangular) distribution

The density function of this distribution is

$$f_X(x) = P[X = x] = \begin{cases} 1/N & \text{if } x = 0,1,2,\ldots,N \\ 0 & \text{otherwise} \end{cases}$$

where $N > 0$.

The uniform distribution gives an equal probability of occurrence to each value of x within the range.

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Properties: $\bar{X}_n = N/2$

Estimator: $\hat{N} = \max_{1 \le i \le N} \{X_i\}$

B. Absolutely continuous distributions

1. The gamma distribution

The gamma distribution has a density function

$$f_X(x) = \begin{cases} \frac{1}{\Gamma(\alpha + 1)\,\beta^{\alpha+1}} x^{\alpha} e^{-x/\beta} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\alpha \geq -1, \beta > 0$ and $\Gamma(\alpha + 1) = \int_0^\infty y^{\alpha} e^{-y} dy = \alpha\Gamma(\alpha)$. If n is an integer ($n > 0$) then $\Gamma(n) = (n - 1)!$ In particular, $\Gamma(1) = 1$ and $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Properties: $\bar{X}_n = \beta(\alpha + 1)$; $s_n^2 = \beta^2(\alpha + 1)$

Estimators: By the method of moments

$$\hat{\alpha} = \frac{\bar{X}_n^2}{s_n^2} - 1; \quad \hat{\beta} = \frac{s_n^2}{\bar{X}_n}$$

These are not necessarily the most efficient estimators (Kendall and Stuart, 1961). Thom (1949) gave approximate solutions to the maximum likelihood estimators of α and β. These are

$$\hat{\alpha} = \frac{1 + \sqrt{1 + \frac{4}{3}(\log \bar{X}_n - \frac{1}{n}\sum_{i=1}^{n} \log x_i)}}{4(\log \bar{X}_n - \frac{1}{n}\sum_{i=1}^{n} \log x_i)} - 1$$

$$\hat{\beta} = \frac{\bar{X}_n}{\hat{\alpha} + 1}$$

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Tables of the gamma function are given in Burington and May (1953).

2. The normal distribution

The normal distribution has density

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

where $-\infty < \mu < \infty$ and $\sigma^2 > 0$. This distribution is often written as $N(\mu, \sigma^2)$, μ and σ^2 being the parameters describing the distribution. Tables of the N(0, 1) distribution function are given in most statistical texts or in Fisher and Yates (1957). Properties: If X is a random variable having a N(0, 1) distribution, then the distribution of X^2 is gamma with $\alpha = -\frac{1}{2}$ and $\beta = 2$ (Tucker, 1962).

If $X_1, \ldots, X_n$ is a sequence of n independent variables each with a N(0, 1) distribution, then the distribution of $X_1^2 + \ldots + X_n^2$ is chi-square with n degrees of freedom.

Estimators: $\hat{\mu} = \overline{X}_n$; $\hat{\sigma^2} = s_n^2$

The following theorem is sometimes found useful when working with distribution problems.

The central limit theorem: Let $X_1, \ldots, X_n$ be a sequence of independent, identically distributed, random variables with finite common expectation, μ, and variance, σ^2. Let

$$Y_n = (X_1 + \ldots + X_n)/\sigma\sqrt{n}$$

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Then

$$F_{Y_n}(x) \longrightarrow \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

as $n \longrightarrow \infty$ uniformly in x (Tucker, 1962).

Because of the limiting nature of this theorem, when n is sufficiently large, the normal distribution may be used to approximate any other distribution.

Pearl-Reed growth curves

Pearl-Reed growth curves, originally developed to describe the growth of the population of the United States (Pearl and Reed, 1920), have been used in forestry to describe diameter distributions (Osborne and Schumacher, 1935). The Pearl-Reed equation is of the form

$$y = \frac{be^{ax}}{1 + ce^{ax}}$$

where a, b, and c are constants having positive values. In the original paper (Pearl and Reed, 1920), y was the population size and x the time. When used to describe diameter distributions, y is the cumulative number of trees and x is the diameter. This equation may be written in the form

$$y = \frac{b}{e^{-ax} + c}$$

This equation has the possible disadvantage that it produces a symmetrical frequency distribution. Osborne and Schumacher (1935) used a modified Pearl-Reed equation of the

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form

$$y = c + \frac{k}{1 + me^{f(x)}}$$

where c is the lower asymptote, k + c is the upper asymptote and m is an arbitrary constant. The term, f(x), is of the form

$$f(x) = b_1x + b_2x^2 + \ldots + b_nx^n$$

This gave asymmetrical distributions which described the distribution of diameters of even-aged stands of red gum (Liquidambar styraciflua L.) with remarkable accuracy.

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APPENDIX II

A DETAILED DESCRIPTION OF THE FORTRAN PROGRAM FOR STAND MODEL II

1. A list of the more important variables in the program with their meanings.

A (also MA)	Age in years.
ACOMP	Age at the beginning of the period in which mortality first occurs.
ASTART	Age at the start of the program.
ASTOP	Age at which the program stops (usually 100 years; ASTOP $\leq$100).
A1	Constant term in the radial growth regression.
A1C, A2C	Constant terms in the two crown width regressions.
BAST	Basal area per acre.
B1,B2,B3,B4	Regression coefficients in the radial growth regression.
B1C, B2C	Regression coefficients in the two crown width regressions.
D	D. b. h. o. b. at the beginning of each five-year period.
DAP5	D. b. h. o. b. at the end of each five-year period.
DINC	Minimum percentage five-year diameter growth for survival.
D10	D. b. h. o. b. at age 10 years.
FLAST	Number of trees per acre at beginning of each five-year period.
FN	Number of trees at end of each five-year period.

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FPLOT	Number of live trees in the matrix at age 0 years.
I	Row number in matrix ($0 < I \leq 20$, usually $I \leq 15$).
J	Column number in matrix ($0 < J \leq 20$, usually $J \leq 15$).
K	Row number of "competitors" in matrix.
L	Column number of "competitors" in matrix.
M	The number of locations that the competing tree in each octant is away from the tree being studied.
MATIO	Number of trees per row and per column in input matrix ($6 \leq MATIO \leq 20$).
NDIST	Number of planting distances (initial spacings).
NMAT	Number of trees per row and per column in working matrix. ($6 \leq NMAT \leq MATIO$).
NT	Number of trees per acre (in input).
NOCT	Number of octant.
PD	Planting distance (initial spacing).
PI	$\pi = 3.141596$.
PS	Plot size in acres (of working matrix).
REDFAC	Reduction factor to reduce calculated crown width to "competitive" crown width.
REDINC	Increment for REDFAC.
SOC(I,J)	Proportion of the circumference of the crown in the I,Jth. position occupied by competitors.
S(M)	Distance of the Mth. competing tree from the tree being studied ($PD \leq S(M) \leq 8 \times PD$).
THETA	Angle subtended at the centre of the crown by the two points of intersection of the "competitive" crowns, divided by two ($0 \leq THETA \leq \pi$).

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2. Description of the FORTRAN program.

Line No.

MAINLINE PROGRAM

4-23	Input section. The parameters, regression coefficients and the basic data are read into the computer.
24-30	Prints out basic matrix of diameters.
31-493	Loop for each planting distance.
37-39	Calculates the distance of each possible competitor from the study tree.
49-374	Loop to calculate five-year periodic diameter growth.
53-369	Loop to calculate the competitive status, SOC, of each tree by octants.
54-97	Values are given to the variables required to calculate the positions in the matrix of "competitors", D(K,L), with respect to each tree, D(I,J). The values of the variables are determined according to the octant being considered (see Fig. 12).
98-104	Calculates the location of the first tree position in the octant. (The function subprogram KFIND(KM) is described below.)
105	Tests to see if there is a tree in this position (live trees >0).
106	If a tree is present in the first position the subroutine CROWN (described below) is called and THETA is calculated. The next statement (107) calculates the competitive status of the tree and the program then branches to the next octant. (To save operating time, THETA in the program is half the angle subtended at the centre of the crown by the intersection of the "competitive" crown perimeters. The proportion of the circumference of the crown occupied by each competitor is therefore divided by π, and not 2π, to obtain the proportion of the crown occupied, SOC).

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Line No.	
109	If there is no tree in the first position the program branches to the second position in the octant and the process is repeated until a tree is found. If no tree is found in any of the fourteen positions in the octant the program branches to the next octant.
370-1	Calculates the d. b. h. of each tree at the end of each five-year period (DAP5).
373	If $D(I,J) \leq 0$, DAP5(I, J) is set equal to zero.
380	Tests for mortality.
381	Sets D(I,J) equal to DAP5(I,J) for all live trees.
383	Sets D(I,J) equal to -DAP5(I,J) for all trees that have "died" during the current five-year period.
385-95	The new diameter matrix is printed out.
396-404	Initialises frequency distribution table loop.
405-18	Calculates the number of trees that have "died" in each one-inch diameter class in the past five years.
420-9	Calculates the number of live trees in each one-inch diameter class.
433-51	Calculates means, standard deviations, basal area, etc.
452-82	Prints out diameter frequency distribution table.
483	Tests to see if mortality has occurred before the current period.
484	Tests to see if mortality has started in the current period.
485	If mortality has occurred during the current period, ACOMP is set equal to the age at the start of the period.
486-8	If mortality has occurred, either before or during the current period, REDFAC is modified.
492	Tests for end of run.

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FUNCTION KFIND(KM)

When determining the position of competitors in the matrix this subprogram ensures that the mainline program does not branch out of the matrix.

SUBROUTINE CROWN

Calculates the value of THETA, the angle subtended at the centre of the crown by the intersection of the crown perimeters divided by two.

4 R = .5* REDFAC so that R1 = .5*(A1C + B1C(D(I,J))* REDFAC

5-12 Calculates the "competitive" crown radius (R1) of the tree being studied and of the potential competitor (R2).

13 Tests to see if the "competitive" crowns overlap.

14 Tests to see if the crown of the tree being studied (the I,Jth) completely overlaps that of the competitor (the K,Lth.).

15 If the answer to 13 is "no" or to 14 is "yes", THETA is set equal to zero and control is returned to the mainline program.

17 Tests to see if the crown of the competitor overlaps that of the tree being studied.

18 If the answer to 17 is "yes", THETA is set equal to π.

20,21 Calculates the ordinates of the point of intersection (in the first quadrant) of the "competitive" crowns.

22 Tests to see if THETA is greater than, equal to or less than $\pi/2$. THETA is calculated accordingly (see Fig. 70) and control returned to the mainline program.

3. Program output

An example of the output given at the end of each five-year period by the FORTRAN program for the I. B. M. 7090 is shown in Fig. 71.

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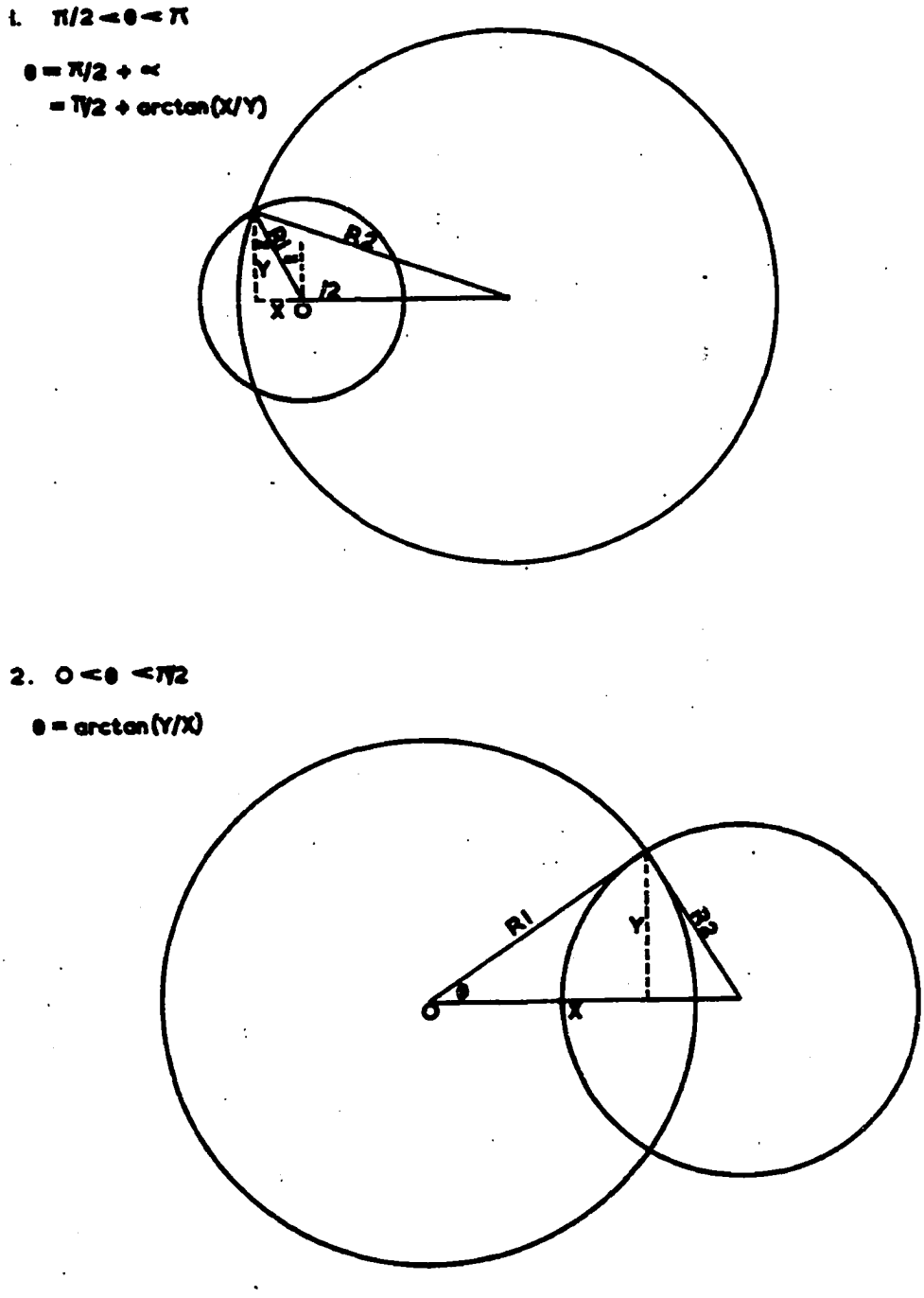

FIG. 70: Calculation of Θ in the FORTRAN program.

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AGE = 30

C.B.H.O.B. FREQUENCY DISTRIBUTION TABLE

D.B.H. CLASS	NO. OF TREES	NO. OF TREES PER AC.	5YR. MORTALITY	5YR. MORTALITY PER AC.
2	0	0.	2	8.9
3	17	75.6	2	8.9
4	28	124.4	4	17.8
5	30	133.3	4	17.8
6	38	168.9	6	26.7
7	29	128.9	1	4.4
8	15	66.7	0	0.
9	13	57.8	0	0.
10	5	22.2	0	0.
11	1	4.4	0	0.
12	1	4.4	0	0.
TOTAL =	177	786.7	19	84.4
MEAN =	5.99 INS.		4.72 INS.	
VARIANCE =	3.6005			
STANDARD DEVIATION =	1.90			
TREE OF MEAN d.A. =	6.28			
BASAL AREA PER AC. =	169.4 SQ. FT.		11.1 SQ. FT.	
P.A.I.(BASAL AREA) =	5.1			
M.A.I.(BASAL AREA) =	5.6			

REDFAC = 0.6059

FIG. 71: Example of program output.

```
STAND MODEL II - NEWNHAM                          FORTRAN SOURCE LIST MODFIR
      ISN        SOURCE STATEMENT

        1        DIMENSION PD(5),PS(5),NT(5),DINC(20),D(20,20),D10(20,20),NDB(30)
        2        DIMENSION DAPS(20,20),SOC(20,20),S(50),NNDB(30),DIST(50)
        3        COMMON REDFAC,A1C,A2C,B1C,B2C,I,J,K,L,M,NMAT,S,THETA,D
        4        READ(5,1) NMAT,MAT10,NDIST,ASTART,ASTOP,FACRED,REDINC
       10 1      FORMAT(3I3,4F6.0)
       11        READ(5,2) (PD(I),I=1,NDIST),(PS(I),I=1,NDIST)
       22 2      FORMAT(10F6.0)
       23        READ(5,5) A1C,A2C,B1C,B2C
       24 5      FORMAT(4F10.0)
       25        READ(5,6) (DINC(LA),LA=2,19)
       32 6      FORMAT(18F4.3)
       33        READ(5,1200) (DIST(I),I=1,48)
       40 1200   FORMAT(12F6.4)
       41        READ(5,3) FPLOT,(NT(I),I=1,NDIST)
       46 3      FORMAT(F6.0,5I6)
       47        READ(5,4) A1,B1,B2,B3,B4,BAMAT
       50 4      FORMAT(6F10.0)
       51 200    READ(5,1201)
       52 1201 0FORMAT(30X,42H                                        /30X,25H
              1                                                                 )
       53        DO 7 I=1,MAT10
       54 7      READ(5,8) (D10(I,J),J=1,MAT10)
       62 8      FORMAT(20F3.1)
       63        WRITE(6,9)
       64 9     0FORMAT(1H1,51X,15HSTAND MODEL IIG/1H0,32HORIGINAL D.B.H.O.B. OF EA
              1CH TREE/)
       65        WRITE(6,1201)
       66        DO 10 I=1,NMAT
       67 10     WRITE(6,11) (D10(I,J),J=1,NMAT)
       75 11     FORMAT(1H0,F4.1,19F5.1)
       76        DO 130 II=1,NDIST
       77        WRITE(6,12)NT(II),PS(II),PD(II),MAT10,FACRED,REDINC
      100 12    0FORMAT(1H1,51X,15HSTAND MODEL IIG///47X,22HNO. OF TREES PER AC. =I
              15//49X,11HPLOT SIZE =F8.5,4H AC.//46X,19HPLANTING DISTANCE =F5.1,4
              2H FT.////10X,8HMATRIX =I3,8H SQUARE,,9H REDFAC =F6.3,9H RLDINC =F7
              3.4)
      101        X=PD(II)
      102        DO 1210 I=1,48
      103 1210   S(I)=DIST(I)*X
      105        BAST=BAMAT/PS(II)
      106        REDFAC=FACRED
      107        FLAST=FPLOT
      110        A=ASTART
      111        ACOMP=0.
      112        DO 14 I=1,NMAT
      113        DO 14 J=1,NMAT
      114 14     D(I,J)=D10(I,J)
      117        PI=3.14159
      120 15     DO 93 I=1,NMAT
      121        DO 93 J=1,NMAT
      122        SOC(I,J)=0.
      123        IF(D(I,J)) 92,92,16
      124 16     DO 91 NOCT=1,8
      125        GO TO (17,17,18,18,17,17,18,18),NOCT
      126 17     I1=I
      127        J1=J
      130        GO TO 19
      131 18     I1=J
      132        J1=I
      133 19     GO TO (20,20,21,21,21,21,20,20),NOCT
      134 20     IM1=-1
      135        IM2=-2
      136        IM3=-3
      137        IM4=-4
      140        IM5=-5
      141        IM6=-6
      142        IM7=-7
      143        IM8=-8
      144        IM9=-9
      145        IM10=-10
      146        GO TO 22
      147 21     IM1=1
      150        IM2=2
      151        IM3=3
      152        IM4=4
      153        IM5=5
      154        IM6=6
      155        IM7=7
      156        IM8=8
      157        IM9=9
      160        IM10=10
      161 22     GO TO (23,24,23,24,24,23,24,23),NOCT
      162 23     JM1=-1
      163        JM2=-2
      164        JM3=-3
      165        JM4=-4
      166        JM5=-5
      167        JM6=-6
      170        JM7=-7
      171        GO TO 25
      172 24     JM1=1
      173        JM2=2
      174        JM3=3
      175        JM4=4
      176        JM5=5
      177        JM6=6
      200        JM7=7
      201 25     M=1
      202        IF(I1-I) 27,26,27
      203 26     K=KFIND(I1+IM1)
      204        L=J1
      205        GO TO 28
      206 27     K=J1
      207        L=KFIND(I1+IM1)
      210 28     IF(D(K,L)) 30,30,29
      211 29     CALL CROWN
      212        SOC(I,J)=SOC(I,J)+.5*THETA/PI
      213        GO TO 91
      214 30     M=2
      215        IF(I1-I) 32,31,32
```

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```
ISN       SOURCE STATEMENT

216 31    L=KFIND(J1+JM1)                                          II 111
217       GO TO 33                                                 II 112
220 32    K=KFIND(J1+JM1)                                          II 113
221       IF(D(K,L)) 34,34,33                                      II 114
222 33    CALL CROWN                                               II 115
223       SOC(I,J)=SOC(I,J)+.5*THETA/PI                            II 116
224       GO TO 91                                                 II 117
225 34    M=3                                                      II 118
226       IF(II-1) 36,35,36                                        II 119
227 35    K=KFIND(I1+IM2)                                          II 120
230       L=J1                                                     II 121
231       GO TO 37                                                 II 122
232 36    K=J1                                                     II 123
233       L=KFIND(I1+IM2)                                          II 124
234 37    IF(D(K,L)) 39,39,38                                      II 125
235 38    CALL CROWN                                               II 126
236       SOC(I,J)=SOC(I,J)+.5*THETA/PI                            II 127
237       GO TO 91                                                 II 128
240 39    M=4                                                      II 129
241       IF(II-1) 41,40,41                                        II 130
242 40    L=KFIND(J1+JM1)                                          II 131
243       GO TO 42                                                 II 132
244 41    K=KFIND(J1+JM1)                                          II 133
245 42    IF(D(K,L)) 44,44,43                                      II 134
246 43    CALL CROWN                                               II 135
247       SOC(I,J)=SOC(I,J)+THETA/PI                               II 136
250       GO TO 91                                                 II 137
251 44    M=5                                                      II 138
252       IF(II-1) 46,45,46                                        II 139
253 45    L=KFIND(J1+JM2)                                          II 140
254       GO TO 47                                                 II 141
255 46    K=KFIND(J1+JM2)                                          II 142
256 47    IF(D(K,L)) 49,49,48                                      II 143
257 48    CALL CROWN                                               II 144
260       SOC(I,J)=SOC(I,J)+.5*THETA/PI                            II 145
261       GO TO 91                                                 II 146
262 49    M=6                                                      II 147
263       IF(II-1) 51,50,51                                        II 148
264 50    K=KFIND(I1+IM3)                                          II 149
265       L=J1                                                     II 150
266       GO TO 52                                                 II 151
267 51    K=J1                                                     II 152
270       L=KFIND(I1+IM3)                                          II 153
271 52    IF(D(K,L)) 54,54,53                                      II 154
272 53    CALL CROWN                                               II 155
273       SOC(I,J)=SOC(I,J)+.5*THETA/PI                            II 156
274       GO TO 91                                                 II 157
275 54    M=7                                                      II 158
276       IF(II-1) 56,55,56                                        II 159
277 55    L=KFIND(J1+JM1)                                          II 160
300       GO TO 57                                                 II 161
301 56    K=KFIND(J1+JM1)                                          II 162
302 57    IF(D(K,L)) 59,59,58                                      II 163
303 58    CALL CROWN                                               II 164
304       SOC(I,J)=SOC(I,J)+THETA/PI                               II 165
305       GO TO 91                                                 II 166
306 59    M=8                                                      II 167
307       IF(II-1) 61,60,61                                        II 168
310 60    L=KFIND(J1+JM2)                                          II 169
311       GO TO 62                                                 II 170
312 61    K=KFIND(J1+JM2)                                          II 171
313 62    IF(D(K,L)) 64,64,63                                      II 172
314 63    CALL CROWN                                               II 173
315       SOC(I,J)=SOC(I,J)+THETA/PI                               II 174
316       GO TO 91                                                 II 175
317 64    M=9                                                      II 176
320       IF(II-1) 66,65,66                                        II 177
321 65    K=KFIND(I1+IM4)                                          II 178
322       L=J1                                                     II 179
323       GO TO 67                                                 II 180
324 66    K=J1                                                     II 181
325       L=KFIND(I1+IM4)                                          II 182
326       IF(D(K,L)) 68,68,67                                      II 183
327 67    CALL CROWN                                               II 184
330       SOC(I,J)=SOC(I,J)+.5*THETA/PI                            II 185
331       GO TO 91                                                 II 186
332 68    M=10                                                     II 187
333       IF(II-1) 70,69,70                                        II 188
334 69    L=KFIND(J1+JM1)                                          II 189
335       GO TO 71                                                 II 190
336 70    K=KFIND(J1+JM1)                                          II 191
337 71    IF(D(K,L)) 73,73,72                                      II 192
340 72    CALL CROWN                                               II 193
341       SOC(I,J)=SOC(I,J)+THETA/PI                               II 194
342       GO TO 91                                                 II 195
343 73    M=11                                                     II 196
344       IF(II-1) 75,74,75                                        II 197
345 74    K=KFIND(I1+IM3)                                          II 198
346       L=KFIND(J1+JM3)                                          II 199
347       GO TO 76                                                 II 200
350 75    K=KFIND(J1+JM3)                                          II 201
351       L=KFIND(I1+IM3)                                          II 202
352 76    IF(D(K,L)) 78,78,77                                      II 203
353 77    CALL CROWN                                               II 204
354       SOC(I,J)=SOC(I,J)+.5*THETA/PI                            II 205
355       GO TO 91                                                 II 206
356 78    M=12                                                     II 207
357       IF(II-1) 80,79,80                                        II 208
360 79    K=KFIND(I1+IM4)                                          II 209
361       L=KFIND(J1+JM2)                                          II 210
362       GO TO 180                                                II 211
363 80    K=KFIND(J1+JM2)                                          II 212
364       L=KFIND(I1+IM4)                                          II 213
365 180   IF(D(K,L)) 81,81,181                                     II 214
366 181   CALL CROWN                                               II 215
367       SOC(I,J)=SOC(I,J)+THETA/PI                               II 216
370       GO TO 91                                                 II 217
371 81    M=13                                                     II 218
372       MM=0                                                     II 219
373       IF(II-1) 83,82,83                                        II 220
```

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```
ISN        SOURCE STATEMENT                                              IS 221
374 82     L=KFIND(J1+JM3)                                               IS 222
375        GO TO 84                                                      IS 223
376 83     K=KFIND(J1+JM3)                                               IS 224
377 84     IF(D(K,L)) 86,86,85                                           IS 225
400 85     CALL CROWN                                                    IS 226
401        SOC(I,J)=SOC(I,J)+THETA/P1                                    IS 227
402        MM=1                                                          IS 228
403 86     IF(I-1) 88,87,88                                              IS 229
404 87     K=KFIND(I1+IM5)                                               IS 230
405        L=J1                                                          IS 231
406        GO TO 89                                                      IS 232
407 88     K=J1                                                          IS 233
410        L=KFIND(I1+IM5)                                               IS 234
411 89     IF(D(K,L)) 298,298,90                                         IS 235
412 90     CALL CROWN                                                    IS 236
413        SOC(I,J)=SOC(I,J)+.5*THETA/P1                                 IS 237
414        GO TO 91                                                      IS 238
415 298    IF(MM) 299,299,91                                             IS 239
416 299    M=15                                                          IS 240
417        IF(I-1) 301,300,301                                           IS 241
420 300    K=KFIND(I1+IM5)                                               IS 242
421        L=KFIND(J1+JM1)                                               IS 243
422        GO TO 302                                                     IS 244
423 301    K=KFIND(J1+JM1)                                               IS 245
424        L=KFIND(I1+IM5)                                               IS 246
425 302    IF(D(K,L)) 303,303,72                                         IS 247
426 303    M=16                                                          IS 248
427        IF(I-1) 305,304,305                                           IS 249
430 304    L=KFIND(J1+JM2)                                               IS 250
431        GO TO 306                                                     IS 251
432 305    K=KFIND(J1+JM2)                                               IS 252
433 306    IF(D(K,L)) 307,307,72                                         IS 253
434 307    M=17                                                          IS 254
435        IF(I-1) 309,308,309                                           IS 255
436 308    K=KFIND(I1+IM4)                                               IS 256
437        L=KFIND(J1+JM4)                                               IS 257
440        GO TO 310                                                     IS 258
441 309    K=KFIND(J1+JM4)                                               IS 259
442        L=KFIND(I1+IM4)                                               IS 260
443 310    IF(D(K,L)) 311,311,90                                         IS 261
444 311    M=18                                                          IS 262
445        IF(I-1) 313,312,313                                           IS 263
446 312    K=KFIND(I1+IM5)                                               IS 264
447        L=KFIND(J1+JM3)                                               IS 265
450        GO TO 314                                                     IS 266
451 313    K=KFIND(J1+JM3)                                               IS 267
452        L=KFIND(I1+IM5)                                               IS 268
453 314    IF(D(K,L)) 315,315,72                                         IS 269
454 315    M=19                                                          IS 270
455        IF(I-1) 317,316,317                                           IS 271
456 316    K=KFIND(I1+IM6)                                               IS 272
457        L=J1                                                          IS 273
460        GO TO 318                                                     IS 274
461 317    K=J1                                                          IS 275
462        L=KFIND(I1+IM6)                                               IS 276
463 318    IF(D(K,L)) 319,319,90                                         IS 277
464 319    M=20                                                          IS 278
465        IF(I-1) 321,320,321                                           IS 279
466 320    L=KFIND(J1+JM1)                                               IS 280
467        GO TO 322                                                     IS 281
470 321    K=KFIND(J1+JM1)                                               IS 282
471 322    IF(D(K,L)) 323,323,72                                         IS 283
472 323    M=21                                                          IS 284
473        IF(I-1) 325,324,325                                           IS 285
474 324    L=KFIND(J1+JM2)                                               IS 286
475        GO TO 326                                                     IS 287
476 325    K=KFIND(J1+JM2)                                               IS 288
477 326    IF(D(K,L)) 327,327,72                                         IS 289
500 327    M=22                                                          IS 290
501        IF(I-1) 329,328,329                                           IS 291
502 328    K=KFIND(I1+IM5)                                               IS 292
503        L=KFIND(J1+JM4)                                               IS 293
504        GO TO 3291                                                    IS 294
505 329    K=KFIND(J1+JM4)                                               IS 295
506        L=KFIND(I1+IM5)                                               IS 296
507 3291   IF(D(K,L)) 330,330,72                                         IS 297
510 330    M=23                                                          IS 298
511        IF(I-1) 332,331,332                                           IS 299
512 331    K=KFIND(I1+IM6)                                               IS 300
513        L=KFIND(J1+JM3)                                               IS 301
514        GO TO 333                                                     IS 302
515 332    K=KFIND(J1+JM3)                                               IS 303
516        L=KFIND(I1+IM6)                                               IS 304
517 333    IF(D(K,L)) 334,334,72                                         IS 305
520 334    M=24                                                          IS 306
521        IF(I-1) 336,335,336                                           IS 307
522 335    K=KFIND(I1+IM7)                                               IS 308
523        L=J1                                                          IS 309
524        GO TO 337                                                     IS 310
525 336    K=J1                                                          IS 311
526        L=KFIND(I1+IM7)                                               IS 312
527 337    IF(D(K,L)) 338,338,90                                         IS 313
530 338    M=25                                                          IS 314
531        MM=0                                                          IS 315
532        IF(I-1) 340,339,340                                           IS 316
533 339    L=KFIND(J1+JM1)                                               IS 317
534        GO TO 341                                                     IS 318
535 340    K=KFIND(J1+JM1)                                               IS 319
536 341    IF(D(K,L)) 343,343,342                                        IS 320
537 342    CALL CROWN                                                    IS 321
540        SOC(I,J)=SOC(I,J)+THETA/P1                                    IS 322
541        MM=1                                                          IS 323
542 343    IF(I-1) 345,344,345                                           IS 324
543 344    K=KFIND(I1+IM5)                                               IS 325
544        L=KFIND(J1+JM5)                                               IS 326
545        GO TO 346                                                     IS 327
546 345    K=KFIND(J1+JM5)                                               IS 328
547        L=KFIND(I1+IM5)                                               IS 329
550 346    IF(D(K,L)) 347,347,90                                         IS 330
551 347    IF(MM) 348,348,91                                             IS 331
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ISN        SOURCE STATEMENT

552 348    M=27                                                    II 331
553        IF(I1-1) 350,349,350                                    II 332
554 349    K=KFIND(I1+IM6)                                         II 333
555        L=KFIND(J1+JM6)                                         II 334
556        GO TO 351                                               II 335
557 350    K=KFIND(J1+JM4)                                         II 336
560        L=KFIND(I1+IM6)                                         II 337
561 351    IF(D(K,L)) 352,352,72                                   II 338
562 352    M=28                                                    II 339
563        IF(I1-1) 354,353,354                                    II 340
564 353    K=KFIND(I1+IM7)                                         II 341
565        L=KFIND(J1+JM2)                                         II 342
566        GO TO 355                                               II 343
567 354    K=KFIND(J1+JM2)                                         II 344
570        L=KFIND(I1+IM7)                                         II 345
571 355    IF(D(K,L)) 356,356,72                                   II 346
572 356    M=29                                                    II 347
573        IF(I1-1)-358,357,358                                    II 348
574 357    L=KFIND(J1+JM3)                                         II 349
575        GO TO 359                                               II 350
576 358    K=KFIND(J1+JM3)                                         II 351
577 359    IF(D(K,L)) 360,360,72                                   II 352
600 360    M=30                                                    II 353
601        IF(I1-1) 362,361,362                                    II 354
602 361    K=KFIND(I1+IM6)                                         II 355
603        L=KFIND(J1+JM5)                                         II 356
604        GO TO 363                                               II 357
605 362    K=KFIND(J1+JM5)                                         II 358
606        L=KFIND(I1+IM6)                                         II 359
607 363    IF(D(K,L)) 364,364,72                                   II 360
610 364    M=31                                                    II 361
611        IF(I1-1) 366,365,366                                    II 362
612 365    K=KFIND(I1+IM8)                                         II 363
613        L=J1                                                    II 364
614        GO TO 367                                               II 365
615 366    K=J1                                                    II 366
616        L=KFIND(I1+IM8)                                         II 367
617 367    IF(D(K,L)) 91,91,90                                     II 368
620 91     CONTINUE                                                II 369
622        ODAPS(I,J)=D(I,J)+2.*(A1+B1*D(I,J)+A*(B2+B4*A)+B3*DIG(I,J))*    II 370
           1(1.-SOC(I,J))                                          II 371
623        GO TO 93                                                II 372
624 92     DAPS(I,J)=0.                                            II 373
625 93     CONTINUE                                                II 374
630        LA=.2+A+.5                                              II 375
631        A=A+5.                                                  II 376
632        DO 97 I=1,NMAT                                          II 377
633        DO 97 J=1,NMAT                                          II 378
634        IF(DAPS(I,J)) 95,95,94                                  II 379
635 94     IF((DAPS(I,J)-D(I,J))/D(I,J)-DINC(LA)) 96,95,95         II 380
636 95     D(I,J)=DAPS(I,J)                                        II 381
637        GO TO 97                                                II 382
640 96     D(I,J)=-DAPS(I,J)                                       II 383
641 97     CONTINUE                                                II 384
644        MA=A                                                    II 385
645        MMA=A+.05+.1                                            II 386
646        MMA=MMA+20                                              II 387
647        IF(MA-MMA) 102,98,102                                   II 388
650 98     WRITE(6,99) MA                                          II 389
651 99     FORMAT(1H1,57X,5HAGE =I3/1HO,17HO.B.M.O.B. MATRIX)      II 390
652        DO 100 I=1,NMAT                                         II 391
653 100    WRITE(6,101) (D(I,J),J=1,NMAT)                          II 392
661 101    FORMAT(1HO,F6.2,15F7.2)                                 II 393
662        WRITE(6,1015)                                           II 394
663 1015   FORMAT(1HO,19H(DEAD TREES ARE - 1)                       II 395
664 102    DO 103 I3=1,30                                          II 396
665        NNDB(I3)=0                                              II 397
666 103    NDB(I3)=0                                               II 398
670        SD=0.                                                   II 399
671        SDD=0.                                                  II 400
672        NNDW=0                                                  II 401
673        SND=0.                                                  II 402
674        SNDD=0.                                                 II 403
675        NDEAD=0                                                 II 404
676        DO 112 I=1,NMAT                                         II 405
677        DO 112 J=1,NMAT                                         II 406
700        IF(D(I,J)) 104,112,108                                  II 407
701 104    DN=-D(I,J)                                              II 408
702        NDEAD=NDEAD+1                                           II 409
703        SND=SND+DN                                              II 410
704        SNDD=SNDD+DN+DN                                         II 411
705        I3=1                                                    II 412
706 105    TI3=I3                                                  II 413
707        TI3=TI3+.5                                              II 414
710        IF(DN-TI3) 107,106,106                                  II 415
711 106    I3=I3+1                                                 II 416
712        IF(I3-30) 105,107,107                                   II 417
713 107    NNDB(I3)=NNDB(I3)+1                                     II 418
714        GO TO 112                                               II 419
715 108    NNDW=NNDW+1                                             II 420
716        SD=SD+D(I,J)                                            II 421
717        SDD=SDD+D(I,J)+D(I,J)                                   II 422
720        I3=1                                                    II 423
721 109    TI3=I3                                                  II 424
722        TI3=TI3+.5                                              II 425
723        IF(D(I,J)-TI3) 111,110,110                              II 426
724 110    I3=I3+1                                                 II 427
725        IF(I3-30) 109,111,111                                   II 428
726 111    NDB(I3)=NDB(I3)+1                                       II 429
727 112    CONTINUE                                                II 430
732        IF(NNDW+NDEAD) 130,130,113                              II 431
733 113    IF(NDEAD) 115,115,114                                   II 432
734 114    PDEAD=NDEAD                                             II 433
735        DBARN=SND/PDEAD                                         II 434
736        FDA=PDEAD/PS(1)                                         II 435
737        BAN=PI*SNDD/(576.*PS(1))                                II 436
740        GO TO 1191                                              II 437
741 115    DBARN=0.                                                II 438
742        FDA=0.                                                  II 439
743        BAN=0.                                                  II 440
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ISN      SOURCE STATEMENT

744 1151  IF(*NOW) 116,116,1152                                          II 441
745 1152  FN=NNOW                                                        II 442
746       DBAR=SD/FN                                                     II 443
747       VAR=(SDD-SD*SD/FN)/(FN-1.)                                     II 444
750       SIGMA=SQRT(VAR)                                                II 445
751       DZBAR=SQRT(SDD/FN)                                             II 446
752       BA=PI*SDD/(576.*PS(II))                                        II 447
753       AVAI=BA/A                                                      II 448
754       CAI=(BA-BAST)*.2                                               II 449
755       BAST=BA                                                        II 450
756       FNTA=FN/PS(II)                                                 II 451
757 116   RMA=A*.1*.1                                                    II 452
760       RMA=RMA*10                                                     II 453
761       IF(MA-RMA) 1251,1161,1251                                      II 454
762 1161  WRITE(6,117) MA                                               II 455
763 117   OFORMAT(1H1,57X,9HAGE =I3/1HO,39HD.B.H.O.B. FREQUENCY DISTRIBUTION II 456
          1TABLE/1HO,12X,6HD.B.H.  NO. OF  NO. OF TREES  YR. MORTALITY  YR II 457
          2. MORTALITY/1H ,12X,29HCLASS    TREES      PER AC.,25X,7HPER AC.//) II 458
764       DO 125 I3=1,30                                                 II 459
765       IF(NDB(I3)+NNDB(I3)) 125,125,118                               II 460
766 118   IF(NDB(I3)) 120,120,119                                        II 461
767 119   FNDB=NDB(I3)                                                   II 462
770       FNDAC=FNDB/PS(II)                                              II 463
771       FNNDAC=0.                                                      II 464
772       GO TO 121                                                      II 465
773 120   FNDAC=0.                                                       II 466
774 121   IF(NNDB(I3)) 123,123,122                                       II 467
775 122   FNNDB=NNDB(I3)                                                 II 468
776       FNNDAC=FNNDB/PS(II)                                            II 469
777 123   WRITE(6,124) I3,NDB(I3),FNDAC,NNDB(I3),FNNDAC                  II 470
1000 124  FORMAT(1H ,12X,I4,I9,F12.1,I14,F14.1)                          II 471
1001 125  CONTINUE                                                       II 472
1003      GO TO 1243                                                     II 473
1004 1251 WRITE(6,1252) MA                                               II 474
1005 1252 FORMAT(1H1,57X,9HAGE =I3/)                                     II 475
1006 1253 OWRITE(6,126) NNUM,FNTA,NDEAD,FDA,DBAR,DBARN,VAR,SIGMA,DZBAR,BA,BAI II 476
          1,CAI,AVAI                                                     II 477
1007 126  OFORMAT(1HO,13X,7HTOTAL =I5,F12.1,I14,F14.1/1HO,14X,6HMEAN =F6.2,5HII 478
          1 INS.,15X,F6.2,5H INS./1HO,10X,10HVARIANCE =F9.4/1HO,20HSTANDARD DI II 479
          2EVIATION =F6.2/1HO,20H TREE OF MEAN D.B. =F6.2/1HO,20HBASAL AREA PE II 480
          3ER AC. =F7.1,8H Sq. FT.,11X,F7.1,8H Sq. FT./1HO,20HP.A.I.(BASAL AR II 481
          4EA) =F7.1/1HO,20HP.A.I.(BASAL AREA) =F7.1///)                 II 482
1010      IF(ACOMP) 127,127,1272                                         II 483
1011 127  IF(FLAST-FN) 128,128,1271                                      II 484
1012 1271 ACOMP=A-5.                                                     II 485
1013 1272 X=A-ACOMP                                                      II 486
1014      Y=X-20.                                                        II 487
1015      REDFAC=REDFAC+REDINC*(1.-.01*Y*ABS(Y))                         II 488
1016 128  WRITE(6,129) REDFAC                                           II 489
1017 129  FORMAT(1H ,8HREDFAC =F7.4)                                     II 490
1020      FLAST=FN                                                       II 491
1021      IF(A-ASTOP) 15,130,130                                         II 492
1022 130  CONTINUE                                                       II 493
1024      GO TO 200                                                      II 494
1025      END                                                            II 495
```

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STAND MODEL II - NELSONAM                    FORTRAN SOURCE LIST KFIND
      ISN      SOURCE STATEMENT

        1      FUNCTION KFIND(KM)                                       KFINDG01
        2      DIMENSION S(30),D(20,20)                                 KFINDG02
        3      COMMON REDFAC,A1C,A2C,B1C,B2C,I,J,K,L,M,NMAT,S,THETA,D   KFINDG03
        4      IF(KM) 1,1,2                                             KFINDG04
      > 1      KFIND=KM+NMAT                                            KFINDG05
        4      RETURN                                                   KFINDG06
        7 2    IF(KM-NMAT) 4,4,3                                        KFINDG07
       10 3    KFIND=KM-NMAT                                            KFINDG08
       11      RETURN                                                   KFINDG09
       12 4    KFIND=KM                                                 KFINDG10
       13      RETURN                                                   KFINDG11
       14      END                                                      KFINDG12
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STAND MODEL II - NELSONAM                    FORTRAN SOURCE LIST CROWN
      ISN      SOURCE STATEMENT

        1      SUBROUTINE CROWN                                         CROWNG01
        2      DIMENSION S(30),D(20,20)                                 CROWNG02
        3      COMMON REDFAC,A1C,A2C,B1C,B2C,I,J,K,L,M,NMAT,S,THETA,D   CROWNG03
        4      R=.5*REDFAC                                              CROWNG04
        5      IF(D(I,J)-3.) 1,2,2                                      CROWNG05
        6 1    R1=(A1C+B1C*D(I,J))*R                                    CROWNG06
        7      GO TO 3                                                  CROWNG07
       10 2    R1=(A2C+B2C*D(I,J))*R                                    CROWNG08
       11 3    IF(D(K,L)-3.) 4,5,5                                      CROWNG09
       12 4    R2=(A1C+B1C*D(K,L))*R                                    CROWNG10
       13      GO TO 6                                                  CROWNG11
       14 5    R2=(A2C+B2C*D(K,L))*R                                    CROWNG12
       15 6    IF(R1+R2-S(M)) 8,8,7                                     CROWNG13
       16 7    IF(R1-R2-S(M)) 9,8,8                                     CROWNG14
       17 8    THETA=0.                                                 CROWNG15
       20      RETURN                                                   CROWNG16
       21 9    IF(R2-R1-S(M)) 11,10,10                                  CROWNG17
       22 10   THETA=3.14159                                           CROWNG18
       23      RETURN                                                   CROWNG19
       24 11   X=(R1*R1-R2*R2+S(M)*S(M))/(2.*S(M))                      CROWNG20
       25      Y=SQRT(R1*R1-X*X)                                        CROWNG21
       26      IF(X) 12,13,14                                           CROWNG22
       27 12   X=-X                                                     CROWNG23
       30      THETA=(1.5708D+ATAN(X/Y))*R2/R1                          CROWNG24
       31      RETURN                                                   CROWNG25
       32 13   THETA=1.5708D*R2/R1                                      CROWNG26
       33      RETURN                                                   CROWNG27
       34 14   THETA=ATAN(Y/X)*R2/R1                                    CROWNG28
       35      RETURN                                                   CROWNG29
       36      END                                                      CROWNG30
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