PION PRODUCTION IN NUCLEAR COLLISIONS
SHUSTER, MALCOLM DAVID
ProQuest Dissertations and Theses; 1971; ProQuest Dissertations & Theses Global

72-1678

SHUSTER, Malcolm David, 1943-
PION PRODUCTION IN NUCLEAR
COLLISIONS.

University of Maryland, Ph.D., 1971
Physics, nuclear

University Microfilms, A XEROX Company, Ann Arbor, Michigan

© 1971

Malcolm David Shuster

ALL RIGHTS RESERVED

THIS DISSERTATION HAS BEEN MICROFILMED EXACTLY AS RECEIVED

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

PION PRODUCTION IN

NUCLEAR COLLISIONS

by

Malcolm David Shuster

Dissertation submitted to the Faculty of the Graduate School
of the University of Maryland in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
1971

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

APPROVAL SHEET

Title of Thesis: PION PRODUCTION IN NUCLEAR COLLISIONS

Name of Candidate: Malcolm David Shuster
Doctor of Philosophy, 1971

Thesis and Abstract Approved: Carl A. Levinson

Carl A. Levinson
Professor of Physics
Department of Physics and Astronomy

Date approved: Sept. 16, 1970

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

PLEASE NOTE:

Some Pages have indistinct print. Filmed as received.

UNIVERSITY MICROFILMS

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

ABSTRACT

Title of Thesis: PION PRODUCTION IN NUCLEAR COLLISIONS

Malcolm David Shuster, Doctor of Philosophy, 1971

Thesis directed by: Carl A. Levinson, Professor of Physics

Current Algebra and the equivalence of the pion field and the divergence of the axial-vector current on the pion mass shell are applied to the study of pion production in nucleon-nucleus and nucleus-nucleus collisions. In the soft-pion limit for nucleon-nucleus collisions a simple form for the pion-production cross-section is obtained which when averaged over projectile and target spins is proportional to a single off-mass-shell nuclear amplitude. When all final nuclear states are summed the pion production cross-section is found to be proportional to the forward scattering amplitude of the projectile-target system off the energy shell by an amount essentially equal to the pion energy. Non-relativistic dispersion relations are used to continue the production amplitude back to the pion mass shell. The physical amplitude is shown to be given largely by the DWBA approximation with an effective potential for pion production. This potential is applied to the study of pion production in nucleus-nucleus collisions. Expressions are derived for the pion-production cross-section which turn out to be sensitive to the real well depth of the nuclear optical well. Some numerical results and a comparison with experiment are presented.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Myself when young did eagerly frequent

Doctor and Saint, and heard great Argument

About it and about: but evermore

Came out by the same Door as in I went.

--Edward Fitzgerald (1809-1883)

(after a quatrain by Omar Khayyám

(ob. ca. 1123))

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

PREFACE

A sufficiently large amount of insight into the physics of the nucleus has been gained from the study of the interactions of elementary particles with nuclei that it should be unnecessary to have to plead very strongly for the study of yet another exotic or semi-exotic reaction. But interestingly enough very little experimental work has been done on pion production in nuclei.

There are perhaps several reasons for this. At the experimental end, people with an interest in nuclei generally do not have access to machines able to excite particles above the pion production threshold. Conversely, the people with the machines seldom show any interest in nuclei. From the theoretical standpoint too, there was little reason to study the reaction since a basic understanding of the pion-nucleon interaction had come only recently. But the theoretical and experimental situations are changing and with the building of the meson factory at Los Alamos and several intermediate energy machines such as the cyclotrons at the University of Maryland and the University of Indiana there should become available in the near future a wealth of information on the interaction of pions with nuclei and certainly of pion production.

The great utility of pion production as a probe of nuclear structure lies in the pions substantial mass of 140 MeV. Thus pion production provides information on very highly off-mass-shell nuclear matrix elements. Thus we may hope using pion production to distinguish between various phenomenological potentials--we refer especially to the very

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

ambiguous choice of parameters for the nucleus-nucleus optical potential --which give identical results on the mass shell. Before discussing these in greater detail we review the theoretical developments leading up to the present work.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

ACKNOWLEDGEMENTS

The author wishes to express his heartfelt thanks and appreciation to his thesis supervisor, Carl Levinson, and to Manoj Banerjee, who together directed his research efforts with much thoughtfulness and good humor. During the period of his research he profited from conversations with Wayne Huang, Joe Redish, and Dean Zollman. Sandy Wall and Howel Pugh often kept the author informed of the experimental situation and otherwise in touch with reality. The author received a great deal of advice and assistance in the numerical calculations from Jerry Lerner, who certainly had better ways to spend his time. These computations were performed on an IBM 360 model 44 computer make available through the generosity of the University of Maryland Cyclotron Facility. The manuscript was typed largely by Miss Susan Kaplan, Mrs. Doree Day, and Mrs. Wendy Magi.

During his researches the author was supported by funds supplied through the United States Atomic Energy Commission.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

TABLE OF CONTENTS

Chapter	Page
PREFACE	iii
ACKNOWLEDGEMENTS	v
HISTORICO-PRACTICAL INTRODUCTION	1
I. NUCLEON-NUCLEUS COLLISIONS I	14
II. NUCLEON-NUCLEUS COLLISIONS II	32
III. NUCLEUS-NUCLEUS COLLISIONS	45
IV. NUMERICAL RESULTS	58
V. CONCLUSIONS	67
APPENDIX A. CONVENTIONS	69
APPENDIX B. THE AXIAL-VECTOR CURRENT	72
APPENDIX C. THE PRODUCTION POTENTIAL	77
REFERENCES	80
FIGURE CAPTIONS	83
FIGURES	84

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

LIST OF FIGURES

Figure — Page

1. Kinematic variables for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$.....
2. Contributions to the soft-pion limit.....
3. Composite emission diagrams.....
4. The optical potential.....
5. Total pion-production cross-sections for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$.....
6. Energy differential cross-section for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$.....
7. Angular differential cross-section for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$.....
8. Total pion-production cross-sections for several projectile on Cu^{65}.....
9. Distorted and undistorted wave functions in the optical well.....
10. Total pion-production cross-sections for the reaction alpha + $C^{12} \rightarrow \pi^\circ + ?$ for several choices of the optical well.....

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

HISTORICO-PRACTICAL INTRODUCTION

Understandably, following the spectacular success of renormalized lagrangian quantum field theory in describing relativistic electrodynamics, the first attention given to the pion-nucleon interaction was to determine the nature of the interaction lagrangian.

The desire for mathematical simplicity narrowed the choice to two forms, a pseudoscalar coupling,

$$\mathcal{L}_{ps}(x) = -g_r(\bar{\psi}(x)\gamma_5\vec{\tau}\psi(x))\cdot\vec{\phi}(x)$$

and a pseudovector coupling,

$$\mathcal{L}_{pv}(x) = -g_r^{ii}(\bar{\psi}(x)\gamma_\mu\gamma_5\vec{\tau}\psi(x))\cdot\partial^\mu\vec{\phi}(x)$$

with $\psi(x)$, the nucleon field, $\vec{\phi}(x)$ the pion field, and $\vec{\tau}$ the nucleon isospin operator.

From the first, certain unpleasantnesses were associated with the pseudovector coupling, of which the lack of renormalizability was the most obvious, and so the pseudoscalar coupling gained the wider acceptance. The choice was unmeaningful, however, since the coupling constant was very large ($g_r^2/4\pi = 15$, as compared to $e^2/4\pi = 1/137$ for electrodynamics). A perturbation expansion in the fashion of quantum electrodynamics was, therefore, unthinkable, not that attempts weren't made, of course, and the further study of the strong interaction, of which the pion-nucleon interaction was the most accessable part, was channeled towards regimes which avoided the direct solution of the equations of motion, namely, the study of higher symmetries and dispersion relations.[1]

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

From the practical standpoint the weak interaction of nucleons was better understood. The beta-decay process in nuclei owned a greater mass of information, which, along with the measurement of the neutrino helicity and the study of the beta-decay of polarized nucleons in the late 1950's, unambiguously established the form of the weak interaction known as the V - A theory [2]. The effective lagrangian was determined to be the bilinear product of weak currents, which were the sum (earlier the difference, hence, V "minus" A) of the vector and axial-vector currents, each of which contained hadron and lepton pieces [3]. Thus, the weak decay of the neutron was described by the amplitude

$$\langle pe\bar{\nu}_e|\mathcal{A}(0)|n\rangle = \frac{G}{\sqrt{2}}\langle e\bar{\nu}_e|J^{\mu *}(0)|0\rangle\langle p|J_\mu^+(0)|n\rangle$$

$$= \frac{G}{\sqrt{2}}\left(\bar{u}(e)\gamma^\mu(1+\gamma_5)u(\bar{\nu}_e)\right)\langle p|V_\mu^+(0)+A_\mu^+(0)|n\rangle$$

and, for the small momentum transfers present in beta-decay,

$$\langle p|V_\mu^+(0)|n\rangle \simeq g_V\,\bar{u}(p)\gamma_\mu u(n)$$

$$\langle p|A_\mu^+(0)|n\rangle \simeq g_A\,\bar{u}(p)\gamma_\mu\gamma_5 u(n)$$

With the scale set by the leptonic weak current g_V and g_A have the values $1.00 \pm .02$ and $1.18 \pm .03$, respectively. Since the coupling in beta-decay is very small ($G \simeq 10^{-5}/M^2_{\text{nucleon}}$) one calculates confidently only to first order sobered perhaps by the knowledge that the theory is unrenormalizable.

Most remarkable was the discovery that g_v was unity i.e., that there was no renormalization due to the strong

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

interaction of the nucleon vector coupling constant. This phenomenon was known to occur for one other coupling constant, the electric charge, which is the same for both leptons and hadrons. This lack of renormalization of the electric charge due to the strong interaction was known to be a result of the conservation of the electromagnetic current. Thus it was postulated that the weak vector current was conserved.

Further, it was pointed out that the nucleon weak vector current belonged to an isospin triplet of which there was no known third component while the isovector piece of the electromagnetic current was supposedly the third component of an isospin triplet of which the other two components remained to be discovered. It was tempting, therefore, to combine these three components into the same isospin triplet. This, together with the previous remarks, constitute the hypothesis of the conserved vector current (C.V.C.) [4,5].

Furthermore, the charges of the three isovector components of the non-strangeness-changing vector current (just discussed), along with the strangeness (the charge corresponding to the isoscalar piece of the electromagnetic current) and the charges of the four strangeness-changing components of the vector current(not discussed) generate the algebra SU(3), which was observed some years before to be an approximate symmetry group of the strong interaction (usually called unitary symmetry) [6]. Mathematically speaking, defining these eight vector charges according to

$$Q^i(t) = \int d^3\vec{x}\, V_0^i(t,\vec{x}) , \qquad i = 1,\dots,8$$

they staisfy the equal-time commutation relations

$$[Q^i(t), Q^j(t)] = i f^{ijk} Q^k(t) \qquad i,j = 1,\dots,8$$

with f^{ijk} the (completely antisymmetric) SU(3) structure tensor. For

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

the isospin triplet of non-strangeness-changing charges, this amounts to

$$[Q^i(t), Q^j(t)] = i\epsilon^{ijk} Q^k(t) \qquad i,j = 1,2,3$$

with ϵ^{ijk} the Levi-Civita tensor, which are the familiar commutation relations for the nucleon isotopic charges $T^i(t)$

$$T^i(t) = \int d^3x\, \bar{\psi}(x) \gamma_0 \frac{\tau^i}{2} \psi(x) \quad , \quad i = 1,2,3$$

Assuming that not only the charges of the vector current but also the charges of the total hadronic weak current generate an SU(3) algebra, one was led to postulate further that [7]

$$\left.\begin{aligned} [Q^i(t), Q_5^j(t)] &= i f^{ijk} Q_5^k(t) \\ [Q_5^i(t), Q_5^j(t)] &= i f^{ijk} Q^k(t) \end{aligned}\right\} \quad i,j = 1,\ldots,8$$

where the axial-vector charges $Q_5^i(t)$, more commonly called the chirality, are defined as

$$Q_5^i(t) = \int d^3x\, A_0^i(t,x) \quad , \quad i = 1,\ldots,8$$

For i,j = 1,2,3, these commutations have the expected form in terms of the Levi-Civita tensor.

$$\left.\begin{aligned} [Q^i(t), Q_5^j(t)] &= i\epsilon^{ijk} Q_5^k(t) \\ [Q_5^i(t), Q_5^j(t)] &= i\epsilon^{ijk} Q^k(t) \end{aligned}\right\} \quad i,j = 1,2,3$$

The algebra of these three vector and axial-vector charges is simply the SU(2)⊗SU(2) sub-algebra of the higher SU(3)⊗SU(3) algebra of all eight vector and axial-vector charges. In the present work, of course,

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

our interest need not extend beyond SU(2)⊗SU(2). The above relations are what is called Current Algebra.

The small renormalization of the nucleon axial-vector coupling constant (1.18 for the nucleon as opposed to 1.00 for the leptons.) occasioned the belief that the axial-vector current was somehow "partially" conserved. The source of the axial-vector current

$$\mathcal{D}^i(x) = \partial^\mu A^i_\mu(x) \qquad i = 1, 2, 3$$

had identical quantum numbers to those of the pion and so, assuming that the axial-vector form factor satisfied an unsubtracted dispersion relation, one wrote, unmindful of possible anomalous thresholds,

$$\langle \beta | \mathcal{D}^+(0) | \alpha \rangle = \frac{\langle \beta | j^+_\pi(0) | \alpha \rangle \langle \pi^+ | \mathcal{D}^+(0) | 0 \rangle}{m_\pi^2 - q^2} + \frac{1}{\pi} \int_{(3m_\pi)^2}^{\infty} \frac{\rho^+_{\beta\alpha}(x)}{x - q^2}\, dx$$

with $q = p_\beta - p_\alpha$, $\vec{j}_\pi$ the pion current, and $|\pi\rangle$ the one pion state. Experimentally, one knew the value of the pion axial vector coupling constant

$$f_\pi \equiv -i \langle \pi^+ | \mathcal{D}^+(0) | 0 \rangle$$

from the decay width of the pion for the reaction

$$\pi^+ \to \mu^+ + \nu_\mu$$

From

$$\vec{j}_\pi(x) = (\Box + m_\pi^2)\, \vec{\varphi}_\pi(x)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

it followed, assuming that the pion-pole term dominated the dispersion relation, that one could write, if only approximately,

$$D^{+}(x) \simeq i f_{\pi} \varphi_{\pi}^{+}(x)$$

and similarly for the other isotopic components. This was the content of the hypothesis of the partially conserved axial-vector current (P.C.A.C.) and it was assumed to hold for small momentum transfers, $|q^2| \lesssim m_{\pi}^2$, when the dispersive corrections would likely be surpressed by the large masses occuring in the denominator. [8]

Most interesting was the case when $|\alpha\rangle$ and $|\beta\rangle$ were one-nucleon states. One had then for small momentum transfers that

$$\langle p|D^{+}(0)|n\rangle = i\sqrt{2}M g_A \bar{u}(p)\gamma_5 u(n)$$

and

$$\langle p|J_{\mu}^{+}(0)|n\rangle = \sqrt{2}\, g_r(0)\, \bar{u}(p)\gamma_{\mu}\gamma_5 u(n)$$

with $g_r(0)$ the pionic form factor of the nucleon for vanishing momentum transfer. This number is not known experimentally, of course, but $g_r(m_{\pi}^2)$ is just the rationalized and renormalized coupling constant occuring in the pseudoscalar theory of the pion-nucleon interaction. If we assume that this is a slowly varying function of the momentum transfer we are led to a very remarkable equation

$$f_{\pi} = \frac{\sqrt{2}\, M m_{\pi}^2 g_A}{g_r(0)} \simeq \frac{\sqrt{2}\, M m_{\pi}^2 g_A}{g_r}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

relating the coupling constants of seemingly unrelated weak and strong processes. This is the celebrated relation of Goldberger and Treiman[9] first derived from a totally different standpoint and it was found to be correct within ten per cent.

These new ideas, which related the strong interaction to the weak and electromagnetic interactions, could not be expected to pass by without creating new interest in dynamical calculations of strong interaction processes which had for a decade following the failure of lagrangian perturbation theory in providing the means for such calculations become a bête noire of elementary particle physics. The first venture into this area was the work of Nambu and Lurié[10], who noted that the chirality (Q_5^j (t) above) was necessarily conserved if the pion were massless. This meant, considering the obviously related example of a massless photon and a conserved electric charge, that one could calculate the amplitude for the production of (light-like) pions much as one calculated the amplitude for the production of Bremsstrahlung. This was what Nambu and Lurié did with remarkable agreement with experiment for low energy pions.

More important for the present research is the work of Adler[11] who using P.C.A.C. rather than chirality conservation was able to derive a number of theorems related to processes involving soft pions, i.e., pions with vanishing 4-momentum. Especially important was the demonstration that for soft pions only those Feynman diagrams in which the axial-vector current insertion was made on an external leg were non-vanishing. Those for which the axial-vector current replaced the external pion leg on an internal nucleon line vanished identically.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

This was equivalent to the statement that the soft pion couples to the nucleon only through the divergence of the axial-vector current and that for low energy (light-like) pions only the lowest-order Feynman diagrams, namely, the Born terms, contributed. Strangely enough, the effective pion-nucleon coupling in this instance is pseudovector.

The approach of Adler and, especially, of Adler and Dothan [12], was very much akin to the earlier work of F. E. Low [13], who derived very similar results for the emission of soft photons (Bremsstrahlung). The essential result of Low's work was that the Bremsstrahlung amplitude depended only on on-mass-shell nuclear amplitudes as the photon 4-momentum vanished. This meant, unfortunately, that the study of soft nuclear Bremsstrahlung offered no information on off-mass-shell nuclear amplitudes as had been previously hoped and would, presumably, have provided some information on the nuclear interaction. This, not surprisingly, was the outcome of the Adler soft pion theorems also and it could not have been otherwise since in the Adler formalism only the pion was permitted to go off the mass shell. Thus as the pion 4-momentum was taken to zero the 4-momenta of all the other particles were readjusted in order that energy and momentum were still conserved (as guaranteed, of course, by the translational invariance of the S-matrix). This meant that in the soft-pion limit for pion production all nuclear matrix elements were per force between states of the same momentum and energy.

This particular choice of the soft-pion limit was not unique, however, but one argued that different choices of the limit gave equivalent amplitudes to order m_π^2. But this discrepancy could well be

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

large if the particles in the system had low-lying excitations ($M^* - M \lesssim m_\pi$). This consideration was unimportant for elementary particle processes for which Adler's approach was intended. However, when nuclei enter the stage, it is clear that soft-pion theorems must be used with greater caution.

One drawback of the Adler approach was that working within the formalism of lagrangian perturbation theory there was no practical way to calculate the corrections to the soft-pion limit. This need was filled by the work of Fubini and Furlan [14], who investigated the soft-pion limit for pion-nucleon scattering within the framework of dispersion relations. Here, they were able to establish a prescription for choosing the soft-pion limit which most approximated the physical amplitude for some choice of the kinematic variables. In addition they displayed transparently the corrections to the soft pion limit coming from rescattering of the pion and intermediate excitation of the nucleon. These were calculated explicitly in terms of phenomenological scattering lengths and coupling constants.

At the same time it should be pointed out that Fubini and Furlan examined matrix elements of the commutator of two axial-vector-current divergences. This was equivalent to taking both initial and final pions off the mass shell in the soft-pion limit of pion-nucleon scattering as opposed to the earliest soft pion work of Adler in which only one pion was taken off the energy shell. This permitted a greater choice of the soft pion limit and it turned out that the best choice was to let both pions become soft simultaneously.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

The approach of Fubini and Furlan is impractical for processes involving nuclei because the number of excited states of the constituents of the reaction is forbiddingly large. It remained for M. Ericson, Figureau and Molinari[15] in studying pion-nucleus scattering to note the simularity of the dispersion relation of Fubini and Furlan to the non-relativistic dispersion relation for pion-nucleus scattering in potential theory. The soft-pion limit was then identified with the Born term and the dispersive corrections with the rescattering terms. The soft-pion limit, in fact, agreed with the Born approximation using the charge-exchange potential for pion-nucleus scattering derived from the study of pionic atoms.[16] Most important, however, was the realization that the chief correction to the soft-pion limit was the inclusion of rescattering corrections, i.e., the distortion of the initial and final states.

In the present work we begin with the formalism of Fubini and Furlan and construct the soft-pion amplitude for pion production in nucleon-nucleus collisions. Like the case of pion-nucleon and pion-nucleus scattering we reduce two particles from the S-matrix element describing the process but since there is no initial pion we reduce the final pion and the incoming nucleon. In the limit of vanishing pion 4-momentum we obtain the equal-time-commutation-relation and pole terms of these authors. These are similar to the external axial-vector-current insertions of Adler but since we have reduced two particles from the S-matrix rather than one we may keep all momenta fixed. Thus, as the pion momentum vanishes the nucleon line must go off the mass shell to compensate for the missing pion 4-momentum. (This is appealing

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

from the standpoint of perturbation theory since the incoming nucleon certainly goes off the mass shell after emitting a pion in that formalism.)

Especially advantageous about this method is that no assumption is made that the pion field and the divergence of the axial-vector current are identical (P.C.A.C.). It is only assumed that these are equivalent operators on the mass shell (which is rigorously true) and that the transition from "soft" to "hard" matrix elements of the axial-vector current can be effected so that one eventually recovers the physical amplitude. It turns out that the commutation relations used are those of the σ-model (in which, in fact, P.C.A.C. holds) but no model-dependent assumption is made, however, except, perhaps, that of the linear realization of the equal-time commutation relation of the chirality with the nucleon field.

The soft-pion amplitude provides an approximation to the physical production amplitude. We discuss this in the approximation that only the nucleon is allowed to emit a pion and obtain an "optical" theorem for the production cross-section relating it directly to an off-shell nuclear matrix element, which can be identified with the nuclear optical transition matrix element. This is the essential content of Chapter I.

In Chapter II we study the relation of the soft-pion limit to the physical amplitude. We recast the dispersion relation of Fubini and Furlan in the form of the non-relativistic Lippmann-Schwinger equation by a method described previously.[17] Keeping only all intermediate states of the (A+1)-nucleon system we recover the distorted-wave Born

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

approximation and the effective potential for pion production. The relation between the hard- and soft-pion limits is shown to amount to the distortion of the initial state or the lack thereof.

In Chapter III we apply this effective potential for pion production to the calculation of pion production cross-sections in nucleus-nucleus collisions. The production amplitude is separated into "external" and "internal" emission pieces, corresponding, respectively, to emission generated by the center-of-mass motion of the projectile and emission generated by the Fermi motion of the projectile. An optical theorem is obtained again for the production cross-section and the difficulty of relating this cross-section to an optical potential is discussed. Expressions for the pion-production cross-section in nucleus-nucleus collisions are given in two approximations which are the same near threshold.

Numerical calculations are presented in Chapter IV. The pion production cross-section for proton-C^{12} collisions as calculated within the framework of Current Algebra agrees fairly well with experimental data and is not very sensitive to details of the nucleon-nucleus optical potential. For nucleus-nucleus collisions there are unfortuneately no data available for comparison and it is observed that the calculated cross-sections vary widely for different possible choices of the parameters of the nucleus-nucleus optical potential.

Conclusions and final discussion are presented in Chapter V.

Appendix A presents the conventions used in this work.

Appendix B provides a derivation of the commutation relations used in the text, which have not yet appeared in the literature.

Appendix C discusses the problem of galilean invariance of the

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Production potential. Our discussion here is mostly prescriptive.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

CHAPTER I

Nucleon-Nucleus Collisions I

We begin by studying the reaction

$$N^a(p,s) + Z^A \rightarrow \pi^b(q) + f \qquad (1.1)$$

in which a nucleon of 4-momentum p and spin s collides with a nucleus Z^A to produce a pion of 4-momentum q and a collection of particles f. a and b are the isospin components of the nucleon and pion, respectively.[18] Except for the constraints imposed by the conservation laws f is arbitrary and may include, among other particles, additional pions. Generally we will suppress the spin-or isospin-quantum number of the nucleon. The initial state will often be denoted by a single index i. The reaction is illustrated kinematically in Figure 1.

The S-matrix element describing this process,

$$\langle f\,\pi^b(\text{out}) \,|\, (p,s)^a Z^A(\text{in})\rangle$$

$$= -i(2\pi)^4 \delta^{(4)}(p_f + q - p - p_A)\, \mathcal{M}(i \rightarrow \pi^b f) \qquad (1.2)$$

defines the Lorentz-invariant amplitude $\mathcal{M}(i \rightarrow \pi^b f)$ given by [19]

$$\langle f\,\pi^b(\text{out}) \,|\, J(0) \,|\, Z^A\rangle\, u_a(p,s)$$

Here, $\bar{j}(0) = \bar{\Psi}(x)\,(-i\overleftarrow{\not\partial}_x - M)$ is the nucleon current and $\bar{\Psi}(x)$ the nucleon field. $u_a(p,s)$ is the (eight-component) spinor for the nucleon. The normalization of the nucleon and other fields is given in Appendix A. In deleting the specification (in) from the ket $|Z^A\rangle$ it has been assumed that the target nucleus is in its ground state.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Further reducing the amplitude we obtain

$$\mathcal{M}(i \to \pi^b f) = -i \int d^4x \, e^{i\delta \cdot x} \, (\Box_x + m_\pi^2)$$

$$\cdot \langle f(\text{out}) | R(\varphi^b(x) J(0)) | i^A \rangle u(p,s) \qquad (1.3)$$

with $\phi^b(x)$ the pion field and R the causal commutator. For physical pions ($q^2 = m_\pi^2$) the pion field in the amplitude above may be replaced by the divergence of the axial-vector current according to [20]

$$\frac{\partial A_\mu^b(x)}{\partial x_\mu} = i \frac{f_\pi^b}{\sqrt{2}} \varphi^b(x) \qquad (1.4)$$

with f_π^b the pion-axial-vector coupling constant and b the isopin. This leads us to define the quantity

$$F^b(q) = -\frac{\sqrt{2}}{f_\pi^b} \int d^4x \, e^{i\delta \cdot x} (\Box_x + m_\pi^2)$$

$$\cdot \langle f(\text{out}) | R(\partial_\mu A_\mu^b(x) J(0)) | i^A \rangle u(p,s) \qquad (1.5a)$$

which assumes the value of the pion-production amplitude for physical values of q, and in analogy to the work of Fubini and Furlan [14] the quantities:

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$M_\mu^b(q) = i\frac{\sqrt{2}}{f_\pi^b}\int d^4x\, e^{iq\cdot x}(\Box_x + m_\pi^2)\langle f(out)|R(A_\mu^b(x)J(0))|Z^A\rangle u(p,s) \quad (1.6a)$$

$$R^b(q) = \frac{\sqrt{2}}{f_\pi^b}\int d^4x\, e^{iq\cdot x}(\Box_x + m_\pi^2)\langle f(out)|[A_0^b(x), J(0)]\delta(x_0)|Z^A\rangle u(p,s) \quad (1.6b)$$

These three quantities satisfy the generalized Ward's identity

$$F^b(q) = R^b(q) + q^\mu M_\mu^b(q) \quad (1.7)$$

for all values of q.

It is valuable to study the three quantities above in the frame in which the spatial components of q vanish. In this frame $F^b(q)$, $M^b(q)$, and $R^b(q)$ become

$$F^b(q_0) = -\frac{\sqrt{2}}{f_\pi^b}(m_\pi^2 - q_0^2)\int d^4x\, e^{iq_0x_0}\langle f(out)|R(\partial_\mu A_\mu^b(x)J(0))|Z^A\rangle u(p,s) \quad (1.8a)$$

$$M_\mu^b(q_0) = i\frac{\sqrt{2}}{f_\pi^b}(m_\pi^2 - q_0^2)\int d^4x\, e^{iq_0x_0}\langle f(out)|R(A_\mu^b(x)J(0))|Z^A\rangle u(p,s) \quad (1.8b)$$

$$R^b(q_0) = -\frac{\sqrt{2}}{f_\pi^b}(m_\pi^2 - q_0^2)\int d^4x\, \langle f(out)|[A_0^b(x), J(0)]\delta(x_0)|Z^A\rangle u(p,s) \quad (1.8c)$$

Equation (1.7) permits the evaluation of $F^b(q_0)$ when $q_0 = 0$. Explicitly,

$$F^b(0) = R^b(0) + \lim_{q_0\to 0} q_0 M_0^b(q_0) \quad (1.9)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

We note immediately that $R^b(0)$ has the value

$$-\frac{\sqrt{2}}{f_\pi^b} m_\pi^2 \int d^4x \langle f(out) | [A_o^b(x), J(0)] \delta(x_o) | Z^A \rangle u(p,s) \qquad (1.10)$$

The equal-time commutator is usually taken to be

$$[A_o^b(x), J(0)] \delta(x_o) = (2M\bar{\psi}(0) + J(0)) \gamma_5 \frac{\tau^b}{2} \delta^{(4)}(x) \qquad (1.11)$$

apart from Schwinger terms, which do not contribute to $R^b(0)$. This commutation relation, which follows immediately from the σ-model[21], has been studied by a number of authors[22-24]. In particular, Banerjee and Levinson[25], who have shown that its validity is somewhat more general than that of the σ-model, have been able using it to derive the pion-nucleon scattering lengths of Weinberg[26].

Inserting equation (1.11) into equation (1.10) we obtain

$$R^b(0) = -\frac{\sqrt{2}}{f_\pi^b} m_\pi^2 \langle f(out) | 2M\bar{\psi}(0) + J(0) | Z^A \rangle \gamma_5 \frac{\tau^b}{2} u(p,s)$$

$$= -\frac{\sqrt{2}}{f_\pi^b} m_\pi^2 \langle f(out) | J(0) | Z^A \rangle \frac{\not{p}_f - \not{p}_A + M}{\not{p}_f - \not{p}_A - M} \gamma_5 \frac{\tau^b}{2} u(p,s) \qquad (1.12)$$

The contribution to $F^b(0)$ from the second term in equation (1.7) is more complicated. Inserting a complete set of states in equation (1.8b) we have

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$M_o^b(q_o) = \frac{(2\pi)^3\sqrt{2}\,(m_\pi^2 - q_o^2)}{f_\pi^b} \sum_{|n\rangle} \Big\{ \langle f(out)|A_o^b(0)|n\rangle\langle n|J(0)|Z^A\rangle\, u(p,s)$$

$$\cdot \frac{\delta^{(3)}(\vec{p}_f - \vec{p}_n)}{(p_f + q - p_n)_o + i\eta} - \langle f(out)|J(0)|n\rangle\, u(p,s)\langle n|A_o^b(0)|Z^A\rangle \frac{\delta^{(3)}(\vec{p}_n - \vec{p}_A)}{(p_n + q - p_A)_o + i\eta} \Big\} \quad (1.13)$$

Only those terms in the braces contribute to $F^b(0)$ for which the denominator vanishes as $q_o \longrightarrow 0$, i.e., only if $p_{no} = p_{fo}$ in the first line and $p_{no} = p_{Ao}$ in the second line.

In the second line of equation (1.13) only the ground state of the target can contribute among the intermediate states. Explicitly, its contribution is

$$\lim_{q_o \to 0} q_o M_o^{b,\pi}(q_o) = -\frac{\sqrt{2}}{f_\pi^b} m_\pi^2 \overline{\langle f(out)|J(0)|(Z'^{A_1})'\rangle}\, u(p,s)\langle (Z'^{A_1})'|A_o^b(0)|Z^A\rangle \quad (1.14)$$

where the vinculum in equation (1.14) denotes a connected matrix element. Z' is determined by the charge of the meson; the other prime in the intermediate state denotes a different spin quantum number from Z^A which we understand to be summed. We postpone discussion of the selection rules arising from the axial-vector matrix element in equation (1.14). We note, however, that $\langle (Z'^A)'|A_\mu^b(0)|Z^A\rangle$ is an axial vector and, therefore, must be proportional to the spin 4-vector of the target. Hence the right member of equation (1.14) vanishes trivially if the target has spin zero. We will, in any case, always argue that its value is usually small compared to the contribution from $R^b(0)$,

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

if A is large.

The number of terms contributing to the first line of equation (1.13) will be very large in general since f may be a very complicated collection of particles. In addition, the number of permissable final states increases exponentially with increasing A. Writing $|f(\text{out})\rangle$ explicitly as $|f_1 f_2 \ldots f_n \ldots (\text{out})\rangle$, where f_n is a particular particle in the final state (nucleon, nucleus, meson, etc.) the contribution of the first line of equation (1.13) becomes

$$\lim_{q_0 \to 0} q_0 M_0^{b;I}(\vec{q}, q_0) = \sum_n \frac{-\sqrt{2}\, m_\pi^2}{f_\pi^b} \langle f_n | A_0^b(0) | f_n' \rangle \cdot (-)^n \overline{\langle f_1 \ldots f_n' \ldots (\text{out}) | J^I(0) | Z^A \rangle} u(p,s) \qquad (1.15)$$

where the sum is restricted to final nucleons and nuclei only since diagonal matrix elements of $A_0^b(0)$ vanish between pion states. Again the primes refer to possible spin degrees of freedom, which are always understood to be summed implicitly; as before, $p_{fn} = p_{fn}'$.

We write the complete expression for $F^b(0)$ as

$$F^b(0) = \frac{-\sqrt{2}\, m_\pi^2}{f_\pi^b} \Big\{ \langle f(\text{out}) | J^I(0) | Z^A \rangle \frac{\not{p}_f - \not{p}_i + M}{p_f^2 - p_i^2 - M^2} \gamma_5 \frac{\tau^b}{2} u(p,s) + \overline{\langle f(\text{out}) | J^I(0) | (Z^A)' \rangle} u(p,s) \langle (Z^A)' | A_0^b(0) | Z^A \rangle + \sum_n (-)^n \langle f_n | A_0^b(0) | f_n' \rangle \overline{\langle f_1 \ldots f_n' \ldots (\text{out}) | J^I(0) | Z^A \rangle} u(p,s) \Big\} \qquad (1.16)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Diagramatically, we depict the divers contributions to equation (1.16) in figure 2.

Equation (1.16) gives the value of $F^b(0)$, the physical amplitude being $F^b(m_\pi)$. The difference between these two quantities, which must be calculated in some fashion different from the above, is needed in order to predict the pion-production amplitude. It is well to ask to what extent $F^b(0)$ approximates $F^b(m_\pi)$. The adequacy of this approximation in a much different regime has been the central theme of the soft-pion theorists[8] and for interactions involving only elementary particles this assumption seems justified as evidenced in part by the validity (with 10%) of the famous Goldberger-Treiman relation. For processes involving composite particles (or more accurately stated, for processes in which there are anomalous thresholds present the assumption of the very slow extrapolation of $F(m_\pi)$ to $F(0)$ is no longer tenable. The difficulty is already present to a small degree in the reaction $N\bar{N} \to \pi f$ [17].

A completely relativistic formulation for processes involving composite particles does not at present exist, at least not within a practical calculational framework.[27] For the moment we postpone discussion of the contributions from composite particles until a later chapter when we will have constructed a non-relativistic theory of pion production. For the moment, we will simply exclude such terms from $F^b(0)$ without justification.

Still another unpleasantness is that the contributions from

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$\lim_{q_0 \to 0} q_0 M_0^b(q_0)$$

contain diagonal matrix elements of the axial-vector current while that of $R^b(0)$ does not. At the same time the internal nucleon in the diagram depicting $R^b(0)$ is not on the mass shell while the internal particles in the other diagrams are on the mass shell. The treatment is thus un-symmetrical. It has been shown, in the detailed study of the reaction $NN \to NN\pi$[27] that this lack of symmetry is small and the contribution from $R^b(0)$ may be recast into a form which is identical to the other contributions. For reactions in which the target and projectile are not identical there is no obvious advantage in doing this and the present form is in fact more convenient.

It is instructive to examine the structure of the pion production cross-section as given by the contribution $R^b(0)$ alone. We shall see in succeeding chapters that this is the dominant contribution to the cross-section. For simplicity we study only the production of neutral pions. In this approximation the production amplitude becomes

$$\mathcal{M}(i \to f\pi^0) \simeq -\frac{\sqrt{2}}{f_\pi} m_\pi^2 \langle f(out)|J(0)|i^A\rangle \frac{\not{p}_2 - \not{p}_A + M}{p_2^2 - p_A^2 - M^2} \gamma_5 \frac{\tau^0}{2} u(p,s)$$

$$= -\frac{\sqrt{2}}{f_\pi} m_\pi^2 \langle f(out)|J(0)|i^A\rangle \frac{\not{p} - \not{q} + M}{p^2 - q^2 - M^2} \gamma_5 \frac{\tau^0}{2} u(p,s) \qquad (1.17)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where now q will denote exclusively the physical pion-momentum ($q = (m_\pi, 0, 0, 0)$) and we have used energy-momentum conservation in writing $p_f - p_A = p - q$. Rationalizing equation (1.17) gives

$$\mathcal{M}(i \to f\pi^0) = -\frac{\sqrt{2}}{f_\pi} m_\pi^2 \langle f(\text{out}) | J_\alpha(0) | i^A \rangle \frac{\not{p} - \not{q} + M}{2p\cdot q - m_\pi^2} \not{q} \gamma_5 \frac{\tau^0}{2} u(p,s) \qquad (1.18)$$

Using the algebraic identity

$$\frac{\not{p} - \not{q} + M}{2p\cdot q - m_\pi^2} \not{q} = \frac{\not{p} + M}{2p\cdot q} \not{q} + \frac{m_\pi^2}{2p\cdot q - m_\pi^2} \left[\frac{(\not{p} + M)}{2p\cdot q} \not{q} - 1 \right] \qquad (1.18\tfrac{1}{2})$$

and noting that the contribution from the second term is of order $m_\pi^2/2M^2$ (= .01) compared to that of the first we write

$$\mathcal{M}(i \to f\pi^0) \simeq -\frac{\sqrt{2}}{f_\pi} m_\pi^2 \langle f(\text{out}) | J_\alpha(0) | i^A \rangle \frac{\not{p} + M}{2p\cdot q} \not{q} \gamma_5 \frac{\tau^0}{2} u(p,s) \qquad (1.19)$$

The differential production cross-section when only the final pion is observed is

$$\frac{d^4\sigma^{\pi^0}}{dq^4} = \frac{1}{(2\pi)^3} \delta^{(+)}(q^2 - m_\pi^2) \left[\frac{M}{P_a} \frac{1}{2p_{cm}} \right] \frac{1}{|\vec{v}_p - \vec{v}_A|} \cdot \sum_{|f\rangle} (2\pi)^4 \delta^{(+)}(p + p_A - q - p_f) |\mathcal{M}(i \to f\pi)|^2 \qquad (1.20)$$

where $\vec{v}_p$ and $\vec{v}_A$ are the velocities of the projectile and target, respectively, and $\delta^{(+)}(q^2 - m^2) = \delta(q^2 - m^2)\theta(q_o)$. If we average over the projectile spin we find that

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$\frac{1}{2}\sum_s |\mathcal{M}(i\to f\pi)|^2$$

$$= \frac{m_\pi^4}{2f_\pi^2}\langle f(out)|J(0)|Z^A\rangle \frac{\not p+M}{2p\cdot q}\not q \frac{\not p-M}{2M}\not q \frac{\not p+M}{2p\cdot q}\langle Z^A|J(0)|f(out)\rangle \quad (1.21)$$

Using now the algebraic identity

$$\frac{\not p+M}{2p\cdot q}\not q \frac{\not p-M}{2M}\not q \frac{\not p+M}{2p\cdot q} = \frac{(p\cdot q)^2-M^2m_\pi^2}{(p\cdot q)^2}\frac{\not p+M}{2M} \quad (1.22)$$

we obtain the useful result

$$\frac{1}{2}\sum_s |\mathcal{M}((p,s)Z^A\to \pi^0 f)|^2$$

$$= \frac{m_\pi^4}{2f_\pi^2}\frac{(p\cdot q)^2-M^2m_\pi^2}{(p\cdot q)^2}\frac{1}{2}\sum_s |\mathcal{M}((p,s)Z^A\to f)|^2 \quad (1.23)$$

where

$$\mathcal{M}((p,s)Z^A\to f) = \langle f(out)|J(0)|Z^A\rangle u(p,s) \quad (1.24)$$

is, except that f may contain other pions, a purely "nuclear" amplitude. Thus, averaging equation (1.20) over the projectile spin we have

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$\left\langle \frac{d^4\sigma^{\pi^0}}{dq^4} \right\rangle = \frac{m_\pi^4}{2f_\pi^2} \frac{(P\cdot q)^2 - M^2 m_\pi^2}{(P\cdot q)^2} \frac{\delta^{(+)}(q^2 - m_\pi^2)}{(2\pi)^3} \left[\frac{M}{P_0} \frac{1}{2P_{A0}}\right]$$

$$\cdot \frac{1}{|\vec{v}_p - \vec{v}_A|} \sum_{|f\rangle} (2\pi)^4 \delta^{(4)}(P + P_A - q - P_f) \left\langle |\mathcal{M}(i \to f)|^2 \right\rangle \qquad (1.25)$$

where the brackets denote spin averaging. Note that although the entire derivation has proceeded in the rest frame of the pion, equation (1.25) is manifestly Lorentz-invariant.

The factor

$$\frac{(P\cdot q)^2 - M^2 m_\pi^2}{(P\cdot q)^2}$$

is the square of the relative velocity of the pion and the projectile, which is just the projectile velocity in the pion's rest frame. If we examine the other contributions to $F^0(0)$ we see that these are also linear in the velocity of the particle for which the axial-vector-current insertion has been made. Thus at threshold, certainly, $R^0(0)$ makes the dominant contribution to $F^0(0)$.

To evaluate equation (1.25) we define a "non-relativistic" interaction U according to

$$\langle f^{(-)} | U | i \rangle = (2\pi)^3 \delta^{(3)}(\vec{p}_f - \vec{p}_i) \langle f(out) | J_0(0) | Z^A \rangle u(p,s) \qquad (1.26)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where round brackets denote non-relativistic matrix elements. Because the matrix element of the right member of equation (1.26) is Lorentz-invariant, U does not transform as a scalar under Galilean transformations but rather in some very complicated fashion to insure that the non-relativistic amplitude as calculated from the Schrodinger equation is Lorentz-invariant. This means that if U is known in one frame, say the laboratory frame, than it is not known in other frames. Note that the pion must always be treated as a relativistic particle, i.e. $q_o^2 - |\vec{q}|^2 = m_\pi^2$. Substituting equation (1.26) into equation (1.25) we have suppressing the spin averaging

$$\frac{d^4\sigma^{\pi^0}}{d\Omega^4} = \frac{\delta^{(+)}(\xi^2-m_\pi^2)}{(2\pi)^3}\,\frac{m_\pi^4}{2f_\pi^2}\,\frac{(P\cdot\xi)^2-M^2m_\pi^2}{(P\cdot\xi)^2}\,\frac{1}{|\vec{v}_p-\vec{v}_A|}$$
$$\cdot\sum_{(f)}(2\pi)^4\delta^{(4)}(P+P_A-\xi-P_f)\,|(f^{(-)}|U|i)|^2 \qquad (1.27)$$

or, more compactly,

$$\frac{\partial^2\sigma^{\pi^0}}{\partial\xi_0\partial\Omega_\xi} = \frac{|\vec{\xi}|}{2(2\pi)^3}\,\frac{m_\pi^4}{2f_\pi^2}\,|\vec{V}_{p\pi}|^2\,\frac{1}{|\vec{v}_p-\vec{v}_A|}$$
$$\sum_{(f)}(2\pi)^4\delta^{(4)}(P+P_A-\xi-P_f)\,|(f^{(-)}|U|i)|^2 \qquad (1.28)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where we have performed the integration over the magnitude of the pion 3-momentum, $|\vec{q}|$, to eliminate the δ-function in the pion phase-space factor. The momentum conserving δ-function in equation (1.28) may be integrated immediately to give

$$\frac{\partial^2\sigma^{\pi^0}}{\partial\theta_\pi\partial\Omega_\pi} = \frac{|\vec{q}|}{2(2\pi)^3}\frac{m_\pi^4}{2f_\pi^2}|\vec{V}_{p\pi}|^2\frac{1}{|\vec{V}_p-\vec{V}_A|} \cdot\sum_{(f)}(2\pi)\delta(E_p+E_A-m_\pi-E_f)|(f^{(-)}||U||i)|^2 \qquad (1.29)$$

where the double brackets, (|| ||), denote a relative matrix element. The energies in the remaining δ-function are total (i.e. c.m. plus relative) energies. Using the identity

$$\frac{1}{x+i\eta} = \frac{P}{x} - i\pi\delta(x) \qquad (1.30)$$

where P means principal value we may use closure to sum over the final states in equation (1.29) and obtain

$$\frac{\partial^2\sigma^{\pi^0}}{\partial\theta_\pi\partial\Omega_\pi} = -\frac{|\vec{q}|}{2(2\pi)^3}\frac{m_\pi^2}{2f_\pi^2}|\vec{V}_{p\pi}|^2\frac{1}{|\vec{V}_p-\vec{V}_A|} \cdot 2\,\mathrm{Im}\left(i\|U\frac{1}{E_p+E_A-m_\pi-H+i\eta}U\|i\right) \qquad (1.31)$$

where H is the total Hamiltonian operator. Since U is hermitian by explicit construction we may rewrite equation (1.31) as

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$\frac{\partial^2\sigma^{\pi^0}}{\partial\varepsilon_0\partial\Omega_{\vec{q}}} = -\frac{|\vec{q}|}{(2\pi)^3}\frac{m_\pi^4}{2f_\pi^2}|\vec{V}_{p\pi}|^2\frac{1}{|\vec{V}_p-\vec{V}_A|}\,\mathrm{Im}\left(i\|T(E_p+E_A-m_\pi)\|i\right) \quad (1.32)$$

where

$$T(E) = V + V\frac{i}{E-H+i\eta}V \quad (1.33)$$

is the transition operator. In the center-of-mass frame equation (1.32) becomes

$$\frac{\partial^2\sigma^{\pi^0}}{\partial\varepsilon_0\partial\Omega_{\vec{q}}} = -\frac{|\vec{q}|}{(2\pi)^3}\frac{m_\pi^4}{2f_\pi^2}|\vec{V}_{p\pi}|^2\frac{1}{|\vec{V}_p-\vec{V}_A|}\,\mathrm{Im}\left(i\|T(E_p+E_A-\varepsilon_0-\frac{|\vec{q}|^2}{2(A+1)M})\|i\right) \quad (1.34)$$

where the energies appearing in equation (1.34) are now relative energies, $|\vec{q}|^2/2(A+1)M$ is the difference in the initial and final c.m. energies of the "non-pionic" systems.

Equation (1.34) contains the forward scattering amplitude

$$\left(i\|T(E_p+E_A-\varepsilon_0-\frac{|\vec{q}|^2}{2(A+1)M})\|i\right)$$

which can be calculated knowing only the nuclear optical potential. We define the single-particle transition operator $\mathcal{T}(e)$ according to

$$\mathcal{T}(e) \equiv \left(\Phi_0^A\|T(e)\|\Phi_0^A\right) \quad (1.35)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where ϕ_o^A is the ground state of the target. $\mathcal{J}(e)$ then operates on a single variable, which in coordinate space is just the relative coordinate of the projectile and target centers of mass.

The optical potential, $U_{opt}(e)$ is then defined according to

$$\mathcal{J}(e) = U_{opt}(e) + U_{opt}(e)\frac{1}{e-K+i\eta}\mathcal{J}(e) \tag{1.36}$$

with K the relative kinetic energy operator of the target-projectile system. Inverting equation (1.36) to give

$$U_{opt}(e) = \mathcal{J}(e) - \mathcal{J}(e)\frac{1}{e-K-\mathcal{J}(e)+i\eta}\mathcal{J}(e) \tag{1.37}$$

shows explicitly that the optical potential is a function only of the energy appearing in $\mathcal{J}(e)$. Letting $|i\rangle_{opt}$ denote the single-particle plane-wave function in the projectile-target relative coordinate, equation (1.34) becomes

$$\frac{\partial^2\sigma^{\pi^0}}{\partial\varepsilon_0\partial\Omega_\gamma} = -\frac{|\vec{\varepsilon}|}{(2\pi)^3}\frac{m_\pi^4}{2f_\pi^2}\frac{|\vec{V}_{\rho\pi}|^2}{|\vec{V}_\rho-\vec{V}_A|}\mathrm{Im}_{opt}(i|\mathcal{J}(E_f)|i)_{opt} \tag{1.38}$$

where

$$E_f = E_p + E_A - \varepsilon_0 - \frac{|\vec{\varepsilon}|^2}{2(A+1)M} \tag{1.39}$$

Note that $\mathcal{J}$ in equation (1.38) and hence also U_{opt}, are functions of the final energy of the non-pionic system only and not of the initial energy. Thus, the pion-production cross-section for the most energetic pions allowed is determined solely by phenomenological potentials appropriate to very low energy nuclear processes.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

The only dependence on the angle of the pion is contained in the factor $|\vec{v}_{p\pi}|^2$. This arises because we have included only the contribution $R^o(0)$ to $F^o(0)$ and summed over all final momenta of the nuclear system. The angular integration is thus easily performed to give

$$\frac{\partial \sigma^{\pi^o}}{\partial q_o} = \frac{|\vec{q}|}{(2\pi)^2} \frac{m_\pi^4}{f_\pi^2} \frac{|\vec{p}|^2 m_\pi^2 + M^2 |\vec{q}|^2}{|\vec{p}|^2 m_\pi^2 + M^2 |\vec{q}|^2 + M^2 m_\pi^2} \frac{1}{|\vec{v}_p - \vec{v}_A|}$$

$$\cdot \, \mathrm{Im}_{opt}(i|J(E_f)|i) \tag{1.40}$$

and the total cross-section is given by

$$\sigma^{\pi^o} = \int_{m_\pi}^{q_o^{max}} \frac{\partial \sigma^{\pi^o}}{\partial q_o} dq_o \tag{1.41}$$

where q_o^{max} is the solution of

$$E_f(q_o^{max}) = 0. \tag{1.42}$$

The treatment of the production amplitude becomes less manageable if we include the contributions from other terms to $F^o(0)$. Since these correspond to axial-vector-current insertions in out-going lines we will call them "post-emission" terms. The contribution from $R^o(0)$ and the term containing the axial-vector-current insertion in the in-coming target line we will call "pre-emission." Explicitly from equations (1.16) and (1.19) we write the production amplitude as

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$F^{a}(0) = -\frac{\sqrt{2}m_\pi^2}{f_\pi}\left\{\langle f(out)|J(0)|Z^A\rangle \frac{\not{p}+M}{2p\cdot q}\not{q}\gamma_5\frac{\tau^a}{2}u(p,s)\right.$$
$$\left.+\sum_n (-)^n \langle f_n|A_0^a(0)|f_n'\rangle\langle f_1\cdots f_n'\cdots(out)|J(0)|Z^A\rangle u(p,s)\right\} \quad (1.43)$$

where again the sum is restricted to out-going nucleons.

If we write

$$\not{p}+M = 2M\sum_{s'} u(p,s')\bar{u}(p,s') \quad (1.44)$$

and define the nucleon-axial-vector coupling constant g_A according to

$$\langle f_n|A_\mu^b(0)|f_n'\rangle = \bar{u}(p_{f_n},s_n)\, g_A\gamma_\mu\gamma_5\frac{\tau^b}{2}u(p_{f_n'},s_n') \quad (1.45)$$

then we may recast equation (1.43) in the form

$$F^a(0) = -\frac{\sqrt{2}m_\pi^2}{f_\pi}\sum_{s'}\left\{\langle f(out)|J(0)|Z^A\rangle u(p,s')\left(\bar{u}(p,s')\gamma_0\gamma_5\frac{\tau^a}{2}u(p,s)\right)\right.$$
$$\left.+\sum_n(-)^n\left(\bar{u}(p_{f_n},s_n)g_A\gamma_0\gamma_5\frac{\tau^a}{2}u(p_{f_n'},s')\right)\langle f_1\cdots f_n'\cdots(out)|J(0)|Z^A\rangle u(p,s)\right\} \quad (1.46)$$

where we have used the fact that we are in the pion rest frame in rewriting the contribution from $R^0(0)$ in equation (1.46).

The various contributions to equation (1.46) are identical in

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

form but for the lack of a factor $g_A = 1.18$ in the first line. Thus, as pointed out earlier, the current-algebraic prescription for evaluating the amplitude is not symmetric, although the descrepancy is small owing to the very small renormalization of the nucleon -axial-vector coupling constant. When there is need of nuclear democracy we will arbitrarily insert a factor g_A in the contribution from $R^o(0)$.

The presence of the post-emission terms in equation (1.46) prevents us from using closure since these terms depend explicitly on the momenta of the final-state particles. These terms are always small near threshold because the particles in the final state have much smaller velocities than those in the initial state. We will see in the following chapter that the contributions from "post-emission" will still be quite small compared to those from the terms already discussed even far from threshold.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

CHAPTER II

Nucleon-Nucleus Collisions II

In the preceeding chapter we studied the contributions to the pion-production cross-section from $F^b(0)$; specifically, we studied only the pre-emission term. In this chapter we study the dispersive corrections in going from the 'soft' - pion limit ($q^2 = 0$) to the physical-pion limit ($q^2 = m_\pi^2$). The contribution from both pre- and post-emission terms will be included. A discussion of the contribution from composite particles will be postponed until the following chapter.

The inclusion of those higher-order contributions which vanished in the soft-pion limit is best made within the framework of non-relativistic quantum mechanics. To do this we identify a relativistic dispersion relation for $F^b(k_o)$ with its non-relativistic counterpart, actually just the Lippmann-Schwinger equation, and identify "non-relativistic" potentials which describe the production of pions and other processes. The problem of going on and off the pion mass shell is then accomplished non-relativistically.

To write a dispersion relation for the production amplitude, $F^b(k_o)$ we recast Equation (1.5a) in the following form by bringing the D'Alembertian operators into the matrix element.

$$F^b(k_o) = i\int d^4x\, e^{ik_ox_o} \langle f(out)|R(J_\pi^b(x)\,\bar{J}(o))|Z^A\rangle u(p,s)$$
$$+ i\int d^4x \langle f(out)|[\dot{D}^b(x),\bar{J}(o)]\delta(x_o)|Z^A\rangle u(p,s)$$
$$+ g_o\int d^4x \langle f(out)|[D^b(x),\bar{J}(o)]\delta(x_o)|Z^A\rangle u(p,s) \qquad (2.1)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where we have written

$$D^b(x) = -\frac{i\sqrt{2}}{f_\pi^b}\frac{\partial A_\mu^b(x)}{\partial x_\mu} \tag{2.2a}$$

$$\dot{D}^b(x) = \frac{\partial D^b(x)}{\partial x_0} \tag{2.2b}$$

$$J_\pi^b(x) = (\Box + m_\pi^2) D^b(x) \tag{2.2c}$$

In the σ-model or any lagrangian model not containing pion-nucleon gradient couplings the third line of Equation (2.1) vanishes. This also follows from the strong version [11] of P.C.A.C. (partially-conserved axial-vector current), which identifies $D^b(x)$ of Equation (2.2a) with the canonical pion field. However, it has been shown [17] that the vanishing of

$$\int [D^b(x), J(0)]\,\delta(x_0)\,d^4x$$

follows from the equal-time commutation relation postulated in Equation (1.11) and the added hypothesis of a conserved vector current (C.V.C.). Thus, the vanishing of the third term of Equation (2.1) requires no new dynamical assumption.

Inserting a complete set of states in the first line of Equation (2.1) we have in analogy to Equation (1.13)

$$\begin{aligned} F^b(k_0) = \sum_{|n\rangle} \Big\{ & \langle f(out)|J_\pi^b(0)|n\rangle\langle n|J(0)|Z^A\rangle u(p,s)\frac{\delta^{(3)}(\vec{p}_f-\vec{p}_n)}{(p_f+k-p_n)_0+i\eta} \\ & -\langle f(out)|J(0)|n\rangle u(p,s)\langle n|J_\pi^b(0)|Z^A\rangle\frac{\delta^{(3)}(\vec{p}_A-\vec{p}_n)}{(p_n+k-p_A)_0+i\eta}\Big\} \\ & +i\int d^4x\,\langle f(out)|[\dot{D}^b(x),J(0)]\,\delta(x_0)|Z^A\rangle u(p,s) \end{aligned} \tag{2.3}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

We note that the last line of Equation (2.3), which cannot be evaluated directly, has the value $F^b(\infty)$, as can be seen easily by taking the limit of both sides of the equation.

The two terms in the sum comprise contributions from the right- and left-hand cuts, respectively, as well as poles in the complex q_o-plane. The lowest order contributions from the right-hand cut arise from elastic projectile-target intermediate states followed by states involving excited states of the target and at still higher energies intermediate states in which a meson has been created. The left-hand cut has as its lowest contributions terms involving the production of pions by the target leaving it in an excited state. We will again arbitrarily exclude such terms from the amplitude.

Equation (2.3) can be cast into a non-relativistic form as follows: We define first non-relativistic potentials U and V according to

$$\left(\psi^{(-)}_{\vec{p}_3\vec{p}_4} \,|\, U \,|\, \chi_{\vec{p}_1\vec{p}_2}\right) = (2\pi)^3\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\langle \vec{p}_3\vec{p}_4(out)|J(0)|\vec{p}_2\rangle u(\vec{p}_1,s_1) \qquad (2.4a)$$

$$\left(\psi^{(-)}_{\vec{p}_3\vec{p}_4} \,|\, \alpha_b V \,|\, \psi^{(+)}_{\vec{p}_1\vec{p}_2}\right) = (2\pi)^3\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\langle \vec{p}_3\vec{p}_4(out)|J^b_\pi(0)|\vec{p}_1\vec{p}_2(in)\rangle \qquad (2.4b)$$

where α_b is a destruction operator for the pion, $\chi_{\vec{p}_1\vec{p}_2}$ is a two-nucleon plane-wave state and $\psi^{(+)}_{\vec{p}_1\vec{p}_2}$ is an eigenstate of the total hamiltonian tending to $\chi_{\vec{p}_1\vec{p}_2}$ as $t\to\mp\infty$. We are careful to define the potentials in terms of two nucleon states since the definition is otherwise ambiguous.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

For example, the nucleon current, $\bar{j}(0)$, has non-vanishing matrix elements between the nucleon state and the two-nucleon-one-pion state. We assume that we can extend the definitions of the potential to larger numbers of nucleons in the usual way:

$$U(\vec{x}_1, \ldots, \vec{x}_N) = \sum_{i<j}^{N} U(\vec{x}_i - \vec{x}_j) \tag{2.5}$$

Otherwise, a different potential would have to be defined for each "nuclear" intermediate state. (Notice especially that U is not the potential defined in Equation 2.26). We therefore write Equation (2.3) as

$$F^b(k_a) = F(\infty) + \sum_n \frac{(\psi_f^{(-)} | \alpha_b V | \psi_n^{(+)})(\psi_n^{(+)} | U | \chi_i)}{E_f + k_a - E_n + i\eta} (2\pi)^3 \delta^{(3)}(\vec{P}_n - \vec{P}_i) \tag{2.6}$$

remembering that the simple identification of the potentials U and V is meaningful only if the intermediate states assymptotically contain no pions.

Performing the sum over the center-of-mass momentum, Equation (2.6) becomes

$$F^b(k_a) = F^b(\infty) + \sum_n \frac{(\psi_f^{(-)} \| \alpha_b V \| \psi_n^{(+)})(\psi_n^{(+)} \| U \| \chi_i)}{E_f + k_a - E_n + i\eta} \tag{2.7}$$

where the double bracket again denotes a relative matrix element and E_f and E_n have here been redefined to be relative energies.

Denoting the total relative hamiltonian by

$$H = K + U + V \tag{2.8}$$

with K the relative kinetic energy we have

$$F^b(k_a) = F^b(\infty) + (\psi_f^{(-)} \| \alpha_b V \frac{1}{E_f - k_a - H + i\eta} U \| \chi_i) \tag{2.9}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

The physical pion production amplitude is given non-relativistically by

$$F^b(m_\pi) = (\psi_f^{(-)} \| \alpha_b V \left(1 + \frac{1}{E_i - k - U + i\eta} U\right) \| \chi_i) \qquad (2.10)$$

with $E_i = E_f + m_\pi$. Comparing Equations (2.9) and (2.10) it follows immediately that

$$F^b(\infty) = (\psi_f^{(-)} \| \alpha_b V \| \chi_i) \qquad (2.11)$$

and

$$F^b(k_o) = (\psi_f^{(-)} \| \alpha_b V \left(1 + \frac{1}{E_f + k_o - k - U + i\eta} U\right) \| \chi_i) \qquad (2.12)$$

Notice that the only dependence on k_o is in the operator,

$$1 + \frac{1}{E_f - k_o - k - U + i\eta} U \qquad (2.13)$$

Thus if we are able to determine the potential V from the soft-pion limit, $k_o = 0$, the physical amplitude could then be evaluated by means of this potential.

To determine the potential for pion production we neglect rescattering of the pion and study that part of Equation (2.12) in which V acts only once, which relativistically amounts to considering the contributions from only the lowest energy intermediate states in Equation (2.3). This assumes tacitly that corresponding terms in the respective expansions of Equations (2.3) and (2.12) can be identified. Rigorously, of course, the identification in unambiguous only for these lowest order contributions.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

In this approximation Equation (2.12) becomes

$$F^b(k_o) \simeq (\phi_f^{(-)} \| [a_b, V] \, \Omega_o^{(+)}(E_f + k_o) \| \chi_i) \tag{2.14}$$

with

$$\phi_f^{(\pm)} = \Omega_o^{(\pm)}(E_f) \chi_f \tag{2.15}$$

and

$$\Omega_o^{(\pm)}(e) = 1 + \frac{1}{e + K - U \pm i\eta} U \tag{2.16}$$

is the Møller operator for the hamiltonian

$$H_o = K + U \tag{2.17}$$

We may rewrite Equation(2.14) as

$$F^b(k_o) = \sum_n (\phi_f^{(-)} \| [a_b, V] \, \Omega_o^{(+)}(E_f) \| \chi_n) \cdot (\chi_n \| \Omega_o^{(-)\dagger}(E_f) \Omega_o^{(+)}(E_f + k_o) \| \chi_i) \tag{2.18}$$

The first factor, which is independent of k_o, we recognise as the soft-pion limit for the process $n \to \pi^b f$. The entire dependence on k_o is now contained in the second factor, a sort of "boosting" factor. For $k_o = 0$, this factor has the simple form δ_{ni}. This factor also exhibits a simple form at $k_o = m_\pi$, though not at intermediate energies. Explicitly this is

$$(\chi_n \| \Omega_o^{(+)-1}(E_f) \, \Omega_o^{(+)}(E_i) \| \chi_i) = (\chi_n \| \Omega_o^{(+)-1}(E_f) \| \phi_i^{(+)})$$
$$= (\chi_n \| \frac{1}{E_f - K + i\eta} (E_f - K - U) \| \phi_i^{(+)})$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$= -m_\pi \left(\chi_n \middle\| \frac{1}{E_f - K + i\eta} \middle\| \phi_i^{(+)}\right) \tag{2.19}$$

which is just a matrix element of the free propagator. Thus Equation (2.18) becomes

$$F^b(m_\pi) = -m_\pi \sum_n F^b_{nf}(0) \left(\chi_n \middle\| \frac{1}{E_f - K + i\eta} \middle\| \phi_i^{(+)}\right) \tag{2.20}$$

To further study Equation (2.20) we note that non-relativistically

$$\bar{u}(p',s')\gamma_0\gamma_5 u(p,s) \to \phi_{s'}^{+}(p')\, i\frac{\vec{\sigma}\cdot\vec{\nabla}}{M}\, \phi_s(p) \tag{2.21}$$

with ϕ_s, and $\phi_{s'}$ Pauli spinors. We see then that $F^b_{nf}(0)$ as given by Equation (1.46) becomes non-relativistically

$$F^b_{nf}(0) = -\frac{\sqrt{2}\, m_\pi^2 g_{\pi NN}}{f_\pi^b} \left\{ \left(\phi_f^{(-)} \middle\| U \sum_j{}' i\frac{\vec{\sigma}_j\cdot\vec{\nabla}_j}{M} \frac{\tau_j^b}{2} \middle\| \chi_n\right) - \left(\chi_f \middle\| \sum_j{}' i\frac{\vec{\sigma}_j\cdot\vec{\nabla}_j}{M} \frac{\tau_j^b}{2} \Omega_0^{(-)\dagger}(E_f) U \middle\| \chi_n\right)\right\} \tag{2.22}$$

where the prime on the summation denotes that we have included only the axial-vector couplings to nucleons but not to composite nuclei in keeping with our relativistic approximation.

Substituting Equation (2.22) into Equation (2.20) we obtain

$$F^b(m_\pi) = -\frac{\sqrt{2}\, m_\pi^2 g_{\pi NN}}{f_\pi^b} \left\{ \left(\phi_f^{(-)} \middle\| U \sum_j{}' i\frac{m_\pi}{M} \vec{\sigma}_j\cdot\vec{\nabla}_j \frac{\tau_j^b}{2} \frac{1}{E_f - K + i\eta} \middle\| \phi_i^{(+)}\right) - \left(\chi_f \middle\| \sum_j{}' \frac{m_\pi}{M} \vec{\sigma}_j\cdot\vec{\nabla}_j \frac{\tau_j^b}{2} \Omega_0^{(+)\dagger}(E_f) U \frac{1}{E_f - K + i\eta} \middle\| \phi_i^{(+)}\right)\right\} \tag{2.23}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

For convenience, defining W^b according to

$$W^b = -\frac{\sqrt{2}\, m_\pi^2 g_A}{f_\pi^b} \sum_j^1 \frac{i m_\pi}{M} \vec{\sigma}_j \cdot \vec{\nabla}_j \frac{\tau_j^b}{2} \tag{2.24}$$

and noting that W^b commutes with K, the first term in Equation (2.23) becomes

$$\begin{aligned} &(\phi_f^{(-)} \| U \frac{1}{E_f - K + i\eta} W^b \| \phi_i^{(+)}) \\ &= \left\{ (\phi_f^{(-)} \| - (\chi_f \| \right\} W^b \| \phi_i^{(+)}) \\ &= (\phi_f^{(-)} \| W^b \| \phi_i^{(+)}) - (\chi_f \| W^b \| \phi_i^{(+)}) \end{aligned} \tag{2.25}$$

Likewise, the second term in Equation (2.23) becomes

$$-(\chi_f \| W^b \Omega_o^{(-)+}(E_f) U \frac{1}{E_f - K + i\eta} \| \phi_i^{(+)})$$

Now $\Omega_o^{(-)+}(E_f) U$ is just $T^{(-)+}(E_f)$, the transition operator, which satisfies the integral equation

$$T^{(-)+}(E_f) = U + T^{(-)+}(E_f) \frac{1}{E_f - K + i\eta} U \tag{2.26}$$

and also

$$T^{(-)+}(E_f) \frac{1}{E_f - K + i\eta} = U \frac{1}{E_f - K - U + i\eta} \tag{2.27}$$

so that this term becomes

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$
\begin{aligned}
&-(\chi_f \| W^b U \frac{1}{E_f - K - U + i\eta} \| \phi_i^{(+)}) \\
&= \frac{1}{m_\pi} (\chi_f \| W^b U \| \phi_i^{(+)}) \\
&= (\chi_f \| W^b \frac{1}{E_i - K + i\eta} U \| \phi_i^{(+)}) \\
&= (\chi_f \| W^b \{ \| \phi_i^{(+)}) - \| \chi_i) \} \\
&= (\chi_f \| W \| \phi_i^{(+)}) - (\chi_f \| W \| \chi_i)
\end{aligned}
\qquad (2.28)
$$

The first term in Equation (2.28) cancels a like term in Equation (2.25). The second term vanishes trivially leaving us with

$$
F^b(m_\pi) = (\phi_f^{(-)} \| - \frac{\sqrt{2}\, m_\pi^2 q_{cm}}{f_\pi^b} \sum_j^1 - \frac{i m_\pi}{M} \vec{\sigma}_j \cdot \vec{\nabla}_j \frac{\tau_j^b}{2} \| \phi_i^{(+)}) \qquad (2.29)
$$

We have recovered the DWBA approximation (Distorted-Wave Born Approximation) of non-relativistic scattering theory. Thus, from Equation (2.29), the potential describing pion production is simply

$$
V = \sum_b \frac{-\sqrt{2} m_\pi^2 q_{cm}}{f_\pi^b} \sum_j^1 - \frac{i m_\pi}{M} \vec{\sigma}_j \cdot \vec{\nabla}_j \frac{\tau_j^b}{2} \alpha_b^\dagger + h.c. \qquad (2.30)
$$

Non-relativistically this potential is generalized to include interactions with composite particles by extending the summation to all nucleons whether or not they are the constituents of composite nuclei.

In the present work we will not consider more than the DWBA term. Corrections to Equation (2.29) have been studied in detail in a previous paper for the reaction $NN \rightarrow \pi f$.[17]

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Let us examine what we called the post- and pre-emission terms in the production amplitude. Noting that $E_i = E_f + m_\pi$, we write

$$F^b(m_\pi) = \frac{1}{m_\pi}(\phi_f^{(-)} \| (E_i - E_f) V^b \| \phi_i^{(+)})$$

$$= \frac{1}{m_\pi}(\phi_f^{(-)} \| (E_i - K - U) V^b \| \phi_i^{(+)}) \qquad (2.31)$$

$$V^b \equiv [\alpha_b, V] \qquad (2.32)$$

Since K commutes with V^b by explicit construction this becomes

$$= -\frac{1}{m_\pi}(\phi_f^{(-)} \| U V^b - V^b (E_i - K) \| \phi_i^{(+)})$$

$$= -\frac{1}{m_\pi}(\phi_f^{(-)} \| U V^b - V^b U \| \phi_i^{(+)})$$

$$= -\frac{1}{m_\pi}(\phi_f^{(-)} \| [U, V^b] \| \phi_i^{(+)}) \qquad (2.33)$$

Note that had we chosen to neglect the distortion of the initial state because of its much higher kinetic energy, there would have been no post-emission term because

$$(E_i - K) \| \chi_i) = 0 \qquad (2.34)$$

and the amplitude would have become simply

$$F^b(m_\pi) = \frac{-1}{m_\pi}(\phi_f^{(-)} \| U V^b \| \chi_i) \qquad (2.35)$$

Were we to insert a complete set of states in Equation (2.35) and note again that K and V commute we would obtain the non-relativistic limit of Equation (1.19) above. Thus the pre-emission terms differ in the soft- and hard-pion limits only in the distortion of the initial state. Also,

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

the post-emission terms are still expected to be smaller than the pre-emission terms since we expect final-state interactions to be more important than initial state interactions.

As before we may write the pion-production cross-section as

$$\frac{d^2\sigma^{\pi^b}}{d\theta_0 d\Omega_0} = \frac{|\vec{q}|}{2(2\pi)^3}\frac{1}{|\vec{v}_p - \vec{v}_A|}\sum_{\{f\}}(2\pi)^3\delta(E_f - E_i - m_\pi)$$
$$\cdot |\langle \phi_f^{(-)} || [U, V^b] || \phi_i^{(+)} \rangle|^2 \qquad (2.36)$$

$$= -\frac{|\vec{q}|}{2(2\pi)^3}\frac{1}{|\vec{v}_p - \vec{v}_A|} Im \langle \phi_i^{(+)} || [V^b, U] \frac{1}{E_i - m_\pi - H_0 + i\eta}$$
$$[U, V^b] || \phi_i^{(+)} \rangle \qquad (2.37)$$

where we have used closure in going from Equation (2.36) to Equation (2.37). Although the nuclear matrix element has been written in closed form, it can no longer be replaced by an equivalent optical amplitude because of the presence of the post-emission terms. Neglecting these but still writing the initial wave as being distorted, Equation (2.37) becomes

$$\frac{d^2\sigma^{\pi^b}}{d\theta_0 d\Omega_0} = \frac{-|\vec{q}|}{2(2\pi)^3}\frac{1}{|\vec{v}_p - \vec{v}_A|} Im \langle \phi_i^{(+)} || V^b U \frac{1}{E - m_\pi - H_0 + i\eta} U V^b || \phi_i^{(+)} \rangle$$

(2.38)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

If the Hilbert vector $V^b|\Phi_i^{(+)}\rangle$ has a well-defined energy, i.e., if it is approximately an eigenstate of H_0 we can then write Equation (2.38) as

$$\frac{\partial^2\sigma^{\pi^b}}{\partial\Omega_0\partial\Omega_\vec{q}} = -\frac{|\vec{q}|}{2(2\pi)^3}\frac{1}{|\vec{v}_p-\vec{v}_A|}\mathcal{J}_{fm\ opt}\langle\Phi_i^{(+)}|V^b\mathcal{J}(E_i-m_\pi)V^b|\Phi_i^{(+)}\rangle_{opt} \tag{2.39}$$

with $\mathcal{J}(e)$ the optical transition operator.

Equation(2.39) again has been derived in the pion rest frame. If we transform the equation to the center-of-mass frame, the cross-section is

$$\frac{\partial^2\sigma^{\pi^b}}{\partial\Omega_0\partial\Omega_\vec{q}} = -\frac{|\vec{q}|}{2(2\pi)^3}\frac{1}{|\vec{v}_p-\vec{v}_A|}\mathcal{J}_{fm\ opt}\langle\Phi_i^{(+)}|V^b\mathcal{J}(E_i-q_0-\frac{|\vec{q}|^2}{2(A+1)m})V^b|\Phi_i^{(+)}\rangle_{opt} \tag{2.40}$$

where now

$$V = \sum_b \frac{-\sqrt{2}\,m_\pi^2 g_A}{f_\pi^b}\sum_j e^{i\vec{q}\cdot\vec{x}_j}\vec{\sigma}_j\cdot(-i\frac{m_\pi}{M}\vec{\nabla}_j-\vec{q})\frac{\tau_j^b}{2}\alpha_b^+ + h.c. \tag{2.41}$$

and where $\vec{x}_j$ is the individual nucleon coordinate.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

The potential for pion production defined above is not manifestly galilean invariant. This lack of galilean invariance is associated with the description through potentials of any inelastic process and is not a disease of the problem of pion production alone. However, from the practical standpoint the discrepancy is more serious since the inelasticity amounts to at least 140 MeV.

It is not clear how serious the lack of galilean invariance is since the non-relativistic description of the nuclear interaction demands that we work in a frame for which the nucleon velocities are small compared to the speed of light. This greatly restricts the velocities of galilean transformations which we are allowed. A more complete discussion of this problem is given in Appendix C.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

CHAPTER III

NUCLEUS-NUCLEUS COLLISIONS

We consider in this chapter from a purely non-relativistic standpoint the reaction

$$Z_1^A + Z_2^B \longrightarrow \pi^b(q) + f \tag{3.1}$$

In the preceding chapter we constructed the potential which describes the production of a pion of momentum q by a system of N nucleons as

$$V = \sum_{j=1}^{N} \sum_{b} -\frac{\sqrt{2}\, m_\pi^2 g_A}{f_\pi^b} e^{-i\vec{q}\cdot\vec{x}_j} \vec{\sigma}_j \cdot \left(-\frac{i m_\pi}{M}\vec{\nabla}_j - \frac{\vec{q}}{2}\right) \frac{\tau_j^b}{2} \alpha_b^\dagger + h.c. \tag{3.2}$$

In applying this potential to the production of pions by composite particles we are employing necessarily an impulse approximation. The current algebra, in fact, has not been used to construct the potential for the production of a pion for any baryon but the nucleon. In writing the production potential for a composite nucleus as a sum of individual nucleon potentials for pion production we are guilty of the same naivity with which we construct the nucleon-nucleus interaction as a similar sum of individual nucleon-nucleon interactions. This approach, however, has had the practical advantage that it is the only one by which processes involving composite particles have been understood.

Concentrating our attention on the production of neutral pions only, for which there are fewer complications due to isospin conservation, we write the transition operator for pion production considering the pre-emission term alone--this is equivalent to neglecting initial state dis-

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

tortion--as

$$T^{o}_{i\to f} = -\frac{1}{m_\pi}\left(\Phi_f^{(-)}\,|\,U\,[V,\alpha^{o}]\,|\,\chi_i\right) \tag{3.3}$$

Here χ_i is the initial undistorted state of the projectile and target and $\Phi_f^{(-)}$ is an exact scattering state of the nuclear hamiltonian

$$H_o = K + U \tag{3.4}$$

for the final system. U is the full nuclear interaction; its identification with the optical potential for the projectile-target system can be made only after all matrix elements have been reduced to elastic-scattering amplitudes.

We define the following quantities:

A (B) is the nucleon number of the projectile (target).

$\vec{r}_A$ ($\vec{r}_B$) is the coordinate of the center of mass of the projectile (target).

$\vec{r}_n$ denotes an individual nucleon coordinate.

$\vec{r}_n^{\,rel} = \vec{r}_n - \vec{r}_A$ ($= \vec{r}_n - \vec{r}_B$) is the relative coordinate for a projectile (target) nucleon.

$\vec{r} = \vec{r}_A - \vec{r}_B$ is the projectile-target relative coordinate

$\vec{R} = (A\vec{r}_A + B\vec{r}_B)/(A + B)$ is the coordinate of the center of mass of the entire nuclear system.

$\vec{\nabla}_A = \sum_{n=1}^{A} \vec{\nabla}_n$ is the gradient operator with respect to the center-of-mass coordinate of the projectile.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$\vec{\nabla}_n^{rel} = \vec{\nabla}_n - \vec{\nabla}_A/A$ is the gradient operator with respect to a relative coordinate of the projectile.

$r_{rel}^{3(A-1)}$ ($r_{rel}^{3(A-1)}$) denotes the internal coordinates of the projectile (target).

Assuming that only the projectile emits the pion--this is, we hope, a reasonable approximation for A << B--equation (3.3) becomes

$$T^{o}_{i\to f} = (\Phi_f^{(-)} | U e^{i\vec{\beta}\cdot\vec{r}_A} \left[\vec{S}^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \cdot \left(-\frac{i\vec{\nabla}_A}{AM} - \frac{\vec{\beta}}{m_\pi}\right) + W^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \right] | \chi_i) \tag{3.5}$$

where

$$\vec{S}^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \equiv \frac{-\sqrt{2}m_\pi^2 g_A}{f_\pi} \sum_{n=1}^{A} e^{-i\vec{\beta}\cdot\vec{r}_n^{rel}} \frac{\vec{\tau}_n^{o}}{2} \vec{\sigma}_n \tag{3.6a}$$

$$W^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \equiv \frac{-\sqrt{2}m_\pi^2 g_A}{f_\pi} \sum_{n=1}^{A} e^{-i\vec{\beta}\cdot\vec{r}_n^{rel}} \frac{\vec{\tau}_n^{o}}{2} \vec{\sigma}_n \left(-\frac{i\vec{\nabla}_n^{rel}}{A}\right) \tag{3.7a}$$

We may insert a complete set of states in equation (3.5) to obtain

$$T^{o}_{i\to f} = -\sum_N (\Phi_f^{(-)} | e^{-i\vec{\beta}\cdot\vec{r}_A} U | \chi_N)(\chi_N | \vec{S}^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \cdot \left(-\frac{i m_\pi \vec{\nabla}_A}{AM} - \frac{\vec{\beta}}{m_\pi}\right) + W^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \Big] | \chi_i) \tag{3.8}$$

Noting that neither $\vec{S}^{o}(\vec{q}; r_{rel}^{3(A-1)})$ nor $W^{o}(q; r_{rel}^{3(A-1)})$ depends on $\vec{R}$ or $\vec{r}$ but only on the internal coordinates of the projectile, equation (3.8) reduces to

$$T^{o}_{i\to f} = -\sum_N (\Phi_f^{(-)} | e^{i\vec{\beta}\cdot\vec{r}_A} U | \chi_N) (\varphi_0(r_{rel}^{3(A-1)}) | \vec{S}^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) \cdot \left(\frac{\vec{\nabla}_A}{AM} - \frac{\vec{\beta}}{m_\pi}\right) + W^{o}(\vec{\beta}; r_{rel}^{3(A-1)}) | \varphi_0(r_{rel}^{3(A-1)}) \tag{3.9}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where we have taken

$$\left(r^{3(A+B)}|\chi_i\right) = e^{i\vec{P}_i\cdot\vec{R}}\, e^{i\vec{p}_i\cdot\vec{r}}\, \varphi_o\left(r_{rel}^{3(A-1)}\right)\, \xi_o\left(r_{rel}^{3(B-1)}\right) \tag{3.10a}$$

$$\left(r^{3(A+B)}|\chi_N\right) = e^{i\vec{P}_N\cdot\vec{R}}\, e^{i\vec{p}_N\cdot\vec{r}}\, \varphi_{N_1}\left(r_{rel}^{3(A-1)}\right)\, \xi_{N_2}\left(r_{rel}^{3(B-1)}\right) \tag{3.10b}$$

but the sum in equation (3.9) is necessarily restricted to states which differ from the initial state only in the internal quantum numbers of the projectile. $\vec{P}_i$ ($\vec{P}_N$) is the total center-of-mass momentum and $\vec{p}_i$ ($\vec{p}_N$) is the relative momentum of the initial and intermediate states, respectively. $\varphi_o\left(r_{rel}^{3(A-1)}\right)$ and $\xi_o\left(r_{rel}^{3(B-1)}\right)$ are internal wave functions for the projectile and target ground states, respectively. Similarly, $\varphi_{N_1}\left(r_{rel}^{3(A-1)}\right)$ and $\xi_{N_2}\left(r_{rel}^{3(B-1)}\right)$ are internal wave functions for excited states of the projectile and target.

The second factor in equation (3.9) becomes

$$\vec{S}^{o}_{N_0}(\vec{\xi}) \cdot \left(\frac{\vec{p}_e}{Am} - \frac{\vec{\xi}}{i\alpha_\pi}\right) + W^{o}_{N_0}(\vec{\xi})$$

where

$$\vec{S}^{o}_{N_0}(\vec{\xi}) \equiv \int d^{3(A-1)}r_{rel}\; \varphi^{*}_{N}\left(\vec{r}^{\,3(A-1)}_{rel}\right) \vec{S}^{o}\left(\vec{\xi};\vec{r}^{\,3(A-1)}_{rel}\right) \varphi_o\left(\vec{r}^{\,3(A-1)}_{rel}\right) \tag{3.11a}$$

$$\vec{W}^{o}_{N_0}(\vec{\xi}) = \int d^{3(A-1)}\vec{r}_{rel}\; \varphi^{*}_{N}\left(\vec{r}^{\,3(A-1)}_{rel}\right) W^{o}\left(\vec{\xi};\vec{r}^{\,3(A-1)}_{rel}\right) \varphi_o\left(\vec{r}^{\,3(A-1)}_{rel}\right) \tag{3.11b}$$

If we wish to identify φ_o and φ_N with shell-model wave functions, φ_o^{SM} and φ_N^{SM}, which depend spuriously on A coordinates, equations (3.11) become

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$\vec{S}^{\circ}_{N\alpha}(\vec{q}) = \frac{1}{A}\int d^{3A}r\ \varphi_N^{3M}(r^{3A})\,\vec{S}^{\circ}(\vec{q};r^{3(A-1)})\,\varphi_{\circ}^{3M}(r^{3A})\,\delta^{(2)}\Big(\sum_{n=1}^{A}\vec{r}_n\Big) \qquad (3.12a)$$

$$W^{\circ}_{N\alpha}(\vec{q}) = \frac{1}{A}\int d^{3A}r\ \varphi_N^{3M}(r^{3A})\,W^{\circ}(\vec{q};r^{3(A-1)})\,\varphi_{\circ}^{3M}(r^{3A})\,\delta^{(2)}\Big(\sum_{n=1}^{A}\vec{r}_n\Big) \qquad (3.12b)$$

Writing $\vec{r}_A = \vec{R} + (B/(A+B))\vec{r}$ and

$$\left(r^{3(A+B)}\middle|\Phi_f^{(-)}\right) = e^{i\vec{P}_f\cdot\vec{R}}\left(r^{3(A+B-1)}\middle|\Phi_f^{(-)}\right) \qquad (3.13)$$

equation (3.9) becomes

$$T^{\circ}_{i\to f} = (2\pi)^3\delta^{(2)}(\vec{P}_i-\vec{P}_f-\vec{q})\sum_N\left(\Phi_f^{(-)}\middle|\middle|e^{i\vec{q}\cdot\frac{B}{A+B}\vec{r}}U\middle|\middle|\chi_N\right)\Gamma^{\circ}_{N\alpha}(\vec{q}) \qquad (3.14)$$

where

$$\Gamma^{\circ}_{N\alpha}(\vec{q}) = \vec{S}^{\circ}_{N\alpha}(\vec{q})\cdot\left(\frac{\vec{P}_A}{AM}-\frac{\vec{q}}{M\sigma}\right)+W^{\circ}_{N\alpha} \qquad (3.15)$$

The quantities appearing in equation (3.14) have a simple interpretation. The nuclear matrix element is simply the nuclear final-state interaction following pre-emission of the pion for which the vertex function is $\Gamma^{\circ}_{NO}(\vec{q})$. The factor $\exp(-i\vec{q}.\vec{r}(B/(A+B)))$ expresses the recoil of thr relative system after the pion is emitted.

The contributions to the vertex are of two kinds. The first term in equation (3.15) is very much like the vertex encountered in the case of nucleon-nucleus collisions in the preceding chapter. Since the scale of this term is determined by the total momentum of the projectile, we will find it convenient to denote it as the "external emission" term.

The second contribution to $\Gamma_{NO}(\vec{q})$ vanishes if the projectile is a single nucleon and thus describes a phenomenon not encountered in the case of nucleon-nucleus collisions. The gradient operators in $W^{\circ}_{NO}(\vec{q})$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

act only on the internal wave function of the projectile. Thus the scale of $W^{o}_{NO}(\vec{q})$ is set by the internal (Fermi) momentum of the projectile. For this reason it is convenient to denote the contribution from this term as "internal emission."

From equation (3.14) the production cross section is

$$\frac{\partial^2 \sigma^{\pi^0}}{\partial q_0 \partial \Omega_q} = \frac{|\vec{q}|}{2(2\pi)^3} \frac{1}{|\vec{V}_A - \vec{V}_B|} \sum_{|\vec{q}|} \sum_{|N|} |\Gamma^{o}_{NO}(\vec{q})|^2 \cdot |(\Phi_f^{(-)} \| e^{i\frac{B}{A+B}\vec{q}\cdot\vec{r}} \mathcal{U} \| \chi_N)|^2 2\pi \delta(E_i - E_f - q_0) \quad (3.16)$$

Summing over all final states and using closure in the usual way this becomes

$$\frac{\partial^2 \sigma^{\pi^0}}{\partial q_0 \partial \Omega_q} = -\frac{|\vec{q}|}{(2\pi)^3} \frac{1}{|\vec{V}_A - \vec{V}_B|} \sum_{|N|} |\Gamma^{o}_{NO}(\vec{q})|^2 \cdot \mathrm{Im}(\chi_N \| \mathcal{U} \frac{1}{E_i - q_0 - \frac{|\vec{q}|^2}{2(A+B)M} - H_0 + i\eta} \mathcal{U} \| \chi_N)$$

$$= -\frac{|\vec{q}|}{(2\pi)^3} \frac{1}{|\vec{V}_A - \vec{V}_B|} \sum_{|N|} |\Gamma^{o}_{NO}(\vec{q})|^2 \mathrm{Im}(\chi_N \| T(E_f) \| \chi_N) \quad (3.17)$$

with T(E), the nuclear transition operator, again given by

$$T(E) = \mathcal{U} + \mathcal{U} \frac{1}{E - H_0 + i\eta} \mathcal{U} \quad (3.18)$$

and

$$E_f = E_A + E_B - q_0 - \frac{|\vec{q}|^2}{2(A+B)M} \quad (3.19)$$

It is tempting at his point to identify $T(E_f)$ with an optical transition operator $\mathcal{T}(E_f)$ as was done in previous chapters. Rigorously, this is possible only for the term N = 0, i.e., the ground state of the projectile, since only for stable particles (whether elementary or

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

composite) can assymptotic states be defined. If the excited state of the projectile has a narrow width compared with a typical energy level spacing, then it may still be meaningful to speak of the projectile as being scattered in the usual way. Thus, we expect the optical description of equation (3.18) to be most reasonable for light projectiles (He^3, α, t) for which the spacing of excited states is large (though these states are unbound) than for heavy nuclei for which the level spacing is exceedingly small.

Simultaneously, a phenomenological optical potential describing the elastic scattering of a composite projectile in an excited state is unknown since the cross section for such a process is not readily available to experiment. One might argue equally well that the optical potential is either the same for the excited state as for the ground state of the projectile or that it is much "weaker."

The first premise rests on the validity of the impulse approximation for calculating the optical potential. Since most of the nuclear material will be concentrated within the same volume for both the ground and excited states of a nucleus, expectation values of the nuclear interaction will not differ greatly in these two cases. The optical potential in this approximation would then be largely independent of the projectile state and the off-shell transition matrix elements appearing in equation (3.17) would all be equal.

The second premise argues, perhaps less naively, that for an excited nucleus some nucleons will be very weakly bound or even unbound and their wave functions very extended spatially. Since absorption from the elastic channel in the projectile-target scattering procedes largely through the excitation of these particles,

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

we would expect that the optical potential would reflect mostly the shape of the wave function of these most highly excited nucleons. With this attitude the optical potential would be very shallow and very wide.

Let us examine the contributions to equation (3.17) for $\vec{q} = 0$, for which the two approximations discussed above are identical, owing to the existence of selection rules. We note that $\vec{S}^0(0)$ and $W^0(0)$ are symmetric operators with opposite parity. The external and internal emission amplitudes then do not interfere. (We shall assume that this is the general case even for non-vanishing $\vec{q}$.) Since $\vec{S}^0(0)$ operates only on the spin coordinates of the nucleus, it can connect the ground state of the projectile only to a member of the same multiplet. Assuming that the target is an even-even nucleus so that the transition operator is rotationally invariant the cross-section for external emission becomes near $\vec{q} = 0$,

$$\left(\frac{\partial^2\sigma^{\pi^0}}{\partial\Omega_0\,\partial\Omega_{\vec{q}}}\right)_{ext} \cong -\frac{|\vec{\delta}|}{(2\pi)^3}\,\frac{1}{|\vec{v}_A - \vec{v}_B|}\left\langle\left|\vec{S}^0_{\alpha\beta}(\vec{\delta})\cdot\left(\frac{\vec{p}_A}{M_A} - \frac{\vec{\delta}}{m_B}\right)\right|^2\right\rangle \cdot \mathrm{Im}\,(\chi_i \| T(E_i) \| \chi_i) \qquad (3.20)$$

where the brackets denote spin averaging. This expression is clearly vanishingly small if the projectile has spin zero, e.g., an alpha-particle. For a spin-(1/2) projectile $\vec{S}^0(0)$ is determined by the spin and isospin of the odd nucleon. Thus for the projectile, say, a triton or He^3 the pion-production cross-section averaged over spins becomes

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$\left(\frac{\partial^2\sigma^{\pi^0}}{\partial\delta_0\,\partial\Omega_\eta}\right)_{ext} \simeq -\frac{|\vec{\delta}|}{(2\pi)^3}\frac{1}{|\vec{v}_A-\vec{v}_\pi|}\,\frac{m_\pi^4 g_A^2}{2f_\pi^2}\,|\vec{V}_{A\pi}|^2 \cdot \mathrm{Im}\,(\chi_i\|T(E_f)\|\chi_i) \tag{3.21}$$

where we have set

$$\vec{V}_{\pi A} = \frac{\vec{P}_A}{AM} - \frac{\vec{\delta}}{m_\pi} \tag{3.21½}$$

the relative pion-projectile velocity. Equation (3.20) is identical in form to equation (1.38) derived earlier.

Because $W^0(0)$ is odd under parity, only states of the projectile with the same total angular momentum but opposite parity contribute to the cross-section for internal emission for small pion momentum. The cross-section for internal emission of the pion is then

$$\left(\frac{\partial^2\sigma}{\partial\delta_0\,\partial\Omega_\eta}\right)_{int} \simeq \frac{-|\vec{\delta}|}{(2\pi)^3}\frac{1}{|\vec{v}_A-\vec{v}_\pi|}\sum_N \left\langle |W^0_{cN}(\omega)|^2\right\rangle \mathrm{Im}\,(\chi_N\|T(E_f)\|\chi_N) \tag{3.22}$$

with the sum restricted to states satisfying the aforementioned selection rules. This expression is insensitive to the details of the internal structure of the projectile if we assume that the summation is exhausted by a single collective state, χ_c. We may then write

$$\begin{aligned}|W^0_{oc}(0)|^2 &= |(\varphi_c\|W^0(\omega)\|\varphi_o)|^2 \\ &= \sum_N (\varphi_o\|W^{0\dagger}(\omega)\|\varphi_N)(\varphi_N\|W^0(\omega)\|\varphi_o) \\ &= (\varphi_o\|\,|W^0(\omega)|^2\,\|\varphi_o)\end{aligned} \tag{3.23}$$

Explicitly, equation (3.23) is

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$|W^{o}_{oc}(o)|^2 = \frac{m_\pi^4 g_A^2}{2f_\pi^2}(\varphi_o \| \sum_{i=1}^{A} \tau_i^o \vec{\sigma}_i \cdot (-i\frac{\vec{\nabla}_i}{M}) \sum_{j=1}^{A} \tau_j \vec{\sigma}_j \cdot (-i\frac{\vec{\nabla}_j}{M}) \| \varphi_o) \tag{3.24}$$

If we ignore correlations in the wave function so that only the terms i = j contribute we have

$$|W^{o}_{oc}(o)|^2 = -\frac{m_\pi^4 g_A^2}{2f_\pi^2}\left(\varphi_o \Big| \sum_{i=1}^{A} \frac{(\vec{\sigma}_i \cdot \vec{\nabla}_i)^2}{M^2} \Big| \varphi_o\right) \tag{3.25}$$

Now

$$\sigma_i^a \sigma_i^b = \tfrac{1}{2}[\sigma_i^a \sigma_i^b + \sigma_i^b \sigma_i^a] - \tfrac{1}{2}[\sigma_i^a \sigma_i^b - \sigma_i^b \sigma_i^a]$$
$$= \delta_{ab} + i\,\epsilon_{abc}\sigma_i^c \tag{3.26}$$

where a,b = 1,2,3. Performing the spin averaging in equation (3.26) only the Kronecker-δ survives and we have then that

$$\left\langle |W^{o}_{oc}(o)|^2 \right\rangle = \frac{m_\pi^4 g_A^2}{M f_\pi^2}\left(\varphi_o \Big| \sum_{i=1}^{A} -\frac{\nabla_i^2}{2M} \Big| \varphi_o\right)$$
$$= \frac{m_\pi^4 g_A^2}{M f_\pi^2} T_{int} \tag{3.27}$$

where T_{int} is the internal kinetic energy of the projectile ground state which is related to the Fermi energy E_F according to

$$T_{int} = \frac{3}{5} A E_F \tag{3.28}$$

The cross-section for internal emission for very small pion momentum is then

$$\left(\frac{\partial^2 \sigma^{\pi^o}}{\partial \Omega_o \partial \Omega_q}\right)_{int} \approx -\frac{|\vec{q}|}{(2\pi)^3} \frac{1}{|\vec{V}_h - \vec{V}_B|} \frac{m_\pi^4 g_A^2}{M f_\pi^2} T_{int}\, Im(\chi_c \| T(E_f) \| \chi_c) \tag{3.29}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

and the pion-production cross-section for both processes for very small pion momentum becomes

$$\frac{\partial^2\sigma^{\pi^+}}{\partial\xi_0\,\partial\Omega_{\vec{q}}} = -\frac{|\vec{q}|}{(2\pi)^3}\frac{1}{|\vec{v}_A-\vec{v}_B|}\,\frac{m_\pi^4 g_A^2}{2f_\pi^2}\left[\,|\vec{v}_{A\pi}|^2\,\mathrm{Im}(\chi_i\|T(E_f)\|\chi_i) + \frac{2T_{int}}{M}\,\mathrm{Im}(\chi_c\|T(E_f)\|\chi_c)\right] \tag{3.30}$$

The two contributions are illustrated in figure 3.

At threshold

$$|\vec{v}_{A\pi}|^2 = 2T_{ext}/AM \tag{3.31}$$

where we have written T_{ext} for the kinetic energy of the projectile. The ratio of the production cross-section for internal and external emission (assuming that the latter does not vanish) is then

$$\frac{\sigma^{\pi^+}(int)}{\sigma^{\pi^+}(ext)} = \frac{A}{4}\,\frac{T_{int}}{T_{ext}} \simeq A^2/30 \tag{3.32}$$

For a triton or He^3 this number is about .3. As A increases the ratio of internal to external emission becomes very large but at the same time the description becomes less and less valid. We have assumed, of course, in constructing the ratio that the off-shell transition matrix elements are equal, which is not obvious.

It is less clear how the cross section is calculated for $\vec{q}$ large. If we assume that only the lowest energy states of the projectile contribute then we may trivially extend equation (3.30) by including form factors of the projectile as

$$\frac{\partial\sigma^{\pi^0}}{\partial\xi_0\,\partial\Omega_{\vec{q}}} = -\frac{|\vec{q}|}{(2\pi)^3}\frac{1}{|\vec{v}_A-\vec{v}_B|}\,\frac{m_\pi^4 g_A^2}{2f_\pi^2}\left[\,|\vec{\rho}_0(\vec{q})|^2\,|\vec{v}_{A\pi}|^2 \cdot \mathrm{Im}(\chi_c\|T(E_f)\|\chi_c) + |\vec{\rho}_0(\vec{q})|^2\,\frac{2T_{int}}{M}\,\mathrm{Im}(\chi_c\|T(E_f)\|\chi_c)\right] \tag{3.33}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where $\rho_o(\vec{x})$ and $\rho_c(\vec{x})$ are hadronic density functions of the projectile in the ground and collective states, respectively, and

$$\tilde{\rho}(\vec{q}) = \int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}} \rho(x) \tag{3.34}$$

On the other hand, if we assume that all the important off-mass-shell transition matrix elements (i.e., those for which $\vec{S}^o_{No}(\vec{q})$ or $W^o_{No}(\vec{q})$ is appreciable) are equal we may then perform closure on the projectile states, namely

$$\left\langle \sum_N |(\varphi_N|\vec{S}^o(\vec{q})|\varphi_o)\cdot\vec{v}_{pA}|^2 \right\rangle = \left\langle (\varphi_o|\vec{S}^o(\vec{q})\cdot\vec{v}_{pA}|^2|\varphi_o) \right\rangle$$

$$\simeq \left\langle (\varphi_o||\vec{S}^o(0)\cdot\vec{v}_{pA}|^2|\varphi_o) \right\rangle = \frac{m_\pi^4 g_A^2}{2f_\pi^2}|\vec{v}_{pA}|^2 \tag{3.35}$$

and

$$\left\langle \sum_N |(\varphi_N|W^o(\vec{q})|\varphi_o)|^2 \right\rangle = \left\langle (\varphi_o||W^o(\vec{q})|^2|\varphi_o) \right\rangle$$

$$\simeq \left\langle (\varphi_o||W^o(0)|^2|\varphi_o) \right\rangle = \frac{m_\pi^4 g_A^2}{2f_\pi^2}\frac{2T_{int}}{M} \tag{3.36}$$

where the brackets denote spin averaging and we have neglected contributions from correlations in the ground state wave function in both equations so that

$$\frac{\partial^2\sigma^{\pi^o}}{\partial\omega\partial\Omega_f} = -\frac{|\vec{q}|}{(2\pi)^3}\frac{1}{|\vec{v}_A-\vec{v}_B|}\frac{m_\pi^4 g_A^2}{2f_\pi^2}\left[|\vec{v}_{A\pi}|^2 + \frac{2T_{int}}{M}\right] \cdot \mathrm{Im}\,(\chi\|T(E_F)\|\chi) \tag{3.37}$$

In this case the approximation gives us an identical expression as

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

at threshold. For $\vec{q} >> \frac{1}{R}$, where R is a characteristic radius of the projectile, the two results, equations (3.33) and (3.37), differ considerably.

This completes our discussion of nucleus-nucleus collisions.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

CHAPTER IV

Numerical Results

In the present chapter we present results obtained by evaluating directly the expressions derived earlier in the text. We limit our discussion to those experiments in which only the final pion is observed. What is more, we assume, owing to the smaller distortion of the incoming nuclear system compared to that of the outgoing nuclear system, that we can ignore post emission diagrams. In this approximation we saw that the pion-production cross-section is related directly to the transition matrix element for forward scattering fully off the energy shell by an amount equal essentially to the pion energy. When we approximated the initial state by a plane wave, the approximation consistent with our neglecting post-emission diagrams this relation became a simple linear dependence (equation (1.34)). The evaluation of the production cross-section then amounts to evaluating this off-energy-shell forward scattering amplitude.

In the case of pion-production in nucleon-nucleus collisions our attention was fixed on the reaction

$$p + C^{12} \rightarrow \pi^{\circ} + ?$$

because of the availability of data (at least for the total pion-production cross-section) and also because the phenomological optical potential describing elastic p-C^{12} scattering is thought to be well known. For the sake of simplicity the optical potential was taken to be an energy-dependent spherical square-well of radius

$$R = r_o A^{1/3} \qquad (4.1)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

with r_o = 1.2 fm and A the mass-number of the target. For C^{12} this radius is 2.8fm. The well depth has been determined from experiments performed by numerous authors. For zero energy we took the well depth to be 52. MeV + i*5. MeV as determined by extrapolating data presented by Bohr and Mottelson [28]. The value at 150 MeV was taken to be 20. MeV + i* 15. MeV from the same source. At 1 BeV there were the experiments of Palevsky et al [29] who give the value 20. MeV + i*100. MeV but a slightly smaller radius, which we did not take into consideration. For an intermediate energy we took the experimental cross-sections at 424 MeV of Hering [30], for which the best fit of a square well gave the well depth of 20. MeV + i*5. MeV. The well depths at other energies were determined from these by linear interpolation. These well depths as a function of the energy are depicted in figure 4.

The values of the well depths at 0,150, and 1000 MeV, which we have taken directly from the literature, were determined from optical model analyses using the Woods-Saxon well. Thus a calculation of the elastic scattering cross-section with our square well with these depths would naturally exceed the experimental values to which the well parameters were supposedly fitted. We expect the off-shell amplitudes to be less sensitive to the diffuseness, however so it is not unlikely that even with a square well the Woods-Saxon well depths are more suitable for calculating off-shell-amplitudes than those determined directly from square well fits to elastic scattering cross-section. We shall see in the case of pion-production in nucleus-nucleus collisions, however, that the choice of a square well can lead to the gross exageration of certain peculiarities of the production cross-section.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Figure 5 shows the calculated total pion production cross-section for the reaction

$$p + C^{12} \rightarrow \pi^{0} + f$$

plotted together with the measurements of Dunaitsev and Prokoshkin.[31] In the calculated curve the following approximations have been made:

1) we have neglected completely all final state interactions of the pion, and

2) we have only considered emission of the pion by the projectile.

The first approximation is perhaps not too serious. We note that for low energies the pions produced will be largely p-wave as indicated by the form of the non-relativistic effective potential for pion production.

$$V = \sum_{b} - \frac{\sqrt{2}\, m_{\pi}^{+} g}{f_{\pi}^{b}} \vec{\sigma} \cdot \left(- \frac{i m_{\pi}}{M} \vec{\nabla} - \vec{q}_{b} \right) \frac{\tau_{-}^{b}}{2} \alpha^{+} + h.c.$$

The p-wave pion may then have important final state interactions with the nuclear system through the (3,3) resonance. We note with Mandelstam[32] that since the pion-nucleon interaction is shorter range than the nucleon-nucleon interaction and the pion is generally moving faster than any other particle we expect that the pion final-state interaction will be largely accomplished before the nuclear final-state interaction sets in. The main effect, then, of the pion final-state interaction is to renormalize the pion vertex. The validity of the optical theorem will, therefore, not be impaired. Also, if the work of Mandelstam is a good indication, although the pion final-state interaction will modify the pion energy distribution substantially, the normalization of the total

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

production cross section should be unchanged.

The second approximation is far more difficult to justify, especially in the light of the discussion of the last chapter. Very close to threshold, say, within 30 MeV, the production will be predominantly s-wave for which the repulsive final-state interaction will greatly reduce the vertex for large nuclei. At higher energies when mostly higher angular momentum pions are produced there is no reason to expect that the production by the target will be very small. In fact, the discrepancy with experiment at high energies may be just this.

Figure 6 shows the energy-differentiated cross-section calculated from equation (1.40). The distribution of pions is dominated largely by the available phase-space. The resonance at 275.7 MeV corresponds to a 4.3 MeV single particle level in the optical well. Since the optical potential is nearly real for the largest possible pion energies the resonance stands out markedly from the rest of the spectrum. For the low energy end of the spectrum the nuclear optical potential is sufficiently absorptive that no structure will be discernable. Properly, there are also small spikes in the pion spectrum for pion energies greater than q_o^{max} given by equation (1.42) corresponding to capture of the proton into a bound state of the (A+1)-particle system following emission of the pion. These processes are not calculable from the optical amplitude since the final nuclear system is in a discrete state, which must be included separately in the sum over states qppearing in equation (1.29). This comment does not apply to the quasi-bound continuum states of the final nuclear system which appear as resonances in the optical amplitude like the 4.3 MeV level above.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Figure 7 shows the angular distribution of pions for an incident energy of 280 MeV, which is given essentially by the factor

$$|\vec{V}_{p\pi}|^2 = \frac{(P\cdot\xi)^2 - M^2\mu^2}{(P\cdot\xi)^2} \tag{4.2}$$

folded over the phase-space (though it has been calculated dynamically here). This factor may be rewritten as

$$|\vec{V}_{p\pi}|^2 = 1 - \frac{(M\mu/P_0\xi_0)^2}{(1-\vec{V}_p\cdot\vec{V}_\xi)^2} \tag{4.3}$$

with

$$\vec{V}_\xi = \frac{\vec{\xi}}{\xi_0} \tag{4.4a}$$

$$\vec{V}_p = \frac{\vec{P}}{P_0} \tag{4.4b}$$

the pion and proton velocities, respectively. The backward peaking will be more pronounced for higher incident energies since v_p will be closer to unity. At threshold, assuming $A \gg 1$, v_p has the value 0.14 so that close to threshold the distribution of pions is nearly isotropic.

The problem of calculating the pion-production cross-section for nucleus-nucleus collisions is more difficult than for nucleon-nucleus collisions because we are hampered by our far more uncertain knowledge of the effective nucleus-nucleus interaction.

It is by now accepted that the scattering of low energy tritons, He^3, and alpha-particles by a nucleus can be described by an optical well whose parameters are

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

$$V_R^{AT} \leq A \cdot V_R^{NT} \qquad (4.5a)$$

$$V_I^{AT} \leq V_I^{NT} \qquad (4.5b)$$

where A is the nucleon number of the projectile, V^{NT} is the optical potential of the nucleon-target system, and R and I denote real and imaginary parts, respectively. The argument for choosing these values is that A nucleon are interacting with target. Since the nuclear force is supposedly not saturated for $A \leq 4$, the real part of the well should be that factor deeper. But since the projectile is usually more tightly bound than the target the effective number of absorptive channels is not much different than for a nucleon projectile. Thus the imaginary part should be largely independent of the projectile mass number. These arguments cannot hold for the heavier projectiles because of the saturation of the nuclear force, nor should it hold for deuteron because of its small binding energy. Not surprisingly, this simple prescription fails in these cases.

Simultaneously, it must be pointed out that these potentials are not unique. One can as well describe the scattering of alpha-particles by heavy nuclei with an optical potential the parameters of which are not much different than those for the scattering of nucleons.[33] This fact is unsupported by recent attempts to calculate the optical potential for composite pro-

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

jectiles from phenomenological nucleon-nucleon potentials within the framework of the impulse approximation.[34]

Figure 8 gives the pion production cross-section for protons, He^3, and alpha-particles on Cu^{65}. The radius of the p-Cu^{65} optical well was taken from equation (4.1) and the well depths were assumed to be given universally by those in figure 4. The well depths for the heavier projectiles were determined according to the prescription given in equation (4.5).

The unusually large cross-section for alpha-particle projectiles (the total reaction cross-section is only 1500 mb) was not totally unexpected and for the most part reflects the choice of well parameters. The large cross-section is a direct result of the possibly unrealistic final state interaction of the nuclear system.

To achieve a better understanding of the cause of these large cross-sections let us decompose the nuclear final-state interaction into its partial-wave amplitudes according to the well known expansion.

$$\mathcal{T}_{fi}(\vec{P}_i|T(E_f)|\vec{P}_f) = 4\pi \sum_l (2l+1) \mathcal{T}_l(P_i,P_f) P_l(\hat{P}_i\cdot\hat{P}_f) \qquad (4.6)$$

For simplicity, taking the optical potential to be real, we have

$$\mathcal{T}_l(P_i,P_f) \propto |\langle \chi_l^{(-)}(P_f,r)|U|\chi_l^{(+)}(P_i,r)\rangle|^2 \qquad (4.7)$$

and since U is a square well this is proportional to

$$\left|\int_0^R dr\, r^2\, \chi_l^{(-)}(P_f,r)\chi_l^{(+)}(P_i,r)\right|^2$$

$$= |A_l(P_f)|^2 \left|\int_0^R dr\, r^2\, j_l(Qr) j_l(P_i r)\right|^2 \qquad (4.8)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

where $A_\ell(p_f)$ is the amplitude of the regular solution inside the well and $\varkappa$ is the momentum of the final nucleon measured from the well bottom. The various momenta are depicted in figure 8.

There are two sources of resonances in the calculation. One, clearly, is the amplitude $A_\ell(p_f)$ whose resonances correspond to quasi-bound continuum states in the well. The other resonances are in the factor

$$\int_0^R dr\, r^2\, j_\ell(\varkappa r)\, j_\ell(p_i r)$$

and these occur when $\varkappa = p_i$. Writing

$$\frac{\varkappa^2}{2\mu} = \frac{p_i^2}{2\mu} + V - m_\pi \qquad (4.9)$$

we see that a resonance will occur when the potential well depth just compensates the pion energy. We call these latter resonances frequency resonances. We note that unlike the amplitude resonances, whose height is very sensitive to the nature of the boundary condition at the well radius, the frequency resonance is rather insensitive to details of the optical well so long as it is largely flat. At the same time we note that the frequency resonances are very wide (on the order of a pion mass) while amplitude resonances are much narrower (for a complex well the half-width is only twice the imaginary well depth). Thus it is the frequency resonance which is responsible for the overall normalization of the production cross-section for alpha-Cu^{65} collisions while the amplitude resonances contribute to the prominent peak. Thus for a more diffuse well, like a Woods-Saxon well, we would expect to see the same cross-section away from the

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

peak but the height of the peak would be much smaller.

As an example of this we have calculated the total pion production cross section for the reaction

$$\alpha + Cu^{65} \longrightarrow \pi^0 + ?$$

using the potential

$$U_{opt}(E) = \lambda V_R(E) + i V_I(E) \qquad (4.10)$$

with the well radius, V_R, and V_I given above and λ varying from 1.0 to 4.0, which is roughly the variation of the phenomenological well depths. The results are plotted in Figure 10.

The magnitude of the amplitude resonance should not be seriously considered since a square well will always exaggerate these. For from this peak, however, we notice a wide variation in the production cross section (an order of magnitude near 275 MeV).

We note that all these calculations have been done in the "closure" approximation, in which there appear no form factors for the projectile.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

CHAPTER V

Conclusions

We have seen how Current Algebra and dispersion relations have been used to calculate the cross-section for the production of physical pions in nucleon-nucleus and nucleus-nucleus collisions. In the case of nucleon-nucleus collisions the soft-pion limit gave a simple expression for the spin averaged cross-section in terms of off-shell amplitudes for purely nuclear processes. When closure was used to sum over all possible final nuclear states the optical theorem gave an expression for the production cross-section in terms of a single off-shell amplitude for forward elastic scattering which could be calculated from the nuclear optical potential. In going from the soft to the hard-pion amplitude we saw that the chief correction was the distortion of the incoming plane wave. This we expected to be only a small correction.

In calculating the corrections to the soft-pion limit we were forced by the complexity of the nuclear interaction to recast the problem within the framework of non-relativistic potential theory. In this process we not only recovered the distorted wave Born approximation for the physical amplitude but also determined the effective potential for pion-production given by current algebra. This we applied to calculating the pion-production cross-section in nucleus-nucleus collisions.

For the reaction

$$p + C^{12} \longrightarrow \pi^{0} + ?$$

we saw that our theory, which depended on no free parameters, described the production cross-section fairly accurately. For pion-production in

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

nucleus-nucleus collisions, however, the results were very uncertain owing to our very poor knowledge of the nucleus-nucleus optical potential. Thus, there is certainly great hope that a study of pion-production in the collisions of nuclei will do much to remove this uncertainty. It is not impossible that our description of the off-shell nuclear transition amplitude using a local potential may fail altogether. Until there are some experiments in this area, however, no statement from us on this subject can be considered appropriate.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

APPENDIX A

CONVENTIONS

Our conventions differ from those of Bjorken and Drell[19] only in the normalization of states and fields.

If A and B are two Lorentz vectors, we adopt a metric such that

$$A \cdot B = A_0 B_0 - \vec{A} \cdot \vec{B} \tag{A.1}$$

Relativistic single-particle states are normalized so that

$$\langle p, s_1 | p_2 s_2 \rangle = (2\pi)^3 \frac{p_0}{M} \delta^{(3)}(\vec{p}_1 - \vec{p}_2)\, \delta_{s_1 s_2} \tag{A.2a}$$

for fermions and

$$\langle q_1 | q_2 \rangle = (2\pi)^3 2q_0 \delta^{(3)}(\vec{q}_1 - \vec{q}_2) \tag{A.2b}$$

for bosons. The summation over states in these two cases is

$$\sum_{|ps\rangle} = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{M}{p_0} \sum_s = 2M \int \frac{d^4p}{(2\pi)^3} \delta^{(+)}(p^2 - M^2) \sum_s \tag{A.3a}$$

for fermions and

$$\sum_{|q\rangle} = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{2q_0} = \int \frac{d^4q}{(2\pi)^3} \delta^{(+)}(q^2 - m^2) \tag{A.3b}$$

for bosons with

$$\delta^{(+)}(p^2 - M^2) = \delta(p^2 - M^2)\, \theta(p_0) \tag{A.4}$$

The closure sum is then

$$1 = \sum_{|ps\rangle} |p,s\rangle \langle p,s| \tag{A.5a}$$

for fermions and

$$1 = \sum_{|q\rangle} |q\rangle \langle q| \tag{A.5b}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

for bosons with the summation being given by equations (A.4a) and (A.4b), respectively.

The scalar field is normalized so that

$$\langle \alpha | \varphi(x) | \beta \rangle = e^{-i\beta \cdot x} \tag{A.6}$$

The components of the charged scalar field are defined in terms of the three hermitian fields $\varphi^1, \varphi^2, \varphi^3$ according to

$$\varphi^a = \varphi^3 \tag{A.7a}$$

$$\varphi^{\pm} = \frac{1}{\sqrt{2}} (\varphi^1 \pm i\varphi^2) \tag{A.7b}$$

The Dirac field is normalized so that

$$\langle \alpha | \psi(x) | (p,s) \rangle = e^{-ip \cdot x} u(p,s) \tag{A.8}$$

Thus, for example, the reduction of the S-matrix according to the LSZ-formalism proceeds as follows:

$$\langle \alpha \beta(\text{out}) | O(\omega) | \beta(\omega) \rangle = \langle \alpha(\text{out}) | O(\omega) | \beta - \beta(i\omega) \rangle - i \int d^4x e^{i\beta \cdot x} (\Box + m^2) \langle \alpha(\text{out}) | R(\varphi(x) O(\omega)) | \beta(i\omega) \rangle \tag{A.9}$$

for outgoing scalar particles and

$$\langle \alpha(\text{out}) | O(\omega) | \beta(p,s) i\omega \rangle = \langle \alpha - (p,s)(\text{out}) | O(\omega) | \beta(i\omega) \rangle - i \int d^4x \langle \alpha(\text{out}) | R(O(\omega) \bar{\psi}(x)) | \beta(i\omega) \rangle (-i\not{\partial} - M) e^{-ip \cdot x} u(p,s) \tag{A.10}$$

for incoming Dirac particles where R denotes the causal commutator,

$$R(A(x) B(y)) = [A(x), B(y)] \Theta(x_4 - y_4) \tag{A.11}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Non-relativistically, we choose the normalization

$$\langle p_1 | p_2 \rangle = (2\pi)^3 \delta^{(3)}(\vec{p}_1 - \vec{p}_2) \tag{A.12}$$

so that the sume over states becomes

$$\sum_{|p\rangle} = \int \frac{d^3\vec{p}}{(2\pi)^3} \tag{A.13}$$

and the closure sum

$$\mathbb{1} = \sum_{|p\rangle} |p\rangle\langle p| \tag{A.14}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

APPENDIX B

THE AXIAL-VECTOR CURRENT

We present in this appendix some conventions regarding the axial-vector current and a derivation of the equal-time commutation relations used in the text.

The pion-axial vector coupling constant, f_π, is defined according to

$$\langle 0 | A_\mu^{(+)}(0) | \pi^+(q) \rangle = \frac{f_\pi}{m_\pi^2} q_\mu \qquad \text{(B.1)}$$

The left member of equation (B.1) is determined from the lifetime of the charged pion. Experimentally, f_π has the value $0.96 m_\pi^2$. Thus, we have for the neutral pion, making the necessary rotation in isospin space,

$$\langle 0 | A_\mu^0(0) | \pi^0(q) \rangle = \frac{f_\pi}{\sqrt{2} m_\pi^2} q_\mu \qquad \text{(B.2)}$$

The hermitian components of the pion field and the axial-vector current then satisfy the relation

$$\langle 0 | \partial_\mu A_\mu^i(x) | \pi^j \rangle = \frac{i f_\pi}{\sqrt{2}} \langle 0 | \varphi^i(x) | \pi^j \rangle \qquad \text{(B.3)}$$

where i=1,2,3. From this it follows within the framework of the LSZ-formalism,[35] as long as the pion is on the mass shell, that matrix elements of the pion field may then be replaced by identical matrix elements of the axial-vector current according to the prescription

$$\partial_\mu A_\mu^i(x) = \frac{i f_\pi}{\sqrt{2}} \varphi^i(x) \qquad i = 1,2,3 \qquad \text{(B.4)}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

For the charge components this becomes

$$\partial_\mu A^a_\mu(x) = \frac{i f_\pi^a}{\sqrt{2}} \varphi^a(x) \qquad a = 0, \pm 1 \tag{B.5}$$

with

$$f_\pi^0 = f_\pi \tag{B.6a}$$

$$f_\pi^\pm = \sqrt{2} f_\pi \tag{B.6b}$$

the different values of the coupling constant resulting from the difference in the definitions of the charge components of the pion field and the axial-vector current,

$$A^0_\mu(x) = A^3_\mu(x) \tag{B.7a}$$

$$A^\pm_\mu(x) = \tfrac{1}{2}\left(A^1_\mu(x) \pm i A^2_\mu(x)\right) \tag{B.7b}$$

(see also equations (A.7) in the preceding appendix).

The non-strangeness-changing charges, Q^i and Q^i_5, are defined in terms of the vector and axial-vector currents, $V^i_\mu(x)$ and $A^i_\mu(x)$, as

$$Q^i(x_0) \equiv \int d^3\vec{x}\, V^i_0(x_0, \vec{x}) \tag{B.8a}$$

$$Q^i_5(x_0) \equiv \int d^3\vec{x}\, A^i_0(x_0, \vec{x}) \tag{B.8b}$$

with $i = 1,2,3$, the index of the hermitian components. The time derivatives of these charges are

$$\dot{Q}^i(x_0) = \int d^3\vec{x}\, \partial_\mu V^i_\mu(x) \tag{B.9a}$$

$$\dot{Q}^i_5(x_0) = \int d^3\vec{x}\, \partial_\mu A^i_\mu(x) \equiv \int d^3\vec{x}\, D^i(x) \tag{B.9b}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

One usually assumes the conservation of the vector current (C.V.C.) so that Q^i is time-independent. One assumes also that $Q^i(x_o)$ and Q_5^i satisfy an SU(2) $\otimes$ SU(2) algebra, namely

$$\left[Q^i(x_o), Q^j(x_o)\right] = i\epsilon_{ijk}Q^k(x_o) \tag{B.10a}$$

$$\left[Q^i(x_o), Q_5^j(x_o)\right] = i\epsilon_{ijk}Q_5^k(x_o) \tag{B.10b}$$

$$\left[Q_5^i(x_o), Q_5^j(x_o)\right] = i\epsilon_{ijk}Q^k(x_o)$$

This algebra may be enlarged to include the nucleon field by introducing the equal-time commutation relation

$$\left[Q^i(x_o), \bar{\Psi}(x)\right] = \bar{\Psi}(x)\frac{\tau^i}{2} \tag{B.11a}$$

$$\left[Q_5^i(x_o), \bar{\Psi}(x)\right] = -\bar{\Psi}(x)\gamma_5\frac{\tau^i}{2} \tag{B.11b}$$

and similarly for $\Psi(y)$. Operating on equations (B.11) with $(-i\overleftarrow{\not\partial}-M)$ yields

$$\left[Q^i(x_o), J(x)\right] = J(x)\frac{\tau^i}{2} \tag{B.12a}$$

$$\left[Q_5^i(x_o), J(x)\right] = \left(J(x)+2M\bar{\Psi}(x)\right)\gamma_5\frac{\tau^i}{2} + i\int d^3\vec{y}\left[D^i(x_o,\vec{y}), \bar{\Psi}(x)\right]\gamma_o \tag{B.12b}$$

We may rewrite equation (B.12b) using equation (B.9b) as

$$\left[Q_5^i(x_o), J(x)\right] = \left(J(x)+2M\bar{\Psi}(x)\right)\gamma_5\frac{\tau^i}{2} + i\left[\dot{Q}_5^i(x_o), \bar{\Psi}(x)\right]\gamma_o \tag{B.12b'}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Crucial to the commutation relation given in equation (1.11) and the dispersion relation, equation (2.3), was

$$[\dot{Q}^{i}_{5}(x_0), \bar{\Psi}(x)] = 0 \tag{B.13}$$

Namely, that $D^{i}(x)$ obeys the same canonical comutation relation with the nucleon field as does the pion field. We will show that this vanishes from the current-algebra hypothesis and C.V.C.

From the Jacobi identity

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0 \tag{B.13$\frac{1}{2}$}$$

it follows that

$$[[Q^{i}_{5}(x_0),Q^{j}_{5}(x_0)], \bar{J}(x)] = [Q^{i}_{5}(x_0),[Q^{j}_{5}(x_0),\bar{J}(x)]] - (i \leftrightarrow j) \tag{B.14}$$

The left member of equation (B.14) is simply

$$i\epsilon_{ijk}[Q^{k}(x_0), \bar{J}(x)] = i\epsilon_{ijk}\bar{J}(x)\frac{\tau^{k}}{2} \tag{B.15}$$

The right member becomes using equation (B.11b)

$$[Q^{i}_{5}(x_0), (\bar{J}(x)+2M\bar{\Psi}(x))\gamma_5\frac{\tau^{j}}{2}+i[\dot{Q}^{j}_{5}(x_0),\bar{\Psi}(x)]\gamma_0] - (i\leftrightarrow j)$$

$$= \bar{J}(x)\frac{\tau^{i}}{2}\frac{\tau^{j}}{2} + i[\dot{Q}^{i}_{5}(x_0),\bar{\Psi}(x)]\gamma_0\gamma_5\frac{\tau^{i}}{2}$$

$$+i[Q^{i}_{5}(x_0),[\dot{Q}^{j}_{5}(x_0),\bar{\Psi}(x)]]\gamma_0 - (i\leftrightarrow j) \tag{B.17}$$

Now

$$\bar{J}(x)\frac{\tau^{i}}{2}\frac{\tau^{j}}{2} - (i\leftrightarrow j) = i\epsilon_{ijk}\bar{J}(x)\frac{\tau^{k}}{2} \tag{B.18}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

so that equation (B.14) becomes

$$\left[\dot{Q}_5^i(x_0), \bar{\Psi}(x)\right]\gamma_0\gamma_5\frac{\tau^j}{2} + \left[Q_5^i(x_0), \left[\dot{Q}_5^j(x_0), \bar{\Psi}(x)\right]\right]\gamma_0 - (i \leftrightarrow j) = 0 \tag{B.19}$$

The second term in equation (B.19) becomes after employing the Jacobi identity

$$-\left[\dot{Q}_5^i(x_0), \left[Q_5^j(x_0), \bar{\Psi}(x)\right]\right]\gamma_0 + \left[\left[Q_5^i(x_0), \dot{Q}_5^j(x_0)\right], \bar{\Psi}(x)\right]\gamma_0 - (i \leftrightarrow j)$$

$$= -\left[\dot{Q}_5^j(x_0), \bar{\Psi}(x)\right]\gamma_0\gamma_5\frac{\tau^i}{2} + \left[\left[Q_5^i(x_0), \dot{Q}_5^j(x_0)\right], \bar{\Psi}(x)\right]\gamma_0 - (i \leftrightarrow j) \tag{B.20}$$

so that equation (B.19) is

$$2\left[\dot{Q}_5^i(x_0), \bar{\Psi}(x)\right]\gamma_0\gamma_5\frac{\tau^j}{2} + \left[\left[Q_5^i(x_0), \dot{Q}_5^j(x_0)\right], \bar{\Psi}(x)\right]\gamma_0 - (i \leftrightarrow j) = 0 \tag{B.21}$$

The second term is equation (B.21), however, is

$$\left[\frac{d}{dt}\left[Q_5^i(x_0), Q_5^j(x_0)\right], \bar{\Psi}(x)\right]\gamma_0$$

$$= i\epsilon_{ijk}\left[\dot{Q}^k(x_0), \bar{\Psi}(x)\right]\gamma_0$$

$$= 0 \tag{B.22}$$

Thus

$$\left[\dot{Q}_5^i(x_0), \bar{\Psi}(x)\right] = 0 \tag{B.23}$$

and the theorem is proven.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

APPENDIX C

THE PRODUCTION POTENTIAL

The pion-production potential defined in equation (2.41) is not galilean invariant owing to the appearance of a factor $\exp(i\vec{q}.\vec{x}_j)$ in every term. Our purpose in the appendix is to present a prescription by which the expression for the production-potential is approximately galilean invariant. We admit ab initio that a galilean invariant expression which is valid both in the center-of-mass frame and pion rest frame cannot be constructed for any but the lowest energy pions. The pion must always be treated relativistically; the nuclear constituents of the reaction, because of their much larger masses than the pion, we assume can be treated in a non-relativistic fashion.

As before, we denote by P_i and P_f the total 4-momenta of the initial system and final system (excluding the pion). Thus,

$$\vec{q} = \vec{P}_f - \vec{P}_i \qquad \text{(C.1)}$$

Now $P_f - P_i$ is a Lorentz covariant but $\vec{P}_f - \vec{P}_i$ interpreted non-relativistically, is not a galilean invariant because the initial and final systems may have different "masses" owing to other inelastic processes accompanying the production. Writing these masses as

$$S_i = \sqrt{P_i \cdot P_i} \qquad \text{(C.2a)}$$

$$S_f = \sqrt{P_f \cdot P_f} \qquad \text{(C.2b)}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

we may write the Lorentz-invariant quantity $(P_f - P_i)^2$ to second order in the 3-momenta as

$$(P_f - P_i)^2 = (S_f - S_i)^2 - S_f S_i \left(\frac{\vec{P}_f}{S_f} - \frac{\vec{P}_i}{S_i}\right)^2 + \cdots \qquad (C.3)$$

The first two terms are clearly galilean invariant if we reinterpret $\vec{P}_f$ and $\vec{P}_i$ as non-relativistic observables. Higher order terms are $O((|\vec{P}|/S)^4)$ and therefore quite small if the nuclear systems are non-relativistic.

If the excitations of the final nuclear system are small compared to S_i we may write equation (C.3) as

$$(P_f - P_i)^2 \simeq (S_f - S_i)^2 - (\vec{P}_f - \vec{P}_i)^2 \qquad (C.4)$$

For pion production

$$(P_f - P_i)^2 = M_\pi^2 \qquad (C.5a)$$

$$(\vec{P}_f - \vec{P}_i)^2 = |\vec{q}|^2 \qquad (C.5b)$$

so that equation (C.4) becomes

$$M_\pi^2 = (S_f - S_i)^2 - |\vec{q}|^2 \qquad (C.6)$$

The pion-momentum in equation (C.6) is surely a Lorentz-invariant and therefore to order $((|\vec{P}|/S)^4)$ a galilean invariant as well. Using this as our value of $|\vec{q}|$ in the exponent immediately renders the potential galilean invariant. We note that equation (C.6) is equivalent to

$$\xi_0 = |S_f - S_i| \qquad (C.7)$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

which says that the pion energy is just the difference of the "masses" of the initial and final system. We note that the difference between this value of q_o and the value which q_o assumes in either the center-of-mass, laboratory, or Breit frame is very small. For convenience, we have always used the center-of-mass value.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

REFERENCES

1. One should not, perhaps,be so quick to discard the static-model calculations of G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956).
2. For a review of these developments see C. S. Wu and S. A. Moszkowski, Beta Decay, John Wiley & Sons, Inc. (New York) 1966.
3. The vector and axial-vector pieces are familiar in nuclear physics as the generators of the Fermi and Gamow-Teller transitions, respectfully.
4. S. S. Gerschtein and J. B. Zel'dovich, Soviet Physics JETP, 2, 576(1957)
5. R. P. Feynman and M. Gell-Mann, Phys. Rev., 109, 193 (1958)
6. M. Gell-Mann, California Institute of Technology Syncrotron Laboratory Report, CTSL-20 (1961)
7. M. Gell-Mann and Y. Ne'eman, Ann. Phys. (N.Y.), 30, 360 (1964)
8. S. L. Adler and R. F. Dashen, Current Algebra and Applications to Particle Physics, W. A. Benjamen, Inc. (New York) 1968
9. M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958)
10. Y. Nambu and D. Lurié, Phys. Rev. 125, 1429 (1962)
11. S. L. Adler, Phys. Rev. 137, B1022 (1965); B139, 1638 (1965).
12. S. L. Adler and Y. Dothan, Phys. Rev. 151, 1267.(1966)
13. F. E. Low, Phys. Rev. 110, 974 (1958)
14. S. Fubini and G. Furlan, Ann. Phys. (N.Y.) 48, 372 (1968)
15. M. Ericson, A. Figureau, and A. Molinari, Nucl. Phys. B10, 501 (1969)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

16. M. Ericson, preprint, Universite de Lyon, LYCEN-7017

17. M. K. Banerjee, C. A. Levinson, M. D. Shuster, and D. A. Zollman, University of Maryland Technical Report No. 70-081 (to appear in Phys. Rev. sec C)

18. We adopt the conventions of the elementary particle physicists, namely, (a = 1/2) denotes a proton and (a = -1/2) denotes a neutron.

19. J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, McGraw-Hill (New York) 1965

20. see Appendix B

21. M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960)

22. M. Genz and J. Katz; and M. Genz, J. Katz and S. Wagner, DESY preprints

23. D. A. Wray, Ann. Phys. (N.Y.) 51, 162 (1969)

24. M. Sugawara, Phys. Rev. 172, 1423 (1968)

25. M. K. Banerjee and C. A. Levinson, University of Maryland Technical Report No. 857 (1968) and private communication.

26. S. Weinberg, Phys. Rev. 166, 1568 (1968)

27. W. Zimmermann, Nuovo Cimento 10, 597 (1958); R. Haag, Phys. Rev. 112, 669 (1958); K. Nishijima, 111, 995 (1958).

28. A. Bohr and B. Mottelson, Nuclear Structure, Vol. 1, W. A. Benjamin Inc. (New York) 1969

29. H. Palevsky et al., Phys. Rev. Letts. 18, 519 (1967)

30. E. Hering, Phys. Rev. 106, 1271 (1957)

31. A. F. Dunaitsev and Yu. D. Prokoshkin, Nucl. Phys. 56, 300 (1964)

32. S. Mandelstam, Proc. Roy. Soc. Lon. A244, 491 (1958)

33. D. F. Jackson and C. G. Morgan, Phys. Rev. 175, 1402 (1968)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

34. A. M. Bernstein, Advances in Nuclear Physics, Vol. 3 325 (1970) and private communication

35. H. Lehmann, K. Symanzik and W. Zimmerman, Nuovo Cimento, I, 205 (1955)

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

FIGURE CAPTIONS

Figure 1: Kinematic variables for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$.

Figure 2: Contributions to the soft-pion limit.

Figure 3: Composite emission diagrams.

Figure 4: The optical potential.

Figure 5: Total pion-production cross-sections for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$. The incident energy is given in the center-of-mass system.

Figure 6: Energy differential cross-section for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$. All quantities are given in the center-of-mass system.

Figure 7: Angular differential cross-section for the reaction $p + C^{12} \rightarrow \pi^\circ + ?$. All quantities are given in the center-of-mass system.

Figure 8: Total pion-production cross-section for several projectiles on Cu^{65}: proton (solid line (x 10)), He^3 (dotted and dashed line), and alpha (dashed line).

Figure 9: Distorted and undistorted wave functions in the optical well.

Figure 10: Total pion-production cross-sections for the reaction alpha + $Cu^{65} \rightarrow \pi^\circ$ + ? for several choices of the optical well.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

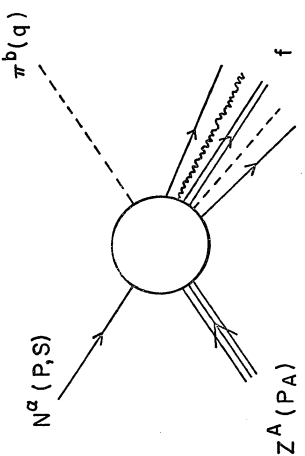

Figure 1

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

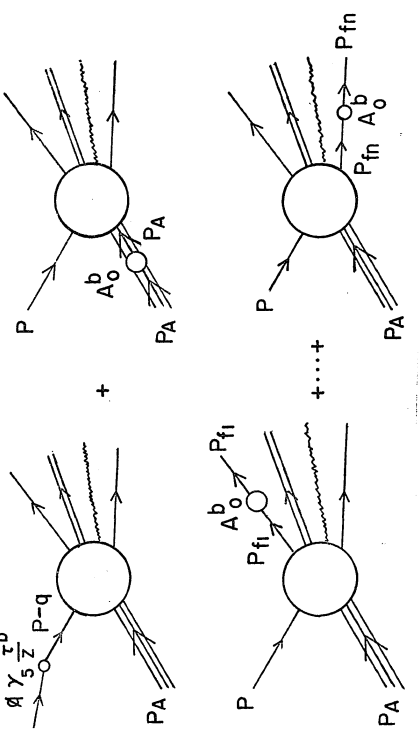

Figure 2

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

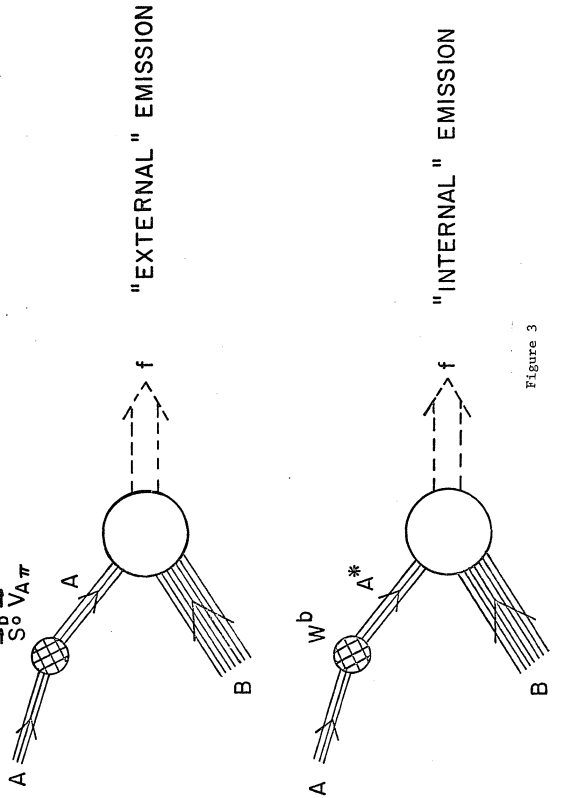

Figure 3

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

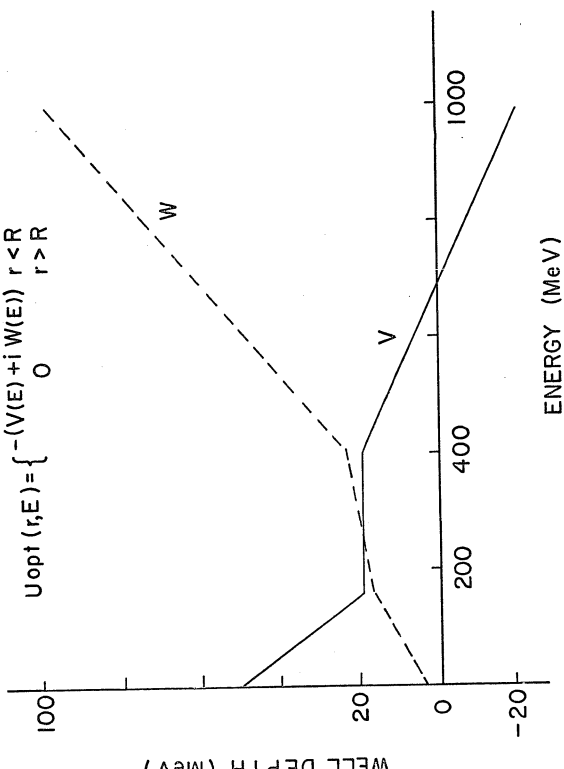

Figure 4

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

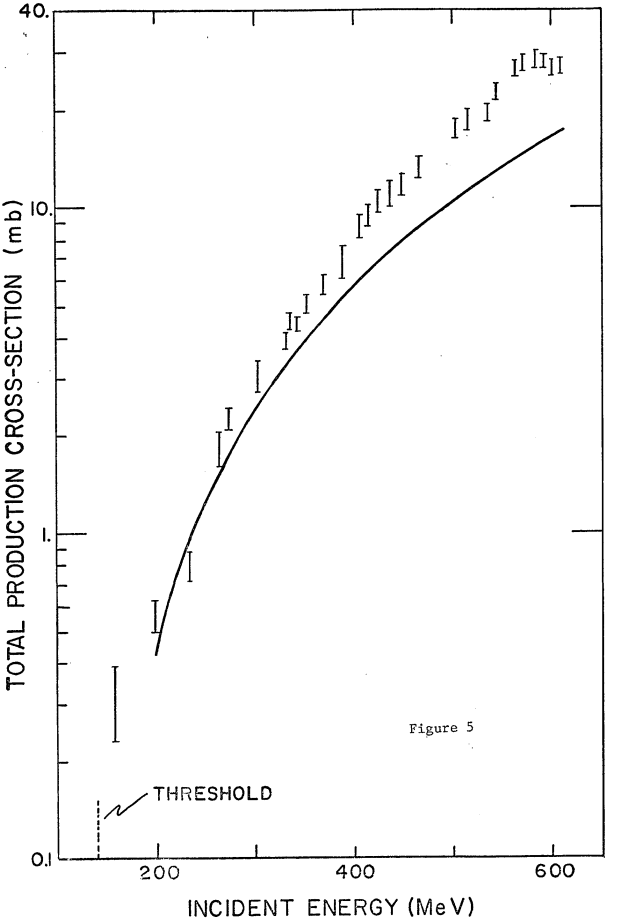

Figure 5

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

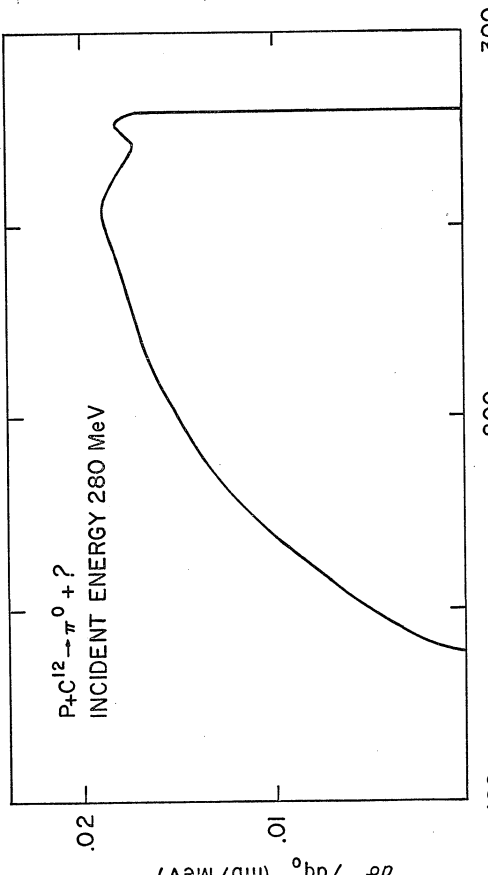

Figure 6

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

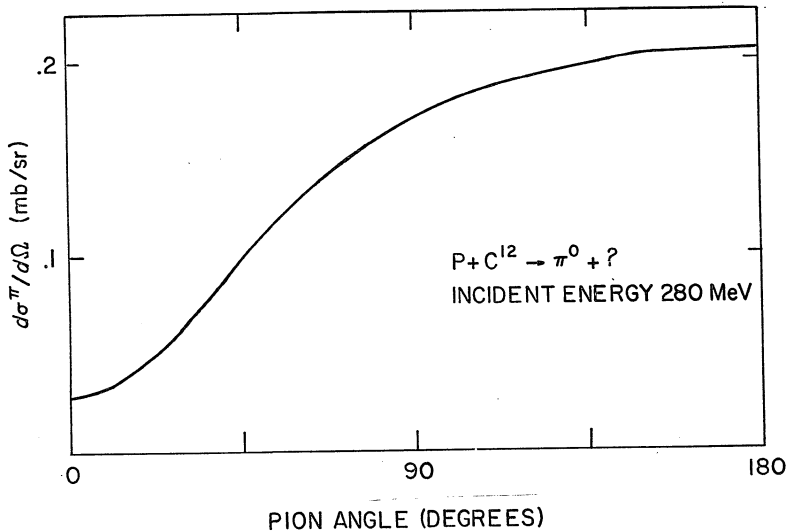

Figure 7

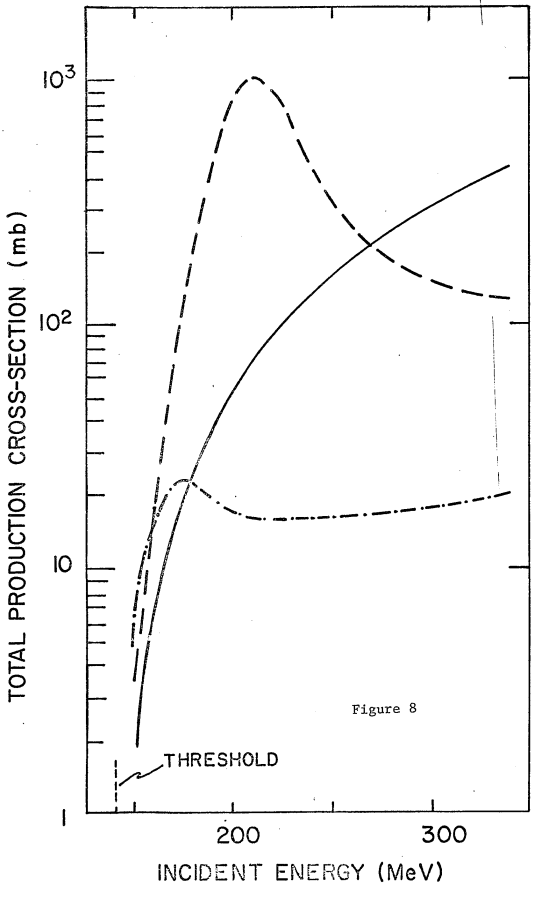

Figure 8

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

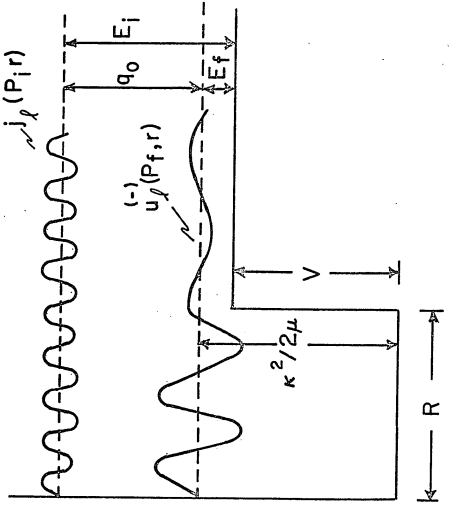

Figure 9

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

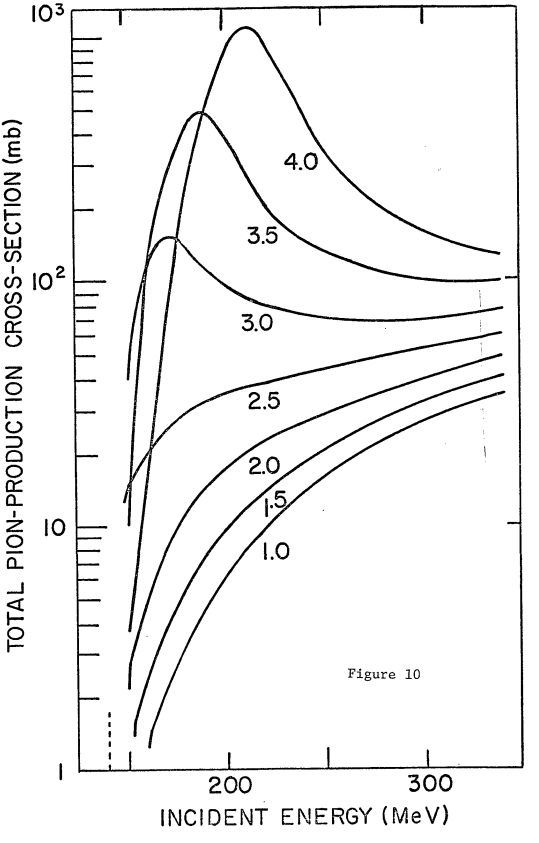

Figure 10

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

VITA

Name: Malcolm David Shuster

Permanent address: 3413 Tulane Drive, Hyattsville, Maryland

Degree and date to be conferred: Ph.D., January, 1971

Date of birth: July 31, 1943

Place of birth: Boston, Massachusetts

Secondary education: Revere High School, 1961

Collegiate institutions attended:	Dates	Degree	Date of Degree
Massachusetts Institute of Technology	1961-65	S.B.	1965
University of Maryland	1965-71	Ph.D.	1971

Major: Theoretical Nuclear Physics

Minors: General Physics, Contemporary Physics

Publications:

OFF-SHELL AMPLITUDES FROM PION PRODUCTION, Bull. Am. Phys. Soc. 15, 525(1970)

PION PRODUCTION IN NUCLEAR COLLISIONS, Bull. Am. Phys. Soc. 15, 525 (1970)

NON-RELATIVISTIC HARD-PION PRODUCTION AND CURRENT-FIELD ALGEBRA, University of Maryland Technical Report No. 70-081, February, 1970 (to appear in Physical Review, sec. C)

Positions held: Graduate Teaching Assistant, University of Maryland, College Park, Maryland, 1965-67

Graduate Research Assistant, University of Maryland, College Park, Maryland, 1967-70

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.