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# THE EFFECTS OF PROVISIONS FOR IMAGERY THROUGH MATERIALS AND DRAWINGS ON TRANSLATING ALGEBRA WORD PROBLEMS, GRADES SEVEN AND NINE

BY
PAUL JOSEPH SHOECRAFT

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## ACKNOWLEDGEMENTS

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# TABLE OF CONTENTS


| | Page |
|---|---|
| ACKNOWLEDGEMENTS | ii |
| LIST OF TABLES | v |
| LIST OF ILLUSTRATIONS | ix |
| LIST OF APPENDICES | x |
| CHAPTER | |
| I. INTRODUCTION TO THE PROBLEM | 1 |
| Statement of the Problem | 3 |
| II. RELATED LITERATURE | 7 |
| Definition of Terms | 8 |
| Psychologies of Problem Solving | 14 |
| The Function of Imagery in Problem Solving | 27 |
| III. RESEARCH DESIGN | 38 |
| Treatments | 38 |
| Instructions to Teachers | 42 |
| Organization of the Study | 44 |
| Timing and Subjects | 46 |
| Instrumentation and Analysis | 48 |
| IV. ANALYSIS OF DATA | 50 |
| Initial Characteristics | 51 |
| Number, Coin, Age | 58 |




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CHAPTER PAGE

Work and Mixture . . . . . 76

Detection of Contradictory Statements . . . . . 83

Retention . . . . . 87

Transfer . . . . . 91

Auxiliary Information and Representation . . . . . 94

Attitudes Toward Mathematics and Algebra Word Problems . . . . . 128

Interviews . . . . . 135

V. SUMMARY AND CONCLUSIONS . . . . . 144

Restatement of the Problem . . . . . 144

Findings and Conclusions . . . . . 145

Discussion . . . . . 155

APPENDICES . . . . . 158

BIBLIOGRAPHY . . . . . 237



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# LIST OF TABLES


| TABLES | | PAGES |
|---|---|---|
| 1- 2 | CORRELATION MATRIX, GRADES SEVEN AND NINE . . . . . | 52- 55 |
| 3- 4 | ANALYSIS OF COMPREHENSIVE TEST OF BASIC SKILLS (LEVELS 3-4) ARITHMETIC ACHIEVEMENT (COMPUTATION, CONCEPTS, APPLICATIONS -- 98 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 56- 57 |
| 5- 6 | ANALYSIS OF INITIAL PROBLEMS TEST (NUMBER, COIN, AGE -- 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . | 59- 60 |
| 7- 8 | ANALYSIS OF FINAL PROBLEMS TEST (NUMBER, COIN, AGE, WORK, MIXTURE -- 20 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 63- 64 |
| 9-10 | ANALYSIS OF VARIABLE TREATMENT PROBLEMS SUBTEST (NUMBER, COIN, AGE -- 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 66- 67 |
| 11-12 | ANALYSIS OF NUMBER PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 68- 69 |
| 13-14 | ANALYSIS OF COIN PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 70- 71 |
| 15-16 | ANALYSIS OF AGE PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 72- 73 |
| 17-18 | ANALYSIS OF IDENTICAL TREATMENT PROBLEMS SUBTEST (WORK, MIXTURE -- 8 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 77- 78 |
| 19-20 | ANALYSIS OF WORK PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 80- 81 |
| 21-22 | ANALYSIS OF MIXTURE PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 82, 84 |
| 23-24 | ANALYSIS OF PROBLEMS RETENTION TEST (NUMBER, COIN, AGE, WORK, MIXTURE -- 20 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADES SEVEN AND NINE . . . . . | 88- 89 |




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TABLES PAGES


25-26 Analysis of Transfer Test (10 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 92- 93

27-28 Analysis of Comprehensive Test of Basic Skills (Levels 3-4) Arithmetic Achievement (Computation, Concepts, Applications -- 98 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 96- 97

29-30 Analysis of Auxiliary Representations Questionnaire (Phrase, Sentence, Coin, Age, Work, Mixture -- 32 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . 99-100

31-32 Analysis of Auxiliary Representations Questionnaire (Phrase, Sentence, Coin, Age, Work, Mixture -- 32 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . 101-102

33-34 Analysis of Phrase Translation Subtest (9 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 103-104

35-36 Analysis of Sentence Translation Subtest (3 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 106-107

37-38 Analysis of Phrase Translation Subtest (9 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 109-110

39-40 Analysis of Sentence Translation Subtest (3 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 111-112

41-42 Analysis of Coin Subquestionnaire (6 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 113-114

43-44 Analysis of Age Subquestionnaire (5 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 115-116

45-46 Analysis of Coin Subquestionnaire (6 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 118-119

47-48 Analysis of Age Subquestionnaire (5 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 120-121




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TABLES


| | | Pages |
|---|---|---|
| 49-50 | Analysis of Work Subquestionnaire (3 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 122-123 |
| 51-52 | Analysis of Work Subquestionnaire (3 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 124-125 |
| 53-54 | Analysis of Mixture Subquestionnaire (6 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 126-127 |
| 55-56 | Analysis of Mixture Subquestionnaire (6 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 129-130 |
| 57-58 | Analysis of Attitude Toward Mathematics Test (80 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 131-132 |
| 59-60 | Analysis of Attitude Toward Mathematics Test (80 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 133-134 |
| 61-62 | Analysis of Affinity Test (2 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 136-137 |
| 63-64 | Analysis of Affinity Test (2 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 138-139 |
| 65-66 | Analysis of Variable Treatment Problems Retention Subtest (Number, Coin, Age -- 12 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 222-223 |
| 67-68 | Analysis of Identical Treatment Problems Retention Subtest (Work, Mixture -- 8 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 224-225 |
| 69-70 | Analysis of Number Problem Retention Subtest (4 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 226-227 |
| 71-72 | Analysis of Coin Problem Retention Subtest (4 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 228-229 |
| 73-74 | Analysis of Age Problem Retention Subtest (4 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine | 230-231 |




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TABLES PAGES

75-76 Analysis of Work Problem Retention Subtest (4 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . . . 232-233

77-78 Analysis of Mixture Problem Retention Subtest (4 Points Possible) By Treatment Groups and Achievement Subgroups, Grades Seven and Nine . . . 234-235



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# LIST OF ILLUSTRATIONS


| | Pages |
|---|---|
| Dewey's Problematic Situations | 9 |
| A Task Hierarchy for Objectives of Assigned Tasks | 19 |
| Maier's Pendulum Problem | 24 |
| Diagram of the Study | 45 |
| Results of Interview by Treatment Groups, Grade Seven | 141-43 |
| Significantly Superior Treatment Subgroups by Variable, Grades Seven and Nine | 151-52 |




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# LIST OF APPENDICES


| APPENDIX | PAGE |
|---|---|
| A. Instructional Materials for Number, Coin, and Age Problems: HIM . . . . . | 158 |
| B. Instructional Materials for Number, Coin, and Age Problems: LO . . . . . | 182 |
| C. Instructional Materials for Work and Mixture Problems . . . . . | 197 |
| D. Instrumentation and Computation of Reliability Coefficients . . . . . | 206 |
| E. Analysis of Retention Subtest Scores and Results by Variable for Two Kennedy Junior High Ninth Grade Algebra Classes Assigned to HIM and HID, Respectively . . . . . | 221 |




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# CHAPTER I

## INTRODUCTION TO THE PROBLEM

Problem solving is well recognized as having great importance in human life, particularly in the realm of productive and creative activity. While used in a slightly different sense, problem solving is central to the activity of a mathematician.

In 1940 a committee of the Progressive Education Association analyzed the problem-solving process and urged that teaching of mathematics stress methods and skills in this area at all levels of instruction (Jones and Coxford, 1970). In way of continued support of problem solving as a major objective of school mathematics, Polya (1968) claims that problem solving should receive top priority in any mathematics class, and Botts (1965) states there is a growing realization of the importance of problem solving in mathematics and an increasing list of books and journals where good problems (at all levels) can be found.

As one facet of problem solving in mathematics, educators have advocated solution of word problems. Lerch (1966), for example, states that a major general objective of elementary mathematics programs is the generation of ability to solve verbal problems, and Brown (1964) claims that this objective should be the major goal of ninth grade mathematics.

Traditionally, when it comes to advice on teaching students to solve word problems, authors have relied heavily on the "step" method (e.g., Ander-



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son, 1965; Coleman, 1964; Ellis, 1943; McLeod and McIntyre, 1937; O'Brien, 1956; Stahl, 1953; Topoly, 1965; Wittrock, 1967; Yudowitch, 1947). Under this approach solution is presented as a sequentially ordered pattern of thought, usually taking one of two forms. In one the directives resemble a checklist (e.g., Draw a picture. Let x = ....); in the other they are analytic (e.g., What is asked? What is given?). In some cases, as with Polya's (1945) "How To Solve It" list, the prompting is a combination of the two.

Although there is evidence that a systematic approach to problem solving is better than none for inculcating problem-solving skill (Kinney, 1959), a number of authors question the helpfulness of the step method. In general, they challenge this apppoach as purchasing generality at the expense of sufficient guidance. For example, in 1930 Ligda listed the following sequence as typical of directives appearing in texts for solution of algebra word problems.

> State what is given and what is to be found.
>
> Represent one of the unknowns by means of some letter; represent all of the other unknowns in terms of the same letter.
>
> After a study of the relations between the parts of the problem, express the verbal statement in an algebraic equation.
>
> Solve and check.

According to Ligda, the major failing with such prescriptions is their powerlessness to render the "parts" and their "relations" to the problem solver. That is, in no way do such directives foretell, say, "This is a part; these are the other parts; and these are the relations among these parts."*

---

*To account for parts and relations a number of educators turned to box-like diagrams (e.g., Haertter, 1931; Henderson, 1954; Mills, 1961) or to leading questions (e.g., Clements, 1955; Lueck, 1945) to result in a listing of the parts. Their relationship was then a matter of formula (e.g., d = rt). This approach is rarely advocated today, however, perhaps because it tends to emphasize the contrived nature of verbal problems.

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At present, it is widely advocated that solution of word problems proceed through "open sentences." These amount to intermediaries between the purely verbal statement of a problem and the final purely mathematical statement (e.g., the value of the nickels + the value of the dimes = the value of all the coins). Their efficacy is applauded by a number of authors (e.g., Dubisch, 1963; Shanok, 1937), and their use is suggested in a number of current beginning algebra texts, including Mathematics for High School: First Course in Algebra (SMSG, 1960) and Modern Algebra, Structure and Method: Book One (Dolciani, et al., 1965).

Whereas the step method tends to emphasize technique at the possible expense of additional factors which may influence problem solving (Ausubel, 1968; Brownell, 1942; MacLatchy, 1941), the open-sentence approach directs attention to the problem at hand. Perhaps this is why the open-sentence approach is so widely acclaimed. Nonetheless, it too skirts the central difficulty in solving word problems, namely, identification of the parts of a problem in their relation to one another. To request of a student an open sentence, which is to derive full-blown from the reading of a problem, is to presuppose his understanding of the structural characteristics of the situation set forth and his awareness of the syntax of the problem statement.

## STATEMENT OF THE PROBLEM

Solution of word problems necessarily involves some sequencing of events, namely, translation, then manipulation. By translation is meant interpreting ideas, natural phenomena, and non-mathematical symbols (such as words) in terms of mathematical symbols. By manipulation is meant responses for dealing with mathematical symbols. Of the two, the recognition and mathematical statement of quantitative relationships (that is, translation), rather than the algorithmic handling of the resultant symbols (that is, man-

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IPULATION), LIES AT THE HEART OF THE PROBLEM-SOLVING PROCESS. EVEN SO, NEITHER THE STEP METHOD NOR THE OPEN-SENTENCE APPROACH GIVES MORE THAN PERFUNCTORY NOTICE TO THE CONSTITUENTS OF TRANSLATION, NAMELY, PARTS AND RELATIONS.

ACCORDINGLY, THE WRITER SOUGHT A METHOD FOR TEACHING TRANSLATION THAT UNDERSCORED THE STRUCTURAL CHARACTERISTICS AND SYNTACTICAL PROPERTIES OF WORD PROBLEMS, IN PARTICULAR, OF THE FOLLOWING TYPES OF ALGEBRA WORD PROBLEMS: NUMBER, COIN, AND AGE. WITH THIS IN MIND, METHODOLOGY WAS DEVISED THAT PRECEDED ALGEBRAIC REPRESENTATION OF STATED MATHEMATICAL RELATIONSHIPS WITH A FORM OF VISUAL REPRESENTATION SO DESIGNED AS TO REFLECT THE INTERDEPENDENCE OF THE QUANTITIES INVOLVED. AS A RESULT OF ENCOUNTERING ALGEBRA WORD PROBLEMS IN THIS MANNER, IT WAS EXPECTED THAT STUDENTS WOULD COME TO REPRESENT THE RELATIONAL ASPECTS OF THESE PROBLEMS IN IMAGE FORM. THE IMAGES WERE TO SERVE TRANSLATION, PERHAPS SPONTANEOUSLY, AS REFERENTS FOR NOTATION BEING APPLIED TO PROBLEMS.

IN THE BROADEST SENSE, THEN, THE TOPIC OF THIS STUDY WAS THE FUNCTION OF MENTAL IMAGERY IN PROBLEM SOLVING. IN RESTRICTED SCALE, THE STUDY DEALT WITH THE EFFECTS OF VARYING INSTRUCTION ON TRANSLATION OF SELECTED ALGEBRA WORD PROBLEMS, AND THE MAIN VARIABLE UNDER MANIPULATION WAS THE INCLUSION OF ACTIVITIES TO RESULT IN A BASIS FOR MENTAL IMAGES EMBODYING THE RELATIONAL ASPECTS OF THE PROBLEM TYPES UNDER CONSIDERATION.

SINCE THE USUAL FOUNDATION FOR MENTAL IMAGES IS OBJECTS AND/OR PICTORIAL REPRESENTATIONS OF OBJECTS, TWO IMAGE-BASED SEQUENCES WERE PREPARED FOR TREATMENT OF NUMBER, COIN, AND AGE PROBLEMS. IN ONE, STATED MATHEMATICAL RELATIONSHIPS WERE DEPICTED THROUGH CONCRETE MATERIALS, IN THE OTHER THROUGH DRAWINGS. IN EITHER, HOWEVER, THE END RESULT WAS THE SAME REGARDLESS OF PROBLEM TYPE: A PAIR OF "STACKS," ONE STACK TALLER THAN THE OTHER (E.G., ). THE DISPARITY BETWEEN THE HEIGHTS OF THE STACKS

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PERMITTED AN ICONIC REPRESENTATION OF CERTAIN PHRASES COMMON TO ALGEBRA WORD PROBLEMS (E.G., "MORE THAN," "LESS THAN," "EXCEEDS BY"), AND EQUALITY COULD BE ENCODED IN TERMS OF ACCOUNTING FOR THIS DISPARITY.

IN ADDITION TO THE TWO IMAGE-BASED SEQUENCES, A THIRD SEQUENCE ON NUMBER, COIN, AND AGE PROBLEMS WAS PREPARED. THIS WAS THE MORE TRADITIONAL OF THE THREE IN ITS REFERRING ALGEBRAIC NOTATION TO NUMBER AND ITS STRICT RELIANCE ON GRAMMATICAL CUES FOR TRANSLATION.* RESULTS ASSOCIATED WITH THIS LATTER TREATMENT WERE USED AS A STANDARD AGAINST WHICH THOSE FOR THE IMAGE-BASED SEQUENCES WERE JUDGED.

THE PROBLEM CAN THEREFORE BE STATED AS FOLLOWS: WHAT ARE THE EFFECTS OF PROVISIONS FOR IMAGERY THROUGH MATERIALS AND DRAWINGS ON TRANSLATING ALGEBRA WORD PROBLEMS AS COMPARED TO INSTRUCTION RELYING PRIMARILY ON VERBAL CUES FOR TRANSLATION? PUTTING THIS IN TERMS OF THE AFOREMENTIONED INSTRUCTIONAL SEQUENCES (THAT IS, TREATMENTS), THE STUDY SOUGHT TO ANSWER THE FOLLOWING QUESTIONS WITH RESPECT TO ACHIEVEMENT, SUBSEQUENT LEARNING OF WORK PROBLEMS AND MIXTURE PROBLEMS, RETENTION, TRANSFER, AND ATTITUDE:

1. WHICH TREATMENT IS THE MOST EFFECTIVE WITH RESPECT TO ACHIEVEMENT ON NUMBER, COIN, AND AGE PROBLEMS?
2. WHICH TREATMENT IS THE MOST EFFECTIVE IN PREPARING FOR SUBSEQUENT LEARNING OF WORK AND MIXTURE PROBLEMS?
3. WHICH TREATMENT TENDS TOWARD GREATER RETENTION?
4. WHICH TREATMENT IS MOST PRODUCTIVE IN WAY OF TRANSFER?
5. WHICH TREATMENT RESULTS IN MOST FAVORABLE ATTITUDE TOWARD MATHEMATICS?
6. WHICH TREATMENT RESULTS IN GREATEST TENDENCY TO SELECT ALGEBRA WORD PROBLEMS FOR SOLUTION?

---

*BY REFERRING NOTATION TO NUMBER IS MEANT THE FOLLOWING: A STATEMENT SUCH AS "ONE NUMBER EXCEEDS ANOTHER BY THREE" WAS TRANSLATED AS $x = y + 3$ WHERE Y WAS TAUGHT AS THE SMALLER NUMBER TO BE BOOSTED BY ADDITION TO EQUAL THE LARGER, X.

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7. Do students categorized on the basis of prior achievement in arithmetic perform differentially with respect to particular treatment?

In addition, instrumentation was so designed as to provide a basis for answering the following more general questions:

8. Does image-based instruction result in greater tendency to attend not only to the verbal description of a situation but to its physical characteristics as well?

9. To what extent does skill in translating phrases determine success in sentence translation?

10. Can it be expected that the study of algebra word problems will result in increased understanding of certain aspects of our environment?

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# CHAPTER II

## RELATED LITERATURE

The review of the literature is done in three parts. First, the meanings most commonly associated with the words "problem" and "problem solving" are delineated since their use is so varied in both ordinary and technical language. Second, in recognition of the complicated nature of problem solving encompassing learning and other complex events, considerable attention is given to developing a conceptual scheme for handling the myriad detail associated with this topic. This was accomplished through categorizing basic notions of how we learn and solve problems to their respective theories which in turn were ordered to a philosophical position on the nature of man. Theory was related to the essence of man in keeping with the pedagogical dictum that as we perceive man, so do we educate him. The writer decided not to include those systems of thought associated with simulated intelligence (e.g., Newell et al., 1958; Simon and Newell, 1964; Ernst and Newell, 1969) since investigations in this area seem more intent on building theory, per se, than in deriving implications for instruction. Finally, selected empirical studies receive notice. Since the field of problem solving is large, only those studies relating imagery to problem solving were developed in any detail.

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# PART I: DEFINITION OF TERMS

## PROBLEM

The popular conception of problem implies the existence of (1) a goal, (2) an obstacle not immediately surmountable in the pathway to the goal, and (3) a felt need to achieve the goal. Duncker (1945) states that "A problem arises when a living creature has a goal but does not know how this goal is to be reached," and Bloom and Broder (1950, p.7) give a similar definition:

> A problem may be regarded as a task which the subject is able to understand but for which he does not have an immediate solution. It is expected that although the subject will be somewhat perplexed, he will not be utterly confused.

Dewey (1910) adds to this "no-path" form of problem with his familiar "forked-path" situations diagramed on the next page.

These descriptions of problem lead to the inference that "felt tension" is a necessary concomitant to "having a problem." That is, a problem has an affective component. The significance of this for education is recognized in statements to the effect that what constitutes a problem for the teacher becomes a problem for the student only in the event that he embraces it (e.g., Henderson and Pingry, 1953). This point of view was particularly prevalent in the writings of the Progressive Education Association which sought a more life-centered curriculum.

A somewhat mechanistic conception of problem that de-emphasizes its affective aspect is advanced by Reitman (1965, p.126):

> A system has a problem when it has or has been given a description of something but does not yet have anything that

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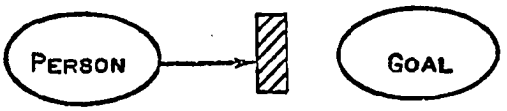
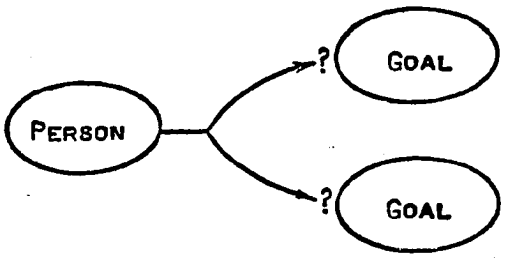
## Dewey's Problematic Situations

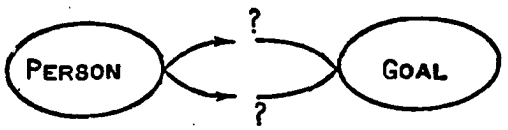
### No-Path Situation
(obstacle appears insurmountable)




### Forked-Path Situation A
(alternate, but equally attractive, goals)




### Forked-Path Situation B
(single goal; alternate, but equally attractive, paths)

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satisfies the description.

As a result of this less restrictive definition, any task or any goal is synonymous with problem. In addition, any idea can be converted into a problem simply by requiring an instance of it.

## PROBLEM SOLVING

According to Duncan (1959), most authors regard problem solving as high on the discovery demension (e.g., Maltzman, et al., 1958):

> The defining characteristics most frequently mentioned are the integration and organization of past experience when the definition refers to all of thinking, and the dimension of discovery of correct response when reference is made to problem solving specifically. (Underlining for emphasis.)

The exalted position of problem solving on the discovery spectrum distinguishes it from conditioning and rote learning, which are presumed to involve relatively little discovery of correct response.

As Duncan points out, nearly all writers concerned with definitions emphasized that they were trying to define thinking or problem solving in such a way as to relate them to simpler processes such as learning, perception, or motivation (e.g., Kendler and Kendler, 1962). Gagne (1964) serves to exemplify the extreme of this trend: "problem solving, regardless of what may have preceded it, is itself a form of learning."

Gagne (1965, p.132) defines problem solving as follows:

> Problem solving is an inferred change in human capability that results in the acquisition of a generalizable rule which is novel to the individual, which cannot have been established by direct recall, and which can manifest itself in applicability to the solution of a class of problems.

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He states (Gagne, 1966) that the kind of human capability acquired in problem solving seems to be one of <u>applying a rule</u> to any number of specific instances.

Newell (1966, pp.172-74) believes that Gagne's adherance to the notion of generalizable rule as a concomitant to problem solving excludes certain complex instances of problem solving. He gives as an example the Firing Squad Problem* with the accompanying explanation:

> There is little doubt that it is a problem....One shows he has solved the problem by giving the solution; there is no need to generalize, nor to show transfer to some other task.

Skinner (1966, p.375) contends that problem solving is a behavioral event:

> A problem is defined when a reinforcing event is contingent upon something, either a property of the response or a property of the environment. Problem solving is the behavior that brings about the condition under which reinforcement will occur.

As Travers (1967) points out, this is essentially the Associationistic viewpoint on problem solving.

Dewey is cited by Andersen (1967) as describing problem solving as a five-phase, thinking-behavioral process:

---

*A long line of soldiers in a dense fog is to fire simultaneously upon command of the General, who stands at the end of the line. However, each soldier is only aware of the men immediately to the right and left of him. The General gives the command "Fire when ready." (Note that only the man next to the General is aware initially that this command is given.) What system of communication is to be used by the soldiers so that they can all fire at the same time? It is also assumed (and this is what makes the problem hard) that not only is the length of the line unknown, but that neither the soldiers nor the General can count above some number, say 1,000, whereas the line can be many times as long, say 1,000,000. (This problem has only recently been solved.)

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1. The student is involved in a situation which is interesting to him.

2. A genuine problem develops within the situation which challenges the student.

3. The student constructs hypotheses and collects data.

4. The student arrives at several solutions.

5. The student has the opportunity to test his solution.

General Problem Solving theory (Ernst and Newell, 1969) describes problem solving as a means-end analysis wherein a desired outcome and the current state of a system are examined for differences which are then reduced (hopefully) until a state of equilibrium is reached. In this theory the desired outcome is a significant determinant of the problem-solving behavior emitted. That is, "the end suggests the means" (Polya, 1968).

Ausubel (1968, p.533), a cognitive psychologist, defines problem solving as follows:

> Problem solving refers to any activity in which both the cognitive representation of prior experience and the components of a current problem situation are reorganized in order to achieve a designated objective....When the activity is limited to the manipulation of images, symbols, and symbolically formulated propositions, and does not involve overt manipulation of objects, it is conventional to use the term <u>thinking</u>.

In this context problem solving might well be summarized as <u>effective thinking</u>.

Bruner (1968, pp.50-51) states that all descriptions of problem solving (and learning) agree on one essential feature:

> that there is a <u>cycle</u> involving the (1) formulation of a testing procedure or trial, (2) the operation of this testing procedure, and (3) the comparison of the results of the test

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WITH SOME CRITERION. (UNDERLINING FOR EMPHASIS, NUMBERING FOR CLARITY.)

ACCORDING TO BRUNER, FOR KNOWLEDGE OF RESULTS TO FACILITATE THE PROBLEM-SOLVING PROCESS IT SHOULD COME AT THAT POINT IN A PROBLEM-SOLVING EPISODE WHEN THE PERSON IS COMPARING THE RESULTS OF HIS TRYOUT WITH SOME CRITERION OF WHAT HE SEEKS TO ACHIEVE (STEP 3).

FINALLY, PIAGET (TRAVERS, 1967) INTERPRETS PROBLEM SOLVING AS THAT ACTIVITY BY WHICH AN ORGANISM COPES WITH ITS ENVIRONMENT. IN THIS THEORY PROBLEM SOLVING IS CONSIDERED TO BE THE CENTRAL FUNCTION OF THE INTELLECT; THEREFORE, PIAGET'S LIFETIME WORK ON ATTEMPTING TO DESCRIBE THE DEVELOPMENT OF THE HUMAN INTELLECT HAS BEEN DEVOTED TO DESCRIPTIONS OF THE DEVELOPMENT OF PROBLEM-SOLVING ABILITY.

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# PART II: PSYCHOLOGIES OF PROBLEM SOLVING

While the role of theories is to bring order among facts without needless proliferation of concepts, oversimplification in an area such as problem solving may succeed only in caricaturing human nature. Starting with this conviction, the systems of thought that unfold call for conceptual openmindedness and reasoned eclecticism.

In order to lay the groundwork for the development of a conceptual framework for human problem solving, it will be helpful to have in mind two broadly contrasting approaches to a psychology of problem solving. Virtually all modern psychological theories seem oriented toward one of two polar conceptions of man, which, at the risk of historical oversimplification, Allport (1955) refers to as the Lockean and Leibnitzian traditions, respectively. It is not the total philosophy of Locke or of Leibnitz that provide the basis for such a polarization, but their positions on one aspect of man's mind: its essentially passive nature (Locke) or its essentially active nature (Leibnitz).

## THE LOCKEAN TRADITION

In the seventeenth century John Locke challenged the notion of innate faculties or ideas and with it the conception of learning as development of innate potentialities or faculties. To Locke the mind at birth was a <u>TABULA RASA</u>, a blank tablet upon which experience etched the substance of intellect. The intellect itself was a passive thing acquiring content and structure only through the impact of sensations and the crisscross of associations. Nothing could exist in the intellect without having first existed in the senses (<u>NIHIL EST IN</u>

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INTELLECTU QUOD NON FUERIT IN SENSU).

LEIBNITZ PROPOSED A CHALLENGING SUPPLEMENT TO THIS FORMULA: NOTHING--SAVE ONLY THE INTELLECT ITSELF (EXCIPE: NISI IPSE INTELLECTUS). TO LEIBNITZ THE INTELLECT WAS A GOING CONCERN IN ITS OWN RIGHT: PERPETUALLY ACTIVE, ADDICTED TO RATIONAL PROBLEM SOLVING, AND BENT ON MANIPULATING SENSORY DATA ACCORDING TO ITS OWN INHERENT NATURE. FOR LOCKE THE INTELLECT WAS REACTIVE WHEN STIMULATED, A RECEPTACLE FOR EXPERIENCE; FOR LEIBNITZ IT WAS SELF-PROPELLED, A PROGENITOR OF EXPERIENCE.

AS WILL BECOME APPARENT, THE TWO DISCIPLINES OF SCIENTIFIC PSYCHOLOGY, WHAT CRONBACH (1963) REFERS TO AS EXPERIMENTAL PSYCHOLOGY AND CORRELATIONAL PSYCHOLOGY, RESPECTIVELY, DIFFER ON JUST THIS POINT AS THEY CONCEPTUALIZE MAN. EXPERIMENTAL PSYCHOLOGY, INCLUDING ASSOCIATIONISM OF ALL TYPES (ENVIRONMENTALISM, BEHAVIORISM, STIMULUS-RESPONSE OR S-R PSYCHOLOGY, AND OTHER STIMULUS-ORIENTED PSYCHOLOGIES), IS DECIDEDLY LOCKEAN IN EMPHASIS. CORRELATIONAL PSYCHOLOGY, ENCOMPASSING CONTEMPORARY COGNITIVE PSYCHOLOGY AND GESTALT PSYCHOLOGY, IS DECIDEDLY LEIBNITZIAN.

OF COURSE, NEITHER EXPERIMENTAL PSYCHOLOGY NOR CORRELATIONAL PSYCHOLOGY CAN BE NEATLY ORDERED TO THIS SIMPLE, BASIC DICHOTOMY. ANY GIVEN THEORY FROM EITHER WILL LIKELY CONTAIN ELEMENTS OF BOTH HISTORICAL MODELS, OF WHICH BOTH ARE TO A DEGREE CORRECT AND USEFUL. HOWEVER, CERTAIN FUNDAMENTAL TENETS, WHETHER TACIT OR OTHERWISE, IN SUCH THEORIES TEND TO SEGREGATE THEM INTO LOCKEAN OR LEIBNITZIAN CAMPS.

BY EXPERIMENTAL PSYCHOLOGY IS MEANT NOT ONLY A METHOD OF INQUIRY, NAMELY, THAT OF CHANGING CONDITIONS IN ORDER TO OBSERVE THEIR CONSEQUENCES, BUT A SET OF FUNDAMENTAL PRESUPPOSITIONS THAT INFLUENCES THE INTERPRETATION OF RESULTS. AMONG THOSE COMMON TO LOCKEAN EMPIRICISM -- THE BELIEF THAT ALL OF A PERSON'S IDEAS MUST COME TO HIM THROUGH HIS

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SENSES -- IS THE CONVICTION THAT THE MOST FUNDAMENTAL OF ALL IS THAT WHICH IS EXTERNAL AND VISIBLE (ALLPORT, 1955). SINCE THE MIND, BY NATURE A TABULA RASA, IS AS A SPONGE -- THIRSTY, PERHAPS; BUT NONE THE LESS INACTIVE -- IT IS THE RAPPINGS FROM WITHOUT RATHER THAN THE HEARTH WITHIN THAT IS IMPORTANT. EVEN MOTIVES ARE INTERPRETED PHYSIOLOGICALLY IN TERMS OF THE CONDITION OF PERIPHERAL TISSUES AS DETERMINED BY EXCESS OR DEFICIT STIMULATION IN THE BODY CAVITIES.

NOR IS THE ADMISSION IN RECENT YEARS OF SO-CALLED "INTERVENING VARIABLES" OR Os (FOR ORGANISM -- WHAT LEIBNITZ WOULD CALL THE INTELLECT) BETWEEN THE Ss AND Rs OF ASSOCIATION THEORY (E.G., MALTZMAN, 1955) MORE THAN A MINIMAL DEPARTURE FROM THE NOTION THAT THE ORGANISM PLAYS A SECONDARY ROLE TO ENVIRONMENT. LEARNING, THEN, IS REGARDED AS THE SUBSTITUTION OF ONE EFFECTIVE RESPONSE FOR ANOTHER. MOREOVER, THE STIMULUS-BASED MODEL OFFERS A PHYSIOLOGICAL DESCRIPTION OF COGNITIVE ACTIVITY.

A FURTHER PRESUPPOSITION COMMON TO EXPERIMENTAL PSYCHOLOGY AND LOCKEAN EMPIRICISM IS THE BELIEF THAT WHAT IS SMALL AND MOLECULAR IS MORE FUNDAMENTAL THAN WHAT IS LARGE AND MOLAR (ALLPORT, 1955). THE HABIT UNIT IS THE FOCUS OF ATTENTION FOR THOSE WORKING IN THE FIELD OF LEARNING, GROWTH, AND DEVELOPMENT. SKINNER (ANDERSEN, 1967), CONCERNED WITH HOW WE LEARN AND THE PRACTICAL CONDITIONS WHICH PROMOTE LEARNING, EXEMPLIFIES THIS VIEWPOINT. HE FEELS THAT PROBLEM SOLVING MUST BE REDUCED TO ITS SPECIFIC COMPONENT BEHAVIORS BEFORE WE CAN TEACH IT.

PREOCCUPATION WITH MOLECULAR UNITS TENDS TO EVOKE A STRONG COMMITMENT TO EQUIVALENCE OF SPECIES (ALLPORT, 1955), THE BELIEF THAT EVERY BASIC FEATURE OF HUMAN NATURE CAN BE STUDIED WITHOUT ESSENTIAL LOSS AMONG ANIMALS OTHER THAN MAN. SINCE MAN IS AN ANIMAL,

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WHY NOT LOOK AT MORE SIMPLE ANIMALS AS PROTOTYPES? ACCORDINGLY, THORNDIKE (KOFFKA, 1924) FORMULATED HIS FAMOUS "LAWS OF LEARNING" ON THE BASIS OF OBSERVATIONS OF HUNGRY, CAGED CATS AND DOGS. SKINNER'S (BUGELSKI, 1964) ACCOMPLISHMENTS WITH HUNGRY, BOXED RATS AND PIGEONS LED HIM TO ADVOCATE WHAT HE CONSIDERS THE BASIC PRINCIPLES OF SUCCESSFUL TEACHING. ONE THING SEEMS CERTAIN FROM SUCH STUDIES: A CONDITIONED ENVIRONMENT HAS A PRONOUNCED EFFECT ON AN ORGANISM. THIS OBSERVATION IS REFLECTED IN EDUCATION IN ITS SHIFT IN EMPHASIS FROM NATURE TO NURTURE.

A FINAL PRESUPPOSITION UNDERLYING EXPERIMENTAL PSYCHOLOGY AND LOCKEAN EMPIRICISM IS THE ASSUMPTION THAT WHAT IS EARLY IN DEVELOPMENT IS MORE FUNDAMENTAL THAN WHAT IS LATE (ALLPORT, 1955). THE EARLY SCRATCHINGS ON THE SLATE OF THE MIND ARE IMPORTANT. ALTHOUGH FIRST IMPRESSIONS MAY LATER BE COMPOUNDED AND CRISSCROSSED, THEY REMAIN THE ELEMENTS OF LATER MENTAL LIFE. IN SCHOOL THIS MEANS CONTENT-ORIENTED, SYSTEMATIC INSTRUCTIONAL PROGRAMS, PROCEEDING EVER FROM THE SIMPLE TO THE MORE COMPLEX. THE CURRICULUM BECOMES A STUDY IN SEQUENCING OF DISCIPLINES.

ALTHOUGH BIGGE (1964) LISTS MANY MEN PROMINENT IN THE FIELD OF LEARNING WHO HAVE ESPOUSED THEORIES AKIN TO LOCKE'S CONCEPTION OF MAN (THORNDIKE, STEPHENS, GATES, WATSON, GUTHERIE, HULL, SKINNER, SPENCE), THE FOLLOWING DISCUSSION IS LIMITED TO ONE MAN'S CONCEPTION OF LEARNING FOUNDED IN THE TRADITION OF TABULA RASA, NAMELY, THAT OF THE SO-CALLED NEO-BEHAVORIST ROBERT GAGNE. THROUGH NUMEROUS ARTICLES AND HIS BOOK CONDITIONS OF LEARNING, GAGNE'S NAME HAS BECOME A HOUSEHOLD WORD AMONG PROFESSIONAL EDUCATORS -- PARTICULARLY WHEN REFERENCE IS MADE TO GUIDED INSTRUCTION. HIS NEATLY ORDERED, CONTEMPORARY LEARNING THEORY PROVIDES A CONCISE FORMULA TO PROBLEM SOLVING.

AS SHULMAN (1970) RELATES, GAGNE FIRST ASKS OF A LEARNER, "WHAT

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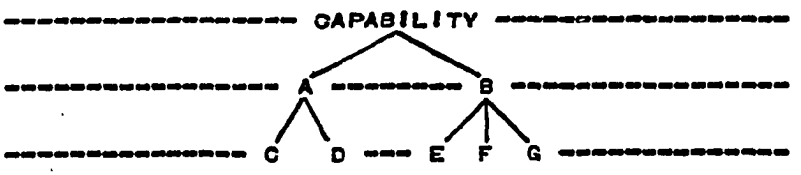
IS IT HE IS TO BE ABLE TO DO?" HE THEN ANSWERS THIS QUESTION IN BEHAVIORAL TERMS. THAT IS, THE SPECIFIC FUNCTIONS TO BE PERFORMED AND THE CONDITIONS UNDER WHICH THEY ARE TO OBTAIN ARE DELINEATED. FINALLY, THE SO-DESCRIBED CAPABILITY IS TAKEN AS TERMINAL BEHAVIOR AND BECOMES THE BASIS FOR A COMPLEX PYRAMID GROUNDED ON TASK ANALYSIS.

AFTER ANALYZING THE DESIRED CAPABILITY GAGNE ASKS, "WHAT WOULD THE LEARNER NEED TO KNOW IN ORDER TO DO THAT?" TASKS A AND B BEING REQUISITE TO THE TASK AT HAND, A PYRAMID BEGINS.

BUT REQUISITE TO A ARE C AND D, AND REQUISITE TO B ARE E, F, AND G.




SO IS BUILT A HIERARCHY OF CAPABILITIES OF PREREQUISITES TO PREREQUISITES TO PREREQUISITES TO THE OBJECTIVE, WHICH IS THE DESIRED CAPABILITY. TO GAGNE (1965, P.60), THEN, "THE MOST IMPORTANT CLASS OR CONDITION THAT DISTINGUISHES ONE FORM OF LEARNING FROM ANOTHER IS ITS INITIAL STATE; IN OTHER WORDS, ITS PREREQUISITES."

GAGNE HAS DEVELOPED A MODEL FOR DISCUSSING THE VARIOUS LEVELS OF PREREQUISITES IN HIS HIERARCHY. IF THE FINAL CAPABILITY DESIRED IS ONE OF PROBLEM SOLVING, THE LEARNER MUST HAVE FIRST AT HIS COMMAND CERTAIN PRINCIPLES. BUT TO BE UNDERSTOOD THESE REQUIRE A KNOWLEDGE OF SPECIFIC CONCEPTS, AND PREREQUISITE TO CONCEPTS ARE PARTICULAR MULTIPLE DISCRIMINATIONS. CONTINUING THIS BRICK-OFF-BRICK PROCESS RESULTS IN THE FUNDAMENTAL BUILDING BLOCKS OF LEARNING -- CONDITIONED RESPONSES. A LEARNING STRUCTURE PROVIDED BY GAGNE FOR TWO ASSIGNED TASKS APPEARING ON THE NEXT PAGE ILLUSTRATES HIS MODEL.

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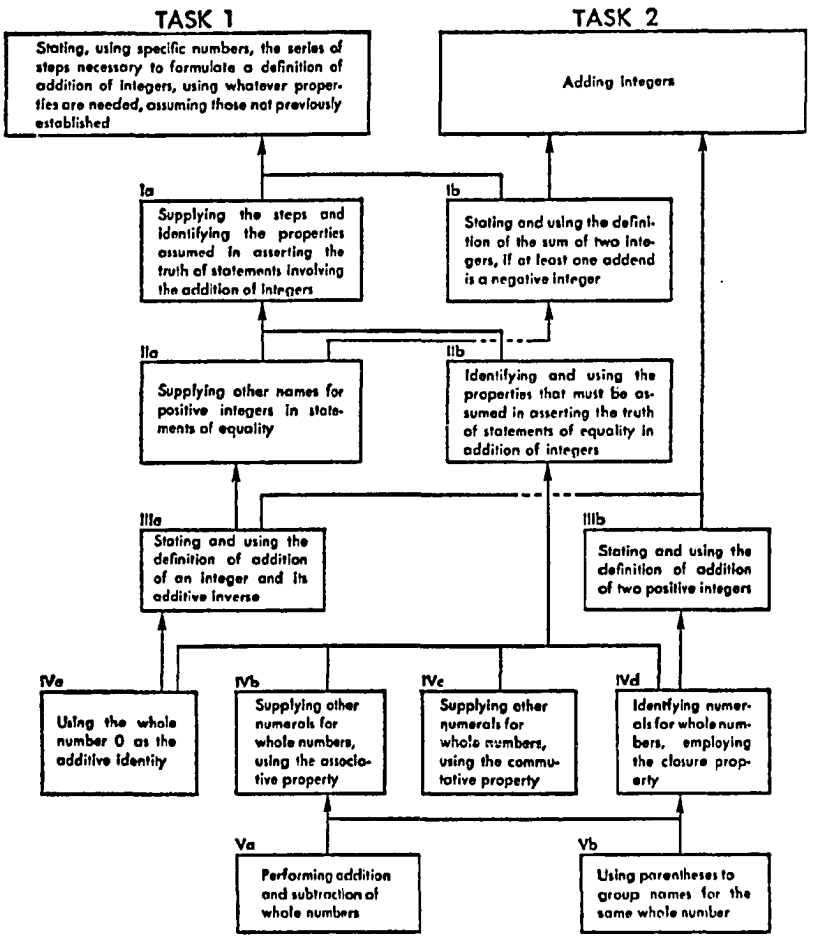
## A Task Hierarchy for Objectives of Assigned Tasks

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So the lesson to be taken from Gagne on problem solving is that it does not take place in a vacuum devoid of content knowledge. Rather such cognition depends primarily upon the availability of prerequisite knowledge. In addition, the order of acquisition of knowledge to problem solving is crucial. That is, for a response to a problem to be other than a simple association it must derive from knowing relevant principles which in turn derive from knowing relevant concepts, and so on (for supportive research see Gagne, 1962; King, 1970). Thus problem solving is the progeny of considerable ancestry: signal learning begets (possibly) stimulus-response connections which beget chains such as verbal associations which beget multiple discriminations which beget concepts which beget principles which beget problem solving.*

## THE LEIBNITZIAN TRADITION

The Leibnitzian tradition, by contrast, alleges that a man is neither a collection of acts nor simply the locus of acts; he is the source of acts. Human activity is purposive, not agitation attuned to internal or external stimulation. As suggested in a passage by Ortega y Gasset cited by McLuhan (1968, p.176), a man is forever in the act of becoming, so one looks to the future rather than the refuse of past stimuli to understand him:

> In contrast to the rest of creation, man, in existing, has to make his existence. He has to solve the practical problem of transferring into reality the program that is himself.

---

*Gagne's theory is that knowledge, regardless of the method evoked to foster it, obtains in the bottom-to-top hierarchical manner he has described. This does not mean that instruction must be sequenced similarly to be most effective. Motivational considerations alone suggest one reason for believing that the best way to teach certain lower order cognitive skills is to start with a problem situation and, in essence, work down Gagne's hierarchy.

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> FOR THIS REASON "MY LIFE" IS PURE TASK, A THING INEXORABLY TO BE MADE.

IN EXISTING, THEN, MAN NOT ONLY REACTS TO HIS ENVIRONMENT, HE MEETS IT HEAD ON.

THE MOST WIDE-SPREAD CONTEMPORARY LEARNING THEORY REPRESENTATIVE OF THIS CONCEPTION OF MAN IS COGNITIVE-FIELD PSYCHOLOGY. BIGGE (1964) DESCRIBES THIS LEARNING MODEL IN TERMS OF ITS CONSTITUENTS, COGNITIVE THEORY AND FIELD THEORY, RESPECTIVELY. COGNITIVE THEORY IS CONCERNED PRIMARILY WITH HOW PEOPLE GAIN AN UNDERSTANDING OF THEMSELVES AND THEIR ENVIRONMENTS AND HOW, USING THEIR COGNITIONS, THEY ACT IN RELATION TO THEIR ENVIRONMENTS. FIELD THEORY CENTERS ON THE IDEA THAT ALL PSYCHOLOGICAL ACTIVITY OCCURS IN A FIELD, THE CONCURRENT INTERRELATIONSHIPS IN ANY ONE SITUATION. SO COGNITIVE-FIELD THEORY DESCRIBES HOW A PERSON GAINS UNDERSTANDING OF HIMSELF AND HIS WORLD IN A SITUATION WHERE HIS SELF AND HIS ENVIRONMENT COMPOSE A TOTALITY OF MUTUALLY INTERDEPENDENT COEXISTING EVENTS.

ADVOCATES OF COGNITIVE-FIELD THEORY SUCH AS JEROME BRUNER SEE LEARNING AS A PURPOSIVE, EXPLORATIVE, IMAGINATIVE, AND CREATIVE ENTERPRISE WHEREIN A LEARNER DEVELOPS INSIGHT, WHICH BIGGE (1964, P.103) DEFINES AS "A SENSE OF, OR FEELING FOR, PATTERN OR RELATIONSHIPS." AS NEWELL, ET AL. (1958) AND MAIER (1931) POINT OUT, INSIGHT IS AN "AHA!" PHENOMENON, CHARACTERIZED BY THE "SUDDEN" APPEARANCE IN CONSCIOUSNESS OF A GRASP OF THE "STRUCTURE" OF A PROBLEM AS EVIDENCED BY ABSENCE OF TRIAL AND ERROR. INSIGHTS ARE GENERATED (THAT IS, A PERSON LEARNS) AS ONE DIFFERENTIATES, GENERALIZES, AND RESTRUCTURES BOTH HIMSELF AND HIS PSYCHOLOGICAL ENVIRONMENT. THUS PROBLEM SOLVING IS SEEN AS GROWING OUT OF ONE'S CONTEMPORARY LIFE SPACE -- THE PSYCHOLOGICAL FORCES

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obtaining in the present -- rather than as emerging from an accumulation of past conditioning.

Additional theoretical support for the Leibnitzian tradition can be inferred from Piaget's (Flavell, 1963) assimilation-accomodation equilibrium model, and the denial of tabula rasa is clearly evidenced in Gestalt psychology as expounded by men such as Wertheimer, Koffka, and Kohler. Whereas stimulus-oriented psychologies assume that learning starts with irreducible elements and consists of combinations of these, Gestalt-field theorists contend that learning is a process of gaining or changing insights, outlooks, or thought patterns through reorganization of perceptual or cognitive fields. They see a person, his environment, and his interaction with this environment as occurring simultaneously. As man is essentially passive in a determining environment in the one, so is he interactive with a psychological environment in the other.

The first formal statement of the position of Gestalt psychology is attributed to the German philosopher-psychologist Max Wertheimer. Central to his point of view is the idea expressed with the German word Gestalt, which Furth (1968) translates as a totality inherent in the perceptually given data to which an organism responds (that is, an organized whole in contrast to a collection of parts). Thus learning and thinking are closely related to perception in Gestalt theory, and they proceed at their "best," as a posthumous publication of Wertheimer's (1945) points out, whenever they proceed in full view of the structural features and structural requirements of a situation. In the event that the structural properties of a situation are deemed inadequate, a problem of a psychological nature arises which the viewer is motivated

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TO SOLVE BY REARRANGING THE PARTS IN RELATION TO ONE ANOTHER AND TO THE WHOLE SO AS TO ACHIEVE GREATER STABILITY, SIMPLICITY, REGULARITY, AND SYMMETRY. SUCH REORGANIZATION TENDS TO BE AS SIMPLE AND CLEAR AS THE CONDITIONS OF THE SITUATION ALLOW.

ALTHOUGH GESTALT-FIELD THEORY DOES NOT DENY THE NECESSITY OF PREREQUISITE KNOWLEDGE FOR SOLVING CERTAIN PROBLEMS, IT CHALLENGES THE IMPORTANCE GRANTED IT BY STIMULUS-ORIENTED PSYCHOLOGIES. THIS THEME IS PARTICULARLY PREVALENT IN MAIER'S (1930) "PENDULUM" EXPERIMENT (NOW A CLASSIC IN THE FIELD) CONDUCTED, IN PART, AT THE UNIVERSITY OF MICHIGAN.

MAIER QUESTIONED THE PREMISE THAT A PROBLEM CALLS UP PAST EXPERIENCES WHICH ARE MANIPULATED MENTALLY IN TRIAL AND ERROR FASHION UNTIL SOME COMBINATION IS FOUND TO WORK. HE FELT THAT A MAJOR OMISSION IN SUCH THEORIES WAS TO EXPLAIN WHY ONLY THE MORE PERTINENT THINGS ARE TRIED OUT. ACCORDINGLY, HE SET OUT TO DETERMINE THE EXTENT TO WHICH EXPERIENCE IS THE SUFFICIENT FUNDAMENTAL FACTOR IN PROBLEM SOLVING AND THE CONDITIONS UNDER WHICH IT FUNCTIONS BEST.

TO TEST THE INFLUENCE OF EXPERIENCE IN PROBLEM SOLVING, MAIER DEVISED THE FOLLOWING EXPERIMENT. HIS SUBJECTS WERE TO CHALK AN "X" ON EACH OF TWO SPOTS ON THE FLOOR. THIS WOULD HAVE BEEN EASY EXCEPT FOR THE RESTRICTION THAT THEY WERE NOT TO HOLD THE CHALK FOR MARKING PURPOSES. INSTEAD, THEY HAD TO CONSTRUCT TWO PENDULUMS FROM WHICH THE CHALK COULD BE SUSPENDED AND SWUNG OVER THE SPOTS (SEE NEXT PAGE). THE VARIABLE WAS THE PLACEMENT OF A RELATIONAL ENTITY WHICH MAIER REFERRED TO AS "DIRECTION."

IN ONE GROUP THE SOLUTION TO THE PROBLEM (THAT IS, THE METHOD OF CONSTRUCTION) WAS PRESENTED IN THREE SEPARATE PARTS. PART A INCLUDED INSTRUCTIONS ON HOW TO MAKE A PLUMB LINE WITH THE MATERIALS AT HAND,

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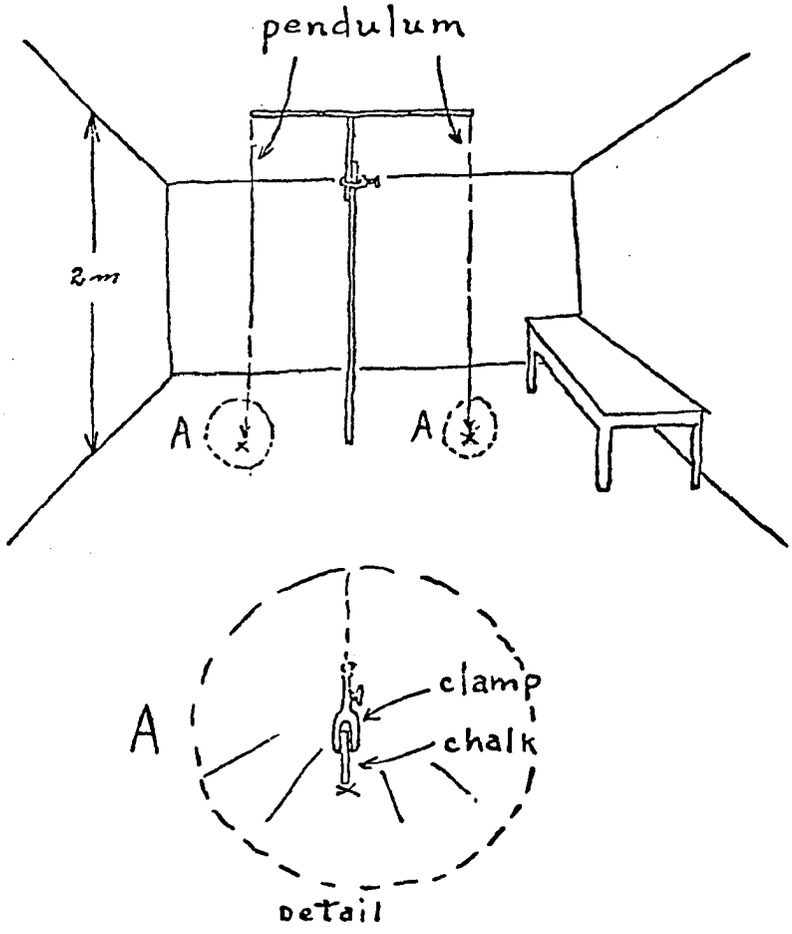
MAIER'S PENDULUM PROBLEM

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PART B ON HOW TO CLAMP TWO POLES TOGETHER TO MAKE A LARGER ONE, AND PART C ON HOW TO BUILD A SUPPORTIVE "T" BY PLACING A STICK AGAINST A DOORWAY AND WEDGING IT IN PLACE WITH ANOTHER STICK RESTING AGAINST THE OPPOSITE WALL. A SUBJECT WITH THIS TREATMENT HAD ONLY TO RECOMBINE THE PARTS IN THE APPROPRIATE ORDER (ABC) TO ACHIEVE SOLUTION.

IN SOME CASES A SUBJECT WAS ALSO GIVEN THE FOLLOWING EXPERIENCE.

> DIRECTION: I SHOULD LIKE TO HAVE YOU APPRECIATE HOW SIMPLE THIS PROBLEM WOULD BE IF WE COULD JUST HANG THE PENDULUMS FROM A NAIL IN THE CEILING. OF COURSE, THAT IT IS NOT A POSSIBLE SOLUTION BUT I JUST WANT YOU TO APPRECIATE HOW SIMPLE THE PROBLEM WOULD BE IF THAT WERE POSSIBLE. NOW THAT IT IS NOT POSSIBLE THE PROBLEM IS, AS YOU MAY FIND, REALLY QUITE DIFFICULT.

THE RESULTS OF THIS INVESTIGATION SHOWED THAT AN APPROPRIATE CONSTRUCTION WAS LIKELY TO APPEAR ONLY WHEN BOTH PARTS AND DIRECTION WERE GIVEN. IN OTHER WORDS, A SELECTED PRESENTATION OF PREREQUISITE KNOWLEDGE WAS INSUFFICIENT FOR SOLUTION. THE PARTS OR EXPERIENCES ALSO HAD TO BE COMBINED IN A CERTAIN WAY W I T H AN ORGANIZING PRINCIPLE SUCH AS DIRECTION.

ALTHOUGH A NUMBER OF AUTHORS HAVE QUESTIONED MAIER'S RESULTS (E.G., GAGNE, 1964; WEAVER AND MADDEN, 1949), IT SEEMS CLEAR THAT THE MANNER IN WHICH A PROBLEM IS PRESENTED INFLUENCES RESULTS. THIS THEME IS RECURRENT IN THE LITERATURE. FOR INSTANCE, A STUDY BY YOUNG AND McISAAC (1941) POINTS OUT THAT SEQUENCE OF REQUIRED OPERATION IS A FACTOR IN THE INTERPRETATION OF VERBAL PROBLEMS IN ARITHMETIC. AND JOHN AND MILLER (1957) HAVE OBSERVED THAT SLIGHT STRUCTURAL DIFFERENCES IN OTHERWISE SIMILAR PROBLEMS RESULT IN LARGE FUNCTIONAL DIFFERENCE IN SUBJECTIVE DIFFICULTY.

IN CONCLUSION, A TEACHER WHO ACCEPTS THE CONCEPT OF AN ACTIVE INTELLECT AS EMBODIED IN COGNITIVE-FIELD PSYCHOLOGY OR GESTALT PSYCHOLOGY

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IS LIKELY TO APPROACH PROBLEM SOLVING DIFFERENTLY FROM ONE WHO OPERATES WITHIN AN "EMPTY ORGANISM," STIMULUS-ORIENTED FRAMEWORK. FOR ONE THING, A TEACHER ORIENTED TOWARD THE FORMER IS CONCERNED ALWAYS WITH THE PROBLEM OF PERSONAL INVOLVEMENT, THAT IS, HELPING STUDENTS SEE A NEED TO LEARN. THEIR PERSONAL GOALS ARE ALWAYS RELEVANT. FOR ANOTHER, ALTHOUGH THE IMPACT OF PREVIOUS EXPERIENCE ON A STUDENT'S LIFE SPACE IS NOT IGNORED, HE FOCUSES ON THE PRESENT SCENE AS THE STUDENT EXPERIENCES IT. THE RESPONSIBILITY FOR DEVELOPMENT RESTS NEITHER WITH THE STUDENT ALONE NOR WITH THE ENVIRONMENT, BUT RATHER IN THE STUDENT AND HIS ENVIRONMENT COMING TOGETHER IN A PSYCHOLOGICAL FIELD. HIS APPROACH TO PROBLEM SOLVING IS <u>SITUATIONAL</u> RATHER THAN <u>HISTORICAL</u>.

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# PART III: THE FUNCTION OF IMAGERY IN PROBLEM SOLVING

Imagery and problem solving are related through the medium of thought. Problem solving is usually defined as thought that results in discovery of correct response (Duncan, 1959), and it has been argued since the time of Aristotle that imagery is the stuff of thinking. To the extent, then, that imagery facilitates thought, imagery is functional in problem solving. Just how functional, however, is a question surrounded with considerable controversy.

Many authors contend that words are the prime mediators of thought. Cofer (1957), for one, states that those instances where thinking proceeds in the absence of verbal processes are infrequent and insignificant in terms of the overall problem. But there is reason to believe otherwise. It would seem that imagery is helpful in the processes of discrimination (Carey, 1915) and memory (Carey, 1915; Ranken, 1963a, 1963b), that visual images serve a facilitating role in evaluating the truth or falsity of statements low on meaning (Fox, 1914), and that persons with high imagery ratings (Barratt, 1953) or who have encoded certain information visually (Ranken, 1963a, 1963b) tend to have the advantage on spatial-medium tasks. In addition, first-hand accounts of the use of imagery by eminent scientists lend support to the notion that imagery is relevant to thought. This latter point is exemplified in a passage from a letter by Albert Einstein to Hadamard (1945, p.142):

> The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be 'voluntarily' reproduced and combined.

At the very least, then, imagery would seem to be illustrative of

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THOUGHT (AVELING, 1927).

TO SAY, HOWEVER, THAT IMAGES ARE RELEVANT TO THOUGHT IS NOT TO SAY THAT THE LATTER IS SIMPLY THE UTILIZATION OF THE FORMER. THAT IS, THE INTELLECT IS NOT NECESSARILY "A LOGICIAN WORKING OVER DATA TURNED OUT BY A COPYING MACHINE" (THORNDIKE, 1907). BUT WHAT IS THIS THING CALLED IMAGERY? ACCORDING TO MCKELLAR (1965), SOME 10% OF THE 500 SCIENTIFIC PERSONNEL COMPRISING HIS SURVEY REPORTED THEY SPENT THEIR NORMAL WAKING LIVES ALMOST TOTALLY FREE OF IMAGES -- VISUAL, AUDITORY, TACTILE, OR OF ANY OTHER KIND. SO A DEFINITION OF IMAGE WILL PRECEDE FURTHER INVESTIGATION OF THE ROLE OF IMAGERY IN PROBLEM SOLVING.

TO DEFINE IMAGE IT IS NECESSARY TO DETERMINE CRITERIA FOR DISTINGUISHING IT FROM PERCEPT, AN IMPRESSION RECEIVED BY THE MIND THROUGH THE SENSES. HENCE KULPE IS CITED BY RICHARDSON (1969) AS REFERRING TO IMAGE AS A "CENTRALLY EXCITED SENSATION," AND LEUBA (1940) INTERPRETS IMAGE AS A "CONDITIONED SENSATION." ONE DIFFICULTY WITH THESE DEFINITIONS IS THEIR INABILITY TO DESCRIBE OR ACCOUNT FOR CERTAIN LIMITED FORMS OF MENTAL IMAGERY. AN IMAGE AS A CENTRALLY EXCITED SENSATION EXCLUDES AFTER-IMAGES CAUSED BY PERIPHERAL STIMULATION (E.G., THE "FELT" PRESSURE EXPERIENCED UPON REMOVING ONE'S HAT), AND AN IMAGE AS A CONDITIONED SENSATION UNSATISFACTORILY EXPLAINS IMAGINATION IMAGES.

A DEFINITION OF IMAGE THAT AVOIDS THIS SHORTCOMING IS PROVIDED BY RICHARDSON (1969, PP.2-3):

> MENTAL IMAGERY REFERS TO (1) ALL THOSE QUASI-SENSORY OR QUASI-PERCEPTUAL EXPERIENCES OF WHICH (2) WE ARE SELF-CONSCIOUSLY AWARE, AND WHICH (3) EXIST FOR US IN THE ABSENCE OF THOSE STIMULUS CONDITIONS THAT ARE KNOWN TO PRODUCE THEIR GENUINE SENSORY OR PERCEPTUAL COUNTERPARTS, AND WHICH (4) MAY BE EXPECTED TO HAVE DIFFERENT CONSEQUENCES FROM THEIR SENSORY OR PERCEPTUAL COUNTERPARTS.

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By "quasi-sensory or quasi-perceptual experiences" is meant any concrete re-presentation of sensory, perceptual, affective, or other experiential states such as hunger or fatigue. To be "self-consciously aware" of these re-presentations means that the individual should be able to report on what he has "seen," "heard," "touched," "tasted," "smelled," or "felt." Moreover, for these experiences to be classified as images rather than as sensations or percepts, they must occur "in the absence of those stimulus conditions that are known to produce their genuine sensory or perceptual counterparts." Finally, image-based behavior typically has "different consequences" from that based upon percepts. For example, "mental fire" might warm one's backside, but it will not burn real sticks. In the "real" world consequences always accrue. Thus image is a generic term for all conscious subjective presentations of a quasi-sensory but nonperceptual character (Holt, 1964).

The subject covered under the heading of mental imagery spans the entire history of experimental psychology. References to this topic date to 1860 (Fechner), and in 1880 Sir Francis Galton administered his classic "breakfast table" questionnaire of imagery vividness to school boys, artists, scientists, and statesmen.* As Galton discovered in his early inquiries, for some the "mind's eye" was blind and talk of imagery was so much nonsense.

> To my astonishment, I found that the great majority of men of science to whom I first applied, protested that mental imagery was unknown to them, and they looked on me as fanciful and fantastic in supposing that the words 'mental imagery'

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*Galton's Questionnaire: Before addressing yourself to the Questions... think of some definite object -- suppose it is your breakfast-table as you sat down to it this morning -- and consider carefully the picture that rises before your mind's eye.

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REALLY EXPRESSED WHAT I BELIEVED EVERYBODY SUPPOSED THEM TO MEAN.

HE CONCLUDED THAT AN OVER-READINESS TO PERCEIVE CLEAR MENTAL PICTURES WAS ANTAGONISTIC TO THE ACQUIREMENT OF HABITS OF HIGHLY GENERALIZED AND ABSTRACT THOUGHT, AND THAT IF THE FACULTY OF PRODUCING THEM WAS EVER POSSESSED BY MEN WHO THINK HARD, IT WAS APT TO BE LOST BY DISUSE.

NOR IS GALTON'S DISCOVERY MERELY AN HISTORICAL CURIOSITY. ROE'S (1951) STUDY OF RESEARCH SCIENTISTS INDICATED THAT IMAGERY IS RELEVANT TO CHOICE OF VOCATION. SHE FOUND THAT THE MORE THEORETICALLY ORIENTED PROFESSIONALS SUCH AS PSYCHOLOGISTS, ANTHROPOLOGISTS, AND THEORETICAL PHYSICISTS WERE PRIMARILY VERBALIZERS, WHEREAS BIOLOGISTS AND EXPERIMENTAL PHYSICISTS WERE CONCENTRATED IN THE VISUAL IMAGERY GROUP. A STATEMENT BY HOROWITZ (1967) SUGGESTS A REASON FOR THIS: "NOT TO BE ABLE TO VISUALIZE MEANS, TO SOME EXTENT, NOT TO BECOME EMOTIONALLY AROUSED."

THE FINDING THAT IMAGERY IS NOTED LESS FREQUENTLY AMONG THEORETICIANS IS CONSISTENT WITH THE NOTION THAT IMAGING AND TASKS OF A HIGHER MENTAL ORDER SUCH AS LEARNING AND PROBLEM SOLVING ARE IN DISCORD. PROPONENTS OF THIS VIEWPOINT ARE NUMEROUS. IN 1908 THORNDIKE CONDUCTED AN EXPERIMENT ON PERFORMANCE OF MENTAL MULTIPLICATION OF ONE THREE PLACE NUMBER BY ANOTHER. HE FOUND THAT MORE INDIVIDUALS REPORTED A DECREASE THAN AN INCREASE IN THE STRENGTH OF THEIR VISUAL IMAGES

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1. ILLUMINATION. -- IS THE IMAGE DIM OR FAIRLY CLEAR? IS ITS BRIGHTNESS COMPARABLE TO THAT OF THE ACTUAL SCENE?

2. DEFINITION. -- ARE ALL THE OBJECTS PRETTY WELL DEFINED AT THE SAME TIME, OR IS THE PLACE OF SHARPEST DEFINITION AT ANY ONE MOMENT MORE CONTRACTED THAN IT IS IN A REAL SCENE?

3. COLOURING -- ARE THE COLOURS OF THE CHINA, OF THE TOAST, BREADCRUST, MUSTARD, MEAT, PARSLEY, OR WHATEVER MAY HAVE BEEN ON THE TABLE, QUITE DISTINCT AND NATURAL? (AND SO ON.)

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OF THE NUMBERS DURING THE COURSE OF PRACTICE AND THAT THOSE WHO REPORTED A DECREASE IMPROVED SOMEWHAT MORE. THIS FINDING IS COMPATIBLE WITH ONE BY CAREY (1915) WHO NOTED A TENDENCY TO INVERSE CORRELATION BETWEEN STRENGTH OF VISUAL AND AUDITORY IMAGERY AND SOME OF THE HIGHER MENTAL PROCESSES OF CHILDREN. CAREY'S CONCLUSION THAT IMAGERY MAY BE DETRIMENTAL TO SCHOOL STUDIES, HOWEVER, HAS BEEN SOMEWHAT SOFTENED BY LATER STUDIES. THE RUSSIAN SCIENTIST KRUTETSKI (1961), FOR EXAMPLE, HAS DEMONSTRATED THAT RELATIVE INCAPACITY FOR SCHOOL MATHEMATICS IS DUE MORE TO AN INSUFFICIENT DEVELOPMENT OF THE VERBAL-LOGICAL COMPONENT OF INTELLECTUAL ACTIVITY THAN AN UNFAVORABLE INTERRELATION OF THE VISUAL-IMAGE AND VERBAL-LOGICAL COMPONENTS OF INTELLECTUAL ACTIVITY.

OTHER STUDIES HAVE EXAMINED THE CORRELATION BETWEEN INTENSITY OF IMAGERY AND MENTAL ABILITY AS MEASURED BY STANDARD INTELLIGENCE TESTS. THE EVIDENCE, HOWEVER, IS CONTRADICTORY. BROWER (1947A, 1947B) FOUND PRACTICALLY NO RELATIONSHIP BETWEEN THE TWO, BUT DAVIS (1932) OBTAINED A LOW POSITIVE CORRELATION FOR IMAGERY SCORES ON A TEST OF HIS MAKING AND INTELLIGENCE AS MEASURED BY ARMY ALPHA. AND SCHMEIDLER (1965) REPORTS A SIMILAR FINDING FOR SCORES ON QUESTIONNAIRES OF VISUAL IMAGERY AND OF CREATIVITY.

ALTHOUGH FEW AUTHORS TODAY WOULD ADVOCATE THAT "IMAGES, ALONG WITH SENSATIONS CONSTITUTE THE MATERIAL OF ALL INTELLECTUAL OPERATIONS: MEMORY, REASONING, IMAGINATION..." AS BINET ONCE DID (AND LATER RENOUNCED -- SEE BETTS, P.95), MANY WRITERS SUCH AS PEAR (1927) ARE UNWILLING TO BELIEVE THAT THEY ARE NECESSARILY IRRELEVANT TO THOUGHT. AS HOROWITZ (1967) HAS POINTED OUT, IMAGES HAVE POSSIBLE PSYCHOLOGICAL UTILITY AS CARRIERS OF AFFECTIVELY CHARGED MEMORIES, IDEAS, AND IMPULSES. THAT IS, IMAGERY WOULD SEEM TO GIVE A "TANG OF AFFECTIVE REAL-

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ITY" TO OUR THINKING (PEAR, 1937). THUS IMAGERY MAY SERVE A MOTIVATIONAL PURPOSE IN INTELLECTUAL FUNCTIONING THROUGH PROVIDING FOR CONTINUITY OF INTEREST.

ANOTHER REASON FOR REFUSING TO BELIEVE THAT IMAGES ARE OF LITTLE UTILITY IN THOUGHT IS THAT MOST STUDIES ATTEMPTING TO CORRELATE IMAGERY AND THINKING HAVE RELIED UPON QUESTIONNAIRES. AS SHORT (1953) OBSERVES,

> IMAGERY QUESTIONNAIRES ARE POOR INDICATORS OF THE FUNCTION OF IMAGES IN MENTAL LIFE. THEY FAIL PARTLY BECAUSE INDIVIDUAL'S SELF SCORINGS CANNOT BE STANDARDIZED, AND PARTLY BECAUSE, IN THINKING, IT IS THE IMAGES THAT OCCUR MOST READILY AND HABITUALLY THAT ARE IMPORTANT, NOT THE ONES THOUGHT TO BE THE MOST 'INTENSE' OR 'VIVID' AT A GIVEN MOMENT. THE MERE EMERGENCE OF VERY VIVID IMAGES MAY NOT BE ASSOCIATED AT ALL WITH THE TENDENCY TO HAVE AND TO USE IMAGES.

THUS AN UNDERSTANDING OF THE ROLE OF IMAGERY IN THOUGHT CANNOT BE HAD FROM AN INVESTIGATION LIMITED TO VOLUNTARY IMAGERY, THE ABILITY TO CALL UP CERTAIN SPECIFIED IMAGES AT WILL AS MEASURED BY IMAGERY QUESTIONNAIRES. INSTEAD, PROVISIONS MUST BE MADE TO ACCOUNT FOR SPONTANEOUS IMAGERY, THE NORMAL FUNCTIONING OF IMAGERY IN THINKING, REMEMBERING, AND IMAGINING.

AMONG THE FIRST OF THE MORE SYSTEMATIC STUDIES TO CONSIDER BOTH VOLUNTARY IMAGERY AND SPONTANEOUS IMAGERY WAS THAT OF BETTS. IN 1909 BETTS EXPANDED GALTON'S QUESTIONNAIRE TO 150 ITEMS TO PROVIDE FOR RESPONSES OF EVERY MODALITY. IN THE VISUAL (AUDITORY) MODALITY SUBJECTS WERE ASKED TO THINK OF SEEING (HEARING) "SHEET LIGHTNING LOW ON THE HORIZON" ("THE CLINK OF GLASSES"), FOR EXAMPLE, AND TO CONSIDER CAREFULLY THE IMAGE WHICH CAME TO THEIR MIND'S EYE (EAR). STIMULUS ITEMS FOR THE OTHER MODALITIES WERE "THE TICKLING OF A FLY AT THE NOSE" (CUTANEOUS), "RISING OUT OF A LOW CHAIR" (KINAESTHETIC), "WATERMELON" (GUSTATORY), "NEWMOWN GRASS" (OLFACTORY), AND "FATIGUE" (ORGANIC).

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Subjects were to classify each image to a seven-point scale with respect to degree of clearness and vividness.

Betts' questionnaire was directed toward answering three questions related to voluntary imagery: (1) the general ability to image in the various modalities, including the degree of clearness and vividness of the images, (2) the correlation between imagery types, and (3) the correlation of ability in voluntary imagery with scholarly ability in general. Data obtained indicated the following: First, the ability to image was greater in college students than in specialists more advanced in years who dealt primarily with abstract lines of thought. Second, few persons were unable to image; however, large individual differences occurred in the dimension of subjective clarity and vividness. Third, the ability to image seemed evenly distributed among the various modalities, providing no support for such classifications as "eye-minded," "ear-minded," and muscle-minded." Finally, there was a slight negative correlation between ability to image voluntarily and ability in college studies as measured by teachers' markings. Similar findings of more recent vintage on the general ability to image and the lack of correlation between imagery types have been reported by Sheehan (1967a, 1967b) using a shortened form of Betts' questionnaire.

To test for the use of spontaneous imagery in thinking and related activities, Betts employed two methods. The first was the method of interrupted thinking. Under this method a subject is pursuing some line of thought without expecting interruption. At some point his thought is broken into by the investigator and he is asked to introspect on his mental proceedings. The second method involved setting a definite problem to be solved without interruption. Upon completion,

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THE SUBJECT WAS TO DESCRIBE THE COURSE OF HIS MENTAL PROCESSES.

A MAJOR DIFFICULTY WITH THE AFOREMENTIONED MODES OF INQUIRY IS THAT BOTH REQUIRE TESTIMONY ONLY AS TO THE PRESENCE OF IMAGERY IN THOUGHT, NOT ITS FUNCTION. HOWEVER, BY REFERRING TO THE CHARACTER OF THE IMAGE REPORTED, ITS TIME OF APPEARANCE, AND ITS ABSENCE IN THE THOUGHT CONTENT OF A CERTAIN PROPORTION OF PERSONS, BETTS BELIEVED THAT REASONABLE INFERENCES COULD BE MADE ON THE USE OF IMAGERY IN THINKING. A PORTION OF HIS "SUMMARY AND CONCLUSIONS" FOLLOWS:

> THERE ARE TWO POINTS IN OUR THINKING AT WHICH IMAGERY HAS THE GREATEST TENDENCY TO EMERGE. (1) AT POINTS WHERE OUR THINKING IS BAFFLED; (2) AT POINTS WHERE PERCEPTS WOULD BE OF GREAT ASSISTANCE. IF THE BAFFLED POINTS ARE NOT DEPENDENT ON PERCEPTS, THE IMAGES WHICH OFFER ARE USUALLY FOR THE MOST PART IRRELEVANT, AND HENCE OF NO POSSIBLE SERVICE IN REACHING THE SOLUTION. THE MOST EFFICIENT AND SUCCESSFUL THINKING, AT LEAST OF LOGICAL AND ABSTRACT NATURE, IS WITH MOST PERSONS ACCOMPANIED BY THE LEAST IMAGERY.

BETTS CONCLUDED, THEN, THAT THINKING CAN AND DOES GO ON WITHOUT THE INTERVENTION OF IMAGERY, THE MENTAL CONTENT BEING MADE UP OF FEELINGS OF MEANING, RELATION, INTENTION, EFFORT, IDENTITY, INTEREST, PLEASURE, DISPLEASURE, AND SO ON. ALTHOUGH IMAGERY OFTEN SERVES AS A FAMILIAR BACKGROUND FOR THE MEANING WITH WHICH WE ARE DEALING, IT IS NOT ESSENTIAL TO MEANING, EXCEPT TO THE EXTENT THAT MEANING MAY INHERE IN A GIVEN PERCEPT AS SUCH (E.G., THE MEANING OF A BEAUTIFUL SUNSET IS CHIEFLY THAT SAME BEAUTIFUL SUNSET).

WITH THE RISE OF STIMULUS-ORIENTED PSYCHOLOGY AFTER THE TURN OF THE CENTURY, THE STUDY OF IMAGERY LOST ITS POPULARITY. BARRATT (1953) GIVES TWO REASONS FOR THIS: FIRST, THE DRIVE FOR "OBJECTIVITY" ACCOMPANYING THE ASCENDANCE OF BEHAVIORISM LED TO THE REJECTION OF SUCH "SUBJECTIVE" PROCESSES AS IMAGERY. EVEN THE PARAPSYCHOLOGICAL LITERATURE

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OF THE 1920s, '30s, AND '40s SHOWED A MARKED DECLINE IN ITS REFERENCE TO GHOSTS AND OTHER IMAGED APPARITIONS (HOLT, 1964). SECOND, ATTEMPTS TO FIND SOME RELIABLE METHOD OF EXAMINING THE EXTENT AND ROLE OF IMAGERY IN THINKING PROVED SO DIFFICULT AS TO LEAD TO THE EXPEDIENT OF "QUITTING THE FIELD."

BUT ALL THIS IS HISTORY. WHAT OF THE STATUS OF IMAGERY TODAY? HOLT CONTRIBUTES THE RESURRECTION OF INTEREST IN IMAGERY WITHIN THE PAST TWO DECADES TO A VARIETY OF FACTORS: (1) PRACTICAL PROBLEMS IN ENGINEERING PSYCHOLOGY ASSOCIATED WITH IMAGERY IN ITS MORE DRAMATIC FORMS SUCH AS HALLUCINATION (E.G., "HIGHWAY HYPNOSIS" AS EXPERIENCED BY LONG-DISTANCE DRIVERS IN NIGHT RUNS OVER TURNPIKES), (2) AN INTEREST IN PSEUDOHALLUCINATORY IMAGERY AS RELAYED FIRST-HAND BY PRISONERS OF WAR WHO SUFFERED PROLONGED ISOLATION, SLEEP DEPRIVATION, AND THE MULTIPLE REGRESSIVE PRESSURES OF FORCIBLE INDOCTRINATION OR THOUGHT REFORM, (3) THE DEVELOPMENT OF MYRIAD SYNTHETIC HALLUCINOGENS BEGINNING WITH THE DISCOVERY OF LYSERGIC ACID, AND (4) THE GREAT ADVANCES BEING MADE IN BRAIN RESEARCH THROUGH THE DEVELOPMENT OF ELECTROENCEPHALOGRAPHY, DIRECT STIMULATION OF THE BRAIN, AND THE WORK OF VARIOUS PERSONS ON THE RETICULAR ACTIVATION SYSTEM OF THE BRAIN STEM.

FOR EXAMPLE, SEVERAL PERSONS HAVE EXAMINED IMAGERY TYPES WITH RESPECT TO ALPHA RECORDS, CERTAIN ELECTRICAL ACTIVITY OF THE POSTERIOR AREAS OF THE BRAIN AS APPEAR ON ELECTROENCEPHALOGRAM (EEG) READINGS (E.G., GOLLA, ET AL., 1943; CHOWDHURY, 1964). SHORT (1953), FOR ONE, USED AN EEG AMPLIFIER TO RECORD THE REGULARITY OF BREATHING OF SUBJECTS INVOLVED IN PROBLEM SOLVING BY PLACING A THERMOCOUPLE OVER THEIR NOSTRILS TO CONVERT VARIATIONS IN BREATH TEMPERATURE INTO ELECTRICAL IMPULSES. AS SUBJECTS COMPLETED A MENTAL TASK THEY WERE CATEGORIZED AS

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EITHER VISUAL OR VERBAL-MOTOR ON THE BASIS OF INTROSPECTIVE REPORTS REPORTED VERBATIM ON THE EEG PAPER. IN THIS WAY SHORT WAS ABLE TO ASSOCIATE TENDENCY TO VISUALIZE OR VERBALIZE WHILE THINKING TO ALPHA TYPES AND BREATHING BEHAVIOR AS WELL. RESULTS INDICATED THAT VISUALISTS BREATHED REGULARLY AND SHOWED FREQUENT BLOCKING OF ALPHA RHYTHMS DURING PROBLEM SOLVING. VERBALISTS, ON THE OTHER HAND, BREATHED IRREGULARLY AND SHOWED ALPHA PERSISTENCE WHILE THINKING OUT PROBLEMS.

SINCE ALPHA RHYTHMS PROVIDE A RELIABLE BASIS FOR CONTRASTING PERSONS (HOLT, 1964), SHORT'S FINDING SUGGESTS AN OBJECTIVE METHOD FOR DISTINGUISHING BETWEEN VISUALISTS AND VERBALISTS. UNFORTUNATELY, THIS APPEALING PICTURE HAS NOT BEEN WHOLLY REPLICABLE. IN A LATER STUDY BARRATT (1956) SUBMITTED THAT ACTIVE VISUALIZING IS ONLY ONE OF MANY FACTORS POSSIBLY PRODUCING SUPPRESSION OF ALPHA RHYTHMS.

ANOTHER SOURCE OF THE RENEWED INTEREST IN IMAGERY HAS ITS ROOTS IN DEVELOPMENTAL PSYCHOLOGY. CHILDREN ARE KNOWN TO BE MORE SUBJECT THAN ADULTS TO EIDETIC IMAGES -- VIVID, PERCEPT-LIKE SUBJECTIVE EXPERIENCES OF A QUASI-HALLUCINATORY KIND (JAENSH, 1930; JENKIN, 1935). THUS PICTORIAL COGNITION WOULD SEEM TO BE A DEVELOPMENTALLY MORE PRIMITIVE SYSTEM THAN VERBAL-CONCEPTUAL THINKING. ACCORDINGLY, A NUMBER OF AUTHORS SUCH AS BRUNER, OLVER, AND GREENFIELD (1966) AND PIAGET (FURTH, 1968) HAVE GRANTED IMAGERY THE SIGNIFICANT BASIS OF COGNITIVE OPERATIONS IN CHILDREN. AS BETTS (P.93) POINTS OUT,

> THE CHILD IS MUCH NEARER THE WORLD OF THE CONCRETE. THE MEANINGS OF THINGS ARE CHIEFLY IN THE OBJECTS THEMSELVES, AND BUT LITTLE IN THEIR RELATIONS TO OTHER OBJECTS. THE CHILD'S MENTAL WORLD IS RELATIVELY A WORLD OF PERCEPTS, COVERING THE RANGE OF ALL THE SENSES. EACH PERCEPT IS THE BASIS FOR AN IMAGE, WHICH COMES TO SUPPLEMENT THE PERCEPT, AND MAKE POSSIBLE A GREATER COMPLEXITY OF EXPERIENCE. THE IMAGES TAKE ON THE SAME MEANINGS AS THE PERCEPTS IN A DEGREE, AND SOMETIMES BECOME ALMOST AS REAL.

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Betts goes on to say that imagery begins to lose its function as meaning comes to inhere more in the relationship of objects than in the objects themselves.

The predominance of imagery in children's cognition and the dependence of imagery upon experience for its development suggest a pedagogical sequence as exhorted by Bruner. According to Bruner (1968), an optimum sequence of instruction will likely move students from an enactive (experiential) mode of representation of whatever they are to learn through an iconic (image) mode to a symbolic representation. The function of imagery in this sequence of events, insofar as mathematics is concerned, is to provide a referent for the notation being applied to a problem (Bruner and Kenney, 1965).

To varying degrees, Bruner's enactive-iconic-symbolic instructional sequence describes two of the three experimental treatments the writer designed for this study. The high-imagery instructional sequences -- materials and drawings -- differed from the low-imagery sequence in their provisions for a basis for visual images embodying the quantitative nature of phrases common to algebra word problems. This study differs from others that have attempted to relate imagery to problem solving in its observation of the effects of providing for specific imagery for a specific mental task.

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# CHAPTER III

# RESEARCH DESIGN

The design and implementation of the study is organized for presentation in this chapter as follows: The instructional sequences or treatments are outlined first, after which an overview of the directives supplied teachers is given. Then the organization of the study is delineated, followed by location and dates and a description of subjects. Finally, the chapter concludes with a discussion of the evaluation instruments and analytic procedures employed.

## TREATMENTS

Three instructional sequences were designed for the study to be applied to three types of algebra word problems common to ninth grade algebra: number, coin, and age. Translation rather than solution, per se, was the objective of each. Two of the sequences were characterized by activities to result in a basis for the formation of mental images to serve as referents for notation being applied to problems. In one, images were to result from representing problems concretely, in the other from illustrating problems pictorially. The remaining treatment was the more traditional of the three in referring notation to number and progressing directly from translation of phrases and sentences to translation of two conditions on



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TWO UNKNOWNS.

PRIOR TO DEVELOPMENT OF THESE MATERIALS, THE WRITER CONDUCTED A PRELIMINARY STUDY ON TRANSLATION OF ALGEBRA WORD PROBLEMS AT UNIVERSITY SCHOOL, UNIVERSITY OF MICHIGAN, ANN ARBOR. FIRST APPROXIMATIONS OF THE IMAGE-BASED SEQUENCES WERE THEN PREPARED FROM WHICH THE WRITER TAUGHT AGE PROBLEMS TO TWO SEVENTH GRADE MATHEMATICS CLASSES WHILE THEIR TEACHERS OBSERVED. THE ROLES WERE THEN REVERSED AS THE REGULAR TEACHERS PROCEEDED THROUGH NUMBER AND COIN AS PRESCRIBED WHILE THE WRITER OBSERVED. FOLLOWING THIS, THE IMAGE-BASED SEQUENCES WERE REVISED TO FINAL FORM.

FOR DISCUSSION PURPOSES THE SEQUENCES CHARACTERIZED BY PROVISIONS FOR MENTAL IMAGERY ARE REFERRED TO AS HIGH-IMAGERY TREATMENTS, THE ONE FOUNDED ON MATERIALS HAVING BEEN ABBREVIATED HIM, THE OTHER BASED ON DRAWINGS HID. CORRESPONDINGLY, THE SEQUENCE AVOIDING VISUAL REFERENTS IS REFERRED TO AS LOW IMAGERY AND HAS BEEN REFERENCED BY LO.

IN THE INITIAL TREATMENT OF NUMBER, COIN, AND AGE; HIM, HID, AND LO WERE SIMILAR. EACH DEALT WITH PROBLEMS AS STATED AND AS PERCEIVED THROUGH (1) INSTRUCTION ON TRANSLATION OF PERTINENT PHRASES AND SENTENCES, AND (2) EXPLICIT MENTION OF INFORMATION AND PHYSICAL ASSUMPTIONS SURROUNDING CERTAIN SITUATIONS YET AUXILIARY TO THEIR WRITTEN DESCRIPTIONS. THIS INTRODUCTORY WORK WAS REGARDED AS SUPPLYING SKILLS AND KNOWLEDGE PREREQUISITE TO EFFECTIVE TRANSLATION.

FOLLOWING THE INTRODUCTION TO PROBLEM TYPE, HIM AND HID DIFFERED SUBSTANTIALLY FROM LO. FROM THIS POINT ON THE CONSTRUCT OF THE HIGH-IMAGERY SEQUENCES IS PERHAPS BEST DISCLOSED IN TERMS OF AN ENACTIVE-ICONIC-SYMBOLIC FRAMEWORK AS SPONSORED BY BRUNER (1966, PP.44-45):

> ANY DOMAIN OF KNOWLEDGE (OR ANY PROBLEM WITHIN THAT DOMAIN OF KNOWLEDGE) CAN BE REPRESENTED IN THREE WAYS: BY A SET OF ACTIONS APPROPRIATE FOR ACHIEVING A CERTAIN RESULT (ENACTIVE REPRESENTATION); BY A SET OF SUMMARY IMAGES OR GRAPHICS THAT STAND FOR

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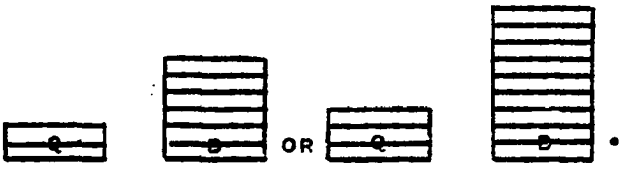
> A CONCEPT WITHOUT DEFINING IT FULLY (ICONIC REPRESENTATION); AND BY A SET OF SYMBOLIC OR LOGICAL PROPOSITIONS DRAWN FROM A SYMBOLIC SYSTEM THAT IS GOVERNED BY RULES OR LAWS FOR FORMING AND TRANSFORMING PROPOSITIONS (SYMBOLIC REPRESENTATION).

WITHIN THIS SCHEME STUDENTS IN HIM AND HID BEGAN WITH AN ENACTIVE REPRESENTATION OF AN ALGEBRA WORD PROBLEM, SOMETHING THAT COULD LITERALLY BE "DONE," PROGRESSED TO AN ICONIC REPRESENTATION, HOWEVER RESTRICTED, AND THEN REPRESENTED THE PROBLEM SYMBOLICALLY AS A SET OF EQUATIONS IN TWO UNKNOWNS. FOR NOTATION, THEN, THERE WAS A VISUAL REFERENT.

TO ILLUSTRATE, HIM AND HID WILL BE APPLIED TO THE FOLLOWING COIN PROBLEM, BEGINNING WITH THE FIRST SENTENCE IRRESPECTIVE OF THE OTHER AND CONCLUDING WITH THE PROBLEM AS STATED.

> A MAN HAS THREE TIMES AS MANY DIMES AS QUARTERS. IF THE DIMES AND QUARTERS TOGETHER ARE WORTH $1.65, HOW MANY DIMES AND QUARTERS DOES HE HAVE?

FOR THE SENTENCE RELATING THE NUMBER OF COINS, STUDENTS IN HIM WOULD HAVE CONSTRUCTED TWO "STACKS" SUCH THAT THE NUMBER OF "RODS" IN EACH MET THE QUANTITATIVE CONDITION. FOR THIS PURPOSE EACH STUDENT IN HIM WAS SUPPLIED THIRTY RECTANGULAR PIECES OF HEAVY CARDBOARD 3/8" x 3/2" IN DIMENSION. IN CONTRAST, STUDENTS IN HID WOULD HAVE DRAWN THE STACKS. IN EITHER CASE, HOWEVER, THE END RESULT WOULD HAVE BEEN THE SAME: ONE OF MANY POSSIBLE ILLUSTRATIONS OF THE CONDITION SUCH AS




SUBSEQUENT TO ILLUSTRATING ONE CONDITION ON TWO UNKNOWNS, THE RESTRICTION OF AN ADDITIONAL CONDITION ON TWO UNKNOWNS WAS CONSIDERED. IN HIM THIS MEANT "BUILDING" A SENTENCE AS BEFORE AND THEN "CHECKING" THE CONSTRUCTION

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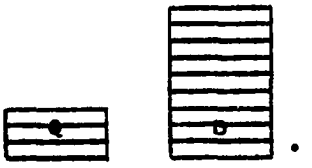
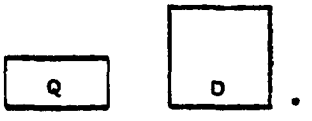
AGAINST THE SECOND SENTENCE TO SEE IF IT REPRESENTED IT AS WELL. IN HiU THIS MEANT SELECTING FROM FOUR POSSIBLE CHOICES THE PAIR OF STACKS THAT MET BOTH CONDITIONS. IN EITHER TREATMENT, THEN, THE COIN PROBLEM WOULD HAVE APPEARED AS




SINCE A NUMBER OF AUTHORS CAUTION THAT PROLONGED USAGE OF REAL OBJECTS MAY INHIBIT GENERALIZATION AND FORMATION OF HIGHER FORMS OF ANALYSIS AND SYNTHESIS (E.G., KALMYKOVA, 1955), THE MANNER OF REPRESENTING PROBLEMS AS IN HIM AND HID WAS ABSTRACTED TO SOME EXTENT. FOLLOWING THE DEPICTION OF PROBLEMS IN TERMS OF STACKS WITH EXACTLY SO MANY RODS -- WHETHER LAID OUT, DRAWN, OR SELECTED -- "VARIABLE" STACKS WERE USED. THESE WERE SIMPLY TWO RECTANGLES; HOWEVER, THE TALLER WAS LABELED SO AS TO CORRESPOND TO THE "LARGER" OF TWO UNKNOWNS. IN THIS WAY EQUALITY BETWEEN, SAY, QUANTITIES OF




COINS, WAS ASSOCIATED WITH DISPARITY BETWEEN HEIGHTS. FOR EXAMPLE, AS LABELED FOR THE COIN PROBLEM STUDENTS COULD "SEE" THAT THE VARIABLE Q RATHER THAN D HAD TO BE MULTIPLIED BY THREE IN ORDER TO BRING THE HEIGHT OF THE SMALLER "Q" STACK UP TO THAT OF THE "D" STACK. ACCORDINGLY, THEY WERE DIRECTED TO WRITE $3Q = D$.

FOLLOWING TREATMENT OF NUMBER, COIN, AND AGE, A SEQUENCE ON WORK PROBLEMS AND MIXTURE PROBLEMS WAS DESIGNED TO BE USED BY TREATMENT GROUPS HIM, HID, AND LO ALIKE. FOR THE MOST PART, THIS SEQUENCE WAS A CONTINUATION OF LO, PARTICULARLY IN ITS TREATMENT OF MIXTURE. ITS TREATMENT OF WORK, HOWEVER, WAS A COMBINATION OF HIGH- AND LO-IMAGERY. FOR WORK, NOTATION WAS REFERRED TO DIAGRAMS SIMILAR TO THOSE EMPLOYED IN HIM AND HID, BUT THIS WAS

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FOR PURPOSE OF EXPLANATION RATHER THAN REPRESENTATION. THIS SEQUENCE, AS WELL AS HIM AND LO, ARE INCLUDED IN THE APPENDIX. (HID CAN BE RECONSTRUCTED FROM HIM BY SUBSTITUTING "DRAW" OR "CHOOSE" FOR "CONSTRUCT.")

IN SUMMARY, THE OBJECTIVE OF HIGH- AND LOW-IMAGERY TREATMENTS WAS TO INCULCATE TRANSLATIVE SKILL AS RELATED TO ALGEBRA WORD PROBLEMS. THIS WAS ACCOMPLISHED THROUGH ATTENDING FIRST TO PREREQUISITE SKILLS AND KNOWLEDGE. FOLLOWING THIS, HIGH-IMAGERY TREATMENTS PROGRESSED DIFFERENTLY FROM LOW-IMAGERY TREATMENT. WHEREAS LO PROCEEDED DIRECTLY FROM TRANSLATION OF PHRASES AND SENTENCES TO TRANSLATION OF VERBAL PROBLEMS, HIM AND HID TOOK A MORE CIRCUITOUS ROUTE THROUGH CONCRETE AND PICTORIAL REPRESENTATIONS OF PROBLEMS PRIOR TO THEIR SYMBOLIZATION.

## INSTRUCTIONS TO TEACHERS

PRIOR TO THE STUDY, PARTICIPATING TEACHERS WORKED FOR NINETY MINUTES WITH THE WRITER ON THE USE OF THE INSTRUCTIONAL MATERIALS. THEY WERE THEN GIVEN A SET OF LESSON PLANS GOVERNING EACH DAYS ACTIVITIES. IN GENERAL, THE DAILY DIRECTIVES PROCEEDED IN KEEPING WITH THE ONE BELOW FOR HIM AND HID ON NUMBER (DAY 1). EXCEPT FOR THE "INSERT" REFERENCE AND THE VISUAL DEVELOPMENT OF THE EXAMPLE, THIS SAMPLE LESSON PLAN TYPIFIES THE ONES PROVIDED TEACHERS ASSIGNED TO LO.

> HAVE YOUR STUDENTS SILENTLY READ THE FIRST PAGE. THEN FILL IN 4 TO 5 BLANKS ON THE NEXT PAGE WITH THEM AND HAVE THEM FILL IN THE REST. AFTER 4 OR 5 MINUTES ASK THEM TO CHECK AND CORRECT THEIR WORK AND FILL IN THE REMAINING BLANKS AS YOU READ THEM THE ANSWERS. (ALL FILL-INS ARE TO BE COMPLETED CORRECTLY BY YOUR STUDENTS, EITHER THROUGH REFERENCE TO CONSENSUS ANSWERS OR GIVEN ANSWERS.) WHEN THE CHECKING IS COMPLETED, HAVE THEM READ TO THE INSERT.
>
> FOR THE INSERT BUILD TWO STACKS AT THE OVERHEAD WITH THE ACETATE RODS, ONE WITH 2 AND THE OTHER WITH 8. THEN ASK YOUR STUDENTS TO USE SOME OF THE PHRASES (OUTLOUD) THEY HAVE JUST TRANSLATED TO DESCRIBE THE VARIOUS RELATIONSHIPS BETWEEN THE TWO

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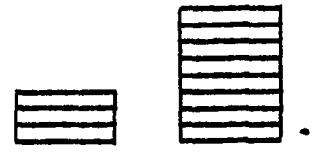
STACKS. THESE ARE TO BE RECORDED BY YOUR STUDENTS IN SENTENCE FORM IN THE BLANKS AND TRANSLATED INTO EQUATIONS. (AGAIN, BE SURE THAT WHAT THEY EVENTUALLY END UP WITH IN THE BLANKS IS CORRECT.)

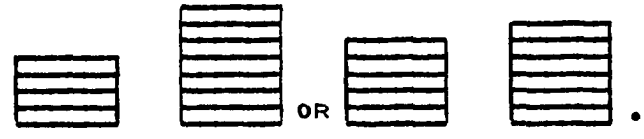
AS SOON AS YOU ARE FINISHED WITH THE INSERT, TURN TO THE PROBLEM SET AND DEVELOP THE EXAMPLE AS INDICATED BELOW. THEN PROCEED THROUGH ONE OF THE PROBLEMS WITH THEM IN THE SAME WAY. IF TIME IS RUNNING SHORT, OMIT THE EXPLANATION OF THE EXAMPLE IN ORDER TO DO ONE OF THE PROBLEMS.

PACE YOURSELF SO THAT AT LEAST 25 MINUTES OF PROBLEM-SOLVING TIME REMAINS FOR YOUR STUDENTS. HAVE YOUR STUDENTS SIGN AND RETURN THEIR PAPERS BY THE END OF THE PERIOD.

$N_1$ PROBLEM SET: ASK AND ILLUSTRATE, "THE CONDITION ON THE NUMBERS IN THE FIRST EXAMPLE IS ILLUSTRATED ON YOUR PAPER AS



WHAT ARE SOME OTHER POSSIBLE ILLUSTRATIONS?" (SOLICIT RESPONSES SUCH AS




AND POINT OUT THAT ANY ONE OF THESE REPRESENTATIONS OF THE CONDITIONS COULD HAVE BEEN DRAWN EVEN THOUGH THERE IS ESSENTIALLY ONLY ONE EQUATION IN TWO UNKNOWNS THAT ALSO DESCRIBES IT (AND THE ILLUSTRATIONS), NAMELY, $x + y = 11$.)

FOR ANY ONE DAY, THEN, ACTIVITIES WERE DIVIDED EQUALLY BETWEEN EXPOSITION AND PROBLEM SOLVING. THE TEACHERS DIRECTED DISCUSSION, WENT OVER EXAMPLES, AND SOLVED ONE OR TWO PROBLEMS WITH THEIR STUDENTS. THE STUDENTS THEN WORKED THE REMAINING PROBLEMS. THE INSERTS SUPPLIED TEACHERS IN HIM AND HID ARE INCLUDED IN THE APPENDIX ALONG WITH THE STUDENT MATERIALS.

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## ORGANIZATION OF THE STUDY

The study was modeled after the orthodox pretest-posttest control group design (Campbell and Stanley, 1963). This form of organization permitted determination of effect of particular treatment on selected items (that is, did learning result?) and ascertainment of relative effect of differential treatment (that is, which treatment resulted in the most learning?). Since learning for most dependent variables was presupposed, analysis dealt primarily with relative superiority of treatments.

Pretest-Posttest Control Group Design

| | | | | |
|---|---|---|---|---|
| R | O | X | O | |
| R | O | | O | |

R: random assignment to separate treatment groups
X: some agent or treatment presumed to cause change
O: pretest or posttest, depending on position

The study progressed through three stages. In stage I varying instruction by treatment group was applied to number, coin, and age. In HIM and HID two days were allotted for number, three for coin, and three for age for eight days in all. In LO, number, coin, and age received two days instruction each for a total of six days, and an additional two days were spent in review to control for time. In stage II low-imagery instruction of sorts was applied equally across treatment groups to work and mixture to permit examination of the respective effects of HIM, HID, and LO on subsequent learning. Two days were given to work and two to mixture for a total of four days. For the diagram above, then, "X" may be replaced with HIM-"LO," HID-"LO," and LO-"LO" (see next page). In stage III treatment groups resumed the original curriculum for twelve days before being tested for retention of number, coin, age, work, and mixture.

In a sense the study was two in one. Since it is generally conceded

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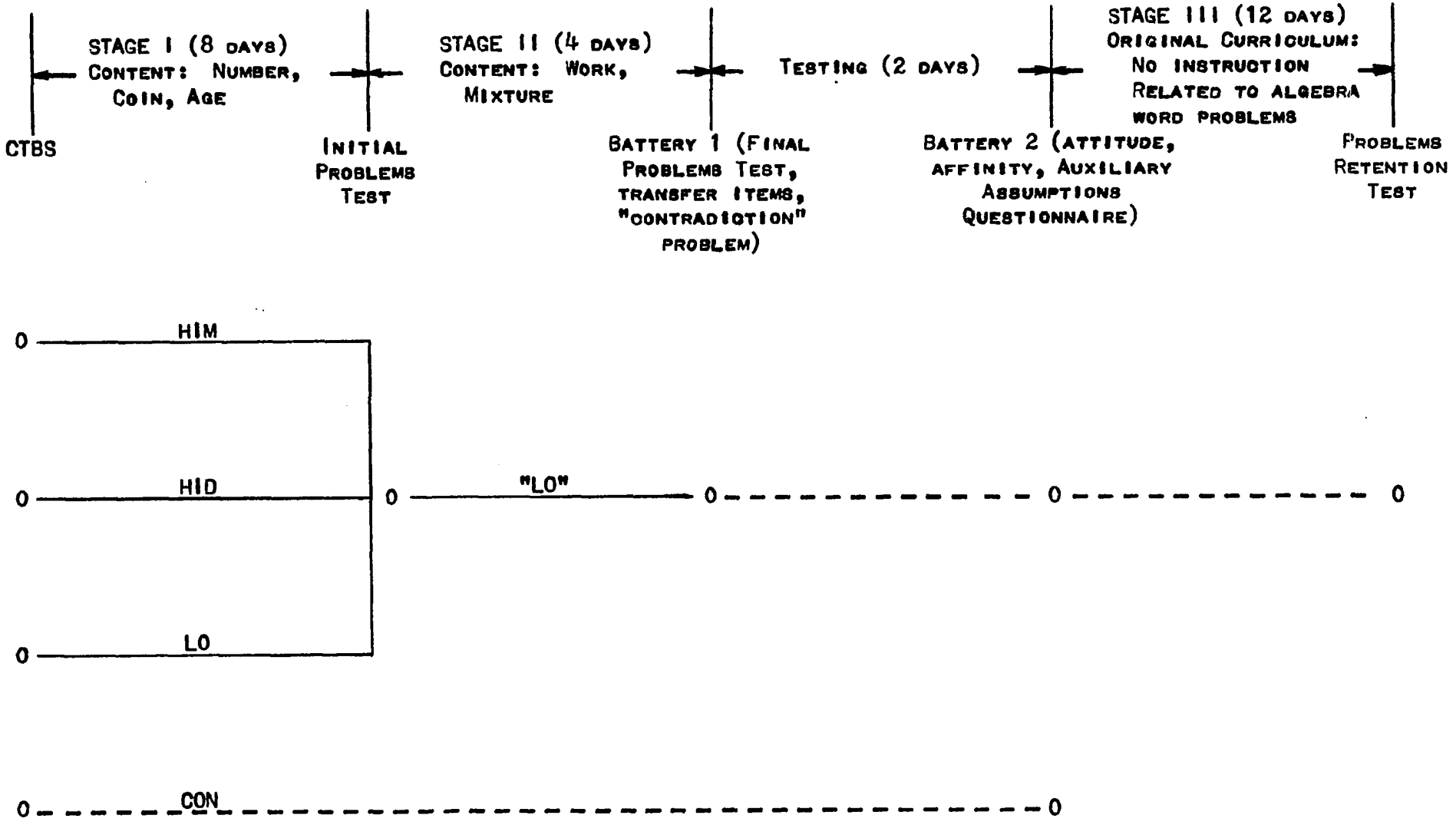
DIAGRAM OF THE STUDY



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THAT TENDENCY TO IMAGE RECEDES WITH AGE (E.G., WERNER, 1948), IT WAS THOUGHT THAT THE EFFECTS OF THE PLACEMENT OF THE IMAGERY VARIABLE MIGHT DIFFER CONSIDERABLY WITH THE AGE OF SUBJECTS. ACCORDINGLY, TWO AGE LEVELS WERE REPRESENTED IN THE STUDY, AND EXPERIMENTATION AT GRADE NINE WAS REPLICATED AT GRADE SEVEN WITH RESULTS FOR EACH GRADE BEING ANALYZED INDEPENDENTLY OF ONE ANOTHER.

## TIMING AND SUBJECTS

THE STUDY WAS CONDUCTED BEGINNING NOVEMBER 2, 1970 IN THE PUBLIC SCHOOLS OF ROCHESTER, MICHIGAN, A TOWN OF SOME 6,000, RURAL IN SETTING YET ECONOMICALLY DEPENDENT ON METROPOLITAN DETROIT. TREATMENT TERMINATED SOME TWO WEEKS LATER ON NOVEMBER 17, AND A TEST ON RETENTION MARKING THE END OF THE STUDY WAS ADMINISTERED DECEMBER 2, 1970.

PARTICIPATING IN THE STUDY WERE SIXTEEN TEACHERS AND TWENTY-TWO OF THEIR CLASSES FROM ROCHESTER'S TWO JUNIOR HIGH SCHOOLS AND TWO HIGH SCHOOLS. TWELVE OF THESE CLASSES WERE SEVENTH GRADE MATHEMATICS CLASSES INVOLVING 366 STUDENTS, AND THE REMAINING TEN WERE NINTH GRADE ALGEBRA CLASSES REPRESENTING 336 STUDENTS. OF THE TWELVE SEVENTH GRADE CLASSES, NINE WITH A TOTAL OF 287 STUDENTS WERE ASSIGNED TO TREATMENT, AND THE REMAINING THREE WITH 79 STUDENTS SERVED AS A QUASI CONTROL GROUP WHOSE PARTICIPATION IN THE STUDY WAS LIMITED TO THE TAKING OF SELECTED TESTS, NAMELY, THE ARITHMETIC PORTION OF THE Comprehensive Test of Basic Skills AND BATTERY 2 (ATTITUDE, AFFINITY, AUXILIARY REPRESENTATIONS QUESTIONNAIRE) ON 11/13/70 AND 12/2/70, RESPECTIVELY. SIMILARLY, OF THE TEN NINTH GRADE CLASSES, SEVEN REPRESENTING 245 STUDENTS WERE ASSIGNED TO TREATMENT, AND THE OTHER THREE WITH 91 STUDENTS WERE ASSIGNED TO NON-TREATMENT (OR CON FOR "CONTROL").

THE WRITER APPORTIONED THESE CLASSES AND THEIR TEACHERS TO TREATMENT AND NON-TREATMENT GROUPS AS INDICATED BELOW. THE CLASSES ARE REFER-

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enced by C, the teachers by T, and the class subscripts agree with those appearing in the tables included in analysis of data in Chapter IV. The teacher subscripts refer to teacher number.

| GRADE 7 | | | | GRADE 9 | | | |
|---|---|---|---|---|---|---|---|
| HIM | HID | LO | CON | HIM | HID | LO | CON |
| $C_1$-$T_1$ | $C_1$-$T_3$ | $C_1$ \ $T_6$ / $C_2$ | $C_1$ \ $C_2$-$T_8$ / $C_3$ | $C_1$-$T_9$ | $C_1$ \ $T_{11}$ / $C_2$ | $C_1$ \ $T_{12}$ / $C_2$ | $C_1$-$T_{14}$ |
| $C_2$ \ $T_2$ / $C_3$ | $C_2$-$T_4$ | | | $C_2$-$T_{10}$ | | | $C_2$-$T_{15}$ |
| | $C_3$-$T_5$ | $C_3$-$T_7$ | | | | $C_3$-$T_{13}$ | $C_3$-$T_{16}$ |

Whereas the study originally involved 702 (366 + 336) students, analysis was in terms of reduced sample size. Data for eleven seventh graders and thirteen ninth graders under treatment were discarded because, as determined by their teachers, these students were absent to such an extent as to prohibit completing work missed. In addition, sample size varied from test to test since some students were unable to make up tests they had missed.

Also under treatment but omitted from analysis for the most part were two ninth grade algebra classes from Kennedy Junior High, Pontiac, Michigan. One was assigned to HIM, the other to HID, and they were taught by the two teachers who had assisted in piloting the study. Since these classes were biased in the sense that their teachers had once taught the high-imagery sequences, they were not reflected in final analysis except for contradictory statements. They were included in experimentation, however, to provide an indication of sequence effectiveness with teachers more familiar in their use.

Results for the two Kennedy Junior High classes on selected variables appear in the appendix. The low scores for HIM in this case are perhaps misleading since school testing (some five days worth) scheduled for the same

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CLASS PERIOD AS HIM INTERFERED CONSIDERABLY WITH THE CONTINUITY OF THE SEQUENCE.

## INSTRUMENTATION AND ANALYSIS

AS A PRETEST SEVENTH GRADERS TOOK LEVEL THREE OF THE ARITHMETIC PORTION OF THE 1968 EDITION OF THE COMPREHENSIVE TEST OF BASIC SKILLS (CTBS), AND NINTH GRADERS TOOK LEVEL FOUR. SCORES ON THE MOST RECENTLY GIVEN TEST OF MENTAL MATURITY WERE ALSO COLLECTED FROM AVAILABLE FILES.

THROUGHOUT THE STUDY THREE ACHIEVEMENT TESTS WERE GIVEN: THE INITIAL PROBLEMS TEST AT THE END OF STAGE I, THE FINAL PROBLEMS TEST AT THE END OF STAGE II, AND THE PROBLEMS RETENTION TEST AT THE CONCLUSION OF THE STUDY. THE FINAL PROBLEMS TEST WAS PART OF A BATTERY INCLUDING TRANSFER ITEMS AND A PROBLEM DESCRIBING A PHYSICALLY IMPOSSIBLE SITUATION.

ON THE DAY FOLLOWING THE ADMINISTRATION OF THE FINAL PROBLEMS TEST AND ITS CONCOMITANTS, A SECOND BATTERY OF TESTS WAS GIVEN, NAMELY, A MATHEMATICS OPINIONNAIRE AS DEVISED BY AIKEN AND DREGER (1961, $r = .94$), A PROBLEMS "AFFINITY" TEST, AND THE AUXILIARY REPRESENTATIONS QUESTIONNAIRE.

WITH THE EXCEPTION OF THE ARITHMETIC PRETEST AND THE MATHEMATICS OPINIONNAIRE, THE TESTS USED IN THIS STUDY WERE CONSTRUCTED BY THE WRITER. ALL BUT THE CTBS INSTRUMENT APPEAR IN THE APPENDIX, WITH THE CONSTRUCT OF EACH TEST, ITS OBJECTIVE, AND SCORING BEING DISCUSSED IN CHAPTER IV. RELIABILITY COEFFICIENTS FOR THE FINAL PROBLEMS TEST AND AUXILIARY REPRESENTATIONS QUESTIONNAIRE WERE DETERMINED FOR EACH GRADE USING THE KUDER-RICHARDSON FORMULA (STANLEY, 1956). AS SHOWN IN THE APPENDIX, THE RELIABILITY INDICES FOR THE FINAL PROBLEMS TEST WERE .88 AND .74 FOR GRADES SEVEN AND NINE, RESPECTIVELY. FOR THE AUXILIARY REPRESENTATIONS QUESTIONNAIRE THEY WERE .59 AND .76, RESPECTIVELY.

TWO QUESTIONS, ESSENTIALLY, WERE TO BE ANSWERED THROUGH ANALYSIS OF

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DATA OBTAINED WITH THE AFOREMENTIONED INSTRUMENTS. FIRST, WHICH TREATMENT -- HIM, HID, OR LO -- FOR EACH GRADE WAS MOST EFFECTIVE WITH RESPECT TO A GIVEN TEST. SECOND, WHICH TYPE OF STUDENT -- ONE HIGH ON THE ACHIEVEMENT SPECTRUM IN ARITHMETIC, ONE SOMEWHAT IN BETWEEN, OR ONE TOWARD THE BOTTOM -- BENEFITED MOST FROM SPECIFIC TREATMENT FOR A GIVEN TEST.

IN ORDER TO ANSWER THE FIRST QUESTION MULTIVARIATE ANALYSIS OF COVARIANCE WAS APPLIED TO STUDENT SCORES BY TREATMENT GROUP. THE COVARIATES WERE CTBS ARITHMETIC COMPUTATION, ARITHMETIC CONCEPTS, AND ARITHMETIC APPLICATIONS SCORES AS OBTAINED DURING PRETESTING. SEVENTH GRADE MEANS WERE ADJUSTED WITH RESPECT TO LEVEL THREE CTBS SCORES, AND NINTH GRADE MEANS WITH RESPECT TO LEVEL FOUR SCORES. IQ PERCENTILES WERE NOT INCLUDED AS COVARIATES SINCE MANY WERE UNAVAILABLE FROM SCHOOL RECORDS.

IN ADDITION TO USING ANALYSIS OF COVARIANCE FOR COMPARING TREATMENT GROUPS, SIMILAR ANALYSIS WAS APPLIED TO CLASSES WITHIN TREATMENTS. IF NO SIGNIFICANT DIFFERENCES OCCURRED BETWEEN CLASSES IN THIS LATTER ANALYSIS, SIGNIFICANCE NOTED IN THE FORMER WAS ATTRIBUTED TO TREATMENT.

TO PROVIDE DATA FOR ANALYSIS WITH RESPECT TO THE SECOND QUESTION, STUDENTS WERE STRATIFIED ACCORDING TO UPPER 1/3, MIDDLE 1/3, AND LOWER 1/3 ON THE BASIS OF TOTAL ARITHMETIC ACHIEVEMENT (=ARITHMETIC COMPUTATION + ARITHMETIC CONCEPTS + ARITHMETIC APPLICATIONS).

FOLLOWING THIS, ONE-WAY ANALYSIS OF VARIANCE WAS APPLIED TO STUDENT SCORES WITHIN ACHIEVEMENT LEVELS BY TREATMENT GROUP. SINCE LINEAR CORRELATION OF VARIABLES (CHAPTER IV, TABLES 1-2) INDICATED HIGH CORRELATION BETWEEN ARITHMETIC ACHIEVEMENT AND DEPENDENT VARIABLES, INTERACTION EFFECTS BETWEEN ACHIEVEMENT STRATA SEEMED UNLIKELY. THEREFORE, ONE-WAY ANALYSIS OF VARIANCE WAS EMPLOYED RATHER THAN TWO-WAY ANALYSIS WHICH WOULD HAVE TESTED FOR THE EFFECT OF ASSIGNMENT TO ACHIEVEMENT LEVELS AS WELL AS TREATMENT EFFECTS.

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# CHAPTER IV

## ANALYSIS OF DATA

Subsequent to analysis of certain initial characteristics of treatment groups, treatment subgroups, and classes, data are organized for presentation as follows:

1. Achievement on problem types receiving differential treatment (number, coin, age)
2. Achievement on problem types for which instruction remained invariant across treatment groups (work, mixture)
3. Detection of contradictory statements
4. Retention
5. Transfer
6. Knowledge of auxiliary information and physical assumptions representative of specific problem types
7. Attitude toward mathematics and toward algebra word problems

The chapter then concludes with a report of personal interviews conducted by the writer with selected seventh grade students.

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## INITIAL CHARACTERISTICS

Two measures were available for judging comparability of treatment groups, subgroups, and classes: IQ and arithmetic achievement. Arithmetic achievement was used because more scores were available and because, as illustrated in tables 1-2* for grades seven and nine, respectively, it correlated more highly with more dependent variables than did IQ. In referring to tables 3-4, then, it is seen that treatment groups and subgroups for both grades were roughly comparable at the outset of the study. That is, as determined by analysis of variance means in each case differed as would be expected on the basis of chance alone ($p > .05$).

Also in tables 3-4 are the results of analysis of variance of classes by treatments. As indicated, the only treatment group with significant initial bias between classes was LO at grade seven (see table 3). The mean for class 3 (72.16) differed from that of class 2 (63.13) at the .05 level of significance, and from that of class 1 (59.89) at the .01 level as given in the concomitant T-test matrix. So significant within-treatment differences, which complicate between-treatment comparisons, would be anticipated for this treatment group on dependent variables correlating highly with arithmetic achievement. However, since further analysis included covarying on the part scores of arithmetic achievement (computation, concepts, and applications), this expectant class variability was controlled to some extent.

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*In every case odd-numbered tables refer to data obtained at the seventh grade and even-numbered ones to data for the ninth grade. Except for this reference to specific grade level, two consecutively ordered odd-even tables refer to identical testing instruments.

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TABLE 1: CORRELATION MATRIX
(SAMPLE SIZES ABOVE DIAGONAL),
GRADE 7

| | Sex | Treatment | Language IQ | Non-Language IQ | Total IQ | CTBS Computation | CTBS Concepts | CTBS Applications | CTBS Arith Achiev | Initial Prob Test | Final Prob Test | Num-Coin-Age$_{FPT}$ | Work-Mixture$_{FPT}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sex | 1.0 | 276 | 214 | 214 | 214 | 261 | 261 | 261 | 261 | 270 | 266 | 266 | 266 |
| Treatment | -.04 | 1.0 | 214 | 214 | 214 | 261 | 261 | 261 | 261 | 270 | 266 | 266 | 266 |
| Language IQ | .05 | .02 | 1.0 | 214 | 214 | 203 | 203 | 203 | 203 | 209 | 205 | 205 | 205 |
| Non-Language IQ | .08 | .04 | .51 | 1.0 | 214 | 203 | 203 | 203 | 203 | 209 | 205 | 205 | 205 |
| Total IQ | .08 | .05 | .91 | .81 | 1.0 | 203 | 203 | 203 | 203 | 209 | 205 | 205 | 205 |
| CTBS Computation | .08 | .05 | .50 | .44 | .55 | 1.0 | 261 | 261 | 261 | 256 | 253 | 253 | 253 |
| CTBS Concepts | .01 | .02 | .59 | .52 | .65 | .73 | 1.0 | 261 | 261 | 256 | 253 | 253 | 253 |
| CTBS Applications | -.04 | .08 | .56 | .51 | .62 | .75 | .78 | 1.0 | 261 | 256 | 253 | 253 | 253 |
| CTBS Arith Achiev | .03 | .05 | .59 | .52 | .65 | .94 | .89 | .89 | 1.0 | 256 | 253 | 253 | 253 |
| Initial Prob Test | .10 | .18 | .45 | .39 | .50 | .60 | .58 | .54 | .63 | 1.0 | 260 | 260 | 260 |
| Final Prob Test | .07 | -.05 | .49 | .51 | .58 | .66 | .64 | .58 | .69 | .70 | 1.0 | 266 | 266 |
| Num-Coin-Age$_{FPT}$ | .06 | -.00 | .47 | .50 | .56 | .67 | .65 | .61 | .71 | .71 | .93 | 1.0 | 266 |
| Work-Mixture$_{FPT}$ | .06 | -.09 | .41 | .42 | .48 | .51 | .49 | .43 | .53 | .54 | .88 | .65 | 1.0 |
| Retention Test | .04 | .09 | .50 | .48 | .57 | .69 | .66 | .61 | .72 | .70 | .80 | .76 | .68 |
| Aux Rep Ques | .08 | .12 | .53 | .46 | .58 | .67 | .68 | .67 | .74 | .64 | .70 | .67 | .58 |
| Phrase$_{ARQ}$ | .10 | .06 | .41 | .28 | .41 | .48 | .52 | .49 | .54 | .47 | .51 | .46 | .47 |
| Sentence$_{ARQ}$ | -.00 | .11 | .43 | .31 | .43 | .53 | .54 | .56 | .59 | .50 | .58 | .56 | .47 |
| Coin$_{ARQ}$ | .08 | -.02 | .33 | .32 | .38 | .45 | .42 | .39 | .47 | .36 | .46 | .46 | .36 |
| Age$_{ARQ}$ | .03 | .12 | .36 | .43 | .44 | .47 | .46 | .47 | .51 | .44 | .42 | .42 | .34 |
| Work$_{ARQ}$ | -.08 | .14 | .19 | .11 | .19 | .24 | .28 | .29 | .29 | .27 | .28 | .30 | .18 |
| Mixture$_{ARQ}$ | .09 | .13 | .35 | .36 | .40 | .45 | .41 | .44 | .48 | .44 | .46 | .45 | .37 |
| Transfer Test | .10 | -.05 | .47 | .42 | .51 | .63 | .63 | .58 | .67 | .62 | .77 | .76 | .61 |
| "Contradiction" | -.03 | -.05 | .11 | .15 | .13 | .20 | .17 | .16 | .20 | .18 | .23 | .21 | .21 |
| Attitude Scale | -.04 | .01 | .14 | .23 | .20 | .42 | .38 | .35 | .43 | .41 | .47 | .44 | .42 |
| Affinity Test | .14 | -.12 | .27 | .26 | .31 | .38 | .38 | .34 | .41 | .42 | .43 | .42 | .35 |

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TABLE 1: CONT.

| | Retention Test | Aux Rep Ques | Phrase ARQ | Sentence ARQ | Coin ARQ | Age ARQ | Work ARQ | Mixture ARQ | Transfer Test | "Contradiction" | Attitude Scale | Affinity Test |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sex | 266 | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 266 | 266 | 260 | 260 |
| Treatment | 266 | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 266 | 266 | 260 | 260 |
| Language IQ | 208 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 205 | 205 | 202 | 202 |
| Non-Language IQ | 208 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 205 | 205 | 202 | 202 |
| Total IQ | 208 | 202 | 202 | 202 | 202 | 202 | 202 | 202 | 205 | 205 | 202 | 202 |
| CTBS Computation | 252 | 247 | 247 | 247 | 247 | 247 | 247 | 247 | 253 | 253 | 247 | 247 |
| CTBS Concepts | 252 | 247 | 247 | 247 | 247 | 247 | 247 | 247 | 253 | 253 | 247 | 247 |
| CTBS Applications | 252 | 247 | 247 | 247 | 247 | 247 | 247 | 247 | 253 | 253 | 247 | 247 |
| CTBS Arith Achiev | 252 | 247 | 247 | 247 | 247 | 247 | 247 | 247 | 253 | 253 | 247 | 247 |
| Initial Prob Test | 260 | 254 | 254 | 254 | 254 | 254 | 254 | 254 | 260 | 260 | 254 | 254 |
| Final Prob Test | 256 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 266 | 266 | 255 | 255 |
| Num-Coin-Age$_{FPT}$ | 256 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 266 | 266 | 255 | 255 |
| Work Mixture$_{FPT}$ | 256 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 266 | 266 | 255 | 255 |
| Retention Test | 1.0 | 251 | 251 | 251 | 251 | 251 | 251 | 251 | 256 | 256 | 251 | 251 |
| Aux Rep Ques | .71 | 1.0 | 260 | 260 | 260 | 260 | 260 | 260 | 255 | 255 | 260 | 260 |
| Phrase$_{ARQ}$ | .55 | .77 | 1.0 | 260 | 260 | 260 | 260 | 260 | 255 | 255 | 260 | 260 |
| Sentence$_{ARQ}$ | .58 | .68 | .44 | 1.0 | 260 | 260 | 260 | 260 | 255 | 255 | 260 | 260 |
| Coin$_{ARQ}$ | .41 | .68 | .37 | .39 | 1.0 | 260 | 260 | 260 | 255 | 255 | 260 | 260 |
| Age$_{ARQ}$ | .46 | .61 | .31 | .28 | .31 | 1.0 | 260 | 260 | 255 | 255 | 260 | 260 |
| Work$_{ARQ}$ | .28 | .43 | .17 | .21 | .22 | .15 | 1.0 | 260 | 255 | 255 | 260 | 260 |
| Mixture$_{ARQ}$ | .41 | .66 | .31 | .37 | .33 | .39 | .27 | 1.0 | 255 | 255 | 260 | 260 |
| Transfer Test | .69 | .68 | .52 | .57 | .41 | .41 | .30 | .44 | 1.0 | 266 | 255 | 255 |
| "Contradiction" | .19 | .18 | .09 | .16 | .14 | .12 | .03 | .18 | .24 | 1.0 | 255 | 255 |
| Attitude Scale | .44 | .44 | .36 | .28 | .32 | .33 | .14 | .21 | .34 | .17 | 1.0 | 260 |
| Affinity Test | .37 | .42 | .31 | .30 | .29 | .30 | .09 | .30 | .34 | .14 | .30 | 1.0 |

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TABLE 2: CORRELATION MATRIX
(SAMPLE SIZES ABOVE DIAGONAL),
GRADE 9

| | Sex | Treatment | Language IQ | Non-Language IQ | Total IQ | CTBS Computation | CTBS Concepts | CTBS Applications | CTBS Arith Achiev | Initial Prob Test | Final Prob Test | Num-Coin-Age$_{FPT}$ | Work-Mixture$_{FPT}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sex | 1.0 | 232 | 74 | 74 | 124 | 214 | 214 | 214 | 214 | 223 | 228 | 228 | 228 |
| Treatment | .01 | 1.0 | 74 | 74 | 124 | 214 | 214 | 214 | 214 | 223 | 228 | 228 | 228 |
| Language IQ | -.26 | -.02 | 1.0 | 74 | 74 | 69 | 69 | 69 | 69 | 69 | 73 | 73 | 73 |
| Non-Language IQ | -.07 | .02 | .64 | 1.0 | 74 | 69 | 69 | 69 | 69 | 69 | 73 | 73 | 73 |
| Total IQ | -.07 | -.00 | .92 | .87 | 1.0 | 118 | 118 | 118 | 118 | 119 | 123 | 123 | 123 |
| CTBS Computation | .13 | -.05 | .43 | .36 | .42 | 1.0 | 214 | 214 | 214 | 208 | 211 | 211 | 211 |
| CTBS Concepts | -.02 | -.12 | .44 | .38 | .39 | .57 | 1.0 | 214 | 214 | 208 | 211 | 211 | 211 |
| CTBS Applications | -.12 | .01 | .40 | .36 | .40 | .52 | .61 | 1.0 | 214 | 208 | 211 | 211 | 211 |
| CTBS Arith Achiev | .03 | -.06 | .50 | .42 | .48 | .90 | .82 | .79 | 1.0 | 208 | 211 | 211 | 211 |
| Initial Prob Test | -.00 | -.05 | .30 | .23 | .33 | .40 | .35 | .31 | .43 | 1.0 | 219 | 219 | 219 |
| Final Prob Test | .02 | .05 | .26 | .15 | .26 | .46 | .35 | .27 | .45 | .62 | 1.0 | 228 | 228 |
| Num-Coin-Age$_{FPT}$ | .02 | .03 | .28 | .21 | .29 | .46 | .36 | .31 | .46 | .67 | .86 | 1.0 | 228 |
| Work-Mixture$_{FPT}$ | .02 | .05 | .13 | .01 | .13 | .30 | .22 | .14 | .28 | .33 | .81 | .38 | 1.0 |
| Retention Test | .08 | -.07 | .18 | .15 | .20 | .49 | .38 | .30 | .48 | .61 | .74 | .67 | .55 |
| Aux Rep Ques | -.00 | -.07 | .35 | .26 | .36 | .55 | .50 | .48 | .61 | .49 | .57 | .61 | .32 |
| Phrase$_{AQR}$ | .16 | .17 | .13 | .17 | .17 | .42 | .27 | .24 | .39 | .35 | .34 | .40 | .16 |
| Sentence$_{AQR}$ | -.06 | -.02 | .14 | .00 | .09 | .34 | .27 | .32 | .37 | .41 | .52 | .55 | .30 |
| Coin$_{AQR}$ | .03 | .03 | .35 | .28 | .36 | .43 | .37 | .39 | .48 | .25 | .35 | .40 | .16 |
| Age$_{AQR}$ | .00 | -.08 | .19 | .18 | .22 | .41 | .43 | .37 | .48 | .30 | .43 | .48 | .21 |
| Work$_{AQR}$ | -.12 | -.27 | .07 | .11 | .08 | .14 | .20 | .18 | .20 | .26 | .14 | .11 | .12 |
| Mixture$_{AQR}$ | -.02 | -.12 | .29 | .16 | .32 | .34 | .33 | .31 | .39 | .33 | .39 | .38 | .27 |
| Transfer Test | -.03 | -.15 | .37 | .14 | .31 | .39 | .36 | .30 | .42 | .60 | .69 | .68 | .45 |
| "Contradiction" | .03 | -.09 | .14 | .17 | .15 | .03 | .03 | .08 | .05 | .20 | .12 | .13 | .07 |
| Attitude Scale | -.02 | .05 | .01 | .06 | .04 | .40 | .25 | .25 | .38 | .22 | .34 | .30 | .27 |
| Affinity Test | .04 | .07 | .04 | -.02 | -.01 | .03 | .06 | .07 | .06 | .01 | .06 | .06 | .04 |

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TABLE 2: CONT.

| | Retention Test | Aux Rep Ques | Phrase ARQ | Sentence ARQ | Coin ARQ | Age ARQ | Work ARQ | Mixture ARQ | Transfer Test | "Contradiction" | Attitude Scale | Affinity Test |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sex | 226 | 223 | 223 | 223 | 223 | 223 | 223 | 223 | 228 | 228 | 223 | 223 |
| Treatment | 226 | 223 | 223 | 223 | 223 | 223 | 223 | 223 | 228 | 228 | 223 | 223 |
| Language IQ | 73 | 71 | 71 | 71 | 71 | 71 | 71 | 71 | 73 | 73 | 71 | 71 |
| Non-Language IQ | 73 | 71 | 71 | 71 | 71 | 71 | 71 | 71 | 73 | 73 | 71 | 71 |
| Total IQ | 123 | 119 | 119 | 119 | 119 | 119 | 119 | 119 | 123 | 123 | 119 | 119 |
| CTBS Computation | 208 | 206 | 206 | 206 | 206 | 206 | 206 | 206 | 211 | 211 | 206 | 206 |
| CTBS Concepts | 208 | 206 | 206 | 206 | 206 | 206 | 206 | 206 | 211 | 211 | 206 | 206 |
| CTBS Applications | 208 | 206 | 206 | 206 | 206 | 206 | 206 | 206 | 211 | 211 | 206 | 206 |
| CTBS Arith Achiev | 208 | 206 | 206 | 206 | 206 | 206 | 206 | 206 | 211 | 211 | 206 | 206 |
| Initial Prob Test | 218 | 214 | 214 | 214 | 214 | 214 | 214 | 214 | 219 | 219 | 214 | 214 |
| Final Prob Test | 223 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 228 | 228 | 220 | 220 |
| Num-Coin-Age$_{FPT}$ | 223 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 228 | 228 | 220 | 220 |
| Work-Mixture$_{FPT}$ | 223 | 220 | 220 | 220 | 220 | 220 | 220 | 220 | 228 | 228 | 220 | 220 |
| Retention Test | 1.0 | 218 | 218 | 218 | 218 | 218 | 218 | 218 | 223 | 223 | 218 | 218 |
| Aux Rep Ques | .56 | 1.0 | 223 | 223 | 223 | 223 | 223 | 223 | 220 | 220 | 223 | 223 |
| Phrase$_{AQR}$ | .39 | .60 | 1.0 | 223 | 223 | 223 | 223 | 223 | 220 | 220 | 223 | 223 |
| Sentence$_{AQR}$ | .47 | .67 | .37 | 1.0 | 223 | 223 | 223 | 223 | 220 | 220 | 223 | 223 |
| Coin$_{AQR}$ | .32 | .70 | .31 | .37 | 1.0 | 223 | 223 | 223 | 220 | 220 | 223 | 223 |
| Age$_{AQR}$ | .35 | .72 | .32 | .41 | .45 | 1.0 | 223 | 223 | 220 | 220 | 223 | 223 |
| Work$_{AQR}$ | .25 | .38 | .05 | .15 | .05 | .12 | 1.0 | 223 | 220 | 220 | 223 | 223 |
| Mixture$_{AQR}$ | .38 | .70 | .32 | .30 | .37 | .34 | .20 | 1.0 | 220 | 220 | 223 | 223 |
| Transfer Test | .62 | .55 | .34 | .43 | .34 | .35 | .17 | .43 | 1.0 | 228 | 220 | 220 |
| "Contradiction" | .13 | .14 | .14 | .08 | .11 | .08 | -.05 | .15 | .14 | 1.0 | 220 | 220 |
| Attitude Scale | .26 | .36 | .22 | .19 | .30 | .35 | .12 | .18 | .28 | .11 | 1.0 | 223 |
| Affinity Test | .07 | -.01 | -.02 | .06 | -.02 | .02 | -.05 | -.03 | .14 | -.09 | -.01 | 1.0 |

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TABLE 3: ANALYSIS OF COMPREHENSIVE TEST OF BASIC SKILLS (LEVEL 3) ARITHMETIC ACHIEVEMENT (COMPUTATION, CONCEPTS, APPLICATIONS — 98 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

ANAVARA BETWEEN TREATMENT GROUPS

| TREATMENT | N | MEAN | SD | F-RATIO | T-TEST MATRIX FOR GROUP MEANS | | |
|---|---|---|---|---|---|---|---|
| HIM | 84 | 63.25 | 16.26 | 1.67 | | HIM | HID |
| HID | 86 | 60.59 | 18.57 | P < .19 | HID | NS | |
| LO | 91 | 65.34 | 17.02 | | LO | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO |
| HIM | 27 | 82.63 | 6.77 | .67 | HIM | 30 | 61.83 | 5.60 | .24 | HIM | 27 | 45.44 | 6.44 | .91 |
| HID | 26 | 84.58 | 6.79 | P < .51 | HID | 23 | 61.00 | 4.61 | P < .79 | HID | 37 | 43.49 | 7.29 | P < .41 |
| LO | 35 | 82.74 | 7.27 | | LO | 34 | 61.94 | 5.49 | | LO | 22 | 42.91 | 7.62 | |

ANAVARA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | MEAN | SD | F-RATIO | C | N | MEAN | SD | F-RATIO | C | N | MEAN | SD | F-RATIO |
| 1 | 23 | 58.87 | 15.99 | 1.85 | 1 | 29 | 60.69 | 15.49 | .84 | 1 | 27 | 59.89 | 17.98 | 4.56 |
| 2 | 30 | 67.33 | 15.03 | P < .16 | 2 | 28 | 57.29 | 19.41 | P < .43 | 2 | 32 | 63.13 | 17.02 | P < .01 |
| 3 | 31 | 62.55 | 17.13 | | 3 | 29 | 63.69 | 20.57 | | 3 | 32 | 72.16 | 14.19 | |

T-TEST MATRICES FOR CLASS MEANS

| HIM | | | HID | | | LO | | |
|---|---|---|---|---|---|---|---|---|
| | $C_1$ | $C_2$ | | $C_1$ | $C_2$ | | $C_1$ | $C_2$ |
| $C_2$ | NS | | $C_2$ | NS | | $C_2$ | NS | |
| $C_3$ | NS | NS | $C_3$ | NS | NS | $C_3$ | .01 | .05 |

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TABLE 4: ANALYSIS OF COMPREHENSIVE TEST OF BASIC SKILLS (LEVEL 4) ARITHMETIC ACHIEVEMENT (COMPUTATION, CONCEPTS, APPLICATIONS -- 98 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

ANAVARA BETWEEN TREATMENT GROUPS

| TREATMENT | N | MEAN | SD | F-RATIO | T-TEST MATRIX FOR GROUP MEANS | | |
|---|---|---|---|---|---|---|---|
| | | | | | | HIM | HID |
| HIM | 60 | 71.37 | 12.14 | .77 | | | |
| HID | 63 | 72.16 | 11.69 | $P < .47$ | HID | NS | |
| LO | 90 | 69.83 | 11.78 | | LO | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO |
| HIM | 23 | 82.83 | 3.68 | .38 | HIM | 19 | 71.79 | 3.31 | .60 | HIM | 18 | 56.28 | 7.96 | .22 |
| HID | 26 | 82.89 | 4.71 | $P < .69$ | HID | 18 | 71.61 | 3.01 | $P < .56$ | HID | 19 | 57.90 | 7.57 | $P < .81$ |
| LO | 29 | 82.00 | 4.13 | | LO | 27 | 72.56 | 3.03 | | LO | 34 | 57.29 | 7.39 | |

ANAVARA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | MEAN | SD | F-RATIO | C | N | MEAN | SD | F-RATIO | C | N | MEAN | SD | F-RATIO |
| 1 | 25 | 69.64 | 13.83 | DID NOT COMPUTE | 1 | 31 | 71.13 | 12.95 | DID NOT COMPUTE | 1 | 28 | 69.00 | 12.99 | .12 |
| 2 | 35 | 72.60 | 10.82 | | 2 | 32 | 73.12 | 10.48 | | 2 | 32 | 69.94 | 11.78 | $P < .89$ |
| | | | | | | | | | | 3 | 30 | 70.50 | 10.94 | |

T-TEST MATRICES FOR CLASS MEANS

| HIM | | HID | | LO | | |
|---|---|---|---|---|---|---|
| | $C_1$ | | $C_1$ | | $C_1$ | $C_2$ |
| $C_2$ | DID NOT COMPUTE | $C_2$ | DID NOT COMPUTE | $C_2$ | NS | |
| | | | | $C_3$ | NS | NS |

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## NUMBER, COIN, AGE

The instructional variable in this study, that of provisions for imagery, was applied to three types of word problems common to beginning algebra: number, coin, and age. In the high-imagery sequences that included notational referents of a visual sort, two days were spent on number, three on coin, and three on age for a total of eight days. In the low-imagery sequence where notation applied to problems was referred to number, each problem type received two days of instruction for a total of six days and, in order to equalize time, an additional two days were given to review.

At the end of the eighth or last day of initial instruction (that is, at the end of variable treatment) an Initial Problems Test was given to measure achievement on the various problem types dealt with up to this point. The test consisted of six word problems: two on number, two on coin, and two on age, with each problem allotted two points (one point for each equation in two unknowns) for a possible score of twelve.

Results for this test were analyzed first by treatment groups. Among seventh graders (table 5), adjusted means significantly favored LO: HIM-5.73, HID-6.06, LO-7.19 ($p < .001$). However, since students in LO had just completed a two-day review before taking the test, it cannot be determined with data at hand which contributed most to initial success, treatment or review. Whichever, it had little effect among ninth graders (table 6) insofar as differential achievement is concerned: HIM-9.47, HID-9.42, LO-9.20 ($p < .78$).

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TABLE 5: ANALYSIS OF INITIAL PROBLEMS TEST (NUMBER, COIN, AGE — 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | 5.82 | 3.68 | 5.73 | 6.99 | | | |
| HID | 84 | 5.76 | 3.53 | 6.06 | p < .001 | HID | NS | |
| LO | 89 | 7.38 | 3.15 | 7.19 | | LO | .001 | .01 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 4.55 | 4.95 | 2.36 | 1 | 29 | 5.14 | 5.18 | 1.26 | 1 | 27 | 6.93 | 7.67 | 2.74 |
| 2 | 30 | 7.17 | 6.77 | p < .10 | 2 | 27 | 5.56 | 5.74 | p < .29 | 2 | 31 | 7.58 | 7.94 | p < .07 |
| 3 | 31 | 5.42 | 5.52 | | 3 | 28 | 6.61 | 6.38 | | 3 | 31 | 7.58 | 6.57 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 8.70 | 2.67 | 1.01 | HIM | 30 | 5.33 | 3.35 | 3.35 | HIM | 26 | 3.15 | 2.75 | 1.08 |
| HID | 25 | 8.24 | 3.37 | p < .37 | HID | 23 | 6.39 | 3.41 | p < .04 | HID | 36 | 3.64 | 2.28 | p < .34 |
| LO | 34 | 9.26 | 2.30 | | LO | 33 | 7.55 | 2.59 | | LO | 22 | 4.23 | 2.60 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | .05 | NS | LO | NS | NS |

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TABLE 6: ANALYSIS OF INITIAL PROBLEMS TEST (NUMBER, COIN, AGE — 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 9.52 | 2.39 | 9.47 | .25 | | HIM | HID |
| HID | 63 | 9.55 | 2.84 | 9.42 | P < .78 | HID | NS | |
| LO | 86 | 9.08 | 2.72 | 9.20 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 23 | 8.91 | 9.20 | .71 | 1 | 31 | 10.32 | 10.43 | 6.58 | 1 | 24 | 8.79 | 9.00 | 5.35 |
| 2 | 35 | 9.91 | 9.72 | P < .40 | 2 | 32 | 8.82 | 8.72 | P < .01 | 2 | 32 | 8.25 | 8.14 | P < .007 |
| | | | | | | | | | | 3 | 30 | 10.20 | 10.15 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 10.52 | 1.81 | .02 | HIM | 19 | 9.53 | 2.06 | .05 | HIM | 18 | 8.33 | 2.85 | .30 |
| HID | 26 | 10.62 | 1.75 | P < .98 | HID | 18 | 9.39 | 3.15 | P < .95 | HID | 19 | 8.21 | 3.33 | P < .74 |
| LO | 28 | 10.54 | 1.91 | | LO | 24 | 9.25 | 3.15 | | LO | 34 | 7.76 | 2.36 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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Included in tables 5-6 are the results of analysis of classes by treatments. As indicated, within-treatment analysis of covariance revealed near significance between seventh grade classes comprising HIM ($p < .10$) and LO ($p < .07$), and significance between ninth grade classes comprising HID ($p < .01$) and LO ($p < .007$). So performance on initial achievement was most likely under the influence of factors additional to the defining characteristics of treatment.

A possible source of class variability within treatments is the extent to which teachers adhered to treatment, that is, presented each days work as requested in accompanying lesson plans and provided the suggested time for students to work on the problems. To account for this the writer visited each class at least once during the initial instructional phase of the study and, as a result, concluded that teachers were indeed following instructions. So within-treatment differences cannot knowingly be attributed to teachers failing to comply with experimental constraints.

An additional area for examination in this respect is the teacher himself. Since every teacher brings to a topic certain content knowledge, teaching strategies, and a set of mannerisms, it is probably more correct to say that treatment went through teachers instead of the other way around. But even here the evidence is contradictory. If the teacher, per se, were the major constituent in determination of results, then similar results should have obtained for ninth grade classes comprising HID since the same teacher taught both. However, the adjusted means for these classes (10.43 and 8.72) differed at the .01 level of significance (table 6). Similarly, the difference between classes 2 (6.77) and 3 (5.52) within seventh-grade HIM was considerably more than for 1 (4.95) and 3 even though the same teacher taught classes 2 and 3 (table 5). And in

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NEITHER CASE WAS VARIABILITY ASCRIBABLE TO THE PRACTICE EFFECT OF TEACHING THE SAME MATERIALS TWICE SINCE THE HIGHER MEANS WERE FOR THOSE CLASSES RECEIVING INSTRUCTION FIRST. IN CONTRAST, RESULTS WERE FAIRLY COMPARABLE FOR CLASSES 1 (7.67) AND 2 (7.94) WITHIN SEVENTH GRADE LO (TABLE 5) AND FOR CLASSES 1 (9.00) AND 2 (8.14) WITHIN NINTH GRADE LO (TABLE 6) WHERE THE SAME TEACHER FOR EACH GRADE TAUGHT BOTH CLASSES.

SINCE RESULTS FOR STUDENTS IN LO IN DIFFERENT CLASSES YET UNDER THE SAME TEACHER DIFFERED LESS THAN RESULTS FOR STUDENTS IN A SIMILAR SITUATION IN HIM OR HID, IT IS POSSIBLE THAT WITHIN-TREATMENT DIFFERENCES WERE A FUNCTION OF TEACHERS' LACK OF FAMILIARITY WITH THE METHODOLOGY SUGGESTED FOR THE HIGH-IMAGERY SEQUENCES. THAT IS, SINCE LO MOST NEARLY APPROXIMATED THE TYPICAL APPROACH TO TRANSLATION OF ALGEBRA WORD PROBLEMS, IT WOULD BE EXPECTED THAT TEACHERS HERE WOULD PERFORM WITH SOME CONSTANCY ON THE BASIS OF PREVIOUS EXPERIENCE. THIS WOULD ALSO ACCOUNT FOR THE VARIABILITY WITHIN LO WHERE CERTAIN TEACHERS HAD PERHAPS DEVELOPED MORE SUCCESSFUL TECHNIQUES FOR PRESENTING TRANSLATION IN A MANNER SIMILAR TO THAT PRESCRIBED FOR LO.

AT THE END OF THE INSTRUCTIONAL PHASE OF THE STUDY, THAT IS, AFTER ALL FIVE PROBLEM TYPES HAD BEEN TREATED, A FINAL PROBLEMS TEST WAS GIVEN TO MEASURE POST-TREATMENT ACHIEVEMENT. THE TEST CONSISTED OF TEN WORD PROBLEMS, AND THE GRADING WAS AS BEFORE: TWO POINTS FOR EACH FOR A POSSIBLE SCORE OF TWENTY. FOR THIS TEST THERE WERE NO WITHIN-TREATMENT DIFFERENCES (TABLES 7-8), SO WHATEVER FACTORS ASIDE FROM TREATMENT THAT CAUSED CLASSES TO PERFORM WITH SUCH VARIABILITY ON THE PRECEDING TEST SEEM TO HAVE WANED IN INFLUENCE AS THE STUDY PROGRESSED.

SINCE THE FINAL PROBLEMS TEST WAS TWO TESTS IN ONE -- A TEST ON PROBLEM TYPES RECEIVING VARIABLE TREATMENT (NUMBER, COIN, AGE) AND A TEST ON THOSE UNDER IDENTICAL TREATMENT (WORK, MIXTURE) -- RESULTS FOR ITEMS PER-

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TABLE 7: ANALYSIS OF FINAL PROBLEMS TEST (NUMBER, COIN, AGE, WORK, MIXTURE -- 20 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 11.11 | 4.53 | 11.07 | 8.28 | | | |
| HID | 83 | 8.43 | 5.29 | 8.84 | $p < .0003$ | HID | .001 | |
| LO | 89 | 10.54 | 4.91 | 10.20 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 9.29 | 10.18 | 1.08 | 1 | 29 | 8.34 | 8.54 | 1.05 | 1 | 26 | 9.38 | 10.55 | .04 |
| 2 | 30 | 11.83 | 11.19 | $p < .34$ | 2 | 27 | 8.85 | 9.16 | $p < .36$ | 2 | 32 | 9.97 | 10.41 | $p < .96$ |
| 3 | 30 | 11.67 | 11.68 | | 3 | 27 | 8.11 | 7.59 | | 3 | 31 | 12.10 | 10.66 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 14.63 | 3.09 | 1.18 | HIM | 27 | 10.74 | 3.84 | 3.66 | HIM | 27 | 7.96 | 3.95 | 5.60 |
| HID | 26 | 13.08 | 4.27 | $p < .31$ | HID | 22 | 8.18 | 4.10 | $p < .03$ | HID | 35 | 5.14 | 3.99 | $p < .005$ |
| LO | 35 | 13.83 | 3.63 | | LO | 32 | 10.81 | 3.67 | | LO | 22 | 4.91 | 2.94 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | .01 | |
| LO | NS | NS | LO | NS | .05 | LO | .01 | NS |

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TABLE 8: ANALYSIS OF FINAL PROBLEMS TEST (NUMBER, COIN, AGE, WORK, MIXTURE -- 20 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 13.95 | 3.14 | 13.80 | 3.66 | | | |
| HID | 63 | 12.83 | 4.27 | 12.68 | P<.03 | HID | NS | |
| LO | 89 | 14.03 | 4.36 | 14.24 | | LO | NS | .01 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 14.13 | 14.47 | 1.02 | 1 | 31 | 13.13 | 13.51 | 2.11 | 1 | 27 | 13.48 | 13.67 | .59 |
| 2 | 34 | 13.82 | 13.58 | P<.32 | 2 | 32 | 12.55 | 12.19 | P<.15 | 2 | 32 | 13.91 | 13.74 | P<.56 |
| | | | | | | | | | | 3 | 30 | 14.67 | 14.68 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 14.50 | 3.18 | 3.10 | HIM | 19 | 14.58 | 3.57 | 1.56 | HIM | 17 | 12.53 | 3.02 | 1.51 |
| HID | 26 | 14.65 | 2.95 | P<.05 | HID | 18 | 12.94 | 4.52 | P<.22 | HID | 19 | 10.11 | 4.42 | P<.23 |
| LO | 28 | 16.39 | 3.07 | | LO | 27 | 14.85 | 2.93 | | LO | 34 | 11.44 | 4.55 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | .05 | .05 | LO | NS | NS | LO | NS | NS |

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taining to number, coin, and age (test questions 1-3, 6-8) were examined independently of those for work and mixture (test questions 4-5, 9-10).

For the number-coin-age items analysis supported two conclusions. First, the drawings sequence tended to produce poorer results than either HIM or LO whose outcomes were about the same. Among seventh graders (table 9), adjusted means for HIM (7.42) and LO (7.15) differed from HID's (6.40) at the .01 and .05 levels of significance, respectively. And results for ninth graders (table 10) were similar: HIM-9.72, HID-9.08, LO-9.76 ($p < .14$). Second, the materials approach to translation of algebra word problems was particularly beneficent for low achievers in arithmetic. Here, results for seventh graders (table 9) clearly favored HIM for students in the lower 1/3 treatment subgroup: HIM-5.41, HID-3.86, LO-4.05 ($p < .04$). And among ninth graders (table 10) in the lower 1/3, analysis portrayed a similar phenomenon: HIM-9.12, HID-7.16, LO-7.85 ($p < .14$).

In addition to examining number, coin, and age collectively, the contribution of each to total score was analyzed. Tables 11-16 deal with these by grade, with 11-12 referring to number problems (test items 1 and 6), 13-14 to coin problems (test items 2 and 7), and 15-16 to age problems (test items 3 and 8).

Differences with respect to problem type were most pronounced among low achievers in arithmetic. As shown in table 11 lower 1/3 means favoring HIM differed significantly: HIM-2.52, HID-1.46, LO-1.82 ($p < .0006$). Unfortunately, the .001 level of significance within HIM tends to discredit treatment alone as producing such results. However, a similar pattern obtained for coin problems at the ninth grade (table 14) for the same type of student in the absence of significant within-treatment differences: HIM-3.06, HID-2.11, LO-2.26 ($p < .04$).

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TABLE 9: ANALYSIS OF VARIABLE TREATMENT PROBLEMS SUBTEST (NUMBER, COIN, AGE -- 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 7.44 | 2.77 | 7.42 | 4.80 | | | |
| HID | 83 | 6.13 | 3.33 | 6.40 | $p < .009$ | HID | .01 | |
| LO | 89 | 7.37 | 3.15 | 7.15 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 6.33 | 6.80 | 1.96 | 1 | 29 | 5.76 | 5.87 | .29 | 1 | 26 | 6.58 | 7.29 | .02 |
| 2 | 30 | 7.73 | 7.38 | $p < .15$ | 2 | 27 | 6.07 | 6.32 | $p < .75$ | 2 | 32 | 7.13 | 7.42 | $p < .98$ |
| 3 | 30 | 7.93 | 7.96 | | 3 | 27 | 6.59 | 6.22 | | 3 | 31 | 8.29 | 7.39 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 9.56 | 1.53 | .23 | HIM | 27 | 7.37 | 2.40 | 1.86 | HIM | 27 | 5.41 | 2.55 | 3.42 |
| HID | 26 | 9.19 | 2.26 | $p < .80$ | HID | 22 | 6.14 | 2.68 | $p < .16$ | HID | 35 | 3.86 | 2.46 | $p < .04$ |
| LO | 35 | 9.43 | 2.10 | | LO | 32 | 7.41 | 2.71 | | LO | 22 | 4.05 | 2.24 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | .05 | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 10: ANALYSIS OF VARIABLE TREATMENT PROBLEMS SUBTEST (NUMBER, COIN, AGE -- 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 9.79 | 1.89 | 9.72 | 1.98 | | HIM | HID |
| HID | 63 | 9.19 | 2.83 | 9.08 | P < .14 | HID | NS | |
| LO | 89 | 9.64 | 2.68 | 9.76 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 10.08 | 10.13 | 1.07 | 1 | 31 | 9.32 | 9.56 | 1.48 | 1 | 27 | 9.63 | 9.76 | .25 |
| 2 | 34 | 9.59 | 9.56 | P < .31 | 2 | 32 | 9.06 | 8.84 | P < .23 | 2 | 32 | 9.50 | 9.40 | P < .78 |
| | | | | | | | | | | 3 | 30 | 9.80 | 9.79 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 10.18 | 1.87 | 2.80 | HIM | 19 | 9.95 | 1.58 | 1.24 | HIM | 17 | 9.12 | 2.15 | 2.06 |
| HID | 26 | 10.62 | 1.72 | P < .07 | HID | 18 | 9.28 | 2.67 | P < .30 | HID | 19 | 7.16 | 3.15 | P < .14 |
| LO | 28 | 11.21 | 1.03 | | LO | 27 | 10.26 | 1.87 | | LO | 34 | 7.85 | 3.13 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | .05 | |
| LO | .05 | NS | LO | NS | NS | LO | NS | NS |

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TABLE 11: ANALYSIS OF NUMBER PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 2.98 | .96 | 2.97 | 13.19 | | | |
| HID | 83 | 2.18 | 1.26 | 2.24 | P = .0000 | HID | .001 | |
| LO | 89 | 2.72 | 1.03 | 2.67 | | LO | .05 | .01 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 2.19 | 2.38 | 7.63 | 1 | 29 | 2.14 | 2.17 | .13 | 1 | 26 | 2.58 | 2.75 | .53 |
| 2 | 30 | 3.23 | 3.14 | P<.001 | 2 | 27 | 2.15 | 2.26 | P<.88 | 2 | 32 | 2.56 | 2.60 | P<.59 |
| 3 | 30 | 3.27 | 3.23 | | 3 | 27 | 2.26 | 2.11 | | 3 | 31 | 3.00 | 2.82 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.41 | .64 | 1.05 | HIM | 27 | 3.00 | 1.00 | 3.39 | HIM | 27 | 2.52 | 1.01 | 8.10 |
| HID | 26 | 3.12 | .82 | P<.36 | HID | 22 | 2.23 | 1.19 | P<.04 | HID | 35 | 1.46 | 1.12 | P<.0006 |
| LO | 35 | 3.29 | .75 | | LO | 32 | 2.72 | .96 | | LO | 22 | 1.82 | .91 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | .001 | |
| LO | NS | NS | LO | NS | NS | LO | .05 | NS |

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TABLE 12: ANALYSIS OF NUMBER PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 3.21 | .87 | 3.17 | 6.55 | | | |
| HID | 63 | 2.97 | 1.07 | 2.94 | P < .002 | HID | NS | |
| LO | 89 | 3.39 | .82 | 3.44 | | LO | NS | .001 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 3.33 | 3.30 | .43 | 1 | 31 | 2.97 | 3.05 | .52 | 1 | 27 | 3.37 | 3.40 | .17 |
| 2 | 34 | 3.12 | 3.14 | P < .52 | 2 | 32 | 2.97 | 2.89 | P < .47 | 2 | 32 | 3.38 | 3.33 | P < .84 |
| | | | | | | | | | | 3 | 30 | 3.43 | 3.45 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.27 | .88 | 2.46 | HIM | 19 | 3.37 | .68 | 2.97 | HIM | 17 | 2.94 | 1.02 | 3.16 |
| HID | 26 | 3.46 | .76 | P < .06 | HID | 18 | 3.00 | 1.03 | P < .06 | HID | 19 | 2.26 | 1.14 | P < .05 |
| LO | 28 | 3.71 | .46 | | LO | 27 | 3.59 | .69 | | LO | 34 | 2.97 | .97 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | .05 | NS | LO | NS | .05 | LO | NS | .05 |

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TABLE 13: ANALYSIS OF COIN PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 81 | 1.88 | 1.26 | 1.87 | 1.77 | | HIM | HID |
| HID | 83 | 1.66 | 1.20 | 1.74 | $p < .17$ | HID | NS | |
| LO | 89 | 2.11 | 1.27 | 2.05 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 1.76 | 1.96 | .94 | 1 | 29 | 1.48 | 1.53 | .60 | 1 | 26 | 1.85 | 2.10 | .70 |
| 2 | 30 | 1.83 | 1.66 | $p < .40$ | 2 | 27 | 1.81 | 1.84 | $p < .55$ | 2 | 32 | 2.16 | 2.28 | $p < .50$ |
| 3 | 30 | 2.00 | 2.04 | | 3 | 27 | 1.70 | 1.64 | | 3 | 31 | 2.30 | 1.95 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 2.67 | 1.07 | .54 | HIM | 27 | 1.74 | 1.13 | 1.84 | HIM | 27 | 1.22 | 1.54 | .11 |
| HID | 26 | 2.54 | 1.17 | $p < .59$ | HID | 22 | 1.41 | 1.14 | $p < .17$ | HID | 35 | 1.17 | .89 | $p < .90$ |
| LO | 35 | 2.83 | 1.04 | | LO | 32 | 2.03 | 1.23 | | LO | 22 | 1.09 | .92 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 14: ANALYSIS OF COIN PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 3.02 | 1.08 | 3.00 | .59 | | | |
| HID | 63 | 2.86 | 1.26 | 2.82 | $p < .56$ | HID | NS | |
| LO | 89 | 2.94 | 1.14 | 2.99 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 3.00 | 3.02 | .0007 | 1 | 31 | 2.87 | 2.94 | .32 | 1 | 27 | 2.89 | 2.94 | .45 |
| 2 | 34 | 3.03 | 3.01 | $p < .98$ | 2 | 32 | 2.85 | 2.78 | $p < .57$ | 2 | 32 | 2.88 | 2.82 | $p < .64$ |
| | | | | | | | | | | 3 | 30 | 3.07 | 3.07 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.18 | 1.18 | 1.32 | HIM | 19 | 2.78 | 1.08 | .53 | HIM | 17 | 3.06 | .97 | 3.39 |
| HID | 26 | 3.31 | 1.09 | $p < .27$ | HID | 18 | 3.06 | 1.21 | $p < .59$ | HID | 19 | 2.11 | 1.24 | $p < .04$ |
| LO | 28 | 3.61 | .57 | | LO | 27 | 3.11 | .97 | | LO | 34 | 2.26 | 1.26 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | .05 | |
| LO | NS | NS | LO | NS | NS | LO | .05 | NS |

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TABLE 15: ANALYSIS OF AGE PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 81 | 2.59 | 1.39 | 2.59 | .54 | | HIM | HID |
| HID | 83 | 2.31 | 1.51 | 2.43 | P < .58 | HID | NS | |
| LO | 89 | 2.54 | 1.40 | 2.43 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 2.38 | 2.46 | .24 | 1 | 29 | 2.21 | 2.24 | .46 | 1 | 26 | 2.15 | 2.44 | .15 |
| 2 | 30 | 2.67 | 2.59 | P < .79 | 2 | 27 | 2.11 | 2.22 | P < .63 | 2 | 32 | 2.41 | 2.54 | P < .86 |
| 3 | 30 | 2.67 | 2.69 | | 3 | 27 | 2.63 | 2.49 | | 3 | 31 | 3.00 | 2.62 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.48 | .85 | .52 | HIM | 27 | 2.63 | 1.31 | .10 | HIM | 27 | 1.67 | 1.36 | 1.19 |
| HID | 26 | 3.54 | .90 | P < .59 | HID | 22 | 2.50 | 1.26 | P < .90 | HID | 35 | 1.29 | 1.27 | P < .31 |
| LO | 35 | 3.31 | .93 | | LO | 32 | 2.66 | 1.29 | | LO | 22 | 1.14 | 1.13 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 16: ANALYSIS OF AGE PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 3.57 | .80 | 3.55 | 1.10 | | HIM | HID |
| HID | 63 | 3.34 | 1.12 | 3.31 | p < .33 | HID | NS | |
| LO | 89 | 3.30 | 1.14 | 3.34 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 3.75 | 3.80 | 3.08 | 1 | 31 | 3.45 | 3.53 | 1.88 | 1 | 27 | 3.37 | 3.41 | .21 |
| 2 | 34 | 3.44 | 3.41 | p < .09 | 2 | 32 | 3.24 | 3.17 | p < .18 | 2 | 32 | 3.25 | 3.24 | p < .81 |
| | | | | | | | | | | 3 | 30 | 3.30 | 3.27 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.73 | .55 | .73 | HIM | 19 | 3.79 | .42 | 2.24 | HIM | 17 | 3.12 | 1.17 | .75 |
| HID | 26 | 3.81 | .49 | p < .49 | HID | 18 | 3.22 | 1.11 | p < .12 | HID | 19 | 2.79 | 1.47 | p < .48 |
| LO | 28 | 3.89 | .42 | | LO | 27 | 3.56 | .80 | | LO | 34 | 2.62 | 1.41 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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Just why differences occurred as they did is a matter for conjecture. The relatively poor showing of HID to HIM on post-treatment achievement was particularly surprising since the two sequences were identical except for their handling of certain problem sections: in HIM the directive was to construct and then draw a picture of the construction, whereas in HID it was to either draw or select the appropriate drawing. But there were extenuating circumstances.

As was expected constructions required more time to complete than drawings, so students in HIM took upwards to twice as much time to complete assigned problems. It might be argued, then, that the superiority of HIM to HID arose from the chronometry of the situation. But students in LO took even less time than HID's to complete a problem section, and results for HIM and LO were nearly equal.

It is the writer's guess that HIM's superiority resulted primarily from greater attendance to verbal detail, particularly when two conditions on two unknowns were to be represented. Considerable strategy is required to depict problems with the type of construction materials provided, and students involved in this activity appeared to refer again and again to the statement of a problem as they seemingly checked an illustration of one of its verbal conditions against the other condition. In contrast, students in HID seemed to proceed directly to identification of the illustration of the four given that was representative of both conditions. If this interpretation of the situation is correct, then it is not surprising that low achievers in arithmetic assigned to HIM did better since, as noted by Balow (1964), students who perform poorly in arithmetic typically read poorly as well.

Of course, there is still the fact that LO, as well as HIM, effected

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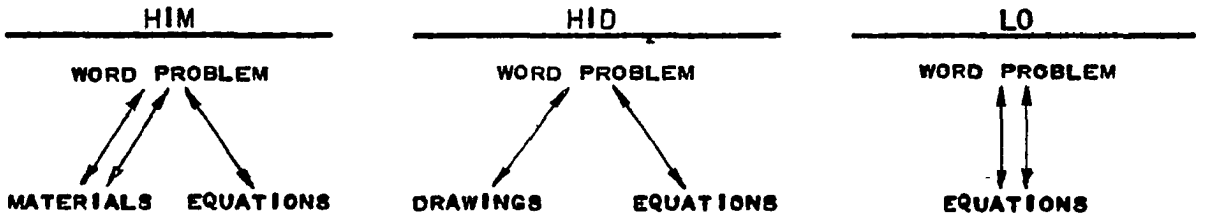
RESULTS SUPERIOR TO HID, EVEN THOUGH OBSERVATIONS OF STUDENTS IN LO AT WORK INDICATED THAT THEY WERE LIKELY TO MOVE FROM WORD PROBLEMS TO EQUATIONS AS DIRECTLY AS THOSE IN HID. HOWEVER, THE TYPE OF ASSOCIATIONS REINFORCED DURING THIS PROCESS WERE QUITE DIFFERENT FOR THE TWO SEQUENCES. IN LO EQUATIONS WERE TO DERIVE DIRECTLY FROM GRAMMATICAL CUES, WHEREAS IN HID THEY WERE TO ARISE FROM A MORE CIRCUITOUS ROUTING: VERBAL CUES TO ICONIC REPRESENTATION TO SYMBOLIC DEPICTION. IN LO, THEN, STUDENTS SPENT MOST OF THEIR TIME WORKING TOWARD THAT VERY SKILL WHICH THE FINAL PROBLEMS TEST MEASURED, NAMELY, ABILITY TO TRANSLATE DIRECTLY. IN CONTRAST, STUDENTS IN HID WERE REQUIRED TO DEPICT VERBAL CONDITIONS PICTORIALLY AS WELL, AND IT IS THE WRITER'S GUESS THAT THE TENDENCY AMONG THESE STUDENTS WAS TO MAKE OF A PROBLEM TWO SOMEWHAT UNRELATED SUBPROBLEMS: (1) REPRESENTATION OF THE WRITTEN CONDITIONS PICTORIALLY, AND (2) REPRESENTATION OF THE WRITTEN CONDITIONS MATHEMATICALLY. IF THIS WERE INDEED THE CASE, THEN THE FINAL PROBLEMS TEST EXPLORED ONLY THE TRANSLATIVE PORTION OF WHAT THESE STUDENTS WERE ABLE TO DO.

IN SUMMARY, THE WRITER BELIEVES THAT STUDENTS, REGARDLESS OF TREATMENT GROUP, WERE PRONE TO TRANSLATE DIRECTLY AS DIAGRAMED BELOW. THIS




CONCLUSION IS CONTRARY TO WHAT WAS EXPECTED IN THAT IT HAD BEEN HYPOTHESIZED THAT STUDENTS IN HIM AND HID WOULD PROCEED FROM THE STATEMENT OF A PROBLEM TO AN ILLUSTRATION OF THE PROBLEM AND FROM BOTH TO A SET OF EQUATIONS. HOWEVER, THE SUPERIORITY OF HIM TO HID IS MORE EASILY ASCRIBED TO GREATER ATTENTION TO VERBAL DETAIL WHICH SEEMED TO ACCOMPANY

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MANIPULATIVE EFFORT THAN TO THE FRUIT OF SUCH, PER SE. AND LO'S RELATIVE SUCCESS WAS PROBABLY THE RESULT OF ITS STRAIGHTFORWARD STRESS ON DIRECT TRANSLATION AS COMPARED TO THE DIVISION OF EFFORT CHARACTERIZING HID.

## WORK AND MIXTURE

FOLLOWING THE FIRST EIGHT DAYS OF VARIABLE TREATMENT OF NUMBER, COIN, AND AGE, FOUR DAYS OF IDENTICAL TREATMENT WERE GIVEN TO WORK PROBLEMS AND MIXTURE PROBLEMS, WITH EACH RECEIVING TWO DAYS OF INSTRUCTION. TO TEST FOR ACHIEVEMENT IN THIS AREA, TWO PROBLEMS OF EACH TYPE WERE INCLUDED IN THE FINAL PROBLEMS TEST (ITEMS 4-5, 9-10) AND ANALYZED SEPARATELY FROM THE OTHERS. IN THIS WAY THE EFFECT OF TYPE OF INSTRUCTION RECEIVED INITIALLY ON SUBSEQUENT LEARNING COULD BE EXAMINED.

FOR SEVENTH GRADERS (TABLE 17), ANALYSIS INDICATED THAT HIM WAS MOST EFFECTIVE IN PREPARING STUDENTS TO LEARN TO TRANSLATE WORK AND MIXTURE: HIM-3.64, HID-2.42, LO-3.06 ($p < .0006$). IN PARTICULAR, THIS TREATMENT WAS VERY HELPFUL AMONG THE LOWER 1/3 ON ARITHMETIC ACHIEVEMENT: HIM-2.56, HID-1.23, LO-.86 ($p < .005$). RESULTS FOR NINTH GRADERS ON THIS VARIABLE PORTRAYED A DIFFERENT PICTURE. AS SHOWN IN TABLE 18, ADJUSTED MEANS FAVORED LO: HIM-4.08, HID-3.60, LO-4.48 ($p < .05$). AND THE MAJOR INFLUENCE OF THIS TREATMENT WAS WITH STUDENTS IN THE UPPER 1/3 TREATMENT SUBGROUP: HIM-4.32, HID-4.04, LO-5.18 ($p < .16$).

INTERPRETATION OF RESULTS SO FAR HAS BEEN IN TERMS OF ACHIEVEMENT ON TWO TYPES OF WORD PROBLEMS THAT DIFFER SUBSTANTIALLY IN THE WAY THEY ARE HANDLED MATHEMATICALLY. ACCORDINGLY, IN ORDER TO DETERMINE THE EXTENT TO WHICH SCORES WERE RELATED TO <u>WHAT</u> WAS TO BE LEARNED AS WELL AS TO PREVIOUS INSTRUCTION, RESULTS FROM THE FINAL PROBLEMS TEST WERE FURTHER SUBDIVIDED INTO THOSE FOR WORK AND THOSE FOR MIXTURE.

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TABLE 17: ANALYSIS OF IDENTICAL TREATMENT PROBLEMS SUBTEST (WORK, MIXTURE — 8 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 3.67 | 2.24 | 3.64 | 7.63 | | | |
| HID | 83 | 2.28 | 2.56 | 2.42 | $p < .0006$ | HID | .001 | |
| LO | 89 | 3.17 | 2.28 | 3.06 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 2.95 | 3.38 | .26 | 1 | 29 | 2.52 | 2.60 | 3.35 | 1 | 26 | 2.81 | 3.26 | .25 |
| 2 | 30 | 4.10 | 3.81 | $p < .77$ | 2 | 27 | 2.78 | 2.85 | $p < .04$ | 2 | 32 | 2.84 | 2.99 | $p < .78$ |
| 3 | 30 | 3.73 | 3.72 | | 3 | 27 | 1.52 | 1.36 | | 3 | 31 | 3.81 | 3.28 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 5.07 | 2.09 | 1.87 | HIM | 27 | 3.37 | 1.96 | 3.81 | HIM | 27 | 2.56 | 1.93 | 5.58 |
| HID | 26 | 3.88 | 2.45 | $p < .16$ | HID | 22 | 2.05 | 2.24 | $p < .03$ | HID | 35 | 1.23 | 2.26 | $p < .005$ |
| LO | 35 | 4.40 | 2.21 | | LO | 32 | 3.41 | 1.72 | | LO | 22 | .86 | 1.17 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | .01 | |
| LO | NS | NS | LO | NS | .05 | LO | .01 | NS |

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TABLE 18: ANALYSIS OF IDENTICAL TREATMENT PROBLEMS SUBTEST (WORK, MIXTURE — 8 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 4.16 | 2.28 | 4.08 | 3.00 | | | |
| HID | 63 | 3.64 | 2.15 | 3.60 | $p < .05$ | HID | NS | |
| LO | 89 | 4.39 | 2.40 | 4.48 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 4.04 | 4.34 | .24 | 1 | 31 | 3.81 | 3.94 | 1.30 | 1 | 27 | 3.85 | 3.91 | 1.23 |
| 2 | 34 | 4.24 | 4.02 | $p < .63$ | 2 | 32 | 3.48 | 3.35 | $p < .26$ | 2 | 32 | 4.41 | 4.33 | $p < .30$ |
| | | | | | | | | | | 3 | 30 | 4.87 | 4.89 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 4.32 | 2.23 | 1.90 | HIM | 19 | 4.63 | 2.22 | 1.11 | HIM | 17 | 3.41 | 2.35 | .50 |
| HID | 26 | 4.04 | 2.01 | $p < .16$ | HID | 18 | 3.67 | 2.43 | $p < .34$ | HID | 19 | 2.95 | 1.99 | $p < .61$ |
| LO | 28 | 5.18 | 2.44 | | LO | 27 | 4.59 | 2.22 | | LO | 34 | 3.59 | 2.31 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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As shown in tables 19-20, scores were generally low for work problems. Except for LO at grade nine, treatment groups averaged only about one of four points possible. It is tempting, then, to conclude that students experienced considerable difficulty with translation of work as compared to number, coin, and age. However, this is necessarily conjecture since results are qualified by the all-or-none manner in which work problems were graded. Whereas partial credit in the form of one point for each of two equations in two unknowns was given for number, coin, and age, a work problem received either a score of zero or two in keeping with its single mathematical correspondent.

As illustrated in table 19, seventh grade treatment groups achieved comparably on work, although scores tended to favor the high-imagery sequences: HIM-1.03, HID-.88, LO-.74 ($p < .32$). The slight superiority of HIM and HID to LO was particularly evident among the lower 1/3 where every student in LO scored zero: HIM-.59, HID-.40, LO-.00 ($p < .08$). In contrast, table 20 shows that ninth graders in LO achieved significantly better on work than those under high imagery, regardless of level of achievement in arithmetic: HIM-1.06, HID-1.04, LO-2.16 ($p = .0000$).

For ninth graders, then, the direct translative approach was superior in way of preparing them for learning to translate work problems. This was unexpected inasmuch as explanations within the instructional sequence designed for work were couched in terms of "stacks" so as to favor those having received high-imagery instruction.

Turning now to mixture problems, it is evident once again that HID fared poorly with seventh graders (table 21), with treatment groups HIM and LO performing on a par basis at a significantly higher level: HIM-2.61, HID-1.54, LO-2.32 ($p = .0000$). Typical also is the greater achievement recorded for those in HIM in the lower 1/3 on arithmetic achievement:

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TABLE 19: ANALYSIS OF WORK PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 81 | 1.04 | 1.27 | 1.03 | 1.15 | | HIM | HID |
| HID | 83 | .82 | 1.50 | .88 | $p < .32$ | HID | NS | |
| LO | 89 | .79 | 1.34 | .74 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | .67 | .84 | .35 | 1 | 29 | .76 | .80 | .17 | 1 | 26 | .77 | .92 | .24 |
| 2 | 30 | 1.20 | 1.08 | $p < .71$ | 2 | 27 | .89 | .94 | $p < .84$ | 2 | 32 | .75 | .78 | $p < .79$ |
| 3 | 30 | 1.13 | 1.14 | | 3 | 27 | .81 | .72 | | 3 | 31 | .84 | .68 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.78 | 1.50 | 1.65 | HIM | 27 | .74 | .98 | 1.53 | HIM | 27 | .59 | .93 | 2.57 |
| HID | 26 | 1.77 | 1.82 | $p < .20$ | HID | 22 | .36 | 1.00 | $p < .22$ | HID | 35 | .40 | 1.17 | $p < .08$ |
| LO | 35 | 1.15 | 1.48 | | LO | 32 | .94 | 1.44 | | LO | 22 | .00 | .00 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 20: ANALYSIS OF WORK PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 1.14 | 1.55 | 1.06 | 13.86 | | HIM | HID |
| HID | 63 | 1.03 | 1.51 | 1.04 | P = .0000 | HID | NS | |
| LO | 89 | 2.11 | 1.52 | 2.16 | | LO | .001 | .001 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 1.08 | 1.25 | .17 | 1 | 31 | 1.23 | 1.31 | 2.21 | 1 | 27 | 2.15 | 2.18 | .09 |
| 2 | 34 | 1.18 | 1.06 | P < .68 | 2 | 32 | .85 | .77 | P < .14 | 2 | 32 | 2.19 | 2.15 | P < .91 |
| | | | | | | | | | | 3 | 30 | 2.00 | 2.01 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 1.00 | 1.60 | 6.74 | HIM | 19 | 1.37 | 1.64 | 2.20 | HIM | 17 | 1.06 | 1.43 | 4.71 |
| HID | 26 | 1.00 | 1.52 | P < .002 | HID | 18 | 1.44 | 1.80 | P < .12 | HID | 19 | .63 | 1.16 | P < .01 |
| LO | 28 | 2.36 | 1.54 | | LO | 27 | 2.30 | 1.64 | | LO | 34 | 1.76 | 1.37 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | .01 | .01 | LO | NS | NS | LO | NS | .01 |

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TABLE 21: ANALYSIS OF MIXTURE PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 2.63 | 1.52 | 2.61 | 12.92 | | | |
| HID | 83 | 1.46 | 1.62 | 1.54 | p= .0000 | HID | .001 | |
| LO | 89 | 2.38 | 1.55 | 2.32 | | LO | NS | .001 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 2.29 | 2.54 | .12 | 1 | 29 | 1.76 | 1.80 | 6.04 | 1 | 26 | 2.04 | 2.34 | .73 |
| 2 | 30 | 2.90 | 2.74 | p < .89 | 2 | 27 | 1.89 | 1.91 | p<.004 | 2 | 32 | 2.09 | 2.21 | p < .49 |
| 3 | 30 | 2.60 | 2.59 | | 3 | 27 | .70 | .65 | | 3 | 31 | 2.97 | 2.60 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.30 | 1.30 | 6.12 | HIM | 27 | 2.63 | 1.42 | 2.86 | HIM | 27 | 1.96 | 1.58 | 6.45 |
| HID | 26 | 2.12 | 1.73 | p<.003 | HID | 22 | 1.68 | 1.73 | p < .06 | HID | 35 | .82 | 1.22 | p<.003 |
| LO | 35 | 3.26 | 1.24 | | LO | 32 | 2.47 | 1.29 | | LO | 22 | .86 | 1.17 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | .01 | | HID | .05 | | HID | .001 | |
| LO | NS | .01 | LO | NS | NS | LO | .01 | NS |

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HIM-1.96, HID-.82, LO-.86 ($p < .003$).

On the other hand, at the ninth grade (table 22) LO resulted in lowest performance, with HIM superior to LO at the .01 level of significance: HIM-3.01, HID-2.56, LO-2.32 ($p < .02$). Within-treatment differences occurred for LO, but since its highest adjusted class mean (2.88) was less than HIM's lowest (2.96), it seems reasonable to conclude that the materials approach to translation rather than the direct approach was more helpful in subsequent learning of mixture problems.

As with work this result was unexpected insofar as the instruction designed for mixture was void of visual referents in keeping with LO's treatment of number, coin, and age. However, since every mixture problem used in this study involved the concept of value, perhaps HIM's relative success was due to the transfer effect of value as learned via the materials when coin problems were treated. This seems reasonable in light of how well the lower 1/3 of the ninth grade in HIM did on coins (table 14) as compared to students in either HID or LO.

## DETECTION OF CONTRADICTORY STATEMENTS

Included among the items accompanying the Final Problems Test was the following coin problem:

> A man has three times as many quarters as he has dimes. The value of the dimes exceeds the value of the quarters by 40¢. How many has he of each coin?

Since having three times as many quarters as dimes is inconsistent with the dimes being worth more than the quarters, this problem describes a physically impossible situation.

As with the other test items students were requested to translate the coin problem. However, they were cautioned that this and the five

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TABLE 22: ANALYSIS OF MIXTURE PROBLEM SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 3.02 | 1.33 | 3.01 | 4.13 | | | |
| HID | 63 | 2.61 | 1.44 | 2.56 | P<.02 | HID | NS | |
| LO | 89 | 2.28 | 1.60 | 2.32 | | LO | .01 | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 2.96 | 3.09 | .13 | 1 | 31 | 2.58 | 2.63 | .02 | 1 | 27 | 1.70 | 1.74 | 3.96 |
| 2 | 34 | 3.06 | 2.96 | P<.72 | 2 | 32 | 2.64 | 2.59 | P<.90 | 2 | 32 | 2.22 | 2.18 | P<.02 |
| | | | | | | | | | | 3 | 30 | 2.87 | 2.88 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.32 | 1.17 | .72 | HIM | 19 | 3.26 | 1.19 | 3.55 | HIM | 17 | 2.35 | 1.50 | 1.01 |
| HID | 26 | 3.04 | 1.46 | P<.49 | HID | 18 | 2.22 | 1.31 | P<.03 | HID | 19 | 2.32 | 1.46 | P<.37 |
| LO | 28 | 2.82 | 1.63 | | LO | 27 | 2.30 | 1.51 | | LO | 34 | 1.82 | 1.53 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | .05 | NS | LO | NS | NS |

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TRANSFER PROBLEMS PRECEDING IT WERE SOMEWHAT DIFFERENT FROM THOSE THEY HAD BEEN WORKING ON. IN ADDITION, THEY WERE ADVISED TO COMMENT ON ANY OF THESE PROBLEMS IN THE MARGIN IF THEY SO DESIRED. THE RAISON D'ETRE FOR THE COIN PROBLEM WILL NOW BE EXPLAINED.

IN FACE OF A PROBLEM THAT REPRESENTS PHYSICALLY IMPOSSIBLE CIRCUMSTANCES, THREE RESPONSES ARE POSSIBLE: (1) A SET OF EQUATIONS DESCRIBING THE PROBLEM LITERALLY, (2) A SET OF EQUATIONS DESCRIBING A RELATED, PHYSICALLY POSSIBLE SITUATION, OR (3) THE PHYSICAL INCONGRUITY OF THE SITUATION CAN BE RECOGNIZED. ACCORDING TO PAIGE AND SIMON (1966), IF A STUDENT IS TRANSLATING DIRECTLY, THAT IS, ATTENDING PRIMARILY TO VERBAL CUES, HE WILL LIKELY MAKE THE FIRST RESPONSE. IF, ON THE OTHER HAND, HE IS REFERRING CHIEFLY TO THE PHYSICAL CHARACTERISTICS OF THE SITUATION, HE WILL MAKE THE SECOND. AND IF HE IS ATTENDING TO BOTH, THE PROBLEM DESCRIPTION WILL BE REDUNDANT, AND HE MAY DETECT THE CONTRADICTION.

ON THE BASIS OF THIS CLASSIFICATION OF RESPONSE PAIGE AND SIMON WERE ABLE TO CATEGORIZE HIGH SCHOOL STUDENTS AS THOSE WHO WERE PRIMARILY "VERBAL" IN THEIR RESPONSES, AND THOSE WHO WERE PRIMARILY "PHYSICAL." THEY THEN NOTED THAT THESE TWO GROUPS DIFFERED QUITE CONSISTENTLY IN THEIR RESPECTIVE DETECTION OF CONTRADICTIONS: THE "VERBAL" PROBLEM SOLVERS DID NOT, AND THE "PHYSICAL" SOMETIMES DID.

RETURNING TO THE COIN PROBLEM, TESTS WERE EXAMINED TO SEE IF STUDENTS HAD RECOGNIZED THE PHYSICAL IMPOSSIBILITY OF THE SITUATION AS DESCRIBED. IN MOST CASES THEY HAD MERELY TRANSLATED. BUT SEVERAL STUDENTS HAD INDICATED AS EXEMPLIFIED BELOW THAT THEY HAD OBSERVED THE FALLACIOUSNESS OF THE PROBLEM.

"THE PROBLEM IS STATED WRONG."

"IF THE MAN HAS MORE QUARTERS THAN DIMES, HOW CAN THE VALUE

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OF THE DIMES EXCEED THAT OF THE QUARTERS?"

"HOW IS IT POSSIBLE? VERY STUPID PROBLEM. THERE'S MORE VALUE IN THE QUARTERS THAN IN THE DIMES."

"THE FIRST PART OF THE PROBLEM SHOULD BE D = 3Q BECAUSE IT'S NOT RIGHT THE WAY IT IS."

WHEN THIS OCCURRED, IT WAS ASSUMED THAT THESE STUDENTS HAD BEEN ATTENDING NOT ONLY TO THE VERBAL DESCRIPTION OF THE SITUATION BUT TO ITS PHYSICAL CHARACTERISTICS AS WELL.

AS SHOWN BELOW, THE NUMBER OF STUDENTS RESPONDING AFFIRMATIVELY TO THE PHYSICAL IMPOSSIBILITY OF THE COIN PROBLEM WAS ABOUT THE SAME FOR EACH TREATMENT GROUP AT THE SEVENTH GRADE. HOWEVER, AMONG NINTH GRADERS CONSIDERABLY MORE IN HID DETECTED THE CONTRADICTION, IN SPITE OF THE HALF-AGAIN AS MANY STUDENTS IN LO.

| GRADE 7 | | |
|---|---|---|
| TREATMENT | N | NUMBER DETECTING CONTRADICTION |
| HIM | 88 | 7 |
| HID | 93 | 3 |
| LO | 95 | 5 |

| GRADE 9 | | |
|---|---|---|
| TREATMENT | N | NUMBER DETECTING CONTRADICTION |
| HIM | 67 | 3 |
| HID | 66 | 9 |
| LO | 99 | 1 |

AND IF RESULTS FOR THE TWO NINTH GRADE CLASSES EXCLUDED FROM ANALYSIS SO FAR ARE INCLUDED, THE DISPARITY INCREASES.

| GRADE 9 | | |
|---|---|---|
| TREATMENT | N | NUMBER DETECTING CONTRADICTION |
| HIM | 93 | 4 |
| HID | 97 | 14 |
| LO | 99 | 1 |

CLEARLY, THEN, THERE IS REASON TO BELIEVE THAT NINTH GRADE STUDENTS IN HID ATTENDED DIFFERENTLY TO WORD PROBLEMS THAN DID THOSE IN HIM OR LO. SPECIFICALLY, IN TERMS OF THE VERBAL-VISUAL FRAMEWORK PROVIDED BY PAIGE AND SIMON, IT SEEMS THAT THESE STUDENTS WERE PRONE TO NOT ONLY DECIPHER WORDS BUT TO REPRESENT PROBLEMS INTERNALLY AS WELL. IN OTHER WORDS, THEIR CON-

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CERN APPARENTLY WENT BEYOND THE PROBLEM-SOLVING PROCESS TO INCLUDE THE PROBLEM ITSELF.

AT ANY RATE, THIS WOULD SEEM TO BE AN AREA FOR FURTHER INVESTIGATION SINCE TRANSLATIVE SKILL AND TENDENCY TO DETECT PHYSICALLY CONTRADICTORY STATEMENTS APPEAR TO BE RELATED. TO ILLUSTRATE, OF THE THIRTEEN NINTH GRADERS WHO NOTED THE CONTRADICTION IN THE COIN PROBLEM, SEVEN WERE IN THE MIDDLE 1/3 WITH RESPECT TO ARITHMETIC ACHIEVEMENT AND FOUR WERE IN THE LOWER 1/3. NEVERTHELESS, THESE STUDENTS AVERAGED AS FOLLOWS ON THE FINAL PROBLEMS TEST: HIM-16.67, HID-14.67, LO-20. IN COMPARISON, MEANS FOR THE GENERAL POPULATION WERE CONSIDERABLY LESS: HIM-13.95, HID-12.83, LO-14.03.

## RETENTION

FOLLOWING THE TWO DAYS ALLOTTED FOR TESTING AT THE END OF THE INSTRUCTIONAL FACET OF THE STUDY, TEACHERS RESUMED THE ORIGINAL CURRICULUM. THIS RETURN TO THE USUAL COURSE OF EVENTS LASTED FOR TWELVE DAYS (INCLUDING WEEKENDS AND HOLIDAYS), DURING WHICH TEACHERS WERE CAUTIONED TO AVOID THE TOPIC OF TRANSLATION. THE STUDY THEN CONCLUDED WITH A PROBLEMS RETENTION TEST ON NUMBER, COIN, AGE, WORK, AND MIXTURE.* EXCEPT FOR MINOR DIFFERENCES THIS INSTRUMENT WAS THE SAME AS THE FINAL PROBLEMS TEST.

AS TABLES 23-24 INDICATE, TREATMENT GROUPS WITHIN EACH GRADE ACHIEVED COMPARABLY ON TOTAL RETENTION. THERE WERE, HOWEVER, SIGNIFICANT WITHIN-TREATMENT DIFFERENCES FOR HIM AT GRADE SEVEN AND FOR HID AT BOTH GRADES, SO THE SEEMING ABSENCE OF RELATIVE EFFECT OF TREATMENTS IS PERHAPS MISLEAD-

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*IN SOME CASES THE TWELVE-DAY, "NO-INSTRUCTIONAL" PERIOD WAS EXCEEDED AT THE NINTH GRADE LEVEL. ONE OF THE CLASSES COMPRISING HIM DID NOT TAKE THE RETENTION TEST UNTIL SEVENTEEN DAYS AFTER EXPERIMENTAL INSTRUCTION HAD TERMINATED, AND TWO CLASSES UNDER LO WERE NOT ABLE TO TAKE THIS TEST UNTIL TWENTY DAYS HAD ELAPSED.

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TABLE 23: ANALYSIS OF PROBLEMS RETENTION TEST (NUMBER, COIN, AGE, WORK, MIXTURE -- 20 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | 9.59 | 4.85 | 9.54 | 1.51 | | | |
| HID | 84 | 9.56 | 5.61 | 10.06 | $p < .22$ | HID | NS | |
| LO | 85 | 10.98 | 5.47 | 10.53 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 7.05 | 7.79 | 4.50 | 1 | 28 | 8.29 | 8.32 | 5.51 | 1 | 23 | 10.39 | 11.80 | .65 |
| 2 | 30 | 11.17 | 10.44 | $p < .01$ | 2 | 28 | 10.89 | 11.49 | $p < .006$ | 2 | 32 | 10.03 | 10.60 | $p < .52$ |
| 3 | 31 | 9.87 | 10.04 | | 3 | 28 | 9.50 | 8.87 | | 3 | 30 | 12.43 | 10.75 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 13.85 | 3.59 | .45 | HIM | 29 | 9.31 | 3.30 | 2.10 | HIM | 27 | 5.63 | 3.77 | .21 |
| HID | 26 | 14.85 | 3.67 | $p < .64$ | HID | 22 | 9.18 | 4.43 | $p < .13$ | HID | 36 | 5.97 | 4.37 | $p < .81$ |
| LO | 33 | 14.58 | 4.44 | | LO | 31 | 11.03 | 3.81 | | LO | 21 | 5.24 | 4.07 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 24: ANALYSIS OF PROBLEMS RETENTION TEST (NUMBER, COIN, AGE, WORK, MIXTURE — 20 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 13.12 | 3.93 | 13.02 | .62 | | HIM | HID |
| HID | 62 | 13.22 | 3.72 | 13.04 | $p < .54$ | HID | NS | |
| LO | 87 | 12.29 | 4.25 | 12.48 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 12.13 | 12.65 | .62 | 1 | 30 | 14.03 | 14.27 | 5.44 | 1 | 25 | 11.32 | 11.56 | 2.91 |
| 2 | 34 | 13.82 | 13.45 | $p < .43$ | 2 | 32 | 12.48 | 12.27 | $p < .02$ | 2 | 32 | 11.81 | 11.63 | $p < .06$ |
| | | | | | | | | | | 3 | 30 | 13.60 | 13.60 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 14.05 | 3.70 | .23 | HIM | 18 | 14.50 | 2.81 | 1.44 | HIM | 18 | 10.61 | 4.15 | .74 |
| HID | 26 | 14.23 | 2.98 | $p < .79$ | HID | 17 | 13.47 | 3.54 | $p < .25$ | HID | 19 | 11.47 | 4.36 | $p < .48$ |
| LO | 28 | 14.68 | 3.59 | | LO | 25 | 12.64 | 3.99 | | LO | 34 | 10.06 | 3.85 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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ING. QUITE POSSIBLY, THE VARIED TOPICS TEACHERS PURSUED DURING THE TWELVE-DAY NON-TREATMENT PERIOD PRIOR TO THE RETENTION TEST INFLUENCED DIFFERENTIAL ACHIEVEMENT MORE THAN VARYING TREATMENT DURING THE INSTRUCTIONAL PERIOD WAS ABLE TO.

IN ITSELF, LITTLE CAN BE CONCLUDED FROM ANALYSIS OF TOTAL RETENTION SCORES, PARTICULARLY IN LIGHT OF WITHIN-TREATMENT SIGNIFICANCE FOR HIM AND HID. HOWEVER, IN COMPARING TABLES 23-24 TO 7-8 ON TOTAL ACHIEVEMENT, IT IS SEEN THAT LITTLE, IF ANY, FORGETTING OCCURRED DURING THE TWELVE DAYS FOLLOWING TREATMENT.

GRADE 7

| TOTAL ACHIEVEMENT (FROM TABLE 7) | | | TOTAL RETENTION (FROM TABLE 23) | | |
|---|---|---|---|---|---|
| TREATMENT | ADJ MEAN | F-RATIO | TREATMENT | ADJ MEAN | F-RATIO |
| HIM | 11.07 | 8.28 | HIM | 9.54 | 1.51 |
| HID | 8.84 | $p<.0003$ | HID | 10.06 | $p<.22$ |
| LO | 10.20 | | LO | 10.53 | |

GRADE 9

| TOTAL ACHIEVEMENT (FROM TABLE 8) | | | TOTAL RETENTION (FROM TABLE 24) | | |
|---|---|---|---|---|---|
| TREATMENT | ADJ MEAN | F-RATIO | TREATMENT | ADJ MEAN | F-RATIO |
| HIM | 13.80 | 3.66 | HIM | 13.02 | .62 |
| HID | 12.68 | $p<.03$ | HID | 13.04 | $p<.54$ |
| LO | 14.24 | | LO | 12.48 | |

IN FACT, TWO SEVENTH GRADE TREATMENT GROUPS AND ONE NINTH GRADE TREATMENT GROUP GAINED WITH RESPECT TO POST-TREATMENT ACHIEVEMENT. IN PARTICULAR, HID GAINED AT BOTH GRADE LEVELS AND, IN CONTRAST TO ITS POOR SHOWING EARLIER, COMPARED FAVORABLY WITH HIM AND LO AT THE CONCLUSION OF THE STUDY. OF COURSE, THE SIGNIFICANT DIFFERENCES WITHIN HID DO NOT PERMIT THE CONCLU-

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SION THAT TREATMENT ALONE ATTRIBUTED TO SUCH GAINS. NEVERTHELESS, IT SEEMS THAT TRANSLATIVE SKILL IS NOT ONLY RESISTANT TO FORGETTING, BUT IS POSSIBLY REINFORCED THROUGH DAILY EVENTS ONCE STUDENTS HAVE REACHED AN INITIAL STAGE OF PERCEPTIVENESS TO QUANTITATIVE STATEMENTS.

## TRANSFER

AS AN ACCOMPANIMENT TO THE FINAL PROBLEMS TEST STUDENTS WERE TO WRITE EQUATIONS FOR FIVE WORD PROBLEMS SOMEWHAT DIFFERENT FROM THOSE TREATED. IN THE FIRST (ITEM 11) THE CONCEPT OF WEIGHT REPLACED THAT OF VALUE, IN THE SECOND (ITEM 12) THREE EQUATIONS IN THREE UNKNOWNS WERE REQUIRED TO DESCRIBE THE STATED MATHEMATICAL RELATIONSHIPS, IN THE THIRD (ITEM 13) THE VARIABLES USED TO SYMBOLIZE CERTAIN VALUE CONDITIONS HAD TO BE INTERPRETED IN TERMS OF THE QUALITATIVE NATURE OF THE OBJECTS IN QUESTION RATHER THAN THEIR QUANTITATIVE ASPECTS, AND FOUR AND FIVE (ITEMS 14-15) WERE GEOMETRIC IN NATURE. THE PURPOSE OF THESE ITEMS WAS TO PROVIDE A MEASURE OF ABILITY TO TRANSFER PREVIOUS LEARNINGS.

EXCEPT FOR ITEM 12 INVOLVING THREE UNKNOWNS, EACH PROBLEM WAS WEIGHTED AND GRADED AS BEFORE. IN SCORING ITEM 12 ONE POINT WAS GIVEN FOR A PROPER TRANSLATION OF ANY TWO OF THE THREE CONDITIONS ON THE UNKNOWNS, AND AN ADDITIONAL POINT WAS RESERVED FOR THE REMAINING EQUATION. IN TOTAL, THEN, THE FIVE PROBLEMS WERE WORTH TEN POINTS.

FOR SEVENTH GRADERS (TABLE 25) THERE APPEARED TO BE LITTLE RELATIONSHIP BETWEEN TYPE OF TREATMENT AND TRANSFER: HIM-5.09, HID-4.42, LO-4.56 ($p < .10$). HOWEVER, THE SITUATION IS PERHAPS MISREPRESENTED AS A RESULT OF SIGNIFICANT VARIANCE WITHIN HIM ($p < .01$). HAD CLASS 1 OF THIS TREATMENT GROUP ACHIEVED ANYWHERE NEAR THAT OF CLASSES 2 AND 3, IT COULD HAVE BEEN CONCLUDED THAT HIM LED TO GREATER TRANSFER AMONG SEVENTH GRADERS. AT THE NINTH GRADE (TABLE 26), ADJUSTED MEANS CLEARLY FAVORED HIM: HIM-7.45,

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TABLE 25: ANALYSIS OF TRANSFER TEST (10 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 5.11 | 2.74 | 5.09 | 2.37 | | | |
| HID | 83 | 4.19 | 2.92 | 4.42 | P<.10 | HID | .05 | |
| LO | 89 | 4.75 | 2.84 | 4.56 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 3.38 | 3.81 | 4.84 | 1 | 29 | 3.93 | 4.02 | .16 | 1 | 26 | 4.62 | 5.26 | 1.63 |
| 2 | 30 | 5.87 | 5.55 | P<.01 | 2 | 27 | 4.07 | 4.29 | P<.85 | 2 | 32 | 4.66 | 4.87 | P<.20 |
| 3 | 30 | 5.57 | 5.58 | | 3 | 27 | 4.59 | 4.29 | | 3 | 31 | 4.97 | 4.20 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 7.11 | 2.15 | 1.04 | HIM | 27 | 5.11 | 2.36 | .97 | HIM | 27 | 3.11 | 2.14 | 1.74 |
| HID | 26 | 7.04 | 2.29 | P<.36 | HID | 22 | 4.18 | 2.17 | P<.38 | HID | 35 | 2.09 | 1.74 | P<.18 |
| LO | 35 | 6.40 | 2.05 | | LO | 32 | 4.59 | 2.43 | | LO | 22 | 2.36 | 2.79 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 26: ANALYSIS OF TRANSFER TEST (10 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 7.50 | 1.75 | 7.45 | 3.92 | | | |
| HID | 63 | 6.59 | 2.38 | 6.51 | p < .02 | HID | .01 | |
| LO | 89 | 6.55 | 2.27 | 6.65 | | LO | .05 | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 7.67 | 7.67 | .33 | 1 | 31 | 7.00 | 7.16 | 4.32 | 1 | 27 | 6.04 | 6.12 | .85 |
| 2 | 34 | 7.38 | 7.38 | p < .57 | 2 | 32 | 6.21 | 6.06 | p < .04 | 2 | 32 | 6.88 | 6.77 | p < .43 |
| | | | | | | | | | | 3 | 30 | 6.67 | 6.70 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 7.91 | 1.63 | .28 | HIM | 19 | 7.84 | 1.12 | 3.46 | HIM | 17 | 6.59 | 2.18 | 2.36 |
| HID | 26 | 7.65 | 1.90 | p < .76 | HID | 18 | 6.33 | 2.70 | p < .04 | HID | 19 | 5.37 | 2.17 | p < .10 |
| LO | 28 | 7.50 | 2.19 | | LO | 27 | 7.30 | 1.30 | | LO | 34 | 5.18 | 2.30 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | .05 | NS |

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HID-6.51, LO-6.65 ($p < .02$). There were, however, significant within-treatment differences for HID ($p < .04$), but the highest adjusted mean for this treatment group (7.16) was less than the lowest for HIM (7.38), so results favoring HIM among ninth graders were attributed to treatment.

## AUXILIARY INFORMATION AND REPRESENTATION

As Paige and Simon (1966) have pointed out, a translation of an algebra word problem depicts two inputs. One is the statement of the problem. The other is supplied by the problem solver in the form of information and assumptions governing the situation yet auxiliary to its written description. For example, an equation such as $5N + 10D = 150$ portrays not only the total value of some nickels and dimes, but also the fact that nickels are worth five cents, dimes ten cents. Similarly, a job-completed equation such as $M/4 + M/12 = 1$ illustrates not only the time each of two persons takes to complete a job, but also the assumption that work is additive, that is, that persons working together continue to work at the same rate as they did apart.

Of course, translation of algebra word problems can proceed in the absence of this latter input. That is, correct responses in this area are possible without apparent recourse to auxiliary information and physical assumptions representative of a situation. To illustrate, suppose a student translated the following sentence as $K + 3 = 2(M + 3)$.

In three years Kimberly will be twice as old as Micah.

Since three was added to Micah's present age (M) as well as Kim's (K), it would seem that this student utilized the information auxiliary to the statement that both Kim and Micah would have three birthdays in three years. However, suppose that in answer to the question on the next page

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THIS SAME STUDENT SAID, "FIFTEEN."

THE SUM OF KIM'S AGE AND MICAH'S AGE IS 12. WHAT WILL THE SUM OF THEIR AGES BE IN THREE YEARS?

THE TRANSLATION, THEN, PROBABLY HAD MEANING PRIMARILY WITHIN ITSELF AS A ROTE BUT CERTAIN RESPONSE. FOR HAD MEANING DERIVED FROM THE PHYSICAL SITUATION, A NUMERIC RESPONSE OF EIGHTEEN WOULD HAVE BEEN ANTICIPATED.

IN ORDER TO TEST THE EXTENT TO WHICH TRANSLATIONS WERE OTHER THAN ROTE RESPONSES, THAT IS, THE EXTENT TO WHICH THEY REFLECTED AN AWARENESS OF THE CONSEQUENTIAL NATURE OF PROBLEM STATEMENTS, A QUESTIONNAIRE COMPOSED OF ITEMS SIMILAR TO THE ONE ABOVE WAS GIVEN ON THE DAY FOLLOWING THE ADMINISTRATION OF THE FINAL PROBLEMS TEST. STUDENTS WERE FIRST ASKED TO TRANSLATE SIMPLE PHRASES AND SENTENCES, AND RESULTS HERE WERE TAKEN AS A MEASURE OF ABILITY TO MOVE, MECHANICALLY OR OTHERWISE, FROM NATURAL LANGUAGE TO ALGEBRAIC SYMBOLS. FOLLOWING THIS, THEY WERE TO RESPOND TO A NUMBER OF "COMMON SENSE" QUESTIONS PERTAINING TO COIN, AGE, WORK, AND MIXTURE. A LOW SCORE ON THESE LATTER ITEMS WAS INTERPRETED AS EVIDENCE THAT TRANSLATIONS WERE NOT REFLECTIVE OF THE SITUATION SURROUNDING PROBLEM STATEMENTS.

SINCE MANY OF THE ITEMS COMPRISING THIS TEST DID NOT REQUIRE ANY KNOWLEDGE OF ALGEBRA, IT WAS ALSO GIVEN TO THREE CLASSES OF SEVENTH GRADERS AND THREE CLASSES OF NINTH GRADERS WHO HAD NOT RECEIVED ANY INSTRUCTION WHATSOEVER ON WRITING EQUATIONS FOR ALGEBRA WORD PROBLEMS. RESULTS FOR THESE CLASSES WERE GROUPED BY GRADE LEVEL AND ANALYZED ALONG WITH RESPECTIVE DATA FOR HIM, HID, AND LO.

AS SHOWN IN TABLES 27-28, THE NO-INSTRUCTIONAL OR "CONTROL" GROUPS (REFERENCED BY CON) WERE SUITABLE FOR COMPARISONS WITH TREATMENT GROUPS. THAT IS, THE ABSENCE OF STATISTICAL SIGNIFICANCE IN THESE TABLES WITH RESPECT TO ARITHMETIC ACHIEVEMENT INDICATED NO INITIAL BIAS BETWEEN THE

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TABLE 27: ANALYSIS OF COMPREHENSIVE TEST OF BASIC SKILLS (LEVEL 3) ARITHMETIC ACHIEVEMENT (COMPUTATION, CONCEPTS, APPLICATIONS -- 98 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

ANAVARA BETWEEN TREATMENT GROUPS

| TREATMENT | N | MEAN | SD | F-RATIO | T-TEST MATRIX FOR ADJ GROUP MEANS | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | HIM | HID | LO |
| HIM | 84 | 63.25 | 16.26 | 1.30 | | | | |
| HID | 86 | 60.59 | 18.57 | $P < .28$ | HID | NS | | |
| LO | 91 | 65.34 | 17.02 | | LO | NS | NS | |
| CON | 79 | 61.68 | 15.83 | | CON | NS | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO |
| HIM | 27 | 82.63 | 6.77 | .56 | HIM | 30 | 61.83 | 5.60 | .64 | HIM | 27 | 45.44 | 6.44 | .65 |
| HID | 26 | 84.58 | 6.79 | $P < .64$ | HID | 23 | 61.00 | 4.61 | $P < .59$ | HID | 37 | 43.49 | 7.29 | $P < .59$ |
| LO | 35 | 82.74 | 7.27 | | LO | 34 | 61.94 | 5.49 | | LO | 22 | 42.91 | 7.62 | |
| CON | 20 | 82.40 | 5.76 | | CON | 34 | 62.91 | 4.96 | | CON | 25 | 43.44 | 6.87 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | NS | NS | NS |

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TABLE 28: ANALYSIS OF COMPREHENSIVE TEST OF BASIC SKILLS (LEVEL 4) ARITHMETIC ACHIEVEMENT (COMPUTATION, CONCEPTS, APPLICATIONS — 98 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

ANAVARA BETWEEN TREATMENT GROUPS

| TREATMENT | N | MEAN | SD | F-RATIO | T-TEST MATRIX FOR GROUP MEANS | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | HIM | HID | LO |
| HIM | 60 | 71.37 | 12.14 | 1.54 | | | | |
| HID | 63 | 72.16 | 11.69 | P<.20 | HID | NS | | |
| LO | 90 | 69.83 | 11.78 | | LO | NS | NS | |
| CON | 91 | 73.64 | 12.58 | | CON | NS | NS | .05 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO | T | N | MEAN | SD | F-RATIO |
| HIM | 23 | 82.83 | 3.68 | 1.16 | HIM | 19 | 71.79 | 3.31 | .39 | HIM | 18 | 56.28 | 7.96 | .87 |
| HID | 26 | 82.89 | 4.71 | P<.33 | HID | 18 | 71.61 | 3.31 | P<.76 | HID | 19 | 57.90 | 7.57 | P<.46 |
| LO | 29 | 82.00 | 4.13 | | LO | 27 | 72.56 | 3.03 | | LO | 34 | 57.29 | 7.39 | |
| CON | 43 | 83.95 | 4.83 | | CON | 27 | 72.04 | 3.29 | | CON | 21 | 54.57 | 5.96 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | NS | NS | NS |

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four student groups for either grade. Accordingly, the significant differences favoring treatment groups to non-treatment groups for the questionnaire (tables 29-30) were attributed to experimental instruction, and it was concluded that treatment -- whether HIM, HID, or LO -- resulted in learning.

Since tables 29-30 depicting results for the questionnaire do so with respect to all four student groups taken simultaneously, significance levels in these tables have no meaning for HIM, HID, and LO apart from CON. Therefore, in order to determine which treatment was most efficacious, the same data but restricted to HIM, HID, and LO were analyzed as appears in tables 31-32. According to table 31, then, LO was most beneficent among seventh graders. HIM-16.86, HID-16-65, LO-18.20 ($p < .01$). In contrast, at the ninth grade (table 32) results were fairly comparable regardless of treatment assignment: HIM-23.44, HID-22.22, LO-22.77 ($p < .14$).

Turning now to the constituents of the questionnaire, tables 33-34 contain analysis of results for nine phrases to be translated. As indicated, achievement on phrase translation was clearly a function of experimental instruction for both grades. Differences on phrase translation were particularly striking at the seventh grade (table 33) where treatment group means differed as much as two points from CON's: HIM-6.81, HID-6.54, LO-7.12, CON-5.02 ($p = .0000$). Differences favoring treatment groups were also large at grade nine (table 34), but the point to be made here is not how poorly CON did with respect to HIM, HID, and LO, but how well CON did with respect to a total possible score of nine: HIM-8.17, HID-8.41, LO-8.57, CON-6.95 ($p = .0000$).

Of course, considered in itself phrase translation is a minor educational objective. But perhaps, as stated in one of SMSG's algebra texts (1960, p.82), "It is a natural step from translation of phrases to translation of sentences." This being the case, the sequence to translation of algebra word problems would be clear-cut: translation of phrases, then translation of sentences. There is reason to believe, however, that this

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TABLE 29: ANALYSIS OF AUXILIARY REPRESENTATIONS QUESTIONNAIRE (PHRASE, SENTENCE, COIN, AGE, WORK, MIXTURE -- 32 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means |
|---|---|---|---|---|---|---|
| HIM | 81 | 16.89 | 4.93 | 16.78 | 22.57 | HIM HID LO |
| HID | 80 | 16.15 | 5.33 | 16.59 | p = .0000 | HID NS |
| LO | 86 | 18.64 | 5.38 | 18.13 | | LO .05 .01 |
| CON | 79 | 13.34 | 4.96 | 13.56 | | CON .001 .001 .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 21.07 | 4.18 | 6.56 | HIM | 27 | 17.00 | 2.47 | 10.07 | HIM | 27 | 12.59 | 3.74 | 4.30 |
| HID | 25 | 21.68 | 2.73 | p<.0004 | HID | 21 | 14.86 | 4.91 | p=.0000 | HID | 34 | 12.88 | 3.55 | p<.007 |
| LO | 32 | 22.66 | 3.67 | | LO | 33 | 17.79 | 3.92 | | LO | 21 | 13.86 | 5.17 | |
| CON | 20 | 17.85 | 4.91 | | CON | 34 | 13.24 | 3.39 | | CON | 25 | 9.88 | 3.94 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .01 | .01 | .001 |

| MIDDLE 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | .05 | | |
| LO | NS | .01 | |
| CON | .001 | NS | .001 |

| LOWER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .05 | .01 | .01 |

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TABLE 30: ANALYSIS OF AUXILIARY REPRESENTATIONS QUESTIONNAIRE (PHRASE, SENTENCE, COIN, AGE, WORK, MIXTURE -- 32 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 57 | 23.44 | 3.71 | 23.63 | 47.73 | | | | |
| HID | 61 | 22.45 | 4.86 | 22.40 | p = .0000 | HID | NS | | |
| LO | 87 | 22.61 | 3.93 | 23.00 | | LO | NS | NS | |
| CON | 91 | 18.32 | 4.91 | 17.86 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 24.62 | 3.34 | 13.47 | HIM | 18 | 24.00 | 2.83 | 19.24 | HIM | 18 | 21.50 | 4.27 | 14.27 |
| HID | 25 | 25.40 | 2.89 | p=.0000 | HID | 17 | 23.18 | 4.20 | p=.0000 | HID | 19 | 18.11 | 4.50 | p=.0000 |
| LO | 28 | 25.46 | 2.59 | | LO | 27 | 23.19 | 2.96 | | LO | 32 | 19.63 | 3.59 | |
| CON | 43 | 21.07 | 4.15 | | CON | 27 | 17.59 | 3.43 | | CON | 21 | 13.62 | 4.09 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | .05 | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | .001 | .001 | .001 | CON | .001 | .001 | .001 | CON | .001 | .001 | .001 |

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TABLE 31: ANALYSIS OF AUXILIARY REPRESENTATIONS QUESTIONNAIRE (PHRASE, SENTENCE, COIN, AGE, WORK, MIXTURE -- 32 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 16.89 | 4.93 | 16.86 | 4.63 | | | |
| HID | 80 | 16.15 | 5.33 | 16.65 | $p < .01$ | HID | NS | |
| LO | 86 | 18.64 | 5.38 | 18.20 | | LO | .05 | .01 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 16.05 | 16.85 | .02 | 1 | 27 | 16.11 | 16.09 | .09 | 1 | 23 | 17.04 | 18.39 | 2.07 |
| 2 | 30 | 17.40 | 16.82 | $p < .98$ | 2 | 25 | 15.76 | 16.41 | $p < .92$ | 2 | 32 | 19.31 | 19.65 | $p < .13$ |
| 3 | 30 | 16.97 | 16.99 | | 3 | 28 | 16.54 | 15.98 | | 3 | 31 | 19.13 | 17.78 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 21.07 | 4.18 | 1.46 | HIM | 27 | 17.00 | 2.47 | 3.87 | HIM | 27 | 12.59 | 3.74 | .61 |
| HID | 25 | 21.68 | 2.73 | $p < .24$ | HID | 21 | 14.86 | 4.91 | $p < .02$ | HID | 34 | 12.88 | 3.55 | $p < .54$ |
| LO | 32 | 22.66 | 3.67 | | LO | 33 | 17.79 | 3.92 | | LO | 21 | 13.86 | 5.17 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | NS | .01 | LO | NS | NS |

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TABLE 32: ANALYSIS OF AUXILIARY REPRESENTATIONS QUESTIONNAIRE (PHRASE, SENTENCE, COIN, AGE, WORK, MIXTURE -- 32 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 23.44 | 3.71 | 23.44 | 1.97 | | | |
| HID | 61 | 22.45 | 4.86 | 22.22 | $p < .14$ | HID | .05 | |
| LO | 87 | 22.61 | 3.93 | 22.77 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 23.00 | 23.30 | .06 | 1 | 29 | 22.34 | 22.71 | .24 | 1 | 25 | 22.48 | 22.56 | .007 |
| 2 | 32 | 23.78 | 23.55 | $p < .81$ | 2 | 32 | 22.55 | 22.22 | $p < .63$ | 2 | 32 | 22.56 | 22.65 | $p < .99$ |
| | | | | | | | | | | 3 | 30 | 22.77 | 22.60 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 24.62 | 3.34 | .59 | HIM | 18 | 24.00 | 2.83 | .39 | HIM | 18 | 21.50 | 4.27 | 3.28 |
| HID | 25 | 25.40 | 2.89 | $p < .56$ | HID | 17 | 23.18 | 4.20 | $p < .68$ | HID | 19 | 18.11 | 4.50 | $p < .04$ |
| LO | 28 | 25.46 | 2.59 | | LO | 27 | 23.19 | 2.96 | | LO | 32 | 19.63 | 3.59 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | .05 | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 33: ANALYSIS OF PHRASE TRANSLATION SUBTEST (9 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 81 | 6.84 | 2.20 | 6.81 | 16.70 | | | | |
| HID | 80 | 6.40 | 2.40 | 6.54 | P = .0000 | HID | NS | | |
| LO | 86 | 7.26 | 1.93 | 7.12 | | LO | NS | NS | |
| CON | 79 | 4.97 | 2.77 | 5.02 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 8.22 | 1.01 | 7.51 | HIM | 27 | 7.00 | 1.64 | 7.34 | HIM | 27 | 5.30 | 2.58 | 5.04 |
| HID | 25 | 8.12 | 1.13 | P<.0001 | HID | 21 | 5.71 | 2.88 | P<.0002 | HID | 34 | 5.56 | 2.12 | P<.003 |
| LO | 32 | 8.44 | .76 | | LO | 33 | 7.39 | 1.56 | | LO | 21 | 5.24 | 2.12 | |
| CON | 20 | 6.65 | 2.52 | | CON | 34 | 5.21 | 2.41 | | CON | 25 | 3.32 | 2.58 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | .05 | | | HID | NS | | |
| LO | NS | NS | | LO | NS | .01 | | LO | NS | NS | |
| CON | .001 | .001 | .001 | CON | .01 | NS | .001 | CON | .01 | .001 | .01 |

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TABLE 34 : ANALYSIS OF PHRASE TRANSLATION SUBTEST (9 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| HIM | 57 | 8.16 | .82 | 8.17 | 27.73 | | HIM | HID | LO |
| HID | 61 | 8.40 | .86 | 8.41 | P = .0000 | HID | NS | | |
| LO | 87 | 8.48 | .89 | 8.57 | | LO | NS | NS | |
| CON | 91 | 7.04 | 2.14 | 6.95 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 8.43 | .68 | 5.07 | HIM | 18 | 7.94 | .87 | 11.22 | HIM | 18 | 8.06 | .87 | 14.62 |
| HID | 25 | 8.76 | .52 | P<.002 | HID | 17 | 8.53 | .72 | P=.0000 | HID | 19 | 7.84 | 1.07 | P=.0000 |
| LO | 28 | 8.79 | .69 | | LO | 27 | 8.63 | .63 | | LO | 32 | 8.09 | 1.09 | |
| CON | 43 | 7.79 | 1.85 | | CON | 27 | 7.04 | 1.65 | | CON | 21 | 5.52 | 2.52 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | .05 | NS | | LO | NS | NS | |
| CON | NS | .01 | .01 | CON | .01 | .001 | .001 | CON | .001 | .001 | .001 |

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succession of events omits a sizable intermediate step.

Included in the questionnaire were three sentences to be translated into equations in two unknowns. As tables 35-36 illustrate, students in both grades under treatment averaged significantly better on sentence translation than those in the control groups. For instance, among seventh graders (table 35) treatment group adjusted means were in the neighborhood of five times as large as CON's on this variable: HIM-.91, HID-1.09, LO-1.21, CON-.19 ($p = .0000$). And at the ninth grade (table 36) the treatment groups did about twice as well: HIM-2.20, HID-1.81, LO-2.08, CON-1.04 ($p = .0000$).

From the preceding statistics it can be argued that there is more to translating sentences than knowing how to translate phrases. For if phrase translation were the key to sentence translation, the control groups should have performed nearly as well on sentences as the treatment groups in light of the control groups relatively high performance on phrases. But at the ninth grade, whereas CON achieved nearly eighty per cent as well on phrases as those under treatment, it did only half as well on sentences. And the argument is strengthened if cast in terms of seventh grade results.

The difficulty in learning to translate algebra word problems, then, seems to depend more on recognition of the way phrases are related grammatically than on mastery of the fundamentals of translation restricted to phrases. Putting this another way, what is difficult is probably not so much a matter of expressing mathematically what is, but of expressing what is <u>equal</u>.

Even though achievement on phrase and sentence translation was certainly influenced by experimental instruction, tables 37-40 indicate that no striking advantage accrued to being under specific treatment. For

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TABLE 35: ANALYSIS OF SENTENCE TRANSLATION SUBTEST (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 81 | .93 | 1.07 | .91 | 23.34 | | | | |
| HID | 80 | 1.03 | 1.10 | 1.09 | P = .0000 | HID | NS | | |
| LO | 86 | 1.29 | 1.06 | 1.21 | | LO | .05 | NS | |
| CON | 79 | .16 | .63 | .19 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.63 | 1.08 | 11.31 | HIM | 27 | .89 | 1.05 | 10.44 | HIM | 27 | .26 | .53 | 28.23 |
| HID | 25 | 2.04 | 1.02 | P=.0000 | HID | 21 | .95 | .97 | P=.0000 | HID | 34 | .32 | .53 | P=.0000 |
| LO | 32 | 1.84 | .99 | | LO | 33 | 1.30 | .98 | | LO | 21 | .43 | .68 | |
| CON | 20 | .40 | .99 | | CON | 34 | .12 | .54 | | CON | 25 | .04 | .20 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | .001 | .001 | .001 | CON | .01 | .01 | .001 | CON | NS | .05 | .05 |

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TABLE 36: ANALYSIS OF SENTENCE TRANSLATION SUBTEST (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 57 | 2.16 | .92 | 2.20 | 24.34 | | | | |
| HID | 61 | 1.82 | 1.11 | 1.81 | p = .0000 | HID | .05 | | |
| LO | 87 | 2.03 | 1.01 | 2.08 | | LO | NS | NS | |
| CON | 91 | 1.10 | 1.10 | 1.04 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 2.33 | .73 | 10.73 | HIM | 18 | 2.17 | 1.15 | 5.26 | HIM | 18 | 1.94 | .87 | 10.60 |
| HID | 25 | 2.28 | .84 | p=.0000 | HID | 17 | 2.12 | 1.05 | p<.002 | HID | 19 | 1.05 | 1.03 | p=.0000 |
| LO | 28 | 2.43 | .74 | | LO | 27 | 2.15 | .99 | | LO | 32 | 1.59 | 1.07 | |
| CON | 43 | 1.37 | 1.09 | | CON | 27 | 1.22 | .89 | | CON | 21 | .38 | .74 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .001 | .001 | .001 |

| MIDDLE 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .01 | .01 | .01 |

| LOWER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | .01 | | |
| LO | NS | NS | |
| CON | .001 | .05 | .001 |

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PHRASES, SEVENTH GRADE (TABLE 37) RESULTS DIFFERED SIGNIFICANTLY ONLY AMONG THE MIDDLE 1/3 WHERE HID, WITH A MEAN OF 5.71, DID POORLY IN COMPARISON TO HIM (7.00) AND LO (7.39). AND AT GRADE NINE (TABLE 38) DIFFERENCES WERE HARDLY NOTABLE IN EDUCATIONAL TERMS REGARDLESS OF STATISTICAL SIGNIFICANCE: HIM-8.15, HID-8.38, LO-8.51 ($p<.03$). FOR SENTENCES, SEVENTH GRADERS (TABLE 39) IN LO TENDED TOWARD GREATER ACHIEVEMENT, BUT THE SIGNIFICANT DIFFERENCES BETWEEN CLASSES COMPRISING LO ($p<.05$) MAKE IT DIFFICULT TO TIE RESULTS TO TREATMENT. AND ALTHOUGH RESULTS TEND TO FAVOR HIM AT GRADE NINE (TABLE 40), ONLY THE LOWER 1/3 IN HIM AND HID WERE INFLUENCED TO THE POINT OF SIGNIFICANCE: HIM-1.94, HID-1.05, LO-1.59 ($p<.03$).

FOLLOWING TRANSLATION OF PHRASES AND SENTENCES, QUESTIONS WERE POSED PERTAINING TO CIRCUMSTANCES NECESSARILY SURROUNDING COIN, AGE, WORK, AND MIXTURE. THESE WERE INTENDED TO REFLECT STUDENTS' ABILITY TO GO BEYOND VERBAL STATEMENTS TO A REPRESENTATION OF THE SITUATION SET FORTH. FOR COIN AND AGE -- TOPICS RECEIVING DIFFERENTIAL TREATMENT -- TABLES 41 AND 43 REVEAL THE SURPRISING RESULT THAT TREATMENT WAS WITHOUT APPARENT EFFECT AMONG SEVENTH GRADERS. THAT IS, STUDENTS UNDER EXPERIMENTAL INSTRUCTION WERE SEEMINGLY READING NO MORE INTO THESE TOPICS THAN BEFORE THE STUDY. AT THE NINTH GRADE (TABLES 42 AND 44), HOWEVER, STUDENTS' PERCEPTION OF COIN AND AGE WAS CLEARLY INFLUENCED BY EXPERIMENTAL INSTRUCTION.

IN COMPARING GRADE SEVEN'S ADJUSTED CONTROL GROUP MEAN ON COIN (3.24 -- TABLE 41) TO THAT FOR GRADE NINE (3.90 -- TABLE 42), THE TWO ARE NOTED TO DIFFER ONLY SLIGHTLY. AND FOR AGE THESE TWO MEANS MORE NEARLY APPROXIMATE EACH OTHER: 2.79 (TABLE 43), 2.90 (TABLE 44). SO, IN LIGHT OF TREATMENT GROUP SUPERIORITY ON COIN AND AGE AT GRADE NINE, IT SEEMS THAT LITTLE GROWTH IN THESE AREAS CAN BE EXPECTED IN THE ABSENCE OF RELEVANT INSTRUCTION. JUST WHY INSTRUCTION MUST APPARENTLY WAIT UNTIL GRADE NINE, HOWEVER, IS A QUESTION THE WRITER CANNOT ANSWER. NONETHELESS, THE

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TABLE 37: ANALYSIS OF PHRASE TRANSLATION SUBTEST (9 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 6.84 | 2.20 | 6.83 | 2.14 | | | |
| HID | 80 | 6.40 | 2.40 | 6.54 | $p < .12$ | HID | NS | |
| LO | 86 | 7.26 | 1.93 | 7.14 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 6.29 | 6.57 | .42 | 1 | 27 | 5.89 | 5.87 | 1.50 | 1 | 23 | 6.87 | 7.28 | .003 |
| 2 | 30 | 6.97 | 6.79 | $p < .66$ | 2 | 25 | 6.68 | 6.92 | $p < .23$ | 2 | 32 | 7.16 | 7.25 | $p < 1$ |
| 3 | 30 | 7.10 | 7.08 | | 3 | 28 | 6.64 | 6.44 | | 3 | 31 | 7.65 | 7.25 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 8.22 | 1.01 | .82 | HIM | 27 | 7.00 | 1.64 | 4.64 | HIM | 27 | 5.30 | 2.58 | .16 |
| HID | 25 | 8.12 | 1.13 | $p < .44$ | HID | 21 | 5.71 | 2.88 | $p < .01$ | HID | 34 | 5.56 | 2.12 | $p < .85$ |
| LO | 32 | 8.44 | .76 | | LO | 33 | 7.39 | 1.56 | | LO | 21 | 5.24 | 2.12 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | NS | .01 | LO | NS | NS |

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TABLE 38: ANALYSIS OF PHRASE TRANSLATION SUBTEST (9 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 8.16 | .82 | 8.15 | 3.49 | | | |
| HID | 61 | 8.40 | .86 | 8.38 | $p < .03$ | HID | NS | |
| LO | 87 | 8.48 | .89 | 8.51 | | LO | .01 | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 8.00 | 8.05 | .61 | 1 | 29 | 8.34 | 8.42 | .01 | 1 | 25 | 8.48 | 8.48 | .02 |
| 2 | 32 | 8.28 | 8.24 | $p < .44$ | 2 | 32 | 8.45 | 8.39 | $p < .90$ | 2 | 32 | 8.53 | 8.51 | $p < .98$ |
| | | | | | | | | | | 3 | 30 | 8.43 | 8.46 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 8.43 | .68 | 2.25 | HIM | 18 | 7.94 | .87 | 5.10 | HIM | 18 | 8.06 | .87 | .37 |
| HID | 25 | 8.76 | .52 | $p < .11$ | HID | 17 | 8.53 | .72 | $p < .09$ | HID | 19 | 7.84 | 1.07 | $p < .69$ |
| LO | 28 | 8.79 | .69 | | LO | 27 | 8.63 | .63 | | LO | 32 | 8.09 | 1.09 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | .01 | NS | LO | NS | NS |

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TABLE 39: ANALYSIS OF SENTENCE TRANSLATION SUBTEST (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 81 | .93 | 1.07 | .92 | 2.29 | | HIM | HID |
| HID | 80 | 1.03 | 1.10 | 1.10 | $p < .10$ | HID | NS | |
| LO | 86 | 1.29 | 1.06 | 1.21 | | LO | .05 | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | .95 | 1.03 | .64 | 1 | 27 | 1.11 | 1.10 | .47 | 1 | 23 | 1.48 | 1.67 | 3.12 |
| 2 | 30 | 1.10 | 1.01 | $p < .53$ | 2 | 25 | .76 | .89 | $p < .63$ | 2 | 32 | 1.19 | 1.22 | $p < .05$ |
| 3 | 30 | .73 | .77 | | 3 | 28 | 1.18 | 1.07 | | 3 | 31 | 1.26 | 1.08 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.63 | 1.08 | 1.04 | HIM | 27 | .89 | 1.05 | 1.47 | HIM | 27 | .26 | .53 | .52 |
| HID | 25 | 2.04 | 1.02 | $p < .36$ | HID | 21 | .95 | .97 | $p < .24$ | HID | 34 | .32 | .53 | $p < .60$ |
| LO | 32 | 1.84 | .99 | | LO | 33 | 1.30 | .98 | | LO | 21 | .43 | .68 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 40: ANALYSIS OF SENTENCE TRANSLATION SUBTEST (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 2.16 | .92 | 2.18 | 2.77 | | | |
| HID | 61 | 1.82 | 1.11 | 1.78 | $p < .06$ | HID | .05 | |
| LO | 87 | 2.03 | 1.01 | 2.05 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 1.88 | 1.89 | 3.39 | 1 | 29 | 1.83 | 1.90 | .30 | 1 | 25 | 1.96 | 1.97 | .23 |
| 2 | 32 | 2.38 | 2.37 | $p < .07$ | 2 | 32 | 1.82 | 1.76 | $p < .59$ | 2 | 32 | 1.97 | 2.00 | $p < .79$ |
| | | | | | | | | | | 3 | 30 | 2.17 | 2.13 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 2.33 | .73 | .25 | HIM | 18 | 2.17 | 1.15 | .01 | HIM | 18 | 1.94 | .87 | 3.68 |
| HID | 25 | 2.28 | .84 | $p < .78$ | HID | 17 | 2.12 | 1.05 | $p < .99$ | HID | 19 | 1.05 | 1.03 | $p < .03$ |
| LO | 28 | 2.43 | .74 | | LO | 27 | 2.15 | .99 | | LO | 32 | 1.59 | 1.07 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | .01 | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 41: ANALYSIS OF COIN SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| HIM | 81 | 3.54 | 1.32 | 3.52 | .74 | | HIM | HID | LO |
| HID | 80 | 3.26 | 1.33 | 3.34 | P < .53 | HID | NS | | |
| LO | 86 | 3.48 | 1.51 | 3.39 | | LO | NS | NS | |
| CON | 79 | 3.19 | 1.21 | 3.24 | | CON | NS | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 4.41 | 1.31 | .42 | HIM | 27 | 3.41 | 1.01 | .91 | HIM | 27 | 2.81 | 1.14 | .40 |
| HID | 25 | 4.00 | 1.29 | P < .74 | HID | 21 | 2.90 | 1.45 | P < .44 | HID | 34 | 2.94 | 1.07 | P < .76 |
| LO | 32 | 4.19 | 1.49 | | LO | 33 | 3.24 | 1.28 | | LO | 21 | 2.76 | 1.48 | |
| CON | 20 | 4.20 | .95 | | CON | 34 | 3.03 | 1.03 | | CON | 25 | 2.60 | 1.15 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | NS | NS | NS |

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TABLE 42: ANALYSIS OF COIN SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| HIM | 57 | 4.74 | 1.06 | 4.79 | 13.82 | | HIM | HID | LO |
| HID | 61 | 4.45 | 1.47 | 4.43 | p = .0000 | HID | NS | | |
| LO | 87 | 4.82 | 1.21 | 4.89 | | LO | NS | .05 | |
| CON | 91 | 3.98 | 1.21 | 3.90 | | CON | .001 | .01 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 5.00 | .71 | 7.44 | HIM | 18 | 5.06 | .64 | 9.85 | HIM | 18 | 4.11 | 1.45 | 2.39 |
| HID | 25 | 5.36 | .76 | p<.0001 | HID | 17 | 4.35 | 1.46 | p=.0000 | HID | 19 | 3.47 | 1.47 | p < .07 |
| LO | 28 | 5.54 | 1.04 | | LO | 27 | 4.85 | .95 | | LO | 32 | 4.16 | 1.19 | |
| CON | 43 | 4.60 | .93 | | CON | 27 | 3.44 | 1.31 | | CON | 21 | 3.38 | .97 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | .05 | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | .001 | .001 | CON | .001 | .05 | .001 | CON | NS | NS | .05 |

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TABLE 43: ANALYSIS OF AGE SUBQUESTIONNAIRE (5 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 81 | 2.65 | 1.10 | 2.64 | 1.39 | | | | |
| HID | 80 | 2.70 | 1.25 | 2.77 | $p < .25$ | HID | NS | | |
| LO | 86 | 3.05 | 1.17 | 2.95 | | LO | .05 | NS | |
| CON | 79 | 2.75 | 1.09 | 2.79 | | CON | NS | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.30 | 1.03 | .70 | HIM | 27 | 2.78 | .85 | .24 | HIM | 27 | 1.89 | .93 | 2.22 |
| HID | 25 | 3.40 | 1.12 | $p < .56$ | HID | 21 | 2.95 | 1.24 | $p < .87$ | HID | 34 | 2.03 | 1.00 | $p < .09$ |
| LO | 32 | 3.69 | .93 | | LO | 33 | 2.73 | 1.21 | | LO | 21 | 2.57 | 1.03 | |
| CON | 20 | 3.45 | 1.32 | | CON | 34 | 2.74 | .86 | | CON | 25 | 2.20 | .87 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | .05 | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | NS | NS | NS |

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TABLE 44a ANALYSIS OF AGE SUBQUESTIONNAIRE (5 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 57 | 3.58 | 1.07 | 3.61 | 6.66 | | | | |
| HID | 61 | 3.24 | 1.20 | 3.24 | $p < .0002$ | HID | NS | | |
| LO | 87 | 3.41 | 1.29 | 3.49 | | LO | NS | NS | |
| CON | 91 | 2.99 | 1.18 | 2.90 | | CON | .001 | NS | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 3.71 | 1.06 | 4.32 | HIM | 18 | 3.94 | .94 | 3.89 | HIM | 18 | 3.06 | 1.06 | 1.19 |
| HID | 25 | 3.88 | 1.05 | $p < .006$ | HID | 17 | 3.18 | 1.01 | $p < .01$ | HID | 19 | 2.53 | 1.12 | $p < .32$ |
| LO | 28 | 4.18 | .72 | | LO | 27 | 3.67 | 1.18 | | LO | 32 | 2.53 | 1.27 | |
| CON | 43 | 3.30 | 1.19 | | CON | 27 | 2.93 | 1.17 | | CON | 21 | 2.43 | .98 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | .05 | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | .05 | .001 | CON | .01 | NS | .05 | CON | NS | NS | NS |

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GREATER ACHIEVEMENT ASSOCIATED WITH TREATMENT AMONG NINTH GRADERS PROVIDES A BASIS FOR ARGUING THAT ALGEBRA WORD PROBLEMS ARE APPROPRIATE CONTENT FOR SECONDARY SCHOOL MATHEMATICS. THAT IS, IT SEEMS THAT EFFORT DEVOTED TO THEIR SOLUTION LEADS TO GREATER FACILITY IN DEALING QUANTITATIVELY WITH TOPICS SUCH AS COIN AND AGE IN MORE GENERAL SETTINGS.

SINCE NINTH GRADE ACHIEVEMENT ON QUESTIONS CONCERNED WITH COIN AND AGE WERE ATTRIBUTED TO EXPERIMENTAL INSTRUCTION, RESULTS WERE EXAMINED FOR DIFFERENTIAL TREATMENT EFFECT. AS ILLUSTRATED IN TABLE 46, HIM AND LO RESULTED IN COMPARABLE PERFORMANCE FOR COIN (4.75 AND 4.85, RESPECTIVELY), WITH LO SUPERIOR TO HID (4.39) AT THE .04 LEVEL OF SIGNIFICANCE. FOR AGE (TABLE 48) HID WAS AGAIN THE LOWEST OF THE THREE, BUT NOT UNEXPECTEDLY SO: HIM-3.56, HID-3.19, LO-3.46 ($p < .15$). (TABLES 45 AND 47 CONTAIN A SIMILAR ANALYSIS FOR SEVENTH GRADE RESULTS. ALTHOUGH TREATMENT AT GRADE SEVEN WAS AN APPARENT PLACEBO FOR THESE ITEMS, THE INCLUSION OF THESE TABLES PERMITS EXAMINATION OF CLASS AND TREATMENT SUBGROUP BEHAVIOR ON THIS PORTION OF THE QUESTIONNAIRE.).

FOR QUESTIONS PERTAINING TO WORK, TREATMENT WAS WITHOUT APPRECIABLE EFFECT AT BOTH GRADES. FOR GRADE SEVEN (TABLE 49) IT APPEARED THAT LO RESULTED IN LEARNING, BUT THE SIGNIFICANT VARIABILITY WITHIN LO (TABLE 51) TENDS TO STIFLE THIS CONCLUSION. AND FOR GRADE NINE (TABLES 50 AND 52) SUPERIOR ACHIEVEMENT WAS AN APPARENT CONCOMITANT OF HIM ONLY. SO PERHAPS WORK PROBLEMS SHOULD BE RESERVED FOR LATER ON IN THE SECONDARY CURRICULUM. THAT IS, IF INCREASED PERCEPTIVENESS TO THE MATHEMATICAL NATURE OF WORK IS A MAJOR CRITERION FOR POSING SUCH PROBLEMS, THEIR INCLUSION IS AT BEST JUSTIFIED ONLY AFTER GRADE NINE.

CONTRARY TO RESULTS FOR WORK, TREATMENT HAD A MARKED EFFECT FOR THE MIXTURE PORTION OF THE QUESTIONNAIRE. AS INDICATED IN TABLES 53 AND 54, TREATMENT GROUPS ACHIEVED SIGNIFICANTLY BETTER THAN CONTROL GROUPS AT BOTH

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TABLE 45: ANALYSIS OF COIN SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 3.54 | 1.32 | 3.53 | .46 | | | |
| HID | 80 | 3.26 | 1.33 | 3.35 | $p < .63$ | HID | NS | |
| LO | 86 | 3.48 | 1.51 | 3.41 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 3.52 | 3.69 | .80 | 1 | 27 | 3.26 | 3.28 | .33 | 1 | 23 | 2.78 | 3.02 | 1.74 |
| 2 | 30 | 3.47 | 3.33 | $p < .46$ | 2 | 25 | 3.36 | 3.41 | $p < .72$ | 2 | 32 | 3.59 | 3.68 | $p < .18$ |
| 3 | 30 | 3.63 | 3.65 | | 3 | 28 | 3.18 | 3.12 | | 3 | 31 | 3.87 | 3.61 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 4.41 | 1.31 | .57 | HIM | 27 | 3.41 | 1.01 | .98 | HIM | 27 | 2.81 | 1.14 | .16 |
| HID | 25 | 4.00 | 1.29 | $p < .57$ | HID | 21 | 2.90 | 1.45 | $p < .38$ | HID | 34 | 2.94 | 1.07 | $p < .85$ |
| LO | 32 | 4.19 | 1.49 | | LO | 33 | 3.24 | 1.28 | | LO | 21 | 2.76 | 1.48 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 46: ANALYSIS OF COIN SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 4.74 | 1.06 | 4.75 | 3.25 | | | |
| HID | 61 | 4.45 | 1.47 | 4.39 | $p < .04$ | HID | NS | |
| LO | 87 | 4.82 | 1.21 | 4.85 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 4.84 | 4.87 | .61 | 1 | 29 | 4.48 | 4.58 | .57 | 1 | 25 | 4.88 | 4.90 | 1.03 |
| 2 | 32 | 4.66 | 4.64 | $p < .44$ | 2 | 32 | 4.42 | 4.34 | $p < .45$ | 2 | 32 | 4.94 | 4.97 | $p < .36$ |
| | | | | | | | | | | 3 | 30 | 4.63 | 4.58 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 5.00 | .71 | 2.35 | HIM | 18 | 5.06 | .64 | 2.11 | HIM | 18 | 4.11 | 1.45 | 1.71 |
| HID | 25 | 5.36 | .76 | $p < .10$ | HID | 17 | 4.35 | 1.46 | $p < .13$ | HID | 19 | 3.47 | 1.47 | $p < .19$ |
| LO | 28 | 5.54 | 1.04 | | LO | 27 | 4.85 | .95 | | LO | 32 | 4.16 | 1.19 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | .05 | NS | LO | NS | NS | LO | NS | NS |

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TABLE 47: ANALYSIS OF AGE SUBQUESTIONNAIRE (5 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 2.65 | 1.10 | 2.65 | 2.15 | | | |
| HID | 80 | 2.70 | 1.25 | 2.78 | $p < .12$ | HID | NS | |
| LO | 86 | 3.05 | 1.17 | 2.98 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 2.24 | 2.45 | 1.42 | 1 | 27 | 2.78 | 2.75 | 1.40 | 1 | 23 | 2.61 | 2.81 | 4.29 |
| 2 | 30 | 2.70 | 2.58 | $p < .25$ | 2 | 25 | 2.80 | 2.95 | $p < .25$ | 2 | 32 | 3.41 | 3.46 | $p < .02$ |
| 3 | 30 | 2.90 | 2.87 | | 3 | 28 | 2.54 | 2.43 | | 3 | 31 | 3.00 | 2.80 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.30 | 1.03 | 1.17 | HIM | 27 | 2.78 | .85 | .27 | HIM | 27 | 1.89 | .93 | 3.09 |
| HID | 25 | 3.40 | 1.12 | $p < .31$ | HID | 21 | 2.95 | 1.24 | $p < .76$ | HID | 34 | 2.03 | 1.00 | $p < .05$ |
| LO | 32 | 3.69 | .93 | | LO | 33 | 2.73 | 1.21 | | LO | 21 | 2.57 | 1.03 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | .05 | NS |

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TABLE 48: ANALYSIS OF AGE SUBQUESTIONNAIRE (5 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 3.58 | 1.07 | 3.56 | 1.93 | | | |
| HID | 61 | 3.24 | 1.20 | 3.19 | $p < .15$ | HID | NS | |
| LO | 87 | 3.41 | 1.29 | 3.46 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 3.56 | 3.67 | .31 | 1 | 29 | 3.00 | 3.05 | 1.71 | 1 | 25 | 3.64 | 3.67 | 4.39 |
| 2 | 32 | 3.59 | 3.50 | $p < .58$ | 2 | 32 | 3.45 | 3.41 | $p < .20$ | 2 | 32 | 3.63 | 3.65 | $p < .02$ |
| 3 | | | | | 3 | | | | | 3 | 30 | 3.00 | 2.94 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 3.71 | 1.06 | 1.55 | HIM | 18 | 3.94 | .94 | 2.32 | HIM | 18 | 3.06 | 1.06 | 1.33 |
| HID | 25 | 3.88 | 1.05 | $p < .22$ | HID | 17 | 3.18 | 1.01 | $p < .11$ | HID | 19 | 2.53 | 1.12 | $p < .27$ |
| LO | 28 | 4.18 | .72 | | LO | 27 | 3.67 | 1.18 | | LO | 32 | 2.53 | 1.27 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | .05 | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 49: ANALYSIS OF WORK SUBQUESTIONNAIRE (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 81 | 1.06 | .89 | 1.06 | 3.67 | | | | |
| HID | 80 | 1.08 | .76 | 1.10 | $p < .01$ | HID | NS | | |
| LO | 86 | 1.33 | .87 | 1.30 | | LO | .05 | NS | |
| CON | 79 | .90 | .65 | .90 | | CON | NS | NS | .01 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.15 | .91 | 1.44 | HIM | 27 | 1.15 | .91 | 1.03 | HIM | 27 | .89 | .85 | 2.11 |
| HID | 25 | 1.40 | .87 | $p < .24$ | HID | 21 | .95 | .67 | $p < .38$ | HID | 34 | .91 | .67 | $p < .10$ |
| LO | 32 | 1.59 | .91 | | LO | 33 | 1.15 | .80 | | LO | 21 | 1.19 | .87 | |
| CON | 20 | 1.25 | .72 | | CON | 34 | .88 | .59 | | CON | 25 | .64 | .57 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | NS | NS | .05 |

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TABLE 50 : ANALYSIS OF WORK SUBQUESTIONNAIRE (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 57 | 1.67 | .85 | 1.69 | 5.57 | | | | |
| HID | 61 | 1.47 | .99 | 1.46 | $p < .001$ | HID | NS | | |
| LO | 87 | 1.09 | .87 | 1.12 | | LO | .001 | .05 | |
| CON | 91 | 1.33 | .79 | 1.29 | | CON | .01 | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 1.90 | 1.00 | 1.62 | HIM | 18 | 1.56 | .78 | 2.09 | HIM | 18 | 1.50 | .71 | 3.68 |
| HID | 25 | 1.48 | 1.05 | $p < .19$ | HID | 17 | 1.71 | 1.16 | $p < .11$ | HID | 19 | 1.21 | .71 | $p < .02$ |
| LO | 28 | 1.32 | .98 | | LO | 27 | 1.07 | .78 | | LO | 32 | .91 | .82 | |
| CON | 43 | 1.51 | .80 | | CON | 27 | 1.41 | .84 | | CON | 21 | .86 | .48 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | .05 | NS | | LO | NS | .05 | | LO | .01 | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | .01 | NS | NS |

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TABLE 51: ANALYSIS OF WORK SUBQUESTIONNAIRE (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 1.06 | .89 | 1.06 | 1.93 | | | |
| HID | 80 | 1.08 | .76 | 1.10 | P<.15 | HID | NS | |
| LO | 86 | 1.33 | .87 | 1.30 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 1.38 | 1.42 | 2.21 | 1 | 27 | 1.22 | 1.23 | .95 | 1 | 23 | 1.35 | 1.46 | 8.44 |
| 2 | 30 | .97 | .96 | P<.12 | 2 | 25 | .92 | .95 | P<.39 | 2 | 32 | 1.66 | 1.69 | P<.0005 |
| 3 | 30 | .93 | .92 | | 3 | 28 | 1.07 | 1.04 | | 3 | 31 | .97 | .85 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.15 | .91 | 1.81 | HIM | 27 | 1.15 | .91 | .47 | HIM | 27 | .89 | .85 | 1.06 |
| HID | 25 | 1.40 | .87 | P<.17 | HID | 21 | .95 | .67 | P<.63 | HID | 34 | .91 | .67 | P<.35 |
| LO | 32 | 1.59 | .91 | | LO | 33 | 1.15 | .80 | | LO | 21 | 1.19 | .87 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 52: ANALYSIS OF WORK SUBQUESTIONNAIRE (3 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 1.67 | .85 | 1.67 | 7.22 | | | |
| HID | 61 | 1.47 | .99 | 1.45 | $p < .0009$ | HID | NS | |
| LO | 87 | 1.09 | .87 | 1.10 | | LO | .001 | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 1.60 | 1.68 | .006 | 1 | 29 | 1.59 | 1.58 | .63 | 1 | 25 | 1.24 | 1.25 | 10.91 |
| 2 | 32 | 1.72 | 1.66 | $p < .94$ | 2 | 32 | 1.36 | 1.37 | $p < .43$ | 2 | 32 | .56 | .60 | $p < .0001$ |
| | | | | | | | | | | 3 | 30 | 1.53 | 1.49 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 1.90 | 1.00 | 2.08 | HIM | 18 | 1.56 | .78 | 3.01 | HIM | 18 | 1.50 | .71 | 3.59 |
| HID | 25 | 1.48 | 1.05 | $p < .13$ | HID | 17 | 1.71 | 1.16 | $p < .06$ | HID | 19 | 1.21 | .71 | $p < .03$ |
| LO | 28 | 1.32 | .98 | | LO | 27 | 1.07 | .78 | | LO | 32 | .91 | .82 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | .05 | NS | LO | NS | .05 | LO | .05 | NS |

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TABLE 53: ANALYSIS OF MIXTURE SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 81 | 1.86 | 1.21 | 1.85 | 7.27 | | | | |
| HID | 80 | 1.69 | 1.21 | 1.75 | $p < .0001$ | HID | NS | | |
| LO | 86 | 2.24 | 1.24 | 2.16 | | LO | .05 | .05 | |
| CON | 79 | 1.37 | .89 | 1.41 | | CON | .01 | .05 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 2.37 | 1.52 | 3.50 | HIM | 27 | 1.78 | 1.01 | 3.27 | HIM | 27 | 1.44 | .85 | 2.12 |
| HID | 25 | 2.72 | .79 | $p < .02$ | HID | 21 | 1.38 | 1.12 | $p < .02$ | HID | 34 | 1.12 | 1.04 | $p < .10$ |
| LO | 32 | 2.91 | 1.12 | | LO | 33 | 1.97 | 1.16 | | LO | 21 | 1.67 | 1.11 | |
| CON | 20 | 1.90 | 1.02 | | CON | 34 | 1.26 | .79 | | CON | 25 | 1.08 | .76 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | .05 | | LO | NS | .05 | |
| CON | NS | .05 | .01 | CON | NS | NS | .01 | CON | NS | NS | .05 |

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TABLE 54: ANALYSIS OF MIXTURE SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 57 | 3.14 | 1.09 | 3.17 | 23.40 | | | | |
| HID | 61 | 3.06 | 1.39 | 3.06 | p = .0000 | HID | NS | | |
| LO | 87 | 2.77 | 1.21 | 2.85 | | LO | NS | NS | |
| CON | 91 | 1.88 | 1.40 | 1.79 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 3.24 | 1.26 | 4.89 | HIM | 18 | 3.33 | .69 | 12.24 | HIM | 18 | 2.83 | 1.20 | 9.21 |
| HID | 25 | 3.64 | 1.03 | p<.003 | HID | 17 | 3.29 | 1.31 | p=.0000 | HID | 19 | 2.00 | 1.29 | p=.0000 |
| LO | 28 | 3.21 | 1.03 | | LO | 27 | 2.81 | 1.30 | | LO | 32 | 2.34 | 1.15 | |
| CON | 43 | 2.49 | 1.51 | | CON | 27 | 1.56 | 1.12 | | CON | 21 | 1.05 | .80 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .05 | .001 | .05 |

| MIDDLE 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .001 | .001 | .001 |

| LOWER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | .05 | | |
| LO | NS | NS | |
| CON | .001 | .05 | .001 |

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grades. Among seventh graders (table 55), results favored LO: HIM-1.66, HID-1.77, LO-2.17 ($p < .05$). In contrast, ninth grade (table 56) results were statistically comparable: HIM-3.14, HID-3.02, LO-2.80 ($p < .21$).

## ATTITUDES TOWARD MATHEMATICS AND ALGEBRA WORD PROBLEMS

To test for attitude toward mathematics an opinionnaire constructed by Aiken and Dreger (1961) was used. Twenty statements comprise this survey, each to be rated on a five-point scale to indicate extent of agreement. For positively phrased statements a "strongly disagree" response received a score of zero and, at the other extreme, a "strongly agree" a score of four. For negatively phrased statements the scoring was reversed. "Strongly disagree" received a score of four and "strongly agree" a score of zero. So four points were achievable for each statement for a total possible score of eighty.

At both grades treatment groups HIM and LO were associated with more favorable attitudes as measured by this instrument. Among seventh graders (tables 57 and 59) both differed significantly from CON: HIM-51.86, HID-47.95, LO-52.33, CON-45.97 ($p < .03$). At grade nine (tables 58 and 60), however, the disparity was with respect to HID: HIM-48.03, HID-41.30, LO-49.63, CON-45.84 ($p < .02$).

For treatment subgroups marked differences occurred at the seventh grade. Among the upper 1/3 differences attained significant proportions in favor of LO: HIM-56.96, HID-54.72, LO-66.22, CON-51.56 ($p < .001$). Among the lower 1/3 HIM was favored at near significance: HIM-48.93, HID-42.12, LO-39.95, CON-37.24 ($p < .09$).

In addition to testing for attitudes, students were examined for tendency to select algebra word problems rather than non-verbal problems

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TABLE 55: ANALYSIS OF MIXTURE SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 81 | 1.86 | 1.21 | 1.86 | 3.10 | | | |
| HID | 80 | 1.69 | 1.21 | 1.77 | $p < .05$ | HID | NS | |
| LO | 86 | 2.24 | 1.24 | 2.17 | | LO | NS | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 1.67 | 1.69 | 1.45 | 1 | 27 | 1.85 | 1.86 | 2.62 | 1 | 23 | 1.96 | 2.15 | .28 |
| 2 | 30 | 2.20 | 2.16 | $p < .24$ | 2 | 25 | 1.24 | 1.29 | $p < .08$ | 2 | 32 | 2.31 | 2.36 | $p < .76$ |
| 3 | 30 | 1.67 | 1.69 | | 3 | 28 | 1.93 | 1.87 | | 3 | 31 | 2.39 | 2.20 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 2.37 | 1.52 | 1.52 | HIM | 27 | 1.78 | 1.01 | 1.84 | HIM | 27 | 1.44 | .85 | 2.08 |
| HID | 25 | 2.72 | .79 | $p < .23$ | HID | 21 | 1.38 | 1.12 | $p < .17$ | HID | 34 | 1.12 | 1.04 | $p < .13$ |
| LO | 32 | 2.91 | 1.12 | | LO | 33 | 1.97 | 1.16 | | LO | 21 | 1.67 | 1.11 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | .05 | LO | NS | .05 |

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TABLE 56: ANALYSIS OF MIXTURE SUBQUESTIONNAIRE (6 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 57 | 3.14 | 1.09 | 3.14 | 1.55 | | HIM | HID |
| HID | 61 | 3.06 | 1.39 | 3.02 | $p < .21$ | HID | NS | |
| LO | 87 | 2.77 | 1.21 | 2.80 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 3.12 | 3.14 | .00 | 1 | 29 | 3.10 | 3.19 | .53 | 1 | 25 | 2.28 | 2.29 | 3.23 |
| 2 | 32 | 3.16 | 3.14 | $p < 1$ | 2 | 32 | 3.03 | 2.96 | $p < .47$ | 2 | 32 | 2.94 | 2.93 | $p < .04$ |
| | | | | | | | | | | 3 | 30 | 3.00 | 3.00 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 3.24 | 1.26 | 1.18 | HIM | 18 | 3.33 | .69 | 1.41 | HIM | 18 | 2.83 | 1.20 | 2.24 |
| HID | 25 | 3.64 | 1.04 | $p < .31$ | HID | 17 | 3.29 | 1.31 | $p < .25$ | HID | 19 | 2.00 | 1.29 | $p < .12$ |
| LO | 28 | 3.21 | 1.03 | | LO | 27 | 2.81 | 1.30 | | LO | 32 | 2.34 | 1.15 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | .05 | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 57: ANALYSIS OF ATTITUDE TOWARD MATHEMATICS TEST (80 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| HIM | 81 | 52.21 | 16.31 | 51.86 | 3.10 | | HIM | HID | LO |
| HID | 80 | 47.21 | 15.76 | 47.95 | $p < .03$ | HID | NS | | |
| LO | 86 | 53.24 | 18.01 | 52.33 | | LO | NS | NS | |
| CON | 79 | 45.35 | 17.20 | 45.97 | | CON | .05 | NS | .01 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 56.96 | 16.34 | 5.76 | HIM | 27 | 50.74 | 17.28 | .34 | HIM | 27 | 48.93 | 14.71 | 2.24 |
| HID | 25 | 54.72 | 14.32 | $p < .001$ | HID | 21 | 46.52 | 15.19 | $p < .79$ | HID | 34 | 42.12 | 15.36 | $p < .09$ |
| LO | 32 | 66.22 | 10.39 | | LO | 33 | 49.12 | 14.76 | | LO | 21 | 39.95 | 19.47 | |
| CON | 20 | 51.56 | 14.15 | | CON | 34 | 47.68 | 15.36 | | CON | 25 | 37.24 | 19.19 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | .05 | .01 | |
| CON | NS | NS | .001 |

| MIDDLE 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | NS | NS | NS |

| LOWER 1/3 | | | |
|---|---|---|---|
| | HIM | HID | LO |
| HID | NS | | |
| LO | NS | NS | |
| CON | .05 | NS | NS |

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TABLE 58: ANALYSIS OF ATTITUDE TOWARD MATHEMATICS TEST (80 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| HIM | 57 | 47.56 | 16.52 | 48.03 | 3.40 | | HIM | HID | LO |
| HID | 61 | 41.47 | 20.40 | 41.30 | $p < .02$ | HID | .05 | | |
| LO | 87 | 48.85 | 17.32 | 49.63 | | LO | NS | .01 | |
| CON | 91 | 46.76 | 16.61 | 45.84 | | CON | NS | NS | NS |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 51.86 | 11.59 | 2.21 | HIM | 18 | 53.28 | 15.65 | 1.41 | HIM | 18 | 36.83 | 17.80 | 1.11 |
| HID | 25 | 45.36 | 17.94 | $p < .09$ | HID | 17 | 50.41 | 15.16 | $p < .25$ | HID | 19 | 28.58 | 22.44 | $p < .35$ |
| LO | 28 | 55.79 | 13.20 | | LO | 27 | 54.93 | 15.14 | | LO | 32 | 37.66 | 16.84 | |
| CON | 43 | 52.19 | 15.54 | | CON | 27 | 46.74 | 15.60 | | CON | 21 | 35.67 | 15.01 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | .05 | | LO | NS | NS | | LO | NS | NS | |
| CON | NS | NS | NS | CON | NS | NS | NS | CON | NS | NS | NS |

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TABLE 59: ANALYSIS OF ATTITUDE TOWARD MATHEMATICS TEST (80 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 81 | 52.21 | 16.31 | 52.04 | 2.03 | | HIM | HID |
| HID | 80 | 47.21 | 15.76 | 48.12 | $p < .13$ | HID | NS | |
| LO | 86 | 53.24 | 18.01 | 52.56 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | 56.14 | 59.84 | 2.93 | 1 | 27 | 44.30 | 44.29 | 1.82 | 1 | 23 | 49.17 | 53.07 | .13 |
| 2 | 30 | 51.93 | 49.91 | $p < .06$ | 2 | 25 | 51.00 | 51.93 | $p < .17$ | 2 | 32 | 51.22 | 52.36 | $p < .88$ |
| 3 | 30 | 49.73 | 49.17 | | 3 | 28 | 46.64 | 45.82 | | 3 | 31 | 58.35 | 54.29 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 56.96 | 16.34 | 5.81 | HIM | 27 | 50.74 | 17.28 | .43 | HIM | 27 | 48.93 | 14.71 | 2.10 |
| HID | 25 | 54.72 | 14.32 | $p < .004$ | HID | 21 | 46.52 | 15.19 | $p < .66$ | HID | 34 | 42.12 | 15.36 | $p < .13$ |
| LO | 32 | 66.22 | 10.39 | | LO | 33 | 49.12 | 14.76 | | LO | 21 | 39.95 | 19.47 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | .05 | .01 | LO | NS | NS | LO | NS | NS |

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TABLE 60: ANALYSIS OF ATTITUDE TOWARD MATHEMATICS TEST (80 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 57 | 47.56 | 16.52 | 47.67 | 4.84 | | | |
| HID | 61 | 41.47 | 20.40 | 40.87 | $p < .009$ | HID | .05 | |
| LO | 87 | 48.85 | 17.32 | 49.21 | | LO | NS | .01 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 46.12 | 45.76 | .54 | 1 | 29 | 40.79 | 42.04 | .05 | 1 | 25 | 48.28 | 48.44 | 3.20 |
| 2 | 32 | 48.69 | 48.97 | $p < .47$ | 2 | 32 | 42.06 | 40.97 | $p < .83$ | 2 | 32 | 54.03 | 54.06 | $p < .05$ |
| | | | | | | | | | | 3 | 30 | 43.80 | 43.64 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 51.86 | 11.59 | 3.41 | HIM | 18 | 53.28 | 15.65 | .45 | HIM | 18 | 36.83 | 17.80 | 1.52 |
| HID | 25 | 45.36 | 17.94 | $p < .04$ | HID | 17 | 50.41 | 15.16 | $p < .64$ | HID | 19 | 28.58 | 22.44 | $p < .23$ |
| LO | 28 | 55.79 | 13.20 | | LO | 27 | 54.93 | 15.14 | | LO | 32 | 37.66 | 16.84 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | .05 | LO | NS | NS | LO | NS | NS |

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FOR SOLUTION. FOR THIS PURPOSE AN AFFINITY TEST PATTERNED AFTER ONE DEVISED BY SWART (1968) WAS USED. IN THIS TEST STUDENTS WERE REQUESTED TO CHOOSE ANY TWO OF FOUR PROBLEMS TO SOLVE, AND THEY RECEIVED A SCORE OF ZERO, ONE, OR TWO DEPENDING ON THE NUMBER OF WORD PROBLEMS CHOSEN.

AS CAN BE INFERRED FROM TABLES 61-62, STUDENTS WITHOUT INSTRUCTION IN TRANSLATION ARE NOT LIKELY TO ATTEMPT SOLUTION OF ALGEBRA WORD PROBLEMS. WHETHER THAT INSTRUCTION BE HIM, HID, OR LO, HOWEVER, SEEMS TO BE OF LITTLE CONSEQUENCE IN THIS RESPECT (TABLES 63-64).

## INTERVIEWS

DURING THE NON-TREATMENT FACET OF THE STUDY THE WRITER CONDUCTED INDIVIDUAL INTERVIEWS WITH EIGHTEEN SEVENTH GRADE STUDENTS. THE CRITERION FOR SELECTION FOR INTERVIEW WAS ACHIEVEMENT ON THE FINAL PROBLEMS TEST: THREE STUDENTS FROM EACH TREATMENT GROUP WERE CHOSEN FOR HAVING LOST 4-5 POINTS, AND THREE STUDENTS FROM EACH TREATMENT GROUP WERE CHOSEN FOR HAVING LOST 13-14 POINTS.

THOSE SELECTED FOR INTERVIEW WERE FIRST ASKED FOR THEIR "FAVORITE" AMONG THE PROBLEM TYPES UNDER STUDY. THEY WERE THEN PRESENTED WITH A WRITTEN DESCRIPTION OF THE PROBLEMS BELOW WHICH THEY WERE TO SOLVE (THE FIRST ALGEBRAICALLY, THE SECOND NUMERICALLY) BY DIRECTING THE WRITER TO INDICATE OR PERFORM CERTAIN OPERATIONS. (TO CONTROL FOR READING ABILITY, THE WRITER READ THE PROBLEMS WITH THE STUDENTS AND POINTED OUT THAT WORDS SUCH AS "SPANS" MEANS "GOES OVER.")

> A JAR CONTAINS $4.15 IN DIMES AND QUARTERS. IF THERE ARE 25 COINS IN ALL, HOW MANY OF EACH ARE THERE?
>
> A BRIDGE SPANS A RIVER THAT IS 720 FEET WIDE. ONE-SIXTH OF THE BRIDGE STANDS ON ONE SIDE OF THE RIVER AND 1/9 ON THE OTHER. HOW LONG IS THE BRIDGE?

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TABLE 61: ANALYSIS OF AFFINITY TEST (2 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 81 | .95 | .88 | .94 | 15.36 | | | | |
| HID | 80 | .80 | .80 | .83 | p = .0000 | HID | NS | | |
| LO | 86 | .70 | .77 | .67 | | LO | .05 | NS | |
| CON | 79 | .23 | .45 | .24 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.41 | .80 | 9.15 | HIM | 27 | 1.00 | .92 | 7.62 | HIM | 27 | .44 | .64 | .96 |
| HID | 25 | 1.28 | .74 | p=.0000 | HID | 21 | .76 | .83 | p<.0001 | HID | 34 | .47 | .66 | p < .41 |
| LO | 32 | 1.03 | .74 | | LO | 33 | .64 | .74 | | LO | 21 | .29 | .64 | |
| CON | 20 | .35 | .59 | | CON | 34 | .15 | .36 | | CON | 25 | .24 | .44 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | NS | | | HID | NS | | |
| LO | NS | NS | | LO | NS | NS | | LO | NS | NS | |
| CON | .001 | .001 | .01 | CON | .001 | .01 | .01 | CON | NS | NS | NS |

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TABLE 62: ANALYSIS OF AFFINITY TEST (2 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID | LO |
| HIM | 57 | 1.12 | .71 | 1.13 | 43.47 | | | | |
| HID | 61 | 1.02 | .78 | 1.01 | P = .0000 | HID | NS | | |
| LO | 87 | 1.17 | .70 | 1.17 | | LO | NS | NS | |
| CON | 91 | .18 | .44 | .17 | | CON | .001 | .001 | .001 |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 1.10 | .70 | 21.43 | HIM | 18 | 1.22 | .73 | 13.73 | HIM | 18 | 1.06 | .73 | 11.69 |
| HID | 25 | 1.20 | .71 | P=.0000 | HID | 17 | .76 | .83 | P=.0000 | HID | 19 | .95 | .78 | P=.0000 |
| LO | 28 | 1.04 | .64 | | LO | 27 | 1.30 | .67 | | LO | 32 | 1.19 | .78 | |
| CON | 43 | .19 | .45 | | CON | 27 | .22 | .51 | | CON | 21 | .95 | .30 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | | MIDDLE 1/3 | | | | LOWER 1/3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | HIM | HID | LO | | HIM | HID | LO | | HIM | HID | LO |
| HID | NS | | | HID | .05 | | | HID | NS | | |
| LO | NS | NS | | LO | NS | .05 | | LO | NS | NS | |
| CON | .001 | .001 | .001 | CON | .001 | .05 | .001 | CON | .001 | .001 | .001 |

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TABLE 63: ANALYSIS OF AFFINITY TEST (2 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 81 | .95 | .88 | .94 | 2.94 | | HIM | HID |
| HID | 80 | .80 | .80 | .83 | $p < .05$ | HID | NS | |
| LO | 86 | .70 | .77 | .67 | | LO | .05 | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 21 | .29 | .36 | 8.75 | 1 | 27 | .81 | .81 | 2.62 | 1 | 23 | .43 | .54 | .70 |
| 2 | 30 | 1.10 | 1.03 | $p < .0004$ | 2 | 25 | .48 | .54 | $p < .08$ | 2 | 32 | .72 | .76 | $p < .50$ |
| 3 | 30 | 1.27 | 1.28 | | 3 | 28 | 1.07 | 1.02 | | 3 | 31 | .87 | .75 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 1.41 | .80 | 1.89 | HIM | 27 | 1.00 | .92 | 1.45 | HIM | 27 | .44 | .64 | .57 |
| HID | 25 | 1.28 | .74 | $p < .16$ | HID | 21 | .76 | .83 | $p < .24$ | HID | 34 | .47 | .66 | $p < .57$ |
| LO | 32 | 1.03 | .74 | | LO | 33 | .64 | .74 | | LO | 21 | .29 | .64 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 64: ANALYSIS OF AFFINITY TEST (2 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 57 | 1.12 | .71 | 1.13 | .94 | | HIM | HID |
| HID | 61 | 1.02 | .78 | 1.01 | P < .39 | HID | NS | |
| LO | 87 | 1.17 | .70 | 1.18 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 25 | 1.08 | .99 | 1.24 | 1 | 29 | 1.17 | 1.18 | 2.65 | 1 | 25 | 1.32 | 1.32 | 2.05 |
| 2 | 32 | 1.16 | 1.23 | P < .27 | 2 | 32 | .88 | .87 | P < .11 | 2 | 32 | 1.28 | 1.26 | P < .14 |
| | | | | | | | | | | 3 | 30 | .93 | .96 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 21 | 1.10 | .70 | .39 | HIM | 18 | 1.22 | .73 | 2.94 | HIM | 18 | 1.06 | .73 | .61 |
| HID | 25 | 1.20 | .71 | P < .68 | HID | 17 | .76 | .83 | P < .06 | HID | 19 | .95 | .78 | P < .55 |
| LO | 28 | 1.04 | .64 | | LO | 27 | 1.30 | .67 | | LO | 32 | 1.19 | .78 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | .05 | LO | NS | NS |

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THEIR ANSWERS HAVING BEEN RECORDED, THEY WERE DIRECTED TO DEPICT THE PROBLEMS IN PICTURE FORM, AFTER WHICH THEY WERE ASKED IF THEY WISHED TO CHANGE THEIR ANSWERS. FINALLY, THEY WERE ASKED IF THEY HAD BEEN AWARE OF ANY "PICTURES" IN THEIR HEADS DURING SOLUTION AND THE EXTENT TO WHICH THEIR DRAWINGS ILLUSTRATED THESE MENTAL IMAGES.

RESULTS AS TABLED ON THE FOLLOWING PAGES SUPPORT A NUMBER OF CONCLUSIONS. FIRST, THE STUDENTS FROM HIM AND HID APPARENTLY PREFERRED HIGH-IMAGERY TREATMENT OF TRANSLATION TO LOW-IMAGERY TREATMENT. THAT IS, EVEN THOUGH ALL TREATMENT GROUPS WORKED THE SAME PROBLEMS, ONLY THE STUDENTS FROM HIM AND HID CONSISTENTLY REPORTED PROBLEM PREFERENCE, AND THEIR CHOICES WERE RESTRICTED TO NUMBER, COIN, AND AGE FOR THE MOST PART. THEREFORE, PREFERENCE MORE LIKELY PERTAINED TO TREATMENT AS APPLIED DIFFERENTIALLY TO THESE PROBLEMS THAN TO THE PROBLEMS THEMSELVES. SECOND, THE BETTER TRANSLATORS FROM HIM AND HID WERE PRONE TO VISUALIZE FOR COIN AS COMPARED TO THOSE FROM LO. RARELY, HOWEVER, WERE VISUALIZATIONS IN TERMS OF STACKS. INSTEAD, THEY TYPICALLY IMAGED WITH RESPECT TO MORE CONCRETE REPRESENTATIONS (E.G., JAR FULL OF COINS). THIRD, STUDENTS WHO TRANSLATED INCORRECTLY TYPICALLY IMAGED DURING SOLUTION. HOWEVER, CORRECT SOLUTIONS WERE JUST AS OFTEN ACCOMPANIED WITH IMAGERY AS NOT. FOURTH, CORRECT TRANSLATIONS WERE LESS LIKELY TO BE CHANGED AFTER PICTORIAL REPRESENTATION THAN INCORRECT TRANSLATIONS. FINALLY, THE STUDENTS INTERVIEWED SEEMED NOT TO RELATE NOTATIONAL AND PICTORIAL REPRESENTATIONS OF PROBLEMS. IN THE "BRIDGE" PROBLEM, FOR EXAMPLE, MOST STUDENTS PICTURED THE BRIDGE AS RESTING ON LAND AND SPANNING THE ENTIRE RIVER. NONETHELESS, ELEVEN STUDENTS RESPONDED WITH A BRIDGE LENGTH SHORTER THAN THE WIDTH OF THE RIVER. THUS THE HEURISTIC VALUE OF PICTORIAL REPRESENTATION OF PROBLEMS SEEMS QUESTIONABLE.

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TREATMENT GROUP HIM, GRADE 7

| | NUMBER MISSED ON FINAL PROBLEMS TEST | PROBLEM TYPE PREFERENCE | COIN PROBLEM: EQUATIONS | COIN PROBLEM: DESCRIPTION OF DRAWING | COIN PROBLEM: ANSWER CORRECT? | COIN PROBLEM: IMAGE SIMILAR TO DRAWING? | BRIDGE PROBLEM: NUMERICAL RESPONSE | BRIDGE PROBLEM: DESCRIPTION OF DRAWING | BRIDGE PROBLEM: ANSWER CORRECT? | BRIDGE PROBLEM: IMAGE SIMILAR TO DRAWING? |
|---|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | 4 | COIN | D+Q=25<br>10D+25Q=415 | JAR OF 25 COINS | YES | YES | $720\frac{5}{18}$ | BRIDGE WITH FOUNDATION ON SHORES | NO, BUT NO CHANGE | YES |
| $S_2$ | 5 | COIN | D+Q=25<br>D10+Q25=415 | TWO LABELED STACKS | YES | NO IMAGE | $720\frac{1}{54}$ | BRIDGE WITH FOUNDATION ON SHORES | NO, SHOULD BE $720\frac{5}{18}$ | YES |
| $S_3$ | 5 | AGE | D+Q=25<br>10D+25Q=415 | STRING OF 25 DIMES AND QUARTERS | YES | YES | $\frac{5}{18}$ | BRIDGE WITH FOUNDATION ON SHORES | NO, BUT NO CHANGE | YES |
| $S_4$ | 13 | AGE | D+Q=415 | TWO LABELED STACKS | NO, INCLUDED D+Q=25 | NO, "SAW" JAR FULL OF COINS | 720 | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |
| $S_5$ | 13 | NONE | DQ=415<br>D=15, Q=10 | JAR OF 25 COINS | YES | YES | 120 | BRIDGE WITH FOUNDATION IN RIVER | NO, SHOULD BE $\frac{5}{18}$ | YES |
| $S_6$ | 13 | MIXTURE | DQ=415<br>DQ=25 | JAR OF 25 COINS | NO, SHOULD BE D+Q=415 | YES | 200 | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |

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TREATMENT GROUP HID, GRADE 7

| | NUMBER MISSED ON FINAL PROBLEMS TEST | PROBLEM TYPE PREFERENCE | COIN PROBLEM: EQUATIONS | COIN PROBLEM: DESCRIPTION OF DRAWING | COIN PROBLEM: ANSWER CORRECT? | COIN PROBLEM: IMAGE SIMILAR TO DRAWING? | BRIDGE PROBLEM: NUMERICAL RESPONSE | BRIDGE PROBLEM: DESCRIPTION OF DRAWING | BRIDGE PROBLEM: ANSWER CORRECT? | BRIDGE PROBLEM: IMAGE SIMILAR TO DRAWING? |
|---|---|---|---|---|---|---|---|---|---|---|
| $S_1$ | 4 | AGE | D+Q=25<br>D10+Q25=415 | JAR OF 25 COINS | NO, SUBSTITUTED D+Q=415 FOR D+Q=25 | YES | 920 | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |
| $S_2$ | 4 | NONE | 10D+25Q=415<br>D+Q=25 | TWO LABELED STACKS | YES | NO IMAGE | 200 | BRIDGE WITH FOUNDATION ON SHORES | NO, SHOULD BE 520 | YES |
| $S_3$ | 4 | AGE | 10D+25Q=415<br>D+Q=25 | TWO LABELED STACKS | YES | NO IMAGE | 920 | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |
| $S_4$ | 14 | NUMBER | D+Q=415 | TWO LABELED STACKS | YES | YES | 200 | FOUNDATION OF BRIDGE ONLY (ON SHORES) | YES | YES |
| $S_5$ | 13 | AGE | D10+Q25=415<br>D+Q=25 | TWO LABELED STACKS | YES | YES | $120\frac{1}{9}$ | BRIDGE WITH FOUNDATION ON SHORES | NO, SHOULD BE 200 | YES |
| $S_6$ | 13 | MIXTURE | D+Q=$4.15<br>D10+Q25=$4.15 | TWO LABELED STACKS | YES | NO IMAGE | $\frac{1}{54}$ | BRIDGE WITH FOUNDATION ON SHORES | NO, PERHAPS $720\frac{15}{54}$ | YES |

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TREATMENT GROUP LO, GRADE 7

| | NUMBER MISSED ON FINAL PROBLEMS TEST | PROBLEM TYPE PREFERENCE | COIN PROBLEM | | | | BRIDGE PROBLEM | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | EQUATIONS | DESCRIPTION OF DRAWING | ANSWER CORRECT? | IMAGE SIMILAR TO DRAWING? | NUMERICAL RESPONSE | DESCRIPTION OF DRAWING | ANSWER CORRECT? | IMAGE SIMILAR TO DRAWING? |
| $S_1$ | 4 | NONE | $d+q=25$<br>$10d+25q=415$ | JAR OF 25 COINS | YES | NO IMAGE | 200 | FOUNDATION OF BRIDGE ONLY (ON SHORES) | NO, BUT NO CHANGE | YES |
| $S_2$ | 4 | NONE | $d+q=415$<br>$10d+25q=25$ | JAR OF 25 COINS | NO, SHOULD BE $J=25$, $10d+25q=415$ | NO IMAGE | $\frac{5}{18}$ | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |
| $S_3$ | 4 | NONE | $10d+25q=415$<br>$d+q=25$ | TWO EQUATIONS | YES | NO IMAGE | 200 | FOUNDATION OF BRIDGE ONLY (IN RIVER) | MAYBE 920 | YES |
| $S_4$ | 14 | WORK FILL-INS | $A+B=415$ | JAR OF 25 COINS | YES | YES | $\frac{1}{9}=720$<br>$+\frac{1}{6}$ (sic) | BRIDGE WITH FOUNDATION IN RIVER | NO, SHOULD BE $\frac{1}{9}+\frac{1}{6}=720$ | NO IMAGE |
| $S_5$ | 14 | NONE | $d+q=415$<br>$415c=25$ COINS | JAR OF 25 COINS | NO, LAST EQUATION WRONG | YES | $720\frac{5}{18}$ | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |
| $S_6$ | 14 | COIN | $d+q=415$ | JAR OF COINS WITH 4 IN ALL | YES | NO IMAGE | $\frac{1}{54}$ | BRIDGE WITH FOUNDATION ON SHORES | YES | YES |

# CHAPTER V

# SUMMARY AND CONCLUSIONS

This final chapter contains a review of the problem followed by a summary of findings and conclusions. The chapter concludes with a discussion of results.

## RESTATEMENT OF THE PROBLEM

The study was formulated to investigate the effects of image-based instruction on translation of selected types of algebra word problems. The problem was to determine the effects of provisions for imagery through materials and drawings relative to achievement, subsequent learning of work problems and mixture problems, retention, transfer, and attitudes as compared to instruction relying primarily on grammatical cues for translation.

The problem was investigated using twelve seventh grade mathematics classes (N = 366) and ten ninth grade algebra classes (N = 336). Three instructional sequences were prepared for use by teachers. Of these, one referred algebraic notation to number and sequenced directly from problem statements to their mathematical expression. In contrast, the remaining two included provisions for visual representation of problems prior to symbolization. Problems were encountered in visual form before translation for the purpose of providing a basis for the formation of mental images, which in turn were to serve translation as internalized referents for notation being



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APPLIED TO PROBLEMS. FOR PURPOSE OF DISCUSSION, THEN, THE PROBLEM CAN BE CAST IN TERMS OF THESE INSTRUCTIONAL SEQUENCES (THAT IS, TREATMENTS) AS A SERIES OF QUESTIONS. SPECIFICALLY, THE PROBLEM CAN BE STATED IN THE FORM OF TEN QUESTIONS AS POSED EARLIER IN CHAPTER I. EACH OF THESE WILL NOW BE RESTATED WITH A SUMMARY OF FINDINGS AND CONCLUSIONS.

## FINDINGS AND CONCLUSIONS

QUESTION 1: WHICH TREATMENT IS THE MOST EFFECTIVE WITH RESPECT TO ACHIEVEMENT ON NUMBER, COIN, AND AGE PROBLEMS?

FINDINGS: ACHIEVEMENT ON NUMBER, COIN, AND AGE PROBLEMS WAS TESTED FOR AT THE END OF STAGE I AND AGAIN AT THE END OF STAGE II. FOR THE FIRST OR INITIAL PROBLEMS TEST, RESULTS SIGNIFICANTLY FAVORED LO AMONG SEVENTH GRADERS. IN CONTRAST, TREATMENT GROUPS AT THE NINTH GRADE PRODUCED NEAR EQUAL RESULTS. THERE WERE, HOWEVER, NEAR SIGNIFICANT WITHIN-TREATMENT DIFFERENCES FOR HIM AND LO AT GRADE SEVEN AND SIGNIFICANT WITHIN-TREATMENT DIFFERENCES FOR HID AND LO AT GRADE NINE.

FOR THE SECOND OR FINAL PROBLEMS TEST, RESULTS FOR HIM AND LO ON THE NUMBER-COIN-AGE ITEMS WERE ROUGHLY COMPARABLE AMONG SEVENTH GRADERS, WITH HID FALLING CONSIDERABLY SHORT OF THESE TWO AT .01 AND .05 LEVELS OF SIGNIFICANCE, RESPECTIVELY. THIS PATTERN WAS REPEATED AT THE NINTH GRADE; HOWEVER, DIFFERENCES WERE NOT STATISTICALLY SIGNIFICANT.

IN ADDITION TO ANALYSIS OF COMBINED SCORES FOR NUMBER, COIN, AND AGE FOR THE FINAL PROBLEMS TEST, PART SCORES PERTAINING TO SPECIFIC PROBLEM TYPES WERE EXAMINED. FOR COIN AND AGE PROBLEMS TREATMENT GROUPS AT BOTH GRADES PERFORMED COMPARABLY. FOR NUMBER PROBLEMS AT GRADE SEVEN, PERFORMANCE OF STUDENTS IN HIM WAS SIGNIFICANTLY SUPERIOR TO PERFORMANCE OF THOSE IN LO, WHICH IN TURN WAS SIGNIFICANTLY SUPERIOR TO PERFORMANCE OF THOSE IN HID. FOR NUMBER PROBLEMS AT GRADE NINE, RESULTS FAVORED LO; HOWEVER, STATISTICAL SIGNIFICANCE WAS RE-

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stricted to HID and LO.

Conclusions: Insofar as initial achievement is concerned, the low-imagery sequence was most effective among seventh graders. However, the superiority of LO to HIM and HID at this grade cannot be attributed entirely to treatment since students in LO had just completed a two-day review prior to taking the Initial Problems Test. For ninth graders no conclusion can be made.

For post-treatment achievement as measured by the Final Problems Test, HIM and LO appeared equally effective at both grades, except that HIM was more productive with respect to number problems at grade seven. The drawings treatment was particularly ineffective among seventh graders, as was the case to a lesser extent among ninth graders.

Question 2: Which treatment is the most effective in preparing for subsequent learning of work and Mixture problems?

Findings: Achievement on work and mixture problems was measured at the end of Stage II as part of the Final Problems Test. For work problems seventh grade performance favored high-imagery treatment to low, but differences were not statistically significant. In sharp contrast, ninth grade performance for students in LO was more than twice that of students in either HIM or HID at the .001 level of significance.

For mixture problems seventh grade students in HIM and LO performed on a statistically par basis at a significantly higher level than students in HID. At the ninth grade students in HIM did considerably better than those in either HID or LO, but significance was restricted to HIM and LO.

Conclusions: At the seventh grade HIM was consistently associated with superior achievement on work and mixture problems. For seventh graders, then, material representation of number, coin, and age problems was most productive in preparing for learning to translate work problems and mixture

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problems.

At the ninth grade no one treatment resulted in greatest performance on both work problems and mixture problems. On the one hand, low-imagery instruction received initially was clearly most productive with respect to subsequent learning of work. On the other hand, high-imagery instruction was most productive in this respect for mixture. For this grade, then, a propitious blend of high- and low-imagery instruction received initially would have most likely effected greatest achievement with respect to subsequent learning of work and mixture problems.

Question 3: Which treatment tends toward greater retention?

Findings: Retention of number, coin, age, work, and mixture problems was measured at the end of Stage III with the Problems Retention Test. For this test adjusted means for all three treatment groups for each grade comprised ranges easily within the realm of chance. There were, however, significant within-treatment differences for HIM at grade seven and for HID at grades seven and nine.

In comparing retention test scores to achievement on the Final Problems Test, it was noted that performance among seventh graders in HID and LO was greater on the retention test. This was also the case for HID at grade nine.

Conclusions: Although treatment groups at each grade performed comparably with respect to retention, the effect of treatment on this variable is perhaps misrepresented as suggested by the considerable within-treatment differences for three of the treatment groups. Quite possibly, the varied topics teachers pursued during the twelve-day, non-treatment period prior to the retention test influenced achievement on this test more than varying treatment during Stage I was able to. Accordingly, no conclusion can be made with respect to retention with reference to particular treatment for

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EITHER GRADE.

AS FOR THE GAINS RECORDED FOR THREE OF THE TREATMENT GROUPS, IT SEEMS THAT TRANSLATIVE SKILL IS NOT ONLY RESISTANT TO FORGETTING BUT IS POSSIBLY REINFORCED THROUGH DAILY EVENTS ONCE STUDENTS HAVE REACHED AN INITIAL STATE OF PERCEPTIVENESS TO QUANTITATIVE RELATIONSHIPS. THIS BEING THE CASE, CURRICULUM DEVOTED TO THE DEVELOPMENT OF TRANSLATIVE SKILL WOULD SEEM WELL SPENT.

QUESTION 4: WHICH TREATMENT IS MOST PRODUCTIVE IN WAY OF TRANSFER?

FINDINGS: TRANSFER WAS MEASURED WITH RESPECT TO FIVE PROBLEMS ACCOMPANYING THE FINAL PROBLEMS TEST. RESULTS AT GRADE SEVEN FAVORED HIM, BUT THERE WERE SIGNIFICANT WITHIN-TREATMENT DIFFERENCES FOR THIS TREATMENT GROUP. FOR GRADE NINE, RESULTS FAVORED HIM TO THE POINT OF SIGNIFICANCE WITH RESPECT TO HID AND LO.

CONCLUSIONS: HIM APPEARED MOST GENERATIVE WITH RESPECT TO APPLICATION OF PREVIOUS LEARNINGS.

QUESTION 5: WHICH TREATMENT RESULTS IN MOST FAVORABLE ATTITUDE TOWARD MATHEMATICS?

FINDINGS: A MEASURE OF ATTITUDE WAS OBTAINED THROUGH ADMINISTRATION OF THE AIKEN AND DREGER (1961) MATHEMATICS OPINIONNAIRE. IN ADDITION TO SURVEYING TREATMENT GROUPS, THIS INSTRUMENT WAS GIVEN TO THREE CLASSES OF SEVENTH GRADERS AND THREE CLASSES OF NINTH GRADERS NOT UNDER TREATMENT. THESE HAVE BEEN REFERENCED BY CON FOR CONTROL.

AT BOTH GRADES HIM AND LO WERE ASSOCIATED WITH MORE FAVORABLE ATTITUDES. AMONG SEVENTH GRADERS, RATINGS BY TREATMENT GROUPS HIM AND LO DIFFERED SIGNIFICANTLY WITH RESPECT TO CON. AT GRADE NINE SIGNIFICANT DISPARITY IN THEIR FAVOR OCCURRED WITH RESPECT TO HID. THERE WAS, HOWEVER, CONSIDERABLE VARIANCE BETWEEN CLASSES FOR CERTAIN TREATMENTS. AT GRADE SEVEN

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VARIABILITY WITHIN HIM AND HID ATTAINED NEAR SIGNIFICANT PROPORTIONS, AND AT GRADE NINE VARIABILITY WITHIN HID WAS SIGNIFICANT.

CONCLUSIONS: SINCE CLASSES COMPRISING TREATMENTS DIFFERED CONSIDERABLY IN THEIR SCORES FOR ATTITUDE, FACTORS ASIDE FROM TREATMENT WERE PERHAPS MORE OPERATIVE WITH RESPECT TO THIS VARIABLE. THEREFORE, IN THE ABSENCE OF ADEQUATE COVARIATES TO CONTROL FOR POSSIBLE BIAS, THE WRITER OFFERS THE FOLLOWING CONCLUSION WITH RESERVATION: HIM AND LO WERE MOST PRODUCTIVE OF FAVORABLE ATTITUDE TOWARD MATHEMATICS.

QUESTION 6: WHICH TREATMENT RESULTS IN GREATEST TENDENCY TO SELECT ALGEGRA WORD PROBLEMS FOR SOLUTION?

FINDINGS: STUDENTS IN CON AS WELL AS IN HIM, HID, AND LO WERE PRESENTED WITH TWO ALGEBRA WORD PROBLEMS AND TWO PROBLEMS SOLUBLE BY MANIPULATION AND ASKED TO CHOOSE TWO OF THE FOUR FOR SOLUTION. RESPONSE BY CONTROL GROUPS FOR EACH GRADE WAS ABOUT .2 WORD PROBLEMS CHOSEN. RESPONSE BY TREATMENT GROUPS FOR EACH GRADE WAS ABOUT ONE WORD PROBLEM CHOSEN, WITH THE EXCEPTION OF SEVENTH GRADE LO WHICH AVERAGED ONLY .67 IN THIS RESPECT.

THE DISPARITY BETWEEN CONTROL GROUPS AND TREATMENT GROUPS IN NUMBER OF WORD PROBLEMS CHOSEN WAS SIGNIFICANT AT THE .001 LEVEL FOR EACH GRADE. THE DISPARITY BETWEEN TREATMENT GROUPS BY GRADE WAS INSIGNIFICANT WITH THE EXCEPTION OF HIM AND LO AT GRADE SEVEN. THERE WERE, HOWEVER, SIGNIFICANT WITHIN-TREATMENT DIFFERENCES FOR HIM AT THIS GRADE.

CONCLUSIONS: COMPARED TO RESPONSE FOR STUDENTS UNDER TREATMENT, STUDENTS WITHOUT INSTRUCTION IN TRANSLATION OF ALGEBRA WORD PROBLEMS WERE MUCH LESS LIKELY TO CHOOSE THEM FOR SOLUTION. THEREFORE, IF SOLUTION OF VERBAL PROBLEMS IS DESIRED BEHAVIOR, CURRICULUM MUST BE DEVOTED TO THEIR SOLUTION. WHETHER THAT INSTRUCTION BE HIM, HID, OR LO, HOWEVER, SEEMS TO BE OF LITTLE CONSEQUENCE WITH RESPECT TO EFFECTING PARTICULAR TENDENCY TO SOLVE WORD PROBLEMS.

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QUESTION 7: DO STUDENTS CATEGORIZED ON THE BASIS OF PRIOR ACHIEVEMENT IN ARITHMETIC PERFORM DIFFERENTIALLY WITH RESPECT TO PARTICULAR TREATMENT?

FINDINGS: THROUGHOUT THE STUDY THE FOLLOWING TESTS WERE GIVEN: INITIAL PROBLEMS TEST, BATTERY 1 (FINAL PROBLEMS TEST, TRANSFER ITEMS, CONTRADICTION PROBLEM), BATTERY 2 (ATTITUDE, AFFINITY, AUXILIARY ASSUMPTIONS QUESTIONNAIRE), PROBLEMS RETENTION TEST. FOR EACH, WHETHER WITH RESPECT TO TOTAL SCORES OR SUBSCORES, ANALYSIS INCLUDED EXAMINATION OF TREATMENT EFFECTS FOR STUDENTS STRATIFIED ON THE BASIS OF ARITHMETIC ACHIEVEMENT AS UPPER 1/3, MIDDLE 1/3, AND LOWER 1/3. IN ALL, THEN, STUDENT PERFORMANCE BY ACHIEVEMENT LEVEL BY TREATMENT GROUP (THAT IS, BY TREATMENT SUBGROUP) WAS ANALYZED WITH RESPECT TO NINETEEN VARIABLES. RESULTS IN THE FORM OF SIGNIFICANTLY FAVORED TREATMENT SUBGROUPS FOR EACH VARIABLE ARE RECORDED ON THE FOLLOWING TWO PAGES.

CONCLUSIONS: AT THE SEVENTH GRADE, STUDENTS IN THE UPPER 1/3 ACHIEVED ABOUT THE SAME REGARDLESS OF TREATMENT ASSIGNMENT, WHEREAS THOSE IN THE MIDDLE AND LOWER STRATA ACHIEVED DIFFERENTIALLY IN THIS RESPECT. IN EACH OF THE SIX INSTANCES WHERE SIGNIFICANCE OBTAINED WITHIN THE MIDDLE 1/3, HID WAS INFERIOR, AND IN FOUR OF THE SIX INSTANCES WHERE SIGNIFICANCE OBTAINED WITHIN THE LOWER 1/3, HIM WAS SUPERIOR TO BOTH HID AND LO. FOR SEVENTH GRADERS, THEN, HID WAS PARTICULARLY INEFFECTUAL FOR AVERAGE ACHIEVERS IN ARITHMETIC, AND HIM WAS PARTICULARLY WELL SUITED TO THE LOW ACHIEVER.

AT THE NINTH GRADE THE UPPER AND LOWER ACHIEVEMENT STRATA WERE INFLUENCED MOST BY TREATMENT ASSIGNMENT. IN THE EVENT OF SIGNIFICANCE AMONG THE UPPER 1/3, LO WAS FAVORED. IN THE SIX INSTANCES OF SIGNIFICANCE AMONG THE LOWER 1/3, EITHER HID OR LO WAS INFERIOR. PUTTING THIS ANOTHER WAY, HIM WAS AT LEAST AS EFFECTIVE AS EITHER HID OR LO AMONG THE LOWER 1/3. FOR NINTH GRADERS, THEN, LO WAS MOST PRODUCTIVE AMONG HIGH ACHIEVERS IN ARITHMETIC, AND HIM WAS MOST PRODUCTIVE AMONG AVERAGE AND LOW ACHIEVERS.

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SIGNIFICANTLY SUPERIOR TREATMENT SUBGROUPS BY VARIABLE, GRADE 7

| VARIABLE | UPPER 1/3 | MIDDLE 1/3 | LOWER 1/3 |
|---|---|---|---|
| INITIAL PROBLEMS TEST | | LO/HIM* | |
| FINAL PROBLEMS TEST | | HIM/HID, LO/HID | HIM/HID, HIM/LO |
| NUMBER, COIN, AGE (COMBINED) | | | HIM/HID |
| WORK AND MIXTURE (COMBINED) | | HIM/HID, LO/HID | HIM/HID, HIM/LO |
| $NUMBER_{FPT}$ | | HIM/HID | HIM/HID, HIM/LO |
| $COIN_{FPT}$ | | | |
| $AGE_{FPT}$ | | | |
| $WORK_{FPT}$ | | | |
| $MIXTURE_{FPT}$ | HIM/HID, LO/HID | | HIM/HID, HIM/LO |
| TRANSFER | | | |
| ATTITUDE | LO/HIM, LO/HID | | |
| AFFINITY | | | |
| AUXILIARY REPRESENTATIONS QUESTIONNAIRE | | HIM/HID, LO/HID | |
| $PHRASE_{ARQ}$ | | HIM/HID, LO/HID | |
| $SENTENCE_{ARQ}$ | | | |
| $COIN_{ARQ}$ | | | |
| $AGE_{ARQ}$ | | | LO/HIM |
| $WORK_{ARQ}$ | | | |
| $MIXTURE_{ARQ}$ | | | |
| PROBLEMS RETENTION TEST | | | |

*LO SUPERIOR TO HIM AT THE .05 LEVEL OR BETTER

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SIGNIFICANTLY SUPERIOR TREATMENT SUBGROUPS BY VARIABLE, GRADE 9

| VARIABLE | UPPER 1/3 | MIDDLE 1/3 | LOWER 1/3 |
|---|---|---|---|
| INITIAL PROBLEMS TEST | | | |
| FINAL PROBLEMS TEST | LO/HIM,* LO/HID | | |
| NUMBER, COIN, AGE (COMBINED) | | | |
| WORK AND MIXTURE (COMBINED) | | | |
| $NUMBER_{FPT}$ | | | LO/HID |
| $COIN_{FPT}$ | | | HIM/HID, HIM/LO |
| $AGE_{FPT}$ | | | |
| $WORK_{FPT}$ | LO/HIM, LO/HID | | LO/HID |
| $MIXTURE_{FPT}$ | | HIM/HID, HIM/LO | |
| TRANSFER | | HIM/HID | |
| ATTITUDE | LO/HID | | |
| AFFINITY | | | |
| AUXILIARY REPRESENTATIONS QUESTIONNAIRE | | | HIM/HID |
| $PHRASE_{ARQ}$ | | | |
| $SENTENCE_{ARQ}$ | | | HIM/HID |
| $COIN_{ARQ}$ | | | |
| $AGE_{ARQ}$ | | | |
| $WORK_{ARQ}$ | | | HIM/LO |
| $MIXTURE_{ARQ}$ | | | |
| PROBLEMS RETENTION TEST | | | |

*LO SUPERIOR TO HIM AT THE .05 LEVEL OR BETTER

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QUESTION 8: DOES IMAGE-BASED INSTRUCTION RESULT IN GREATER TENDENCY TO ATTEND NOT ONLY TO THE VERBAL DESCRIPTION OF A SITUATION BUT TO ITS PHYSICAL CHARACTERISTICS AS WELL?

FINDINGS: AS PART OF BATTERY 1 STUDENTS WERE PRESENTED WITH A PROBLEM THAT DESCRIBED PHYSICALLY IMPOSSIBLE CIRCUMSTANCES. ALONG WITH THE DIRECTIVE TO TRANSLATE THEY WERE ADVISED TO COMMENT IN THE MARGIN ON THIS OR ANY OF THE FIVE TRANSFER PROBLEMS THAT PRECEDED IT. THEIR PAPERS WERE THEN EXAMINED TO SEE IF ANY HAD RECOGNIZED THE INCONGRUITY OF THE SITUATION SURROUNDING THE PROBLEM STATEMENT.

FOR THE MOST PART, STUDENTS AT BOTH GRADES HAD MERELY TRANSLATED. FOR GRADE SEVEN FIFTEEN STUDENTS HAD DETECTED THE CONTRADICTION, APPORTIONED AMONG TREATMENT GROUPS AS FOLLOWS: HIM-7, HID-3, LO-5. FOR GRADE NINE THE NUMBER WAS THIRTEEN: HIM-3, HID-9, LO-1. DISPARITY OF RESULTS FOR GRADE NINE WAS PARTICULARLY STRIKING IN VIEW OF THE SUBSTANTIALLY LARGER STUDENT POPULATION COMPRISING LO: HIM (N = 67), HID (N = 66), LO (N = 99).

CONCLUSIONS: IN FACE OF A PROBLEM THAT REPRESENTS PHYSICALLY IMPOSSIBLE CIRCUMSTANCES, THREE RESPONSES ARE POSSIBLE: (1) A SET OF EQUATIONS DESCRIBING THE PROBLEM LITERALLY, (2) A SET OF EQUATIONS DESCRIBING A RELATED, PHYSICALLY POSSIBLE SITUATION, OR (3) THE PHYSICAL INCONGRUITY OF THE SITUATION CAN BE RECOGNIZED. AS CONCLUDED IN A STUDY BY PAIGE AND SIMON (1966), STUDENTS WHO CHOOSE THE THIRD RESPONSE, THAT IS, WHO DETECT THE CONTRADICTORY CIRCUMSTANCES, ARE MOST LIKELY ATTENDING TO A PHYSICAL REPRESENTATION OF THE PROBLEM AS WELL AS TO VERBAL CUES. CORRESPONDINGLY, THE WRITER CONCLUDED THAT AMONG NINTH GRADERS HIGH-IMAGERY INSTRUCTION AS PRESENTED IN HID RESULTED IN GREATER ATTENTIVENESS TO PROBLEMS AS REPRESENTED AS WELL AS TO PROBLEMS AS DESCRIBED. EVEN SO, THE NUMBER OF STUDENTS DETECTING THE CONTRADICTION BY TREATMENT GROUP WAS LOW IN COMPARISON TO THE TOTAL NUMBER OF STUDENTS FOR THAT TREATMENT: HIM (4%), HID (14%), LO (1%).

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Question 9: To what extent does skill in translating phrases determine success in sentence translation?

Findings: Control groups and treatment groups alike were presented with nine phrases and three sentences to translate as part of the Auxiliary Representations Questionnaire. For the phrases control groups at each grade achieved nearly 80% as well as their respective treatment groups. For the sentences control group means compared less favorably with treatment group means: among seventh graders the figure was about 20%, among ninth graders about 50%.

Conclusions: Although phrase translation is necessary for sentence translation, it seems hardly sufficient. Nor is this surprising. Sentences involve not only phrases but considerable syntax as well. Thus sentence translation requires not only the mathematical expression of phrases but of the way in which the phrases are related.

Question 10: Can it be expected that the study of algebra word problems will result in increased understanding of certain aspects of our environment?

Findings: Included in the Auxiliary Representations Questionnaire were a number of questions pertaining to coin, age, work, and mixture. In general, answers to these questions were not dependent upon translative skill. Rather they required an awareness of the circumstantial nature of the topics under question.

Among seventh graders, previous instruction in translation, regardless of treatment assignment, was of little consequence with respect to selection of correct response. Only for the mixture questions was superior performance linked to treatment ($p < .05$ or better). For the coin and age questions the control group performed in keeping with the treatment groups, and for the work questions significance with respect to treatment versus non-treatment was restricted to L0.

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Among ninth graders, treatment had more impact. Correct response for the coin and mixture portion of the questionnaire favored treatment to non-treatment at the .01 level or better. For the age questions significance levels were at the .001 level in favor of treatment, except for HID which failed to attain significance with respect to CON. For the work questions HIM and HID were associated with best performance, but significance was restricted to HIM and CON.

Conclusions: It seems that effort devoted to the study of coin, age, work, and mixture through the medium of algebra word problems leads to greater facility in dealing with these topics in a general setting. This statement is qualified, however, with respect to grade level, problem type, and treatment received.

Treatment groups at the seventh grade were apparently reading more into the mixture questions than was the control group for this grade, but this was not the case for the number, coin, and age questions. At the ninth grade, treatment groups were clearly at an advantage with respect to all questions, but significance between treatment and non-treatment throughout was restricted to HIM.

## DISCUSSION

The facts having been reported, conclusions stated, and conjectures made, this much seems clear: no striking advantage accrued to image-based instruction at either grade. Except for the general beneficence of HIM among low achievers in arithmetic, performance of students in LO was typically comparable to performance of students in HIM, and performance of students in HID was substandard throughout.

To account for the near equal results for HIM and LO and the inferior results for HID, the writer conjectured in Chapter IV that the major

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DETERMINANT OF SUCCESS WAS ATTENDANCE TO VERBAL DETAIL. ACCORDINGLY, STUDENTS UNDER HIGH IMAGERY WERE ACCREDITED WITH TRANSLATING DIRECTLY JUST AS THOSE IN LO, AND VISUAL REPRESENTATION OF PROBLEMS AS IN HIM AND HID WAS BELIEVED TO HAVE HAD AT MOST AN INDIRECT EFFECT ON TRANSLATION.

IN THE WRITER'S OPINION, IF THERE IS ANY ONE REASON FOR THE HIGH-IMAGERY SEQUENCES EFFECTING LITTLE MORE THAN TRADITIONAL SUCCESS, IT IS THAT THEIR DESIGN PLACED TOO MUCH FAITH ON STUDENTS BEING ABLE TO RELATE ALGEBRAIC NOTATION TO TWO STACKS PURELY ON THE BASIS OF EFFORT DEVOTED TO THEIR DERIVATION. AT THE OUTSET OF THE STUDY IT WAS CONJECTURED THAT THROUGH TRANSLATING PROBLEMS AT THE ENACTIVE OR ICONIC LEVEL, THEIR TRANSLATION IN SYMBOL FORM WOULD AMOUNT TO LITTLE MORE THAN THE ACT OF DESCRIBING WHAT HAD GONE ON BEFORE, A TASK COMPARABLE TO RELAYING YESTERDAYS EXPERIENCES IN VERBAL FORM. HOWEVER, TO ASSUME THAT ENACTIVE AND ICONIC REPRESENTATION OF PROBLEMS FOSTERS SUCH A DESCRIPTIVE PROCESS IS TO ASSUME THAT THE APPROPRIATE SYMBOLISM AND MATHEMATICAL MEANING IS APPARENT WITHIN SUCH REPRESENTATIONS. FOR IF IT IS NOT, THEN MATHEMATICS WOULD HARDLY SUGGEST ITSELF AS THE MEDIUM FOR DESCRIPTION.

WITH THE ADVANTAGE OF HINDSIGHT THE WRITER WOULD HAVE GIVEN MORE ATTENTION IN THE HIGH-IMAGERY SEQUENCES TO RELATING THE MODEL TO THE NOTATION WHICH IT WAS DESIGNED TO REPRESENT. FOR EXAMPLE, ALONG WITH LABELING THE STACKS ALGEBRAICALLY, QUESTIONS SUCH AS THE FOLLOWING COULD HAVE BEEN POSED: CAN YOU EQUATE THE SMALLER TO THE TALLER USING ADDITION? MULTIPLICATION? ADDITION AND MULTIPLICATION? PERHAPS THROUGH ADROIT QUESTIONING, RESULTS WOULD HAVE MERITED THE CONCLUSION THAT IMAGES, RATHER THAN WORDS, WERE COMMONLY EMPLOYED AS NOTATIONAL REFERENTS.

IF THE EFFECTIVENESS OF EITHER ENACTIVE OR ICONIC REPRESENTATION IS INDEED DEPENDENT ON THE EXTENT TO WHICH STUDENTS PERCEIVE THESE REPRESENTATIONS AS MATHEMATICS IN ALTERED FORM, THEN FUTURE STUDIES ON IMAGE-BASED

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INSTRUCTION WOULD DO WELL TO STRESS THE CONNECTION BETWEEN WHATEVER MODEL IS EMPLOYED AND THE MATHEMATICS IT IS INTENDED TO REPRESENT. IT IS ONE THING TO SAY THAT STUDENTS ARE EXHIBITING MATHEMATICS IN WORKING WITH A VISUAL MODEL; IT IS QUITE ANOTHER TO SAY THAT SUCH MATHEMATICS IS DESCRIPTIVE OF WHAT IS GOING ON IN THEIR HEADS DURING EXHIBITION.

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# APPENDIX A

INSTRUCTIONAL MATERIALS FOR NUMBER, COIN, AND AGE PROBLEMS: HIM.*

THE UNITS ARE LABELED N, C, A FOR NUMBER, COIN, AND AGE, RESPECTIVELY. THE ORDER OF PRESENTATION FOR EACH UNIT IS INDICATED BY SUBSCRIPTS: DAY 1, $N_1$; DAY 2, $N_2$; DAY 3, $C_1$; DAY 4, $C_2$; DAY 5, $C_3$; DAY 6, $A_1$; DAY 7, $A_2$; DAY 8, $A_3$. AN OVERHEAD WAS REQUIRED FOR ALL EXCEPT $C_3$ AND $A_3$.

---

*HID CAN BE RECONSTRUCTED FROM HIM BY SUBSTITUTING "DRAW" OR "CHOOSE" FOR "CONSTRUCT."



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NAME_____ TEACHER_____ $N_1$

## INTRODUCTION

For the next few days you will be learning to translate stated mathematical relationships into mathematical symbols, a system of equations. In arithmetic you exchanged numbers and operations such as plus or times for words in story problems, and the ability to describe verbal conditions mathematically becomes increasingly important in science, more advanced mathematics, and daily events. Therefore, the instruction that follows is aimed toward increasing your translative skill.

The setting for this endeavor will be algebra word problems. These mathematically relate concepts such as number, value, and age. Algebra word problems usually include a question requiring a numerical answer (e.g., What are the two numbers?). In general, the answer to such a question derives from a system of equations that mathematically describes the relationships stated in the problem. Thus solving algebra word problems may be viewed as a two-step process: (1) translation, and (2) solution of equations.

An examination of computer programs for solving algebra word problems quickly reveals the difficulty machines experience with the first step, even though their speed and accuracy with the second is without equal. Thus the ability to move from words to mathematical symbols seems peculiarly human. Therefore, of the two parts of the solution process for algebra word problems, only translation will concern us for the present. The solution of equations, being somewhat mechanical and better suited to machines, is a topic for another day.

## NUMBER PROBLEMS

The verbage you will be working with today relates numbers. Only a few words and phrases are commonly used to express relationships between numbers (e.g., "sum," "difference," "product," "quotient," "more than," "less than," "increased by," "decreased by," and so on), and they serve as keys to translation. Since they appear again and again in all types of algebra word problems, their meaning and algebraic equivalents will be given special attention at this time.

The following examples give some indication of the many ways mathematical symbols can be interpreted verbally. In each case the mathematical expression contains either an addition (+), subtraction (-), multiplication (x), or division (÷) sign; yet each has a number of corresponding combinations of words. Your task, given one of the phrases on the next page, is to express it algebraically using one or more of the four operations: +, -, x, ÷.

| WORDS | SYMBOLS |
|---|---|
| The sum of a and b, b more than a, a increased by b, exceeds a by b | $a + b$ |
| The difference between m and n, m less n, n less than m, m decreased by n | $m - n$ |
| The product of p and q, p times q | $pq$ |
| The quotient of r and s, r divided by s | $r \div s$ |

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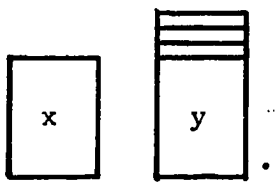
Express the following phrases in algebraic symbols:

1. The sum of a and b _____
2. 5 more than a _____
3. a more than 5 _____
4. a increased by 3 _____
5. 3 increased by a _____
6. a exceeded by 7 _____
7. The difference between m and n _____
8. m less 4 _____
9. m less than 4 _____
10. m decreased by 18 _____
11. 18 decreased by m _____
12. The difference between m and n, added to a _____
13. The product of p and q _____
14. 17 times p _____
15. Twice q _____
16. 3 times the sum of a and b _____
17. 6 times the difference between m and n _____
18. The product of p and q, increased by 3 times the product of p and q _____
19. Twice the sum of a and b less the product of p and q _____
20. r divided by s _____

Sometimes it is difficult when translating phrases to determine what gets done to what. For example, does "x less than 7" translate as x - 7 or 7 - x? And as one begins to translate sentences into equations, the difficulty increases. For instance, given the following sentence to translate, "Three times one number is 1 more than twice another," where should one put the "+ 1" associated with the phrase "1 more than"? With 3m? With 2n?

A convenient devise for keeping track of the notation one applies to verbal statements is a pair of stacks, one taller than the other. The stacks can be marked in such a way as to represent various numbers, and by referring to their heighths, one can easily decide on the placement of indicated operations.

For example, suppose you were asked to translate "one number exceeds another by 3." Letting a rod count as 1, this relationship between two numbers can be illustrated as




And with the stacks labeled as they are you can see that 3 must be added to x in order to equalize the heights. Otherwise, the taller stack, y, would be made even taller. Thus you write x + 3 = y.

(Insert $N_{1_1}$)

1. _____ (eq. )
2. _____ (eq. )
3. _____ (eq. )
4. _____ (eq. )
5. _____ (eq. )

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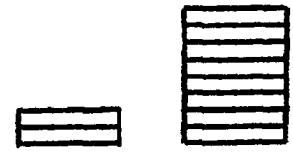
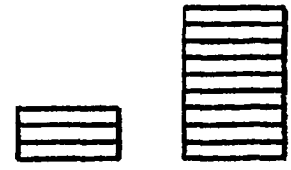
INSERT $N_{1_1}$

ASK AND ILLUSTRATE, "HOW DO THE FOLLOWING STACKS RELATE TO EACH OTHER IN TERMS OF THE PHRASES YOU HAVE JUST TRANSLATED?"



(HAVE YOUR STUDENTS LIST THEIR RESPONSES IN SENTENCE FORM IN THE BLANKS. POSSIBLE ANSWERS WOULD INCLUDE SENTENCES CONTAINING PHRASES SUCH AS "6 MORE THAN," "EXCEEDS BY 6," "SUM IS 10," "DIFFERENCE IS 6," "4 TIMES," AND SO ON.)

ASK AND ILLUSTRATE, "DO ANY OF THE RELATIONSHIPS YOU HAVE RECORDED APPLY TO THESE STACKS?"



(SOLICIT ANSWERS CONTAINING PHRASES SUCH AS "6 MORE THAN," "EXCEEDS BY 6," "DIFFERENCE IS 6," AND MAKE A POINT OF THE ARBITRARY NATURE OF SUCH RELATIONSHIPS. THAT IS, MAKE EXPLICIT THE NOTION THAT ANY <u>STATED MATHEMATICAL RELATIONSHIP</u> CAN BE ILLUSTRATED IN MANY WAYS. THEN SAY THAT TO INDICATE THIS WE WRITE EQUATIONS IN, SAY, M AND N, WHERE THE LETTERS STAND FOR ARBITRARY NUMBERS. FINALLY, LABEL THE STACKS M AND N, RESPECTIVELY, AND HAVE YOUR STUDENTS WRITE EQUATIONS IN M AND N CORRESPONDING TO THE SENTENCES RECORDED IN THE BLANKS (POSSIBLE ANSWERS: $M+6=N$, $M+N=10$, $N-M=6$, $4M=N$).)

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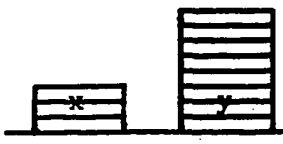
DIRECTIONS: For each sentence use the construction rods to build 2 stacks that are related visually just as the numbers in the sentences are verbally. Then, on this paper (1) draw and label a picture of your construction, and (2) write an equation in 2 unknowns that describes the relationship.

| | STACKS | CONDITION TRANSLATED |
|---|---|---|
| EXAMPLE$_1$: The sum of two numbers is 11. |   | $x + y = 11$ |
| 1. The difference between two numbers is 4. | | |
| 2. Separate 16 into two parts. | | |
| 3. The smaller of two numbers is 4 less than the larger. | | |
| 4. Three times one number is twice the other. | | |
| 5. Adding 7 to one number gives the same as adding 5 to another. | | |
| 6. One number is 4 less than three times another. | | |

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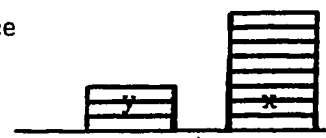
NAME_____ TEACHER_____ $N_2$

DIRECTIONS: For each problem use the construction rods to build 2 stacks that are related visually just as the numbers in the problems are verbally. Then, on this paper (1) draw and label a picture of your construction, and (2) write 2 equations in 2 unknowns that describe the relationships. There isn't time to solve each pair of equations to find the numbers, so don't worry about answering the question asked in the problems—the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

| | STACKS | RELATIONSHIPS TRANSLATED |
|---|---|---|
| EXAMPLE 1: The difference between two numbers is 5. Twice the larger exceeds five times the smaller by 1. Find the smaller number. |   | $x - y = 5$<br>$2x = 5y + 1$ |

1. The sum of two numbers is 8. If adding 7 to one of them gives the same result as adding 5 to the other, what are they? _____

2. One number is 8 more than another. Their sum is 20. What is the smaller number? _____

3. One number is 3 less than another. Three times the smaller number is the same as twice the larger. What are they? _____

4. One number exceeds another by 3. The larger number decreased by 1 equals twice the smaller. What is the smaller number? _____

5. One number increased by another is 10. If their difference is 4, what is their sum? _____

6. The difference between two numbers is 2. Three times one number is one more than twice another. What are they? _____

7. Twice one number is one less than another. Their difference is 4. What are they? _____

8. The difference between two numbers is 6. The larger number is 5 more than three times the smaller. Find the numbers. _____

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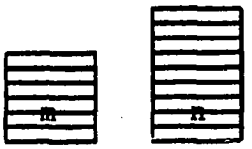
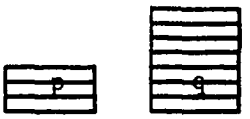
1.


$x+y=8$
$x+7=y+5$

3. 


$m=n-3$ [or $m+3=n$]
$3m=2n$

5. 


$p+q=10$
$q-p=4$

7. same as 5

$2p=q-1$
$q-p=4$

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NAME_____ TEACHER_____ $C_1$

Imagine someone with a bucketful of nickels and dimes who wants to find out how much the coins in the bucket are worth. He could take the whole mess to a bank and say to the manager, "Sir, I will give you this fine bucket if you will tell me how much all these coins are worth." Or he could weigh the nickels and dimes separately and compute their value in terms of nickel-weight and dime-weight.

An easy way to find the value of a lot of coins—without the help of a bank manager or a scale--is to first sort them in accordance with their denomination (pennies, nickels, dimes, and so on) and stack them up. Then the coins in each stack can be counted quickly, and the value of a stack can be found by multiplying the number of coins in it by the value of one of its coins. The value of all the coins is then simply the sum of the values for each stack.

For example, suppose than when the nickels and dimes from the bucket of coins mentioned earlier were sorted, stacked, and counted; there were 50 nickels in the nickel stack and 70 dimes in the dime stack. Then the value of the nickel stack would be 50 x 5 = 250 pennies, where 5 pennies is the value of a nickel; and 70 x 10 = 700 pennies would be the value of the dime stack, where 10 pennies is the value of a dime. The total value of the coins would then be 250 + 700 = 950 pennies, or $9.50.

Coin problems in algebra are also fairly easy to solve if one begins by mentally sorting and stacking the coins involved and computes total value as the sum of values of stacked coins. However, before one can add the values of coins of the same denomination he must know how to find the value of a single stack of coins. Whenever the number of coins in a stack is known, as in the example, this is easy. Just multiply. Whenever the number of coins is unknown, as is usually the case in coin problems, it is even easier (though more confusing). Just indicate multiplication.

(Insert $C_{1_1}$)

| | NUMBER AND KIND OF COINS | ¢ VALUE | $ VALUE |
|---|---|---|---|
| 1. | | | |
| 2. | | | |
| 3. | | | |
| 4. | | | |
| 5. | | | |
| 6. | | | |
| 7. | | | |
| 8. | | | |
| 9. | | | |
| 10. | | | |
| 11. | 37¢ | | |
| 12. | $1.15 | | |

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As you can see from your table, value, as it relates to coins, is a measurement in terms of pennies or dollars. Since the ¢ column has the easiest numbers to work with, only penny measure will be used while solving a coin problem. (If dollar measure were used, fractions would creep into the work making computation difficult.) After a coin problem has been solved, value can be expressed in terms of either pennies or dollars, depending on your choice.

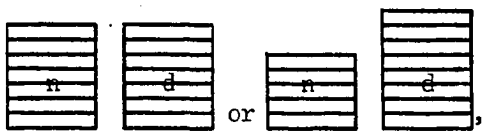
Once the values of stacks of coins that make up a total assortment have been computed or indicated, their total value is easily found. If the values are arithmetic (e.g., 250 and 700), just add. If they are algebraic (e.g., 10d and 25q), just indicate addition.

(Insert $C_{1_2}$)

| | NUMBER AND KIND OF COINS | VALUE | NUMBER AND KIND OF COINS | VALUE | TOTAL VALUE |
|---|---|---|---|---|---|
| 1. | | | | | |
| 2. | | | | | |
| 3. | | | | | |
| 4. | | | | | |
| 5. | | | | | |

Even though we seldom know the number of pennies, nickels, dimes, and so on that make up an assortment of coins, we often know their total value in coin problems. Whenever this occurs, an equation can be written which mathematically describes the relationship between the coins and their total value.

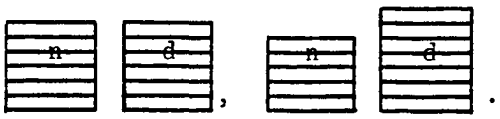
For example, suppose that the total value of some nickels and dimes was $1.05. This value condition could be illustrated as




, or in many other ways.

To indicate the arbitrary nature of such a condition we write 5n + 10d = 105, where 5n + 10d represents the total value of any amount of nickels and dimes.

Coin problems usually contain a numerical condition, also. For example, one might read that a piggy bank had 12 dimes and quarters in it. This condition can also be illustrated in a variety of ways:




In this case we write d + q = 12 to indicate the arbitrary nature of the condition, where d denotes the number of dimes, q the number of quarters.

With these ideas in mind you are now ready to write equations corresponding to coin problems and coin-type problems that involve stamps and other things like coins that can be separated, stacked, and assigned values. Read the directions on the next page carefully and solve one of the problems with your teacher. Then you're on your own.

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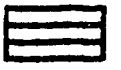
INSERT $C_{1_1}$

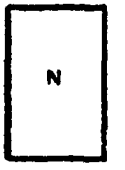
ASK AND ILLUSTRATE, "SUPPOSE THE FOLLOWING STACK STANDS FOR SOME NICKELS:



HOW MANY ARE THERE?" (HAVE YOUR STUDENTS ENTER "3 NICKELS" ON THEIR PAPERS.)

ASK, "WHAT IS THEIR VALUE?" (HAVE YOUR STUDENTS ENTER 15 AND 15/100 ON THEIR PAPERS FOR FILL-IN NUMBER 1. THEN, BUILD A STACK WITH EIGHT RODS AND GO THROUGH THE SECOND EXERCISE IN THE SAME WAY. EXERCISE THREE IS TREATED DIFFERENTLY AS SHOWN BELOW.

DRAW A RECTANGLE LABELED N AT THE OVERHEAD OR ON THE BLACKBOARD AS FOLLOWS:




THEN ASK THE "HOW MANY?" AND "WHAT IS THEIR VALUE?" QUESTIONS AS BEFORE AND HAVE YOUR STUDENTS ENTER "N NICKELS," "5N," AND "N/20" ON THEIR PAPERS FOR FILL-IN NUMBER 3.

THE FOLLOWING IS A LISTING OF THE NUMBER AND KIND OF COINS YOU ARE TO ILLUSTRATE AT THE OVERHEAD FOR YOUR STUDENTS:

| | | | |
|---|---|---|---|
| 1. 3 NICKELS | 2. 8 NICKELS | 3. N NICKELS | 4. 7 DIMES |
| 5. D DIMES | 6. D+1 DIMES | 7. 6 QUARTERS | 8. Q QUARTERS |
| 9. 7 PENNIES | 10. P PENNIES | | |

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INSERT $C_{12}$

ASK AND ILLUSTRATE, "SUPPOSE THE FOLLOWING PAIR OF STACKS STAND FOR AN ASSORTMENT OF NICKELS AND DIMES:

(THESE SHOULD BE LABELED AT THE OVERHEAD WITH A MARKER, EITHER ON ONE OF THE RODS OR BENEATH THEM.)

HOW MANY NICKELS ARE THERE IN THE FIRST STACK? (HAVE YOUR STUDENTS ENTER "6 NICKELS" ON THEIR PAPERS FOR FILL-IN NUMBER 1.)

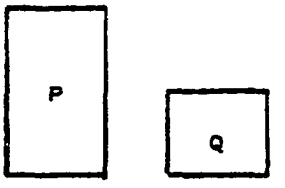
WHAT IS THE VALUE OF THE NICKELS? (HAVE YOUR STUDENTS ENTER 30 ON THEIR PAPERS AS BEFORE.)

HOW MANY DIMES ARE THERE IN THE SECOND STACK? (HAVE YOUR STUDENTS ENTER "5 DIMES" ON THEIR PAPERS AS BEFORE.)

WHAT IS THE VALUE OF THE DIMES? (HAVE YOUR STUDENTS ENTER 50 ON THEIR PAPERS AS BEFORE.)

WHAT IS THE TOTAL VALUE OF THE COINS?" (HAVE YOUR STUDENTS ENTER 80 ON THEIR PAPERS AS BEFORE. THEN, DO EXERCISES TWO AND THREE IN A SIMILAR FASHION. EXERCISE FOUR IS TREATED DIFFERENTLY AS SHOWN BELOW.)

DRAW TWO RECTANGLES LABELED P AND Q, RESPECTIVELY, AT THE OVERHEAD OR ON THE BLACKBOARD AS FOLLOWS:




THEN ASK THE SAME QUESTIONS AS BEFORE. HAVE YOUR STUDENTS ENTER "P PENNIES," "P," "Q QUARTERS," "25Q," AND "P+25Q" ON THEIR PAPERS FOR FILL-IN NUMBER 4.

THE FOLLOWING IS A LISTING OF THE NUMBER AND KIND OF COINS YOU ARE TO ILLUSTRATE AT THE OVERHEAD FOR YOUR STUDENTS:

1. 6 NICKELS, 5 DIMES
2. 4 DIMES, 4 QUARTERS
3. 8 PENNIES, 7 NICKELS
4. P PENNIES, Q QUARTERS
5. N NICKELS, D DIMES

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DIRECTIONS: For each sentence use the construction rods to build 2 stacks that meet the condition on the coins, stamps, or whatever. Then, on this paper (1) draw and label a picture of your construction, and (2) write an equation in 2 unknowns that describes the condition. Some of the sentences contain value conditions, others numerical.

| | | STACKS | CONDITION TRANSLATED |
|---|---|---|---|
| $EXAMPLE_1$: | John has 2 less nickels than dimes. | | n = d - 2 [or n+2=d] |
| $EXAMPLE_2$: | A boy has a pocketful of quarters and nickels worth $1.05. | | 25q + 5n = 105 |
| 1. | A jar contains dimes and quarters worth a total of $1.70. | | |
| 2. | Some change in nickels and dimes is worth 90¢. | | |
| 3. | The value of some 6-cent (regular) stamps and 10-cent (air mail) stamps together is $1.00 | | |
| 4. | The value of some dimes exceeds the value of some quarters by 30¢. | | |
| 5. | There are 3 more pennies than nickels in a drawer. | | |
| 6. | A man put 9 pennies and nickels in a parking meter. | | |
| 7. | A boy bought 4 less 7-cent (regular) packs of gum than 10-cent (sugarless) packs. | | |

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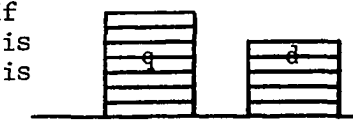
NAME_____ TEACHER_____ $C_2$

REVIEW: There are several key ideas crucial to solving coin problems. First, all coins have value, which is a measurement in terms of pennies or dollars. Second, value is usually expressed in terms of pennies rather than dollars to keep fractions out of the work. Third, the total value of an assortment of coins is found by adding the values associated with those stacks of coins (pennies, nickels, dimes, and so on) that make up the assortment. (The value of each stack is the product of the number of coins in it and the value of one of its coins.) Finally, the words "worth" and "value" are used interchangeably in coin problems.

DIRECTIONS: For each problem use the construction rods to build 2 stacks that meet both conditions on the coins, stamps, etc. Then, on this paper (1) draw and label a picture of your construction, and (2) write 2 equations in 2 unknowns that describe the conditions. There isn't time to solve each pair of equations to find the number of coins or whatever, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

| | STACKS | CONDITIONS TRANSLATED |
|---|---|---|
| EXAMPLE 1: A piggy bank has 12 dimes and quarters in it. If the total value of the coins is \$2.25, how many of each coin is in the pig? |   | $q + d = 12$<br>$25q + 10d = 225$ |
| 1. There are 3 more pennies than nickels in a drawer. If the pennies and nickels together are worth 21¢, how many coins are in the drawer? | | |
| 2. Some nickels and dimes are worth 90¢. If there are just as many nickels as dimes, how many of each are there? | | |
| 3. A boy has a pocketful of quarters and nickels worth \$1.05. If he has twice as many nickels as quarters, how many of each coin does he have in his pocket? | | |
| 4. A post office sold 12 stamps to a small boy for \$1.00. If the boy bought 6-cent (regular) stamps and 10-cent (air mail) stamps, how many air mail stamps did he buy? | | |
| 5. A girl bought equal amounts of 10-cent (air mail) stamps and 45-cent (special delivery) stamps for \$2.20. How many of each did she buy? | | |
| 6. The admission price to a school play was 75¢ for adults and 25¢ for students. If a man paid \$5.00 for 12 tickets, how many student tickets did he buy? | | |

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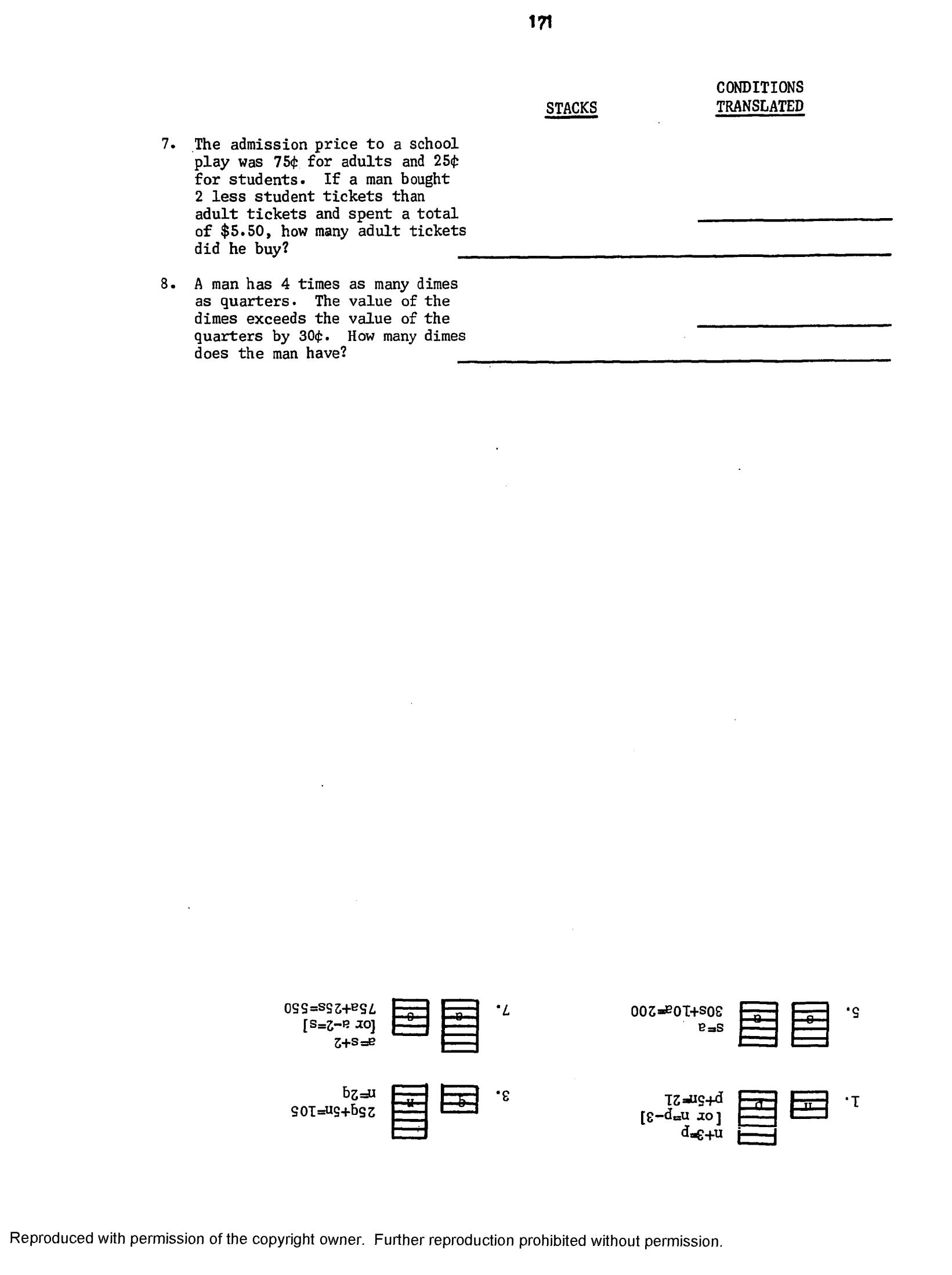
| | STACKS | CONDITIONS TRANSLATED |
|---|---|---|
| 7. The admission price to a school play was 75¢ for adults and 25¢ for students. If a man bought 2 less student tickets than adult tickets and spent a total of $5.50, how many adult tickets did he buy? | | |
| 8. A man has 4 times as many dimes as quarters. The value of the dimes exceeds the value of the quarters by 30¢. How many dimes does the man have? | | |

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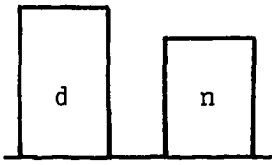
NAME_____ TEACHER_____ $C_3$

In most of the problems below the amount of money involved is so large that it would take a great many rods to illustrate the conditions on the coins and coin-type objects. Therefore, variable stacks will be drawn instead to represent the conditions (your teacher will show you how to draw these). By referring to both the verbal statement of a coin problem and its pictorial representation, you should be able to write 2 equations that describe the conditions on the coins, stamps, etc.

DIRECTIONS: For each problem draw and label variable stacks and write 2 equations in 2 unknowns that describe the conditions on the coins, stamps, etc. In drawing variable stacks label them so that the taller stack corresponds to the denomination with the most coins or coin-type objects. Sometimes it isn't possible from reading a problem to tell which stack should be the taller (e.g., problem 4). Whenever this happens draw the variable stacks in any way you wish. There isn't time to solve each pair of equations to find the number of coins or whatever, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

| | VARIABLE STACKS | CONDITIONS TRANSLATED |
|---|---|---|
| EXAMPLE 1: A boy has a pocketful of nickels and dimes worth $5.25. If he has 3 more dimes than nickels, how many of each coin does he have in his pocket? |   | 10d + 5n = 525<br>d = n + 3 |
| 1. A boy has a pocketful of dimes and nickels whose value is $3.55. He has 7 more dimes than nickels. How many dimes and how many nickels does he have? | | _____ |
| 2. A jar contains quarters and dimes worth a total of $149.75. There are 4 less dimes than quarters in the jar. How many coins are in the jar? | | _____ |
| 3. An assortment of pennies and dimes is worth $2.64. If there are just as many pennies as dimes, how many of each kind are there? | | _____ |
| 4. In one day a meter maid collected 1104 coins in pennies and nickels. If the total value of the coins was $35.00, how many pennies did she collect? | | _____ |
| 5. In one day the post office sold 763 stamps for $63.27. Some were 6-cent stamps, and the rest were 10-cent stamps. How many of each kind did the post office sell? | | _____ |
| 6. A boy spent $1.47 on gum. He bought 4 more 7-cent (regular) packs than 10-cent (sugarless) packs. How many packs of sugarless gum did he buy? | | _____ |

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| | VARIABLE STACKS | CONDITIONS TRANSLATED |
|---|---|---|
| 7. The admission price to a local high school football game was 60¢ for adults and 15¢ for students. If the gate receipts from 7240 paid admissions amounted to $3394.50, how many adults attended the game? | | |
| 8. A man has 7 times as many nickels as he has quarters. The value of the nickels exceeds the value of the quarters by 70¢. How many has he of each coin? | | |
| 9. Rick has twice as many pennies as Tick has nickels, but Tick has $2.55 more than Rick. How much money does Tick have? | | |
| 10. Jill has 2 more dimes than Jack has nickels. If the value of Jill's coins exceeds the value of Jack's bv $1.10, how many dimes does Jill have? | | |

1. $\boxed{d}$ $\boxed{n}$ d=n+7 [or d-7=n] 10d+5n=355

3. $\boxed{p}$ $\boxed{d}$ p+10d=264 p=d

5. $\boxed{s}$ $\boxed{t}$ s+t=763 6s+10t=6327

7. $\boxed{a}$ $\boxed{s}$ 60a+15s=339450 a+s=7240

9. $\boxed{p}$ $\boxed{n}$ p=2n 5n=p+255 [or 5n-255=p]

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NAME_____ TEACHER_____ $A_1$

It is customary to indicate one's age on his birthday by the number of candles on his birthday cake. For example, the candles on an eight year old's birthday cake might appear as

Another way to illustrate age on one's birthday would be with just one candle whose heighth told the story. Then the candle for an eight year old's birthday cake would appear as

And in this form his age "looks" like a stack of numbers or coins, so the imagery associated with number problems and coin problems can also be used with age problems.

Age--a measurement of existence--is something that all of us have grown up with from day one. Nevertheless, many of us are unaware of some of the mathematical properties of age. For example, differences between ages always remain the same regardless of how much time has passed--even though the ratio of two unequal ages, or the way in which they compare, does change with time.

(Insert $A_{1_1}$)

| | | |
|---|---|---|
| AGES NOW_____ | AGES IN 3 YEARS_____ | AGES 1 YEAR AGO _____ |
| DIFFERENCE NOW_____ | DIFFERENCE IN 3 YEARS_____ | DIFFERENCE 1 YEAR AGO _____ |
| COMPARISON NOW_____ | COMPARISON IN 3 YEARS_____ | COMPARISON 1 YEAR AGO _____ |

Differences between ages remaining constant while their ratio changes has some interesting consequences. For instance, the difference between the ages of a 30 year old man and a 10 year old little girl is 20, and he is three times as old as she (that is, the ratio of 30 to 10 is 30/10 = 3). In, say, 10 years, the difference between their ages is still 20; however, he is now only twice as old as she (that is, the ratio of 40 to 20 is 40/20 = 2), and the law would permit them to marry! In a sense, then, the little girl "caught up" with the man as they grew older, even though he remained 20 years her senior.

Aside from the above, much of the mathematical nature of age is familiar to most of us. For example, most of us know that phrases like "in 3 years" (or "3 years from now") and "2 years ago" refer to addition and subtraction, respectively. That is, we would know to add 3 to a person's present age to find his age "in 3 years." Similarly, we would know to subtract 2 to get his age "2 years ago." (You used this knowledge when you filled in the blanks after the insert.)

The mathematical nature of phrases containing the words "older" and "younger" is also known to most. For example, if Graham is 14 and Kathy is 12, we say Graham is 2 years older than Kathy (or Kathy is 2 years younger than Graham). By this we mean that 2 years added to Kathy's age is Graham's age (or 2 years subtracted from Graham's age is Kathy's age).

(Insert $A_{1_2}$)

TWO-OLDER SENTENCE_____

\+ SENTENCE_____(eq. )

TWO-YOUNGER SENTENCE_____

\- SENTENCE_____(eq. )

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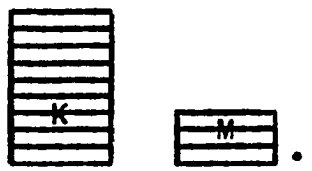
INSERT $A_{1_1}$

ASK AND ILLUSTRATE, "SUPPOSE KIM IS 9 AND MICAH IS 3. THEN THEIR AGES NOW "LOOK" LIKE




(THESE "CANDLES" SHOULD BE LABLED AT THE OVERHEAD WITH A MARKER, EITHER ON ONE OF THE RODS OR BENEATH THEM.)

WHAT IS THE DIFFERENCE BETWEEN THE TWO AGES NOW? (6)

HOW DO THEIR AGES COMPARE NOW? (WHENEVER A COMPARISON IS ASKED FOR, HAVE YOUR STUDENTS RESPOND USING THE WORD "TIMES." FOR EXAMPLE, KIM IS 3 TIMES AS OLD AS MICAH. THEN, HAVE THEM EXPRESS THIS IN ALGEBRAIC FORM AS $K = 3M$ ON THEIR PAPER.)

HOW WILL THEIR AGES APPEAR 3 YEARS FROM NOW? (ADD 3 RODS TO EACH STACK SINCE EVERYONE WILL HAVE 3 MORE BIRTHDAYS AND HAVE YOUR STUDENTS ENTER K+3, M+3 IN THE "AGES IN 3 YEARS" BLANK. YOU MIGHT WANT TO COLOR SOME RODS WITH A MARKER TO USE AS "IN" RODS SO THAT KIM'S AND MICAH'S PRESENT AGES CAN BE EASILY DIFFERENTIATED FROM THEIR AGES IN 3 YEARS.)

WHAT WILL THE DIFFERENCE BETWEEN THE TWO AGES BE THEN? (STILL 6)

HOW WILL THEIR AGES COMPARE THEN? ($K = 2M$)

HOW WOULD THEIR AGES HAVE APPEARED A YEAR AGO? (REMOVE 1 ROD FROM EACH STACK SINCE EVERYONE WOULD HAVE BEEN A YEAR YOUNGER AND HAVE YOUR STUDENTS ENTER K-1, M-1 IN THE "AGES 1 YEAR AGO" BLANK. THE WORD "REMOVE" IS TO BE TAKEN GENTLY. THAT IS, MOVE THE "AGO" RODS ONLY A SHORT DISTANCE SO THAT KIM'S AND MICAH'S PRESENT AGES ARE STILL VISIBLE, EVEN THOUGH DISJOINTED.)

WHAT WOULD THE DIFFERENCE BETWEEN THE TWO AGES HAVE BEEN THEN? (STILL 6)

HOW WOULD THEIR AGES HAVE COMPARED THEN?" ($K = 4M$)

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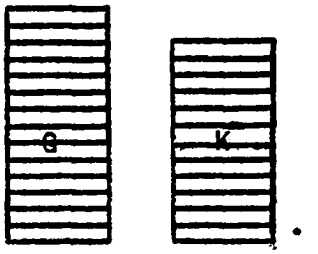
INSERT $A_{12}$

ASK AND ILLUSTRATE, "SUPPOSE GRAHAM IS 14 AND KATHY IS 12. THEN THEIR AGES NOW "LOOK" LIKE




WHAT CAN BE SAID ABOUT THE AGES OF GRAHAM AND KATHY USING THE WORDS "TWO" AND "OLDER"? (POSSIBLE ANSWER: GRAHAM IS 2 YEARS OLDER THAN KATHY.)

USE A WORD RELATED TO ADDITION IN A SENTENCE TO TELL WHAT IT MEANS TO SAY THAT GRAHAM IS 2 YEARS OLDER THAN KATHY. (POSSIBLE ANSWER: TWO ADDED TO KATHY'S AGE GIVES GRAHAM'S AGE. ACCOMPANYING EQUATION: $K + 2 = G$)

WHAT CAN BE SAID ABOUT THE AGES OF GRAHAM AND KATHY USING THE WORDS "TWO" AND "YOUNGER"? (POSSIBLE ANSWER: KATHY IS 2 YEARS YOUNGER THAN GRAHAM.)

USE A WORD RELATED TO SUBTRACTION IN A SENTENCE TO TELL WHAT IT MEANS TO SAY THAT KATHY IS 2 YEARS YOUNGER THAN GRAHAM." (POSSIBLE ANSWER: TWO SUBTRACTED FROM GRAHAM'S AGE GIVES KATHY'S AGE. ACCOMPANYING EQUATION: $G - 2 = K$)

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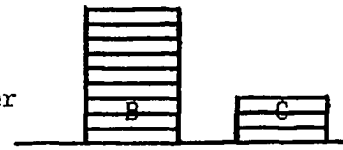
DIRECTIONS: For each sentence use the construction rods to build 2 "candles" that meet the condition on the ages. Then, on this paper (1) draw and label a picture of your construction, and (2) write an equation in 2 unknowns that describes the condition.

| | "CANDLES" | CONDITION TRANSLATED |
|---|---|---|
| EXAMPLE$_1$: Clyde is 6 years younger than Bonnie. |   | C + 6 = B or C = B - 6 |
| EXAMPLE$_2$: Two years ago Enoch was 5 times as old as Frog. | | (E-2) = 5(F-2) |
| 1. Nathan is 4 years younger than Kimberly. | | |
| 2. Luap is 8 years older than Ettolrahc. | | |
| 3. The sum of Trebor's and Hi-Ho's ages is 10. | | |
| 4. One year ago Trebor was 3 times as old as Hi-Ho. | | |
| 5. A year ago Paul was 4 times as old as Charlotte. | | |
| 6. In 4 years Nathan will be twice as old as Micah. | | |

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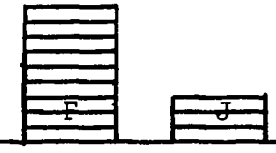
NAME_____ TEACHER_____ $A_2$

REVIEW: There are several key ideas crucial to solving age problems. First, phrases like "in 3 years" (or "3 years from now") and "5 years ago" refer to addition and subtraction, respectively. This is also the case for the words "younger" and "older." Second, each year everyone has a birthday, which means that differences between ages always remain the same regardless of how much time has passed. Third, the way in which ages compare does change with time. (Another way to say this is that the ratio of 2 unequal ages changes as time passes.)

DIRECTIONS: For each problem use the construction rods to build 2 "candles" that meet both conditions on the ages. Then, on this paper (1) draw and label a picture of your construction, and (2) write 2 equations in 2 unknowns that describe the conditions. There isn't time to solve each pair of equations to find the ages, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

| | "CANDLES" | CONDITIONS TRANSLATED |
|---|---|---|
| EXAMPLE 1: Frankie is 3 times as old as Johnny. In 3 years Frankie will be twice as old as Johnny. How old is Frankie? |   | F = 3J<br>(F+3) = 2(J+3) |
| 1. Paul is 4 years older than Charlotte. In 1 year Paul will be twice as old as Charlotte. How old is Charlotte? | | |
| 2. Thin is 3 years older than Skin. A year ago Thin was twice as old as Skin. How old is Skin? | | |
| 3. Micah is 3 years younger than Kimberly. In 1 year Kimberly will be twice as old as Micah. How old is Kimberly? | | |
| 4. Clyde is 6 years younger than Bonnie. A year ago Bonnie was 4 times as old as Clyde. How old is Bonnie? | | |
| 5. Frog is 8 years younger than Enoch. Two years ago Enoch was 5 times as old as Frog. How old is Frog? | | |
| 6. Nathan is 3 times as old as Micah. In 4 years Nathan will be twice as old as Micah. How old is Nathan? | | |
| 7. Luap is 4 times as old as Ettolrahc. In 4 years Luap will be twice as old as Ettolrahc. How old is Ettolrahc? | | |

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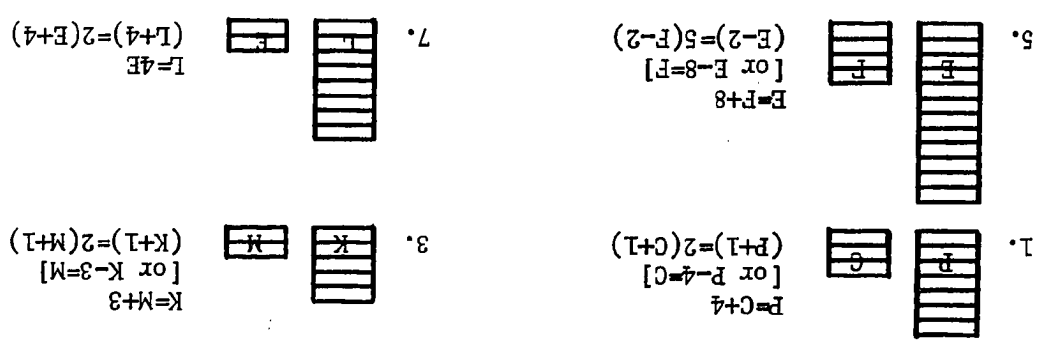
| | "CANDLES" | CONDITIONS TRANSLATED |
|---|---|---|
| 8. The sum of Trebor's and Hi-Ho's ages is 10. One year ago Trebor was 3 times as old as Hi-Ho. How old is Trebor? | | |

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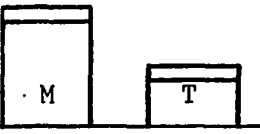
NAME_____ TEACHER_____ $A_3$

In most of the problems below the people are so old that it would take a great many rods to illustrate their ages. Therefore, variable stacks will be drawn instead to represent the conditions on the ages (your teacher will show you how to draw these). By referring to both the verbal statement of an age problem and its pictorial representation, you should be able to write 2 equations that describe the conditions on the ages.

DIRECTIONS: For each problem draw and label a variable candle picture and write 2 equations in 2 unknowns that describe the conditions on the ages. There are several acceptable ways to add or subtract rods from variable stacks; however, be careful to label the taller stack with the older person's initial. There isn't time to solve each pair of equations to find the ages, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

| | VARIABLE "CANDLES" | CONDITIONS TRANSLATED |
|---|---|---|
| EXAMPLE 1: The sum of Mot's and Tap's ages is 88. In 1 year Mot will be twice as old as Tap. How old is Mot? |   | M + T = 88<br>(M+1) = 2(T+1) |
| 1. A man is 5 times as old as his son. In 5 years he will be 3 times as old as his son. Find the ages of the man and his son. | | |
| 2. A father is 3 times as old as his son is. Eleven years from now he will be only twice as old as his son. How old is the man's son? | | |
| 3. A man is 4 times as old as his son. Sixteen years from now he will be only twice as old as his son. How old is the man now? | | |
| 4. Professor Kool is 3 times as old as Rad. In 20 years Professor Kool will be twice as old as Rad. How old is Professor Kool? | | |
| 5. John is twice as old as his brother Bob. Five years ago John was 3 times as old as Bob. What are the ages of the two brothers? | | |
| 6. Karen's mother is 7 times as old as Karen is. One year ago she was 9 times as old as Karen was. Find Karen's age. | | |

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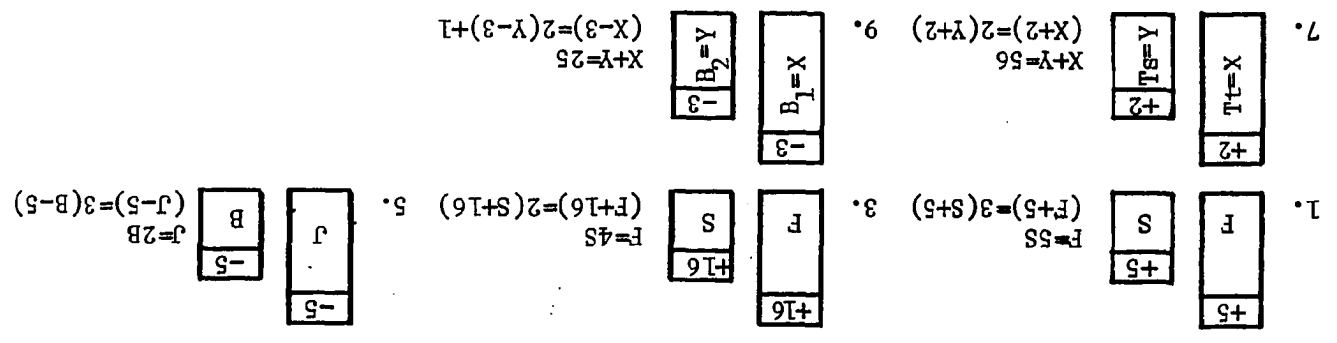
| | VARIABLE "CANDLES" | CONDITIONS TRANSLATED |
|---|---|---|
| 7. The sum of This's age and That's age is 56. In 2 years That will be twice as old as This. Find the ages of This and That. | | |
| 8. In 3 years Kim will be twice as old as Micah. One year ago Kim was 4 times as old as Micah. How old is Kim? | | |
| 9. The combined ages of two boys is 25. Three years ago the age of one boy exceeded twice the age of the other by 1 year. Find the ages of both boys. | | |
| 10. Three years ago Jack's age was 1 year more than twice Jill's age. Six years from now Jack's age will be 10 more than half Jill's age. How old is Jack? | | |

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# APPENDIX B

INSTRUCTIONAL MATERIALS FOR NUMBER, COIN, AND AGE PROBLEMS: LO. THE UNITS ARE LABELED N, C, A, R FOR NUMBER, COIN, AGE, AND REVIEW, RESPECTIVELY. THE ORDER OF PRESENTATION FOR EACH UNIT IS INDICATED BY SUBSCRIPTS: DAY 1, $N_1$; DAY 2, $N_2$; DAY 3, $C_1$; DAY 4, $C_2$; DAY 5, $A_1$; DAY 6, $A_2$; DAY 7, $R_1$; DAY 8, $R_2$.

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NAME_____ TEACHER_____ $N_1$

## INTRODUCTION

For the next few days you will be learning to translate stated mathematical relationships into mathematical symbols, a system of equations. In arithmetic you exchanged numbers and operations such as plus or times for words in story problems, and the ability to describe verbal conditions mathematically becomes increasingly important in science, more advanced mathematics, and daily events. Therefore, the instruction that follows is aimed toward increasing your translative skill.

The setting for this endeavor will be algebra word problems. These mathematically relate concepts such as number, value, and age. Algebra word problems usually include a question requiring a numerical answer (e.g., What are the two numbers?). In general, the answer to such a question derives from a system of equations that mathematically describes the relationships stated in the problem. Thus solving algebra word problems may be viewed as a two-step process: (1) translation, and (2) solution of equations.

An examination of computer programs for solving algebra word problems quickly reveals the difficulty machines experience with the first step, even though their speed and accuracy with the second is without equal. Thus the ability to move from words to mathematical symbols seems peculiarly human. Therefore, of the two parts of the solution process for algebra word problems, only translation will concern us for the present. The solution of equations, being somewhat mechanical and better suited to machines, is a topic for another day.

## NUMBER PROBLEMS

The verbage you will be working with today relates numbers. Only a few words and phrases are commonly used to express relationships between numbers (e.g., "sum," "difference," "product," "quotient," "more than," "less than," "increased by," "decreased by," and so on), and they serve as keys to translation. Since they appear again and again in all types of algebra word problems, their meaning and algebraic equivalents will be given special attention at this time.

The following examples give some indication of the many ways mathematical symbols can be interpreted verbally. In each case the mathematical expression contains either an addition (+), subtraction (-), multiplication (x), or division (÷) sign; yet each has a number of corresponding combinations of words. Your task, given one of the phrases on the next page, is to express it algebraically using one or more of the four operations: +, -, x, ÷ .

| WORDS | SYMBOLS |
|---|---|
| The sum of a and b, b more than a, a increased by b, exceeds a by b | a + b |
| The difference between m and n, m less n, n less than m, m decreased by n | m - n |
| The product of p and q, p times q | pq |
| The quotient of r and s, r divided by s | r ÷ s |

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Express the following phrases in algebraic symbols:

1. The sum of a and b _____
2. 5 more than a _____
3. a more than 5 _____
4. a increased by 3 _____
5. 3 increased by a _____
6. a exceeded by 7 _____
7. The difference between m and n _____
8. m less 4 _____
9. m less than 4 _____
10. m decreased by 18 _____
11. 18 decreased by m _____
12. The difference between m and n, added to a _____
13. The product of p and q _____
14. 17 times p _____
15. Twice q _____
16. 3 times the sum of a and b _____
17. 6 times the difference between m and n _____
18. The product of p and q, increased by 3 times the product of p and q _____
19. Twice the sum of a and b less the product of p and q _____
20. r divided by s _____

Whenever two numbers are given, such as 4 and 12, it is always possible to write sentences containing phrases like those you have just translated that describe their mathematical relationship to each other [e.g., Three times 4 is 12, Four increased by 8 is 12, and so on]. These sentences can then be translated into equations, which also describe the way the numbers are mathematically related [e.g., 3(4) = (12), (4) + 8 = (12), and so on]. See if you can write five such sentences and their corresponding equations for the numbers 2 and 8.

1. _____ (eq. )
2. _____ (eq. )
3. _____ (eq. )
4. _____ (eq. )
5. _____ (eq. )

For the most part, the numbers referred to in algebra word problems are unknown. In this case the equations describing their relationship to each other mathematically is algebraic [e.g., $3x = y$, $x + 8 = y$, and so on]. In the following sentences there are many possible pairs of numbers that satisfy each stated mathematical relationship, so in a sense the numbers are unknown. Therefore, represent them with variables such as m,n,x,y and translate each sentence into an equation in two unknowns. The answers to the odd numbered exercises have been provided as a check.

1. The sum of two numbers is 11. (eq. )
2. The difference between two numbers is 4. (eq. )
3. Separate 16 into two parts. (eq. )
4. One number exceeds another by 17. (eq. )
5. The smaller of two numbers is 4 less than the larger. (eq. )
6. Three times one number is twice the other. (eq. )
7. Adding 7 to one number gives the same as adding 5 to another. (eq. )
8. One number is 4 less than three times another. (eq. )
9. The larger of two numbers is 2 more than three times the smaller. (eq. )
10. Five times one number decreased by three times another is 7. (eq. )

1. $x+y=11$ 3. $m+n=16$ 5. $p=q-4$ 7. $r+7=s+5$ 9. $w=3z+2$

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NAME_____ TEACHER_____ $N_2$

DIRECTIONS: For each problem write 2 equations in 2 unknowns that describe the relationships between the numbers. There isn't time to solve each pair of equations to find the numbers, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

EXAMPLE 1: The difference between two numbers is 5. Twice the larger exceeds 5 times the smaller by 1. Find the smaller number. $x - y = 5$; $2x = 5y + 1$

1. The sum of two numbers is 112. Their difference is 34. What are they?

2. One number is five times another. Their sum is 42. What is the larger number?

3. One number is 3 less than another. Their product is 54. What is the smaller number?

4. The difference between two numbers is 16. Seven times the smaller is 3 times the larger. What are the two numbers?

5. The larger of two numbers is four times the smaller. Their difference is 27. What is their product?

6. The sum of two numbers is 8. If adding 7 to one of them gives the same result as adding 5 to the other, what are they?

7. Separate 55 into two parts such that the larger exceeds the smaller by 7.

8. One number is 8 more than another. Their sum is 20. What is the smaller number?

9. The difference between two numbers is 2. Three times one number is 1 more than twice another. What are they?

10. The larger of two numbers is 6 more than three times the smaller. If their sum is 102, what are they?

11. One number exceeds three times another by 18. Their sum is 102. Find the smaller number.

12. The larger of two numbers exceeds the smaller by 62. If 5 times the smaller is decreased by twice the larger, the difference is 155. What is their sum?

1. $a+b=112$, $a-b=34$ 3. $m-n=3$ [or $m-3=n$, $m=n+3$], $mn=54$ 5. $x=4y$, $x-y=27$
7. $p+q=55$, $p=q+7$ 9. $w-z=2$, $3w=2z+1$ 11. $r=3s+18$, $r+s=102$

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NAME_____ TEACHER_____ $C_1$

Imagine someone with a bucketful of nickels and dimes who wants to find out how much the coins in the bucket are worth. He could take the whole mess to a bank and say to the manager, "Sir, I will give you this fine bucket if you will tell me how much all these coins are worth." Or he could weigh the nickels and dimes separately and compute their value in terms of nickel-weight and dime-weight.

An easy way to find the value of a lot of coins--without the help of a bank manager or a scale--is to first sort them into groups: pennies, nickels, dimes, and so on. The value of each group can then be found by multiplying the number of coins in it by the value of one of its coins, and the value of all the coins is simply the sum of the values associated with each group.

For example, suppose that when the nickels and dimes from the bucket of coins mentioned earlier were sorted and counted there were 50 nickels and 70 dimes. Then the value of the nickels would be 50 x 5 = 250 pennies, where 5 is the value of a nickel measured in pennies; and 70 x 10 = 700 pennies would be the value of the dimes, where 10 is the value of a dime measured in pennies. The total value of the coins would then be 250 + 700 = 950 pennies, or $9.50.

Coin problems in algebra are also fairly easy to solve if one thinks of total value as the sum of values of sorted coins. Before one can add the values of coins of the same denomination, however, he must know how to find the value of a single group of coins. Whenever the number of coins in a group is known, as in the example above, this is easy. Just multiply. Whenever the number of coins is unknown, as is usually the case in coin problems, it is even easier (though more confusing). Just indicate multiplication.

| NUMBER AND KIND OF COINS | ¢ VALUE | $ VALUE |
|---|---|---|
| 1 penny | | |
| 7 pennies | | |
| 37¢ | | |
| p pennies | | |
| 3 nickels | | |
| 14 nickels | | |
| n nickels | | |
| 7 dimes | | |
| d dimes | | |
| d+1 dimes | | |
| 6 quarters | | |
| q quarters | | |
| q-3 quarters | | |
| 5 dollars | | |
| $1.15 | | |

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As you can see from your table, value, as it relates to coins, is a measurement in terms of pennies or dollars. Since the ¢ column has the easiest numbers to work with, only penny measure will be used while solving a coin problem. (If dollar measure were used, fractions would creep into the work making computation difficult.) After a coin problem has been solved, value can be expressed in terms of either pennies or dollars, depending on your choice.

Once the values of groups of coins that make up a total assortment have been computed or indicated, their total value is easily found. If the values are arithmetic (e.g., 250 and 700), just add. If they are algebraic (e.g., 10d and 25q), just indicate addition.

| NUMBER AND KIND OF COINS | VALUE | NUMBER AND KIND OF COINS | VALUE | TOTAL VALUE |
|---|---|---|---|---|
| 3 nickels | ____ | 5 dimes | ____ | _____ |
| 4 dimes | ____ | 4 quarters | ____ | _____ |
| 8 pennies | ____ | 7 nickels | ____ | _____ |
| n nickels | ____ | d dimes | ____ | _____ |
| p pennies | ____ | q quarters | ____ | _____ |
| n-3 nickels | ____ | d+1 dimes | ____ | _____ |
| p pennies | ____ | q+2 quarters | ____ | _____ |

Even though we seldom know the number of pennies, nickels, dimes, and so on contained in an assortment of coins, we often know their total value in coin problems. Whenever this occurs, an equation can be written which mathematically describes the relationship between the coins and their total value.

For example, suppose that the total value of some nickels and dimes was $1.05. This could be expressed mathematically as $5n + 10d = 105$, since the total value of n nickels and d dimes is $5n + 10d$. See if you can write total-value equations for the following sentences, some of which involve stamps and other things like coins that can be sorted and assigned values.

1. Some change in nickels and dimes is worth 90¢. _____
2. There are $2.47 in pennies and quarters on a dresser. _____
3. A boy has a pocketful of quarters and nickels worth $5.80. _____
4. The value of some 6-cent (regular) stamps and 10-cent (air mail) stamps together is $1.00. _____
5. A man paid $5.00 for some 25-cent (student) tickets and 75-cent (adult) tickets for a school play. _____

In each of the above sentences, the condition on the coins is one of value. Coin problems usually contain a numerical condition, also. For example, one might read that a piggy bank had 12 dimes and quarters in it. In this case it would not be possible to write an equation involving value, since we do not know what the total value--$10d + 25q$--is equal to. Instead, one writes $d + q = 12$, where d and q stand for the number of dimes and quarters, respectively. Some sentences with numerical conditions on coins follow. See if you can translate them into equations.

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1. There are 3 more pennies than nickels in a drawer. _____

2. A man bought 2 less 25-cent student tickets than 75-cent adult tickets for a school play. _____

3. A boy has twice as many nickels as quarters in his pocket. _____

4. Paul received just as many nickels as dimes in change. _____

5. It took 15 stamps to mail a package. Some were 6-cent (regular) stamps, and the rest were 10-cent (air mail) stamps. _____

In the remaining time write mathematical equivalents (that is, equations) for the following sentences--some containing value conditions, others numerical. The answers to the odd numbered problems have been provided as a check.

$EXAMPLE_1$: The value of an assortment of nickels and dimes is \$3.75. $5n + 10d = 375$

$EXAMPLE_2$: There are 4 less 6-cent (regular) stamps than 10-cent (air mail) stamps on this letter. _____ $r + 4 = a$ [or $r = a - 4$]

1. A jar contains dimes and quarters worth a total of \$149.80. _____

2. The total value of some pennies and nickels is \$2.30. _____

3. A man has 4 times as many dimes as quarters. _____

4. In one day a meter maid collected 1104 coins in pennies and nickels. _____

5. The value of some dimes exceeds the value of some quarters by 30¢. _____

6. A boy bought 4 more 7-cent (regular) packs of gum than 10-cent (sugarless) packs. _____

1. $10d + 25q = 14980$ 3. $d = 4q$ 5. $10d = 25q + 30$

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NAME_____ TEACHER_____ $C_2$

Review: There are several key ideas crucial to solving coin problems. First, all coins have value, which is a measurement in terms of pennies or dollars. Second, value is usually expressed in terms of pennies rather than dollars to keep fractions out of the work. Third, the total value of an assortment of coins is found by adding the values associated with those groups of coins (pennies, nickels, dimes, and so on) that make up the assortment. (The value of each group is the product of the number of coins in it and the value of one of its coins.) Finally, the words "worth" and "value" are used interchangeably in coin problems.

DIRECTIONS: For each problem write 2 equations in 2 unknowns that describe the conditions on the coins or coin-type objects. There isn't time to solve each pair of equations to find the number of coins or whatever, so don't worry about answering the question asked in the problems—the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

EXAMPLE 1: A piggy bank has 12 dimes and quarters in it. If the total value of the coins is \$2.25, how many of each coin is in the pig? $d + q = 12$; $10d + 25q = 225$

1. There are 3 more pennies than nickels in a drawer. If the pennies and nickels together are worth 21¢, how many coins are in the drawer? _____

2. A jar contains dimes and quarters worth a total of \$149.75. There are 4 less dimes than quarters in the jar. How many coins are in the jar? _____

3. A boy spent \$1.47 on gum. He bought 4 more 7-cent (regular) packs than 10-cent (sugarless) packs. How many packs of sugarless gum did he buy? _____

4. A girl has \$4.20 in nickels and quarters in her purse. If she has twice as many nickels as quarters, how many of each coin does she have in the purse? _____

5. Some nickels and dimes are worth 90¢. If there are just as many nickels as dimes, how many of each are there? _____

6. A girl bought equal amounts of 10-cent (air mail) stamps and 45-cent (special delivery) stamps for \$2.20. How many of each did she buy? _____

7. A post office sold 12 stamps to a small boy for \$1.00. If the boy bought 6-cent (regular) stamps and 10-cent (air mail) stamps, how many air mail stamps did he buy? _____

8. A man has 4 times as many dimes as quarters. The value of the dimes exceeds the value of the quarters by 30¢. How many dimes does the man have? _____

9. The admission price to a school play was 25¢ for students and 75¢ for adults. If a man paid \$5.00 for 12 tickets, how many student tickets did he buy? _____

10. In one day a post office sold 763 stamps for \$63.27. Some were 6-cent stamps, and the rest were 10-cent stamps. How many of each kind did the post office sell? _____

1. $p=n+3$ [or $p-3=n$], $p+5n=21$ 3. $r=s+4$ [or $r-4=s$], $7r+10s=147$ 5. $5n+10d=90$, $n=d$
7. $r+s=12$, $6r+10s=100$ 9. $s+a=12$, $25s+75a=500$

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NAME_____ TEACHER_____ $A_1$

Age--a measurement of existence—is something that all of us have grown up with from day one. Nevertheless, many of us are unaware of some of the mathematical properties of age. For example, differences between ages always remain the same regardless of how much time has passed--even though the ratio of two unequal ages, or the way in which they compare, does change with time. To illustrate, if two boys were 8 and 2, one would be 6 (= 8-2) years older than the other. And in 4 years he would still be 6 (= 12-6) years older. On the other hand, the 8 year old is four times as old as the 2 year old (that is, the ratio of 8 to 2 is 8/2 = 4). In 4 years, however, he would be only twice as old as the younger since the ratio of their ages then, 12 to 6, would be 2 (= 12/6).

Differences between ages remaining constant while their ratio changes has some interesting consequences. For instance, the difference between the ages of a 30 year old man and a 10 year old little girl is 20, and he is three times as old as she (that is, the ratio of 30 to 10 is 30/10 = 3). In, say, 10 years, the difference between their ages is still 20 (= 40-20); however, he is now only twice as old as she (that is, the ratio of 40 to 20 is 40/20 = 2), and the law would permit them to marry! In a sense, then, the little girl "caught up" with the man as they grew older, even though he remained 20 years her senior.

Aside from the above, much of the mathematical nature of age is familiar to most of us. For example, most of us know that phrases like "in 3 years" (or "3 years from now") and "5 years ago" refer to addition and subtraction, respectively. That is, we would know to add 3 to a person's present age to find his age "in 3 years." Similarly, we would know to subtract 5 to get his age "5 years ago." Completing the following with your teacher will illustrate this.

| AGE NOW FOR ONE PERSON | AGE "IN 3 YEARS" | AGE "5 YEARS AGO" |
|---|---|---|
| 5 | ____ | ____ |
| 12 | ____ | ____ |
| P | ____ | ____ |
| C | ____ | ____ |

| AGES NOW FOR TWO PERSONS | AGES "IN 7 YEARS" * | AGES "1 YEAR AGO" |
|---|---|---|
| (3,6) | (____,____) | (____,____) |
| (12,13) | (____,____) | (____,____) |
| (E, F) | (_____,_____) | (_____,_____) |
| (M,N) | (_____,_____) | (_____,_____) |

The mathematical nature of phrases containing the words "older" and "younger" is also known to most. For example, if Graham is 14 and Kathy is 12, we say Graham is 2 years older than Kathy (or Kathy is 2 years younger than Graham). By this we mean that 2 years added to Kathy's age is Graham's age (or 2 years subtracted from Graham's age is Kathy's age).

* Remember, everyone will have seven birthdays.

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In the following pairs of ages the first person is <u>older</u> than the second. Write an equation using <u>addition</u> for each pair that expresses how much older the first person is.

( 8,6) _____ (13,12) _____

(12,3) _____ (47,20) _____

Referring to the same pairs, one could also say that the second person is <u>younger</u> than the first. Write an equation using <u>subtraction</u> for each pair that expresses how much younger the second person is.

_____ _____

_____ _____

As illustrated above, whenever several ages are mentioned it is often possible to write an equation that mathematically describes their relationship to each other. If the ages are known, like 12 and 3, then it is arithmetic [e.g., (12) = 4(3), (12) = (3) + 9, (12) - 9 = (3), and so on]. If, as is usually the case, the ages are not known, it is algebraic [e.g., P = 4C, P = C + 9, P - 9 = C, and so on]. Your teacher will work with you for the remainder of the period on writing <u>algebraic</u> equations that represent the condition on (or relationship between) the ages of the persons in the following sentences.

DIRECTIONS: For each sentence write an equation in two unknowns that describes the condition on the ages. The answers to the odd numbered problems have been provided as a <u>check</u>.

EXAMPLE$_1$: Bonnie is 6 years younger than Clyde. C - 6 = B [or C = B + 6]

EXAMPLE$_2$: Two years ago Enoch was 5 times as old as Frog. (E-2) = 5(F-2)

1. Nathan is 4 years younger than Kimberly. _____
2. Luap is 8 years older than Ettolrahc. _____
3. Nathan is 3 times as old as old as Micah. _____
4. John is twice as old as Mary. _____
5. The sum of Trebor's and Hi-Ho's ages is 10. _____
6. The difference between Mot's and Tap's ages is 17. _____
7. In 4 years Nathan will be twice as old as Micah. _____
8. In 12 years Thin will be 5 times as old as Skin. _____
9. A year ago Paul was 4 times as old as Charlotte. _____
10. Five years ago Trebor was 3 times as old as Hi-Ho. _____

1. K-4=N [or K=N+4] 3. N=3M 5. T+H=10 7. N+4=2(M+4) 9. P-1=4(C-1)

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NAME _____ TEACHER _____ $A_2$

REVIEW: There are several key ideas crucial to solving age problems. First, phrases like "in 3 years" (or "3 years from now") and "5 years ago" refer to addition and subtraction, respectively. This is also the case for the words "younger" and "older." Second, each year everyone has a birthday, which means that differences between ages always remain the same regardless of how much time has passed. Third, the way in which ages compare does change with time. (Another way to say this is that the ratio of 2 unequal ages changes as time passes.)

DIRECTIONS: For each problem write 2 equations in 2 unknowns that describe the conditions on the ages. There isn't time to solve each pair of equations to find the ages, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

EXAMPLE 1: Enoch is 8 years older than Frog. Two years ago Enoch was 5 times as old as Frog. How old is Frog? _____ E = F + 8 [or E - 8 = F]; (E-2) = 5(F-2)

1. Thin is 3 years older than Skin. A year ago Thin was twice as old as Skin. How old is Skin? _____

2. Paul is 4 years older than Charlotte. In 1 year Paul will be twice as old as Charlotte. How old is Charlotte? _____

3. Clyde is 6 years younger than Bonnie. A year ago Bonnie was 4 times as old as Clyde. How old is Bonnie? _____

4. Nathan is 3 times as old as Micah. In 4 years Nathan will be twice as old as Micah. How old is Nathan? _____

5. A man is 4 times as old as his son. Sixteen years from now he will be only twice as old as his son. How old is the man now? _____

6. Frankie is 3 times as old as Johnny. A year ago Frankie was 4 times as old as Johnny. How old is Frankie? _____

7. Karen's mother is 7 times as old as Karen is. One year ago she was 9 times as old as Karen was. Find Karen's age. _____

8. The sum of Trebor's and Hi-Ho's ages is 10. One year ago Trebor was 3 times as old as Hi-Ho. How old is Trebor? _____

9. The sum of Mot's and Tap's ages is 88. In 1 year Mot will be twice as old as Tap. How old is Mot? _____

10. The combined ages of two boys is 25. Three years ago the age of one boy exceeded twice the age of the other by 1 year. Find the ages of both boys. _____

1. T=S+3 [or T-3=S], (T-1)=2(S-1) 3. B-6=C [or B=C+6], (B-1)=4(C-1)
5. M=4S, (M+16)=2(S+16) 7. M=7K, (M-1)=9(K-1) 9. M+T=88, (M+1)=2(T+1)

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NAME_____ TEACHER_____ $R_1$

REVIEW

Certain combinations of words such as "more than," "difference," "times," and "divided by" appear in practically every algebra word problem. Since their appearance always refers to one of the four arithmetical operations--addition, subtraction, multiplication, or division--they serve as keys to translation. That is, one can write instead of such phrases +, -, x, or ÷.

Use at least one of the arithmetic signs of operation to express the following mathematically:

1. The sum of a and 47_____
2. 18 more than b_____
3. a increased by b_____
4. The difference between m and 56_____
5. 75 less 50_____
6. 50 less than 75_____
7. m decreased by 63_____
8. Twice 15_____
9. The product of p and 29_____
10. 13 times the difference of of m and 56_____
11. The product of p and 29, increased by twice the sum of a and 47_____

Whenever phrases like the above occur in sentences, equations can be written that describe the manner in which they relate numbers, coins, age, and so on. See if you can translate each of the following into an equation in two unknowns.

1. The sum of two numbers is 17. _____
2. The larger of two numbers is 7 more than the smaller. _____
3. One number is 16 less than another. _____
4. Adding 8 to one number gives the same result as subtracting 104 from another. _____
5. The larger of two numbers increased by 3 times the smaller is 118. _____
6. A man put 4 more pennies than nickels in a parking meter. _____
7. Seven stamps were used to mail a package. Some were 6-cent (regular) stamps, and the others were 10-cent (air mail) stamps. _____
8. Nathan received 3 times as many nickels as quarters in change. _____
9. Four less $2.00 10-yard line tickets were sold than $5.00 50-yard line tickets. _____
10. The value of some nickels exceeds the value of some dimes by 15¢. _____
11. Kimberly is 3 times as old as Micah. _____
12. The difference between Kimberly's and Nathan's ages is 4. _____
13. The sum of Enoch's and Frog's ages is 47. _____
14. Nhoj's age exceeds Trebor's age by 10. _____
15. Luap's age decreased by 14 is Trebor's age. _____

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In addition to including phrases that denote one of the arithmetic signs of operation, many algebra word problems involve concepts or include words and phrases of a special nature (e.g., the meaning of value as it relates to coins in coin problems, the words "older" and "younger" as they are interpreted mathematically in age problems). Since one of the two equations in two unknowns that describe the mathematical relationships stated in, say, a coin or age problem, usually derives from such concepts, words, or phrases; a thorough understanding of their mathematical meaning is necessary to solving these types of algebra word problems. (The other equation is usually obtained from a sentence similar to those translated on the preceding page.) Therefore, some review exercises related to value and age follow:

| NUMBER AND KIND OF COINS | VALUE | NUMBER AND KIND OF COINS | VALUE | TOTAL VALUE |
|---|---|---|---|---|
| 3 pennies | ____ | 4 nickels | ____ | _____ |
| 5 dimes | ____ | 6 quarters | ____ | _____ |
| 83¢ | ____ | $4.15 | ____ | _____ |
| p pennies | ____ | q quarters | ____ | _____ |
| n nickels | ____ | d-1 dimes | ____ | _____ |

| AGES NOW FOR TWO PERSONS | AGES "IN 12 YEARS" | AGES "6 YEARS AGO" |
|---|---|---|
| (14, 7) | (____, ____) | (____, ____) |
| (38,13) | (____, ____) | (____, ____) |
| (P,C) | (_____, _____) | (_____, _____) |
| (K,N) | (_____, _____) | (_____, _____) |

Translate the following sentences into equations in two unknowns:

1. An accumulation of dimes and quarters in a jar is worth $7.85. _____
2. The value of some rolls of pennies and nickels is $5.50. _____
3. In one day a meter maid collected $97.73 in pennies and nickels. _____
4. Some 5-cent candy bars and 7-cent packs of gum cost $1.14. _____
5. It took 96¢ in 6-cent and 10-cent stamps to mail this package. _____
6. A club spent $79.00 on 2-dollar 10-yard line tickets and 3-dollar 50-yard line tickets. _____
7. Skin is 2 years younger than Thin. _____
8. A father is 23 years older than his son. _____
9. A year ago Trebor was 5 times as old as Hi-Ho. _____
10. Three years ago Nathan's sister was twice as old as he was. _____
11. In 17 years Mr. Betta will be twice as old as his son Alpha. _____

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DIRECTIONS: For each problem write 2 equations in 2 unknowns that describe the conditions on the numbers, coins, ages, or whatever. There isn't time to solve each pair of equations to find the numbers, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

$EXAMPLE_1$: One number is 3 less than another. Three times the smaller number is the same as twice the larger. What are they? x=y-3 [or x+3=y]; 3x = 2y

$EXAMPLE_2$: A boy has a pocketful of quarters and nickels worth $1.05. If he has twice as many nickels as quarters, how many of each coin does he have in his pocket? 5n + 25q = 105; n = 2q

$EXAMPLE_3$: Micah is 3 years younger than Kimberly. In 1 year Kimberly will be twice as old as Micah. How old is Kimberly? K-3=M [or K=M+3]; K+1 = 2(M+1)

1. One number exceeds another by 3. The larger number decreased by 1 equals twice the smaller. What is the smaller number? _____

2. Twice one number is one less than another. Their difference is 4. What are they? _____

3. The difference between two numbers is 6. The larger number is 4 more than three times the smaller. Find the numbers. _____

4. A boy has a pocketful of dimes and nickels whose value is $3.55. He has 7 more dimes than nickels. How many dimes and how many nickels does he have? _____

5. An assortment of pennies and dimes is worth $2.64. If there are just as many pennies as dimes, how many of each kind are there? _____

6. In one day a meter maid collected 1104 coins in pennies and nickels. If the total value of the coins was $35.00, how many pennies did she collect? _____

7. The admission price to a local high school football game was 60¢ for adults and 15¢ for students. If the gate receipts from 7240 paid admissions amounted to $3394.50, how many adults attended the game? _____

8. Rick has twice as many pennies as Tick has nickels, but Tick has $2.55 more than Rick. How much money does Tick have? _____

9. Jill has 2 more dimes than Jack has nickels. If the value of Jill's coins exceeds the value of Jack's by $1.10, how many dimes does Jill have? _____

10. A man is five times as old as his son. In 5 years he will be three times as old as his son. Find the ages of the man and his son. _____

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11. A father is 23 years older than his son. Eleven years from now he will be twice as old as his son. How old is the man's son? _____

12. Professor Kool is three times as old as Rad. In 20 years Professor Kool will be twice as old as Rad. How old is Professor Kool? _____

13. John is twice as old as his brother Bob. Five years ago John was three times as old as Bob. What are the ages of the two brothers? _____

14. The sum of This's age and That's age is 56. In 2 years That will be twice as old as This. Find the ages of This and That. _____

15. Three years ago Jack's age was 1 year more than twice Jill's age. Six years from now Jack's age will be 10 more than half Jill's age. How old is Jack? _____

1. $x=y+3$ [or $x-3=y$]
   $x-1=2y$

3. $m-n=6$
   $m=3n+4$

5. $p+10d=264$
   $p=d$

7. $s+d=7240$
   $15s+60d=33945O$

9. $d=n+2$
   $10d=5n+11O$

11. $F=S+23$
    $F+11=2(S+11)$

13. $J=2B$
    $J-5=3(B-5)$

15. $J_a-3=2(J_i-3)+1$
    $J_a+6=1/2(J_i+6)+10$

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# APPENDIX C

INSTRUCTIONAL MATERIALS FOR WORK AND MIXTURE PROBLEMS. THE UNITS ARE LABELED W, M FOR WORK AND MIXTURE, RESPECTIVELY. THE UNIT ON WORK PRECEDED THAT FOR MIXTURE. TWO DAYS WERE ALLOTTED FOR EACH UNIT.

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NAME_____ TEACHER_____ $W_{1-2}$

Consider the following problems, their corresponding equations, and the numbers obtained from the equations:

| PROBLEM | EQUATIONS | EQUATIONS SOLVED |
|---|---|---|
| Number: The sum of two numbers is 7. Their product is 10. What are they? | $m + n = 7$<br>$mn = 10$ | $m = 2, n = 5$ |
| Coin: An assortment of 7 coins is worth \$1.15. If the coins are dimes and quarters, how many of each are there? | $d + q = 7$<br>$10d+25q=115$ | $d = 4, q = 3$ |
| Age: Kim is four times as old as Micah. In 1 year Kim will be three times as old as Micah. How old are they now? | $K = 4M$<br>$K+1 = 3(M+1)$ | $K = 8, M = 2$ |

By substituting the numbers from the "equations solved" column for the indicated letters in the pairs of equations, you can see that they satisfy the conditions stated in the problems and indicated in the equations. For example, substituting 2 and 5 for m and n, respectively, in the first pair of equations; we see that $(2) + (5) = 7$ and $(2)(5) = 10$, as required. Continue this checking process for the remaining pairs of equations. Coin:_____
Age:_____

Not only do these numbers answer the questions in the problems, but they do so exactly. That is, for the number problem the smaller number really is 2, and not something like 1.999; and the larger number really is 5, and not something like 5.001. Similarly, the numbers of dimes and quarters for the coin problem really are 4 and 3, respectively; and so on.

In contrast, the answers to some algebra word problems only approximate the unknowns. For example, certain so called "work" problems in algebra cannot be answered exactly. To illustrate, suppose either of two boys can mow a lawn in 2 hours. Working together, then, one might expect them to complete the job in half the time, or 1 hour. However, in practice the two would probably spend some time visiting with each other, thus taking more than an hour; or make a contest of it, thus taking less than an hour. Therefore, contrary to numbers, coins, and ages; human effort is rarely additive (that is, the rate at which a group works is seldom the sum of the work rates of the individuals that make up the group). Nevertheless, in order to deal with work algebraically we assume that individuals' work rates are additive, which is all right so long as an answer to a work problem is regarded as an approximation of the real situation.

With the premise that work rates are additive, equations corresponding to work problems are easily written. Examine the following examples carefully and complete the accompanying fill-ins with your teacher.

$EXAMPLE_1$: Winston can shovel the snow from a sidewalk in 4 minutes, and Winner can do it in 12 minutes. How long will it take them to shovel the snow from the sidewalk if they work together?

The job-completed equation corresponding to example 1 is simply $m/4 + m/12 = 1$, where m stands for the number of minutes both boys working together would need to complete the job. In other words, if you were asked to translate this problem, all you would have to write is $m/4 + m/12 = 1$. However, since the form of this translation is somewhat different from what you have encountered so far, what with the fractions and all, an explanation of its derivation follows.

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The mathematical characteristics of a work problem can be illustrated with shaded rectangles where the rectangles designate the job and the shaded areas the portion of the work still to be completed. To illustrate, for the snow shoveling problem (example 1) the rectangles portray the sidewalk to be cleared, and their darkened regions illustrate the snow remaining on the sidewalk (see figures 1-6). By referring to such visual interpretations given below for example 1, you should find it easy to answer the questions that follow.

Since it takes Winston 4 minutes to clear the sidewalk, he would clear 1/4 of it <u>in 1 minute</u> (see figure 2).

How much of the job would Winston complete in 2 minutes?_____
3 minutes?_____ 4 minutes?_____ m minutes?_____

Winner, requiring 12 minutes to shovel the snow from the sidewalk, would clear 1/12 of it <u>in 1 minute</u> (see figure 3).

How much of the job would Winner complete in 2 minutes?_____
3 minutes?_____ 4 minutes?_____ 6 minutes?_____
8 minutes?_____ 9 minutes?_____ 10 minutes?_____
12 minutes?_____ m minutes?_____

Therefore, if we assume that human work rates are additive, Winston and Winner working together would clear 1/4 + 1/12 (=4/12) of the sidewalk <u>in 1 minute</u> (see figure 4).

How much of the job would Winston and Winner working together complete in 2 minutes? (see figure 5)_____(=?/12)
3 minutes? (see figure 6)_____(=?/12) m minutes?
_____

Snow-covered sidewalk

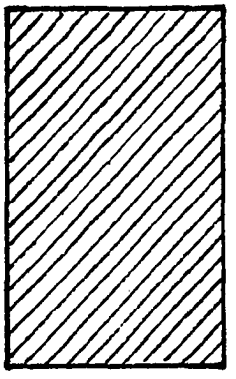

figure 1

Amount of snow Winston can remove <u>in 1 minute</u> if he can complete the job in 4 minutes

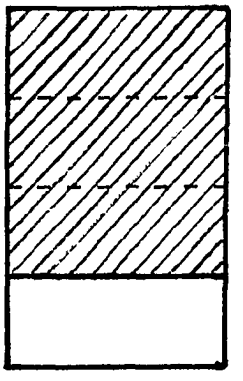

figure 2

Amount of snow Winner can remove <u>in 1 minute</u> if he can complete the job in 12 minutes

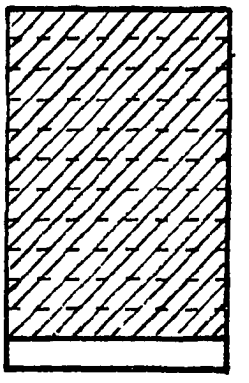

figure 3

Amount of snow both can remove <u>in 1 minute</u>

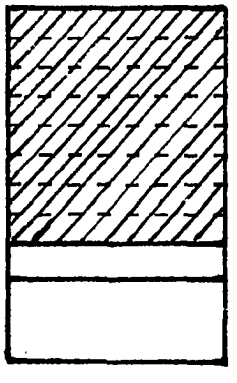

figure 4

Amount of snow both can remove <u>in 2 minutes</u>

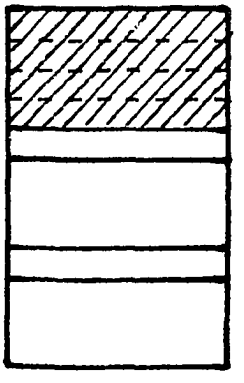

figure 5

Amount of snow both can remove <u>in 3 minutes</u>

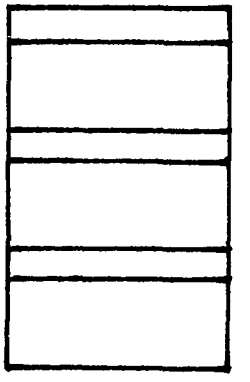

figure 6

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Now examine the following equations that can be written for the snow shoveling exercise as the minutes tick off with both boys working together. (These equations indicate the portion of the job completed for a given interval of time.)

After 1 minute: $1/4 + 1/12 = 4/12$

After 2 minutes: $2/4 + 2/12 = 8/12$

After 3 minutes: $3/4 + 3/12 = 12/12 = 1$, and the job is completed.

What do the numerators 1 and 1 in the first equation tell us?_____

the numerators 2 and 2 in the second equation?_____

the numerators 3 and 3 in the third equation?_____

What does the denominator 4 in each equation tell us?_____

the denominator 12 on the left-hand side of each equation?_____

Choose one of the fractions on the left-hand side of one of the equations and explain its meaning. Fraction_____ Meaning_____

Choose one of the fractions on the right-hand side of one of the equations and explain its meaning. Fraction_____ Meaning_____

What is the significance of the $12/12 = 1$ in the third equation? _____

Now suppose that we did not know that Winston and Winner could complete the job together in 3 minutes. Then, letting m represent the time in minutes it would take them working together to complete the job, do you see why the job-completed equation for example 1 is $m/4 + m/12 = 1$?

$$m/4 + m/12 = 1$$

Portion of job Winston completes in m minutes

Portion of job Winner completes in m minutes

Since m represents the total time required for both boys working together to complete the job, the sum of their efforts in m minutes would leave no part of the job unfinished.

EXAMPLE 2: A large pipe can fill a tank with water in 5 hours, and a smaller one can drain it in 8 hours. With the drain open and the tank empty, how long will it take to fill the tank?

Just as before, the job in example 2 can be visualized in terms of shaded rectangles—except this time the shaded areas will represent that portion of the work already completed. For the tank problem, then, the rectangles stand for the tank to be filled, and the darkened regions of the rectangles illustrate the amount of water in the tank (see figures 1-4).

Since the larger pipe fills the entire tank in 5 hours, it would fill 1/5 of it in 1 hour (see figure 2).

How much of the tank would the larger pipe fill in 2 hours?_____
3 hours?_____ 4 hours?_____ 5 hours?_____ h hours?_____

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The smaller pipe, requiring 8 hours to drain the tank, would empty 1/8 of it in 1 hour (see figure 3).

How much of the tank would the smaller pipe drain in 2 hours?_____
4 hours?_____ 5 hours?_____ 8 hours?_____ h hours?_____

Therefore, if we assume that drain rates are additive (or subtractive if you prefer), the two pipes operating together would fill 1/5 - 1/8 (=3/40) of the tank in 1 hour (see figure 4).

How much of the tank would be filled with both pipes operating for 2 hours?_____(=?/40) 5 hours?_____(=?/40)
10 hours?_____(=?/40) 13 hours?_____(=?/40)
h hours?_____

And if we let h represent the number of hours it would take to fill the tank with the "faucet" and drain working simultaneously, the job-completed equation would be_____. Why?

Empty tank

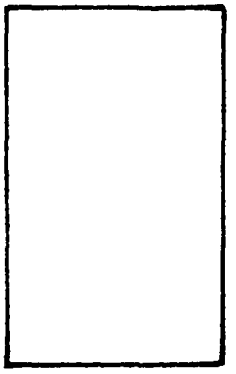

figure 1

Amount of water large pipe can put in tank in 1 hour

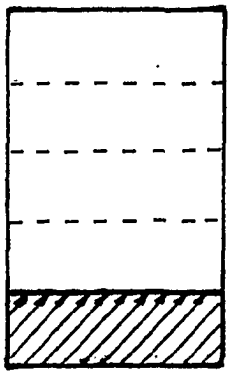

figure 2

Amount of water small pipe can drain in 1 hour

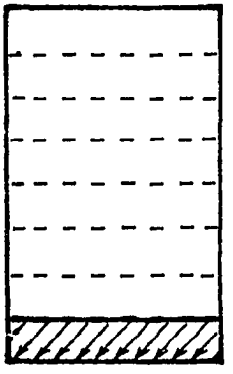

figure 3

Amount of water in tank in 1 hour with the "faucet" on and the drain open

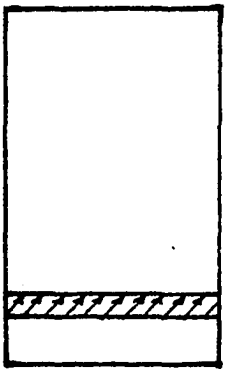

figure 4

DIRECTIONS: For each problem write a job-completed equation. There isn't time to solve each of these equations in order to find the amount of time required to complete a job, so don't worry about answering the question asked in the problems—the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

1. Mot can mow a lawn in 2 hours. Tap, who is younger and uses a smaller mower, requires 5 hours to mow the same lawn. In how many hours will the boys finish the lawn if they work together?_____

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2. Jefe can complete an order in 7 hours, and Peon can do it in 5 hours. How long will it take Jefe and Peon working together to fill the order?_____

3. How long would it take Enoch and Frog to perform a job together if, when working alone, Enoch takes 12 weeks and Frog 8 weeks?_____

4. Mr. Fallin is building a basement. To remove the dirt he and Ole Mole are using a large diesel shovel and a small one, respectively. If Mr. Fallin can do the job in 6 days and Ole Mole can do it in 9, how long will it take them together?_____

5. Mr. Foyer estimates he can paint Mrs. Hall's fence alone in 6 hours. Mr. Cubby, one of Mr. Foyer's assistants, can paint it alone in 8 hours. How long will it take Mr. Foyer and Mr. Cubby working together to paint Mrs. Hall's fence?

_____

6. A high-speed press can print one day's newspapers in 4 hours. Another press takes twice as much time to do the same job. How fast can both presses together do the printing?_____

7. A bathtub can be filled in 4 minutes and emptied in 5. If the tub is empty and the drain is left open, how long will it take to fill the tub?_____

8. The hot water faucet fills a tub in 40 minutes, and the cold water faucet fills it in 30 minutes. The tub can be drained in 20 minutes. If both faucets are on while the drain is open, how soon will the tub be full?

_____

9. A swimming pool has two inlet pipes. One fills the pool in 3 hours, the other in 6 hours. The outlet pipe empties the pool in 4 hours. Once the outlet pipe was left open while both inlet pipes were being used to fill the pool. In how many hours was the pool full?_____

10. Farmer Jack has two tractors. With the larger he can plow a field in 12 days. If he and a neighbor run both tractors, they can plow the same field in 8 days. How long would it take Farmer Jack to plow the field with the smaller tractor?_____

11. A pool empties in 6 hours through a newly installed copper drain. When this drain is used along with the factory installed cast-iron model, the pool empties in 4 hours. How long would it take the cast-iron drain alone to empty the pool?_____

12. A refrigeration unit lowers the temperature 10 degrees in 12 minutes. With a second unit also working, this change takes 4 minutes. How long would it take the second unit working alone to lower the temperature 10 degrees?

_____

13. One pump fills a tank twice as fast as another. If together they fill the tank in 16 minutes, how long does the smaller pump take?_____

14. One bulldozer clears land three times as fast as another. Together they clear a field in 5 hours. How much time would the larger bulldozer need to clear the field alone?_____

1. $h/2 + h/5 = 1$ 3. $w/12 + w/8 = 1$ 5. $h/6 + h/8 = 1$ 7. $m/4 - m/5 = 1$
9. $h/3 + h/6 - h/4 = 1$ 11. $4/6 + 4/h = 1$ 13. $16/m + 16/2m = 1$

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NAME_____ TEACHER_____ $M_{1-2}$

The problems you will be working with for the next two days involve mixtures of various quantities. In each case the mixture is a blend whose value differs from the values of the original quantities. Since the value of this mixture is the sum of the values of the quantities comprising it, these problems are very similar to coin problems.

For example, compare the translations of the mixture problem and coin problem that follow:

| | |
|---|---|
| How many pounds of 5-cent nudge should be mixed with 10-cent divinity to give a 15 pound grab-bag mixture worth 7¢ a pound? | n + d = 15 (n represents the number of pounds of nudge, d the number of pounds of divinity)<br>5n + 10d = 7(15) = 105 |
| Fifteen nickels and dimes are worth $1.30. How many of each are there? | n + d = 15 (n represents the number of nickels, d the number of dimes)<br>5n + 10d = 130 |

As you can see, the translations are identical except for the underlined portions; so much of what you have learned already from working with coin problems will apply to mixture problems.

Compute or indicate algebraically:

1. Suppose that Welmax coffee sells for 75¢ a pound. What would 3 pounds of Welmax coffee cost?_____ 10 pounds?_____ 100 pounds?_____ W pounds?_____

2. If cashews sell for $1.90 a pound, how much would 2 pounds cost?_____ 10 pounds?_____ 100 pounds?_____ c pounds?_____

3. If Suggitt tea is valued at 80 cents per ounce, how much are 5 ounces worth?_____ 50 ounces?_____ S ounces?_____

4. An almond-cashew mixture weighs 38 pounds. If it contains 15 pounds of almonds, how many pounds of cashews does it contain?_____

5. If a mixture contains 25 pounds of candy worth 70¢ a pound and 75 pounds of candy worth 90¢ a pound, the _____ pound mixture is worth $85.00.

6. A grocer makes 70 pounds of a coffee blend to sell at 85¢ per pound by mixing 30 pounds of a 90-cents-a-pound grade with _____ pounds of an 81 and 1/4-cents-a-pound grade.

Compute and write equations, both numerical and algebraic, as indicated:

1. A mixture contains 25 pounds of coffee worth 70 cents a pound and 75 pounds of coffee worth 90 cents a pound. How many pounds are in the mixture?_____ Write the equation you used to compute this number._____ How much is the mixture worth?_____ What is the value of one pound of this mixture?_____ Use this number to write a total value equation._____ Rewrite your two numerical equations algebraically using c for the number of pounds of cheaper coffee and e for the number of pounds of expensive coffee._____ _____

2. A mixture contains 40 pounds of tea worth 70 cents a pound and 10 pounds of tea worth 90 cents a pound. How many pounds are in the mixture?_____ Write the equation you used to compute this number._____ How much is the mixture worth?_____ What is the value of one pound of

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this mixture?_____ Use this number to write a total value equation._____ Rewrite your two numerical equations algebraically using c for the number of pounds of cheaper tea and e for the number of pounds of expensive tea._____ _____

3. A coffee blend is made using 30 pounds of an 80-cents-a-pound grade and 20 pounds of an 85-cents-a-pound grade. How many pounds are in the mixture?_____ Write the equation you used to compute this number._____ How much is the mixture worth?_____ What is the value of one pound of this mixture?_____ Use this number to write a total value equation._____ Rewrite your two numerical equations algebraically using c for the number of pounds of cheaper coffee and e for the number of pounds of expensive coffee._____ _____

4. Eight hundred gallons of gasoline valued at 31 cents per gallon are mixed with 200 gallons of gasoline valued at 36 cents per gallon. How many gallons are in the mixture?_____ Write the equation you used to compute this number._____ How much is the mixture worth?_____ What is the value of one gallon of this mixture?_____ Use this number to write a total value equation._____ Rewrite your two numerical equations algebraically using c for the number of gallons of cheaper gasoline and e for the number of gallons of expensive gasoline. _____ _____

5. A blend of "delicious herbs and spices" is made by a Southern gentleman who mixes 100 pounds of a grade of grass worth 90¢ a pound with 100 pounds of a grade of bark worth $1.60 a pound. How many pounds are in the mixture?_____ Write the equation you used to compute this number._____ How much is the mixture worth?_____ What is the value of one pound of this mixture?_____ Use this number to write a total value equation._____ Rewrite your two numerical equations algebraically using g for the number of pounds of grass and b for the number of pounds of bark. _____ _____

DIRECTIONS: For each problem write 2 equations in 2 unknowns that describe the conditions on the quantities. There isn't time to solve each pair of equations to find the number of pounds of coffee or whatever, so don't worry about answering the question asked in the problems--the equations are enough of an answer. The answers to the odd numbered problems have been provided as a check.

1. Mr. Gruel mixes coffee worth 55¢ a pound with coffee worth 95¢ a pound. How much of each must he use to get 10 pounds of coffee that will sell for 85¢ per pound? _____ _____

2. Coz wants to make a mixture of 60 pounds of candy to sell at 35¢ a pound. He has candy worth 30¢ a pound and some worth 45¢ a pound. How many pounds of each does he need to use? _____ _____

3. How many pounds of sugar at 60¢ a pound must be mixed with sugar at 75¢ a pound to make a 50 pound mixture worth 70¢ a pound? _____ _____

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4. Some ancespitorian people decide to raise exactly $220 for their general fund by selling 1000 posies of two kinds. If some are to sell for 20¢ apiece and the rest for 25¢ apiece, how many of each kind are needed?* _____ _____

5. A school bookstore sold 348 notebooks the first day of school, some at 25 cents each, the rest at 38 cents each. The total receipts for notebooks was $100.91. How many of each kind were sold? _____ _____

6. Kimberly purchased 100 items in a stationery store for 95¢. She bought erasers at 5¢ each and clips at two-for-a-penny. How many of each did Kimberly buy? _____ _____

7. A nut shop sells almonds for $1.80 a pound and peanuts for 65¢ a pound. The shopkeeper makes a mixture of 30 pounds of these nuts to sell for $1.00 a pound. How much of each does he use? _____ _____

8. How many gallons of apple cider at $1.12 a gallon and how many gallons of another brand at 79¢ a gallon must be mixed to make 100 gallons to sell for 89¢ a gallon? _____ _____

9. A confectioner is making a mixture of almonds and cashews. The cashews are worth $1.90 a pound, and the almonds are worth $2.10 a pound. How many pounds of each kind of nut should he use to make 30 pounds of a mixture worth $2.00 a pound? _____ _____

10. A grocer sells two kinds of tea, the first for 60¢ an ounce and the other for 90¢ an ounce. Since he is often bothered with the question, "What's the difference?"; he decides to mix the two and have just one price. How much of each kind would he have to use in order to get 120 pounds of tea worth 80¢ an ounce? _____ _____

*This is a coin problem. Can you find two other coin problems in this problem set?

1. $c+e=10,\ 55c + 95e = 85(10)$ 3. $c+e=50,\ 60c + 75e = 70(50)$
5. $c+e=348,\ 25c + 38e = 10091$ 7. $p+a=30,\ 65p + 180a = 100(30)$
9. $c+a=30,\ 190c + 210a = 200(30)$

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# APPENDIX D

## INSTRUMENTATION AND COMPUTATION OF RELIABILITY COEFFICIENTS.



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NAME_____ TEACHER_____ $T_1$

This is a timed 8-minute test to see how well you remember how to translate certain types of algebra word problems into mathematical symbols. When your teacher tells you to begin, write 2 equations in 2 unknowns for each problem that correspond to the stated conditions on the numbers, coins, ages, or whatever. Do not solve the equations or attempt to answer any of the questions asked in the problems. Use the bottom of this page for scratchwork.

1. One number is 7 more than another. Their sum is 52. What are they? _____ _____

2. The difference between two numbers is 3. Three times one of them is 2 less than twice the other. What are they? _____ _____

3. A boy received four times as many dimes as quarters in change. If their total value was $1.95, how many coins did he receive? _____ _____

4. A post office sold 70 stamps to a lady for $5.12. If some were 6-cent (regular) stamps and the rest were 10-cent (air mail) stamps, how many of each did she buy? _____ _____

5. The sum of Luap's and Trebor's ages is 43. In 1 year Luap will be twice as old as Trebor. How old is Luap? _____ _____

6. Hi is 6 years older than Fi. Three years ago Hi was three times as old as Fi. How old is Hi? _____ _____

INITIAL PROBLEMS TEST

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NAME_____ TEACHER_____ $T_2$

Except for the work problems, translate each of the following into 2 equations in 2 unknowns. (Translate the work problems into job-completed equations.) Do not attempt to answer the questions asked in the problems. After completing this page, read the directions on the next one and continue working. When you are finished turn your paper over on your desk until your teacher asks for it. Use the margins and the space provided for scratch work.

1. One number is three times another. Their difference is 26. What are they? _____ _____

2. Some dimes and quarters are worth $4.20. If there are just as many dimes as quarters, how many of each are there? _____ _____

3. Luap is 11 years older than Mot. In 5 years he will be twice as old as Mot. How old is Luap? _____ _____

4. Jefe can paint a fence in 10 hours, and Peon can paint the same fence in 8 hours. How long will it take them working together to paint the fence? _____

5. Mr. Toil wishes to blend coffee worth 60¢ a pound with coffee worth 85¢ a pound. How much of each must he use to get 50 pounds of coffee worth 75¢ a pound? _____ _____

6. The sum of two numbers is 33. The larger is 3 more than four times the smaller. Find the numbers. _____ _____

7. A post office sold 7 less 10-cent (air mail) stamps than 6-cent (regular) stamps. If the stamps cost $1.70, how many of each were sold? _____ _____

8. Bonnie is 5 times as old as Clyde. Two years ago she was 7 times as old as Clyde. How old is Bonnie? _____ _____

9. A bathtub can be filled in 6 minutes and emptied in 9. If the tub is empty and the drain is left open, how long will it take to fill the tub? _____

10. A nut shop sells cashews for $1.90 a pound and peanuts for 65¢ a pound. The shopkeeper makes a mixture of 25 pounds of these nuts to sell for $1.20 a pound. How many pounds of each does he use? _____ _____

FINAL PROBLEMS TEST

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FINAL PROBLEMS TEST RELIABILITY COEFFICIENT, GRADE 7

| ITEM NUMBER | NUMBER WRONG OR OMITTED IN LOWEST 27 PER CENT GROUP: $W_L$ | NUMBER WRONG OR OMITTED IN HIGHEST 27 PER CENT GROUP: $W_H$ | $W_L - W_H$ | $W_L + W_H$ | DISCRIMINATION INDEX: $\frac{W_L - W_H}{N^*}$ | DIFFICULTY INDEX: $\frac{W_L + W_H}{2N}$ | DIFFICULTY INDICES SQUARED |
|---|---|---|---|---|---|---|---|
| 1(A) | 7 | 0 | 7 | 7 | 7/13 | 7/26 | 49/676 |
| 1(B) | 8 | 0 | 8 | 8 | 8/13 | 8/26 | 64/676 |
| 2(A) | 9 | 4 | 5 | 13 | 5/13 | 13/26 | 169/676 |
| 2(B) | 11 | 0 | 11 | 11 | 11/13 | 11/26 | 121/676 |
| 3(A) | 9 | 0 | 9 | 9 | 9/13 | 9/26 | 81/676 |
| 3(B) | 11 | 1 | 10 | 12 | 10/13 | 12/26 | 144/676 |
| 4 | 13 | 3 | 10 | 16 | 10/13 | 16/26 | 256/676 |
| 5(A) | 13 | 0 | 13 | 13 | 13/13 | 13/26 | 169/676 |
| 5(B) | 13 | 2 | 11 | 15 | 11/13 | 15/26 | 225/676 |
| 6(A) | 7 | 0 | 7 | 7 | 7/13 | 7/26 | 49/676 |
| 6(B) | 13 | 10 | 3 | 23 | 3/13 | 23/26 | 529/676 |
| 7(A) | 12 | 4 | 8 | 16 | 8/13 | 16/26 | 256/676 |
| 7(B) | 12 | 2 | 10 | 14 | 10/13 | 14/26 | 196/676 |
| 8(A) | 6 | 2 | 4 | 8 | 4/13 | 8/26 | 64/676 |
| 8(B) | 12 | 2 | 10 | 14 | 10/13 | 14/26 | 196/676 |
| 9 | 13 | 8 | 5 | 21 | 5/13 | 21/26 | 441/676 |
| 10(A) | 13 | 0 | 13 | 13 | 13/13 | 13/26 | 169/676 |
| 10(B) | 13 | 1 | 12 | 14 | 12/13 | 14/26 | 196/676 |
| SUM | | | | | 156/13 | 234/26 | 3374/676 |

*FOR THE KUDER-RICHARDSON FORMULA N IS 27% OF THE NUMBER OF STUDENTS TAKING THE TEST. IN THIS CASE, HOWEVER, N WAS FIRST REDUCED BY 5/6 ON THE BASIS OF FINAL PROBLEMS TEST SCORES IN THE FOLLOWING MANNER: IF, FOR EXAMPLE, EIGHTEEN STUDENTS MISSED TEN POINTS AND TWELVE MISSED NINE, THEN THREE (=18/6) AND TWO (=12/6), RESPECTIVELY, WERE SELECTED RANDOMLY FOR THE SAMPLE POPULATION. THEREFORE, N = 0.27(N/6). (K = NUMBER OF TEST ITEMS)

$$KR = \frac{K}{K-1}\left[1 - \frac{6(\text{SUM OF DIFFICULTY INDICES} - \text{SUM OF SQUARED DIFFICULTY INDICES})}{(\text{SUM OF DISCRIMINATION INDICES})^2}\right] = .88$$

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FINAL PROBLEMS TEST RELIABILITY COEFFICIENT, GRADE 9

| ITEM NUMBER | NUMBER WRONG OR OMITTED IN LOWEST 27 PER CENT GROUP: $W_L$ | NUMBER WRONG OR OMITTED IN HIGHEST 27 PER CENT GROUP: $W_H$ | $W_L - W_H$ | $W_L + W_H$ | DISCRIMINATION INDEX: $\frac{W_L - W_H}{N^*}$ | DIFFICULTY INDEX: $\frac{W_L + W_H}{2N}$ | DIFFICULTY INDICES SQUARED |
|---|---|---|---|---|---|---|---|
| 1(A) | 4 | 0 | 4 | 4 | 4/13 | 4/26 | 16/676 |
| 1(B) | 9 | 2 | 7 | 11 | 7/13 | 11/26 | 121/676 |
| 2(A) | 8 | 1 | 7 | 9 | 7/13 | 9/26 | 81/676 |
| 2(B) | 4 | 1 | 3 | 5 | 3/13 | 5/26 | 25/676 |
| 3(A) | 5 | 0 | 5 | 5 | 5/13 | 5/26 | 25/676 |
| 3(B) | 6 | 0 | 6 | 6 | 6/13 | 6/26 | 36/676 |
| 4 | 11 | 2 | 9 | 13 | 9/13 | 13/26 | 169/676 |
| 5(A) | 6 | 0 | 6 | 6 | 6/13 | 6/26 | 36/676 |
| 5(B) | 6 | 0 | 6 | 6 | 6/13 | 6/26 | 36/676 |
| 6(A) | 3 | 0 | 3 | 3 | 3/13 | 3/26 | 9/676 |
| 6(B) | 10 | 2 | 8 | 12 | 8/13 | 12/26 | 144/676 |
| 7(A) | 8 | 3 | 5 | 11 | 5/13 | 11/26 | 121/676 |
| 7(B) | 7 | 1 | 6 | 8 | 6/13 | 8/26 | 64/676 |
| 8(A) | 3 | 0 | 3 | 3 | 3/13 | 3/26 | 9/676 |
| 8(B) | 8 | 0 | 8 | 8 | 8/13 | 8/26 | 64/676 |
| 9 | 13 | 6 | 7 | 19 | 7/13 | 19/26 | 361/676 |
| 10(A) | 7 | 0 | 7 | 7 | 7/13 | 7/26 | 49/676 |
| 10(B) | 7 | 0 | 7 | 7 | 7/13 | 7/26 | 49/676 |
| SUM | | | | | 107/13 | 143/26 | 1415/676 |

*FOR THE KUDER-RICHARDSON FORMULA N IS 27% OF THE NUMBER OF STUDENTS TAKING THE TEST. IN THIS CASE, HOWEVER, N WAS FIRST REDUCED BY 4/5 ON THE BASIS OF FINAL PROBLEMS TEST SCORES IN THE FOLLOWING MANNER: IF, FOR EXAMPLE, FIFTEEN STUDENTS MISSED TWELVE POINTS AND TEN MISSED ELEVEN, THEN THREE (=15/5) AND TWO (=10/5), RESPECTIVELY, WERE SELECTED RANDOMLY FOR THE SAMPLE POPULATION. THEREFORE, $N = 0.27(N/6)$. (K = NUMBER OF TEST ITEMS)

$$KR = \frac{K}{K-1}\left[1 - \frac{6(\text{SUM OF DIFFICULTY INDICES} - \text{SUM OF SQUARED DIFFICULTY INDICES})}{(\text{SUM OF DISCRIMINATION INDICES})^2}\right] = .74$$

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Write equations for each of the following. These problems are somewhat different from the preceding ones, but they can be translated in pretty much the same fashion as those you have been working with. As before, do not attempt to answer the questions asked in the problems. If you wish to comment on any of the problems, do so to the right of the "equations" box.

11. Some gold bars weigh 30 pounds each, and some silver bars weigh 25 pounds each. There are 4 more silver bars than gold bars, and the total weight of the bars is 1035 pounds. How many of each are there?

scratch work | equations

12. The sum of the ages of Luap, Mot, and Nhoj is 28. Two years ago Nhoj's age was equal to the sum of Luap's and Mot's ages then. In 1 year Nhoj will be twice as old as Luap will be. How old are Luap, Mot, and Nhoj?

scratch work | equations

13. A bottle and a cork together cost $1.05. The bottle costs exactly $1.00 more than the cork. How much does the cork cost?

scratch work | equations

14. A board was cut into two pieces. One piece was three times as long as the other, and the difference between their lengths was 4 feet. How long was each piece?

scratch work | equations

15. The length of a rectangular field is 38 feet more than its width. It takes 728 feet of fencing to enclose the field. Find the length and width of the field.

scratch work | equations

16. A man has three times as many quarters as he has dimes. The value of the dimes exceeds the value of the quarters by 40¢. How many has he of each coin?

scratch work | equations



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NAME_____ TEACHER_____ $T_3$

# OPINIONNAIRE

DIRECTIONS: Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to express on a five-point scale the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are: Strongly disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly agree (SA). You are to encircle the letter which best indicates how closely you agree or disagree with the feeling expressed by each statement as it concerns you.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | I do not like mathematics. I am always under a terible strain in a math class. | SD | D | U | A | SA |
| 2. | I do not like mathematics, and it scares me to have to take it. | SD | D | U | A | SA |
| 3. | Mathematics is very interesting to me. I enjoy math courses. | SD | D | U | A | SA |
| 4. | Mathematics is fascinating and fun. | SD | D | U | A | SA |
| 5. | Mathematics makes me feel secure, and at the same time it is stimulating. | SD | D | U | A | SA |
| 6. | I do not like mathematics. My mind goes blank, and I am unable to think clearly when working math. | SD | D | U | A | SA |
| 7. | I feel a sense of insecurity when attempting mathematics. | SD | D | U | A | SA |
| 8. | Mathematics makes me feel uncomfortable, restless, irritable, and impatient. | SD | D | U | A | SA |
| 9. | The feeling that I have toward mathematics is a good feeling. | SD | D | U | A | SA |
| 10. | Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out. | SD | D | U | A | SA |

ATTITUDE

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| | | | | | | |
|---|---|---|---|---|---|---|
| 11. | Mathematics is something which I enjoy a great deal. | SD | D | U | A | SA |
| 12. | When I hear the word math, I have a feeling of dislike. | SD | D | U | A | SA |
| 13. | I approach math with a feeling of hesitation — hesitation resulting from a fear of not being able to do math. | SD | D | U | A | SA |
| 14. | I really like mathematics. | SD | D | U | A | SA |
| 15. | Mathematics is a course in school which I have always liked and enjoyed studying. | SD | D | U | A | SA |
| 16. | I don't like mathematics. It makes me nervous to even think about having to do a math problem. | SD | D | U | A | SA |
| 17. | I have never liked math, and it is my most dreaded subject. | SD | D | U | A | SA |
| 18. | I love mathematics. I am happier in a math class than in any other class. | SD | D | U | A | SA |
| 19. | I feel at ease in mathematics, and I like it very much. | SD | D | U | A | SA |
| 20. | I feel a definite positive reaction to mathematics; it's enjoyable. | SD | D | U | A | SA |

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Select any two of the four problems below and solve them. Show all of your work within the boxes.

Multiply:

$$\begin{array}{r} 937 \\ \times\ 548 \\ \hline \end{array}$$

Translate into 2 equations in 2 unknowns: Hi-Ho is 8 years older than Trebor. Two years ago he was five times as old as Trebor. How old is Hi-Ho?

Translate into 2 equations in 2 unknowns: A piggy bank contains 4 less dimes than nickels. If the coins are worth $2.15, how many of each are in the bank?

Add:

$$\begin{array}{r} 74956 \\ +\ 54628 \\ \hline \end{array}$$



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Select any two of the four problems below and solve them. Show all of your work within the boxes.

Solve for x:

$$13x + 18 = 96$$

Translate into 2 equations in 2 unknowns: Hi-Ho is 8 years older than Trebor. Two years ago he was five times as old as Trebor. How old is Hi-Ho?

Translate into 2 equations in 2 unknowns: A piggy bank contains 4 less dimes than nickels. If the coins are worth $2.15, how many of each are in the bank?

Combine like terms:

$$79p - 42q - 26r + 15q + 37r$$



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Answer as many of the following as you can in the remaining time. Do not spend too much time on any one question. When in doubt, guess. (There is no penalty for guessing.)

Express the following phrases algebraically:

1. The sum of 5n and 10d _____
2. 7 more than x _____
3. x increased by 7 _____
4. The difference between x and 7 _____
5. x less than 7 _____
6. x decreased by 7 _____
7. The product of x and 7 _____
8. 7 times x _____
9. Twice x _____

Write an equation in 2 unknowns for each of the following sentences:

10. Seven times one number is 4 more than another. _____
11. The larger of two numbers is 7 less than 4 times the smaller. _____
12. Four times one number exceeds 7 times another by 1. _____

Either circle the correct answer or indicate the answer algebraically:

13. What is the total value in cents of d dimes and q quarters? _____
14. How many stamps in all do you have if you have r 6-cent regular stamps and s 45-cent special delivery stamps? _____
15. Which is worth more, d dimes or q quarters? d dimes / q quarters / can't tell
16. One boy has n nickels and another boy has d dimes. If the value of the dimes is equal to the value of the nickels, who has the most coins? boy with n nickels / boy with d dimes / can't tell
17. A girl has 3 times as many nickels as dimes. Which are worth the most? all the nickels / all the dimes / can't tell
18. A girl has 4 times as many nickels as quarters. Which are worth the most? all the nickels / all the quarters / can't tell

Either compute or circle the correct answer:

19. The difference between the ages of a father and son is 24. What will the difference between their ages be in 10 years? _____
20. The sum of the ages of Enoch and Frog is 17. What was the sum of their ages 3 years ago? _____
21. A man who is 29 years old has a 6-year-old son. Will the father ever be just twice as old as the son? yes / no / can't tell
22. Thin is 12 and Skin is 6. Was Thin ever four times as old as Skin? yes / no / can't tell
23. Nathan is 5 and his brother Micah is 3. Will Nathan always be older than Micah? yes / no / can't tell



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Either compute, indicate the answer algebraically, or circle the correct answer:

24. Enoch can mow a lawn in 4 hours and Frog can mow the same lawn in 8 hours. How much of the lawn can both working together mow in 1 hour? _____

25. Trebor can clear a sidewalk of snow in 12 minutes. How much of the sidewalk can he clear in m minutes? _____

26. The hot and cold water faucets working together can fill a bathtub in 5 minutes. When completely full, the drain empties it in 4 minutes. If both faucets are on while the drain is open, will the bathtub ever fill up? yes no can't tell

27. A mixture contains c pounds of coffee worth 70¢ a pound and e pounds of coffee worth $1.10 a pound. How many pounds are in the mixture? _____

28. A tea blend is made using 3 pounds of an 80-cents-a-pound grade and e pounds of an 85-cents-a-pound grade. What is the total value in cents of this mixture? _____

29. A mixture of peanuts and cashews sells for $1.20 a pound. The peanuts sell for 60¢ a pound and the cashews for $1.70 a pound. Which does the mixture contain the most of? peanuts cashews can't tell

30. Are h ounces of herbs at 60¢ a pound worth more than s ounces of spices at 70¢ a pound? yes no can't tell

31. Ettolrahc mixed c gallons of gas that sells for 31 cents per gallon with e gallons of gas that sells for 36 cents per gallon. The blend sells for 33 cents per gallon. Find two ways to express the cost of all c + e gallons. _____ _____

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AUXILIARY REPRESENTATIONS QUESTIONNAIRE RELIABILITY COEFFICIENT, GRADE 7

| ITEM NUMBER | NUMBER WRONG OR OMITTED IN LOWEST 27 PER CENT GROUP: $W_L$ | NUMBER WRONG OR OMITTED IN HIGHEST 27 PER CENT GROUP: $W_H$ | $W_L - W_H$ | $W_L + W_H$ | DISCRIMINATION INDEX: $\frac{W_L - W_H}{N^*}$ | DIFFICULTY INDEX: $\frac{W_L + W_H}{2N}$ | DIFFICULTY INDICES SQUARED |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 0 | 3 | 3 | 3/12 | 3/24 | 9/576 |
| 2 | 6 | 1 | 5 | 7 | 5/12 | 7/24 | 49/576 |
| 3 | 4 | 1 | 3 | 5 | 3/12 | 5/24 | 25/576 |
| 4 | 7 | 0 | 7 | 7 | 7/12 | 7/24 | 49/576 |
| 5 | 9 | 2 | 7 | 11 | 7/12 | 11/24 | 121/576 |
| 6 | 8 | 0 | 8 | 8 | 8/12 | 8/24 | 64/576 |
| 7 | 4 | 1 | 3 | 5 | 3/12 | 5/24 | 25/576 |
| 8 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 9 | 6 | 1 | 5 | 7 | 5/12 | 7/24 | 49/576 |
| 10 | 9 | 3 | 6 | 12 | 6/12 | 12/24 | 144/576 |
| 11 | 11 | 6 | 5 | 17 | 5/12 | 17/24 | 289/576 |
| 12 | 10 | 3 | 7 | 13 | 7/12 | 13/24 | 169/576 |
| 13 | 8 | 3 | 5 | 11 | 5/12 | 11/24 | 121/576 |
| 14 | 8 | 4 | 4 | 12 | 4/12 | 12/24 | 144/576 |
| 15 | 7 | 1 | 6 | 8 | 6/12 | 8/24 | 64/576 |
| 16 | 4 | 0 | 4 | 4 | 4/12 | 4/24 | 16/576 |
| 17 | 3 | 2 | 1 | 5 | 1/12 | 5/24 | 25/576 |
| 18 | 3 | 1 | 2 | 4 | 2/12 | 4/24 | 16/576 |
| 19 | 9 | 3 | 6 | 12 | 6/12 | 12/24 | 144/576 |
| 20 | 9 | 7 | 2 | 16 | 2/12 | 16/24 | 256/576 |
| 21 | 9 | 9 | 0 | 18 | 0 | 18/24 | 324/576 |
| 22 | 9 | 1 | 8 | 10 | 8/12 | 10/24 | 100/576 |
| 23 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 24 | 7 | 10 | -3 | 17 | -3/12 | 17/24 | 289/576 |
| 25 | 10 | 3 | 7 | 13 | 7/12 | 13/24 | 169/576 |
| 26 | 7 | 5 | 2 | 12 | 2/12 | 12/24 | 144/576 |
| 27 | 6 | 3 | 3 | 9 | 3/12 | 9/24 | 81/576 |
| 28 | 11 | 5 | 6 | 16 | 6/12 | 16/24 | 256/576 |
| 29 | 6 | 10 | -4 | 16 | -4/12 | 16/24 | 256/576 |
| 30 | 6 | 5 | 1 | 11 | 1/12 | 11/24 | 121/576 |
| $31_A$ | 11 | 8 | 3 | 19 | 3/12 | 19/24 | 361/576 |
| $31_B$ | 8 | 9 | -1 | 17 | -1/12 | 17/24 | 289/576 |
| SUM | ---------- | ---------- | ---------- | ---------- | 113/12 | 327/24 | 4171/576 |

* FOR THE KUDER-RICHARDSON FORMULA N IS 27% OF THE NUMBER OF STUDENTS TAKING THE TEST. IN THIS CASE, HOWEVER, N WAS FIRST REDUCED BY 5/6 ON THE BASIS OF SCORES FOR THE AUXILIARY REPRESENTATIONS QUESTIONNAIRE IN THE FOLLOWING MANNER: IF, FOR EXAMPLE, EIGHTEEN STUDENTS MISSED TEN POINTS AND TWELVE MISSED NINE, THEN THREE (=18/6) AND TWO (=12/6), RESPECTIVELY, WERE SELECTED RANDOMLY FOR THE SAMPLE POPULATION. THEREFORE, N = 0.27(N/6).

$$KR = \frac{K}{K-1}\left[1 - \frac{6(\text{SUM OF DIFFICULTY INDICES} - \text{SUM OF SQUARED DIFFICULTY INDICES})}{(\text{SUM OF DISCRIMINATION INDICES})^2}\right] = .59$$

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AUXILIARY REPRESENTATIONS QUESTIONNAIRE RELIABILITY COEFFICIENT, GRADE 9

| ITEM NUMBER | NUMBER WRONG OR OMITTED IN LOWEST 27 PER CENT GROUP: $W_L$ | NUMBER WRONG OR OMITTED IN HIGHEST 27 PER CENT GROUP: $W_H$ | $W_L - W_H$ | $W_L + W_H$ | DISCRIMINATION INDEX: $\frac{W_L - W_H}{N^*}$ | DIFFICULTY INDEX: $\frac{W_L + W_H}{2N}$ | DIFFICULTY INDICES SQUARED |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 2 | 3 | 0 | 3 | 3 | 3/12 | 3/24 | 9/576 |
| 3 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 4 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 5 | 7 | 1 | 6 | 8 | 6/12 | 8/24 | 64/576 |
| 6 | 2 | 0 | 2 | 2 | 2/12 | 2/24 | 4/576 |
| 7 | 3 | 0 | 3 | 3 | 3/12 | 3/24 | 9/576 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 8 | 0 | 8 | 8 | 8/12 | 8/24 | 64/576 |
| 10 | 7 | 1 | 6 | 8 | 6/12 | 8/24 | 64/576 |
| 11 | 9 | 3 | 6 | 12 | 6/12 | 12/24 | 144/576 |
| 12 | 8 | 0 | 8 | 8 | 8/12 | 8/24 | 64/576 |
| 13 | 5 | 1 | 4 | 6 | 4/12 | 6/24 | 36/576 |
| 14 | 6 | 2 | 4 | 8 | 4/12 | 8/24 | 64/576 |
| 15 | 5 | 0 | 5 | 5 | 5/12 | 5/24 | 25/576 |
| 16 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 17 | 5 | 0 | 5 | 5 | 5/12 | 5/24 | 25/576 |
| 18 | 1 | 0 | 1 | 1 | 1/12 | 1/24 | 1/576 |
| 19 | 7 | 1 | 6 | 8 | 6/12 | 8/24 | 64/576 |
| 20 | 8 | 2 | 6 | 10 | 6/12 | 10/24 | 100/576 |
| 21 | 11 | 6 | 5 | 17 | 5/12 | 17/24 | 289/576 |
| 22 | 9 | 1 | 8 | 10 | 8/12 | 10/24 | 100/576 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 12 | 5 | 7 | 17 | 7/12 | 17/24 | 289/576 |
| 25 | 5 | 2 | 3 | 7 | 3/12 | 7/24 | 49/576 |
| 26 | 6 | 6 | 0 | 12 | 0 | 12/24 | 144/576 |
| 27 | 3 | 1 | 2 | 4 | 2/12 | 4/24 | 16/576 |
| 28 | 8 | 1 | 7 | 9 | 7/12 | 9/24 | 81/576 |
| 29 | 8 | 8 | 0 | 16 | 0 | 16/24 | 256/576 |
| 30 | 7 | 0 | 7 | 7 | 7/12 | 7/24 | 49/576 |
| $31_A$ | 11 | 2 | 9 | 13 | 9/12 | 13/24 | 169/576 |
| $31_B$ | 10 | 3 | 7 | 13 | 7/12 | 13/24 | 169/576 |
| SUM | ---------- | ---------- | ---------- | ---------- | 132/12 | 224/24 | 2352/576 |

* FOR THE KUDER-RICHARDSON FORMULA N IS 27% OF THE NUMBER OF STUDENTS TAKING THE TEST. IN THIS CASE, HOWEVER, N WAS FIRST REDUCED BY 4/5 ON THE BASIS OF SCORES FOR THE AUXILIARY REPRESENTATIONS QUESTIONNAIRE IN THE FOLLOWING MANNER: IF, FOR EXAMPLE, FIFTEEN STUDENTS MISSED TWELVE POINTS AND TEN MISSED ELEVEN, THEN THREE (=15/5) AND TWO (=10/5), RESPECTIVELY, WERE SELECTED RANDOMLY FOR THE SAMPLE POPULATION. THEREFORE, N = 0.27(N/6).

$$KR = \frac{K}{K-1}\left[1 - \frac{6(\text{SUM OF DIFFICULTY INDICES} - \text{SUM OF SQUARED DIFFICULTY INDICES})}{(\text{SUM OF DISCRIMINATION INDICES})^2}\right] = .76$$

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NAME_____ TEACHER_____ $T_4$

Except for the work problems, translate each of the following into 2 equations in 2 unknowns. (Translate the work problems into job-completed equations.) Do not attempt to answer the questions asked in the problems. When you are finished turn your paper over on your desk until your teacher asks for it. Use the margins or the back of this paper for scratch work.

1. One number is four times another. Their difference is 39. What are they? _____ _____

2. Some dimes and quarters are worth $4.60. If there are 3 less dimes than quarters, how many of each are there? _____ _____

3. Mot is 18 years older than Nhoj. In 5 years he will be twice as old as Nhoj. How old is Mot? _____ _____

4. Micah can mow a lawn in 7 hours, and Nathan can mow the same lawn in 5 hours. How long will it take them working together to mow the lawn? _____

5. Mr. Doit wishes to blend tea worth 70¢ a pound with tea worth 95¢ a pound. How much of each must he use to get 25 pounds of tea worth 80¢ a pound? _____ _____

6. The sum of two numbers is 37. The larger is 5 more than seven times the smaller. Find the numbers. _____ _____

7. A post office sold 3 less 10-cent (air mail) stamps than 6-cent (regular) stamps. If the stamps cost $2.42, how many of each were sold? _____ _____

8. Luap is 3 times as old as Kimberly. Three years ago he was 4 times as old as Kimberly. How old is Luap? _____ _____

9. A bathtub can be filled in 7 minutes and emptied in 10. If the tub is empty and the drain is left open, how long will it take to fill the tub? _____

10. A nut shop sells almonds for $2.10 a pound and peanuts for 65¢ a pound. The shopkeeper makes a mixture of 20 pounds of these nuts to sell for $1.70 a pound. How many pounds of each does he use? _____ _____

PROBLEMS RETENTION TEST

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# APPENDIX E

ANALYSIS OF RETENTION SUBTEST SCORES AND RESULTS BY VARIABLE FOR TWO KENNEDY JUNIOR HIGH NINTH GRADE ALGEBRA CLASSES ASSIGNED TO HIM AND HID, RESPECTIVELY.

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TABLE 65: ANALYSIS OF VARIABLE TREATMENT PROBLEMS RETENTION SUBTEST (NUMBER, COIN, AGE -- 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | 7.05 | 3.33 | 7.02 | 2.05 | | | |
| HID | 84 | 6.74 | 3.55 | 7.05 | $p < .13$ | HID | NS | |
| LO | 85 | 7.99 | 3.42 | 7.71 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 5.50 | 5.78 | 4.60 | 1 | 28 | 6.00 | 6.01 | 1.86 | 1 | 23 | 7.65 | 8.47 | .66 |
| 2 | 30 | 7.77 | 7.49 | $p < .01$ | 2 | 28 | 6.93 | 7.32 | $p < .16$ | 2 | 32 | 7.34 | 7.66 | $p < .52$ |
| 3 | 31 | 7.45 | 7.52 | | 3 | 28 | 7.29 | 6.89 | | 3 | 30 | 8.93 | 7.97 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 9.81 | 2.20 | .04 | HIM | 29 | 7.03 | 2.44 | 2.79 | HIM | 27 | 4.30 | 2.81 | .03 |
| HID | 26 | 9.96 | 1.93 | $p < .96$ | HID | 22 | 6.64 | 3.32 | $p < .07$ | HID | 36 | 4.47 | 2.76 | $p < .97$ |
| LO | 33 | 9.97 | 2.34 | | LO | 31 | 8.32 | 2.65 | | LO | 21 | 4.38 | 3.07 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | .05 | LO | NS | NS |

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TABLE 66: ANALYSIS OF VARIABLE TREATMENT PROBLEMS RETENTION SUBTEST (NUMBER, COIN, AGE — 12 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 9.62 | 2.40 | 9.62 | .43 | | HIM | HID |
| HID | 62 | 9.43 | 2.72 | 9.30 | $p < .65$ | HID | NS | |
| LO | 87 | 9.18 | 2.51 | 9.28 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 9.63 | 9.80 | .24 | 1 | 30 | 9.93 | 10.06 | 3.38 | 1 | 25 | 8.72 | 8.85 | .47 |
| 2 | 34 | 9.62 | 9.49 | $p < .63$ | 2 | 32 | 8.97 | 8.85 | $p < .07$ | 2 | 32 | 9.25 | 9.22 | $p < .62$ |
| | | | | | | | | | | 3 | 30 | 9.50 | 9.42 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 10.27 | 1.75 | .23 | HIM | 18 | 10.50 | 1.29 | 1.01 | HIM | 18 | 7.94 | 3.08 | .21 |
| HID | 26 | 10.15 | 2.17 | $p < .80$ | HID | 17 | 9.59 | 2.81 | $p < .37$ | HID | 19 | 8.21 | 3.07 | $p < .81$ |
| LO | 28 | 10.50 | 1.77 | | LO | 25 | 9.76 | 1.90 | | LO | 34 | 7.68 | 2.67 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 67: ANALYSIS OF IDENTICAL TREATMENT PROBLEMS RETENTION SUBTEST (WORK, MIXTURE — 8 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 83 | 2.54 | 2.12 | 2.51 | 1.42 | | HIM | HID |
| HID | 84 | 2.82 | 2.58 | 3.02 | $p < .24$ | HID | NS | |
| LO | 85 | 2.99 | 2.57 | 2.83 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 1.55 | 2.01 | 1.78 | 1 | 28 | 2.29 | 2.31 | 10.92 | 1 | 23 | 2.74 | 3.33 | .40 |
| 2 | 30 | 3.40 | 2.96 | $p < .18$ | 2 | 28 | 3.96 | 4.17 | $p < .0001$ | 2 | 32 | 2.69 | 2.94 | $p < .67$ |
| 3 | 31 | 2.42 | 2.52 | | 3 | 28 | 2.21 | 1.98 | | 3 | 30 | 3.50 | 2.78 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 4.04 | 1.91 | .91 | HIM | 29 | 2.28 | 1.69 | .40 | HIM | 27 | 1.33 | 1.88 | .81 |
| HID | 26 | 4.88 | 2.42 | $p < .40$ | HID | 22 | 2.55 | 1.97 | $p < .67$ | HID | 36 | 1.50 | 2.08 | $p < .45$ |
| LO | 33 | 4.61 | 2.57 | | LO | 31 | 2.71 | 2.02 | | LO | 21 | .86 | 1.31 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 68: ANALYSIS OF IDENTICAL TREATMENT PROBLEMS RETENTION SUBTEST (WORK, MIXTURE -- 8 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 3.50 | 2.41 | 3.41 | 1.17 | | | |
| HID | 62 | 3.79 | 2.08 | 3.74 | $p < .31$ | HID | NS | |
| LO | 87 | 3.10 | 2.35 | 3.20 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 2.50 | 2.85 | 2.92 | 1 | 30 | 4.10 | 4.21 | 2.44 | 1 | 25 | 2.60 | 2.70 | 5.79 |
| 2 | 34 | 4.21 | 3.96 | $p < .09$ | 2 | 32 | 3.52 | 3.42 | $p < .12$ | 2 | 32 | 2.56 | 2.41 | $p < .005$ |
| | | | | | | | | | | 3 | 30 | 4.10 | 4.18 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.77 | 2.51 | .20 | HIM | 18 | 4.00 | 2.14 | 1.56 | HIM | 18 | 2.67 | 2.45 | 1.05 |
| HID | 26 | 4.08 | 1.87 | $p < .82$ | HID | 17 | 3.88 | 2.06 | $p < .22$ | HID | 19 | 3.26 | 2.40 | $p < .36$ |
| LO | 28 | 4.18 | 2.48 | | LO | 25 | 2.88 | 2.55 | | LO | 34 | 2.38 | 1.76 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 69: ANALYSIS OF NUMBER PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 83 | 2.72 | 1.18 | 2.72 | 1.14 | | HIM | HID |
| HID | 84 | 2.58 | 1.27 | 2.68 | $p < .32$ | HID | NS | |
| LO | 85 | 2.98 | 1.17 | 2.89 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 2.14 | 2.25 | 5.00 | 1 | 28 | 2.29 | 2.29 | 3.15 | 1 | 23 | 2.91 | 3.16 | .69 |
| 2 | 30 | 2.77 | 2.72 | $p < .009$ | 2 | 28 | 2.82 | 2.94 | $p < .05$ | 2 | 32 | 2.78 | 2.86 | $p < .51$ |
| 3 | 31 | 3.10 | 3.06 | | 3 | 28 | 2.64 | 2.52 | | 3 | 30 | 3.23 | 2.96 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.37 | .69 | .87 | HIM | 29 | 2.90 | .98 | 1.42 | HIM | 27 | 1.89 | 1.31 | .07 |
| HID | 26 | 3.58 | .64 | $p < .42$ | HID | 22 | 2.64 | 1.36 | $p < .25$ | HID | 36 | 1.83 | 1.06 | $p < .93$ |
| LO | 33 | 3.61 | .83 | | LO | 31 | 3.13 | .85 | | LO | 21 | 1.76 | 1.18 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 70: ANALYSIS OF NUMBER PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 3.29 | .97 | 3.30 | 1.96 | | | |
| HID | 62 | 3.10 | .91 | 3.06 | $p < .14$ | HID | NS | |
| LO | 87 | 3.31 | .80 | 3.33 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 3.46 | 3.57 | 2.62 | 1 | 30 | 3.10 | 3.12 | .04 | 1 | 25 | 3.28 | 3.31 | .05 |
| 2 | 34 | 3.18 | 3.10 | $p < .11$ | 2 | 32 | 3.09 | 3.07 | $p < .84$ | 2 | 32 | 3.28 | 3.28 | $p < .95$ |
| | | | | | | | | | | 3 | 30 | 3.37 | 3.34 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.18 | .85 | 1.55 | HIM | 18 | 3.67 | .69 | 5.01 | HIM | 18 | 3.06 | 1.26 | .31 |
| HID | 26 | 3.38 | .70 | $p < .22$ | HID | 17 | 2.94 | .97 | $p < .01$ | HID | 19 | 2.79 | 1.03 | $p < .74$ |
| LO | 28 | 3.54 | .58 | | LO | 25 | 3.60 | .65 | | LO | 34 | 2.91 | .90 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | .01 | LO | NS | NS |

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TABLE 71: ANALYSIS OF COIN PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | 1.76 | 1.34 | 1.75 | 3.58 | | | |
| HID | 84 | 1.68 | 1.32 | 1.76 | $p < .03$ | HID | NS | |
| LO | 85 | 2.25 | 1.46 | 2.18 | | LO | .05 | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 1.59 | 1.72 | .19 | 1 | 28 | 1.29 | 1.30 | 2.23 | 1 | 23 | 1.96 | 2.20 | .32 |
| 2 | 30 | 2.00 | 1.86 | $p < .83$ | 2 | 28 | 1.82 | 1.91 | $p < .11$ | 2 | 32 | 2.03 | 2.13 | $p < .73$ |
| 3 | 31 | 1.65 | 1.69 | | 3 | 28 | 1.93 | 1.83 | | 3 | 30 | 2.70 | 2.41 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 2.81 | 1.30 | .46 | HIM | 29 | 1.59 | 1.09 | 3.85 | HIM | 27 | .89 | .85 | .67 |
| HID | 26 | 2.54 | 1.27 | $p < .63$ | HID | 22 | 1.50 | 1.26 | $p < .03$ | HID | 36 | 1.17 | 1.08 | $p < .51$ |
| LO | 33 | 2.85 | 1.37 | | LO | 31 | 2.32 | 1.33 | | LO | 21 | 1.19 | 1.25 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | .05 | .05 | LO | NS | NS |

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TABLE 72: ANALYSIS OF COIN PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 2.84 | 1.30 | 2.85 | .34 | | HIM | HID |
| HID | 62 | 2.95 | 1.37 | 2.90 | P < .71 | HID | NS | |
| LO | 87 | 2.69 | 1.36 | 2.73 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 2.75 | 2.70 | .44 | 1 | 30 | 3.33 | 3.39 | 6.34 | 1 | 25 | 2.32 | 2.38 | 1.20 |
| 2 | 34 | 2.91 | 2.95 | P < .51 | 2 | 32 | 2.61 | 2.55 | P < .01 | 2 | 32 | 2.91 | 2.88 | P < .31 |
| 3 | | | | | 3 | | | | | 3 | 30 | 2.77 | 2.74 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.27 | 1.16 | .39 | HIM | 18 | 3.00 | .84 | .94 | HIM | 18 | 2.17 | 1.58 | .58 |
| HID | 26 | 3.04 | 1.46 | P < .68 | HID | 17 | 3.35 | 1.11 | P < .40 | HID | 19 | 2.47 | 1.43 | P < .56 |
| LO | 28 | 3.32 | 1.06 | | LO | 25 | 2.88 | 1.27 | | LO | 34 | 2.03 | 1.38 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 73: ANALYSIS OF AGE PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | 2.57 | 1.47 | 2.56 | .11 | | | |
| HID | 84 | 2.48 | 1.46 | 2.60 | $p < .89$ | HID | NS | |
| LO | 85 | 2.76 | 1.40 | 2.64 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 1.77 | 1.80 | 6.50 | 1 | 28 | 2.43 | 2.42 | .11 | 1 | 23 | 2.78 | 3.11 | 1.33 |
| 2 | 30 | 3.00 | 2.90 | $p < .003$ | 2 | 28 | 2.29 | 2.47 | $p < .90$ | 2 | 32 | 2.53 | 2.67 | $p < .27$ |
| 3 | 31 | 2.71 | 2.78 | | 3 | 28 | 2.71 | 2.54 | | 3 | 30 | 3.00 | 2.61 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.63 | .79 | 1.52 | HIM | 29 | 2.55 | 1.30 | .66 | HIM | 27 | 1.52 | 1.42 | .03 |
| HID | 26 | 3.85 | .37 | $p < .22$ | HID | 22 | 2.50 | 1.41 | $p < .52$ | HID | 36 | 1.47 | 1.16 | $p < .97$ |
| LO | 33 | 3.52 | .87 | | LO | 31 | 2.87 | 1.26 | | LO | 21 | 1.43 | 1.36 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 74: ANALYSIS OF AGE PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| HIM | 58 | 3.48 | .92 | 3.47 | 1.14 | | HIM | HID |
| HID | 62 | 3.38 | 1.14 | 3.34 | P < .32 | HID | NS | |
| LO | 87 | 3.18 | 1.03 | 3.22 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 3.42 | 3.53 | .16 | 1 | 30 | 3.50 | 3.55 | 1.31 | 1 | 25 | 3.12 | 3.16 | .54 |
| 2 | 34 | 3.53 | 3.45 | P < .69 | 2 | 32 | 3.27 | 3.23 | P < .26 | 2 | 32 | 3.06 | 3.07 | P < .59 |
| | | | | | | | | | | 3 | 30 | 3.37 | 3.33 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 3.82 | .39 | .45 | HIM | 18 | 3.83 | .38 | 2.39 | HIM | 18 | 2.72 | 1.27 | .20 |
| HID | 26 | 3.73 | .72 | P < .64 | HID | 17 | 3.29 | 1.16 | P < .10 | HID | 19 | 2.95 | 1.47 | P < .82 |
| LO | 28 | 3.64 | .73 | | LO | 25 | 3.28 | .94 | | LO | 34 | 2.74 | 1.14 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | .05 | NS | LO | NS | NS |

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TABLE 75: ANALYSIS OF WORK PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | .72 | 1.06 | .72 | 5.83 | | | |
| HID | 84 | 1.29 | 1.54 | 1.37 | P < .003 | HID | .001 | |
| LO | 85 | 1.13 | 1.46 | 1.05 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | .45 | .56 | .61 | 1 | 28 | .93 | .91 | 6.93 | 1 | 23 | 1.22 | 1.46 | 1.00 |
| 2 | 30 | 1.00 | .89 | P < .55 | 2 | 28 | 1.86 | 2.01 | P < .002 | 2 | 32 | 1.00 | 1.11 | P < .37 |
| 3 | 31 | .65 | .67 | | 3 | 28 | 1.07 | .94 | | 3 | 30 | 1.20 | .90 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | .96 | 1.16 | 6.22 | HIM | 29 | .83 | 1.00 | .34 | HIM | 27 | .37 | .96 | 2.10 |
| HID | 26 | 2.46 | 1.73 | P < .003 | HID | 22 | .82 | 1.01 | P < .71 | HID | 36 | .72 | 1.19 | P < .13 |
| LO | 33 | 1.82 | 1.69 | | LO | 31 | 1.03 | 1.25 | | LO | 21 | .19 | .60 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | .001 | | HID | NS | | HID | NS | |
| LO | .05 | NS | LO | NS | NS | LO | NS | NS |

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TABLE 76: ANALYSIS OF WORK PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 1.10 | 1.31 | 1.04 | 3.57 | | | |
| HID | 62 | 1.14 | 1.33 | 1.15 | p<.03 | HID | NS | |
| LO | 87 | 1.56 | 1.54 | 1.61 | | LO | .05 | .05 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | .67 | .84 | 1.41 | 1 | 30 | 1.20 | 1.27 | .61 | 1 | 25 | 1.60 | 1.64 | 3.47 |
| 2 | 34 | 1.41 | 1.29 | p<.24 | 2 | 32 | 1.09 | 1.03 | p<.44 | 2 | 32 | 1.13 | 1.03 | p<.04 |
| | | | | | | | | | | 3 | 30 | 2.00 | 2.07 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 1.00 | 1.60 | 2.59 | HIM | 18 | 1.44 | 1.15 | .07 | HIM | 18 | .89 | 1.02 | 1.79 |
| HID | 28 | 1.00 | 1.41 | p<.08 | HID | 17 | 1.52 | 1.33 | p<.93 | HID | 19 | .95 | 1.22 | p<.18 |
| LO | 28 | 1.86 | 1.72 | | LO | 25 | 1.36 | 1.60 | | LO | 34 | 1.47 | 1.33 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 77: ANALYSIS OF MIXTURE PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 7

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 83 | 1.82 | 1.62 | 1.80 | .33 | | | |
| HID | 84 | 1.54 | 1.52 | 1.64 | p < .72 | HID | NS | |
| LO | 85 | 1.86 | 1.54 | 1.77 | | LO | NS | NS |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 22 | 1.09 | 1.44 | 1.23 | 1 | 28 | 1.36 | 1.41 | 5.45 | 1 | 23 | 1.52 | 1.87 | .01 |
| 2 | 30 | 2.40 | 2.07 | p < .30 | 2 | 28 | 2.11 | 2.16 | p < .006 | 2 | 32 | 1.69 | 1.83 | p < .99 |
| 3 | 31 | 1.77 | 1.85 | | 3 | 28 | 1.14 | 1.04 | | 3 | 30 | 2.30 | 1.88 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 27 | 3.07 | 1.36 | 1.35 | HIM | 29 | 1.45 | 1.43 | .32 | HIM | 27 | .96 | 1.23 | .39 |
| HID | 26 | 2.42 | 1.53 | p < .27 | HID | 22 | 1.73 | 1.42 | p < .73 | HID | 36 | .78 | 1.20 | p < .73 |
| LO | 33 | 2.79 | 1.45 | | LO | 31 | 1.68 | 1.30 | | LO | 21 | .67 | 1.02 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | NS | NS | LO | NS | NS |

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TABLE 78: ANALYSIS OF MIXTURE PROBLEM RETENTION SUBTEST (4 POINTS POSSIBLE) BY TREATMENT GROUPS AND ACHIEVEMENT SUBGROUPS, GRADE 9

MULTIPLE ANCOVA BETWEEN TREATMENT GROUPS

| Treatment | N | Mean | SD | Adj Mean | F-Ratio | T-Test Matrix For Adj Group Means | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | HIM | HID |
| HIM | 58 | 2.40 | 1.57 | 2.37 | 10.75 | | | |
| HID | 62 | 2.65 | 1.42 | 2.60 | p = .0000 | HID | NS | |
| LO | 87 | 1.54 | 1.48 | 1.60 | | LO | .01 | .001 |

MULTIPLE ANCOVA WITHIN TREATMENT GROUPS

| W/IN HIM | | | | | W/IN HID | | | | | W/IN LO | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio | C | N | Mean | Adj Mean | F-Ratio |
| 1 | 24 | 1.83 | 2.01 | 2.36 | 1 | 30 | 2.90 | 2.94 | 2.47 | 1 | 25 | 1.00 | 1.07 | 4.64 |
| 2 | 34 | 2.79 | 2.67 | p < .13 | 2 | 32 | 2.42 | 2.39 | p < .12 | 2 | 32 | 1.44 | 1.38 | p < .01 |
| | | | | | | | | | | 3 | 30 | 2.10 | 2.11 | |

ANAVARA BETWEEN TREATMENT SUBGROUPS

| UPPER 1/3 | | | | | MIDDLE 1/3 | | | | | LOWER 1/3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio | T | N | Mean | SD | F-Ratio |
| HIM | 22 | 2.77 | 1.38 | 1.90 | HIM | 18 | 2.56 | 1.54 | 3.01 | HIM | 18 | 1.78 | 1.70 | 6.85 |
| HID | 26 | 3.08 | 1.29 | p < .16 | HID | 17 | 2.35 | 1.46 | p < .06 | HID | 19 | 2.32 | 1.49 | p < .002 |
| LO | 28 | 2.32 | 1.59 | | LO | 25 | 1.52 | 1.45 | | LO | 34 | .91 | 1.08 | |

T-TEST MATRICES FOR SUBGROUP MEANS

| UPPER 1/3 | | | MIDDLE 1/3 | | | LOWER 1/3 | | |
|---|---|---|---|---|---|---|---|---|
| | HIM | HID | | HIM | HID | | HIM | HID |
| HID | NS | | HID | NS | | HID | NS | |
| LO | NS | NS | LO | .05 | NS | LO | .05 | .001 |

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Results by Variable for Two Kennedy Junior High Ninth Grade Algebra Classes Assigned to HIM and HID, Respectively

| Variable | HIM | | | HID | | | Rochester Population | | |
|---|---|---|---|---|---|---|---|---|---|
| | N | Mean | SD | N | Mean | SD | N | Mean | SD |
| Total IQ | 24 | 64.25 | 24.10 | 27 | 76.81 | 15.29 | 124 | 81.73 | 16.74 |
| CTBS Computation | 26 | 35.85 | 8.46 | 31 | 35.42 | 4.50 | 214 | 35.43 | 6.72 |
| CTBS Concepts | 26 | 21.96 | 4.07 | 31 | 20.00 | 3.28 | 214 | 21.92 | 3.60 |
| CTBS Applications | 26 | 12.65 | 3.95 | 31 | 13.45 | 2.78 | 214 | 13.62 | 3.63 |
| CTBS Arith Achiev | 26 | 70.46 | 14.70 | 31 | 68.87 | 7.38 | 214 | 70.96 | 11.84 |
| Initial Prob Test | 24 | 6.75 | 2.75 | 31 | 9.84 | 2.13 | 223 | 9.18 | 2.73 |
| Final Prob Test | 25 | 11.28 | 4.15 | 29 | 16.59 | 2.56 | 228 | 13.57 | 4.06 |
| $\text{Number}_{FPT}$ | 25 | 2.64 | .81 | 29 | 3.34 | .72 | 228 | 3.20 | .92 |
| $\text{Coin}_{FPT}$ | 25 | 2.24 | 1.33 | 29 | 2.96 | 1.12 | 228 | 2.89 | 1.20 |
| $\text{Age}_{FPT}$ | 25 | 2.96 | 1.14 | 29 | 3.72 | .70 | 228 | 3.36 | 1.08 |
| $\text{Work}_{FPT}$ | 25 | .72 | 1.28 | 29 | 2.62 | 1.52 | 228 | 1.54 | 1.60 |
| $\text{Mixture}_{FPT}$ | 25 | 2.72 | 1.51 | 29 | 3.93 | .37 | 228 | 2.58 | 1.50 |
| Retention Test | 25 | 8.24 | 5.17 | 29 | 14.17 | 3.34 | 226 | 12.55 | 4.16 |
| Aux Rep Ques | 20 | 18.45 | 4.66 | 30 | 20.83 | 3.58 | 223 | 22.61 | 4.24 |
| $\text{Phrase}_{ARQ}$ | 20 | 6.90 | 1.89 | 30 | 8.20 | .76 | 223 | 8.34 | .92 |
| $\text{Sentence}_{ARQ}$ | 20 | 1.35 | .99 | 30 | 2.10 | .99 | 223 | 1.98 | 1.02 |
| $\text{Coin}_{ARQ}$ | 20 | 3.20 | 1.10 | 30 | 4.33 | 1.02 | 223 | 4.66 | 1.26 |
| $\text{Age}_{ARQ}$ | 20 | 3.60 | 1.05 | 30 | 2.90 | 1.21 | 223 | 3.39 | 1.20 |
| $\text{Work}_{ARQ}$ | 20 | 1.40 | .99 | 30 | 1.00 | .74 | 223 | 1.35 | .92 |
| $\text{Mixture}_{ARQ}$ | 20 | 2.00 | 1.75 | 30 | 2.30 | 1.12 | 223 | 2.91 | 1.29 |
| Transfer Test | 25 | 4.96 | 2.47 | 29 | 6.90 | 2.35 | 228 | 6.72 | 2.29 |
| Attitude Scale | 20 | 46.30 | 15.52 | 30 | 46.10 | 14.23 | 223 | 46.72 | 18.09 |
| Affinity Test | 20 | .95 | .60 | 30 | 1.30 | .60 | 223 | 1.10 | .72 |

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