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THE EFFECTS OF TEXTBOOK INSTRUCTION, MANIPULATIVES AND IMAGERY ON RECALL OF THE BASIC MULTIPLICATION FACTS

by

James H. Babb

A thesis submitted in partial fulfillment of
the requirements for the degree of Doctor
of Philosophy in the College of Education
in
The University of South Florida

August, 1975

Thesis supervisor: Dr. A. Edward Uprichard

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Graduate Council
University of South Florida
Tampa, Florida

CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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with a major in Elementary Mathematics Education
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ACKNOWLEDGEMENTS

The author would like to thank his major advisor, Dr. A. Edward Uprichard, whose guidance, advice and support throughout the doctoral program were invaluable.

Appreciation and thanks are also extended to the other members of the committee--Dr. Richard M. Jaeger, Dr. Daniel M. Purdom, Dr. E. Ray Phillips and Dr. Gerald M. Weeks--for their assistance and guidance.

Finally to Valorie and Lisa, their help and encouragement kept me going.

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THE EFFECTS OF TEXTBOOK INSTRUCTION, MANIPULATIVES AND IMAGERY ON RECALL OF THE BASIC MULTIPLICATION FACTS

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An Abstract

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ABSTRACT

Over the years, four identifiable approaches to multiplication instruction have been taken: 1) rote memorization of the basic multiplication facts, 2) mathematically meaningful instruction employing representative and abstract materials, 3) mathematically meaningful instruction on a concrete level (manipulatives) and 4) instruction using mnemonics to facilitate recall of the facts. Of these four approaches, rote memorization has been determined to be ineffective and mathematically unsound even when recall of the basic multiplication facts is the primary consideration. Two of the three remaining methods, textbook instruction and manipulative instruction, have been compared with respect to their relative effectiveness in facilitating recall of the facts and comprehension of the concept of multiplication. The results of these comparisons have been conflicting. The last method, instruction using mnemonic devices has not been researched. The purpose of this study was to compare three approaches to multiplication instruction--textbook instruction, manipulative instruction and imagery (a mnemonic method)-- and to determine their relative efficacy when recall of the basic multiplication facts was the primary objective.

To effect this comparison, three treatments were designed to be representative of the three approaches to multiplication instruction: Treatment T, textbook instruction, used multiplication pages from the Holt School Mathematics (Nichols, et al., 1974) third-grade text. Only

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representative and abstract stimuli were available to the learner. Treatment M, manipulative instruction, was designed with the learner's developmental level in mind. Concrete materials were available to the child at all times during instruction. Work sheets emphasizing mathematical meaning were used. Treatment I, imagery, consisted of an analytic substitution system to code the digits 0 through 9 into sounds, and a set of pictures (images) for the multiplication facts. The child remembered "two cows jumping rope" to recall the fact "7 x 7 = 49".

Although recall of the basic multiplication facts was the primary concern, comprehension and attitude measures were taken at the end of the study. Treatments were randomly assigned, one to each of three second-grade classes (total n = 76). The investigator conducted all instruction. Treatments were conducted for twenty-three, forty-five minute periods; a recall test was administered weekly. A repeated-measures design was employed.

Three hypotheses were tested in the study: I. There will be no significant difference on mean gain in recall of the basic multiplication facts for students taught by a textbook method, a manipulative method or an imagery method; II. there will be no significant occasion effect or occasion by treatment interactions; III. there will be no significant difference in treatment group centroids of recall, comprehension and attitude for students taught by a textbook method, a manipulative method or an imagery method.

To test the three hypotheses, recent MAT raw scores were used as covariates in both analyses of covariance (ANCOVA) and multivariate analyses of covariance (MANCOVA). Results of the several analyses

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were: Hypothesis I was rejected. There was a significant difference in recall over time between the adjusted mean recall score for manipulatives and that for imagery, the manipulative adjusted mean score being higher. Hypothesis II was accepted for occasions, the effect of the repeated-measures on recall was not significant. Hypothesis II for occasion by treatment interaction was rejected, indicating treatment growth curve interaction. An analysis of trends indicated that treatment-growth curve interaction could be represented by no greater than second-degree polynomials. Hypothesis III was rejected. Final recall and comprehension adjusted mean scores did not differ significantly, but those for attitude did. The adjusted mean score for attitude for manipulatives was significantly more positive than that for textbook instruction.

Recommendations of the study call for the use of more manipulatives in the early-elementary classroom and further investigation of the use of imagery with smaller, or more specialized groups.

Abstract approved: C. Edward Sprinkle, thesis supervisor
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Chapter 1

INTRODUCTION

Morris Kline, with his recent book, Why Johnny Can't Add (1974), has created a stir among critics of "new math". Kline supports the claim that the "new math" is not as successful at developing competence in mathematics as the populace of the United States was originally led to believe. Kline and other critics of new math claim that the abandonment of rote learning associated with traditional methods of teaching mathematics and its replacement by the discovery processes of modern mathematics programs have, in many cases, produced children without the capability for performing basic computation in addition, subtraction, multiplication and division.

The major thrust of the attack against "modern math" by its critics is centered around the apparent lack of basic skill development in learners and its effect on their subsequent learning. Articles critical of "new math" cite statistics indicating a lower level of competency in arithmetic, especially in computation. Newsweek magazine, in an article reviewing Dr. Kline's book, cited a California study showing a twenty percent drop in standardized mathematics scores for sixth-graders (1973, p. 77). In an article for Science Digest, Richard Martin cited other California results: In 1962 fifth-graders scored in the 70th percentile nationally on an arithmetic achievement text. In 1971, following statewide adoption of "new math" in 1968,

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sixth-graders scored in the 38th percentile nationally. Martin also cited a New Hampshire study: In 1969 eighth-graders in a traditional mathematics program achieved average grade equivalent scores of 8.8 on a test of ability to compute quickly. Eighth-graders who were studying "new math" scored an average of 6.8 on the same test (Martin, 1973, p. 66).

Whenever "new math" is discussed, it is inevitable that the discussion will include the pros and cons of memorization of the basic facts of addition, subtraction, multiplication and division. Many people say that while memorization of basic facts was a curse associated with traditional methods in mathematics, it is virtually nonexistent in modern programs. Few critics of modern mathematics wish to return to the rote memorization methods of the past; some, however, do not rule out this alternative. Dr. Oliver Selfridge, a Senior Research Associate in the School of Electrical Engineering, Massachusetts Institute of Technology, in his review of Why Johnny Can't Add, says: "Being able to learn things quickly by rote is, in fact, an enormously valuable skill, and, not just in school, but humanly as an educated person." (1974, p. 32).

Although Kline and his followers infer the opposite, proponents of modern mathematics have not said that memorization of basic facts is unnecessary. They have instead criticized the methods used to bring about this memorization. Kenneth Lovell, a noted leader in the "new mathematics" movement, says: "While it is of the utmost importance that children learn that the operation of multiplication involves the joining of equivalent sets and learn how to build up the multiplication tables, it is also necessary for them to commit the

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results (the products) to memory." (1971, p. 61). Max Beberman, one of the two "Fathers of the new math", says, in the teacher's guide to a mathematics series he co-authored: "The hope for mastery must be limited to the basic facts of arithmetic. Here, mastery can and must be achieved." (1964, p. 4). L. Edwin Hirschi devotes three pages of his text, Building Mathematical Concepts in Grades Kindergarten through Eight, to methods which are designed to make memorization of the multiplication facts palatable to children. Hirschi (1970) claims a child should not have to memorize the tables by rote, but at the same time, he infers that memorization is necessary!

In essence, people who are both for and against modern mathematics appear to be of the same opinion with respect to memorization of the basic facts of addition, subtraction, multiplication and division. There is, however, no consensus on how this memorization is to be accomplished. Over the years many methods have been used, but no one method has emerged as outstanding.

Need for the Study

Instruction in multiplication, more specifically in the basic multiplication facts, has involved four identifiable approaches in this country over the past century: 1) drill has been utilized to facilitate recall of the facts, 2) methods stressing mathematical meaning with representative and abstract materials are in current use, 3) methods stressing mathematical meaning through the use of manipulative materials are recommended to give the concrete-operational third grader more interaction with his environment in learning the facts and 4) mnemonic devices have been used to aid the child in recall of

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the facts through associations.

Drill, when used to facilitate recall of the facts by repetitive practice with no apparent attempt at giving meaning to the process, has been determined to be both ineffective and undesirable for teaching multiplication, or even for facilitating recall of the facts (Brownell, 1943; Smith, 1927).

Current textbooks rely on degrees of mathematical meaningfulness to instruct children in multiplication (Lovell, 1971; Beberman, 1964; Hirschi, 1970). This type of instruction reflects the theories of Ausubel and others that meaningful learning takes place when the material to be learned is "...related in a nonarbitrary, substantive fashion to what the learner already knows." (Ausubel, 1968, p. 24). In mathematics this has come to mean the relating of multiplication to addition, to ideas of grouping and the presentation of mathematics structure to the child. Until recent years, however, mathematically meaningful instruction, at any level, has included only representative and abstract stimuli.

Manipulative instruction also emphasizes mathematical meaning. It is generally accepted that six to twelve year old students profit from the use of manipulatives to facilitate comprehension in mathematics (Copeland, 1974; Piaget, 1952; Dienes, 1967; Nichols, 1971). It appears that the theories of Piaget (1952) which emphasize the value of manipulatives in helping the child tie together abstract ideas may be tenable. The value of utilizing manipulatives to facilitate recall has not, however, been clearly explicated.

Mnemonic devices to facilitate recall of the facts are not currently enjoying widespread use. They have, however, been popular

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in the past (Hatton, 1917; Martyn, 1899). Current psychological research tends to support the value of mnemonics to facilitate recall (Miller, 1967; Cermak, 1972). If the learning of the basic multiplication facts is viewed as stimulus→response learning, then the use of a mnemonic device may be viewed as a way to strengthen the S→R linking by adding mediators (Gagné, 1971).

While attempts have been made to compare the use of abstract and representative materials with the use of manipulatives in the teaching of multiplication, no studies are in evidence which compare these approaches to the use of mnemonic devices designed to facilitate recall of the facts. There is, in fact, little available research comparing any methods with respect to their effect on recall of the facts.

Psychological Relationships

The learning of multiplication, more specifically, the basic multiplication facts, may be discussed in terms of the associated psychological processes related to such learning. The purpose of this section is to make some connections among several methods for teaching multiplication and types of learning related to these methods.

As indicated previously, multiplication instruction has taken four distinct approaches to the teaching of multiplication facts: 1) rote memorization, 2) mathematically meaningful instruction using representative and abstract materials (textbook approach), 3) mathematically meaningful instruction on a concrete level (manipulatives) and 4) instruction using mnemonics to facilitate recall of the facts. The relationships of these four approaches to multiplication to four

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types of learning are displayed in Figure 1.[1]

	S→R Learning	Verbal Association Learning	Meaningful Verbal Learning	Piaget's Developmental Learning
Rote Memorization	X			
Textbook Instruction		X	X	
Manipulative Instruction		X	X	X
Mnemonic Instruction		X		

Figure 1. Relationships among methods for multiplication instruction and types of learning associated with these methods.

As illustrated in the figure, one approach to multiplication instruction, that of rote memorization, may be considered stimulus→response learning in its most basic sense, since the instruction makes no attempt to make the learning meaningful or to supply mediating links to connect the stimulus to the response. In the present study, rote memorization to facilitate recall of the multiplication facts was not considered to be a tenable alternative for multiplication instruction. Gagné (1970) indicates that S→R learning is inferior to the learning of verbal associations, even when recall of paired-associates is the primary concern. Mathematics educators (Brownell, 1943; Smith, 1923)

[1]Although four types of learning are listed, the reader is cautioned that these types are not professed to be the only learning types available. The types are not mutually exclusive (e.g., there is overlap between meaningful verbal learning and verbal association learning).

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have presented valid arguments against rote memorization methods taken alone. For the present study only approaches to multiplication instruction classifiable under more complex types of learning will be considered.

Referring again to Figure 1, we find that the remaining three approaches to multiplication--textbook instruction, manipulative instruction and mnemonic instruction--may be classified as the learning of verbal associations. These three approaches all attempt to supply mediating links between the stimulus and the response to facilitate recall of the multiplication facts. Textbook instruction and manipulative instruction both attempt to supply mathematically meaningful links while mnemonic instruction uses any link that has meaning within the cognitive structure of the learner.

In supplying mathematically meaningful links to learners, these two approaches, textbook instruction and manipulative instruction, are relating multiplication to the child's knowledge of addition. As indicated in Figure 1, this relating of new knowledge to a pre-existing cognitive structure may also be viewed as meaningful verbal learning.

The rationale for the manipulative approach to multiplication instruction has its basis in the developmental theories of Jean Piaget. If the developmental level of the early-elementary child is considered, he will be provided with concrete materials to aid him in concept development. Only one instructional method, manipulative instruction, employs concrete materials, thus it alone can be classified under developmental theory.

In summary, for the four learning processes under

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consideration, approaches to multiplication instruction may be categorized as follows: Rote memorization may be considered S→R learning. Textbook instruction, manipulative instruction and mnemonic instruction may be classified as the learning of verbal associations. Textbook instruction and manipulative instruction may also be considered meaningful verbal learning. Manipulative instruction considers the developmental theory of Piaget as it pertains to early-elementary students. Of these four approaches only textbook instruction, manipulative instruction and mnemonic instruction are considered viable approaches to the learning of the basic multiplication facts. The three types of learning; verbal association learning, meaningful verbal learning and Piaget's developmental theory are described below.

Verbal Associations

The three approaches to multiplication instruction--textbook instruction, manipulative instruction and mnemonic instruction--may all be considered as the learning of verbal associations. Verbal associations are a form of chaining. Gagné (1970) cites the chaining of behavior as a widely occurring event in the learning of motor skills and in learning verbal behavior. In the process of chaining, the individual connects a sequence of individual Ss→Rs. The chain formed may be a motor sequence, such as the series of Ss→Rs necessary to open a door; or it may be verbal, such as memorizing a poem.

In chaining, each preceding event triggers a stimulus for the event to follow. Gagné (1970) gives an example of a chain which a person might go through when told to start an engine:

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> S ("Start the engine") → R(looking forward and to the rear)...
> ...S (sight of clear road) → R (testing for gear in neutral)...
> S (gear in neutral) → R (turning key to activate starter)...
> S (sound of motor catching) → R (release of key)...S (key released) → R (depressing accelerator). (p. 124).

Gagné (1970) specifies necessary conditions for the formation of chains, some within the learner and others within the situation. The necessary conditions within the learner are: 1) each stimulus→ response connection must be previously learned and 2) the mediational connections between the verbal units must be previously learned. The necessary conditions within the situation are: 1) The links must be reinstated in the proper order. 2) The links must be executed in a close time sequence. 3) A satisfying state of affairs must occur at the end.

Numerous studies have been performed where the subjects are asked to recall lists of pairs such as dog-hat, boy-tree, etc. (Underwood and Scholtz, 1960; Cofer, 1961). The results of these studies indicate that the learning of verbal associations is facilitated by the use of an intervening or mediating link between the pairs (see also Gagné, 1970). Jenkins (1963) cites use of these mediating links as producing striking results.

In multiplication instruction, a stimulus, say "2 x 3 =" is to be linked to the response "6". To accomplish this linking, two approaches are taken by the three different forms of multiplication instruction: Both textbook instruction methods and instruction using manipulatives employ a mathematically meaningful link, for "2 x 3 =" the link could be "3 + 3 =" to connect the stimulus to the desired response, "6". For instruction employing mnemonics, the link can be anything that helps the learner connect the stimulus and the

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response. A mnemonic, for example, could be a rhyme similar to "2 x 3 -- Where's the bee? -- The bee is sick -- 6".

Thus, while all three types of instruction are classifiable as the learning of verbal associations, they are subdivided essentially into two types; methods which use mathematical meaning as a basis for the development of links (textbook instruction and manipulatives) and instruction which uses any kind of link that has meaning to the learner (mnemonics). Methods stressing mathematical meaning may also be identifiable as meaningful verbal learning.

Meaningful Verbal Learning

Of the three types of multiplication under consideration for this study, two, textbook instruction and manipulative instruction, can be considered as meaningful verbal learning.

David Ausubel (1968) describes meaningful verbal learning as the subsuming of new ideas into an already existing cognitive structure. For Ausubel, the acquiring of verbal knowledge is accomplished by first finding relevant anchoring ideas. After identifying these anchoring ideas the learner must be able to distinguish these new ideas from a pre-existing structure. For Ausubel, the term meaningful would be defined as mathematically meaningful when discussing the multiplication facts.

The two methods of instruction, textbook instruction and instruction using manipulative materials, relate multiplication to addition. The relating of multiplication to addition can then be viewed as relating multiplication to the pre-existing structure of the learner.

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When the two mathematically meaningful methods, textbook and manipulative, are compared, there is one obvious difference between then: the stimuli available to the learner during the instructional process. Textbook instruction traditionally makes available only representative and abstract stimuli (pictures and numbers) while manipulative instruction provides both of these stimuli and the additional stimulus of concrete materials. This essential difference is based on the developmental theory of Jean Piaget (1952).

Piaget's Theory of Cognitive Development

Piaget distinguished four main stages in the development of mental structures. These stages, and the approximate ages at which they occur are: sensory-motor (0 - 2), pre-operational (3 - 7), concrete-operational (8 - 11) and formal-operational (12 years) (Copeland, 1970). Copeland (1970) stresses that these ages are only approximations, however, and a child may be simultaneously pre-operational with respect to one concept and formal-operational with respect to another.

For Piaget, then, the elementary child would, on the average, be in the concrete-operational stage of development. In his text on multiplication instruction, Copeland (1970) discusses this stage:

> This is the concrete-operational stage because the child is obtaining ideas from operations on such concrete objects as water, clay, etc. At the beginning of the concrete-operational level the ideas of the child are still based on observation and experience with objects in the physical world, but he is beginning to generalize or break away from manipulation of objects as a way of "knowing". When these generalizations are complete and correct the child is at the concrete-operational level. (p. 15).

The concrete-operational child, then, learns concepts through

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the manipulation of concrete materials. After he has learned concepts and made generalizations, he will then move away from the concrete materials, but, according to Piaget's theories, he needs these manipulatives to aid him in learning the concepts and making generalizations.

Only one of the mathematically meaningful treatments, manipulative instruction, has manipulatives included in its instructional package. Textbook instruction does not usually include any manipulative materials, only representative and abstract stimuli.

Summary

Of the four approaches to multiplication instruction--rote memorization, textbook instruction, manipulative instruction and mnemonic instruction--one, rote memorization, is considered ineffective for facilitating recall of the multiplication facts. The other three methods are relatively untested with respect to their effectiveness in promoting recall. The types of learning related to these three approaches indicate they should all be effective; to what extent, however, remains to be determined.

Statement of the Problem

The purpose of this study was to investigate the relative efficacy of three methods for facilitating memorization of the basic multiplication facts; a current textbook approach, a manipulative approach stressing mathematical meaning and a mnemonic approach employing imagery.

The dependent variables in this study were: 1) recall of the basic multiplication facts, 2) comprehension of the process of

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multiplication and 3) attitude toward mathematics. The independent variables were: 1) treatment; textbook, manipulatives or imagery and 2) week of testing.

Significance of the Study

Many critics of the current trend toward stressing mathematical meaning suggest that this approach results in considerable less emphasis on memorization (Kline, 1974; Selfridge, 1974; Martin, 1973). In investigating instructional methods most studies performed in the last decade have tested for comprehension in multiplication rather than ability to recall the facts (Gray, 1964; Nichols, 1971; Punn, 1973). Although the psychological literature includes a multitude of memorization studies, there is little research available which specifically explores the application of memorization theories and techniques to learning the multiplication facts.

This study investigates two techniques designed specifically for teaching multiplication through understanding and at a technique designed specifically to facilitate recall of the facts. Several questions are addressed: Which of the several methods in current use better facilitate recall of the facts? Would it be advantageous to use a memorization technique to facilitate recall? Is there any difference among the three types of instruction under consideration in the comprehension of multiplication? Are any of the three treatments under consideration more effective for maintaining a favorable attitude toward mathematics?

The answers to these questions should be valuable to teachers, parents, curriculum planners and to educational researchers: to

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teachers by adding to their knowledge of what methods best facilitate recall and comprehension in multiplication; to parents by answering some of the questions on whether the "new math" facilitates recall and/or comprehension; the answers should aid curriculum planners in the development of new programs in mathematics; and these answers will raise new questions to be studied by educational researchers concerning memorization and comprehension in general.

Definition of Terms

To facilitate the reading of the review of literature, the following terms are defined:

Attitude toward mathematics - The expressed feeling of the student toward mathematics, as expressed by his responses to multiple choice questions pertaining to math class, math problems, math computation and comparison of math to other school work.

Commutative property for multiplication - For any rea' numbers a and b, $a \times b = b \times a$.

Comprehension of basic multiplication facts - The ability of students to use multiplication facts in a new problem solving situation, given he knows the concept he is to apply is multiplication.

Concrete materials - Manipulable items for concept learning (e.g., blocks, counters).

Distributive property of multiplication over addition - For any real numbers a, b and c, $a \times (b + c) = (a \times b) + (a \times c)$.

Experimental group - Any group of students in the study receiving other than their regular classroom instruction.

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Field properties - A set of mathematical properties which are true for the set of real numbers under the operations of addition and multiplication. They are closure, commutative, associative, identity, inverse and distributive.

Imagery - The use of mental pictures to facilitate organization and recall of data.

Meaningful multiplication instruction - The teaching of multiplication which emphasizes mathematical structure.

Meaningful verbal learning - The relating of potentially meaningful material in a nonarbitrary and nonverbatim fashion to the learner's existing structure of knowledge.

Mnemonic - Any organizational schema designed to facilitate recall which does not involve mathematical meaning.

Multiplication combinations - Multiplication facts.

Multiplication facts - For a, b = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and c = the product of a and b, then a x b = c are the multiplication facts.

Multiplication table - A square array containing the products of all of the combinations of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 taken two at a time.

Recall of basic multiplication facts - To supply the correct products for a set of equations (e.g., 2 x 3 =, 4 x 7 =) within a given time constraint.

Repeated addition - An instructional schema whereby the multiplication facts are taught as an extension of addition (e.g., 4 x 3 = 3 + 3 + 3 + 3).

Representative materials - Pictures and/or illustrations used to relate a concept.

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Verbal association learning - The connecting together in a sequence two (or more) previously learned Ss→Rs where the Ss→Rs connected are verbal (nonmotor) behavior.

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Chapter 2

REVIEW OF LITERATURE

Historically, multiplication instruction has taken four distinct approaches to teaching the multiplication facts: 1) rote memorization, 2) mathematically meaningful instruction using representative and abstract materials (textbook approach), 3) mathematically meaningful instruction on a concrete level (manipulatives) and 4) instruction using mnemonics to facilitate recall of the facts.

Rote memorization of the facts, in and of itself, has been ruled out as an effective approach to learning the multiplication facts (Brownell, 1943). Current mathematics educators find rote memorization both inadequate and mathematically unsound (Beberman, 1964; Hirschi, 1970).

The three remaining approaches to multiplication; textbook methods, manipulative instruction and mnemonic methods all have sound psychological bases and would be expected to exhibit some degree of success in facilitating recall of the basic multiplication facts. Evidence relating to their relative values in facilitating recall of the basic multiplication facts, comprehension of the concept of multiplication and attitude toward mathematics is, however, either inconsistent or unavailable.

In this chapter, literature is reviewed which bears on the following issues central to this study: 1) Does mathematically

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meaningful instruction facilitate recall of the multiplication facts? 2) Should a child's developmental level be considered when he is learning material for recall, or are manipulative materials appropriate only for concept learning? 3) Would a mnemonic system be facilitative for the memorization of the basic multiplication facts? and 4) Is an elementary student's attitude toward mathematics important? The first part of this review deals with the learning of multiplication; the second with attitudes toward mathematics.

Learning of Multiplication

Textbook Instruction

The use of meaningful instruction began in this country about the turn of the century. A 1912 textbook developed at the Normal Training School at the University of Wyoming stressed "... counting as a foundation for multiplication." (1912, p. 6). It began with repeated addition exercises (e.g., 2 + 2 = 4, 2 + 2 + 2 = 6), and from them gradually developed the multiplication tables.

Hatton (1917) used the development of the multiplication table itself as an incentive for learning multiplication. She used what was essentially a repeated addition approach and discovery methods to develop the table. Smith (1923) cited use of the multiplication table as a valuable method for facilitating a child's acquisition of the facts.

Brownell's study of the learning of multiplication facts led to his conclusion that meaning must be included for learning. He found the commutative property valuable for the learning of the facts in pairs (e.g., 2 x 3 = 3 x 2). After meaning was established, Brownell

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recommended drill to fix the facts for retention.

Later studies investigated specific aspects of meaningful instruction. Fullerton (1955) compared the use of pictures and diagrams to teach multiplication to third-graders to a numbers only approach. He tested both recall of the facts and ability to transfer training and concluded that results were better when pictures and diagrams were used. He observed more student involvement in the work when pictures were used. MacShell (1964), in a similar study, found no significant difference between a representative-abstract and an abstract only approach.

Tietz (1968) compared instruction using a repeated addition approach to a ratio-to-one method. Fourth-graders were used in his study. The dependent variable was mastery of the facts. After fifteen programmed lessons, he found no significant difference between the treatments on the mastery variable. Both groups did improve, however.

Other studies explored meaningfulness in relation to the learning of multiplication facts by using the field properties. Schrankler (1966) stressed the structure of the number system through the field properties. His fourth-graders improved in computational skills, understanding of processes and problem solving.

Gray (1964) taught multiplication to third-graders and stressed the distributive property. He found it enabled the children to proceed independently to learn the products of previously unencountered combinations. He also found that children in his study who were not specifically introduced to the distributive property did not discover it themselves.

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Hall (1967), in contrast to Schrankler and Gray, taught third-grade students who had no prior multiplication instruction. One group was taught to use the commutative property, the other was not. After thirty-six lessons, he found no difference between the groups on a test of recall of the facts.

It would appear, then, that to be effective in teaching multiplication, even when recall of the facts is a prime concern, that methods stressing mathematical meaning should be considered. The learner should receive instruction in the concept of multiplication as it relates to addition and instruction in the field properties as they relate to multiplication. Representative and abstract materials should be utilized for instruction.

Manipulative Materials

A second mathematically meaningful approach has drawn interest in the past ten to fifteen years. Multiplication is traditionally a third-grade subject and third-grade students are predominantly concrete-operational in their learning style (Piaget, 1952). Children in the concrete-operational stage need concrete props (materials) to enable them to understand relationships between abstractions. In Piaget's theory, experiences with manipulative materials help the child to internalize the concept of multiplication by putting it on a concrete level. This approach is, then, mathematically meaningful, both in substance and in process.

Although studies have been conducted which look specifically at the need for manipulatives to teach multiplication, the evidence at this time is not sufficient to conclude that manipulatives are

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necessary.

Nichols (1971) compared a manipulative approach utilizing pupil discovery of the facts to a similar approach using only representative and abstract materials. Her sample consisted of third-grade pupils. After fifteen forty-five minute periods, she found differences on attitude, understanding of multiplication and skill in computation all favoring the manipulative group.

Punn (1973) taught multiplication to third-graders using combinations of enactive, iconic and symbolic presentations. After nine weeks of instruction, he found students were better prepared to solve missing factor problems when taught by an enactive-symbolic combination of instruction.

Thompson (1973) observed fourth-grade open classrooms for eleven weeks. He concluded that students need manipulatives to learn multiplication.

Although Nichols, Punn and Thompson found evidence to support the use of manipulatives to teach multiplication, other studies have shown manipulatives are not necessary to learn mathematics when computational skills are the primary concern.

Haynes (1963) compared the use of Cuisenaire rods to a conventional third-grade text. He used beginning of the year third-graders, and after six weeks found no significant differences on tests of achievement or problem solving abilities.

Ekman (1966) compared the use of algorithms, pictures plus algorithms and manipulatives plus pictures plus algorithms in the teaching of addition and subtraction to third-grade students. After eighteen days of instruction, he found the multi-stimuli students who

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had had instruction with manipulatives were superior on tests of understanding and transfer. A text of computational skill yielded no significant differences, however.

Lucas (1966) compared attribute block training to the Greater Cleveland Math Program. He compared first-grade students on ability to understand multiplication relations, ability to conserve and computational skills. After a ten-week treatment period, he found the students taught with attribute blocks to be superior on all but computational skills. On computation there was no significant difference between treatments.

Williams (1967) reported on a three-year study in England comparing traditional mathematics instruction for the early elementary grades with three manipulative approaches: the Cuisenaire, the Stearn and the Dienes. The results of this longitudinal study show that traditional methods are as effective for producing students who can compute well, but a higher number anxiety was exhibited by the students in the traditional group. In tests of mathematical understanding, motivation and attitude toward mathematics, the Dienes method was superior.

Although the studies cited tend to support the use of manipulatives for learning mathematics, or at least for the understanding of mathematics, some studies in mathematics have not shown manipulatives to be necessary. Lucow (1964) taught multiplication and division using both a Cuisenaire and a conventional approach. He found no differences between the two treatments' effect on student understanding.

Sister Aurich (1973) taught first-grade students using

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Cuisenaire and conventional approaches. On a traditional achievement test, she found no differences between the groups after four years of instruction.

Thus, while several research studies indicate the value of teaching multiplication by manipulative approaches, others find no differences. The differences that are found usually pertain to understanding of arithmetic, not recall or computation. It would, then, appear important to test the value of manipulatives when recall of the facts is the primary consideration.

Mnemonic Devices

While instruction may be designed to make the multiplication facts mathematically meaningful to the child, another alternative is available to the teacher. Mnemonic devices have previously been used to facilitate recall of the facts.

Miller (1967) defines a mnemonic device as a translation of new material into a pattern of some previously learned material. This previously learned material is usually in the form of a coding system. One such system specifically designed for coding numbers is called an analytic substitution method.

In an analytic substitution method, substitutions are used to change the numbers into sounds. The sounds are then changed into words and the words into sentences or phrases. Norman (1969) credits Stanislaus Mink Von Winckelmann with developing this scheme in 1648. A. Loisette expanded upon Winckelmann's method in his book, Assimilative Memory, or How to Attend and Never Forget. Loisette established a sound for each of the ten digits (0 through 9) and then

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used the sounds to make the words or phrases. His alphabet is given in Figure 2.

0	1	2	3	4	5	6	7	8	9
s	t	n	m	r	l	sh	s hard	f	b
	th					j	k	v	p
z	d						c hard		
c soft						ch	q		
						g soft	ng		

Figure 2. The Loisette figure alphabet.

Once the alphabet is memorized, a person can, for instance, code the date of the founding of Jamestown, Virginia, 1607, as follows:

1 t, th, d
6 sh, j, ch, g(soft)
0 s, z, c(soft)
7 g(hard), k, c(hard), q, ng

Using the above sounds, 1607 can be translated to "A Dutch song", "Dash a sack", "To wash a sock", etc.

Substitutions such as Loisette's would then be a possibility for the memorization of multiplication facts; "2 x 3 = 6" could be recalled as a phrase.

Mnemonics may also be rhymes or phrases such as "i before e, except after c". Multiplication instruction has used such approaches in the past.

In 1830, Marmaduke Multiply's Merry Methods of Making Minor

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Mathematicians was imported from England. The book contained "... catchy rhymes and engaging woodcuts" such as: "Twice 1 are 2, this book is something new, twice 2 are 4, pray hasten on before." Marmaduke enjoyed a high degree of acceptance and went through at least six printings between 1830 and 1900 (Marmaduke, 1971, p. 3).

The success of Marmaduke prompted other books of rhymes, including LuLu Multiplier, inferior in both verse and illustration to Marmaduke. Another book, The Multiplication Table in Rhyme for Young Arithmeticians, was published in New York. Its rhymes were connected into such stories as:

Little Jane

Four times one are four,
Little Jane was very poor
Four times two are eight,
And on others had to wait
Four times three are twelve
In the garden she would delve...(1970, p. 100).

This fixation with rhymes to facilitate memorization of the tables continued into the 20th century. Lizzie Stanley Martyn, who had previously published a book designed to facilitate memorization of multiplication rules (such as how to calculate the amount of plaster for a room, or sod for an area), followed it with The Multiplication Chant and Gesture Drill. She describes this book as a combination of the multiplication facts, useful information for object lessons and calisthenics. According to Miss Martyn, use of this book would assure that "...a thorough knowledge of the multiplication tables can be easily acquired and ever retained." (Martyn, 1899, p. 6).

The value of mnemonics for memorization was not investigated again until the fifties. Since then, reviews of literature all

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Identify imagery, a special type of mnemonic involving mental pictures, as being one of the most powerful memory factors ever studied (Bower, 1971; Bugelski, 1970; Paivio, 1971; Reese, 1970; Rohwer, 1970). Imagery is an extension of mnemonics in which the learner is asked to picture the association in his mind. This technique has been shown to be more effective than using words alone as mediators.

Miller, Galanter and Pribram, in their book Plans and the Structure of Behavior (1960), give an example of a mnemonic which involves imagery:

> One evening we were entertaining a visiting colleague, a social psychologist of broad interests, and our discussion turned to Plans. "But exactly what is a Plan?", he asked. "How can you say that memorizing depends on Plans?"
>
> "We'll show you," we replied. "Here is a Plan that you can use for memorizing. Remember first that
>
> one is a bun,
> two is a shoe,
> three is a tree,
> four is a door,
> five is a hive,
> six are sticks,
> seven is heaven,
> eight is a gate,
> nine is a line, and
> ten is a hen."
>
> "You know, even though it is only ten-thirty here, my watch says one-thirty. I'm really tired, and I'm sure I'll ruin your experiment."
>
> "Don't worry, we have no real stake in it." We tightened our grip on his lapel. "Just relax and remember the rhyme. Now you have part of the Plan. The second part works like this: when we tell you a word, you must form a ludicrous or bizarre association with the first word in your list, and so on with the ten words we recite you."
>
> "Really, you know, it'll never work. I'm awfully tired.", he replied.
>
> "Have no fear," we answered, "just remember the rhyme and then form the association. Here are the words:

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1. ashtray,
2. firewood,
3. picture,
4. cigarette,
5. table,
6. matchbook,
7. glass,
8. lamp,
9. shoe,
10. phonograph."

The words were read one at a time, and after reading the word, we waited until he announced that he had the association. It took about five seconds on the average to form the connection. After the seventh word he said that he was sure the first six were already forgotten. But we persevered.

After one trial through the list, we waited a minute or two so that he could collect himself and ask any questions that came to mind. Then we said, "What is number eight?"

He stared blankly, and then a smile crossed his face, "I'll be damned," he said. "It's a 'lamp'."

"And what number is cigarette?"

He laughed outright now, and then gave the correct answer.

"And there is no strain," he said, "absolutely no sweat."

We proceeded to demonstrate that he could in fact name every word correctly, and then asked, "Do you think that memorizing consists of piling up increments of response strength that accumulate as the words are repeated?" The question was lost in his amazement. (Miller, Galanter and Pribram, 1960).

Although there is no available research on the value of mnemonics and imagery to facilitate memorization in multiplication, some work has been done outside of mathematics. Miller (1967) studied the value of mnemonic devices for learning the important parts of history chapters. He used four different groups; one group used no mnemonics and the other three used three representative types of mnemonics. He found that all three of the experimental groups remembered the important parts of the chapters better than the group without a mnemonic device. He also determined that students in his study who weren't specifically

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instructed to use mnemonics would not use them. Paivio and Foth (1970) compared subjects on ability to recall lists of thirty paired-associates. They found that students who used imagery methods recalled more pairs than subjects using a verbal mediation technique. Yuille and Paivio (1968), Bugelski, Kidd and Segman (1968) and Martin, Cox and Boersma (1967) all found that subjects using pictures have more success at memorization than those using natural language mediators. Bower (1971), Raser and Bartz (1968), Rimm, Alexander and Eiles (1969) and Paivio and Rowe (1970) have all shown imagery to be consistently superior to verbal mnemonics of the meaningful type.

While there exists no evidence as to the value of mnemonic devices or imagery that deals specifically with multiplication, the literature supports the value of imagery in other applications. The success obtained by using mnemonic links in paired-associate learning suggests their possible value in facilitating recall of the facts.

Summary

In looking at the literature that deals with learning, more specifically, the learning of multiplication facts, some inconsistencies appear. Mathematical meaning is considered important for learning, but the relative values of a textbook (representative and abstract materials) approach and a manipulative (concrete materials) have not been established. Further, the relative values of textbook and meaningful instruction in facilitating recall of the facts have not been examined.

The use of imagery is effective in aiding memorization. Would it be more effective for promoting recall of the facts than

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mathematically meaningful methods? Mathematical meaning could, perhaps, be given either before or after the facts are learned with a mnemonic system.

Attitude Toward Mathematics

A dependent variable receiving increased attention is that of attitude toward mathematics. Current research shows attitude toward a subject appears to be important, especially in the later grades.

In a six-year longitudinal study, Anttonen (1969) examined the relationship between attitude and achievement. He found a high positive correlation between them in secondary students. Brown and Abel (1965) found the correlation between attitude and achievement higher for arithmetic than for spelling, language or reading, among elementary school pupils. Keane (1968) found arithmetic the least liked subject among elementary school pupils. Shapiro (1962) found that perseverance toward solutions of math problems was higher in elementary children who liked math than among those who didn't. Aiken (1970), after reviewing the research on attitudes toward mathematics in the period between 1960 and 1970, said: "It is clear that serious thought must be given to experiments concerned with temporary and more permanent effects of pre-school and early school experiences on attitudes toward and performance in mathematics." (p. 591).

Fedon (1958) and Stright (1960) both concluded that definite attitudes about arithmetic are formed by children as early as the third grade. They found these attitudes toward mathematics are often formed in the period between the third and sixth grades. Sharples (1969) found mathematics consistently rated low by junior high pupils. It would

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appear, then, to be important to strive for methods which promote good attitudes toward math in the lower grades.

Bernstein (1964) says teachers and mathematicians agree that rote learning procedures are a major factor in producing negative attitudes toward mathematics. Aiken (1970) says that attitudes toward the multiplication tables or other rote learning may be more negative than those toward other dimensions of math. Wilson (1961) concluded that a primary cause for avoidance of mathematics in high school was drill beyond the fundamental processes. Thus, it appears that the method used for the teaching of multiplication could influence a child's future participation in mathematics, thus, his mathematics achievement.

In looking at teaching methods, this conclusion is corroborated to some extent. Kaprelian (1961) utilized television instruction to make math more meaningful for fourth-graders. In his study 75% of the pupils stated that their attitude toward arithmetic had changed because the television had helped them to understand the subject. In another fourth-grade study, Lyda and Morse (1963) found positive changes in attitude when a "meaningful" method stressing understanding of the underlying concepts was employed.

The consensus among the studies is, then, that attitude and achievement are interrelated, and that attitude is formed, to some extent, through method. Thus, any method used to help a child to learn multiplication facts should be examined to determine its effect upon students' attitude toward mathematics. Without this information an invalid decision on the treatment's worth could be reached.

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Summary

The literature cited reveals both resolved and unresolved issues on the teaching of multiplication:

Rote memorization of the multiplication facts, without mathematical meaning, appears to be of little value. Studies show that mathematically meaningful instruction facilitates both recall and comprehension of the multiplication facts.

When the use of manipulative materials is compared to the use of representative and abstract materials for teaching multiplication to children in the early primary grades, tests of comprehension generally favor the manipulative approach. The value of manipulatives for facilitating recall has not been clearly explicated, however.

Mnemonic devices, imagery in particular, have been established to be effective for memorization. Paired-associate learning is facilitated by their use. No attempts, however, have been made to explore the value of mnemonics in facilitating recall of the multiplication facts.

Attitude toward mathematics begins forming in the early school years and is firmly established by the high school years, and attitude influences learner achievement. Thus, since attitude may be influenced by method, any early elementary mathematics study should examine the treatments' effects on student attitude.

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Chapter 3

METHODOLOGY

In the review of literature on the learning of multiplication, questions were raised as to the relative efficacy of several methods for teaching multiplication. Of these several methods the most prominent approaches appear to be meaningful instruction using abstract and representative materials (a textbook approach), meaningful instruction using manipulative materials and instruction using mnemonic devices to facilitate recall. Questions were raised about the effectiveness of these several methods for facilitating recall of the basic multiplication facts. In addition, since some methods purport to facilitate comprehension, a comparison of methods should include a comprehension measure. The review also pointed to a need for an attitude toward mathematics measure being taken in any research study on instruction of young children in mathematics.

From these questions and issues the following statistical hypotheses were formulated:

Hypotheses

I. There will be no significant difference in mean total recall achievement among students taught multiplication using a textbook approach, a manipulative approach or an imagery approach.

II. There will be no significant difference in mean recall achievement trend among students taught multiplication using a textbook

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approach, a manipulative approach or an imagery approach.

III. There will be no significant differences in population group centroids for recall, comprehension and attitude toward mathematics among students taught using a textbook approach, a manipulative approach or an imagery approach.

Design of the Study

Investigation of the effects over time of several methods of instruction on the recall of basic multiplication facts may be considered a growth or learning curve problem. Experimental designs in which several measures are taken to determine student learning as a function of time are called repeated-measures designs. Peng (1975) cites the increased use of repeated-measures designs in behavioral research. He attributes this increase to the design's ability to characterize a growth pattern for the subjects or groups under consideration. Repeated-measures designs are especially appropriate in longitudinal studies or when gain over time is being investigated (e.g., pre-post test) (Peng, 1975).

In a repeated-measures design the same group of subjects is measured repeatedly on the same variable. These repeated-measures (within-subject factors) are referred to as occasions. The between-subject factor is referred to as the sampling factor (Peng, 1975).

The design for this study employed six measures on recall of the basic multiplication facts over a period of five weeks. Figure 3 displays the design. In the figure G_1, G_2 and G_3represent three experimental groups, A_1, A_2 and A_3, the three treatments used in the study and B_0, B_1, B_2, B_3, B_4 and B_5 represent the six measures of

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recall.

	Weeks	B_0	B_1	B_2	B_3	B_4	B_5
Treatments	A_1	G_1	G_1	G_1	G_1	G_1	G_1
	A_2	G_2	G_2	G_2	G_2	G_2	G_2
	A_3	G_3	G_3	G_3	G_3	G_3	G_3

Figure 3. Repeated-measures design for recall of the basic multiplication facts.

Independent Variables

The three treatments for this study are designed to be representative of three approaches to multiplication instruction: textbook instruction (T), instruction employing manipulative materials (M) and instruction employing a mnemonic imagery system (I).

The three treatments are identifiable along two dimensions: type of stimuli available to the learner and mathematical structure included in the treatment.

The stimuli available to the child were operationally defined using Functional Analysis of Classroom Tasks (FACT) (Ober and Uprichard, 1971). The FACT system has two dimensions: sensory and cognitional. It was designed to classify, examine or quantify instructional stimuli available to the learner. The sensory dimension of the system has five subcomponents: Visual (V), Auditory (A), Tactile (T), Smell (S) and Taste (T^1) The cognitional dimension has three subcomponents:

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Concrete (C), Representative (R) and Abstract (A).

Using combinations of the two dimensions, thirteen categories of instructional stimuli can be identified. These categories are shown in Figure 4. A description of each of the thirteen classifications is given in Figure 5. Categories within FACT are mutually exclusive within a sensory subcomponent. For the purpose of describing the three treatments only stimuli directed to the learner are classified.

	Cognitional		
	Concrete	Representative	Abstract
Visual	VC	VR	VA
Auditory	AC	AR	AA
Tactile	TC	TR	TA
Smell	SC	SR	
Taste	T^1C	T^1R	

Figure 4. Category classifications for instructional stimuli - FACT.

Mathematical structure, the second dimension for treatment identification, is subdivided into seven parts: Multiplication as grouping (G), multiplication as repeated addition (A), multiplication as arrays (AR), the commutative property for multiplication (CO), the identity property for multiplication (I), the zero-product property for multiplication (Z) and the distributive property of multiplication over addition (D). Figure 6 identifies the instructional stimuli and

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Description of Behavior and/or Stimuli

1. Visual concrete Viewing the real or actual object or thing around which the instruction is centered.

2. Visual representative Viewing a model (two dimension or three-dimensional) or diagram representing an object, thing or idea around which the instruction is centered.

3. Visual abstract Reading a written description of something related to the object, thing or idea around which the instruction is centered.

4. Auditory concrete Hearing the real or actual object or thing around which the instruction is centered.

5. Auditory representative Hearing a representation of the object, thing or idea around which the instruction is centered.

6. Auditory abstract Hearing a verbal description of something related to the object, thing or idea around which the instruction is centered.

7. Tactile concrete Feeling physically and/or kinesthetically the real or actual object or thing around which the instruction is centered.

8. Tactile representative Feeling physically and/or kinesthetically a representation of an object, thing or idea around which the instruction is centered.

9. Tactile abstract Feeling physically and/or kinesthetically a verbal description of something related to the object, thing or idea around which the instruction is centered.

10. Smell concrete Smelling the real or actual object or thing around which the instruction is centered.

11. Smell representative Smelling an artificial scent representative of the real or actual object or thing around which the instruction is centered.

12. Taste concrete Tasting the real or actual object or thing around which the instruction is centered.

13. Taste representative Tasting an artificial substance representative of the real or actual object or thing around which the instruction is centered.

Figure 5. Functional Analysis of Classroom Tasks (FACT), R. L. Ober and A. E. Uprichard, University of South Florida.

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mathematical meaning available to each student in each of the three treatments: textbook instruction (T), instruction employing manipulatives (M) and instruction employing imagery devices (I).

Upon examining Figure 6, the similarities and differences among the treatments become evident. With respect to stimuli available to the learner, the manipulative instruction treatment is the only treatment in which the children either see or touch concrete materials (VC and TC). The other two treatments receive only representative and abstract stimuli. This lack of concrete material is the only major difference between the textbook and the manipulative approaches. Past research suggests that this difference is important (Nichols, 1971; Punn, 1973).

Looking at the mathematical meaning portion of Figure 6, the difference between the two mathematical approaches and the imagery approach becomes evident; the imagery approach has no mathematical meaning attached to its instruction, while the textbook and manipulative approaches differ only on the use of arrays for teaching multiplication. This lack of emphasis on mathematical meaning in the imagery treatment should cause lower comprehension scores if mathematics educators are correct in their assumptions (Brownell, 1943; Hirschl, 1970).

Thus, Treatments T and I differ from M in stimuli used, and Treatment I differs from T and M on mathematical meaning. More complete descriptions of the individual treatments follow with sample materials and sequence of instruction available in Appendix A.

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Treatment Descriptors		Treatments: Textbook Instruction	Manipulative Instruction	Imagery
STIMULI	VC		X	
	TC		X	
	VR	X	X	X
	AR	X	X	X
	TR	X	X	X
	VA	X	X	X
	AA	X	X	X
	TA	X	X	X
MATHEMATICAL MEANINGFULNESS	G	X	X	
	A	X	X	
	AR	X		
	CO	X	X	
	D	X	X	
	I	X	X	
	Z	X	X	

Figure 6. Treatments described by mathematical meaningfulness and stimuli available to learner.

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Textbook Instruction

Treatment T, textbook instruction, consists of pages pertaining to multiplication from one of the five Florida State approved third-grade texts.

Instruction for Treatment T consisted of twenty-three, forty-five minute periods. Time of day for instruction was held constant, with classes from 8:30 - 9:15 every day. The investigator taught all classes.

Treatment T includes the mathematical concepts of grouping, repeated addition, arrays, the commutative property, the identity property, the zero-product property and the distributive property of multiplication over addition. Instruction was on the representative and abstract levels, with no concrete materials used by either the teacher or the children. Instruction included pages 154-165, 184-194, 198-203 and 210-212 in Holt School Mathematics third-grade text (Nichols, et al., 1974). These pages included some word problems, missing factor problems and division problems; they were omitted.

Although Treatment T is a mathematically meaningful treatment, a good deal of drill (practice) with the facts is included in the textbook pages. Some pages, in fact, are devoted to practice with the facts. Examples of pages from the text are inciuded in Appendix A.

Instruction for Treatment T consisted of a sequential progression through the pages. Each day the investigator would present pages (usually two) to the children with any necessary general instructions on their completion. The children then worked individually on the pages while the investigator moved about the room and helped students with any difficulties they were experiencing. Completed

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textbook pages were collected each day and checked by the experimenter to assess individual student progress.

When any sequence of instruction showed that a child or children needed more work on the concept being taught, a work page was designed which was similar to the one in the text on which the child was having difficulty. In all, ten such extra pages were used; they consisted of practice with the distributive property (four pages) and drill pages on recall (six pages).

Manipulative Instruction

The instruction for Treatment M relied on the principles of grouping and repeated addition for the instruction on basic facts of multiplication. The commutative, distributive, identity and zero-product properties were emphasized during instruction when appropriate. Treatment M is essentially a duplication of a treatment used by Nichols (1971). Her study tested comprehension and compared manipulative materials with representative and abstract instruction. She recommended replication of her study with younger children (end-of-year third-graders were used for her sample) and a smaller sample size (her study had a very large one). The present study incorporated both of her recommended changes in its design.

Instruction for Treatment M consisted of twenty-three, forty-five minute periods. Time of day for instruction was held constant, with classes from 9:45 - 11.30 a. m. every day. The investigator taught all classes.

In the manipulative treatment group, children were provided with felt work mats divided into areas. The number of areas on a mat

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was determined by which set of multiplication facts were currently being studied (e.g., for the facts involving two, the pads had two areas, for the threes, three areas, etc.). In addition, each child was given a supply of $\frac{1}{2}$ inch cadmium-plated washers (hereafter referred to as counters) in a plastic container.

Initial instruction to the children defined the vocabulary: the "x" sign may be read "groups of", the "=" sign as "are". A child may then say "two groups of three are six" for "2 x 3 = 6".

Students had work pads and counters available to them at all times. Written exercises asked them to complete both representative and abstract representations of the facts. Stress was always on the correct representation of the facts, not on recall without understanding. Figure 7 is an illustration of a correct representation for 2 x 3 = 6 on the felt pad.

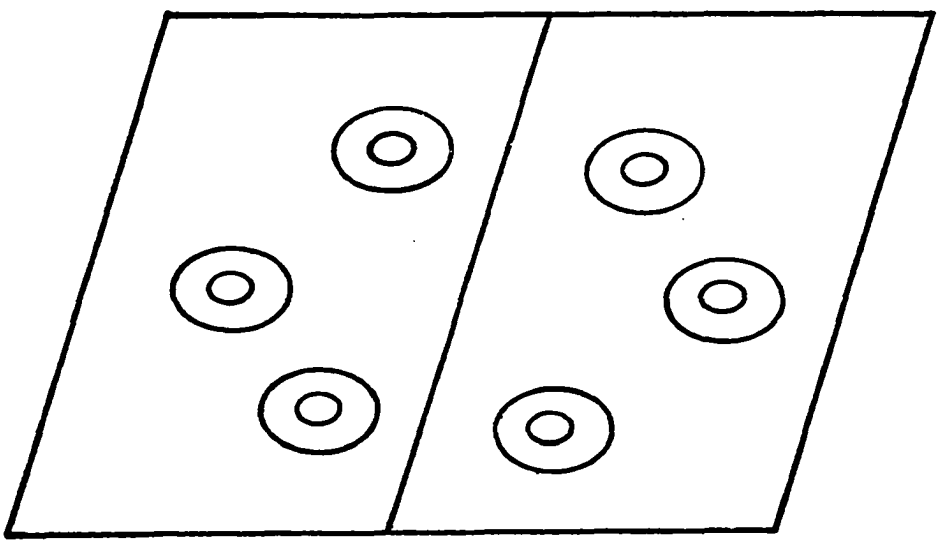

Figure 7. Representation of 2 x 3 = 6 on a work pad.

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The duty of the investigator in Treatment M was very similar to that in Treatment T. At the beginning of each instructional period, the investigator would see to it that each child had work pads and counters. Then, after any preliminary instructions dealing with the completion of the specific work sheets, the investigator would move about the room helping students with individual difficulties. In all cases, the child was referred to his counters for help or examples. He was asked questions such as "Five groups of three are how many?"

As in Treatment T, work sheets for Treatment M were collected and checked on a daily basis. The children who indicated lack of progress were given additional work with a concept or set of facts before moving to more difficult concepts or facts. Sample work sheets for Treatment M are included in Appendix A.

Imagery

Treatment I employed an analytic substitution system and imagery to facilitate recall of the multiplication facts. Instruction for Treatment I consisted of twenty-three, forty-five minute periods. Time of day for instruction was held constant, with classes from 12:30 - 1:15 p. m. each day. The investigator taught all classes.

Second-grade students traditionally have little or no introduction to multiplication; therefore the imagery group was given a one-day introduction to the concept. Pages 154 and 155 from Holt School Mathematics (Nichols, et al., 1974) were used for the introduction. These are the same pages as those used for Treatment T. After this basic introduction, no further attempt was made to make

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the facts mathematically meaningful to the child.[1]

On the second day of instruction, and thereafter, the learner's attention was focused on recall of the facts and how he could facilitate this recall by utilizing a mnemonic system.

The materials for Treatment I were in the form of a booklet, Multi-Memory Math Kit, developed by Growth Factors, Inc., of Tampa, Florida (1975). The booklet provides the learner with a sound association code between the digits 0 through 9 and the consonants. This code is given in Figure 8.

0	1	2	3	4	5	6	7	8	9
s	t	n	m	r	l	sh	s hard	f	b
	th					j	k	v	p
z	d						c hard		
c soft						ch	q		
						g soft	ng		

Figure 8. Sound association code for the digits 0 through 9 (Martin, 1975).

Once the sound association is developed for the ten digits, the system uses this coding to construct words with the correct sounds in them, and then supplies the child with a picture for each

[1]This instruction in multiplication, while mathematically meaningful, accounted for 4% of the total treatment time. Since it was not referred to again during instruction, it was purposely not considered for inclusion in Figure 6.

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word. The representation of the digits 2 through 4 are displayed in Figure 9.

As soon as the children have been introduced to the coding system and the basic pictures, they are presented with pictures for the facts. Figure 10 is a page from the Growth Factors text which is used for the recall of the fact 2 x 3 = 6. To recall the fact, the child forms a mental image of "A hen and a ham struggling with a shoe".

It should be noted in Figure 10 that, although no mathematical meaning is given to the instruction, the commutative property is stressed on every work sheet in the Growth Factors text. Thus, when a child is given "3 x 2", he is to think "ham and hen" or "hen and ham", and then complete the association to find the answer "6". The result of this method of instruction should be essentially the same for students in Treatment I as the emphasis on the commutative property is for students in Treatments T and M. Since no mathematical meaning is emphasized, however, the use of the commutative property by Treatment I is not indicated in Figure 6.

As was the case in both Treatments T and M, group instruction was kept to a minimum in Treatment I. The amount of group instruction was, however, somewhat more for Treatment I than for either T or M. This group instruction lasted about one week and was necessary because of the construction of the Growth Factors book. The purpose of this first week's work was to teach the digit-to-consonant code. For this week, verbal discussion of the codes constituted about 50% of class-time and practice on the codes the other 50%.

Following the first week of instruction on the codes, the Growth Factors test was used for practice on the codes along with work

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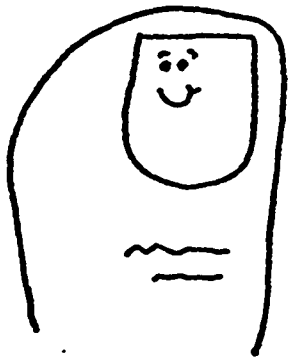
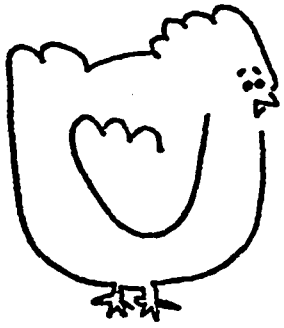
Remember use a, e, i, o, u and w, h, y to help build words.

use the T sound and add two vowels (which have no number value) to form TOE. TOE is a word-picture for the number 1 because T is the sound for 1. Draw a 1 on the TOE and fill in the block.



TOE

2 is equivalent to the sound of N

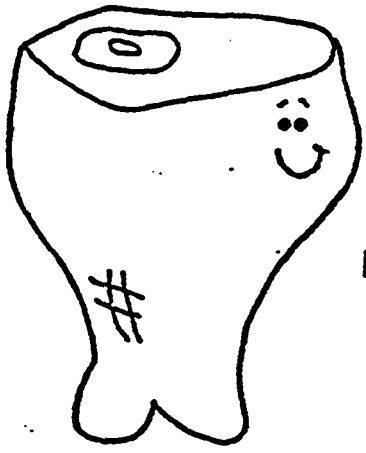


Connect these two lines to make an "N." To make a picture for the number 2, we can add an "H" and an "E" to form HEN. HEN is a word-picture for 2 because N is the sound for the number 2. Draw a 2 on the HEN and fill in the block.

HEN

3 is equivalent to the sound of M

Make an "m" from these three lines. We can use HAM as our word picture for the number 3 because M is the sound for 3 and H and A have no value. Color the 3 in the HAM's feet and translate the letter M in the block.



HAM

Figure 9. Pictorial representations of the digits 1 through 3.

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The hen is struggling with the ham
to tie the shoe.

HEN HAM

□ • □ = ○ SHOE

or

HAM HEN

□ • □

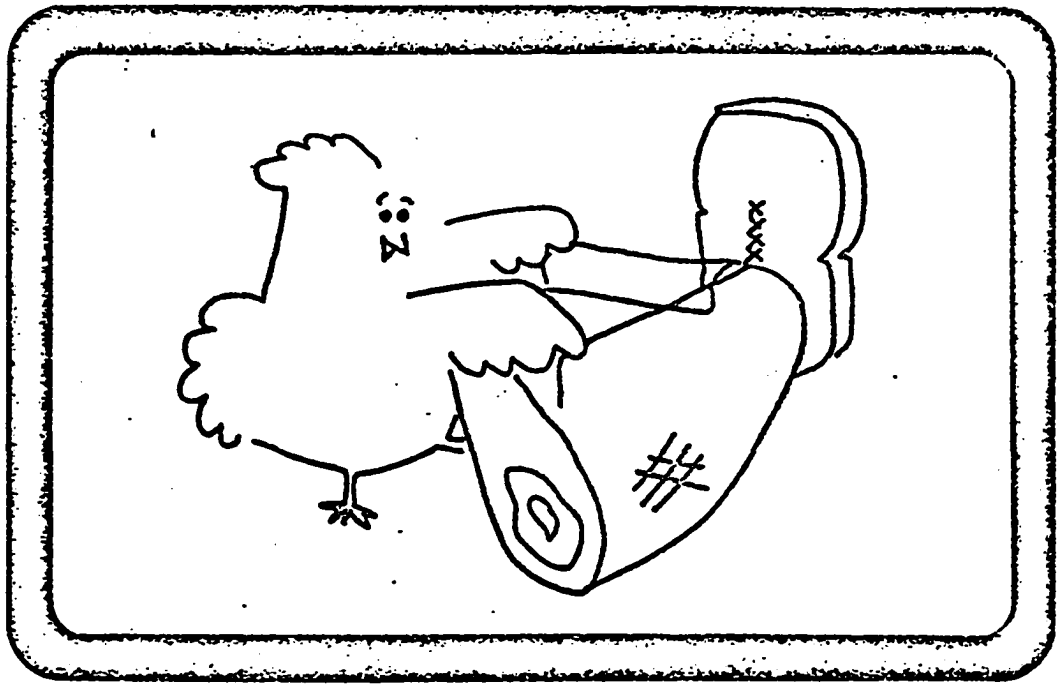

Figure 10. Textbook page for the fact 2 x 3 = 6.

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sheets designed to reinforce the memorization of the codes and allow for recall practice with the facts.

During all instruction in Treatment I, the use of the codes for recall of the facts was encouraged. If a child was having difficulty with the fact "2 x 3", he was encouraged to look for the picture in the book, or asked "What are the HEN and the HAM doing?"

The introduction, pages from the Growth Factors text and work sheets are included in Appendix A.

Summary

Instruction for the three treatments was designed, in all cases, to hold between-student interaction to a minimum. The instructor employed almost no group instruction with the exception of Treatment I, and this instruction took place only during the first week.

The materials to be used in the specific classes were designed to emphasize the particular medium used by that method. For Treatment T only textbook or textbook-like pages were used, for Treatment M the emphasis was put on correct representation of the facts and for Treatment I the emphasis was on knowing the code and using the pictures for recall of the facts.

Dependent Variables

Recall

Recall of the basic facts for multiplication was measured with Test 3, Part C, "Multiplication" of the Stanford Diagnostic Arithmetic Test (1966). The test consists of forty items which constitute a

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representative sample of the 100 basic multiplication facts.

The two forms of the test, W and X, contain the same facts, but their orders have been reversed (e.g., 2 x 3 in Problem 8 on one form is 3 x 2 on the other). For this study, six recall tests were necessary, thus Forms W and X and four additional forms developed by randomly selecting items from Forms W and X were used.

Peng (1975) stresses the requirement that the same, or equivalent, forms of a test be used in a repeated-measures design. Since Forms W and X contained essentially the same items, with the exception of order, and the other four tests contained items from either W or X, this requirement was met.

No reliability values are given for the SDAT "Multiplication" test by its makers. To establish reliability for the test, forty third-grade students similar to those in the study were given Forms W and X on a test-retest basis over a twenty-four hour interval; the reliability obtained was r_{xw} =.90.

The recall tests were administered by a cassette recording of the SDAT items to assure consistency across treatment groups. Students listened to the fact given (e.g., "Number 5, 2 x 4 =") and responded by writing the product in the appropriate blank on their individual answer sheets. Tests were hand scored with one point given for each correct response. The range of possible scores was from 0 to 40 correct. The recall tests were administered on the same day to each treatment group. Make-up testing due to student absence was performed as soon as the student returned.

Forms W, X and the student answer sheet are included in Appendix B.

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Comprehension

Comprehension of basic multiplication was measured at the end of the study. Comprehension was defined as the student's score on a Multiplication Usage Test (Punn, 1973). The test consists of ten missing factor problems and five word problems, two of which may be classified as division or missing factor problems and three as multiplication problems. The tests were hand scored with responses scored either right or wrong. The missing factor problems were graded one point each and the word problems, two points each. Possible scores ranged from 0 to 20.

In the treatments, the children had no instruction involving either missing factor (division) problems or word problems. Bloom (1956) says that comprehension may be tested in a problem solving situation if the student is aware of what abstraction (in this case, multiplication) must be used. Ausubel (1968) also recommends a problem solving approach to determine if students really comprehend meaningful ideas they are able to verbalize. He cautions, however, that not being able to solve the problems does not necessarily infer lack of understanding, but possibly a lack of other abilities (e.g., reading ability, reasoning ability, etc.). The Multiplication Usage Test is included in Appendix B. Punn (1973) found the test to have a test-retest reliability of .94 with third-grade students. A test-retest reliability verification used forty third-graders of backgrounds similar to those of pupils in the current study's sample; the time between the tests was twenty-four hours, and reliability was $r_{xx} = .84$.

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Attitude

Attitude toward mathematics was measured at the end of the study. Attitude is defined operationally as the student's score on the Attitude Toward Arithmetic Scale (Nichols, 1971). Nichols designed her test for use with third-graders. The test consists of 14 multiple choice items. Twelve of the items have two choices, one item has three choices and one item has five choices. The possible total test scores range from 0, indicating extreme dislike for arithmetic, to 30, indicating extreme liking for arithmetic. The attitude test is given in Appendix B.

The reliability of the attitude scale was estimated using a test-retest method on second-grade students with achievement and background similar to those in the current study. Time between testing was twenty-four hours. Forty students were used, and the resulting reliability estimate was $r_{xx} = .82$.

Procedures

Sampling

Students for the study were taken from the second grades of schools in the Pinellas County, Florida School System. Although instruction in multiplication is traditionally begun in the third grade (Brownell, 1943), end-of-year second graders were chosen for this study, since it was desired to have subjects with little or no prior multiplication instruction. The use of second graders was not felt to be a problem, because the instruction, as described in the independent variable section of this chapter, relies only on the student's knowledge of addition. Copeland (1970), in fact, advocates the teaching of

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addition and multiplication concurrently as early as the first grade.

Second-grade students in the county participate in one of two math programs; a conventional textbook approach or the Pinellas County Mathematics Systems (Zeph, 1974) approach. Mathematics Systems is a management system for elementary teachers. It is based on diagnostic and prescriptive procedures. In Math Systems, students are tested for skill mastery and materials are prescribed to reinforce those skills in which the child does not meet pre-established criteria. Since the Math Systems approach is very different from what would be considered a normal textbook approach, it was decided to use either all "Systems" or all "non-Systems" schools for the study. The county math supervisors recommended the use of three "Systems" schools, because they felt that the children were more independent workers and would be less likely to be affected by the change of teachers. As the investigator was to teach all three treatment groups, the selection of schools was limited to Upper Pinellas County.

Using the two criteria of being a Math Systems school and school proximity, and the additional criterion of willingness to participate in the study, three schools were purposefully selected for the study. Classes within schools were selected to allow the instructor travel time among the three schools. The three treatments; textbook, manipulative and imagery, were randomly assigned, one to each school.

To facilitate comparison of the three schools and comparison of the county system to other school systems, the following data on various demographic and facilitative variables were obtained from the book Profile '75 (Tocco, 1975).

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Table 1 Profiles for three treatment schools and total county:

Categories	Total County	Textbook	Manipulative	Imagery
Absentee rate %	6.1	5.8	4.8	5.0
Non-promotion %	2.1	1.0	2.5	1.3
% Non-white pupils	17.9	9.4	11.2	6.6
% Non-white faculty	14.9	10.0	14.3	14.7
% Male faculty	14.9	13.3	8.6	14.7
% Faculty holding Bachelor's degree	75.5	76.7	71.4	61.8
Masters or above	24.5	23.3	28.6	38.2
Teaching experience less than 1 year	5.2	0.0	2.9	0.0
1 - 3 years	23.7	20.0	25.7	23.5
4 - 9 years	30.7	26.7	37.1	17.7
10 or more years	40.4	53.3	34.3	58.8
Pupil/teacher ratio	21.65	23.62	22.02	21.26
Per pupil expenditure	$698.26	$700.02	$645.73	$722.61

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The Pinellas County system is the 30th largest school system in the United States and the fifth largest in the State of Florida. Pinellas County has a population of 731,512 of which 15% attend the public schools. There are 76 elementary schools (K - 6) in the county serving some 44,600 pupils.

Table 1 displays data for each of the three treatment schools selected and for the total county. As evidenced in the table, there is only small variation on most variables among the three treatment schools. The county average non-white pupil percentage is substantially higher than that of the three schools in the study, however.

In addition to the descriptive data, achievement data were obtained from Profile '75 (Tocco, 1975). The percentile ranks for the May, 1974 administration of the Metropolitan Achievement Test (Durost, et al., 1970) are displayed in Table 2.

Table 2 Percentile ranks for second-grade classes on Metropolitan Achievement Test, May, 1974, based on 1970 norms:

Subtest	County %tiles	Textbook School %tiles	Manipulative School %tiles	Imagery School %tiles
Word Knowledge	50	56	50	58
Word Analysis	46	58	54	58
Reading	44	54	44	60
Spelling	38	56	38	52
Math Computation	46	68	52	52
Math Concepts	48	68	47	52
Math Problem Solving	50	68	54	50

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As evidenced in Table 2, the percentile scores for math computation, math concepts and math problem solving are all substantially higher for the textbook instruction school than for the county or either of the other treatment schools.

While county-wide data for the May, 1975 administration of the Metropolitan Achievement Test were unavailable, the research, evaluation and planning division of the schools made available raw-score data for the second-grade students in the three schools used in the study. Table 3 displays the means, standard deviations and sample size (n) for the second-grade students in the schools and for the classes used in the study. Scores were obtained for the mathematics computation, mathematics concepts and problem solving portions of the test.

Inspection of Table 3 reveals consistently higher scores for the textbook treatment school compared to those of the manipulative and imagery treatment schools. This difference is also reflected when comparing the three experimental classes; textbook being 3 - 5 raw-score points higher on computation, 4 raw-score points higher on concepts and 4 - 6 raw-score points higher on problem solving. Subsequent analyses, discussed in the next section, were performed to examine the significance of these differences.

Analyses

The large variation in mean MAT scores across treatments led to an investigation of the statistical hypothesis of no significant differences on MAT test scores in the population. For this analysis the three MAT scores in computation, concepts and problem solving were

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Table 3 Means and standard deviationsfor raw-scores on the Metropolitan Achievement Test, May, 1975:

	TEXTBOOK		MANIPULATIVE		IMAGERY	
	Second Grade	Experimental Class	Second Grade	Experimental Class	Second Grade	Experimental Class
Math Computation Mean	24.28	23.46	20.73	20.38	20.48	18.17
St. Dev.	5.86	6.03	6.75	6.31	6.91	6.62
Math Concepts Mean	30.54	28.29	26.08	24.07	26.59	24.74
St. Dev.	6.27	5.92	7.31	7.88	7.21	7.09
Math Problem Solving Mean	25.64	24.75	21.41	19.97	21.43	18.00
St. Dev.	6.52	6.85	7.52	7.80	6.97	6.54
n	97	24	120	29	111	23

used as dependent variables in a one-way Multivariate Analysis of Variance (MANOVA). Multivariance IV (Finn, 1968) was used for this analysis. Tests of significance revealed that the use of covariance analysis would be necessary.

In all subsequent analyses the three MAT scores for computation, concepts and problem solving were used as multiple covariates.

To test the hypotheses of no significant difference among treatments on recall of the basic multiplication facts and significance of occasion and occasion by treatment interactions, a multivariate analysis of variance for repeated-measures design was employed. The analysis of repeated-measures design has inherent problems when the repeated measures define steps along a growth (learning) curve. Peng (1975) cites the likelihood of a given measure being more highly correlated with those immediately preceding and following it than with others. If this occurs, the assumption of an equal or uniform covariance structure will not be met. The failure to meet this assumption results in the rejection of the null hypotheses for occasions (weeks) and treatment by occasion interactions, with probability higher than the specified level of significance (Box, 1954; Kogan, 1948).

This problem has two possible solutions: using corrective statistics (Greenhouse and Geisser, 1959) or utilizing a multivariate analysis technique where the repeated measures are treated as separate dependent variables. Peng recommends the use of the multivariate approach as it yields a more accurate probability statement and is more precise than are F statistics. The only restriction put on the use of this procedure is that n (number of subjects) exceed k (number of occasions) by 20 or more (Davidson, 1972).

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A brief description of the procedure necessary to use multivariate analysis on the repeated measures follows:

The recall measures on an individual student, y_0, y_1, y_2, y_3, y_4, y_5, must first be transformed into new variables which allow the questions of interest to be examined. These transformed variables are:

$$\hat{y}_1 = -1y_0 + 1y_1 + 1y_2 + 1y_3 + 1y_4 + 1y_5 \qquad \text{GAIN}$$

$$\hat{y}_2 = -5y_0 - 3y_1 - 1y_2 + 1y_3 + 3y_4 + 5y_5 \qquad \text{LINEAR}$$

$$\hat{y}_3 = +5y_0 - 1y_1 - 4y_2 - 4y_3 - 1y_4 + 5y_5 \qquad \text{QUADRATIC}$$

$$\hat{y}_4 = -5y_0 + 7y_1 + 4y_2 - 4y_3 - 7y_4 + 5y_5 \qquad \text{CUBIC}$$

$$\hat{y}_5 = +1y_0 - 3y_1 + 2y_2 + 2y_3 - 3y_4 + 1y_5 \qquad \text{QUARTIC}$$

Coefficients for the four polynomial effects were taken from a table of orthogonal polynomials (Hays, 1973).

For the multivariate tests the model becomes:

$$\underline{\hat{y}}_{ij} = \underline{\mu} + \underline{\alpha}_j + \underline{\beta}\,(x_{1ij} - \underline{x}_{1}..)' + \hat{\underline{\beta}}\,(x_{2ij} - \underline{x}_{2}..)' + \hat{\hat{\underline{\beta}}}\,(x_{3ij} - \underline{x}_{3}..)' + \underline{\varepsilon}_{ij}$$

where the $\underline{\beta}$, $\hat{\underline{\beta}}$, and $\hat{\hat{\underline{\beta}}}$ terms represent the effects of the three MAT covariates, computation, concepts and problem solving on the y_{ij}.

The hypotheses to be tested become:

$H_0 : \alpha_1 = \alpha_2 = \alpha_3$ on $\hat{y}_1$ — TREATMENT

$H_0 : \underline{\mu} = \underline{0}$ on $\hat{y}_2$, $\hat{y}_3$, $\hat{y}_4$, $\hat{y}_5$ — OCCASION

$H_0 : \underline{\alpha}_1 = \underline{\alpha}_2 = \underline{\alpha}_3$ on $\hat{y}_2$, $\hat{y}_3$, $\hat{y}_4$, $\hat{y}_5$ — OCCASION X TREATMENT

Tests of the above hypotheses examine the effects of the three treatments on total recall achievement gain, the overall occasion effect and the occasion x treatment interactions.

To test the hypothesis of no significant differences on centroids of recall, comprehension and attitude toward mathematics, both multivariate and univariate analyses of covariance were used.

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Whenever significance is found through the use of the MANOVA or MANCOVA, the next step is to identify the dependent variables on which the treatments actually differ. To accomplish this, two approaches are available. One is inspection of the univariate results using each of the variables from the multivariate results as a single dependent variable, the other is the use of discriminant function analysis.

Finn (1974) recommends using univariate results to identify significant differences when significance is found on the multivariate tests. Although the three dependent variables are correlated, thus invalidating tests of significance unless sizes of Type I error specifications are adjusted, the univariate F-ratios provide relative strength-of-effect estimates for the outcome measures. The largest F-statistic will be obtained for that variable having the largest between-group variation relative to within-group variation; the smallest F is associated with that variable least affected by group membership.

The second approach, discriminant function analysis, is somewhat different, but usually yields results similar to those of univariate analysis.

In discriminant function analysis, linear combinations of the dependent variables which account for the largest amount of group differentiation are determined. Subsequent to the calculation of the first discriminant function, a second is calculated which has the property that it is uncorrelated with the first, and accounts for the largest group differences not accounted for by the first. This procedure reveals the "dimensions" of group differences (Tatsuoka,

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1971). After the discriminant functions are obtained, they are tested to determine if they contribute significantly to group differentiation. Only those discriminant functions found significant are then investigated further.

Once the significance of one or more discriminant functions has been established, the relative contributions of the variables to each significant function are investigated. Two approaches may be taken; the standardized coefficients of the discriminant functions may be compared for relative magnitude, or the correlations between the variables and each discriminant function may be calculated and compared. The first approach was employed in all analyses in this study. Discriminant analyses were completed following all multivariate analyses of variance and covariance used in this study.

Assumptions

The following assumptions were made to allow statistical analysis of the data:

1. Student responses are independent within treatment groups.

2. The dependent variables have a multivariate normal distribution.

3. The variance σ^2 is assumed to be equal at all levels of the independent variable; this is assumed true for all dependent variables.

4. The recall measures used have the same metric properties and are psychometrically equivalent.

5. Attrition from the experimental treatments occurred in a random manner.

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Constraints on External Validity

The following constraints on external validity were necessary to accomplish this study:

1. Treatments were assigned randomly to groups, not subjects to treatments. Random assignment of subjects to treatments is a necessary assumption of the analysis of covariance. Elashoff (1969) says that the results may be interpreted with care providing the other necessary assumptions are met.

2. Subjects are from an urban school system. Data available on the students will be used to identify the population. No attempts will be made to generalize to dissimilar populations.

3. Time of day was held constant with two treatments receiving instruction in the morning and one in the afternoon.

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Chapter 4

RESULTS

The results reported in this chapter are subdivided with respect to the three hypotheses under consideration in the study: 1) There is no significant difference in mean recall achievement gain among students taught multiplication using a textbook approach, a manipulative approach or an imagery approach. 2) There are no significant differences among students in mean occasion effect or treatment by occasion interactions when they are taught multiplication by a textbook approach, a manipulative approach or an imagery approach. 3) There are no significant differences in population group centroids for recall, comprehension and attitude toward mathematics among students taught multiplication by a textbook approach, a manipulative approach or an imagery approach.

Covariates

The obviously large differences in mean MAT scores among the treatment groups indicated possible differences in population mathematics ability centroids, as measured by the MAT. This concern that the three treatment groups were not, in fact, representative of the same population led to the testing of an additional hypothesis. The hypothesis to be tested was: There are no significant differences among population centroids for the three MAT raw scores, computation, concepts and problem solving. Multivariate analysis of variance

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(MANOVA) was used to test this hypothesis. The multivariate results are displayed in Table 4.

Table 4 Results of MANOVA on MAT raw scores for comprehension, concepts and problem solving:

Source	HYP	df	$F_{(df)}$	
Grand Mean	$\underline{\mu} = \underline{0}$	1	$352.53_{(3,71)}$	
Treatments	$\underline{\alpha}_1 = \underline{\alpha}_2 = \underline{\alpha}_3$	2	$3.01^*_{(6,142)}$	$p < .009$
Error		71		

In Table 4, the F-ratio of 3.01 is significant ($\alpha = .05$), thus the hypothesis of no significant differences in mean MAT scores is rejected. The rejection of the hypothesis of no significant differences among group centroids indicates further analyses are required to identify the dependent variables on which these differences occur. To identify these dependent variables the univariate results were used. The results of the three univariate tests using the MAT variables for computation, concepts and problem solving as separate dependent variables are reported in Table 5.

The univariate F-ratios in Table 5, although not equal in magnitude, do not differ enough to give a clear picture of the relative importance of the three variables. The F of 5.63 for problem solving indicates that this variable has the largest ratio of between-to-within-group variation, while that for concepts, 2.59, is the smallest. Using these differences alone, it would appear that computation (F = 4.16) and problem solving (F = 5.63) would be the most

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important of the three variables. To verify the conclusions of the univariate results, a discriminant function analysis was performed. The results of the analysis are displayed in Table 6.

Table 5 Results of ANOVAs for the MAT raw scores:

Variable	Source	df	MS	F
Computation	Treatments	2	166.17	4.16
	Error	73	39.94	
Concepts	Treatments	2	129.45	2.59
	Error	73	49.98	
Problem Solving	Treatments	2	287.23	5.63
	Error	73	51.02	

In Table 6, the first discriminant function is significant ($\alpha = .05$) and may be considered to describe the only significant relationship among the three variables. To determine the relative importance of the three variables on the first discriminant function, two approaches are available: The standardized coefficients may be compared by relative size, the larger coefficients indicating a larger contribution, or the correlations between each of the three variables and the first discriminant function may be compared. Since Finn (1975) recommends comparison of the standardized coefficients, this approach will be taken here. For the first discriminant function, the standardized coefficients of the MAT scores for computation, concepts and

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problem solving are .55, -1.07 and 1.30, respectively. A comparison of these coefficients indicates that the values for concepts and problem solving are more influential in the description of the first discriminant function than are the computation scores. This conclusion disagrees with the univariate results were the concepts variable appeared least important.

Table 6 Discriminant Analysis for MAT scores:

Variable	Standardized Coefficient	Bartlett's χ^2 test for significance	
First Discriminant Function			
Computation	.55		
Concepts	-1.07	$17.24_{(6)}$*	$p < .009$
Problem Solving	1.30		
Second Discriminant Function			
Computation	-.66		
Concepts	1.47	$3.28_{(2)}$	$p < .19$
Problem Solving	-0.16		

The inconsistency between the univariate analysis and discriminant function analysis results may be explained by looking at the correlations among the MAT scores. These correlations are displayed in Table 7.

In Table 7, the minimum correlation among the three variables is .72. This high intercorrelation among the variables suggests

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underlying psychological connections which are not isolated by either the univariate tests or the discriminant function analysis. Because of the inconsistent results of the univariate tests, the discriminant function analysis and the high correlations among variables, the decision was made to use all three MAT scores as multiple covariates in all subsequent analyses.

Table 7 Sample correlation matrix for MAT raw scores:

	Computation	Comprehension	Problem Solving
Computation	1.00		
Comprehension	.72	1.00	
Problem Solving	.72	.83	1.00

Several assumptions are necessary for the analysis of covariance. Elashoff (1969) cites three assumptions crucial to the underlying rationale for the use of covariance analyses: 1) Assignment to treatments has been random. 2) The covariate is independent of the treatments. 3) There is no treatment-slope interaction.

In the present study, treatments were randomly assigned to intact classes, thus covariance analysis may be used, but the results should be interpreted with caution since we may not be sure that all between-group bias has been accounted for by the covariates (Elashoff, 1969).

The second crucial assumption, that of the covariates being independent of treatment effects, is met in this study. The MAT tests were administered either previous to, or on the first day of, the

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study and, therefore, are independent of treatment effects. The third assumption, the assumption of no treatment-slope interaction was tested for each individual analysis to verify that it had been met.

In addition to the three crucial assumptions, Elashoff (1969) cites three assumptions necessary for statistical simplicity and validity of standard statistical tests: 1) The regression of the criterion variables on the covariates is linear. 2) The residual distributions are normal. The residual variances are homogeneous across treatments.

For these three assumptions, Elashoff (1969) recommends scatter plots be used to verify linear regression and that tests be performed to verify the assumption of homogeneity of variance. Normality of the residuals is difficult to examine. Elashoff (1969) says that non-normal distribution of the y's will not affect the F-test if the distribution of the x's (the covariates) is normal.

For all subsequent ANCOVAs and MANCOVAs the following procedure was employed: 1) The hypothesis was tested that the vectors of polynomial coefficients all equal the null vector (i.e., $\underline{\beta}_i = \underline{0}$, i = subscript for polynomial coefficients). 2) If this hypothesis was accepted, and extension of Bartlett's X^2 test was used to test for homogeneity of variance for all multivariate cases (Morrison, 1967). Hartley's F_{max} test was used for the univariate cases. 3) The hypothesis was tested that the vectors of interaction coefficients are equal to the null vector (i.e., $\underline{\beta}_i = \underline{0}$, i = subscript for interaction coefficients). A normal distribution on the residuals was assumed.

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Hypothesis 1

Hypothesis 1 was tested to determine the overall effects of the three treatments on recall of the basic multiplication facts. A repeated-measures design was employed with the initial recall measure, y_0, being taken on the first day of the study (B_0), and five more measures, y_1 through y_5, taken on subsequent Fridays (B_1 through B_5). Table 8 displays the means, standard deviations and sample sizes (n) for the three treatment groups.

Mean recall scores increased over time for all three treatment groups with Treatment T means ranging from 4.88 to 26.13, Treatment M means from 5.38 to 24.48 and Treatment I means from 5.91 to 19.35. Mean recall scores for the three treatment groups are graphed in Figure 11. The three growth curves reveal some information about the results of the three treatments. As evidenced on the graph, pupils given the two meaningful Treatments T and M, began improving almost immediately, while those given Treatment I were comparatively unchanged on mean recall score for the first few weeks. The improvement in mean recall scores for Treatments T and M almost parallel each other, with the mean scores for T consistently ahead of M. After the second week of instruction, the growth curve for Treatment I runs nearly parallel to that for Treatments T and M, but is approximately seven raw recall-score points below them.

To test the hypothesis of no significant differences in total recall achievement, each individual student's measures on recall for the five weeks of treatment y_1 through y_5, were summed and his entering recall score, y_0, subtracted from this sum. This new variable, $\hat{y}_1$, called Gain, was then used in a univariate analysis of covariance to

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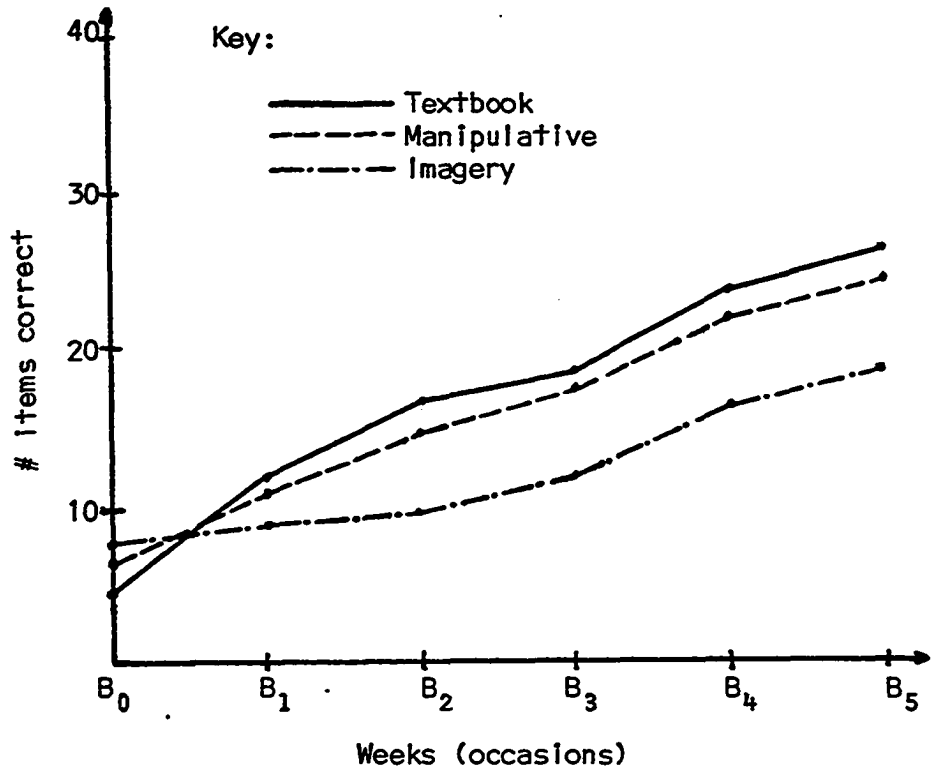

Figure 11. Graphs of treatment group mean scores for repeated-measures on recall of the basic multiplication facts.

Table 8 Means, standard deviations and sample size (n) for recall test scores over occasions (weeks):

		Occasions (weeks)					
	n	B_0	B_1	B_2	B_3	B_4	B_5
Textbook	23						
Mean		4.88	11.75	16.92	18.88	23.83	26.13
St. Dev.		4.57	4.67	6.12	6.11	8.12	8.60
Manipulatives	29						
Mean		5.38	11.28	14.41	18.34	22.00	24.48
St. Dev.		5.01	7.29	7.75	9.57	10.05	9.33
Imagery	24						
Mean		5.91	8.78	9.65	12.13	16.17	19.35
St. Dev.		6.92	7.79	8.69	10.04	9.07	10.00

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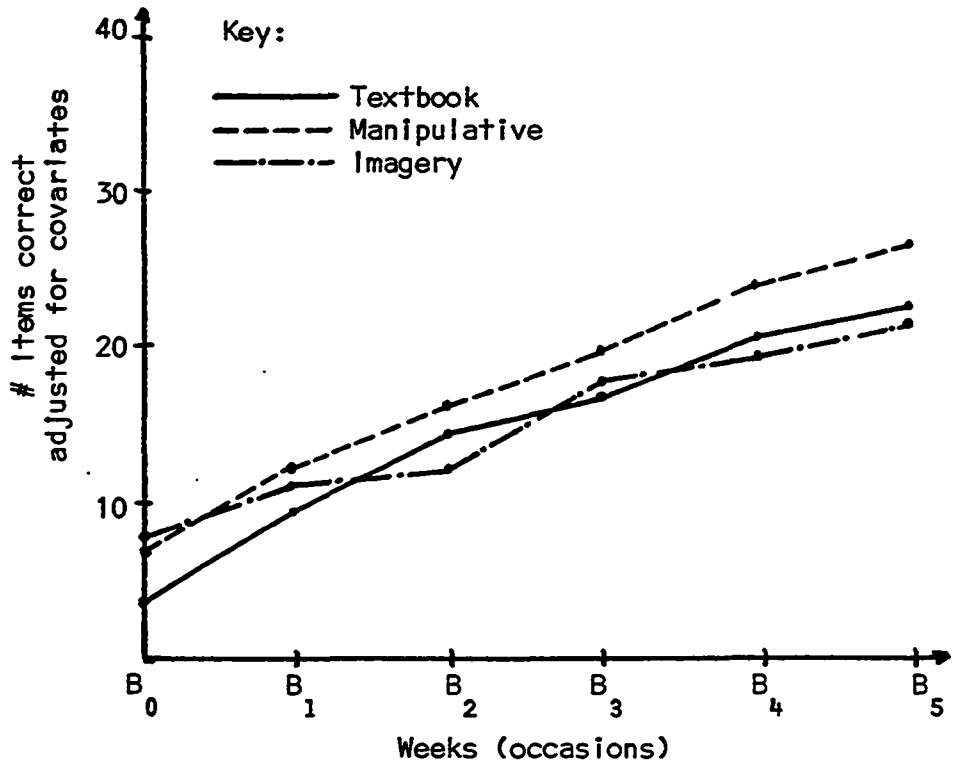

Figure 12. Graph of treatment group adjusted means for repeated-measures on recall of the basic multiplication facts.

Table 9 Group means for the six recall measures, adjusted for MAT covariates:

Treatments	Occasions B_0	B_1	B_2	B_3	B_4	B_5	$\hat{y}1$ Gain
Textbook	3.17	9.41	14.30	15.60	20.01	22.34	78.56
Manipulatives	5.67	11.98	15.27	19.24	22.88	25.45	89.17
Imagery	7.35	10.43	11.42	16.24	19.10	22.15	70.26

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test Hypothesis I.

Results for the tests for linear regression, homogeneity of variance and parallel regression lines were as follows: The test for linear regression yielded an F-ratio of $1.14_{(3,69)}$ ($p < .34$). The homogeneity of variance test yielded an F-ratio of .73 ($p > .05$). The parallel regression lines test yielded an F-ratio of $.64_{(6.64)}$ ($p < .70$). None of these tests were significant, thus the ANCOVA was conducted. The adjusted means for the six recall measures and the Gain scores for the three treatments are presented in Table 9. Figure 12 contains a graph of the adjusted means over the five weeks of treatment.

As evidenced in Table 9 and Figure 12, the adjustment for covariates alters the data considerably. Treatment T adjusted means range from 3.17 to 22.34, Treatment M adjusted means from 5.67 to 25.45 and Treatment I adjusted means from 7.35 to 22.15. The graph of the adjusted recall means still indicates the rapid increase for the first two weeks for Treatments T and M and the small increase in adjusted mean recall scores for Treatment I. The curves are, however, much closer in adjusted raw score points than they were on raw score points in Figure 11. A large B_5 difference now exists between Treatment M adjusted mean score (25.45) and the adjusted mean scores for Treatments T and I (22.34 and 22.15, respectively). The adjusted treatment means for Gain in Table 10 are 78.56, 89.17 and 70.27 for Treatments T, M and I respectively.

The results of the ANCOVA using Gain as a single dependent variable and the three MAT scores as covariates are displayed in Table 10.

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Table 10 Results of ANCOVA using Gain as the dependent variable and the three MAT scores as covariates:

Source	df	MS	F	
Treatments	2	2157.55	4.86*	$p < .01$
Error	70	443.94		

In Table 10, the Gain effect F-ratio of 4.86 is significant ($\alpha = .05$). Subsequent to the determination of the significant difference, post-hoc comparisons were performed using the Scheffé procedure (Hays, 1973). The results of these comparisons are reported in Table 11.

Table 11 Results of post-hoc comparisons (Scheffé method) for Gain, (adjusted mean scores) for recall of the basic multiplication facts:

Gain adjusted mean scores	Differences	Confidence Intervals
T = 78.56	T - M = 10.61	$\hat{y}_g \pm 18.24$ ($\alpha = .05$)
M = 89.17	T - I = 8.30	$\hat{y}_g \pm 15.92$ ($\alpha = .10$)
I = 70.26	M - I = 18.91	

In Table 11, the only significant ($\alpha = .05$) adjusted mean difference is that for M - I, a difference of 18.91. The next largest difference, that of T - M, is not significant at the $\alpha = .10$ level.

In summary, the results of the ANCOVA to test the hypothesis of no significant differences among treatments on total recall score

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is rejected ($\alpha = .05$). Subsequent post-hoc comparisons identify one significant ($\alpha = .05$) difference, M - I.

Hypothesis II

The second hypothesis under consideration in the study deals with the overall effect of occasions and the treatment by occasion interactions on the recall of multiplication facts. To test this hypothesis, the six recall scores (y_{ij}) were transformed, using coefficients of orthogonal polynomials, into four new variables which reflect the linear, quadratic, cubic and quartic contributions of the variables. These four new dependent variables ($\hat{y}_2$, $\hat{y}_3$, $\hat{y}_4$, $\hat{y}_5$) were then analyzed via a one-way multivariate analysis of covariance (MANCOVA) in which the three MAT scores for computation, concepts and problem solving were used as covariates.

Results for the tests for linear regression, homogeneity of variance and parallel regression lines were: The test for linear regression yielded an F-ratio of $1.03_{(12,175)}$ ($p < .42$). The homogeneity of variance test yielded a X^2 of $15.73_{(20)}$ ($p < .70$). The parallel regression planes test yielded an F-ratio of $.73_{(24,214)}$ ($p < .81$). None of these tests were significant, thus the MANCOVA was conducted. The results are displayed in Table 12.

Table 12 Results of MANCOVA for four polynomial dependent variables and three MAT covariates:

Source	HYP	df	$F_{(df)}$	
Occasions	$\underline{\mu} = \underline{0}$	1	$1.20_{(4,67)}$	$p < .32$
Occasion x Treatment	$\underline{\alpha}_1 = \underline{\alpha}_2 = \underline{\alpha}_3$	2	$2.63^*_{(8,134)}$	$p < .01$

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In Table 12 the F of 1.2 to test the effect of occasions is not significant. The F of 2.63 for polynomials, however, is significant ($\alpha = .05$). Accordingly, the univariate results were again examined to determine which polynomials had no effect on the result. The univariate results for polynomials using each polynomial variable as a dependent variable in an ANCOVA are displayed in Table 13.

Tabe 13 Results of four ANCOVAs, one on each of the variables created by use of orthogonal polynomials:

Variable	Source	df	MS	F
Linear	Treatments	2	6289.84	3.34
	Error	70	1883.19	
Quadratic	Treatments	2	7722.25	8.09
	Error	70	954.54	
Cubic	Treatments	2	1212.31	.89
	Error	70	1362.15	
Quartic	Treatments	2	97.55	.51
	Error	70	191.27	

In Table 13, both the linear and the quadratic criterion variables have relatively large F-ratios, 3.34 and 8.09 respectively, while those for cubic and quartic are relatively low, .89 and .51 respectively. The univariate results would, then, indicate that there is interaction between the adjusted growth curves for the respective treatments, and that these respective curves may be described by a regression equation using no higher than a quadratic term.

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Hypothesis III

To test Hypothesis III, the last recall score (measured at time = B_5), the comprehension score and the attitude score were to be used as dependent variables in a MANCOVA using the three MAT scores as covariates. The assumption of homogeneity of variances was tested first, however, using an extension of Bartlett's test. The assumption of homogeneity of variances was not met (X^2 = 50.75, critical value = 21).

An explanation for the failure of the data to meet the homogeneity of variance assumption may be obtained by looking at the correlations between attitude and the two more mathematical dependent variables of recall and comprehension in the several groups. These correlations are displayed in Table 14.

Table 14 Correlations between attitude and comprehension and attitude and recall for the three treatment groups:

	Treatments		
	T Attitude	M Attitude	I Attitude
Recall	.25	.34	-.32
Comprehension	.20	.22	-.09

In Table 14, attitude-recall and attitude-comprehension correlations of -.32 and -.09 respectively for the Treatment I group are quite different from those for Treatments T or M. These differences appear to be the factors influencing the difference in variance-covariance matrices indicated in the Bartlett's X^2 test.

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Subsequent to the determination of the lack of homogeneity of variance for the three dependent variables, and noting the possibility of the attitude correlation being the cause, a test for equality of the variance-covariance matrices for two dependent variables, recall and attitude, was performed. For this test the obtained χ^2 was 2.09. With df = 6 the critical value,(α = .05) is 12.6, thus, the hypothesis of equality of variance-covariance matrices was accepted.

Means, standard deviations and adjusted means for recall (B_5), comprehension and attitude toward mathematics are displayed in Table 15. The introduction of covariates had an appreciable effect on the Treatment T scores, the adjusted mean scores being 4, 1.5 and 1 raw score points lower on recall, comprehension and attitude respectively. The Treatment I adjusted mean scores are each approximately one point higher than their mean score counterparts.

Table 15 Means, standard deviations and adjusted means for recall (B_5), comprehension and attitude.

	Recall (B_5)	Comprehension	Attitude
Textbook			
Mean	26.13	10.33	17.29
St. Dev.	8.60	5.54	10.00
Adjusted Mean	22.34	8.60	16.54
Manipulatives			
Mean	24.48	7.21	24.79
St. Dev.	9.33	5.38	6.54
Adjusted Mean	25.45	7.65	24.53
Imagery			
Mean	19.35	5.35	22.04
St. Dev.	10.00	5.23	7.23
Adjusted Mean	22.15	6.61	22.92

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To analyze the data for the third hypothesis, two analyses were performed: first, a MANCOVA using recall and comprehension as the two dependent variables and the three MAT scores as covariates; second, an ANCOVA using the attitude score as a single dependent variable and the MAT scores as covariates. Since inter-correlations among recall, comprehension and attitude cannot be assumed zero, the desired level of significance ($\alpha = .05$) was halved for each of the two separate tests of significance and levels of $\alpha = .025$ were used in all significance tests.

Results for the tests for linear regression and parallel regression planes for the first analysis were: The F-ratio for linear regression was $2.14_{(6,136)}$ ($p < .052$). The parallel regression planes test yielded an F-ratio of $1.39_{(12,126)}$ ($p < .18$). Since these two F-ratios are significant and the homogeneity of variance hypothesis was found to be tenable above, the MANCOVA was conducted. The results of this MANCOVA using recall and comprehension as dependent variables are displayed in Table 16.

Table 16 Results of MANCOVA on recall and comprehension using MAT covariates:

Source		df	$F_{(df)}$	
Grand Mean	$\underline{\mu} = \underline{0}$	1	$4.39_{(2,69)}$	
Treatments	$\underline{\alpha}_1 = \underline{\alpha}_2 = \underline{\alpha}_3$	2	$2.13_{(4,138)}$	$p < .08$

For the MANCOVA on recall and comprehension using the MAT covariates, the F value of 2.13 is not significant. Thus no difference

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in group centroids may be concluded.

Results for the tests for linear regression, homogeneity of variance and parallel regression lines when attitude is the dependent variable were: The tests for linear regression yielded an F-ratio of $.78_{(3,69)}$ ($p < .51$). The homogeneity of variance test yielded an F-ratio of 2.33 ($p > .05$). The test for parallel regression lines yielded an F-ratio of $1.46_{(6,60)}$ ($p < .21$). None of these values were significant, thus, the ANCOVA was conducted using attitude as the dependent variable. The results of this ANCOVA are displayed in Table 17.

Table 17 Results of ANCOVA on attitude using MAT covariates:

Source	df	MS	F	
Treatments	2	406.86	6.45*	$p < .003$
Error	70	63.08		

For the ANCOVA, the F-ratio of 6.45 is significant at the chosen $\alpha = .025$ level. A Scheffé post-hoc comparison was performed to identify those significant adjusted mean attitude differences. The results of the Scheffé analysis are displayed in Table 18.

In Table 18, the confidence interval indicates the T - M difference of eight adjusted mean score points on attitude to be significant ($\alpha = .05$). The adjusted mean score difference for T - I of six adjusted mean score points is not significant at $\alpha = .10$. Thus, the only difference that may be concluded significant is the difference between Treatments T and M, Treatment M having the higher score.

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Table 18 Results of post-hoc comparisons (Scheffé mathod) on adjusted mean attitude scores:

Attitude Adjusted Mean Scores	Differences	Confidence Intervals
T = 16.54	T - M =-7.99*	$\hat{y}_g \pm 7.67$ $\alpha = .025$
M = 24.53	T - I =-6.38	$\hat{y}_g \pm 6.88$ $\alpha = .05$
I = 22.92	M - I = 1.61	

In Summary, the results of the MANCOVA on recall and comprehension and the ANCOVA on attitude toward mathematics yield only one significant difference: between the adjusted mean score for attitude of Treatment M (24.53) and that for Treatment I (16.54).

Summary

The results of the several tests for the three hypotheses were as follows: Reject Hypothesis I; the total recall adjusted Gain score shows the manipulative treatment to yield a significantly higher adjusted mean score for Gain than that of imagery. The manipulative-textbook difference, while larger than that for textbook-imagery is not significant. Accept Hypothesis II for occasion effect; there is not a significant occasion effect for the mean scores adjusted for covariates. Reject Hypothesis II for interaction; there is a significant occasion by treatment interaction. The treatment growth curves may effectively be described by a linear and a quadratic term in the regression equation. Reject Hypothesis III; while the multivariate test for significance on recall and comprehension adjusted mean scores yielded no significant differences, the tests for significance

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for attitude show a significant difference exists between the adjusted mean attitude score for manipulatives (24.53) and that for textbook (16.54).

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Chapter 5

SUMMARY, CONCLUSIONS AND IMPLICATIONS

Summary

The purpose of this study was to investigate the differential effects of three approaches to the teaching of multiplication--textbook instruction, manipulative instruction and imagery--on recall of the basic multiplication facts. Although students' ability to recall the facts was the major concern of the study, measures were also taken on comprehension of the operation of multiplication and attitude toward mathematics. For this investigation, three treatments were designed to be representative of the three approaches to multiplication instruction.

Treatment T, textbook instruction, used pages on multiplication from the Holt School Mathematics (Nichols, et al., 1974) third-grade text. Only representative and abstract stimuli were available to the learner. Instruction consisted of the student completing the textbook pages on his own, with individual help being provided by the investigator when necessary.

Treatment M, manipulative instruction, was designed with the learner's level of cognitive development in mind. Concrete materials, in the form of counters, were available to the children at every instructional session. Work sheets, emphasizing mathematical meaning, were provided for each student. The investigator supplied individual

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student help when necessary.

Treatment I, imagery, used the text, Multi-Media Math Kit (Martin, 1975). Instruction consisted of first teaching the child an analytic substitution method to code the digits 0 through 9 into sounds, and then teaching the translation of these sounds into pictures. Exercises were then provided to help the child recall each fact by recalling a unique picture associated with it. When a child could not recall a fact, he was always referred to the picture associated with it (e.g., for the fact "2 x 3 =" the child would be asked, "What are the HEN and the HAM doing?").

Subjects for the study consisted of three second-grade classes, one from each of three schools in an urban Florida school system. Second graders were selected since the study was to be conducted in the spring of the school year and it was deemed desirable to have subjects with minimal knowledge of the operation of multiplication. Instruction took place during twenty-three, forty-five minute classes. The investigator taught all classes.

As previously indicated, recall of the basic multiplication facts was the primary concern of the study. Accordingly, recall measures were taken on the first day of the study, and on subsequent Fridays for the five weeks of treatment. The test used was a forty-item sample of the multiplication facts. Administration was accomplished by playing a cassette recording of the facts to the children and having them write the correct product on their individual answer sheets. A comprehension test and an attitude toward mathematics test were administered at the end of the study.

The statistical hypotheses tested were: 1. There will be

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no significant difference in mean recall achievement gain among students taught multiplication using a textbook approach, a manipulative approach or an imagery approach; II. There will be no significant differences among students in mean occasion effect or occasion by treatment interactions when they are taught multiplication using a textbook approach, a manipulative approach or an imagery approach; III. There will be no significant differences in population group centroids of recall, comprehension and attitude toward mathematics, among students taught multiplication by a textbook approach, a manipulative approach or an imagery approach.

To enable the testing of Hypotheses I and II, a repeated-measures design was employed, with six recall measures being taken during the five weeks of the study. These six scores for each subject were transformed to new dependent variables which allowed the questions of interest to be examined. The transformed variables reflected overall recall improvement (Gain) and linear, quadratic, cubic and quartic trend components. These variables were used in multivariate and univariate analyses of covariance to test Hypotheses I and II. The students' current Metropolitan Achievement Test (Durost, et al., 1970) raw scores for computation, concepts and problem solving were used as multiple covariates in all analyses.

To test Hypothesis III, the students' final recall scores and comprehension scores were considered as realizations of multiple dependent variables in a multivariate analysis of covariance and their attitude toward mathematics scores were used as realizations of a single dependent variable in an analysis of covariance. The three MAT scores for the mathematics subtests were again used as multiple

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covariates.

Results of the several analyses were: Hypothesis I was rejected. There was a significant difference on recall Gain over time between the adjusted mean recall Gain score for Treatment M and that for Treatment I; the Treatment M adjusted mean recall Gain score being approximately 19 points greater. Hypothesis II was tenable for occasions; it was rejected, however, for occasion by treatment interaction. An analysis of trends indicated that the adjusted mean recall growth curves could be represented by no higher than a second-degree polynomial term. Hypothesis III was rejected. Although the centroids for the adjusted mean final recall score and the adjusted mean comprehension scores did not differ significantly, the adjusted mean attitude scores did. Post-hoc comparisons identified the Treatment M adjusted mean attitude score as being significantly more positive than that for Treatment T.

Conclusions

Hypothesis I

The rejection of Hypothesis I allows the conclusion that Treatment M was more effective over time in promoting recall of the facts than was Treatment I. The adjusted mean score for Gain for Treatment T is about midway between scores for Treatments M and I, and lends no support to a hypothesis that any mathematically meaningful treatment would be effective in promoting recall of the facts. The recall results do support the conclusion that concrete materials are necessary for early-elementary students even when recall of the facts is the primary concern.

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Hypothesis II

The data in Figure 12 and Table 9 of Chapter 4 (see also Table 8) would indicate that an overall occasion effect was present in the sample; the test of significance, however, did not support this conclusion for the population. The large variation in sample scores appears to have influenced the standard error of estimate to the extent that the mean square errors for linear, quadratic, cubic and quartic effects are very large. Therefore, these effects may not be concluded to differ from zero in the population.

Although the occasion effect was not significant, the analysis involving the transformed variables revealed that the treatment-occasion interactions were significant. This conclusion may be verified by observing the intersections of the three treatments adjusted mean score growth curves. Inspection of the respective mean growth curves for Treatments T, M and I reveals the two mathematically meaningful treatments increased in recall, rapidly at first, and then slowed down while maintaining a positive slope. The growth curve for Treatment I was slower starting, but began to parallel Treatments T and M by the third week of instruction. The smooth curves and lack of third or higher degree terms indicates a continual increase in recall learning with virtually no set-backs from one week to a following week. Were the treatments to be continued for longer than five weeks, it would appear that this trend upward would continue, but the rate of growth would approach zero as the mean recall scores neared 40 items correct.

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Hypothesis III

In the MANCOVA for Hypothesis III, the two dependent variables were recall and comprehension. The finding of no significant difference on group centroids for these two variables is contradictory to the results for Hypothesis I. It would appear that the three treatments differed little in final recall results.

The ANCOVA results for the attitude variable in Hypothesis III are clear. Treatment M produced significantly more positive mean attitude than did Treatment T. This result, coupled with the findings of Hypothesis I would appear to indicate a need for the use of manipulative materials in multiplication instruction for early-elementary children.

The use of manipulatives is supported by the results of both Hypotheses I and III. The use of manipulatives influenced a significantly larger mean Gain score for Treatment M than I, and a significantly more positive attitude for Treatment M than T.

It would appear, then, that the developmental level of a child should be considered, even when the primary purpose of instruction is recall of the multiplication facts. The effectiveness of manipulatives for facilitating recall and the high positive attitude of students in Treatment M support their use for multiplication instruction.

The relatively small differences among treatments on the test of comprehension does not support results cited in the literature review. The results, if they were consistent with the recall results, should have shown Treatment M well above Treatment I on comprehension, with Treatment T somewhere between the two. The measure for

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comprehension of the basic multiplication facts was taken after only five weeks of instruction and was, perhaps, premature. The mean test scores were all well below the possible score of 20, which would lead to the hypothesis that the test of comprehension was too difficult for students who had only five weeks of instruction, even though they were able to produce significant gains in recall of the facts over the same period. The determination of reliability for the comprehension test was with third graders who had upwards of five months of multiplication instruction, thus, they could be expected to perform better.

The relatively poor performance of Treatment I subjects may have been influenced by time of day for instruction, since Treatment I was the only afternoon class taught (12:30 - 1:15, p. m.). The high attitude scores achieved by Treatment I, however, do not lend support to this hypothesis. Another possible cause of the poor performance of the Treatment I group is the need for the students to learn more material during the same time period. While Treatments T and M both make use of a pre-existing structure of addition concepts which the child has been developing for at least two years, Treatment I requires that the student first learn a coding system and then learn images which are tied to the coding. When this involved procedure is attempted with an entire classroom, the teacher has, perhaps, too little feedback on individual student progress. While a longer instructional time sequence might show more recall improvement for Treatment I students, mathematical meaning must, at some time, be given to multiplication. It would appear to be self-defeating, then, to use an imagery system when the manipulative approach was so successful.

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While the imagery treatment is designed to be used either individually or with groups, the amount of control exercised by a teacher upon his class is, necessarily, inversely proportional to group size. Thus, a small group may be necessary for treatment effectiveness. With the imagery treatment, students who didn't learn the coding system and some images had no way to obtain products on the recall tests. These few students, therefore, had consistently low recall scores. In contrast, a student who learned the multiplication to repeated-addition relationship emphasized in the textbook and manipulative treatments was able to score progressively higher each week, thus increasing the group mean. This problem of the disinterested student must be dealt with, and the imagery treatment appears to deal with it poorly.

Recommendations

In this study several assumptions have been made: The adjustment for the MAT covariates was assumed to account for all of the treatment group bias existing at the start of the study. The measures for recall, comprehension and attitude toward mathematics were assumed to be valid measures of the constructs underlying them. Time of day for instruction was assumed to have no effect on recall, comprehension or student attitude toward mathematics. Subjects were assumed to be representative of students from other, similar urban school populations. The reader is reminded that the following recommendations to parents, teachers, curriculum developers and educational researchers should be considered in light of these assumptions.

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Parents

The books and articles critical of modern math have influenced many parents to conclude that mathematical meaning is not as important as drill. The results of this study did not support this conclusion. Parents should verify that mathematical meaning is included in their children's instruction in multiplication and that manipulatives are used when appropriate.

Teachers

The results of this study, when taken in conjunction with previous literature cited, suggest that teachers of early-elementary children should assure that the textbooks provided them do not become the sole source of instruction available to their students. The results of this study indicate a need for manipulative materials when teaching multiplication facts to end-of-year second-grade students, even when the major objective of the instruction is to facilitate recall of the facts. The use of textbook materials with second graders appears to influence student attitude, yielding less positive attitudes toward mathematics.

Curriculum Developers

When developing instructional materials for learning of the multiplication facts it would appear to be desirable for the curriculum developers to assure the material is mathematically meaningful. In addition, it would appear that materials should include manipulatives as an integral part of their treatment. While reference is made to manipulatives and how they can be used to supplement instruction in the teachers' guides to most mathematics texts, the actual obtaining

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and use of the manipulatives is left to the teacher. It would appear that materials designed today for multiplication instruction should assure the use of manipulatives by including them in an instructional package to accompany the textbook. While mathematics laboratory materials do make some attempt to accomplish this, they are not tied specifically to a text, and therefore, may not be used by the teacher. The development of a complete instructional package which includes manipulatives as an integral part of its design would appear to be an acceptable remedy to this difficulty.

Educational Researchers

Several research questions may be raised as a result of this study. The first of these questions deals with the study itself; it should be replicated, or a similar study carried out to either support or contradict the results. The results support meaningful mathematics instruction using manipulatives to facilitate recall, but were inconclusive with respect to comprehension. Further investigations should be designed to identify the reason for this lack of significance. The test of comprehension was, perhaps, too difficult or perhaps administered after too little multiplication instruction. The development of a comprehension test which would be of value in a short-term experiment is suggested.

To further investigate the value of an imagery system for facilitating recall, other studies are recommended. The present study used a cross-section of students. Perhaps the imagery system would be more applicable to students who cannot learn the facts through conventional means. Another area for research in imagery could

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be the mathematical level of students. This study used students with little background in multiplication; perhaps the students with more multiplication knowledge would find more value in an imagery system.

Summary

The results of this study indicate a need in the early-elementary classroom for meaningful mathematics instruction using manipulatives even when recall of the basic multiplication facts is the primary concern. The results of this type of instruction should be an increase in recall of the basic multiplication facts and a more positive attitude toward mathematics.

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APPENDICES

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APPENDIX A

TREATMENTS

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TREATMENTS

Sequence of Instruction for Three Treatments

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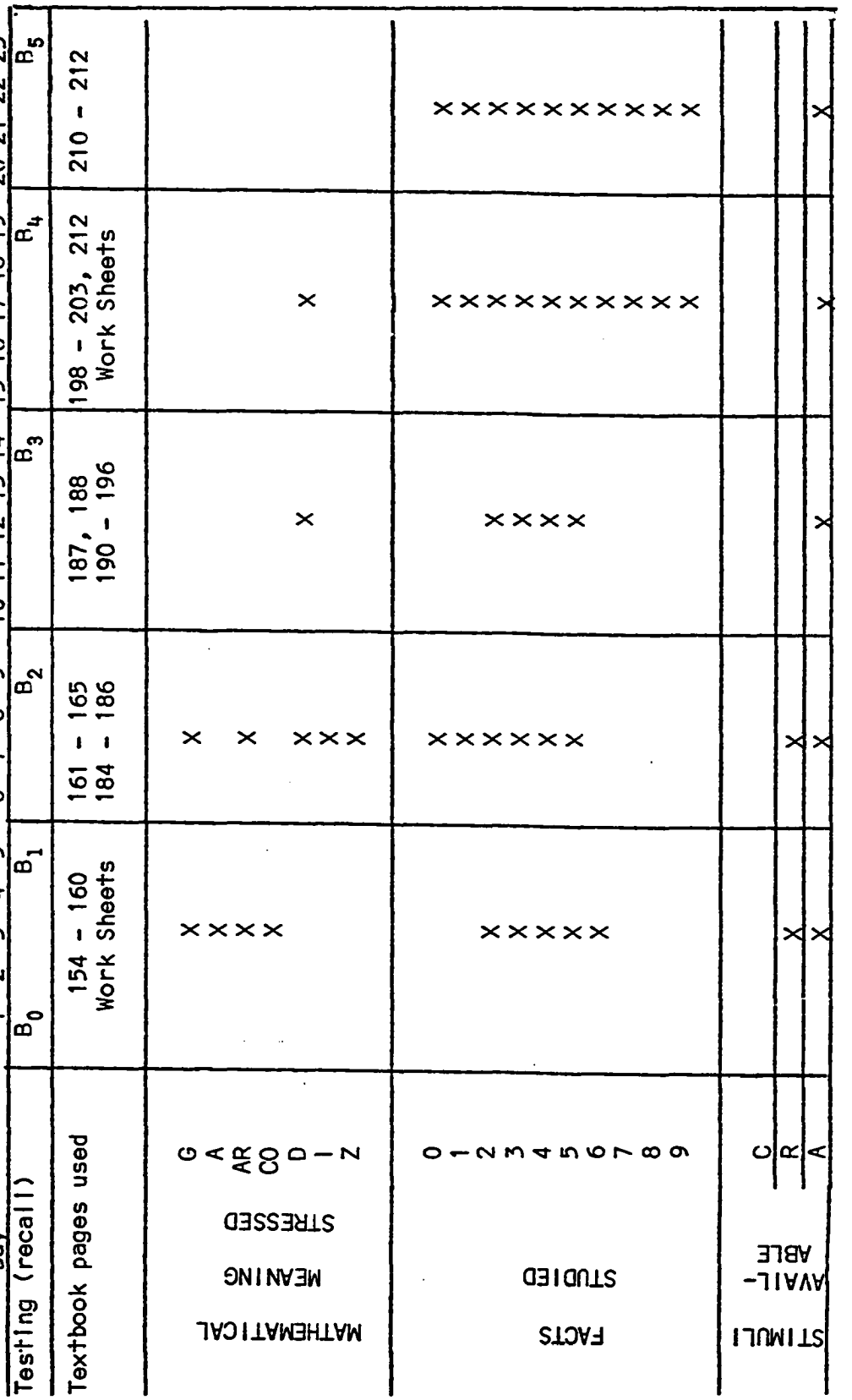
Sequence of Instruction

Textbook Treatment



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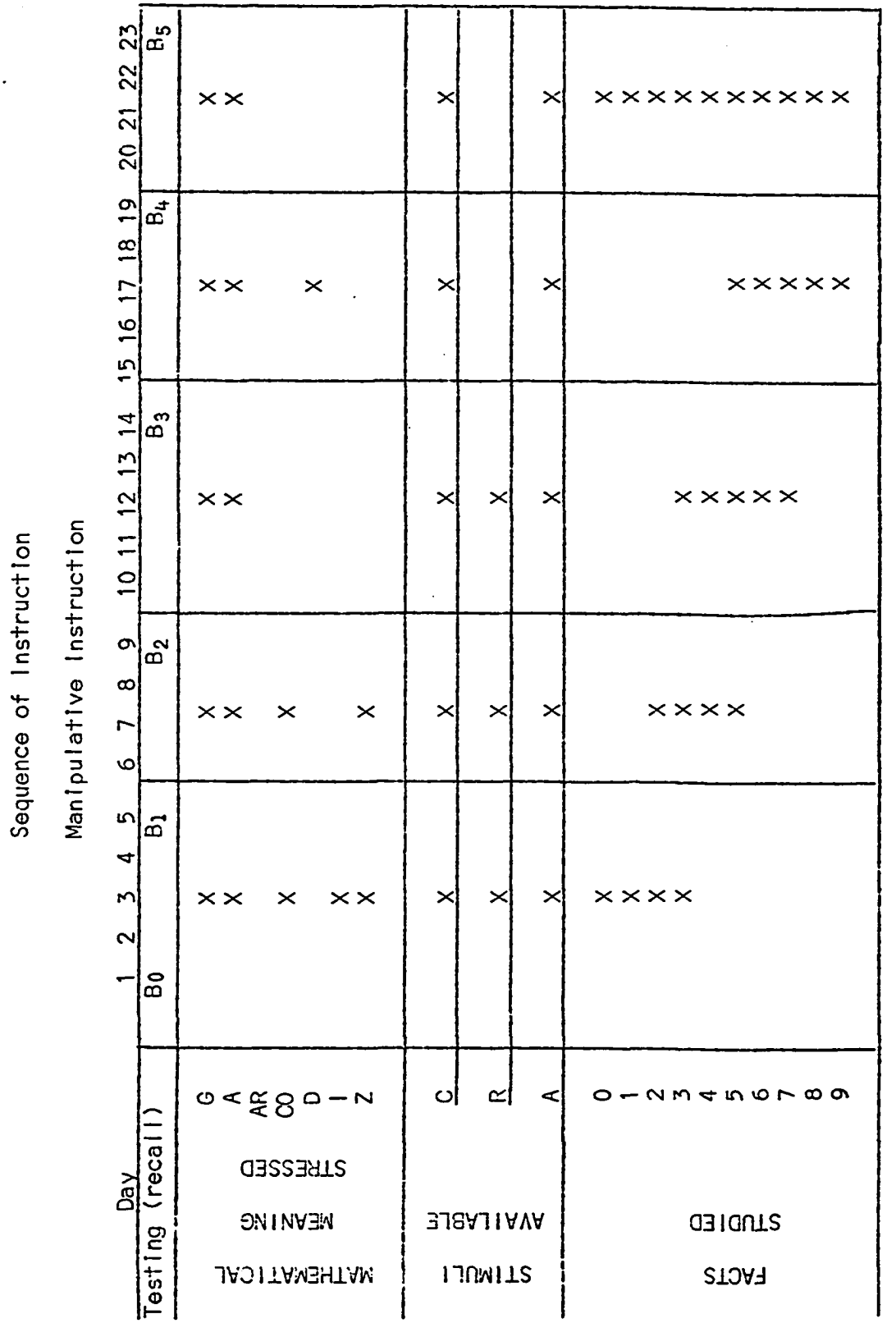

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Sequence of Instruction

Imagery Treatment

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Testing (recall)	B_0				B_1				B_2					B_3					B_4				B_5
Meaningful mathematics instruction	x																						
Analytic coding			X																				
Images			X					X				X					X					X	
Memorization and use of images												X					X					X	
STIMULI AVAILABLE C																							
STIMULI AVAILABLE R																							
STIMULI AVAILABLE A																							
FACTS STUDIED 0			X					X															
FACTS STUDIED 1			X					X															
FACTS STUDIED 2			X					X				X										X	
FACTS STUDIED 3			X					X				X										X	
FACTS STUDIED 4			X					X				X					X					X	
FACTS STUDIED 5			X					X									X					X	
FACTS STUDIED 6			X					X									X					X	
FACTS STUDIED 7			X					X									X					X	
FACTS STUDIED 8			X					X									X					X	
FACTS STUDIED 9			X					X									X					X	

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TREATMENTS

Sample Textbook Pages

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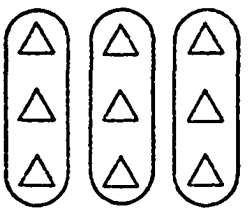
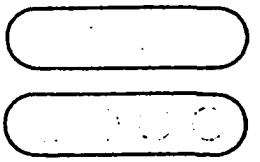
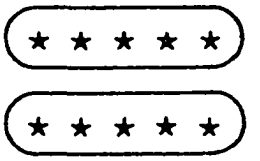
5. Write a multiplication sentence for each.

a. 

b. 

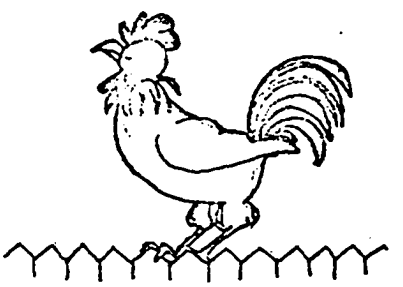
c. 

EXERCISES

Complete.

1. $5+5+5=___$
$3\times5=___$

2. $2+2+2+2+2=___$
$5\times2=___$



Write a multiplication sentence for each.

3. $2+2+2=6$

4. $5+5+5+5=20$

5. $3+3+3=9$

6. $4+4+4+4=16$

7. $1+1+1+1=4$

8. $4+4=8$

Write an addition sentence for each.

9. $3\times2=6$

10. $2\times1=2$

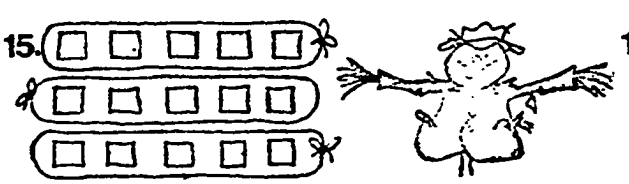
11. $5\times2=10$

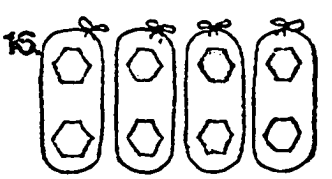
12. $4\times5=20$

13. $5\times3=15$

14. $9\times4=36$

Write a multiplication sentence for each.

15. 

16. 

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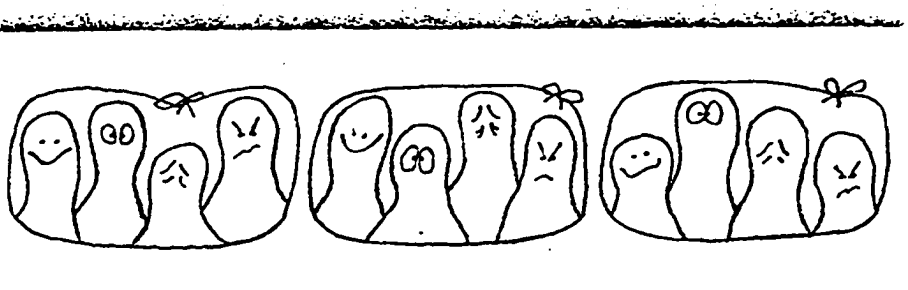
Holt School Mathematics, Textbook Page 158



$3 \times 4 = 12$

factor × factor = product

1. Give the factors. Give the products.

a. $2 \times 4 = 8$ **b.** $4 \times 1 = 4$ **c.** $3 \times 2 = 6$

2. Find the products. Complete.

a. $3 \times 4 =$ ___ **b.** $4 \times 3 =$ ___

c. $3 \times 4 = 4 \times$ ___

Changing the order of the factors does not change the product. $3 \times 4 = 4 \times 3$
This is the **order property of multiplication.**

3. Complete.

a. $2 \times 3 = 6$
$3 \times 2 =$ ___
$2 \times 3 = 3 \times$ ___

b. $2 \times 4 = 8$
$4 \times 2 =$ ___
$2 \times 4 =$ ___ $\times 2$

c. $3 \times 1 =$ ___ $\times 3$

d. $3 \times 6 = 6 \times$ ___

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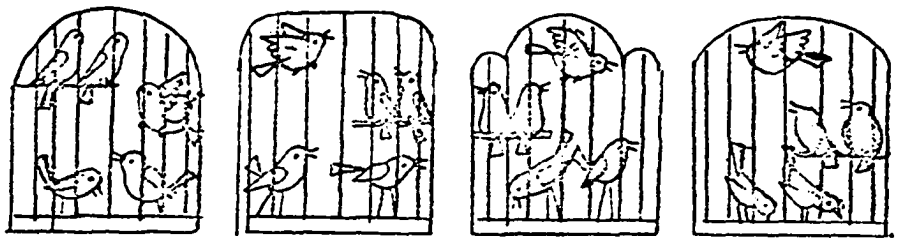
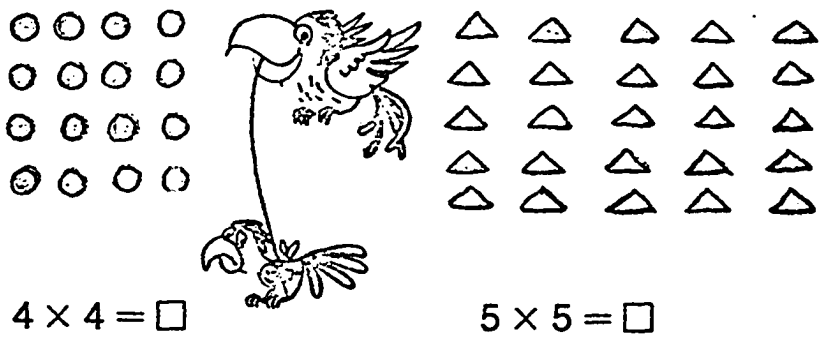


4 cages. 5 birds in each. 20 birds in all.

$5 + 5 + 5 + 5 = 20$

$4 \times 5 = 20$

1. Find the products.



$4 \times 4 = \square$ $\quad$ $5 \times 5 = \square$

2. Copy and complete. Look for patterns.

a.
$1 \times 4 = __$
$2 \times 4 = __$
$3 \times 4 = __$
$4 \times 4 = __$
$5 \times 4 = __$

b.
$1 \times 5 = __$
$2 \times 5 = __$
$3 \times 5 = __$
$4 \times 5 = __$
$5 \times 5 = __$

3. Make true sentences.

a. $3 \times 5 = 15$
$5 \times 3 = \square$

b. $4 \times 5 = 20$
$5 \times 4 = \square$

c. $2 \times 5 = 10$
$5 \times 2 = \square$

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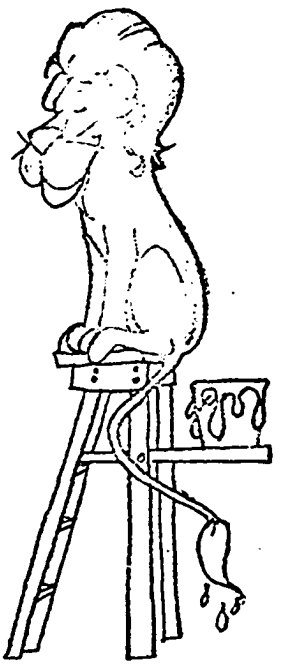
4. A multiplication table helps us to find facts.

X	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25



Let's find 4×3.

(a) Find 4 on green.
(b) Find 3 on pink.
(c) Follow the yellow path from 4 on green until it is under the 3 on pink.

Complete: $4 \times 3 = ___$.

EXERCISES

Multiply. Use the table if you need help.

1. 2×5	**2.** 3×4	**3.** 1×5	**4.** 5×3	**5.** 4×4
6. 4×2	**7.** 5×4	**8.** 3×5	**9.** 0×5	**10.** 2×4
11. 4×0	**12.** 5×5	**13.** 4×1	**14.** 5×2	**15.** 4×5

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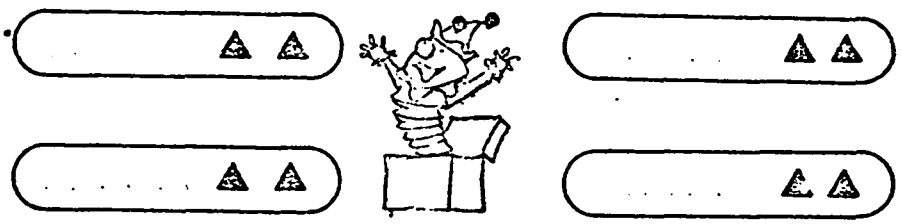
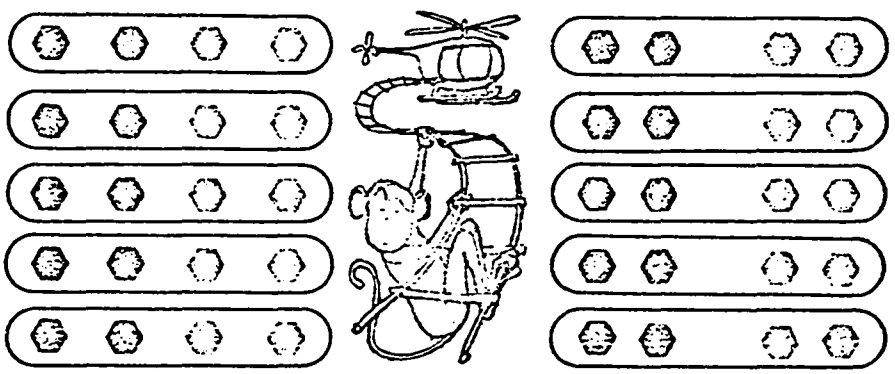
EXERCISES

Complete.

1. 

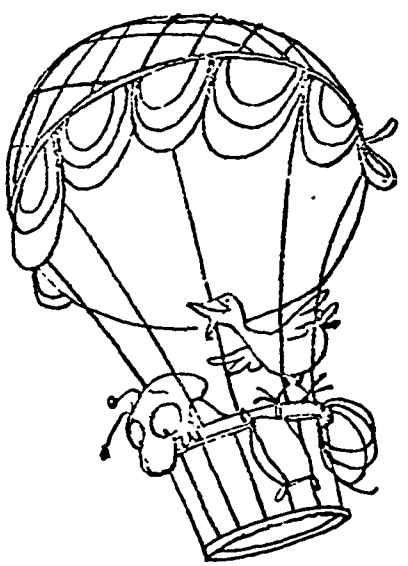
$2 \times 5 = 2 \times (3 + __)$

2. 

$5 \times 4 = 5 \times (__ + 2)$

Complete.

3. $4 \times 3 = 4 \times (2 + __)$
4. $3 \times 2 = 3 \times (__ + 1)$
5. $2 \times 4 = 2 \times (1 + __)$
6. $4 \times 5 = 4 \times (__ + 1)$
7. $5 \times 3 = 5 \times (__ + 2)$
8. $4 \times 4 = 4 \times (3 + __)$



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We may think about 2×5 in these ways.

$2 \times (4 + 1)$
2×5
10

$(2 \times 4) + (2 \times 1)$
$8 + 2$
10

$2 \times (4 + 1) = (2 \times 4) + (2 \times 1)$

Multiplication-Addition Property

1. We may use the multiplication-addition property to find new facts. Copy and complete.

a.
2×8
$2 \times (5 + 3)$
$(2 \times 5) + (2 \times 3)$
$10 + __$
$__$

b.
2×6
$2 \times (4 + 2)$
$(2 \times 4) + (2 \times 2)$
$8 + __$
$__$

c.
2×9
$2 \times (4 + 5)$
$(2 \times 4) + (2 \times __)$
$__ + __$
$__$

d.
2×7
$2 \times (3 + 4)$
$(2 \times 3) + (2 \times __)$
$__ + __$
$__$

2. Find the products.

a. $2 \times 9 = 18$
$9 \times 2 = \square$

b. $2 \times 7 = 14$
$7 \times 2 = \square$

c. $2 \times 8 = 16$
$8 \times 2 = \square$

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9 MULTIPLYING

GROUPING PROPERTY

We can change the grouping of factors.

$(2 \times 2) \times 3$
4×3
12

$2 \times (2 \times 3)$
2×6
12

Changing the grouping of the factors does not change the product. $(2 \times 2) \times 3 = 2 \times (2 \times 3)$
This is the grouping property of multiplication.

1. Complete.

a. $(2 \times 1) \times 3$
$___ \times 3$
$___$

b. $2 \times (1 \times 3)$
$2 \times ___$
$___$

c. $(2 \times 1) \times 3 = 2 \times (1 \times ___)$

2. Complete.

a. $(4 \times 1) \times 6 = ___$
$4 \times (1 \times 6) = ___$

b. $3 \times (2 \times 1) = ___$
$(3 \times 2) \times 1 = ___$

3. Complete.

a. $(3 \times 5) \times 4 = 3 \times (5 \times ___)$

b. $2 \times (6 \times 3) = (2 \times 6) \times ___$

c. $(3 \times 2) \times 7 = 3 \times (___ \times 7)$

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Multiply.

3	9	3	2	0	5	6			
3	1	7	4	3	8	4			
2	8	7	6	7	5	7			
2	4	7	1	2	0	8			
9	9	8	5	0	3	6	7	3	6
2	5	8	6	0	1	8	9	5	6
6	5	1	4	3	7	9	8	5	6
7	3	1	7	9	5	9	3	5	9
8	8	2	9	0	1	7	2	4	3
9	7	6	4	1	7	6	5	3	0
8	3	9	8	9	2	4	7	0	5
6	2	8	0	7	8	2	1	4	7
6	1	9	3	1	1	8	6	3	0
2	8	6	4	0	9	5	0	8	6
1	0	4	5	2	0	2	7	5	4
4	8	6	2	0	7	1	3	4	8
0	3	4	2	9	5	6	2	8	1
2	6	1	9	0	1	5	7	2	3
1	7	4	1	4	0	8	4	0	9
6	4	9	2	4	5	1	0	9	3
4	6	2	5	7	9	1	7	6	8
5	3	3	9	0	6	5	8	5	4

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TREATMENTS

Sample Manipulative Treatment Work Sheets

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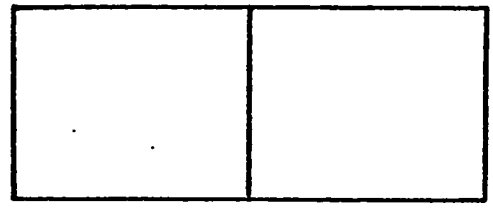
Work Sheet for 2 Facts

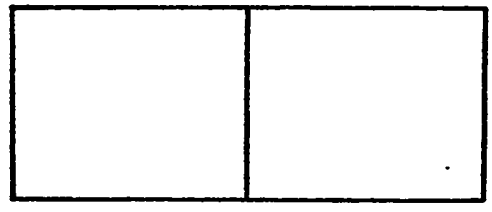
Name_____

How do you show multiplication of?

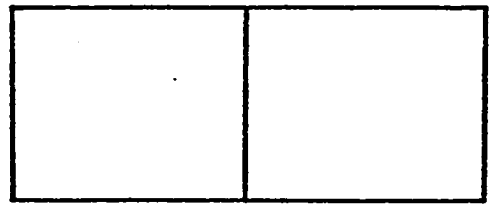
2 x 2



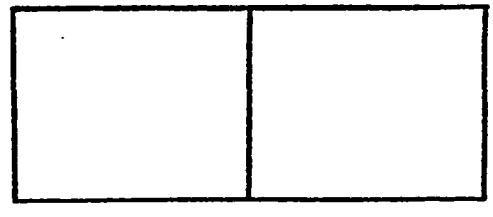
2 x 3



2 x 4



2 x 5



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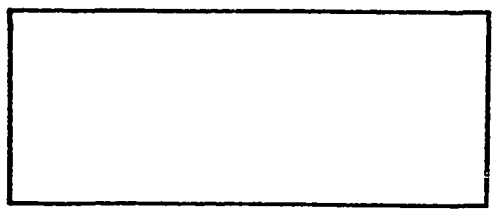
Work Sheet for Facts involving 2

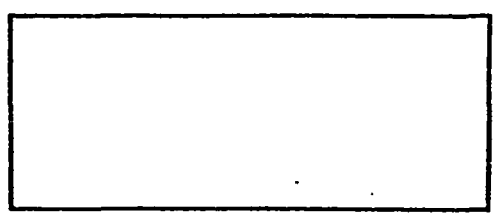
Name_____

Multiply:

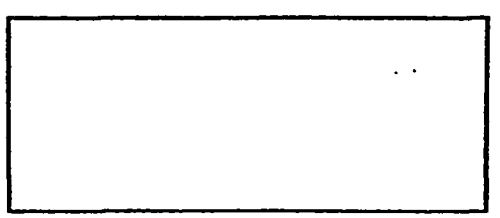
2 x 4 = _____



2 x 4 = _____

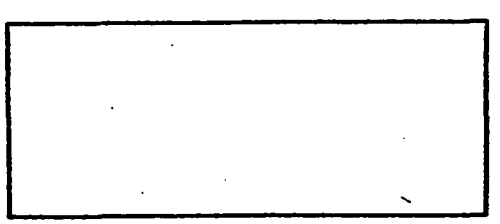


2 x 6 = _____

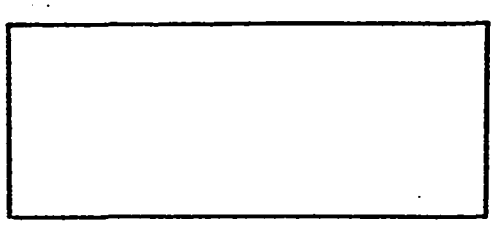


New ones:

2 x 8 = _____



2 x 9 = _____



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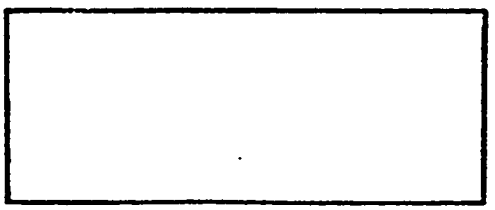
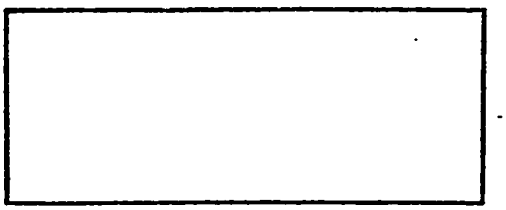
Work Sheet for Facts 1 through 4

Name_____

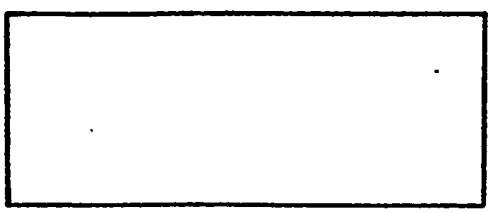
Draw a picture and write the answer:

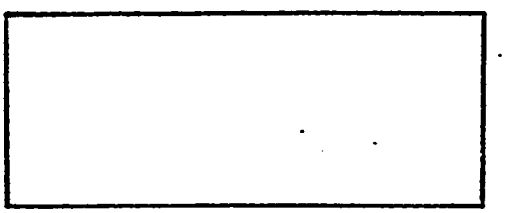




3 x 4 = _____

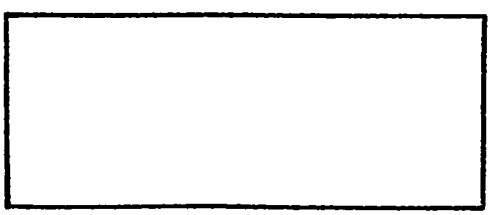
2 x 5 = _____

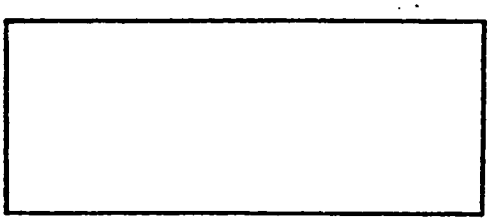




4 x 5 = _____

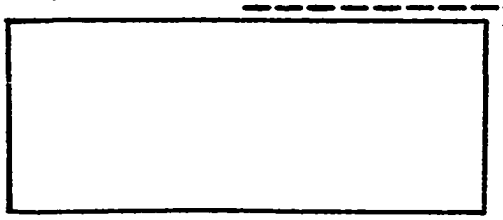
6 x 1 = _____

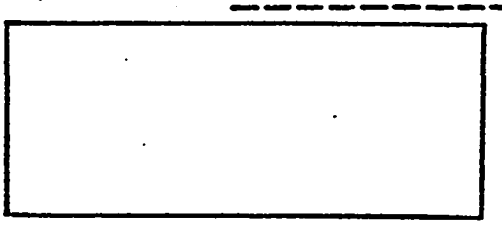




7 x 3 = _____

3 x 7 = _____





5 x 3 = _____

6 x 3 = _____

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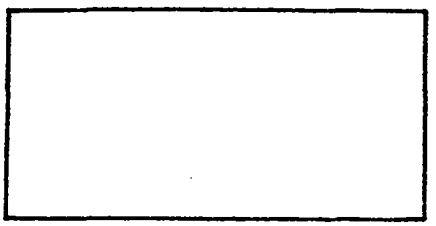
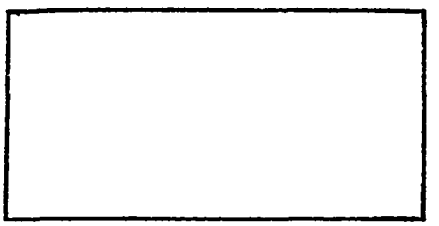
Work Sheet for Facts 0 through 3

Name_____

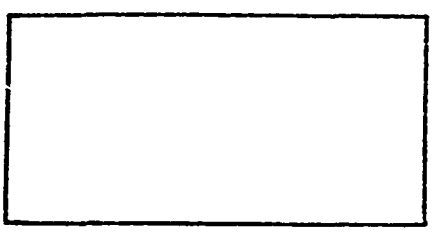
Draw the pictures:



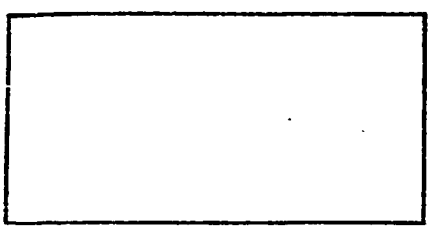
3 x 2 = _____



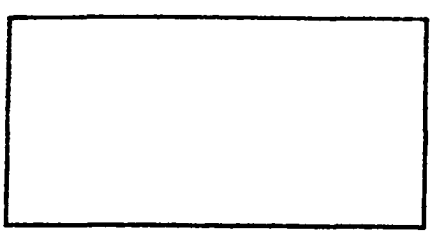
1 x 4 = _____



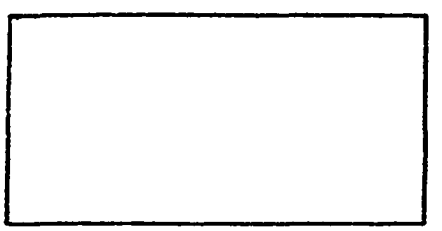
3 x 6 = _____



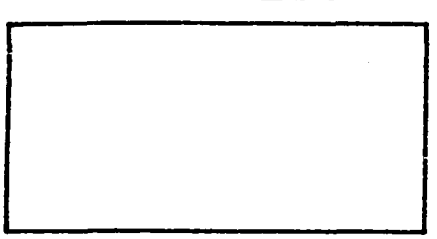
8 x 3 = _____



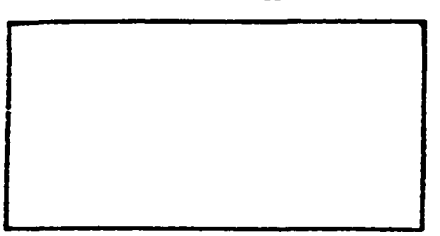
7 x 2 = _____



7 x 0 = _____



1 x 9 = _____



6 x 1 = _____

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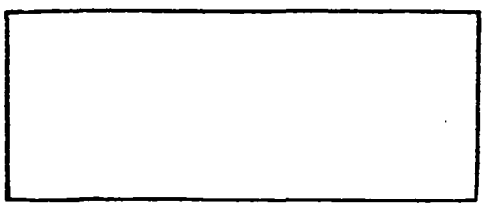
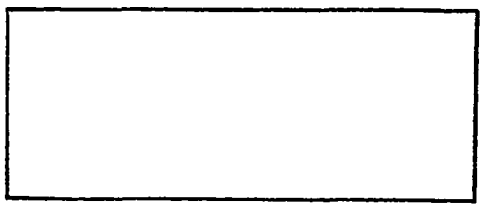
Work Sheet for Facts involving 2 and 3

Name_____

Draw a picture and write the answer:

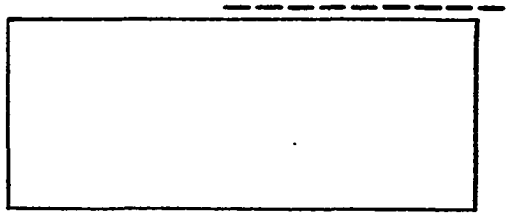


5 x 3 = _____

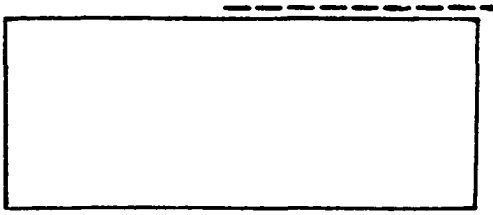


3 x 7 = _____

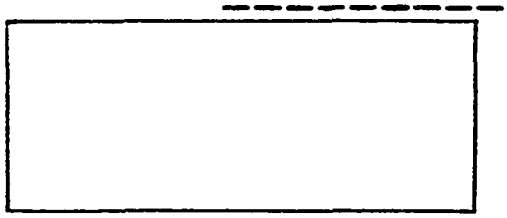
3 x 4 = _____



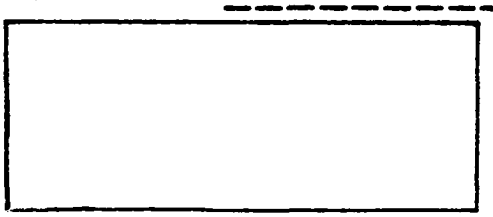
4 x 2 = _____



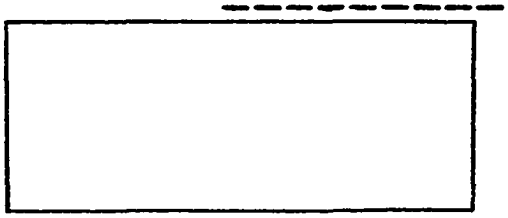
6 x 3 = _____



5 x 2 = _____



3 x 5 = _____



2 x 9 = _____

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TREATMENTS

Sample Imagery Treatment Pages and Work Sheets

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Multi-Media Math Kit

Notes to the Instructor:

Notes to the Instructor

There are several ways to approach this workbook. You may wish to present it over a period of days, or in a single block of time, working with a single child or with a small group of children. Each way is successful, so use your own judgement and gauge it according to the child's mood and attention span. Most children will not want to leave the book and this is fine too, as many children have learned all their multiplication facts in a single afternoon.

The System Explains Itself

Before you begin, we have a few suggestions to help you and your child benefit the most from this workbook. As you use the workbook initially, it is not necessary for the child to be absolutely sure of each multiplication fact before progressing to the next fact. The standard pictures in this system are mutually reinforcing; as the child works with each set of pictures, the standards and images become stronger and more discreet.

The Pace of the Lesson

For this reason, let the child set the pace of the lesson, spending as much or as little time as she or he wishes. This prevents boredom and makes the learning process fun. After completing the workbook, most of the imaged facts will be clear and strong in your child's mind. Now, go through the book again, spending additional time reinforcing any of the facts which are a problem for the child. Use the same procedure found in the sample lesson plan which follows.

Concentrate on the Image

Next, working with the child, there are two important rules which must be followed. First, stress the image -- concentrate on giving the child a vivid mental picture of the action between the standards presented on each page. The verbal message is just for reinforcement; the emphasis must be on the image itself.

Sound Not Symbol

Second, make sure both you and the child know that the phonetic code is translated by the sound of the letter and not the letter itself. Thus, the sounds for the letters k, g, and c could all translate to the number 7, which represents the hard "k" sound, "kuk" (key, go, cow). Six can be especially troublesome if it is translated by letter instead of sound--j, sh, ch, tch, dg, and similar consonant blends with the "juh" sound are all equivalent to 6. So be sure to translate by what sound you hear, not what letter you see.

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Multi-Media Math Kit

Translation code and images for digits 1 through 9:

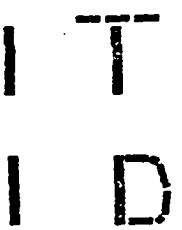
Step 1
Translating Numbers into Pictures--the Sound Code

Numbers are hard to remember because they are intangible. We never have a meaning for them in real life. For example, each of us can picture a tree or a cat or a shoe in our minds because these are things which we can see in real life. Try it and see. Close your eyes and see a tree, now a cat, now a shoe. If we could see numbers in the same way it would be easy to remember them. For this reason we are going to learn a "code" which will help us change every number into a word which will name something like a tree or a cat or a shoe.

We know that all numbers are made up of one or more of the ten single digits 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Therefore, we can choose a sound for each of these ten digits and use the sounds we choose to form a word or words for any number we want. From these words that we form we will then form a picture to represent each number.

The sounds that we choose for the ten digits will all be consonant sounds. We will save the vowel sounds of a, e, i, o, and u and the half-vowel sounds of h, w, and y to help us build words from their sounds. And from the words we build we will make pictures which we can change into numbers.

Standard Sounds and Word Pictures



Draw a line across the top of the number "1" below to make it look like the letter "T." Now, draw a curved line from the top to the bottom of the second "1" to make a "D." Both T and D have a similar sound, "tuh" and "duh."

1 is equivalent to the sound of T or D

To make a word-picture for 1, we can

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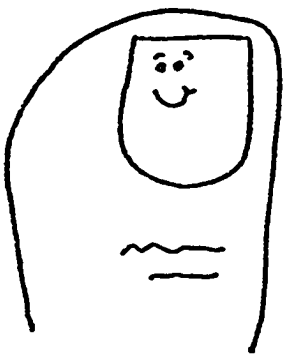
Multi-Media Math Kit

Translation code and images (continued):

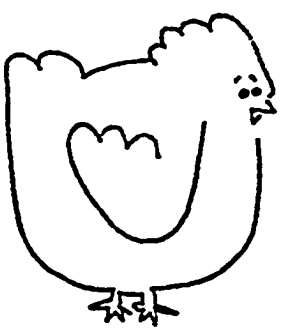
Remember use a, e, i, o, u and w, h, y to help build words.

use the T sound and add two vowels (which have no number value) to form TOE. TOE is a word-picture for the number 1 because T is the sound for 1. Draw a 1 on the TOE and fill in the block.



TOE

2 is equivalent to the sound of N

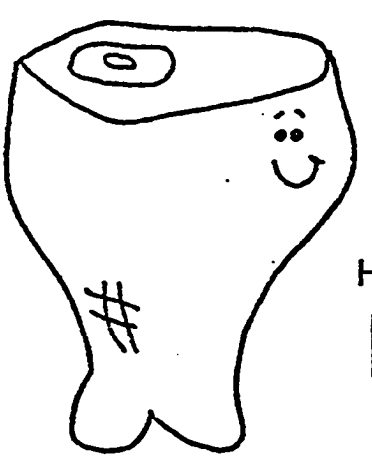


HEN

Connect these two lines to make an "N." To make a picture for the number 2, we can add an "H" and an "E" to form HEN. HEN is a word-picture for 2 because N is the sound for the number 2. Draw a 2 on the HEN and fill in the block.

3 is equivalent to the sound of M

Make an "m" from these three lines. We can use HAM as our word picture for the number 3 because M is the sound for 3 and H and A have no value. Color the 3 in the HAM's feet and translate the letter M in the block.



HAM

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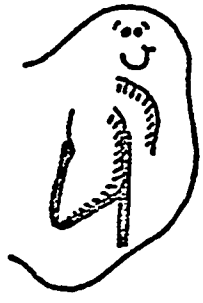
Multi-Media Math Kit

Translation code and images (continued):

FOUR

Circle the fourth letter of the word FOUR.

4 is equivalent to the sound of R



We can use a picture of an EAR to represent 4, because the "rrrrr" sound stands for 4. Outline the 4 on the EAR and translate the letters.

EAR

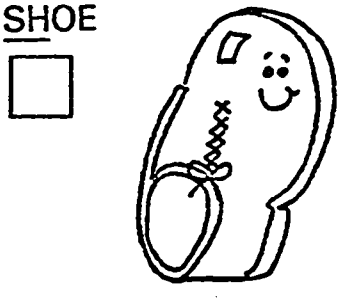
Color the L in the number 5. Now, draw a 5 on the OWL, which will be our picture for 5. Fill in the block under the L. Remember, vowels and half-vowels have no number value; they are just used to form words.

5 is equivalent to the sound of L

OWL

6 is equivalent to the sound of J

SHOE



When you turn a 6 over, it looks almost like a J. There are other letters that have a sound like the "juh" sound of J. SH, CH, TCH, DG, and soft G all have a similar sound, so all these consonant blends will represent 6. Let's use some of the sound alike letters to form a word picture for 6. SHOE will be our picture for 6.

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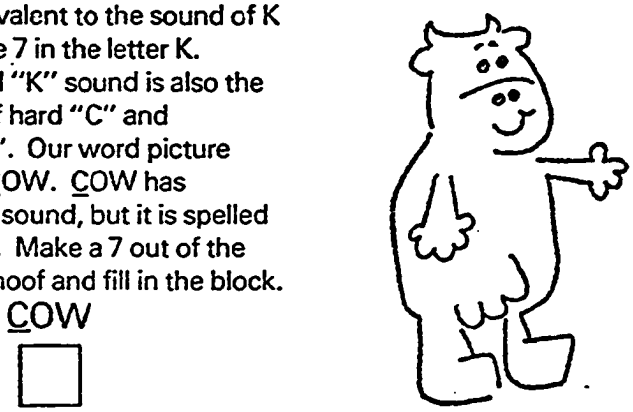
Multi-Media Math Kit

Translation code and images (continued):

7 is equivalent to the sound of K
Color the 7 in the letter K.
The hard "K" sound is also the sound of hard "C" and hard "G". Our word picture for 7 is COW. COW has a hard K sound, but it is spelled with a C. Make a 7 out of the COW's hoof and fill in the block.

COW

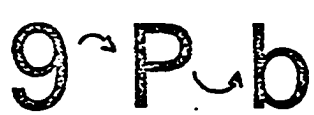


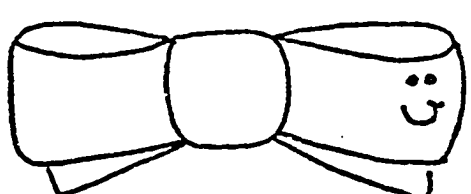
8 is equivalent to the sound of F or V

If we smash this 8, we can get the letter F. The "ffffff" sound of F is a lot like the "vvvvv" sound of V. We'll use V to get a word picture, HIVE. See all the 8's stacked up to make the HIVE? The feet also look like an 8. Color all the 8's and fill in the second-number translation.

HIVE

Draw a 9 and turn it over to make a "P." Now, turn it upside down to make a "b." Both these letters have sounds which stand for 9.





9 is equivalent to the sounds of P or B

BOW

We'll use the B sound to make BOW, our word picture for the number 9. Draw a 9 on the BOW and fill in the block.

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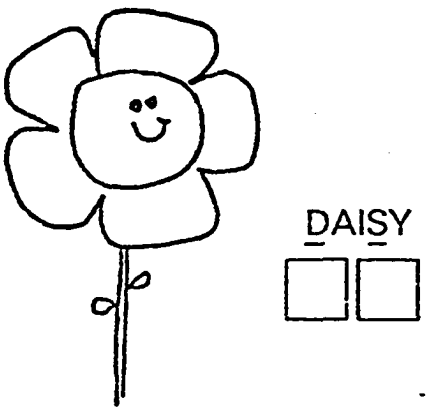
Multi-Media Math Kit

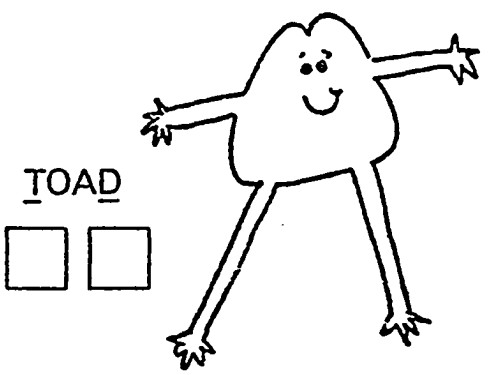
Translation code and images for digits 10, 11 and 12:

The number 10 is made up of two digits. We already have a sound for 1, T or D, so we must find a sound for 0. Say the word "zero" and hear the "zzzz" sound. The "zzzz" sound and the similar "ssss" sound of S will represent o.



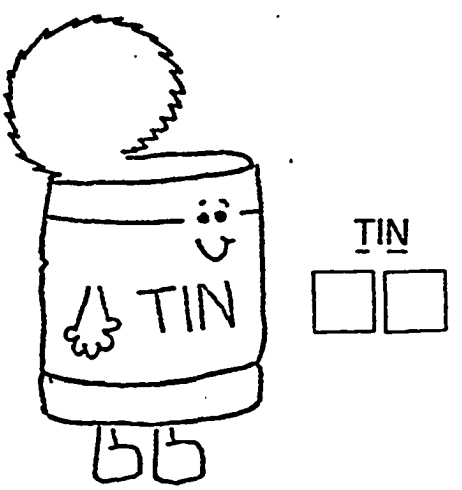
0 is equivalent to the sounds of Z and S

Now, we can take the D sound of 1 and connect with vowels to the sound for 0 to make DAISY, our word picture for the number 10.



TOAD will be our standard picture for 11. Remember, the T sound and the D sound both translate to 1. When you put both 1's side by side, they make an 11, which looks like the TOAD's legs.

The last standard we will use to multiply with is 12. To get a word picture, combine the T sound of 1 and the N sound of 2 to form TIN. A TIN can will be our standard picture for the number 12.



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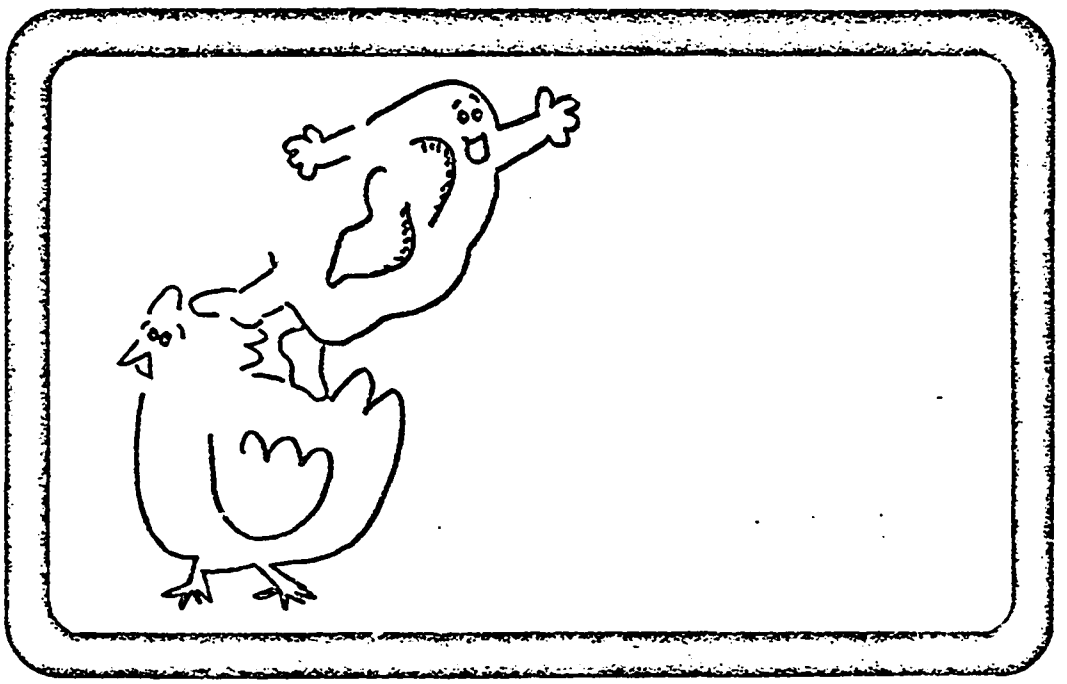
Multi-Media Math Kit

Textbook page for 2 x 4 = ?:



The hen lets the ear stand on her.
Why?

HEN EAR

□ • □

or

EAR HEN

□ • □

= ?

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Multi-Media Math Kit

Textbook page for 2 x 4 = 8:

The hen lets the ear stand on her
to reach the hive.

HEN EAR

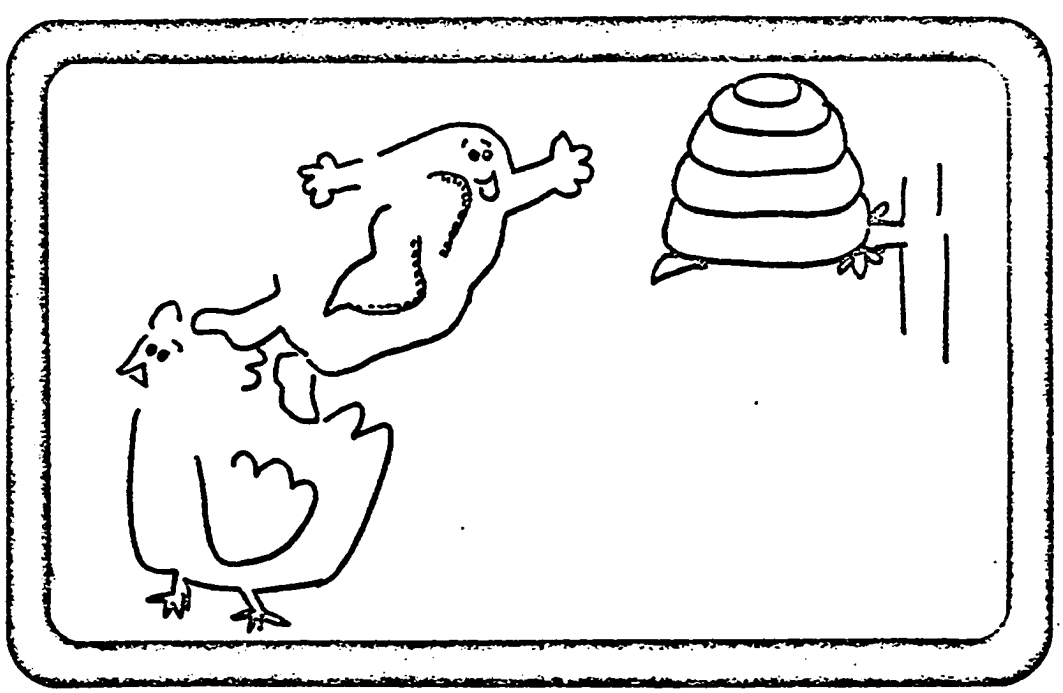
□ • □

or

EAR HEN

□ • □

= ○ HIVE



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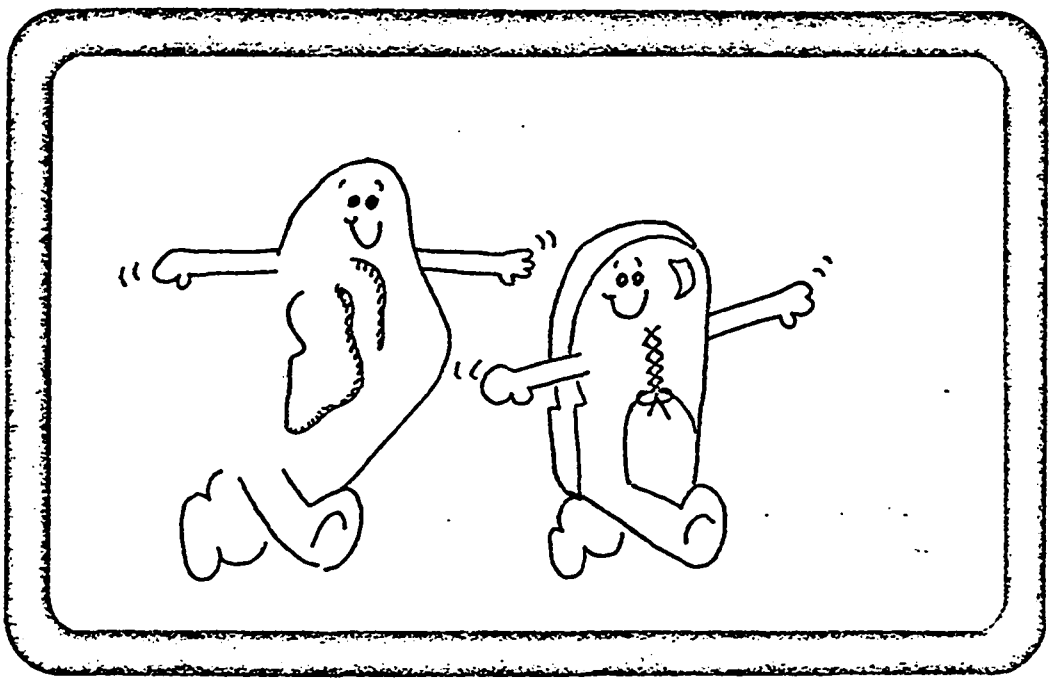
Multi-Media Math Kit

Textbook page for 4 x 6 = ?:



The ear and the shoe are walking on something.

What?

EAR SHOE

□ • □ = ?

or

SHOE EAR

□ • □

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Multi-Media Math Kit

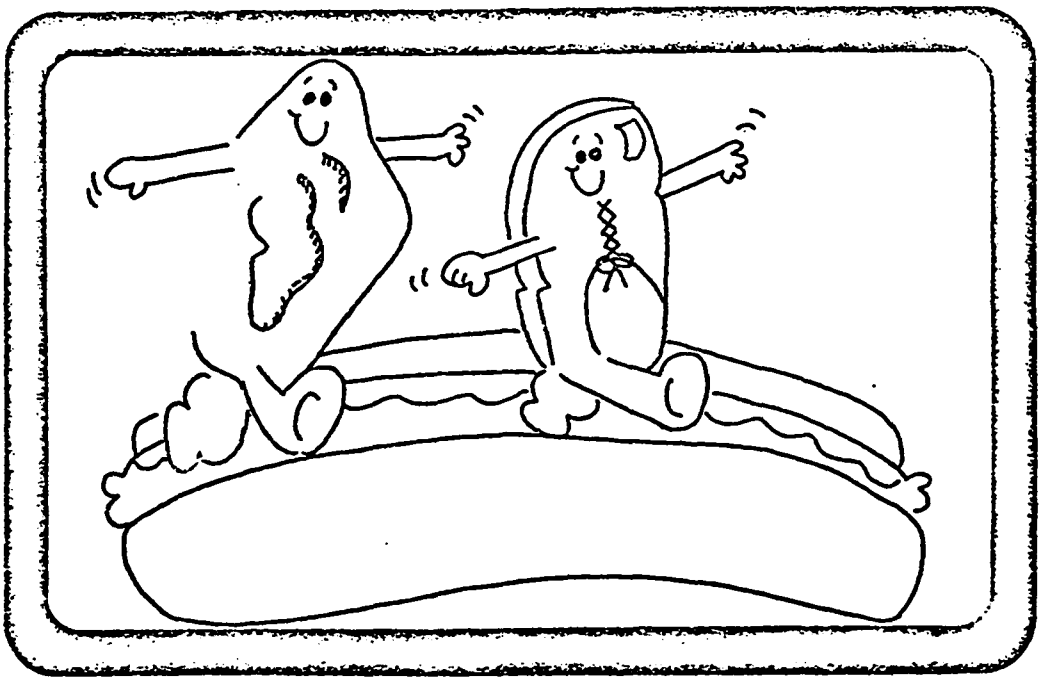
Textbook page for 4 x 6 = 24:

The ear and the shoe are walking on a wiener.

EAR • SHOE = WIENER

or

SHOE • EAR = WIENER



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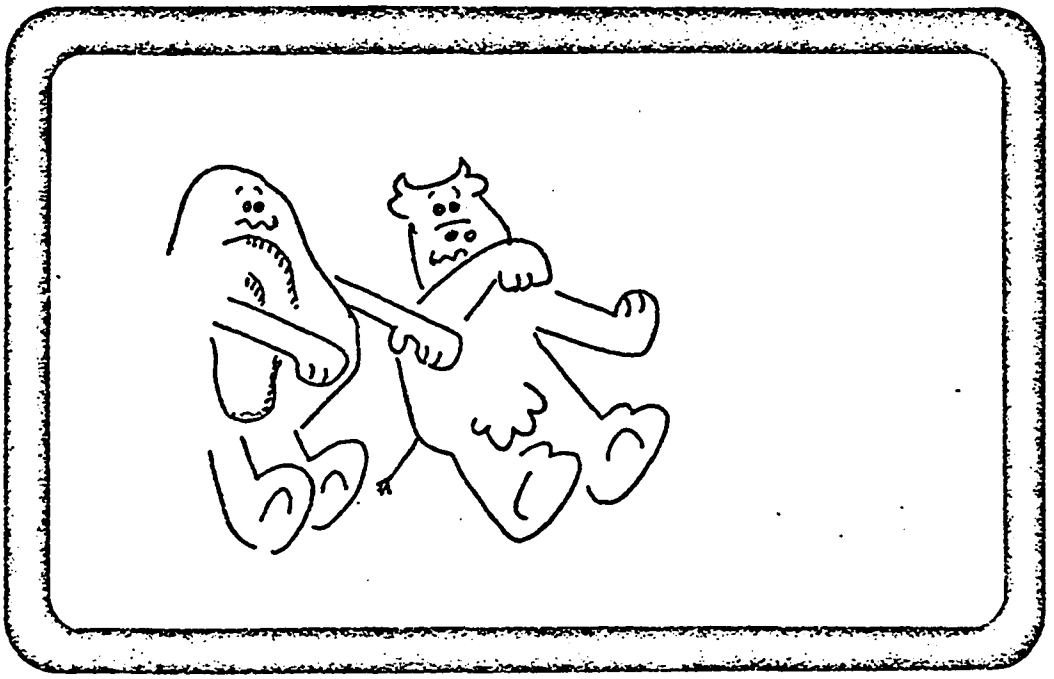
Multi-Media Math Kit

Textbook page for 4 x 7 = ?:



The ear and the cow are struggling to open something.

What?

EAR • COW = ?

or

COW • EAR = ?

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Multi-Media Math Kit

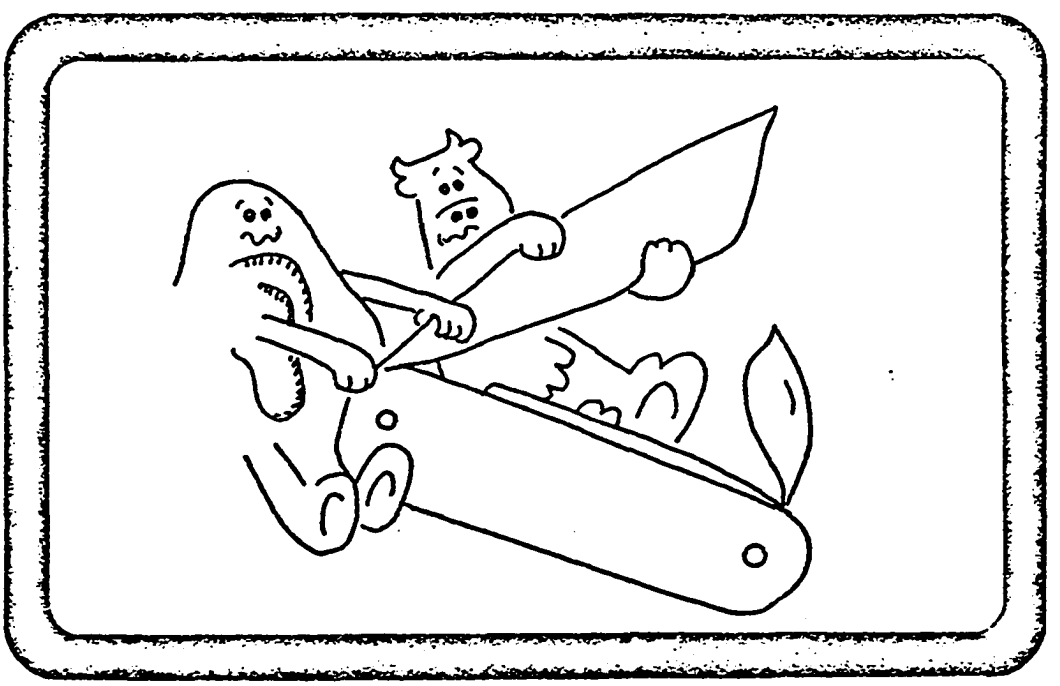
Textbook page for 4 x 7 = 28:

The ear and the cow are struggling to open a knife.

EAR □ • COW □ = KNIFE ○

or

COW □ • EAR □ = KNIFE ○



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Practice Sheet for Word Codes

Name_____

Hen - Hen_____	Owl - Bow_____
Cow - Hive_____	Ear - Ear_____
Bow - Bow_____	Ear - Owl_____
Cow - Bow_____	Owl - Hive_____
Hive - Hive_____	Ear - Cow_____
Hen - Ham_____	Ear - Bow_____
Ham - Ham_____	Ear - Hive_____
Shoe - Hive_____	Ear - Shoe_____
Cow - Cow_____	Hen - Ear_____
Hive - Bow_____	Hen - Shoe_____
Shoe - Shoe_____	Hen - Hive_____
Shoe - Cow_____	Ham - Ear_____
Shoe - Bow_____	Ham - Owl_____
Hen - Bow_____	Ham - Shoe_____
Hen - Cow_____	Ham - Cow_____
Hen - Owl_____	Ham - Hive_____
Owl - Owl_____	Ham - Bow_____
Owl - Cow_____	Shoe - Hive_____

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Practice Sheet for Facts 4, 5, 6, 7, 8 and 9

Name_____

9 x 9	_____	_____
8 x 8	_____	_____
8 x 9	_____	_____
7 x 7	_____	_____
7 x 8	_____	_____
7 x 9	_____	_____
6 x 6	_____	_____
6 x 7	_____	_____
6 x 8	_____	_____
6 x 9	_____	_____
5 x 5	_____	_____
5 x 6	_____	_____
5 x 7	_____	_____
5 x 8	_____	_____
5 x 9	_____	_____
4 x 4	_____	_____
4 x 5	_____	_____
4 x 6	_____	_____
4 x 7	_____	_____
4 x 8	_____	_____

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APPENDIX B

TESTS

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STANFORD DIAGNOSTIC ARITHMETIC TEST

Recall

Form W	Form X
1. 2 x 2	1. 2 x 2
2. 5 x 3	2. 3 x 5
3. 4 x 2	3. 2 x 4
4. 3 x 4	4. 4 x 3
5. 2 x 5	5. 5 x 2
6. 3 x 3	6. 3 x 3
7. 6 x 2	7. 2 x 6
8. 2 x 3	8. 3 x 2
9. 8 x 2	9. 2 x 8
10. 3.x 6	10. 6 x 3
11. 2 x 7	11. 7 x 2
12. 4 x 5	12. 5 x 4
13. 2 x 9	13. 9 x 2
14. 7 x 3	14. 3 x 7
15. 6 x 4	15. 4 x 6
16. 9 x 3	16. 3 x 9
17. 4 x 4	17. 4 x 4
18. 5 x 5	18. 5 x 5
19. 4 x 7	19. 7 x 4
20. 3 x 8	20. 8 x 3
21. 5 x 9	21. 9 x 5
22. 4 x 8	22. 8 x 4
23. 6 x 6	23. 6 x 6
24. 3 x 0	24. 0 x 3
25. 8 x 5	25. 5 x 8
26. 9 x 9	26. 9 x 9
27. 2 x 1	27. 1 x 2
28. 5 x 6	28. 6 x 5
29. 8 x 7	29. 7 x 8
30. 7 x 6	30. 6 x 7
31. 9 x 4	31. 4 x 9
32. 8 x 8	32. 8 x 8
33. 0 x 4	33. 4 x 0
34. 7 x 9	34. 9 x 7
35. 6 x 8	35. 8 x 6
36. 7 x 7	36. 7 x 7
37. 9 x 8	37. 8 x 9
38. 1 x 3	38. 3 x 1
39. 5 x 7	39. 7 x 5
40. 9 x 6	40. 6 x 9

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Recall Test Answer Sheet

Name _____

1. _____	21. _____
2. _____	22. _____
3. _____	23. _____
4. _____	24. _____
5. _____	25. _____
6. _____	26. _____
7. _____	27. _____
8. _____	28. _____
9. _____	29. _____
10. _____	30. _____
11. _____	31. _____
12. _____	32. _____
13. _____	33. _____
14. _____	34. _____
15. _____	35. _____
16. _____	36. _____
17. _____	37. _____
18. _____	38. _____
19. _____	39. _____
20. _____	40. _____

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Comprehension Test

MULTIPLICATION USAGE TEST

Name _____

Date _____

1. 4 x ☐ = 24
2. 7 x ☐ = 35
3. ☐ x 7 = 21
4. 6 x ☐ = 48
5. ☐ x 3 = 27
6. ☐ x 7 = 56
7. ☐ x 5 = 20
8. 9 x ☐ = 45
9. 7 x ☐ = 28
10. ☐ x 2 = 10

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Comprehension Test (continued)

1. Jane reads 4 stories everday. How many stories would she read in 7 days?

_____ Answer

2. If there are 45 pencils to share among 5 girls, how many pencils will each girl get?

_____ Answer

3. Jack rides the bus to and from school everyday. He rides the bus 7 miles a day. How many miles does he ride in 5 days?

_____ Answer

4. 36 apples have to be put in bags. Each bag can hold 6 apples. How many bags can be filled?

_____ Answer

5. 7 men worked for 8 hours each. What was the total amount of hours that they worked?

_____ Answer

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Attitude Test

Name_____

---

1. ☐ I would rather not do arithmetic.

   ☐ I like to do arithmetic.

---

2. ☐ Arithmetic is interesting.

   ☐ Arithmetic is boring.

---

3. ☐ I like to do story problems.

   ☐ I hate to do story problems.

---

4. ☐ I am sort of afraid of arithmetic.

   ☐ I think arithmetic is fun.

---

5. ☐ Working with numbers is fun.

   ☐ Working with numbers is not fun

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Attitude Test (continued)

---

6. ☐ I like arithmetic better than I like other school work.

☐ I like other school work better than I like arithmetic.

---

7. ☐ I have never liked arithmetic.

☐ I have always liked arithmetic.

---

8. ☐ I am not sure I can do arithmetic right.

☐ I think I can do arithmetic right.

---

9. ☐ Sometimes I like to try to do hard arithmetic problems.

☐ I never like to try to do hard arithmetic problems.

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Attitude Test (continued)

---

10. ☐ I don't care if we have arithmetic today or not.

☐ I want to have arithmetic today.

---

11. ☐ I like to do arithmetic problems even if I'm not in school.

☐ I don't like to do arithmetic problems when I'm not in school.

---

12. ☐ I don't get tired of working with numbers.

☐ I get tired of working with numbers.

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Attitude Test (continued)

13. ☐ I like arithmetic more than I like my other school work.

☐ I like arithmetic just as much as I like my other school work.

☐ I don't like arithmetic as much as I like other school work.

14. Of my school work I like arithmetic

☐ best of all.

☐ very much.

☐ a little bit.

☐ not too much.

☐ I hate arithmetic.

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APPENDIX C

RAW DATA

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RAW DATA

Textbook Group
(n = 24)

		Repeated-measures Recall							MAT	
						Comprehension	Attitude	Computation	Concepts	Problem Solving
B_0	B_1	B_2	B_3	B_4	B_5					
0	15	25	29	35	36	20	24	32	34	33
3	7	11	10	24	25	17	18	29	34	33
5	6	14	14	14	20	7	26	22	28	20
10	15	21	22	27	26	14	28	28	27	26
3	11	12	16	22	29	4	10	23	29	22
1	3	9	9	12	4	1	17	12	19	14
4	15	20	21	25	30	14	26	26	30	26
0	9	17	17	27	21	9	4	23	28	24
1	14	15	18	24	26	6	4	25	27	28
0	0	6	9	7	12	2	9	12	17	13
12	14	19	16	18	23	6	29	18	21	17
4	11	11	18	25	24	18	17	26	30	26
4	15	17	14	17	23	8	25	29	35	33
14	16	30	29	36	36	17	27	32	38	33
12	18	24	22	30	36	15	26	28	34	32
6	14	17	22	29	30	9	27	27	26	22
1	12	24	29	38	40	14	6	30	28	31
4	13	18	16	21	26	15	4	17	26	15
2	7	14	19	15	17	11	13	16	21	14
14	20	28	28	31	35	15	6	28	34	30
10	15	10	11	12	16	2	4	20	17	17
4	10	17	20	31	37	11	30	18	31	27
1	12	15	23	27	30	5	29	26	29	31
2	10	12	21	22	25	8	6	16	36	27

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RAW DATA

Manipulative Group
(n = 29

Repeated-measures Recall						MAT				
						Comprehension	Attitude	Computation	Concepts	Problem Solving
B_0	B_1	B_2	B_3	B_4	B_5					
6	16	15	24	27	29	18	24	24	25	27
11	20	27	31	37	37	11	26	26	33	28
13	19	25	29	37	36	13	26	27	32	31
2	3	5	11	11	24	5	30	19	21	13
2	2	3	3	4	8	0	21	13	15	12
1	16	10	19	21	22	5	25	18	22	13
2	5	13	10	6	7	2	8	16	21	15
10	15	18	21	33	35	7	7	26	32	28
0	0	1	1	4	6	0	16	9	8	11
3	13	11	19	22	25	6	30	20	20	18
7	17	25	27	28	31	17	17	6	31	26
2	10	14	14	18	18	3	30	17	19	3
0	1	6	7	15	16	2	30	15	15	17
7	13	14	24	24	26	9	27	27	32	26
14	17	20	25	29	31	14	29	25	30	26
16	25	26	37	38	39	20	30	31	31	27
9	8	15	19	25	28	5	27	20	23	21
5	15	20	16	19	23	6	30	19	31	26
1	16	11	17	26	32	8	30	22	24	17
5	16	21	28	31	29	8	28	25	30	27
14	12	16	20	25	23	7	28	26	33	29
2	5	17	16	20	25	8	29	24	30	20
5	13	11	8	18	26	6	30	23	29	26
0	5	14	19	22	25	0	26	19	15	14
3	11	20	28	28	30	11	29	25	26	30
2	6	11	24	25	29	3	18	20	28	17
13	26	26	29	32	31	9	30	28	26	20
1	2	2	6	10	14	6	18	12	9	10
0	0	1	0	3	5	0	20	9	7	4

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RAW DATA

Imagery Group
(n = 23)

Repeated-measures Recall									MAT	
B_0	B_1	B_2	B_3	B_4	B_5	Comprehension	Attitude	Computation	Concepts	Problem Solving
5	5	5	5	14	19	8	29	16	16	12
7	4	4	5	10	9	1	29	16	14	11
20	22	29	34	36	39	20	23	26	24	19
8	5	5	10	16	18	9	30	19	26	20
1	1	2	5	10	9	0	27	17	22	12
0	1	1	3	7	8	3	28	11	24	13
8	2	7	7	5	10	8	26	23	28	19
20	20	18	25	31	35	16	26	32	34	30
0	12	20	20	24	27	3	29	21	30	21
3	5	2	3	6	6	2	27	13	18	11
3	13	6	17	17	27	7	27	19	32	30
1	3	1	6	10	15	1	21	14	19	16
3	11	13	9	13	15	0	25	17	21	17
2	5	2	8	9	11	0	17	8	18	16
1	1	5	12	15	21	0	21	14	23	14
1	8	9	9	12	17	6	10	22	30	10
0	1	6	7	7	14	0	27	7	11	11
16	23	23	33	30	35	12	14	31	37	26
2	10	9	8	13	18	3	19	12	27	16
14	23	23	15	20	27	7	2	24	35	24
19	20	24	32	34	38	7	13	25	34	32
2	6	7	6	18	11	4	21	13	26	18
0	1	1	0	15	16	3	16	18	20	16

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