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Interactions of cosmic rays with hydromagnetic waves in the galaxy.

A thesis presented

by

Catherine Jeanne Cesarsky

to

The Department of Astronomy

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the subject of

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ABSTRACT

The possibility that ultrarelativistic cosmic rays are confined to the galactic disk as a result of their interactions with hydromagnetic waves is examined.

Cosmic rays themselves, when they stream at velocities in excess of the Alfvèn velocity in the medium, generate waves; the waves are damped by the collisions between the charged and neutral particles in the interstellar medium. If only these two processes controlled the wave spectrum, the diffusion of cosmic rays of different energies would proceed at very different rates -- a result that is difficult to reconcile with cosmic ray observations. A similar difficulty is encountered when cosmic rays are assumed to be confined exclusively by their resonant interactions, through high harmonics, with waves much shorter than their Larmor radii.

Turbulent gas motions in the galaxy are a possible source of long hydromagnetic waves, with wavelengths of the order of a few parsecs. By analogy with hydrodynamic turbulence, we assume that nonlinear effects cause the energy in these long waves to be transmitted to waves of successively smaller size. This process, together with the different mechanisms tending to damp the waves, determines the wave spectrum.

The diffusion of cosmic rays in the presence of given spectrum of hydromagnetic waves is parameterized by their mean free path, λ. Cosmic rays are scattered in pitch

angle by their resonant interactions with hydromagnetic waves; they are also mirror reflected by magnetosonic (or magneto-acoustic) waves. Both effects have to be considered in the evaluation of λ.

Because of charged-neutral collision damping the power needed to sustain a spectrum of waves that extends to the shortest wavelengths of interest in an intercloud medium with currently accepted properties (n_H = 0.2 cm^{-3}, n_e = 0.02cm^{-3}, T = 1000°K), is unacceptably high. In a more diffuse medium, with a density of hydrogen in the range 0.02 - 0.04 cm^{-3}, the power dissipated by the waves is sufficient to maintain the medium at a high temperature, and thus highly ionized; then, the effect of charged-neutral collision damping is of no importance. Short magnetoacoustic waves are damped by thermal conduction, but, under certain assumptions, a spectrum of Alfvèn waves may develop. The mean free path of ultrarelativisitic cosmic rays in the presence of the resulting spectrum of waves, is either constant or approximates a power law in the rigidity, with a small exponent.

A model for the propagation of cosmic rays in the galaxy is proposed, in which cosmic rays traverse the gas disk, of thickness 300 pc, at large speeds, but diffuse slowly in 2 slabs of hot and diffuse gas, 200 pc thick, at each side of the gas disk. It is shown that the predictions of this model agree with the available observations.

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INTRODUCTION

The main objective of this work is to investigate the possibility that ultrarelativistic cosmic rays are confined to the disk of the galaxy as a result of their interactions with hydromagnetic waves. For this purpose, we analyzed the consequences for the cosmic rays of diverse hypothesis on wave sources and wave-wave interactions, and compared them with the available evidence on cosmic rays at the earth -- energy spectrum, isotropy and age -- and on the medium they pervade.

The observed spectrum and isotropy of cosmic rays were naturally explained in the model proposed by Fermi (1949), in which cosmic rays are accelerated in the interstellar medium by their collisions with magnetized clouds moving at random. Morrison et al (1954) based their study of the origin and diffusion of cosmic rays in the galaxy on Fermi's theory; they were able to reproduce the observed properties of cosmic rays, but had for this to resort to extreme parameters for the number and velocity of magnetized clouds. For this reason, the idea that cosmic rays are continuously accelerated in the interstellar space has lost credibility; and in recent years, attention has been given to discrete sources of cosmic rays in the galaxy. The most widely accepted theories postulate that supernovae (Ginzburg 1958, Hayakawa et al 1958,

Colgate and White 1964) or pulsars (Goldreich and Julian 1969, Ostriker and Gunn 1969) are responsible for accelerating the cosmic rays. In this case, the problems of the origin of cosmic rays and of their propagation in the galaxy may be considered separately.

Models of cosmic ray propagation which take into account particle production, spallation, energy losses in the interstellar medium, etc. have been extensively discussed in the literature (Ginzburg and Syrovatskii 1964, Shapiro et al 1970, and references therein). The principal aim of these theories is to explain the observed chemical composition of cosmic rays: for this purpose, diverse assumptions about the diffusion coefficient of cosmic rays (Ramaty et al 1970), the distribution of path lengths in the galaxy (Cowsik et al 1968) or the probability of escape (Gloeckler and Jokipii 1969) are used. The trapping of cosmic rays in the galaxy is generally assumed to be due to the presence of inhomogeneities in the magnetic field, but, in these papers the relation between the field structure and the diffusion of cosmic rays is not considered.

Large scale inhomogeneities in the magnetic field reflect particles of large pitch angle, and accelerate them (Fermi 1954). Cosmic rays are also scattered by magnetic fluctuations of scale comparable to their Larmor radius; the corresponding diffusion coefficient has been evaluated by

Jokipii (1966) and Kulsrud and Pearce (1969, hereinafter referred to as KP). Lerche (1967), Wentzel (1968) and KP showed that, when cosmic rays stream at a velocity in excess of the Alfvèn velocity, hydromagnetic waves are unstable by a two-stream instability. It was then proposed (Wentzel 1969, KP) that cosmic rays regulate their own diffusion by this mechanism. Under this assumption, KP developed a model for the propagation of cosmic rays in the galaxy, comparable to that of Morrison et al (1954); the predictions of this model are in agreement with the observed properties of 10Bev cosmic rays. The behavior of higher energy cosmic rays was not considered.

In the first chapter of this thesis, we show that, if the amplitude of the waves were determined only by cosmic ray plasma instabilities and ambipolar collision damping, the streaming velocity of cosmic rays would increase rapidly with the energy; this result conflicts with cosmic ray observations. Thus, if hydromagnetic waves confine the cosmic rays in the galactic disk they have to be provided by sources other than the cosmic rays.

In the second chapter, after briefly reviewing the observational data on ultrarelativistic cosmic rays, we examine possible sources of waves. We suggest that long waves, with scales of the order of a few parsecs, may be associated with interstellar motions. The decay of these waves into shorter

ones may generate a wave spectrum adequate to confine the cosmic rays. An equation, similar to that proposed by Heisenberg (1948) for hydrodynamic turbulence, is assumed to govern the spectrum of waves. Its solutions, for different forms of the damping coefficient, are discussed.

The third chapter is devoted to establishing a formalism for calculating the mean free path of cosmic rays in the presence of a given spectrum of waves.

In the fourth chapter, we use the procedures described in Chapters II and III to study the propagation of cosmic rays in the interstellar medium. The development of a spectrum of waves in media with different physical properties is considered. A model of cosmic ray propagation is proposed, and compared with the observations. Finally, a summary of the principal results of this work is given.

REFERENCES

Colgate, S. A. and White, R. H., 1964, Astrophys. J., 143, 626.

Cowsik, R., Pal, Y., Tandon, S. N. and Verma, R. P., 1968, Can. J. Phys. 46, S646.

Fermi, E., 1949, Phys. Rev., 75, 1169.

Fermi, E., 1954, Astrophys. J., 119, 1.

Ginzburg, V. L., 1958, Progress in Elementary Particle and Cosmic Ray Physics, 4 (North-Holland, Amsterdam).

Ginzburg, V. L. and Syrovatskii, S. I., 1964, The Origin of Cosmic Rays (Macmillan, New York).

Gloeckler, G. and Jokipii, J. R., 1969, Phys. Rev. Letters 22, 1448.

Goldreich, P. and Julian, W. H., 1969, Astrophys. J., 157, 869.

Hayakawa, S., Ito, K. and Terashima, Y., 1958, Progr. Theoret. Phys., 8, 571.

Heisenberg, W., 1948, Z. Phys., 124, 628.

Jokipii, J. R., 1966, Astrophys. J., 146, 480.

Kulsrud, R. M. and Pearce, W. P., 1969, Astrophys. J., 156, 445.

Lerche, I., 1967, Astrophys. J., 147, 689.

Morrison, P., Olbert, S., and Rossi, B., 1954, Phys. Rev., 94, 440.

Ostriker, J. P. and Gunn, J. E., 1969, Astrophys. J., 157, 1395.

Ramaty, R., Reames, D. V. and Lingenfelter, R. E., 1970, Phys. Rev. Letters, 24, 913.

Shapiro, M. M., Silberberg, R. and Tsao, C. H., 1970, to appear in the George Gamow memorial volume, edited by Frederick Reines.

Wentzel, D., 1968, Astrophys. J., 152, 987.

Wentzel, D. G., 1969, Astrophys. J., 156, 303.

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REFERENCES

Dalgarno, A., 1960, Proc. Phys. Soc. (London), A75, 374.

Field, G.B., Goldsmith, D. W., and Habing, H. J., 1969, Astrophys. J., 155, L149.

Ginzburg, V. L., and Syrovatskii, S. I., 1964, The Origin of Cosmic Rays (Macmillan, New York).

Hjellming, R. M, Gordon, C. P. and Gordon, K. J., 1969, Astron. Astrophys., 2, 202.

Kulsrud, R. M., and Pearce, W. P., 1969, Astrophys. J., 156, 445.

Kulsrud, R. M., and Cesarsky, C. J., 1971, Astrophys. Letters, 8, 189.

Parker, E. N., 1969, Space Sci. Rev., 9, 651.

Peek, J. M., 1966, Phys. Rev., 143, 33.

Ramaty, R., Reames, D. V., and Lingenfelter, R. E., 1970, Phys. Rev. Letters, 24, 913.

Smith, R. J., 1966, Planet. Space Sci., 14, 929.

Spitzer, L., 1968, Diffuse Matter in Space (Interscience, New York).

Tademaru, E., 1969, Astrophys. J., 158, 959.

Wentzel, D., 1969, Astrophys. J., 156, 303.

Wentzel, D., 1971, Astrophys. J., 163, 503.

CHAPTER II

Sources of waves and wave spectrum.

1) Introduction

The next three chapters are devoted to studying the possibility that ultrarelativistic cosmic rays are confined to the galactic disk as a result of their interaction with hydromagnetic waves, which are produced by a mechanism independent of the cosmic rays.

The waves, while exchanging little energy with the cosmic rays, are very effective in scattering them. Consequently, instead of streaming freely along the disk, the cosmic rays diffuse out at a rate proportional to the diffusion coefficient D, or the mean free path λ ($D = \lambda c/3$). A number of constraints on the value of λ and its functional dependence on the rigidity ε, are imposed by the available observational data on high energy cosmic rays (section 2).

Possible sources of waves are discussed in section 3, where we suggest that a considerable amount of hydromagnetic waves with wavelengths of the order of a few parsecs may be associated with the observed interstellar turbulence. The decay of these long waves into shorter ones may then result in the development of a wave spectrum adequate to confine the cosmic rays to the disk.

In section 4, Heisenberg's theory of turbulence is outlined; and a modified version of Heisenberg's equation

is assumed to govern the spectrum of hydromagnetic waves. The general behaviour of the solution of this equation for the case in which the damping rate of the waves is independent of the frequency is investigated in section 5. A numerical solution of the equation for different values of the power input in waves is given in the appendix.

2) Observational constraints on the mean free path of cosmic rays.

Observations of cosmic rays at the earth place some constraints on the value of $\lambda(\varepsilon)$. There are essentially three kinds of experimental evidence on high energy cosmic rays from which direct conclusions on λ can be drawn: the measurements of the energy spectrum, of the cosmic ray composition, and the degree of anisotropy.

The interstellar spectrum of cosmic rays of $\varepsilon \lesssim$ 1Bv is not well known, because of the critical effects of solar modulation. For this reason, our discussion will be restricted to cosmic rays of rigidities in the range 10 Bv $\lesssim \varepsilon \lesssim 10^6$ Bv. In this range of rigidities, the integral spectrum of cosmic rays is a decreasing power law, of index 1.6 (see Figure (II-1)). Since the spectra of non-thermal radio sources tend to follow a power law, it seems natural to assume that the cosmic ray spectrum at the source is also a power law. In this case, and for any reasonable diffusion

model, $\lambda(\varepsilon)$ must be equal to either a constant or a power law, for 10 Bv $\lesssim \varepsilon \lesssim 10^6$Bv.

The cosmic ray spectrum steepens to an index of 2.2 at a rigidity of $\sim 3.10^6$ Bv; it regains its original slope at $\varepsilon \simeq 3.10^9$Bv. The most widely accepted interpretation for the double break in the spectrum (Ginzburg and Syrovatskii 1964; Hayakawa 1969) is that it represents a cut-off of the galactic component of cosmic rays, and that rays of rigidity above 3.10^9 Bv are of extragalactic origin.

The spectrum of cosmic ray electrons with energies between 3 and 50 Gev is parallel to the spectrum of nuclei. What happens at higher energy is the subject of some controversy. Observations prior to 1969 indicate that the power law extends to energies of $\sim$ 300 Gev (Meyer 1969), whereas Anand et al (1969) find that the spectrum becomes steeper at $\sim$ 200 Gev; in Nishimura et al (1969) results, it steepens even earlier, at $\sim$ 50 Gev. If the break in the cosmic ray electron spectrum at an energy between 300 and 50 Gev is caused by synchrotron and Compton losses of electrons in the interstellar medium, the time of residence of such electrons in the galaxy would be limited to about 1 to 10 million years. At these energies, the diffusion of electron and proton cosmic rays is identical ; thus, the age of cosmic ray protons with $\varepsilon \simeq$ 100 Bv should also lie in the range 1-10 million years.

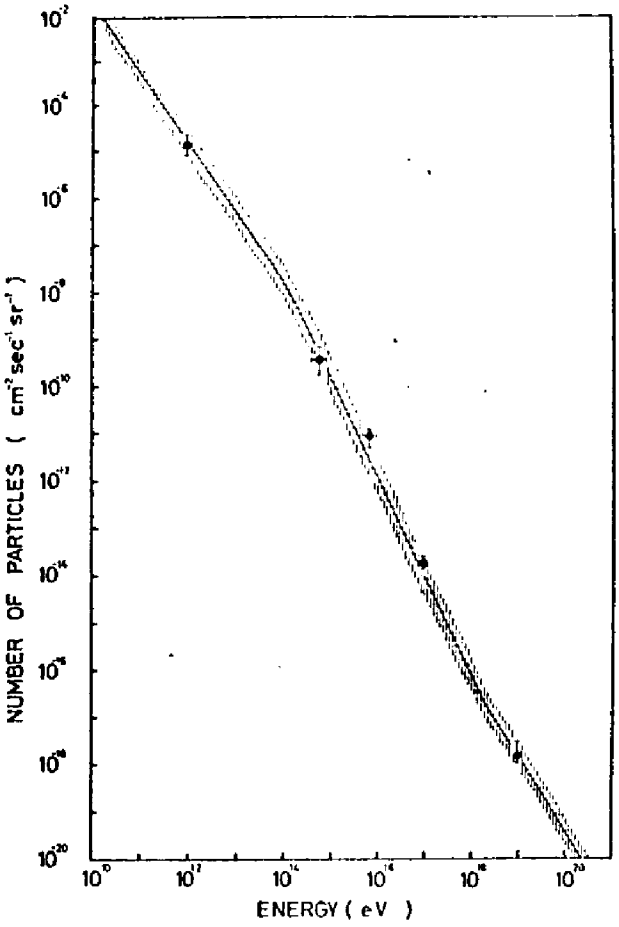


Figure (II-1)

Integral energy spectrum of cosmic rays.

This Figure is from Hayakawa (1969), p.569.

This result is consistent with other estimates of the age of cosmic rays, based on measurements of either the composition or the abundance of radioactive isotopes. Detailed theoretical studies of the transformations suffered by cosmic ray nuclei, due to nuclear collisions in interstellar space, permit to reconstruct their composition at the source, and to compute the amount Σ of interstellar gas they have traversed. The result obtained, for cosmic rays with $\varepsilon \lesssim$ 30 Bv, is: $\Sigma = 3.9 \pm 0.6$ g/cm^2 (Durgaprasad et al 1970). This result implies that such cosmic rays spend no more than $\sim$ 3 million years in the narrow gas disk of mean density $n_H \simeq 1$ cm^{-3}; but, on the basis of the value of Σ alone, they could spend a longer time in more diffuse regions.

Measurements of the abundance of isotopes of mean life τ_o of a few million years can be used to obtain a direct estimate of the cosmic ray age. The evidence -- as yet inconclusive -- on ^{10}Be ($\tau_o \simeq$ 3.9 million years) seems to indicate that the cosmic ray age is < 20 million years (Shapiro 1970).

The degree of anisotropy of cosmic rays is defined as:

$$\delta = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

where I_{max} and I_{min} are the maximum and minimum intensity at a given point, as a function of the direction. No anisotropy in the cosmic ray flux at the earth has been detected. The

upper limits available range from 4.10^{-4} (Elliot et al 1963) at a rigidity of $\sim$ 300 Bv, to 7.10^{-4} at $\sim 2 \times 10^4$ Bv (Cachon 1962) and 10^{-3} at $10^5 - 10^6$ Bv (Cocconi 1961). The fact that the cosmic rays anisotropy is small at all the rigidities of interest seems to imply that the dependence of the mean free path of cosmic rays on ε has to be weak. Thus, the spectrum of cosmic rays, together with the isotropy observations, indicates that λ is either a constant or a power law in ε, with a small index. For a given model of cosmic ray propagation, the magnitude of λ is partly determined by the cosmic rays mean age, $\langle \tau \rangle$.

For example, let us consider the simple model described by Kulsrud and Pearce (1969; hereinafter referred to as KP), in which cosmic rays diffuse along a magnetic tube of force of uniform cross-section, of length 2L, the two ends opening into the halo. The diffusion equation for the distribution function $\mathcal{F}$ of the cosmic ray is:

$$\frac{\partial \mathcal{F}}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \mathcal{F}}{\partial z}\right) + S \qquad \text{(II-1)}$$

where D is the diffusion coefficient and $S(\varepsilon, z)$ represents the cosmic ray sources; z is the distance from the center of the tube. In the time-independent case, if D and S are uniform in space,

$$\mathcal{F} = \frac{S}{2D}\left(L^2 - z^2\right)$$

To evaluate the mean age $\langle \tau \rangle$ of cosmic rays, KP define $\mathcal{F}'$, so that $\mathcal{F}'$ is the number of cosmic rays in the range $d\tau$, and $q = \int_0^\infty \mathcal{F}' \tau \, d\tau$. They show that q satisfies:

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} D \frac{\partial q}{\partial z} + \mathcal{F} \qquad \text{(II-2)}$$

while $\langle \tau \rangle = q / \mathcal{F}$.

When S, D are uniform:

$$\langle \tau \rangle = \frac{L^2}{12 D} \left(5 - \frac{z^2}{L^2} \right) \qquad \text{(II-3)}$$

The anisotropy is:

$$\delta = - \frac{\lambda}{\mathcal{F}} \frac{\partial \mathcal{F}}{\partial z} = \frac{2 \lambda z}{L^2 - z^2} \qquad \text{(II-4)}$$

Thus, in this model, the approximation $\delta \simeq \frac{\lambda}{L}$ is not valid, and the absence of anisotropy at the earth does not necessarily imply that the mean free path of the cosmic rays has to be extremely small. It may instead indicate that the sun is placed near the center of its tube of force. Because the sun lies close to the plane of symmetry of the galaxy, this possibility is not farfetched. Combining equations (II-3) and (II-4), and using the observed limit on δ at 300 Bv, we obtain an upper limit for the distance $z_\odot$ of the sun to the center of the tube:

$$z_0 \simeq \frac{2}{5} \, c \, \delta \langle \tau \rangle \lesssim 150 \, pc$$

where we have assumed $\langle \tau \rangle \simeq$ 3 million years.

The diffusion of cosmic rays along a tube of force, caused by their interaction with hydromagnetic waves, is accompanied by diffusion perpendicular to the magnetic field direction, at a rate given by (Skilling 1970):

$$D_\perp \simeq \left(\frac{r_\ell}{\lambda} \right)^2 D \qquad \text{(II-5)}$$

where $r_\ell = \varepsilon / B_0$ is the Larmor radius.

If $L_\perp$ is the half width of the tube, cosmic rays with rigidity ε larger than ε_B will leave the disk by diffusing across the field rather than along it; ε_B satisfies:

$$\frac{r_\ell(\varepsilon_B)}{\lambda(\varepsilon_B)} \simeq \frac{L_\perp}{L} \qquad \text{(II-6)}$$

The length of the tube of force may be determined by identifying ε_B with the observed break in the cosmic ray spectrum at 3.10^6 Bv. Equation (II-6), together with equation (II-3), then yields:

$$L = \left\{ \frac{4 \lambda c \langle \tau \rangle}{5} \right\}^{1/2} = L_\perp \frac{\lambda_B}{r_{\ell B}} \qquad \text{(II-7)}$$

If, for 10 Bv $\lesssim \varepsilon \lesssim 3.10^6$ Bv, λ is a power law:

$$\lambda \propto \varepsilon^a,$$

we obtain from equation (II-7):

$$\lambda(100\ \text{Bv}) = \frac{4}{5} \langle\tau\rangle_{100} \quad \frac{r_{e_B}^2\, c}{L_\perp^2} \quad \left(\frac{100}{3 \times 10^6}\right)^{2a} \tag{II-8}$$

If the mean age of 100 Bv cosmic rays is $\langle\tau\rangle_{100} \simeq$ 3 million years, the intensity of the magnetic field is $B_0 = 3\mu$ G, and $L_\perp \simeq 100$ pc,

$$L = \frac{7200}{(3\times10^4)^a} \text{ pc}$$

and

$$\lambda = \frac{72}{(3\times10^4)^{2a}} \text{ pc}$$

Since in our model the value of the mean free path is determined by the wave spectrum F, the conditions imposed on λ by the observations will ultimately be requirements that have to be met by F.

3) Sources of waves.

In the first chapter, we have used numerical arguments to show that, although streaming cosmic rays are capable of generating hydromagnetic waves, they cannot regulate their own diffusion in the galactic disk; if they did, the streaming velocity and the anistropy would increase rapidly with the

energy. Thus, since high energy cosmic rays cannot be responsible for their own confinement, other sources of waves have to be present.

The observed scintillation of pulsars indicate that there are large perturbations in the electron density of the interstellar medium: $\Delta n_e \approx 10^{-4\pm1} cm^{-3}$, on a scale $\sim 10^{12} cm$ (Rickett 1970). These fluctuations could be associated with hydromagnetic waves, some of which could even be generated by cosmic rays of $\varepsilon \lesssim 1$ Bv (Wentzel 1969). It has been proposed (Melrose and Wentzel 1970) that these waves may control the diffusion of cosmic rays of all energies, through high harmonic interactions. But a simple argument shows that, if this were the case, the mean free path of cosmic rays would be even more dependent on the energy than in the case discussed and dismissed in Chapter I.

Let us assume that a large amount of energy is present, in hydromagnetic waves with wave number $k_0 \approx 10^{-12} cm$, while larger or shorter waves are absent. The mean free path of ultra relativistic cosmic rays is proportional to (KP):

$$\lambda(\varepsilon) \propto \frac{\varepsilon^2}{\mathcal{E}(k_z = 1/r_c)} \qquad \text{(II-9)}$$

where:

$$\mathcal{E}(k_z) = \sum_n \mathcal{E}_n(nk_z)$$

$$\mathcal{E}_n(nk_z) = 4\int d\vec{k}_\perp \left\{ W_\alpha(\vec{k}_\perp, nk_z) \frac{J_n^2(x) n^2}{x^2} + W_{ms}(\vec{k}_\perp, nk_z) J_n'^2(x) \cos^2\xi \right\} \qquad \text{(II-10)}$$

where $X = \frac{k_\perp v_\perp}{\Omega}$, Ω is the gyrofrequency of the cosmic ray, W_α and W_{ms} are the energy densities per unit $\vec{k}$ in Alfvèn and magnetosonic waves respectively, and ξ is the angle which the direction of propagation of the wave makes with the field.

If $B_0 = 3\mu$ Gauss, the Larmor radius is: $r_\ell = 10^{12} \frac{\varepsilon e}{mc^2} \simeq 10^{12}\ \varepsilon_{Bv}$ cm. Hence, a cosmic ray of rigidity ε interacts with waves of wavenumber 10^{-12}cm through the n^{th} harmonic, where $n \simeq \varepsilon$. Consequently:

$$\left. \xi(k_z = 1/r_\ell) \simeq \xi_n(n k_z) \right|_{n=\varepsilon} \simeq$$

$$\simeq 4 k_0^2 \left\{ W_\alpha(k_0) \int_0^{\beta_n} \frac{J_n^2(x)\,dx}{x} + W_{ms}(k_0) \int_0^{\beta_n} \frac{J_n'^2(x)\ x\,dx}{(n^2 + x^2)} \right\} \Bigg|_{n=\varepsilon}$$

(II-11)

where β_n are constants of order 1, describing the cut-off in the spectrum at long wavelengths.

Both integrals in (II-11) are proportional to $\left(\frac{1}{n}\right)^u$, where $u \gtrsim 1$. Using this result in (II-9), we obtain:

$$\lambda(\varepsilon) \propto \varepsilon^{3+\Delta}, \qquad \Delta \gtrsim 0$$

(II-12)

Such a strong dependence of λ on ε is in contradiction with the isotropy observations.

Hydromagnetic waves long enough to interact through the first harmonic with cosmic rays of ultrarelativistic energy could be emitted by a number of discrete sources in the galaxy, such as high velocity stars with stellar winds (Dokuchaev, 1964), rotating white dwarfs or magnetic stars (Kulsrud 1970), exploding stars, etc. However, it is difficult to accept the possibility that cosmic rays are confined to the galaxy by waves issuing from a number of discrete sources: a) in regions with a sizeable density of neutral particles, the waves would be so efficiently damped by neutral charged particle collisions and by the scattered cosmic rays that they would not be able to propagate very far from their sources; thus, an improbably large number of sources would be required (Kulsrud 1968). b) If waves in different ranges of wavelengths were emitted by different types of sources, the spectrum of waves would probably be very irregular, making it difficult to account for the simple cosmic ray spectrum.

On the other hand, it may be possible to obtain a smooth spectrum if only very long waves, with scales of the order of parsecs, are being generated directly in the interstellar medium. Turbulent gas motions, stirred by expanding H II regions and supernovae explosions in the galaxy, are a possible source of such waves. The energy density present in these motions is comparable to that in cosmic rays or in the

magnetic field. By analogy with hydrodynamic turbulence, we assume that non linear effects cause the energy in these long waves to be transmitted to waves of successively smaller scale. This process, together with the different mechanisms tending to damp the waves, determines the wave spectrum.

4) Wave spectrum.

The problem of the nature of hydromagnetic turbulence is far from solved. Phenomenological theories for a much simpler case, that of stationary, uniform and isotropic hydrodynamic turbulence in an incompressible medium, have been developed by several authors (see review by Saffman 1968). Of these theories, Heisenberg's (1948) has been most generally accepted. An extensive discussion of it is given by Chandrasekhar (1949); we will outline here the principal features of this theory.

The distribution of energy into eddies of wavenumber k is described in terms of the spectral function F(k), expressing the kinetic energy per unit mass in the interval dk. If a source communicates a power E [erg / g sec] to the largest scales ($k < k_0$), for any $k > k_0$, in a steady state, we have: E = energy dissipated per second at all wavenumbers < k + energy transferred per second to all wavenumbers > k.

The energy dissipated at small k is equal to:

$$2\nu \int_{k_0}^{k} F(k') k'^2 \, dk' \qquad \text{(II-13)}$$

where ν is the kinematic viscosity.

Heisenberg writes the energy transferred to smaller scales as:

$$2\,\nu_k \int_{k_o}^{k} F(k')\,k'^2\,dk' \qquad \text{(II-14)}$$

where ν_k is the "eddy viscosity:"

$$\nu_k = \kappa \int_{k}^{\infty} \sqrt{\frac{F(k'')}{k''^3}}\; dk'' \qquad \text{(II-15)}$$

Thus, F(k) must satisfy the equation:

$$E = \text{constant} = 2\left\{\nu + \kappa \int_{k}^{\infty} \sqrt{\frac{F(k'')}{k''^3}}\; dk''\right\} \int_{k_o}^{k} F(k')\,k'^2\,dk' \qquad \text{(II-16)}$$

Chandrasekhar (1955) extended the Heisenberg theory to hydromagnetic turbulence. He defined an "eddy resistivity", analogous to the "eddy viscosity", and derived an integral equation similar to (II-16), but involving two different spectral functions, one for the kinetic energy and the other for the magnetic energy. Because we are interested in hydromagnetic turbulence in the presence of a uniform magnetic field, we infer that, at least when dissipation is not too rapid, the 'eddies' are replaced by a superposition of hydromagnetic waves. (Kraichnan (1965) argues that this should be the case at least in the inertial range of turbulence). Then, in the

framework of our model, the relation between kinetic and magnetic energy is predetermined; and the spectral function $F(k)dk$ represents the total energy, per unit mass, in waves of wavenumber in the range dk. We take the distribution of directions of wave propagation to be isotropic; any different assumption would force us to assign to the rate of energy transfer some arbitrary dependence on the angle of propagation. If Γ is the rate of damping of the waves, the equation -- analogous to Heisenberg's -- that we consider is:

$$E = 2\int_{k_0}^{k} \Gamma(k') F(k')\, dk' + 2\kappa \int_{k}^{\infty} \sqrt{\frac{F(k'')}{k''^3}}\, dk'' \int_{k_0}^{k} F(k')\, k'^2\, dk' \qquad \text{(II-17)}$$

We realize that the Heisenberg equation may not be applicable to the hydromagnetic turbulence under consideration. However, because there is no adequate theory of fully developed hydromagnetic turbulence, it seems advisable to take Heisenberg's as the simplest possible equation to describe the spectrum of turbulence. In any event, it would seem that this equation, which has a strong intuitive appeal, represents an upper limit to the rate of transfer of energy to smaller wavelengths. It is difficult to imagine that a turbulent eddy of energy kF can transfer its entire energy to a shorter length in a time shorter than its "turn-over" time, i.e. the time for its velocity to cross its wavelength. Further, additional processes are known which transfer energy

to larger wavelengths. On the other hand, single hydromagnetic waves have a steepening property, similar to that of sound waves, which effectively generates higher harmonics at about the rate given by Heisenberg transfer function.

With these remarks in mind, we now take the Heisenberg equation as our governing equation, with the expectation that one can get some idea of the resulting wave spectrum or, at least, of an upper limit to it.

5) Wave spectrum for different $\Gamma(k)$.

When Γ is proportional to k^2 ($\Gamma \equiv \beta k^2$), equation (II-17) is identical to the hydrodynamic Heisenberg equation, and has a simple analytical solution (Chandrasekhar, 1949; we re-derive this solution in the appendix II-1):

$$F = \left(\frac{8}{9}\frac{E}{K}\right)^{2/3} \Big/ k^{5/3}\left(1 + \frac{8\beta^3 k^4}{3K^2 E}\right)^{4/3} \qquad \text{(II-18)}$$

At large k, ($k > k_s = (\frac{3K^2 E}{8\beta^3})^{\frac{1}{4}}$) dissipation is important, and the spectrum decays. At small k, the time for energy transfer between waves, $\tau_T \simeq \frac{1}{k\sqrt{kF}}$, is shorter than the dissipation time $\tau_d \simeq \frac{1}{\beta k^2}$, and the spectrum adopts the familiar form of the Kolmogorov spectrum: $F \propto k^{-5/3}$. This result can be inferred from dimensional considerations only: if the dissipation is negligible, F must be solely a function of E and k, and the only dimensionally correct combination is:

$$F = b\, E^{2/3}\, k^{-5/3}$$

(II-19)

Experiments in hydrodynamic turbulence have confirmed this result (Gibson 1963; Grant et al 1962), and the constant b was determined to be in the range 1.45 - 1.6. In the following, we will use: $\kappa = 0.45$, corresponding to $b = 1.57$.

As we mentioned in Chapter I, in an interstellar medium with currently accepted properties, the dominant wave damping mechanism is caused by charged neutral particle collisions (KP; we discuss it in detail in Chapter IV, Appendix 1); the resulting damping rate Γ^* is independent of k over a large range of frequencies. Thus, it is also relevant to consider the solution of the Heisenberg equation when terms of the viscous form, $\Gamma' = \beta k^2$, are small compared to Γ^*.

Since it did not seem possible to find an exact analytical solution to this Heisenberg equation, we solved it numerically (see appendix). To understand the behaviour of the spectrum, we considered an approximate form of the Heisenberg equation, obtained by replacing the integrals in the second term by their values at k, multiplied by k. (This approximation is appropriate if the logarithmic slope of the spectrum is > -3).

$$\frac{d}{dk}\left(F^{3/2}\,k^{5/2}\right) + 2\,\Gamma^{*}\,F = 0 \qquad \text{(II-20)}$$

The physical meaning of this equation is obvious if we write it as:

$$\frac{d(Fk)}{dt} \simeq -\frac{d}{dk}\left(\frac{Fk}{\tau_T}\right) - 2\,F\,k\,\Gamma^{*} = 0 \qquad \text{(II-21)},$$

where $\tau_T = 1/\sqrt{Fk^3}$ is the typical time for transfer of energy between waves.

If Γ^* is independent of k, the solution of equation (II-20) is:

$$F = \left(\frac{A\,\Gamma^{*}\,k^{-5/6}}{k_0^{2/3}} + \Gamma^{*}\,k^{-3/2}\right)^{2} \quad \text{for} \quad k_0 < k < k_1 \qquad \text{(II-22)}$$

where A is an arbitrary constant, to be determined.

The power input to the system, per unit mass, is:

$$E = E_1 + E_2 = 2\,\Gamma^{*}\int_{k_0}^{\infty} F\,dk + 2\int_{k_0}^{\infty} \Gamma' F\,dk \qquad \text{(II-23)}$$

The first term in this sum, E_1, is the power dissipated by Γ^*; the second term, E_2 is that dissipated by other damping mechanisms at shorter wavelengths ($k > k_1$). Equation (II-23), together with equation (II-22), yields:

$$\frac{E_1}{(\Gamma^{*3}/k_0^2)} \equiv \bar{E}_1 = 3A^2 + 3A + 1 \lesssim \bar{E} \qquad \text{(II-24)}$$

where $\bar{E}$ is E in its natural units, $\frac{\Gamma^{*3}}{k_0^2}$.

Four types of solutions are possible, depending on the value of $\bar{E}$:

a) if $\bar{E} >> 1$, the spectrum, at least for $k < k_1$, has the Kolmogorov form. Most of the energy is dissipated at short wavelengths. Using equation (II-19), we find: $\bar{E}_1 \propto \bar{E}_2^{2/3}$.

b) If $\bar{E} \gtrsim 1$: at small k, both terms in (II-22) are important. The spectrum starts off with a negative slope, between -3 and -5/3. As k increases, the first term starts dominating; at very large k, the logarithmic slope is $\simeq$ -5/3. A large fraction of the energy input is dissipated at k around k_0. This situation is exactly opposite to that where $\Gamma = \beta k^2$.

c) If $\bar{E} = 1$: A = 0, and the spectrum has a slope equal to -3. Practically all the energy is dissipated at the longer wavelengths. The corresponding value of $F(k_0)$ is: $F(k_0)_{crit.} = \Gamma^{*2}/k_0^3$. (II-25)

d) If $\bar{E} < 1$, there is not enough energy fed to the system to get any appreciable transfer; the spectrum decays sharply.

A schematic diagram of F, for different values of $\bar{E}$, is given in Figure (II-2).

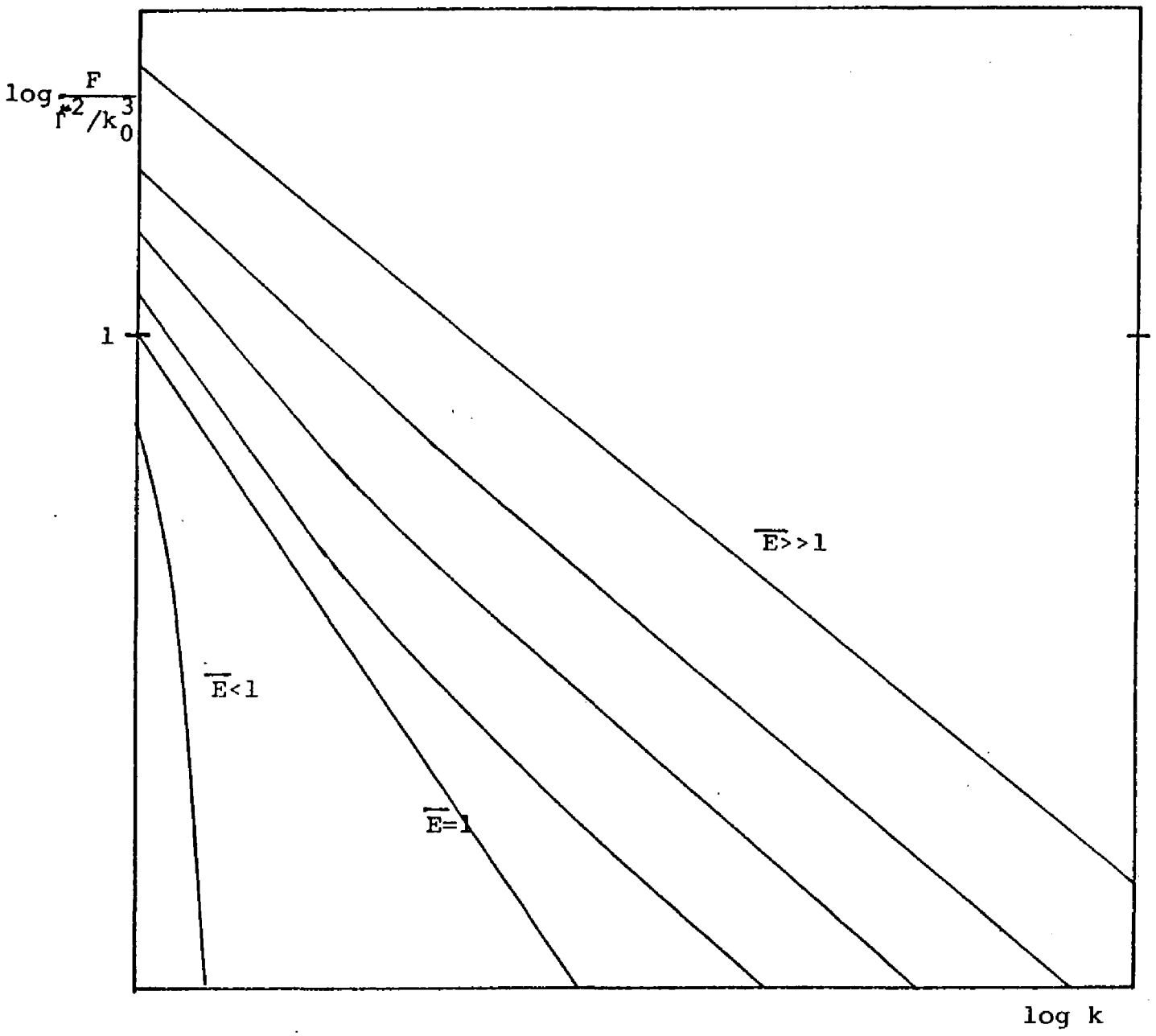


Figure (II-2)

Schematic diagram of the variations of F with k, for different values of the power input E, when $\Gamma = \Gamma^* =$ constant.

Let us point out an interesting feature of F: when $\bar{E}$ is $\gtrsim 1$, a small variation of $\bar{E}$, while leaving $F(k_0)$ virtually unchanged, leads to a large change in $F(k_0)$, at $k >> k_0$. Since the cosmic rays mean free path is, in order of magnitude, proportional to $1/F(k=1/r_\ell)$, very similar power inputs in the critical range ($\bar{E} \simeq 1$) would result in very different cosmic ray mean free paths, at intermediate cosmic ray energies. Also, as the power input can hardly be expected to remain strictly constant in time, the mean free path of cosmic rays would fluctuate considerably.

In the next chapter, we will present a method to calculate the mean free path of cosmic rays in the presence of a given spectrum of waves. In chapter IV, we will apply the results of this chapter and of Chapter III to the interstellar medium.

APPENDIX II-1

Resolution of equation (II-16).

To solve equation (II-16), we define:

$$G = \int_{k_o}^{k} F(k')\, k'^2\, dk' \tag{AII-1}$$

Then,

$$F = \dot{G} / k^2 \tag{AII-2}$$

where $\cdot$ denotes derivative with respect to k.

The equation can be rewritten:

$$\int_{k_o}^{k} \frac{\Gamma(k')\, \dot{G}(k')\, dk'}{k'^2} + \kappa\, G(k) \int_{k}^{\infty} \sqrt{\frac{\dot{G}(k')}{k'^5}}\, dk' = E \tag{AII-3}$$

We differentiate (AII-3) with respect to k, to obtain:

$$\int_{k}^{\infty} \sqrt{\frac{\dot{G}(k')}{k'^5}}\, dk' = -\frac{\Gamma(k)}{\kappa k^2} + \frac{G(k)}{\sqrt{G'(k)\, k^3}} \tag{AII-4}$$

and, differentiating again:

$$\ddot{G}(k) = -\frac{4\, \dot{G}(k)}{G(k)} + \frac{5\, \dot{G}(k)}{k} - \frac{4\, \Gamma(k)\, \dot{G}^{3/2}}{\kappa G \sqrt{k}} + \frac{2\, \dot{\Gamma}(k)\, \dot{G}(k)^{3/2} \sqrt{k}}{\kappa\, G(k)} \tag{AII-5}$$

When Γ is proportional to k^2: $\Gamma = \beta k^2$, equation (AII-5) is reduced to:

$$\ddot{G}(k) - \frac{4\,\dot{G}(k)}{G(k)} + \frac{5\,\dot{G}(k)}{k} = 0 \qquad \text{(AII-6)}$$

which has a simple solution:

$$G = \frac{1}{\left(\frac{3}{4}\frac{A}{k^4} + B\right)^{1/3}} \qquad \text{(AII-7)}$$

The constants A and B can be determined from the boundary conditions. From (AII-4), when $k \to \infty$, we get

$$A = \left(\frac{k}{\beta}\right)^2 B^{2/3} \qquad \text{(AII-8)}$$

and B is obtained from:

$$E = 2\int_{k_0}^{\infty} \beta k^2 F\, dk = 2\beta B^{-1/3} \qquad \text{(AII-9)}$$

and (II-18) follows from the above relations. For an arbitrary Γ, equation (AII-5) may be solved numerically. In particular, we solved it for $\Gamma = \Gamma^* =$ constant over a wide range of k ($k < k_1$). We examined cases corresponding to $\bar{E} > 1$, or $F \frac{(k_0)k_0^3}{\Gamma^{*2}} > 1$, so that a spectrum of waves can develop.

When k is very large (of order k_1), the solution approaches $G = \alpha\, k^{4/3}$, or $F = \frac{4}{3}\,\frac{\alpha}{k^{5/3}}$, where α is a constant. Starting from this solution at large k, equation (AII-5) was integrated towards smaller k, for different values of α. The solutions are presented in Figure (II-3). The power input corresponding to each result may be calculated:

$$E = 2\int_{k_0}^{\infty} F(k)\, T^{*} dk \quad + E_2 \qquad \text{(AII-10)}$$

where

$$E_2 = \left(\frac{4}{3}\,\alpha\right)^{3/2}$$

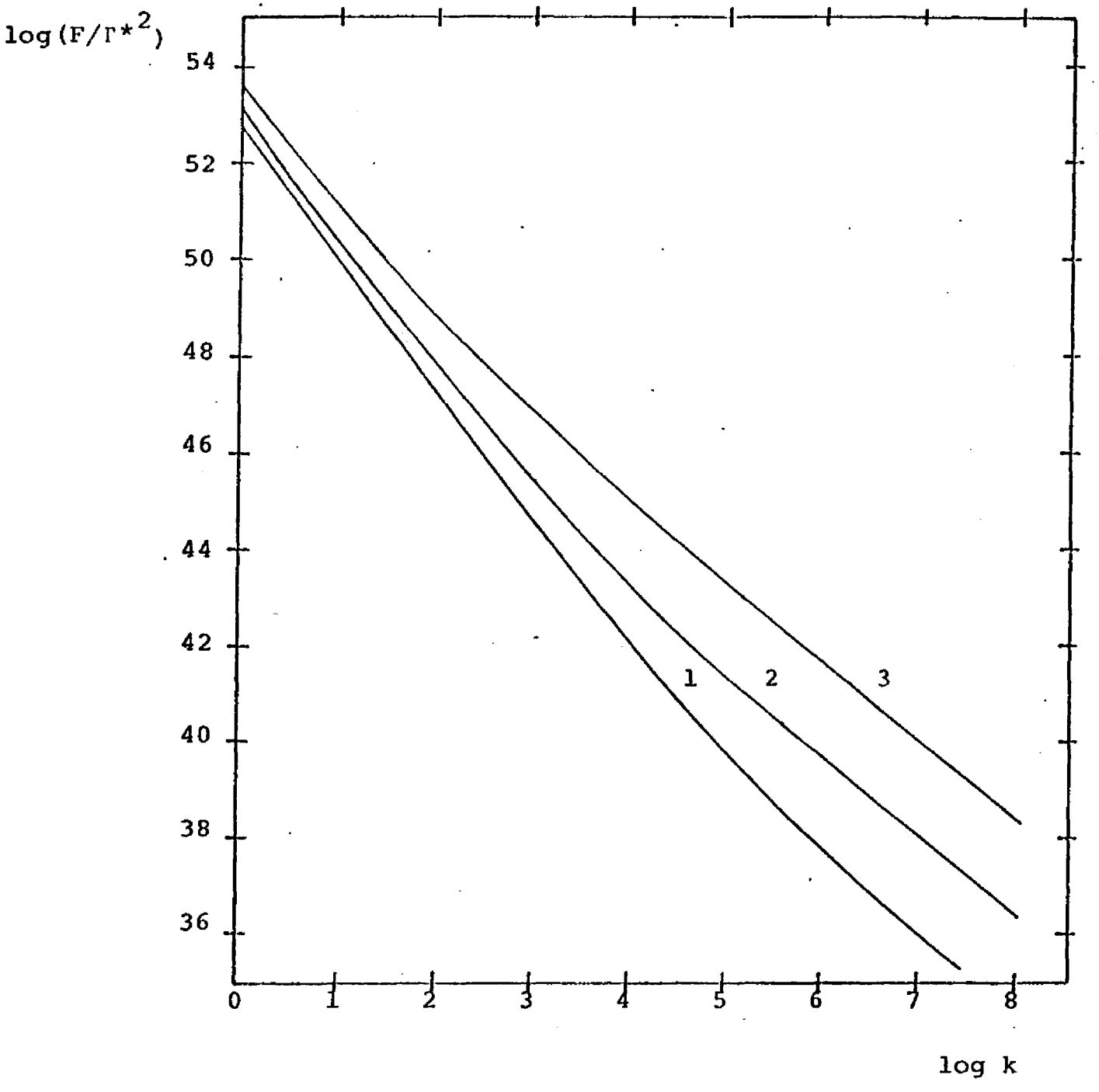


Figure (II-3)

Solutions of the Heisenberg equation (II-16) for $\Gamma = \Gamma^* =$ constant, for different values of α. F/Γ^{*2} and k are in units of cm^{-3} and 10^{-18} cm^{-1} respectively. Curve 1 corresponds to $\alpha = 1.9 \times 10^{6}$ in CGS units; α increases by a factor 100 in every successive curve.

REFERENCES

Anand, K.C., Daniel, R. R. and Stephens, S. A., 1969, Proc. of the 11th International Conference on Cosmic Rays, Budapest, OG41.

Cachon, A., 1962, Fifth Interamerican Seminar on Cosmic Rays, 2, XXXIX.

Chandrasekhar, S., 1949, Astrophys. J., 110, 329.

Chandrasekhar, S., 1955, Proc. Roy. Soc., 233(A), 330.

Cocconi, G., 1961, Handbuck der Physik, PH XLVI/1, 215.

Dokuchaev, V. P., 1964, Soviet Astronomy - A. J., 8, 23.

Durgaprasad, N., Fichtel, C. E., Guss, D. E., Reames, D. V., O'Dell, F. W., Shapiro, M. M., Silberberg, R., Stiller, B. and Tsao, C. H., 1970, Phys. Rev. D, 1, 1021.

Elliot, H. Thambyahpillai, T. and Peacock, D. S., 1969, Proc. of the 11th International Conference on Cosmic Rays, Budapest, OG95.

Grant, H. L., Stewart, R. W. and Moilliet, A., 1962, J. Fluid Mech., 12, 241.

Ginzburg, V. L., and Syrovatskii, S. I., 1964, The Origin of Cosmic Rays,(Macmillan, New York).

Hayakawa, S., 1969, Cosmic Ray Physics, (Wiley-Interscience, New York).

Heisenberg, W., 1948, Z. Phys., 124, 628.

Kraichnan, R. H., 1965, Phys. of Fluids, 8, 1385.

Kulsrud, R. M., 1968, talk at AAS meeting, New York.

Kulsrud, R. M. and Pearce, W. P., 1969, Astrophys. J., 156, 445.

Kulsrud, R. M., 1971, Astrophys. J., 163, 567.

Melrose, D. B. and Wentzel, D. G., 1971, Astrophys. J., 161, 457.

Meyer, P., 1969, Ann. Rev. Astron. and Astrophys., 7, 1.

Nishimura, J., Mikumo, M., Nito, I., Niu, K., Ohta, I. and Taira, I., 1969, Proc. of the 11th International Conference on Cosmic Rays, Budapest, OG43.

Rickett, B. J., 1970, Monthly Not. Roy. Astron. Soc., 150, 67.

Saffman, P. G., 1968, Topics in Nonlinear Physics, edited by N. J. Zabusky , (Springer-Verlag, New York Inc.), p. 485.

Shapiro, M. M., 1970, presented at the XIV General Assembly of the IAU.

Skilling, J., 1970, Mon. Not. R. Astr. Soc., 147, 1.

Wentzel, D. G., 1969, Astrophys. J., 156, L91.

CHAPTER III

Behaviour of cosmic rays in the presence of a spectrum of waves

1) Introduction

In this chapter, we establish a procedure to evaluate the mean free path of cosmic rays in the presence of a given spectrum of hydromagnetic waves.

Cosmic rays of Larmor radius r_ℓ resonate with waves whose wavenumber component k_z is equal to (n/r_ℓ), where n is an integer. In section 2, we show that, when the spectrum of waves has a negative slope, the effect on cosmic rays of the high harmonic interactions ($n >> 1$) can be neglected.

In section 3, it is found that, if the wave spectrum is a power law $F \propto k^{-a}$, with $a > 1$, the rate of pitch angle scattering due to resonant interactions goes to zero when the absolute value $|\mu|$ of the cosine of the pitch angle approaches 0. In this case, the diffusion of cosmic rays with $|\mu|$ less than $|\mu_{\text{cut off}}|$ is not regulated by the waves of short wavelength that resonate with them, but by the longer waves which mirror-reflect them. $|\mu \text{ cut off}|$ is determined by equating the rate of pitch angle scattering due to resonant interactions with that due to mirror reflections. The mean free path is estimated by integrating the inverse of the resonant scattering rate over $|\mu|$, from $|\mu \text{ cut off}|$ to 1.

In section 4, the method described in section 3 is applied for calculating the mean free path of cosmic rays in the presence of a Kolmogorov spectrum of hydromagnetic waves, when the energy is equally divided between Alfvèn and magnetosonic waves. The mean free path obtained exhibits a weak dependence on the cosmic rays rigidity.

In the Appendix (III-1), the treatment of section 3 is extended to the case where the ratio of the sound velocity to the Alfvèn velocity is not small, and the hydromagnetic waves are replaced by MHD waves.

2) Mean free path of cosmic rays; importance of harmonics $n > 1$.

Let $W^{\alpha}_{ms}(\vec{k})$ be the energy per unit volume in hydromagnetic waves of wavenumber $\vec{k}$. α denotes Alfvèn waves, of frequency $\omega = k_z v_A$ (the z axis is parallel to the magnetic field); ms denotes magnetosonic waves, of frequency $\omega = k v_A$. Kulsrud and Pearce (1969) have shown that the diffusion coefficient of cosmic rays of velocity v, streaming along a uniform magnetic field B_0 and interacting resonantly with hydromagnetic waves, is given by:

$$D = \frac{v\lambda}{3} = v^2 \int_0^1 \frac{1-\mu^2}{\nu(\mu,\varepsilon)}\, d\mu \qquad \text{(III - 1)}$$

where λ is the mean free path, μ is the cosine of the pitch

angle, and where the collision frequency $\nu(\mu,\varepsilon)$ is equal to:

$$\nu(\mu,\varepsilon) = \frac{\pi}{2}\ \Omega\ \frac{8\pi}{|\mu| r_\ell B_0^2}\ \mathcal{E}\left(1/|\mu| r_\ell\ ,\ \mu\right) \qquad \text{(III - 2)}$$

where Ω is the cyclotron frequency and $r_\ell = \varepsilon/B_0$ is the Larmor radius of a cosmic ray of rigidity ε. $\mathcal{E}(k_z, |\mu|)$ has been defined in Chapter II, equation (II - 10); let us repeat this definition:

$$\mathcal{E}(k_z, |\mu|) = \mathcal{E}_\alpha(k_z, |\mu|) + \mathcal{E}_{ms}(k_z, |\mu|)$$

$$\mathcal{E}_{\substack{\alpha \\ ms}}(k_z, |\mu|) = \sum_n \mathcal{E}_{\substack{n\ \alpha \\ ms}}(n k_z, |\mu|)$$

$$\mathcal{E}_{n_\alpha}(n k_z, |\mu|) = 4 \int d\vec{k}_\perp\ W_\alpha(\vec{k}_\perp, n k_z)\ \frac{J_n^2(x)\, n^2}{x^2}$$

$$\mathcal{E}_{n_{ms}}(n k_z, |\mu|) = 4 \int d\vec{k}_\perp\ W_{ms}(\vec{k}_\perp, n k_z)\ J_n'^2(x) \cos^2\xi$$

$$x = \frac{k_\perp v_\perp}{\Omega} \qquad \text{(III - 3)}$$

where $\perp$ means perpendicular to the direction of the field and ξ is the angle which the direction of propagation of the wave makes with the field $\left(\cos^2\xi = \frac{n^2 k_z^2}{k_\perp^2 + n^2 k_z^2}\right)$.

Thus, the diffusion of cosmic rays of Larmor radius r_ℓ is not completely controlled by waves with $(1/k_z) = r_\ell$, but also depends on waves with $\frac{1}{k_z} < r_\ell$. Cosmic rays interact with waves shorter than their Larmor radii, and propagating at a finite angle with the magnetic field, through

high-order harmonics. They also interact with short waves, through the first harmonic, when their pitch angle is large. Because of these two effects, the mean free path of cosmic rays is not simply proportional to the inverse of the energy in waves with $k_z = 1/r_\ell$.

Whether higher order harmonics are important evidently depends on the angular distribution of the waves. For instance, we have mentioned in Chapter I that, if high energy cosmic rays were the main source of the waves, most waves would run nearly parallel to the magnetic field; then, the effect of harmonics is negligible.

If, on the other hand, the interaction with cosmic rays does not noticeably affect the wave spectrum, it is reasonable to assume that waves are present at all angles. To be specific, we concentrate on the case of an isotropic distribution of waves, and define the wave spectral function as:

$$F_{\substack{\alpha \\ ms}}(k)\, dk = 4\pi\, \rho\, W_{\substack{\alpha \\ ms}}(k)\, k^2\, dk$$

where ρ is the density; $F(k)\, dk$ represents the energy per unit mass in waves whose wavenumber is in a range dk around k.

Then, for Alfvèn waves:

$$\mathcal{E}_{n_\alpha}(nk_z, |\mu|) = \frac{2\rho}{t^2} \int_0^\infty \frac{F_\alpha\left(nk_z \sqrt{1 + x^2/n^2t^2}\right)}{x\left(1 + x^2/n^2t^2\right)}\, J_n^2(x)\, dx \qquad \text{(III - 4)}$$

where $t = 1-\mu^2/\mu$ is the tangent of the pitch angle.

For magnetosonic waves:

$$\mathcal{E}_{n_{ms}}(nk_z, |\mu|) = \frac{2\rho}{n^2 t^2} \int_0^\infty \frac{F_{ms}\left(nk_z \sqrt{1 + x^2/n^2 t^2}\right)}{\left(1 + x^2/n^2 t^2\right)^2} \; J_n'^2(x) \; x \; dx \qquad \text{(III - 5)}$$

Let us consider a power-law spectrum:

$$F_{\alpha_{ms}}(k) = F_{0\,\alpha_{ms}} \left(\frac{k_0}{k}\right)^a \qquad \text{(III - 6)}$$

Then:

$$\mathcal{E}_{n_\alpha}(nk_z, |\mu|) = 2\rho \, F_{0\alpha} \left(\frac{k_0}{k_z}\right)^a n^2 t^a \int_0^\infty \frac{J_n^2(x)}{x\left(n^2 t^2 + x^2\right)^{1+a/2}} \, dx \qquad \text{(III - 7)}$$

$$\mathcal{E}_{n_{ms}}(nk_z, |\mu|) = 2\rho \, F_{0_{ms}} \left(\frac{k_0}{k_z}\right)^a n^2 t^{a+2} \int_0^\infty \frac{J_n'^2(x) \; x}{\left(n^2 t^2 + x^2\right)^{2+a/2}} \, dx \qquad \text{(III - 8)}$$

For $a > -2$, it is easy to find an upper limit to the integrals in (III - 7) and (III - 8):

$$\int_0^\infty \frac{J_n^2(x)}{x\left(n^2 t^2 + x^2\right)^{1+a/2}} \, dx < \int_0^\infty \frac{J_n^2(x)}{x^{3+a}} \, dx$$

$$\int_0^\infty \frac{J_n'^2(x) \; x}{\left(n^2 t^2 + x^2\right)^{2+a}} \, dx < \int_0^\infty \frac{J_n'^2(x)}{x^{3+a}} \, dx \qquad \text{(III - 9)}$$

The integrals on the right hand side can be evaluated exactly. When n is large , the first is equal to:

$$\left(\frac{1}{2}\right)^{3+a} \frac{\Gamma(3+a)}{[\Gamma(2+a/2)]^2} \quad n^{-3-a}$$

while the second is equal to:

$$\left(\frac{1}{2}\right)^{4+a} \frac{\Gamma(3+a)}{(3+a/2)[\Gamma(2+a/2)]^2} \quad n^{-3-a}$$

Then, we can write:

$$\mathcal{E}_n(nk_z, |\mu|) \lesssim C(|\mu|, k_z) \quad \frac{1}{n^{1+a}} \qquad \text{(III - 10)}$$

where C is a function of $|\mu|$ and k_z only. The contribution of high order harmonics to $\mathcal{E}(k_z, |\mu|)$ is less than:

$$C(k_z, |\mu|) \quad \sum_{n} \frac{1}{n^{1+a}}$$

For a > 0, the sum is convergent. Thus, when the waves are isotropic, and their spectrum is a decreasing power law, the effect of high order harmonic interactions is not important. This result can be readily extended to any spectrum that is a decreasing function of k. Consequently, in the rest of this thesis, we will only take into account interactions through the first harmonic.

3) Calculation of λ.

We now consider the variations of $\xi_{1\,\alpha}$ and $\xi_{1\,ms}$ with μ. If the waves are isotropic, and the spectrum is a power law:

$$\xi_{1\alpha}\left(1/|\mu| r_\ell\,,\,|\mu|\right) = 2\,\rho\,F_{0\alpha}\,(k_0 r_\ell)^a\,|\mu|^a \int_0^\infty \frac{1}{\left(1+x^2/t^2\right)^{1+a/2}} \frac{J_1^2(x)}{x}\,dx \qquad \text{(III - 11)}$$

$$\xi_{1ms}\left(1/|\mu| r_\ell\,,\,|\mu|\right) = 2\,\rho\,F_{0ms}\,(k_0 r_\ell)^a\,\frac{|\mu|^a}{t^2} \int_0^\infty \frac{x\,J_1'(x)}{\left(1+x^2/t^2\right)^{2+a/2}}\,dx \qquad \text{(III - 12)}$$

When $|\mu|$ approaches zero, both $\xi_{1\,\alpha}$ and $\xi_{1\,ms}$ tend to zero; $\xi_{1\,\alpha}$ like $|\mu|^{a+2}$ and $\xi_{1\,ms}$ like $|\mu|^{a+1}$. Thus, at large pitch angles, magnetosonic waves dominate the resonant interactions, and the collision frequency in (III - 2) is proportional to $|\mu|^a$. For $a > 1$, the expression (III - 2) for D and λ exhibits a singularity at $|\mu| = 0$.

The reason for this behaviour is apparent: eq. (III - 1) would imply that, in order to reverse their direction, cosmic rays have to interact with waves of shorter and shorter wavelength, and if $a > 1$, there are very few of these. However, it should be kept in mind that the interactions of a particle with individual waves result in finite -- rather than infinitesimal -- changes in pitch angle; for a sufficiently small value of μ, one interaction is enough for the particle to turn around. Thus, the lower limit of the integral in (III - 1) should actually be finite. To determine at which

value of μ the integral should be cut off, it is necessary to have a more detailed knowledge of the waves than is contained in the wave spectrum.

Another effect, pointed out by Noerdlinger (1968) and by Kulsrud and Pearce (1969), will probably be more significant in removing the singularity in D at $\mu = 0$: particles of large pitch angle are easily mirror-reflected by large scale inhomogeneities in the magnetic field. Thus, at any given time, we may distinguish two types of particles: "resonant" particles, with $|\mu| > |\mu_{\text{cut off}}|$, which are mainly undergoing resonant scatterings, and "mirroring" particles, with $|\mu| < |\mu_{\text{cut off}}|$, which are trapped between mirror points, with an equal probability of emerging at both ends. The equation of diffusion in space, for the "resonant" particles, is similar to equation (31) of Kulsrud and Pearce (1969), with an extra term to account for exchanges with mirroring particles, and with:

$$D = \frac{1}{3} v \lambda = v^2 \int_{|\mu_{cut\,off}|}^{1} \frac{1-\mu^2}{2\nu(|\mu|,\epsilon)} d\mu \qquad \text{(III - 13)}$$

Noerdlinger (1969) proposes to take: $|\mu_{\text{cut off}}| = \left(\sqrt{\frac{B_1}{B_0}}_z\right)_{\max}$, where B_{1_z} is the amplitude of magnitude fluctuations of B. For a decreasing spectrum, this would imply that particles mirroring is due to the longest waves -- of wavelength that may be comparable to λ. In fact, it appears that, in

most cases, resonant interactions prevent cosmic rays from being trapped by such waves. To evaluate $|\mu_{\text{cut off}}|$, we will instead compare the rates of change of $|\mu|$ due to resonant interactions and to mirroring; $|\mu|$ is equal to $|\mu_{\text{cut off}}|$ when both rates are equal.

a) Resonant scattering.

The rate of change of pitch angle, due to resonant scattering, is approximately given by (see equation (III-2); also Jokipii 1966).

$$\frac{1}{\mu^2}\left\langle \frac{\Delta\mu^2}{t} \right\rangle = \frac{\pi}{2}\,\Omega\,\frac{1}{\mu^2}\,\frac{8\pi k_z\,\mathcal{E}(k_z, |\mu|)}{B_0^2}\Bigg|_{k_z = 1/|\mu| r_\ell} \qquad \text{(III - 14)}$$

b) Mirroring.

The force exerted by a wave of wavenumber $\vec{k}$ of components k_z, $k_\perp$ on a particle with $r_\ell << \frac{1}{k}$ is:

$$\dot{p}_z \simeq -\frac{v_\perp p_\perp}{2 B_0} B_{1_z} k_z \cos\phi \qquad \text{(III - 15)}$$

where ϕ is the phase of the wave. In this estimate, we take $\cos\phi = \frac{1}{2}$. Then:

$$\frac{1}{|\mu|}\,\frac{d|\mu|}{dt} \simeq -\frac{c}{4}\,\frac{B_{1_z}}{B_0}\,k_z\,\frac{1-\mu^2}{|\mu|} \qquad \text{(III - 16)}$$

For α waves, $B_{1_z} = 0$; for ms waves, propagating at an angle ξ with B_0:

$$B_{1_z} = |B_1| \sin\xi$$

If the spectrum of ms waves is given by (III - 6), with $a > 1$:

$$\frac{B_{1z}^2(k_z)}{8\pi} \simeq \frac{1}{2a(1+a/2)} F_{o_{ms}} \rho \left(\frac{k_o}{k_z}\right)^a k_z \qquad \text{(III - 17)}$$

so that $\frac{d\mu}{dt} \propto k_z^{\frac{3-a}{2}}$. For $a < 3$, shorter waves mirror reflect particles faster than longer waves. However, the condition $(1/k_z) > r_\ell$ must be satisfied.

A particle may be reflected by a wave of amplitude B_{1_z} only if $\mu^2 \leq \frac{B_{1_z}}{B_0}$. Consequently, for a given μ, the waves that are most effective in reversing the direction of a cosmic ray are those for which $\mu^2 = B_{1z}(k_z)/B_0$, or:

$$\frac{1}{|k_z|} = \left(\frac{V_A^2}{F_{o_{ms}} k_o}\left(1+\frac{a}{2}\right) a\right)^{1/(a-1)} \frac{1}{k_o} |\mu|^{4/(a-1)} \qquad \text{(III - 18)}$$

Let us define: $W \equiv F_{o_{ms}} k_o / V_A^2$; W measures the ratio of the energy in ms waves to the energy in the field. The largest $|\mu|$ for which mirror reflection can occur is:

$$\mu_{max}^2 = \frac{B_{1z}(k_o)}{B_o} = \left(\frac{W}{a(a/2+1)}\right)^{1/4} \qquad \text{(III - 19)}$$

Formula (III - 17) is of course not valid when $(1/k_z)$ becomes shorter than r_ℓ, or when $|\mu|$ is less than $|\mu_{min}|$, where:

$$|\mu|_{min} = |\mu|_{max} (r_\ell k_o)^{(a-1)/4} \qquad \text{(III - 20)}$$

For $|\mu| < |\mu_{min}|$, it is predominately waves with $\frac{1}{|k_z|} \sim r_\ell$ that are responsible for the mirroring, and:

$$\frac{1}{|\mu|}\frac{d|\mu|}{dt} = -\frac{v k_o}{4}\sqrt{\frac{w}{a(a/2+1)}}\;\frac{(1-\mu^2)}{|\mu|}\;(k_o r_\ell)^{(a-3)/2} \qquad \text{(III - 21a)}$$

while, if $|\mu_{min}| < |\mu| < |\mu_{max}|$:

$$\frac{1}{|\mu|}\frac{d|\mu|}{dt} = -\frac{v k_o}{4}\left(\frac{w}{a(a/2+1)}\right)^{1/(a-1)}\;\frac{(1-\mu^2)}{|\mu|^{(5-a)/(a-1)}} \qquad \text{(III - 21b)}$$

To estimate the mean free path of cosmic rays as a function of their rigidity ε, or of their Larmor radius r_ℓ, we compare the rate in equation (III - 14) to that in equation (III - 21); the value of $|\mu|$ for which they become equal is $|\mu_{cut\ off}|$. Then, λ is given by equation (III - 13). In the next paragraph, we illustrate the preceding derivation with an example.

4) Example.

Let us consider a Kolmogorov spectrum of waves ($a = 5/3$). We assume that the energy is equally divided between Alfvèn and magnetosonic waves, and that $w_{ms} = w_\alpha = 1/10$. We write $\mathcal{E}(k_z, |\mu|)$ as:

$$\mathcal{E}_{5/3}(k_z, |\mu|) = \frac{2\rho^2}{1-\mu^2}\;\rho\;(k_o r_\ell)^{5/3}\left[F_{o_\alpha} I'_{5/3_\alpha}(|\mu|) + F_{o_{ms}} I'_{5/3_{ms}}(|\mu|)\right]$$

(III - 22)

where:

$$I'_{a_\alpha}(|\mu|) = \int_0^\infty \frac{|\mu|^a}{(1 + x^2/t^2)^{a/2+1}} \frac{J_1^2(x)}{x} dx$$

$$I'_{a_{ms}}(|\mu|) = \int_0^\infty \frac{|\mu|^a}{(1 + x^2/t^2)^{a/2+2}} \times J_1'^2(x)\, dx$$

(III - 23)

(the inclusion of the second harmonic would have increased I' in about 15% at small μ, and less at large μ). The variation of $I'_{5/3\ \alpha}$, $I'_{5/3\ ms}$ and their sum with $|\mu|$ are shown in Figure (III-1). The corresponding resonant rate of change of $|\mu|$, from equation (III - 14), is represented in Figure (III-2), for different values of $(k_0\ r_\ell)$. In the same figure, we present the mirror rate of change of $|\mu|$, equation (III-21). The intersections of both sets of curves give $\mu_{\text{cut off}}\ (r_\ell)$.

The variations of:

$$\bar{\lambda}(|\mu|) \equiv \int_{|\mu|}^{1} \frac{(1-\mu'^2)}{|\mu|\ |\mu'|\ I'_1(|\mu'|)} d|\mu'|$$

(III - 24)

with $|\mu|$ are shown in Figure (III-3).

The mean free path is given by:

$$\lambda(r_\ell) = \frac{3}{2\pi} \frac{1}{\omega} (k_0 r_\ell)^{1/3} \bar{\lambda}(\mu_{\text{cut off}}(r_\ell)) \frac{1}{k_0}$$

(III - 25)

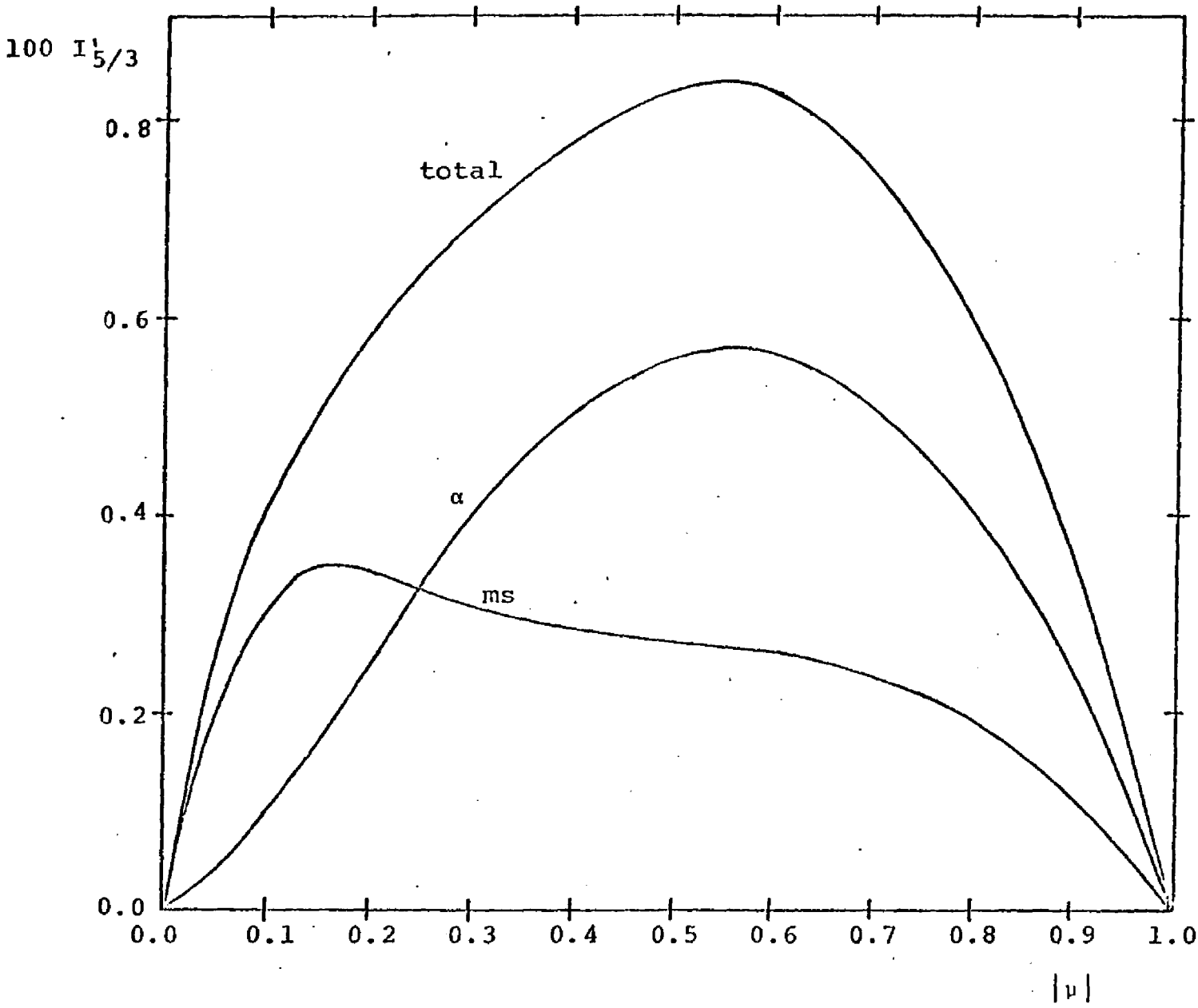


Figure (III-1)

$I'_{5/3_\alpha}$, $I'_{5/3_{ms}}$ and $(I'_{5/3_\alpha} + I'_{5/3_{ms}})$ as a function of $|\mu|$

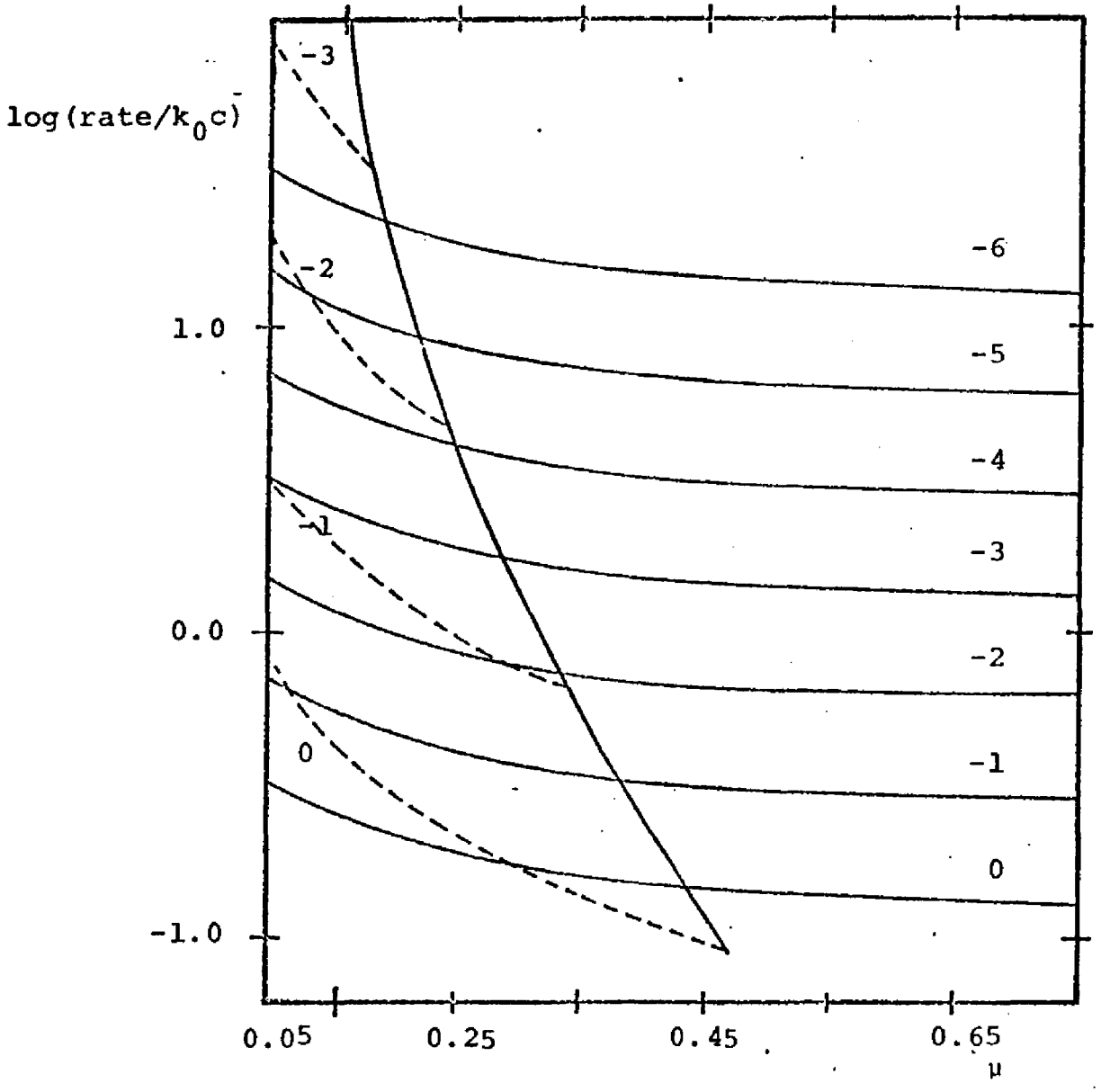


Figure (III-2)

Logarithm of the resonant and mirror rate of change of μ, in units of $(k_0 c)$, as a function of μ , for different values of $\log (k_0 r_\ell)$. The set of horizontal curves represent the resonant rate and are labelled according to the value of $\log(k_0 r_\ell)$. The diagonal curve represents the mirror rate (Eq. III-21 b) ; the dashed curves represent the mirror rate (eq. III-21 a) for particles with the indicated value of $\log(k_0 r_\ell)$.

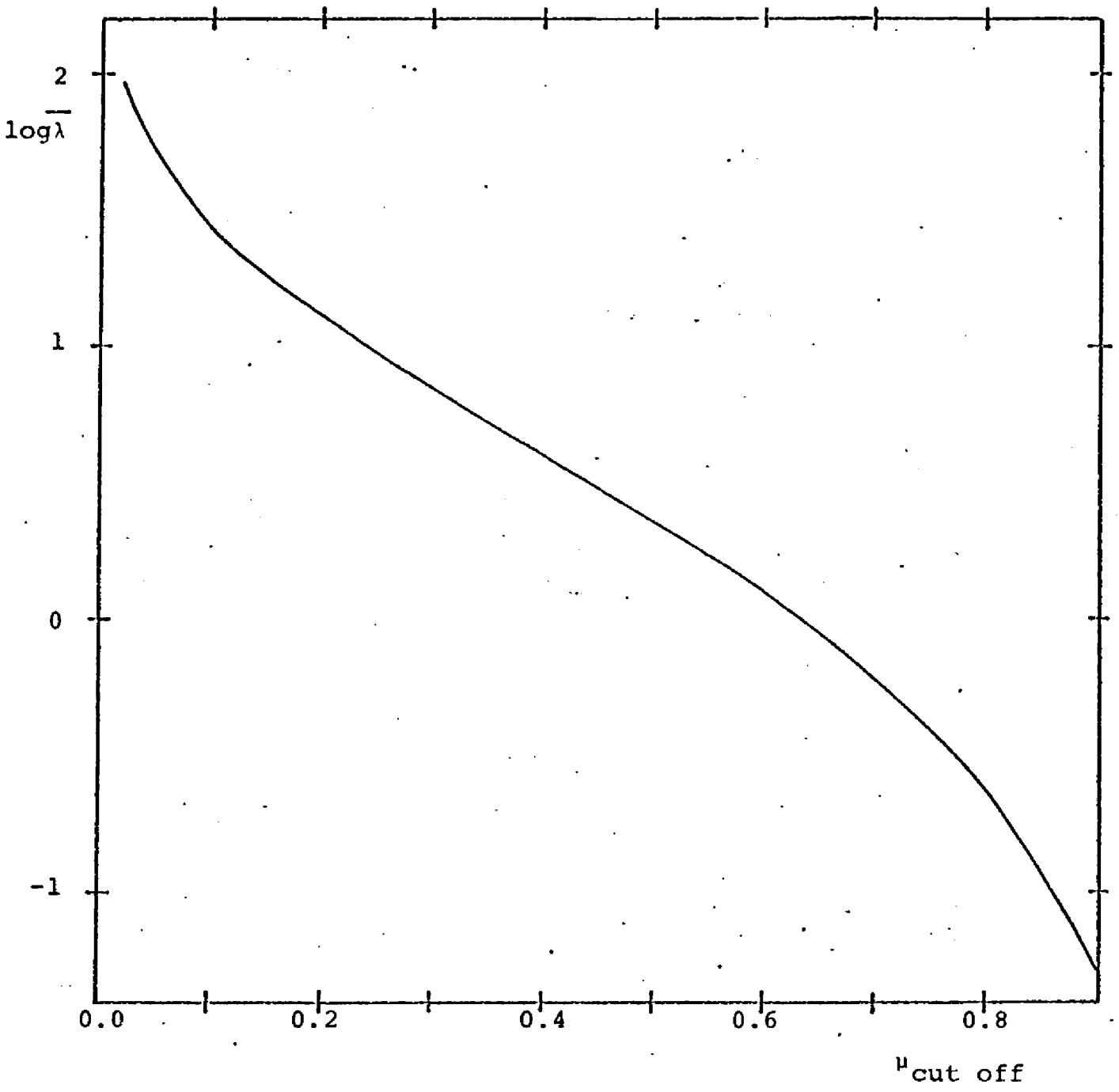


Figure (III-3)

Variations of $\overline{\lambda}$ as a function of $\mu_{\text{cut off}}$.

Log (λk_0) as a function of $\log(k_0\ r_\ell)$ is represented in Figure (III-4). (When $k_0\ r_\ell$) approaches 1, a number of the approximations used are not valid, and the result is not really meaningful.)

For $(k_0\ r_\ell) < 0.5$, the mean free path obtained is approximately proportional to $r_\ell^{0.24}$, i.e.: the dependence of λ on r_ℓ is weaker than if $\mu_{\text{cut off}}$ were independent of the energy. At $(k_0\ r_\ell) \simeq 1/3$, the spectrum of λ has a pronounced break, due to the passage, for $\left(\frac{1}{\mu}\frac{d\mu}{dt}\right)_{\text{mirror}}$, from expression (III - 21a) to expression (III - 21b). When w is smaller, the break is displaced towards a shorter r_ℓ.

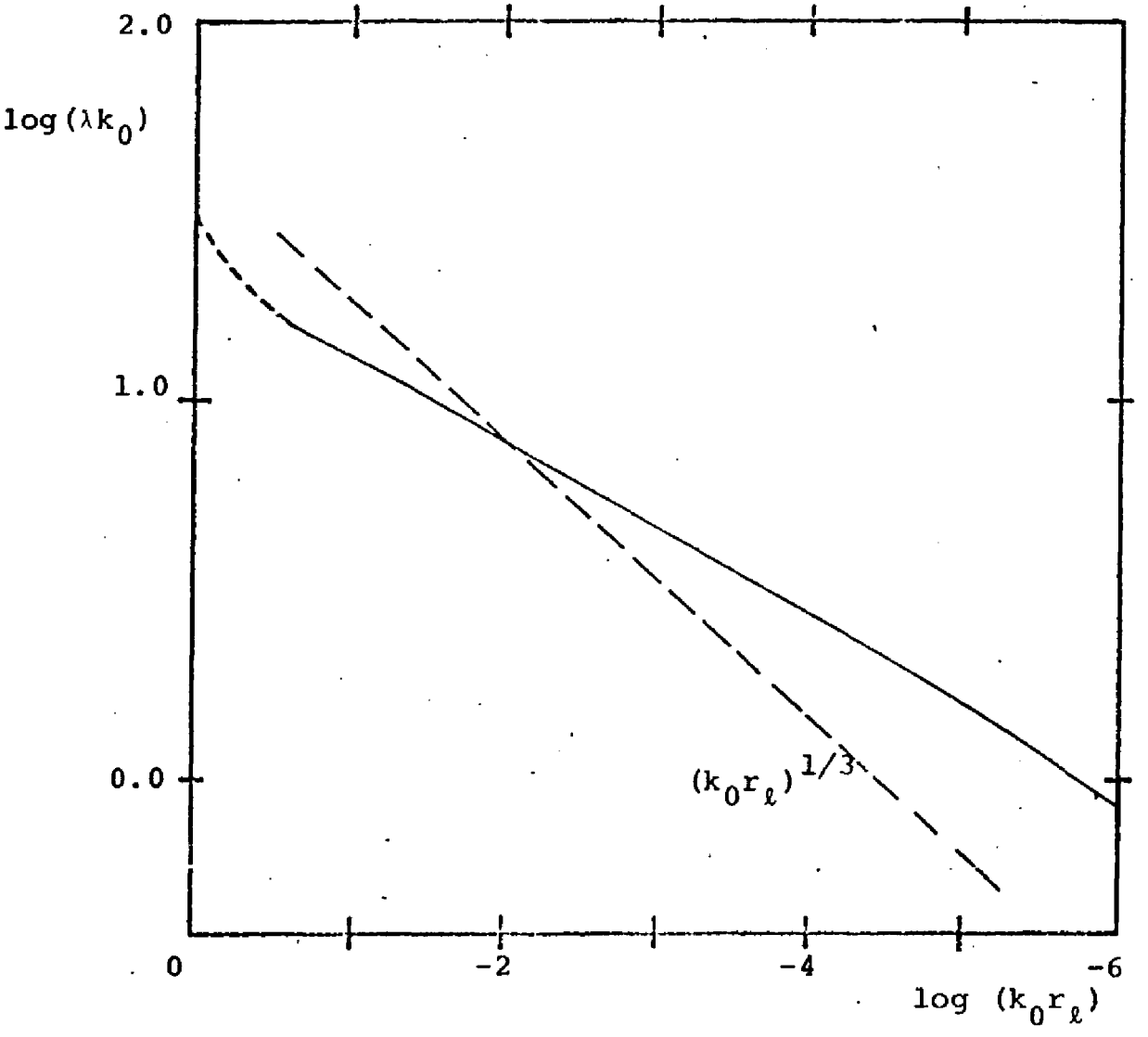


Figure (III-4)

Mean free path of cosmic rays in the presence of a Kolmogorov spectrum of hydromagnetic waves. Log $(k_0\lambda)$ is represented as a function of $\log(k_0 r_\ell)$.

APPENDIX III-1

$\mathcal{E}(k_z, |\mu|)$ in a hot medium.

The development of Chapter III is valid when the ratio α of the sound velocity v_s to the Alfvén velocity v_A is small:

$$\alpha \equiv \frac{v_s}{v_A} \ll 1 \qquad \text{(AIII-1)}$$

In a hot medium, this inequality may not be satisfied; then, instead of 2 "hydromagnetic" modes, 3 "magneto hydrodynamic" (MHD) modes are present: one Alfvén wave, and two magneto-acoustic waves, denominated "fast" and "slow" wave.

The properties of the Alfvén wave are similar in the h.m. and the MHD case, and thus $\mathcal{E}_{n_\alpha}$, in equation (III-3) is unchanged. The magnetoacoustic waves, on the other hand, differ from the magnetosonic waves on two accounts: their phase velocity is different from v_A, and a sizable fraction of their energy is in the form of compression energy. As a consequence, in a hot medium, the part of equation (III-3) concerning the ms waves has to be modified; this appendix is devoted to deriving the necessary corrections.

By means of a derivation parallel to that in equations (3) to (15) of KP, one can show that the rate of scattering of cosmic rays by magnetoacoustic waves is proportional to:

$$\xi_{n_{ma}}(k_z, |\mu|) = 4\int d^2k_\perp \; W_{ma}(\vec{k}_\perp, nk_z) \; e(\xi) \frac{2c^2}{v_\phi^2(\xi)} \; J_n'^2(x) \cos^2\xi \tag{AIII-2}$$

where v_ϕ is the phase velocity, and e is the ratio of electric to total energy, in waves propagating at an angle ξ with the magnetic field. In the hydromagnetic limit (magnetosonic waves), $v_\phi = v_A$, and $e = \frac{v_A^2}{2c^2}$, while for magnetoacoustic waves, both v_ϕ and e are functions of ξ.

The phase velocity of magnetoacoustic waves is given by:

$$v_{\substack{\phi \\ f,s}}^2 = \frac{1}{2} v_A^2 \left\{ (1+\alpha^2) \pm \sqrt{(1+\alpha^2)^2 - 4\alpha^2 \cos^2\xi} \right\} \tag{AIII-3}$$

where the upper sign corresponds to the f (fast) wave and the lower to the s (slow) wave.

To evaluate e (ξ), let us consider a magnetoacoustic wave, whose direction of propagation makes an angle ξ with the magnetic field. Let ρ_1, $\vec{v}_1$, $\vec{E}_1$ and $\vec{B}_1$ be the amplitude of the perturbations in the density, velocity, electric field, and magnetic field, associated with the wave. The kinetic, compression, magnetic and electric energy densities of the wave are respectively equal to:

$$w_K = \frac{\rho_0 v_1^2}{2}$$

$$w_C = \frac{\rho_1^2 v_s^2}{2\rho_0}$$

$$W_M = \frac{B_1^2}{8\pi}$$

$$W_E = \frac{E_1^2}{8\pi}$$

(AIII-4)

These energy densities are related through:

$$W_E = \frac{v_\phi^2}{c^2} W_M \ll W_M$$

$$W_M + W_C + W_E \simeq W_M + W_C = W_K$$

(AIII-5)

Also, the equation of continuity yields:

$$\rho_1 = \rho_0 \frac{\vec{k}.\vec{v}}{\omega}$$

(AIII-6)

Thus:

$$W_C = W_K \frac{v_S^2}{v_\phi^2} \cos^2\psi$$

(AIII-7)

where ψ is the angle between $\vec{k}$ and $\vec{v}$. It can be shown, from geometrical arguments, that (Alfvèn and Fälthammar 1963):

$$tg\,\psi = \frac{\sin\xi\cos\xi}{\cos^2\xi - \frac{v_\phi^2}{v_A^2}}$$

(AIII-8)

Combining equations (AIII-5) and (AIII-7), we obtain:

$$e(\xi) = \frac{v_\phi^2}{2c^2} \quad (1 - f(\xi))$$

(AIII-9)

where

$$f(\xi) = \alpha^2 \frac{v_A^2}{v_\phi^2} \frac{1}{1 + tg^2 \psi}$$

(AIII-10)

Hence:

$$\mathcal{E}_{n_{ma}}(k_z, |\mu|) = 4 \int d^2\vec{k}_\perp \; W_{ma}(\vec{k}_\perp, nk_z) \; J_n'^2(x) \; \cos^2\xi \; (1 - f(\xi))$$

(AIII-11)

Let us consider the case where for a given direction of propagation, equal amounts of fast and slow waves are present:

$$W_{ma_f}(\vec{k}_\perp, k_z) = W_{ma_s}(\vec{k}_\perp, k_z) \equiv W_{ma}(\vec{k}_\perp, k_z)$$

Then:

$$\mathcal{E}_{n_{ma}}(k_z, |\mu|) = \mathcal{E}_{n_{ma_f}}(k_z, |\mu|) + \mathcal{E}_{n_{ma_s}}(k_z, |\mu|) =$$

$$= 4 \int d^2\vec{k}_\perp \; W_{ma}(\vec{k}_\perp, nk_z) \; J_n'^2(x) \; \cos^2\xi \; (2 - f_f(\xi) - f_s(\xi))$$

(AIII-12)

It can be shown, from equations (AIII-3), (AIII-8) and (AIII-10), that:

$$f_s(\xi) + f_f(\xi) = 1 \qquad \text{(AIII-13)}$$

Thus, the effect on cosmic rays of a distribution of magneto-acoustic waves such that $W_{ma}(k_\perp, k_z) = 2W_f(k_\perp, k_z) = 2W_s(k_\perp, k_z)$ is identical to that of a distribution of magnetosonic waves with energy density per unit k:

$$W_{ms}(k_\perp, k_z) = \frac{W_{ma}(k_\perp, k_z)}{2}$$

REFERENCES

Alfvèn, H. and Fälthammar, C. G., 1963, Cosmical Electrodynamics (Clarendon Press, Oxford).

Jokipii, J. R., 1966, Astrophys. J. 146, 480.

Kulsrud, R. M. and Pearce, W. P., 1969, Astrophys. J., 156, 445.

Noerdlinger, P. D., 1968, Phys. Rev. Letters, 20, 1513.

CHAPTER IV

Wave spectrum and cosmic ray confinement in the interstellar medium.

1) Introduction

In this chapter, we make use of the methods described in Chapters II and III to investigate the propagation of cosmic rays in the interstellar medium, and the question of their confinement to the galactic disk.

While a wealth of data on the dense regions of the galactic disk, the "interstellar clouds," is available, the medium filling the space in between clouds -- the "intercloud medium" -- proves itself to be an elusive prey to the observer. It is fair to say that the physical parameters describing the intercloud medium have not been, up to now, unequivocally determined. For this reason, our approach is as follows:

We first take a set of currently accepted values for the intercloud medium (neutral hydrogen density $n_0 = 0.2$, proton density $n^* = 0.02$, temperature $T = 1000^\circ K$). We show that, when the effect on the medium of the dissipation of hydromagnetic waves by charged-neutral collisions is considered, it becomes clear that it is impossible to develop a spectrum of waves in such a region (Section 2).

The importance of the charged-neutral collision damping is considerably diminished if the medium is completely ionized; and, in fact, the waves themselves may supply enough

power to maintain the medium ionized -- and hot ($T \simeq 40000^{\circ}$). In this case, a different problem arises: at high temperatures the magnetoacoustic waves are rapidly eliminated by thermal conduction. If, in the dissipative range of the magnetoacoustic waves, the interaction between the different kinds of waves is weak, a spectrum of Alfvèn waves could develop beyond that point. The resulting spectrum is obtained in Section 3.

When the behaviour of the cosmic rays in the presence of such a spectrum of waves is examined, it is found that the dependence on the rigidity ε of the mean free path of cosmic rays with 10 Bv $\lesssim \varepsilon \lesssim 10^6$ Bv, is very weak (Section 4).

In section 5, we discuss the possible location in the galaxy of a hot, diffuse gas where the high energy cosmic rays could be trapped. We show that the existence of two slabs of hot gas, containing magnetic fields and cosmic rays, at each side of the neutral gas disk, is compatible with the available observational data. High velocity clouds are a possible source of turbulence in these slabs. The over all properties of cosmic ray propagation in this composite galactic disk are studied, and shown to be in agreement with the observational results surveyed in Chapter II, section 1.

Finally, a summary of the principal results in Chapter I to IV is given. (Section 6).

Two appendices are attached to this chapter. In the first, we discuss the propagation of waves in a partially ionized medium. In the second, the effect of the cosmic rays on the wave spectrum is considered.

2) Spectrum of waves in a neutral medium.

In this section, we investigate the development of a wave spectrum in the galactic gas disk. This region comprises the galactic plane, and has a thickness of 200 to 300 pc. Because of the inhibiting effect of charged-neutral collision damping, of rate proportional to the neutral density, the most promising regions for developing a spectrum of waves are the most diffuse ones. Thus, we limit our discussion to the intercloud medium, for which we assume a density of neutral hydrogen $n_0 = 0.2\ cm^{-3}$, a density of protons $n^* = 0.02$ and a temperature $T = 1000^{\circ}K$. We will use simple order of magnitude arguments to show that the power per unit volume P necessary to develop a spectrum of waves that extends to the shortest wavelengths of interest is unacceptably high.

It can be verified that, for the above conditions, the damping rate of hydromagnetic waves with $k < 10^{-13} cm$ by charged-neutral collisions dominates over the other damping mechanisms that are operative, namely: viscous, Joule and thermal damping. (The corresponding damping rates have been derived by Braginskii (1965), and are quoted by Kulsrud and Pearce (1969)).

The propagation of waves in a partially ionized medium with a uniform magnetic field is discussed in Appendix IV-1, where we show that, depending on k, two domains have to be distinguished:

a) if $k << k_c = \frac{\nu_0(1+y)}{v_A}$

(where ν_0 is the frequency of collisions between protons and hydrogen atoms, defined in Chapter I; y is the ratio of charged to neutral hydrogen density: $y = n^*/n_0$, and v_A is the Alfvèn velocity in the charged component only.) The hydromagnetic waves carry the whole medium in their vibrations, and are damped at a rate $\Gamma^* = \omega_k^2/2\nu_0(1+y)^2$. A third mode, purely damped, involves mostly the charged particles; it is supressed by friction, and contains a negligible amount of energy.

b) $k >> k_c$: only the charged particles participate in the motions of the hydromagnetic waves; the damping rate is $\Gamma^* = \nu_0/2$. In addition to the hydromagnetic waves, a third mode, neutral and purely damped, is present; a small amount of magnetic energy is associated to it.

Let E be the power input per unit mass, at $k \lesssim k_0$. The power input per unit volume is $P \equiv (n_0 + n^*)m_H E$, where m_H is the mass of a proton. For $k_0 < k < k_c$, the damping rate is of the viscous type: $\Gamma^* = \beta k^2$, with $\beta = \frac{v_A^2}{2\nu_0(1+y)}$. From the results of Chapter II, we know that the spectrum is

proportional to $k^{-5/3}$ for $k < k_s$, and decays at larger k. Thus, the condition for the spectrum not to decay before it attains k_c is:

$$k_c < k_s = \left(\frac{3\kappa^2 E}{8\beta^3}\right)^{1/4} = \left(\frac{0.6\ \gamma_0^3\ (1+\gamma)^6\ E}{V_A^6}\right)^{1/4} \qquad \text{(IV-1)}$$

or

$$E > E_m \equiv \frac{\gamma_0\ V_A^2}{0.6\ (1+\gamma)^2} \qquad \text{(IV-2)}$$

Once the spectrum reaches k_c, the motions of the charged and neutral components decouple. The power input is divided between the two components; since, at larger wavelengths, the energy density per unit of charged or neutral mass is the same, it is likely that, at $k \approx k_c$, each component will take a share of the power input proportional to its mass. Thus, the charged particles receive a power per unit volume $P' \lesssim Py$, or per unit mass $E' \lesssim E$. The energy of the neutral particles goes to the neutral mode, that of the charged particles into hydromagnetic waves. The spectrum of hydromagnetic waves will drop sharply at $k \gtrsim k_c$, unless: (see Chapter II, section 5)

$$E' > \frac{\gamma_0^3}{k_c^2} = \frac{\gamma_0\ V_A^2}{(1+\gamma)^2} \qquad \text{(IV-3)}$$

(IV-2) is a necessary condition for (IV-3) to be fulfilled. If $E >> E_m$, a negligible fraction of the power input is dissipated at k between k_0 and k_c, and $E' \simeq E$, so that (IV-3) is automatically satisfied. But if E is just slightly above E_m, enough energy could be dissipated at $k \simeq k_c$ that (IV-3) is violated. Hence, we keep as necessary and sufficient condition for the spectrum to develop: $E > 10\ E_m$.

For an intercloud medium described by the parameters we considered: $y = 0.1$, $v_A = 4.7 \times 10^6$cm, $\nu_0 = 1.3 \times 10^{-9}\text{sec}^{-1}$, and $E_m = 4.8 \times 10^4$ erg/g sec. Since the energy fed to the waves is ultimately dissipated as heat, let us compare E to the rate of cooling E_c of the medium. E_c is proportional to the density, $n = n_0 + n^*$, and is generally written:

$$E_c = \frac{n \Lambda}{m_H} \qquad \text{(IV-4)}$$

where Λ depends on the composition, the temperature and the degree of ionization. Over a large range of temperatures, the cooling agents are predominantly trace elements; thus to evaluate the cooling rate, the pure hydrogen plasma approximation is no longer valid. We assume that other elements are present, with concentrations given by their cosmic abundances (Alle 1961). The cooling rate of such a gas, at $T < 10^4\ ^\circ$K, has been calculated by Jura (1971), for several

values of the degree of ionization x (when $y \ll 1$, $x \approx y$). The variations of Λ with T are displayed in Figure (IV-1). At 1000°K for $x = 0.1$, $\Lambda = 3.7 \times 10^{-25}$ erg cm^3/sec; consequently, $E_c = 0.04$ erg/g sec.

About 200 times as much power would be required to bring the gas to a temperature $T \gtrsim 20000^{\circ}$K, where the hydrogen would be completely ionized. Thus, the power needed to maintain a spectrum of hydromagnetic waves in a mostly neutral, diffuse medium is six orders of magnitude above that needed to keep the medium at 1000°K and about four orders of magnitude above that required to collisionally ionize it. Clearly then, it is only in regions where the gas is highly ionized that it may be possible to develop a spectrum of waves that does not decay too early.

This conclusion, which was reached by assuming that the rate of energy transfer between waves is correctly given by the Heisenberg equation, is evidently also valid if the actual rate is slower -- and we remarked in Chapter II, section 3, that it could hardly be faster. Consequently, provided that energy is fed directly only to waves with $k \ll k_c$, it seems impossible that a wave spectrum could develop -- and high energy cosmic rays be trapped -- in the neutral portions of the interstellar medium.

3) Spectrum of waves in a highly ionized medium.

Having ruled out neutral regions as possible trapping regions for the cosmic rays, we now turn our attention to ion-

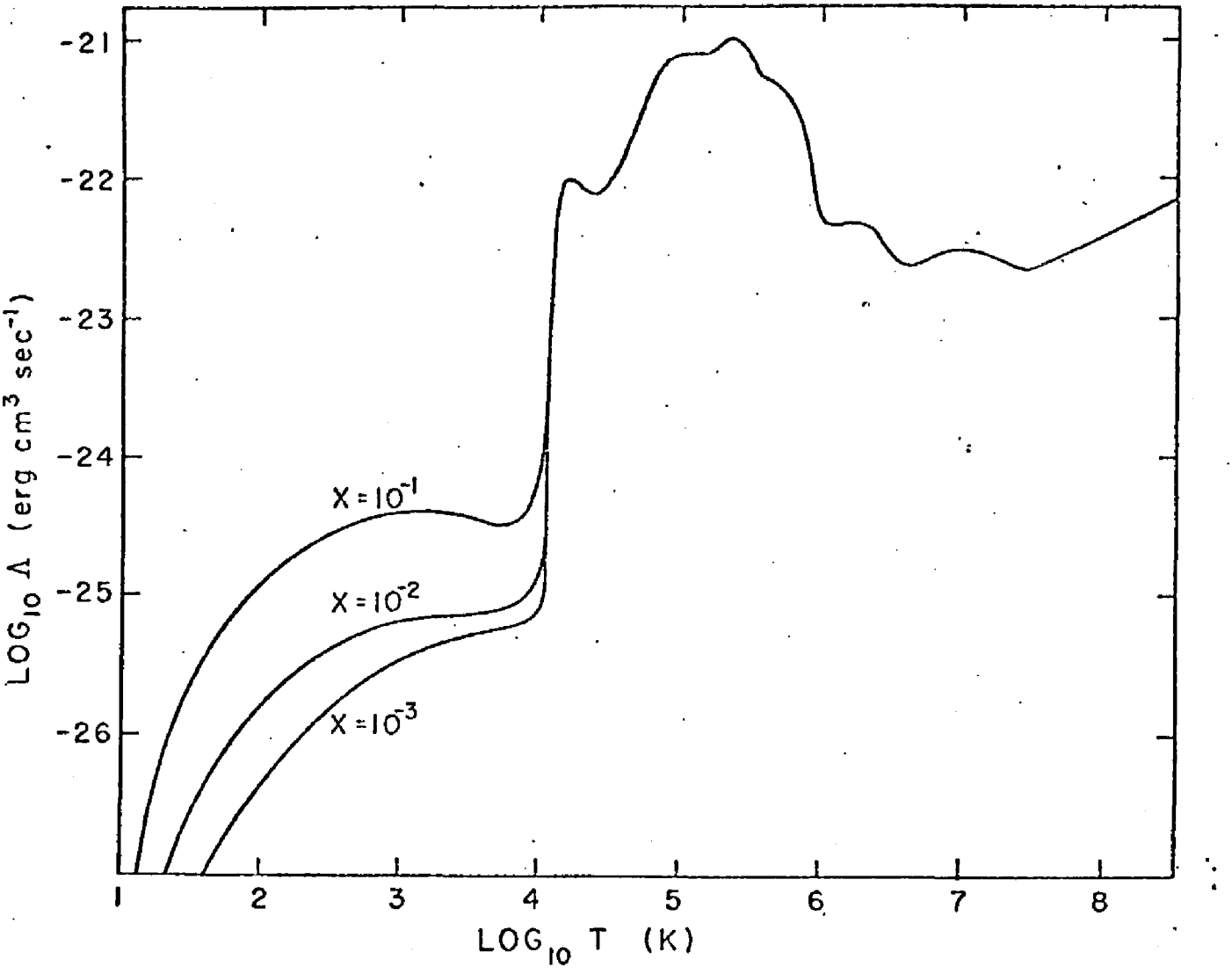


Figure (IV-1)

Λ as a function of T, for a low density plasma of cosmic abundance. x is the ionized fraction n_e/n. This figure is from McCray and Schwarz (1971).

ized regions.

The condition for the waves to overcome charged neutral collisions damping, so that the spectrum can develop beyond k_c, is:

$$E > 10\, E_m = \frac{16\, \gamma_0\, V_A^2}{(1+y)^2} \qquad \text{(IV-5)}$$

In a mostly neutral medium, of total density n ($n \simeq n_H \equiv n_0 + n^*$) this inequality can be written:

$$E > \beta\, \langle \sigma v \rangle\, V_A^2\, n \qquad \text{(IV-6)}$$

where $\langle \sigma v \rangle$, the collision rate between protons and hydrogen atoms, is a slowly increasing function of T (see Chapter I); in a highly ionized medium, where $y \gg 1$ and $n \simeq n^*$, the condition (IV-5) becomes:

$$E > \frac{\beta\, \langle \sigma v \rangle\, V_A^2\, n}{y^3} \qquad \text{(IV-7)}$$

Thus, not surprisingly, we find that, if friction between the charged and neutral particles in the medium is the main source of wave dissipation, the condition to develop a spectrum is considerably relaxed when $y \gg 1$.

Ionized regions in the galaxy are generally thought to be associated with sources of radiation of wavelength shorter than the Lyman limit. For instance, young stars are sur-

rounded by H II regions, with temperatures around 10^4 °K (Spitzer, 1968). Supernovae explosions are accompanied by an ultraviolet flash (Morrison and Sartori 1969); as a consequence, the gas in the vicinity is rapidly heated and ionized; because the cooling time is shorter than the recombination time, the gas is still completely ionized when its temperature has dropped to $\sim 10^4$ °K (Bottcher et al 1970).

But we have found that the dissipation of hydromagnetic waves provides an important heating mechanism, that may even be strong enough to collisionally ionize the medium. For this reason, we assume in the following that the power input E in waves, which is ultimately converted into heat, dominates over any other sources of ionization and heating that may be present. In this case, not only the spectrum of hydromagnetic waves, but also the physical conditions in the medium supporting them are determined by E.

In a steady state, collisional ionization is balanced by radiative and dielectric recombination. The rate of cooling of the gas is $E_c = n\ (T)/m_H$. The function $\Lambda\ (T)$, for a gas of cosmic abundances at $T > 10^4$ °K, has been calculated by Cox and Tucker (1969), and is represented in Figure (IV-1). In equilibrium, the cooling rate E_c is equal to the heating rate E, or:

$$\mathcal{H} \equiv E_c - E = 0 \qquad \text{(IV-8)}$$

If pressure equilibrium prevails, the gas is thermally unstable if (Field 1965):

$$\left(\frac{\partial \mathcal{H}}{\partial T}\right)_p = \left(\frac{\partial \mathcal{H}}{\partial T}\right)_\rho - \frac{\rho}{T}\left(\frac{\partial \mathcal{H}}{\partial \rho}\right)_T < 0 \qquad \text{(IV-9)}$$

When E is independent of P and T, this condition is equivalent to:

$$\frac{d(\log \Lambda)}{d(\log T)} < 1 \qquad \text{(IV-10)}$$

A glance at Figure (IV-1) tells us that the gas is unstable for $16500^\circ K < T < 30000\ ^\circ K$. This happens because, in this range of temperatures, hydrogen is an important cooling agent, and its cooling rate is a rapidly decreasing function of T. At $T > 25000^\circ K$, other cooling agents -- first carbon, then oxygen, helium and neon ions -- take over, and the slope of the cooling curve becomes positive.

When the main ionization mechanism is electron collisions, the hydrogen is mostly ionized when $T \gtrsim 18000^\circ K$. Thus, the gas can only be ionized and thermally stable for $T > 30000^\circ K$. To be specific let us take a given value for the temperature: $T = 40000^\circ K$. Then, $\Lambda = 1.5 \times 10^{-22}$ erg cm^3/sec, and the power input in the medium must be: $E = \frac{n\Lambda}{m_H} = 90.5n$ erg/g sec (IV-11).

With T fixed, there is only one independent parameter left in the problem; rather than taking a value for n or E, we assume a given level of turbulence at k_0. Thus, we can guarantee from the beginning that the energy density in waves remains below that in the uniform magnetic field B_0.

At a temperature of 40000°K, if $B_0 = 3\mu G$ and $n \gtrsim 0.01$, the cold plasma approximation is no longer valid. Hence, in addition to the Alfvèn mode, two kinds of magneto-acoustic waves are present, the fast and slow mode (see Chapter III, Appendix 1). We assume that each of the three modes receives an equal portion of the power input. Using the notation of Chapter III, we measure the energy present in any one mode with the parameter:

$$\sigma = \frac{F(k_0)\, k_0}{v_A^2}$$

If the transfer of energy between waves is much faster than the dissipation rate at $k \approx k_0$, for each mode, we have:

$$F(k_0) = 1.57 \left(\frac{E}{3}\right)^{2/3} k_0^{-5/3} \qquad \text{(IV-12)}$$

Combining the preceding equation with equation (IV-11) and the definition of σ, we obtain:

$$n = \left(\frac{6}{1.57}\, \sigma\, W_m\right)^{3/5} \left(\frac{k_0}{\Lambda \sqrt{m_H}}\right)^{2/5} \qquad \text{(IV-13)}$$

where $W_m = \frac{B_0^2}{8\pi}$ is the energy density of the magnetic field. Let us consider in more detail the case $\omega = \frac{1}{10}$. The total energy density in waves, at k_0, is:

$$k_0 [F_\alpha(k_0) + F_f(k_0) + F_s(k_0)] P = 3 k_0 F(k_0) P = 0.6 W_m \qquad \text{(IV-14)}$$

while the magnetic energy in waves is equal to a fraction (7/18) of this total energy. If $k_0 \simeq 10^{-18}$cm, corresponding to a wavelength of about two parsecs, Equation (IV-13) yields: $n = 0.037$ cm^{-3}; then, from equation (IV-12), $E = 3.3$ erg/g sec. The power input per unit volume is: $P = 2.03 \times 10^{-25}$erg/cm^3 sec.

Let us now examine the rate of damping of the waves. For the Alfvèn waves, the dominant damping mechanism is still charged-neutral collisions; its rate, even in a hot plasma, is as discussed in Appendix IV-1. At 40000°K, the ratio y of charged to neutral hydrogen is ≃ 1850 (Weyman 1967; but a more recent calculation by Bottcher 1970, yields a value of y = 3200; because, as it will become apparent, we are interested in finding an upper limit to ν_0, we will use Weyman's value), and the collision rate between protons and hydrogen atoms is: $\langle\sigma v\rangle = 1.47 \times 10^{-8}$ cm^3 sec^{-1}. (Using cross-section values taken from Peek 1966). Previously, because both the abundances and the cross-sections were lower, we neglected He collisions in the evaluation of ν_0; but in a gas of

cosmic abundance ($n_{He} = 0.16\ n_H$), at 40000°K, most of the neutral particles present are He atoms. Following Weyman again, $y_{He} = \frac{n(He+)}{n(He)} \simeq 40$, so that the density of He atoms is $n_0(He) = 7.4\ n_0(H)$. When the values of the cross-sections for the atoms collisions are examined (H+ - He in Dalgarno and Dickinson 1968, He+ - He in Dalgarno 1958), it turns out that charge-exchange interactions and collisions with the ions are not important, while the contribution of (H+ - He) elastic collisions to ν_0 is of the same order or less than that from (H+ - H) collisions. Taking into account the uncertainties in the degree of ionization and the cross-sections, we can safely write:

$$\nu_o \simeq \frac{n}{1850} \langle \sigma v \rangle_{H^+H} f \qquad \text{with } f < 5 \qquad \text{(IV-15)}$$

Then, condition (IV-5) is, in our case:

$$E \gtrsim 8 \times 10^{-5} \text{ erg/g sec},$$

and is amply satisfied: damping due to charge-neutral collisions will not force the spectrum to decay. Its effect on the spectrum is considered in more detail in the next section.

For the magnetoacoustic waves, the dominant dissipative mechanism is thermal damping. Its rate, when the sound velocity $v_s \ll v_A$, is given by Braginskii (1965). In our case, the ratio α of the sound velocity to the Alfvèn velocity is

equal to 0.97, so that Braginskii's result has to be slightly modified. In the notation of Chapter III, Appendix 1, we get:

$$\Gamma_{ther} = \frac{T(\gamma-1)^2}{2\rho}\left(\frac{\cos^2\psi}{v_\phi^2}\right)\left\{\kappa_{//}\, k_{//}^2 + \kappa_\perp\, k_\perp^2\right\} \qquad \text{(IV-16)}$$

where $\kappa_{//}$ and $\kappa_\perp$ are the thermal conduction coefficients along and across the magnetic field. $\kappa_\perp << \kappa_{//}$, and (Spitzer 1967):

$$\kappa_{//} = 4.5 \times 10^{-5}\frac{T^{5/2}}{\mathfrak{L}} \quad \text{erg} \quad \text{sec}\ ^\circ K\ \text{cm} \qquad \text{(IV-17)}$$

where $\mathfrak{L}$ is the Coulomb logarithm. In our case, $\mathfrak{L} = 27.2$.

Since we neglect the angle dependence of the damping rate (see Chapter II), we adopt an effective value for the damping rate of both the fast and the slow magnetoacoustic wave (because $v_s \approx v_A$, the damping rates are of the same order):

$$\Gamma_{ther} = \frac{(\gamma-1)^2\, T\, \kappa_{//}\, k^2}{2\,\rho\, v_A^2} \equiv \beta_T\, k^2 \qquad \text{(IV-18)}$$

where $\beta_T = 6.4 \times 10^{21}\ cm^2/sec$.

Consequently, the spectrum of the magnetoacoustic waves decays for

$$k > k_s = \left(\frac{3\,\kappa^2\,(E/3)}{8\,\beta_T^3}\right)^{1/4} = 2.4 \times 10^{-17}\ cm. \qquad \text{(IV-19)}$$

Thus, it seems impossible, under any reasonable conditions, to develop a spectrum of magnetoacoustic waves that extends to the shortest wavelengths of interest: dissipation, caused by charged-neutral collisions in a cold medium, and by thermal conduction in a hot medium, sets in after a small rangeof k·On the other hand, in a hot and ionized medium, the damping rate of the Alfvèn mode is very slow. The energy in magnetoacoustic waves is all converted into heat at wavenumbers $k \approx k_s$; the spectrum of the Alfvèn waves depends on how fast they interact with magnetoacoustic waves in this range of k. Assuming that the energy transfer is as fast as in the Heisenberg theory, two opposite situations can be distinguished.

a) the energy in one type of wave is transfered to shorter waves of all three kinds.

b) the energy in one type of wave is transfered preferentially to shorter waves with a similar polarization.

In case a), most of the energy in Alfvèn waves would be transmitted to magnetoacoustic waves, and thus dissipated, at $k \approx k_s$. In this case, a spectrum of waves could not develop -- and high energy cosmic rays could not be trapped in hot regions either.

In case b), the Alfvèn waves lose a small fraction of their energy to magnetoacoustic waves at $k \approx k_s$; at $k >> k_s$, the development of their spectrum proceeds undisturbed. Since

this is the only possibility left for confining high energy cosmic rays with hydromagnetic, or MHD waves, we will examine this case in detail in the next section.

4) Mean free path of cosmic rays in a hot medium.

In case b), the spectra of Alfvèn and magneto-acoustic waves are calculated separately. For the Alfvèn waves, the relevant damping mechanism is charged-neutral particles collisions; its rate, over the whole range of k, is closely approximated by:

$$\Gamma^* = \frac{\nu_0}{2\left(1 + \nu_0^2 \gamma^2 / \omega_k^2\right)} \qquad \text{(IV-20)}$$

The Heisenberg equation (II-17), with this form of Γ, was solved for f = 1 and 5 in ν_0 (equation IV-15), and the power dissipated at different wavelengths was calculated. The results showed that the percentage of the power input dissipated by Γ^* (at $k \simeq k_c$) is 3.3% if f = 1, and 5.4% if f = 5. Consequently, at $k >> k_c$, F(k) is smaller than its Kolmogorov value:

$$F(k) = 1.57 \left(E/3\right)^{2/3} k^{-5/3} \qquad \text{(IV-21)}$$

by 2% if f = 1, and by 3.6% if f = 5. Given the uncertainties involved in the problem, this corrections are of no importance; thus, we adopt for the spectrum of Alfvèn waves the

expression in equation (IV-21).

The spectrum of the magnetoacoustic waves is given by:

$$F_{f,s} = \frac{1.57\ (E/3)^{2/3}\ k^{-5/3}}{(1 + k^4/k_s^4)^{4/3}} \qquad \text{(IV-22)}$$

k_s is given in equation (IV-19). Both spectra are displayed in Figure (IV-2).

The evaluation of the mean free path of cosmic rays in the presence of this spectrum of waves is similar to that in the example at the end of Chapter III, section 4; some modifications have to be introduced, to account for the lack of magnetoacoustic waves with $k > k_s$.

a) Resonant scattering rate of change of μ. The resonant scattering rate is given by equation (III-14). Cosmic rays with Larmor radii $r_\ell > 1/k_s$ are scattered by the three types of waves. We have shown in Chapter III, Appendix 1, that the contribution to ξ of fast and slow waves of equal energy density and similar angular distribution is identical to that of cold magnetosonic waves with the same energy content. Thus, for this range of r_ℓ, $\xi\ (1/r_\ell)$ and the scattering rate are as in Chapter III, section 4. For $r_\ell < 1/k_s$, only the α term remains in ξ : consequently, the scattering rate is lower than in Chapter III, section 4, especially at small μ.

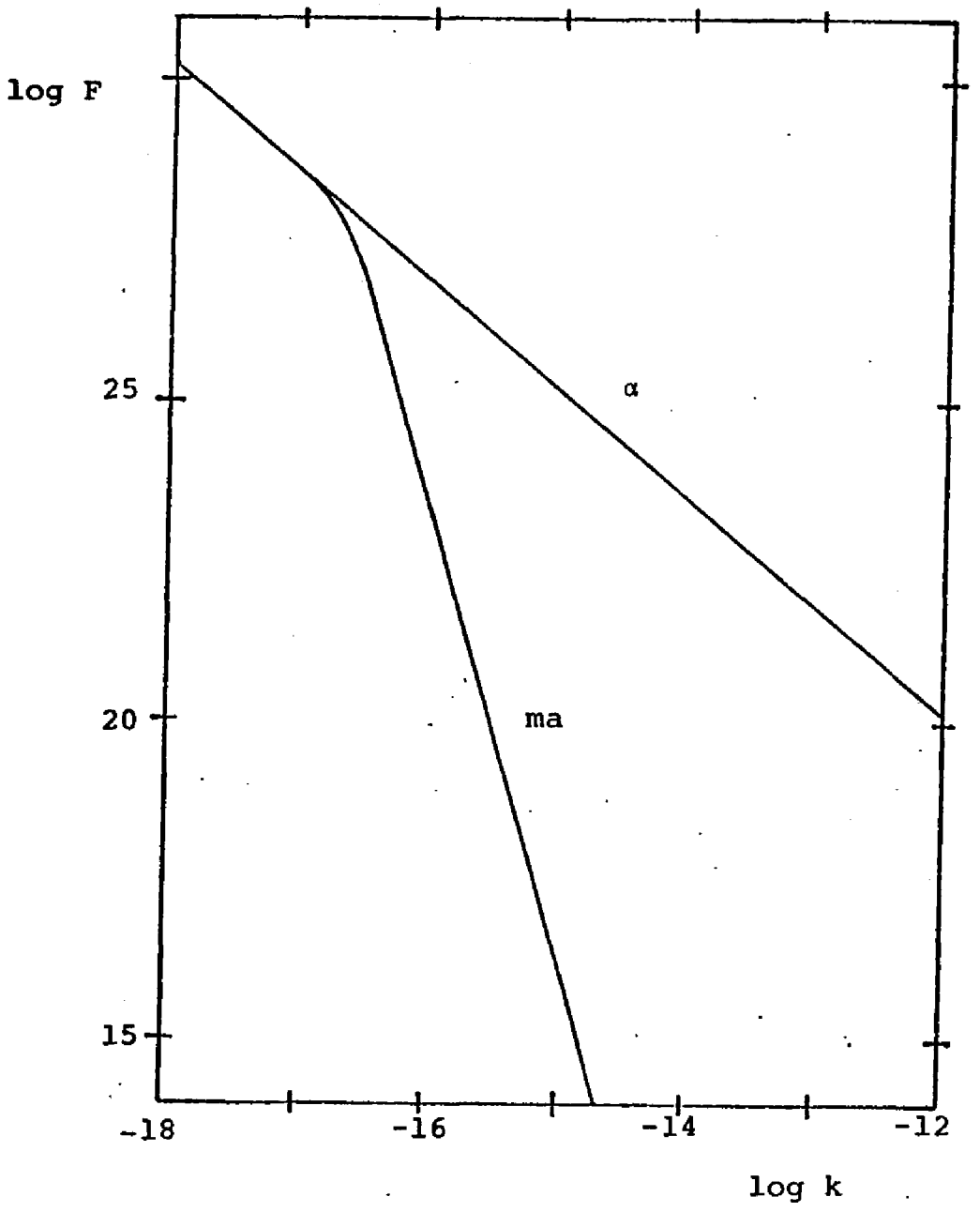


Figure (IV-2)

Spectrum of MHD waves, in a hot medium, in case b, for T=40000°K and n=0.037 cm^{-3}. F is in erg cm/g, and k in cm^{-1}.

The resonant rate as a function of μ, for different values of r_ℓ, is represented in Figure (IV-3).

b) Mirror rate of change of μ. The mirror rate of change of μ is proportional to the amplitude of the component $B_{1_z}(k_z)$ of the magnetic field associated with the waves and parallel to B_0. To evaluate $B_{1_z}(k_z)$, we approximate the spectrum of magnetoacoustic waves to:

$$F_{f,s} = \begin{cases} F_0 \left(\frac{k_0}{k}\right)^{5/3} & k < k_s \\ 0 & k > k_s \end{cases} \qquad \text{(IV-23)}$$

Then, after some algebra, we find:

$$\frac{B_{1_z}^2(k_z)}{8\pi} = \frac{9}{55} F_0 \rho \left(\frac{k_0}{k_z}\right)^{5/3} k_z \; h(k_z) \qquad \text{(IV-24)}$$

with:

$$h(k_z) = 1 - \left(\frac{k_z}{k_s}\right)^{5/3} \left(\frac{11}{6} - \frac{5}{6}\frac{k_z^2}{k_s^2}\right) \qquad \text{(IV-25)}$$

$h(k_z)$ is displayed in Figure (IV-3).

Thus, the rate of change of μ due to mirroring by waves of a given k_z, such that $\frac{B_{1_z}(k_z)}{B_0} \gtrsim \mu^2$, is:

$$\frac{1}{|\mu|}\frac{d|\mu|}{dt} = -\frac{c k_0}{4}\sqrt{\frac{18}{55}\,\omega\, h(k_z)}\;\frac{(1-\mu^2)}{|\mu|}\left(\frac{k_z}{k_0}\right)^{2/3} \qquad \text{(IV-26)}$$

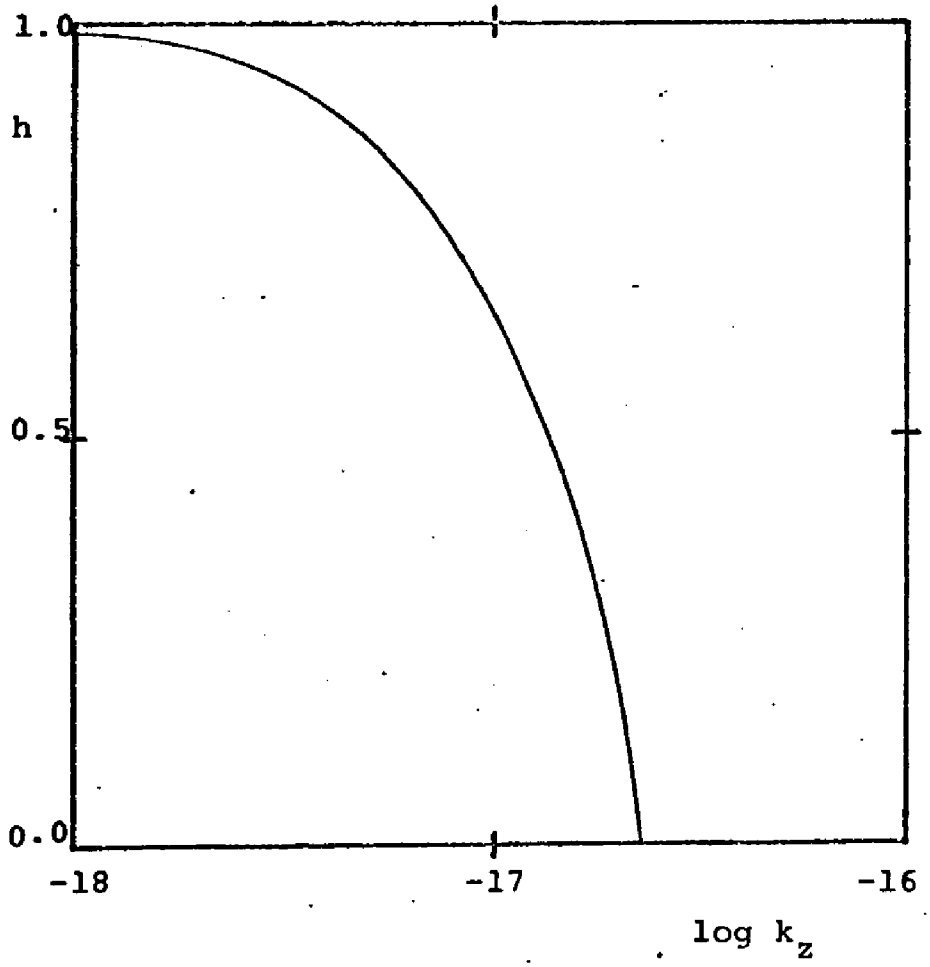


Figure (IV-3)

$h(k_z)$ as a function of log k_z ; k_z is in units of cm^{-1}.

Let k_μ be the value of k_z such that:

$$\frac{B_{1_z}(k_\mu)}{B_0} = \mu^2 \qquad \text{(IV-27)}$$

and k_m the value of k_z for which $\sqrt{h(k_z)}\left(\frac{k_z}{k_0}\right)^{2/3}$ is maximum. In our case, $k_m = 11.5k_0$. The waves that are faster in mirroring cosmic rays of pitch angle ($\cos^{-1}\mu$) are those with:

$$k_z = k_\mu \quad \text{if} \quad k_\mu < k_m \ (\text{and } k_\mu < 1\backslash r_\ell)$$
$$k_z = k_m \quad \text{if} \quad k_\mu > k_m \ (\text{and } k_m < 1\backslash r_\ell) \qquad \text{(IV-28)}$$

The mirroring rate as a function of μ is represented in Figure (IV-4). The intersections of this curve with those representing the resonant rate at different values of r_ℓ define $\mu_{\text{cut-off}}$ as a function of r_ℓ.

For $r_\ell = 10^{18}$cm, or $\varepsilon \simeq 10^6$Bv, $k_z|\mu| > 1/r_\ell$ for $|\mu| < 0.425$; an upper limit to the rate of mirroring is given by equation (IV-6), with $|k_z| = 1/r_\ell$; the result is represented by a dashed line in Figure (IV-4). The resonant rate at this energy has also been represented by a dashed line, to emphasize that it, too, is approximate: the energy density in waves at $k \simeq k_0$ is so large that the pitch angle of a cosmic ray with $r_\ell \simeq 1/k_0$ is considerably altered after one resonant interaction with a wave; in other words, the approximation $\lambda >> r_\ell$ does not hold. Thus, the uncertainty in our estima-

tion of λ at these energies is higher than at the lower energies.

c) Mean free path. For $r_\ell > (1/k_s)$, $\bar{\lambda}\ (|\mu|)$ is as calculated in Chapter III and represented in Figure (III-3). For $r_\ell < (1/k_s)$, only Alfvèn waves are present; the corresponding $\bar{\lambda}\ (|\mu|)$, displayed in Figure (IV-5), is slightly larger -- except at $\mu < 0.1$, where it is much larger.

Finally, the mean free path is given by equation (III-25). The result -- (log λ), in parsecs, as a function of (log r_ℓ) -- is given in Figure (IV-6).

The mean free path obtained is nearly constant: 3 pc < λ < 5 pc, for 10^{12}cm < r_ℓ < 10^{17}cm, or 1 Bv $\lesssim \varepsilon \lesssim 10^5$Bv. At a smaller r_ℓ, the mean free path tends to increase, because $\mu_{\text{cut-off}}$ is small, and $\bar{\lambda}(\mu)$ increases rapidly when $|\mu|$ decreases. At large r_ℓ, λ has also a tendency to increase; at $r_\ell = 3 \times 10^{17}$cm, $\lambda \approx 6$ pc; while at $r_\ell = 10^{18}$cm, we estimate $\lambda \approx 16$ pc.

It is interesting to consider the change in λ produced by a change in the parameters (T, B_0, k_0), while ω is kept constant. An increment in T would result in a higher power input and a lower density. As k_s would be diminished, so would $\mu_{\text{cut-off}}$; so that λ would be increased. On the other hand, lower values of k_0 -- or B_0 -- lead to a lower density, hence relaxing the requirement on the power input, and to a higher mean free path. In this case $\lambda(\varepsilon)$, rather than being constant, approaches a power law in ε of small exponent.

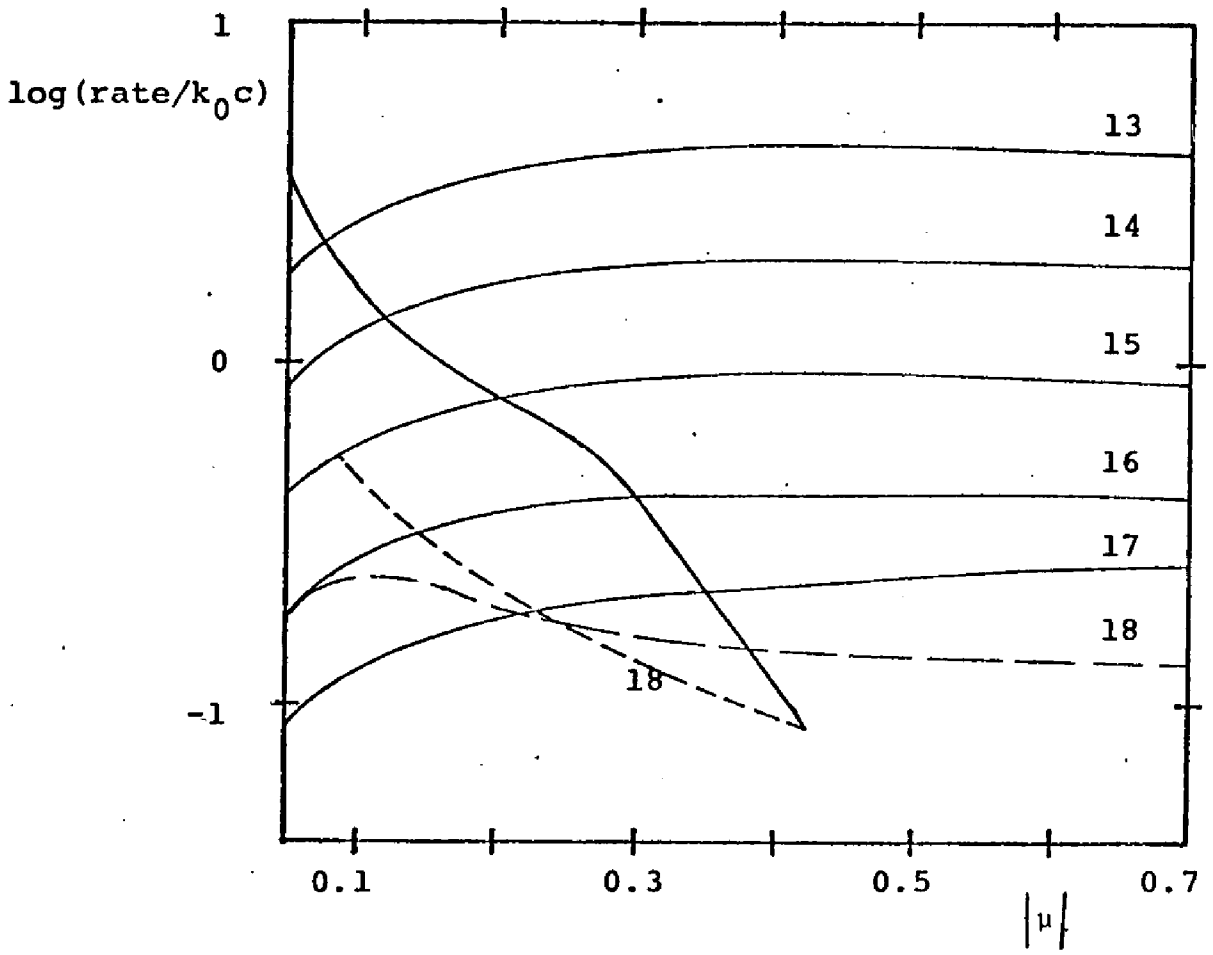


Figure (IV-4)

Resonant and mirror rate of change of $|\mu|$, in units of $(k_0 c)$, when the wave spectrum is as in Figure (IV-2). The horizontal curves represent the resonant rate, and are labelled according to the value of $\log(r_\ell)$. The diagonal curve represents the mirror rate for $r_\ell \lesssim 2\times10^{17}$cm; the dashed curve is an approximation to the mirror rate when $r_\ell \approx 10^{18}$cm.

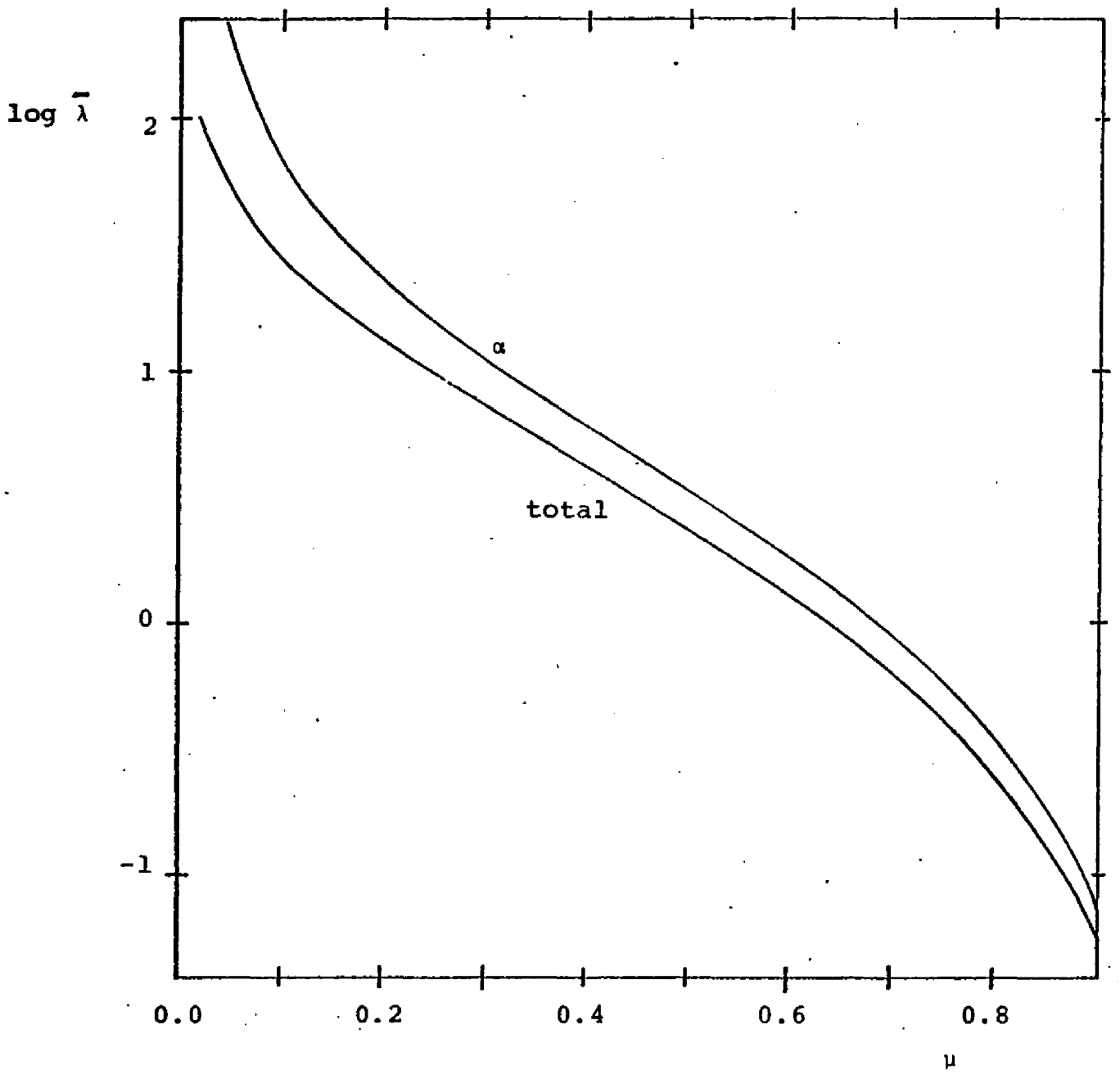


Figure (IV-5)

Log $\bar{\lambda}$, as a function of μ . When $k<<k_s$, both Alfvèn and ma waves are present, and λ is given by the curve labelled "total"; when $k>>k_s$, only Alfvèn waves are present, and λ is given by the curve labelled α.

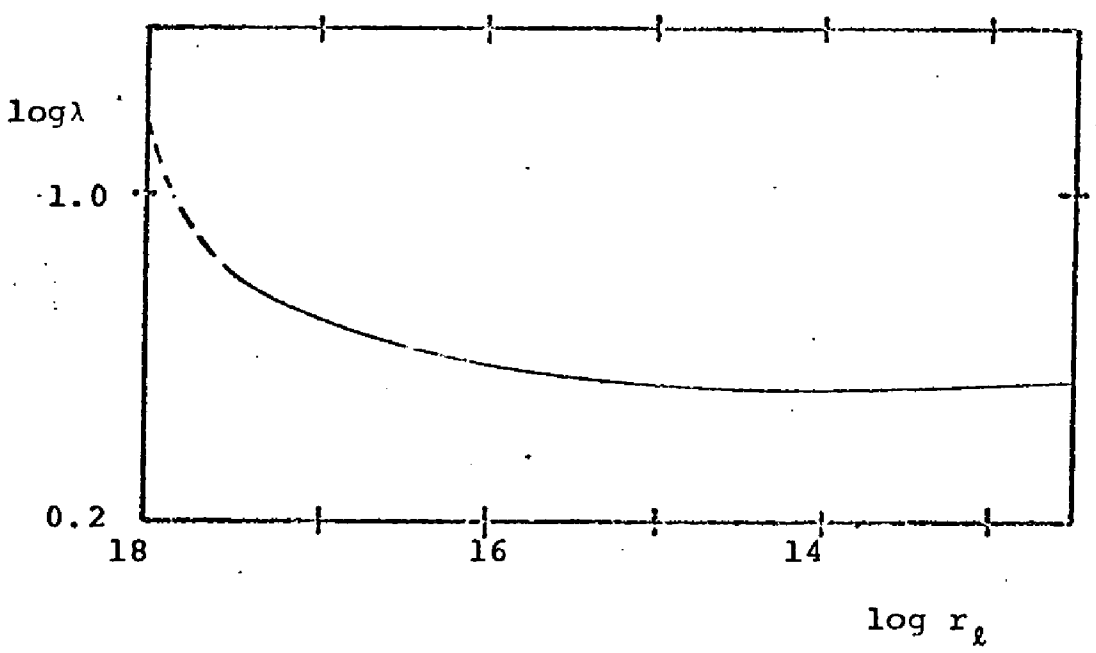


Figure (IV-6)

Log λ , as a function of log r_ℓ, when the wave spectrum is as given in Figure (IV-2). λis in pc, and r_ℓ in cm.

For example, if T, B_0 are unchanged, but $k_0 = 2 \times 10^{-19} cm^{-1}$ -- corresponding to a scalelength of $\sim$ 10 pc -- the density becomes: $n \simeq 0.019\ cm^{-3}$, and the total power needed: $P \simeq 5 \times 10^{-26}\ erg/cm^3\ sec$. The resulting mean free path, for 10 Bv $< \varepsilon <$ 10^6 Bv, can be written:

$$log\ \lambda\ (pc) = (0.65 + 0.11\ log\ \mathcal{E}) \pm 0.1 \qquad \text{(IV-29)}$$

For comparison, the uncertainty in the logarithm of the cosmic ray spectrum, at these rigidities, is $\simeq 0.7$ (see Figure II-1).

5) A model of cosmic ray propagation in the galaxy.

In the preceding sections, we have demonstrated that, although cosmic rays cannot be trapped by waves in neutral regions of the galaxy, they may be contained in highly ionized, hot regions. We will now discuss the possible location of such regions in the galaxy, assess the chances of detecting them, establish a model of cosmic ray propagation in the galactic disk, and verify that the over all results do not conflict with the observational evidence on cosmic rays reviewed in Chapter II, section 1.

The neutral component of the interstellar gas is confined to a thin disk of equivalent width $\lesssim$ 300 pc. (Spitzer 1968). In this "gas disk", the atomic hydrogen tends to

be concentrated in clouds (H I regions), of density ~ 10 cm^{-3} and temperature $\sim 100^{\circ}$K, which occupy about 7% of the volume. Another 2% of the gas disk (Spitzer 1968) contains ionized nebulae (H II regions) of comparatively high density ($n \gtrsim 10^3$), excited by young stars. The rest -- most of the interstellar space in the gas disk -- has been called intercloud medium, and is believed to be hotter and more diffuse than H I clouds. Because most observational techniques discriminate against low density, high temperature regions, the real nature of the intercloud medium is open to speculation.

Currently accepted values for the physical parameters in the intercloud medium are: $T \gtrsim 1000^{\circ}$K, $n \approx 0.2$ cm^{-3}, $n_e \approx 0.02$ cm^{-3}. We have seen that a spectrum of waves cannot develop in these conditions. Could the diffuse regions, heated by waves, needed to detain cosmic rays, be part of the intercloud medium? Observations do not rule out the possibility that at least in part of the intercloud medium the density is $\lesssim 0.04$ cm^{-3}, while the temperature is very high; but, even if the power input per unit volume in waves were sufficient to keep such a low density medium very hot, a large fraction of the power would be dissipated in the clouds that are floating in the diffuse region. The remainder may not be sufficient to develop a spectrum of waves.

Consequently, we now turn our attention to the region immediately above -- and below -- the gas disk. Ob-

servations of dispersion measures of pulsars, together with 21 cm data along the same line of sight, provide us with evidence that the electron density in a slab of $\sim$ 200 pc above the gas disk is comparable to that of the gas disk -- $n_e \approx 0.02$ (Bridle and Venugopal 1969). Bridle and Venugopal further point out that measurements of Faraday rotation of extragalactic radio sources are also consistent with the existence of an electron slab of thickness 200 pc, in which is immersed a magnetic field of $\sim$ 2.5μ Gauss. The proposition that the slabs are trapping regions of cosmic rays is substantiated by determinations of the latitude distribution of the non-thermal continuum radiation at 400 MH_z, which indicate that the emission originates in a region of equivalent width $>750 \pm 100$ pc (Baldwin 1967).

Because the density is low, the temperature in the slabs is difficult to determine observationally. Measurements of the absorption of radio signals, at low frequency, provide with a measure of $\left\langle \frac{n_e^2}{T^{3/2}} \right\rangle$ along the line of sight; thus, a cool condensation, however small, may screen the effects of long path lengths through a hot and diffuse medium. Thus, for a given $\langle n_e \rangle$, all that can be extracted from low frequency absorption data is a lower limit, of questionable significance, for T.

Other observations that could help to confirm or reject the hypothesis that the diffuse gas above the neutral gas disk is very hot are listed below. The estimates have

been made for a slab with $n_e = 0.02$, $T = 40000^\circ$, extending for 200 pc over a gas disk of half-thickness 150 pc.

a) Oxygen forbidden lines. Most of the optical radiation that originates in a diffuse gas at $40000^\circ K$ is in the form of emission of a doublet forbidden line of OII, at a wavelength $3726A^\circ$ (line 1) and $3729A^\circ$ (line 2). For a gas of cosmic abundances, the power radiated per unit volume in each of the lines at $T = 40000^\circ K$ is (Cox and Daltabuit 1971):

$$P_1 = 1.3 \times 10^{-23} \; n_e n_H \quad \text{erg/cm}^3 \text{ sec}$$

$$P_2 = 1.8 \times 10^{-23} \; n_e n_H \quad \text{erg/cm}^3 \text{ sec} \qquad \text{(IV-30)}$$

If we assume an extinction of 1 magnitude/kpc in the gas disk, the maximum values for the surface brightness seen from the earth, which should be obtained when looking at an angle $\sim 8^\circ$ with the galactic plane, are:

$$S_1 \simeq 0.7 \times 10^{-6} \text{ erg/cm}^2 \text{ sec ster}$$

$$S_2 \simeq 10^{-6} \text{ erg/cm}^2 \text{ sec ster}$$

while at high latitudes,

$$S_1 = 0.2 \times 10^{-6} \text{ erg/cm}^2 \text{ sec ster}$$

$$S_2 = 0.3 \times 10^{-6} \text{ erg/cm}^2 \text{ sec ster}$$

These intensities are low, but may not be beyond detection.

b) Balmer line emission. A classical method of detecting faint H II regions is through observations of Balmer line emission. Because the sky background is lower in the region of wavelengths of H_β than in that of H_α (Reynolds et al 1971), we compute the expected intensity of the H_β emission. The surface brightness, for a path length L(cm), can be written (Gould 1971):

$$I_\beta = \frac{1}{4\pi} \alpha_\beta \, n_e \, n_H^+ \, L \quad \text{photons/cm}^2 \text{ sec ster} \qquad \text{(IV-31)}$$

where at 40000°K, $\alpha_\beta = 7.25 \times 10^{-15}$ cm^3/sec. When observing at low latitudes, the emission from the slabs is substantially attenuated, and if any H_β radiation from diffuse regions were detected, it would probably have originated in the gas disk. At high latitudes, the contribution of the slabs to the H_β surface brightness at the earth would be:

$$I_{\beta_{slab}} \approx 130 \text{ photons/ cm}^2 \text{ sec ster;}$$

that of the gas in the disk would be of the same order or larger. On the other hand, the H_β intensity from the geocorona, due to absorption of solar Lyγ, has been estimated by Reynolds et al (1971) to be:

$$I_{\beta \text{ geocorona}} \lesssim 5.5 \times 10^3 \text{ photons/cm}^2 \text{ sec ster.}$$

We conclude that the H_β radiation from the slab is probably too faint to be detected; also, it would be difficult to distinguish between the emission from the slab and that from the gas disk or the geocorona.

c) Radio recombination line emission. The emission of recombination lines of frequency in the radio portion of the electromagnetic spectrum is caused by transitions between levels with large principal quantum number n: $n \gtrsim 60$. If the atomic levels are populated in local thermodynamic equilibrium (LTE), the emissivity of a given line is determined by the mean value of $\left(\frac{n_e^2}{T^{1.5}} \right)$ along the line of sight. Consequently, if LTE prevailed, the diffuse and hot slabs would be very faint in the light of radio recombination lines. But Duprée (1971) has pointed out that, at temperatures $\gtrsim$ 40000 °K, dielectronic recombination may lead to strong departures from LTE in the population of the high levels of the He II ion. If T_L^* is the line temperature in LTE, the line temperature in non LTE of the line corresponding to the transition m → n is (Duprée 1971)

$$T_L = T_L^* \left(b_m - \frac{1}{2} b_m \beta_{nm} \tau_c \right) \equiv T_L^* b^* \qquad \text{(IV-32)}$$

where b_m, b_n are departure coefficients for the m and n levels,

$$\beta_{nm} = \frac{b_m}{b_n} \left[1 - \frac{h T_e}{h \nu} \left(\frac{b_m - b_n}{b_m} \right) \right] \qquad \text{(IV-33)}$$

and τ_e is the optical depth in the continuum at the same frequency.

We considered, as an example, the line corresponding to the transition n = 157 to n = 156 (He 157α), and computed the value of b* necessary to obtain a spectral line power of $10^{\circ}K\ kH_z$. (The faintest lines detected up to date, by Gottesman and Gordon (1971), have a spectral power of $11.2^{\circ}K\ kH_z$). Assuming a path length of 10 kpc, we found: $b^* = 6.5 \times 10^4$. Although the occurrence of such a high value of b* seems quite improbable to us, it cannot be ruled out until calculations of departure coefficients at very low densities become available.

The power required to establish a temperature of $40000^{\circ}K$ in the slab (with $n_e \simeq 0.02$) and to maintain a spectrum of waves of the type of that in Figure (IV-2), is comparable to the power necessary to heat the gas disk (if the intercloud medium has a density $\gtrsim 0.2\ cm^{-3}$ and a temperature of 10000°) or to replenish disk and slab with cosmic rays every million years. It is only equal to $\sim$ 3% of the energy output of supernovae (assuming that every 30 years a supernovae releases 10^{52} ergs in the galaxy); thus, supernovae are a possible source for this power, although, since type II supernovae are concentrated in the gas disk and type I supernovae explosions are much less energetic, the connection between supernovae and turbulence in the slabs is not obvious.

Turbulent energy could be deposited directly in the slabs by high velocity clouds. 21cm observations at high galactic latitudes have demonstrated the presence, above the disk, of clouds of neutral hydrogen that are falling rapidly towards the plane. Oort (1970) estimates their mean column density to be 3×10^{19} cm^{-2}, and their mean velocity 150km/sec. (The amount of infalling material could be larger if part of it is ionized -- a possibility that does not seem too farfetched). If, as Oort assumes, the observed high velocity clouds are within 1 kpc of the galactic plane, and if they are braked in the slabs, they could be the source of power required.

Finally, let us consider the overall picture of cosmic ray propagation which emerges from our discussion. Cosmic rays are trapped in an enlarged galactic disk, composed of the central gas disk of width $\sim$ 300 pc, encased between two slabs of width $\sim$ 200 pc. In the slabs, the energy density of hydromagnetic waves at all wavelengths is sufficient to detain high energy cosmic rays, forcing them to diffuse at a low pace and to become isotropic. In the gas disk, the density of hydromagnetic waves that can interact resonantly with high energy cosmic rays is low, and cosmic rays traverse it at large speeds.

With this picture in mind, we return to the simple model of cosmic ray propagation described in Chapter II, section 2, and to the accompanying discussion on the observ-

ational constraints placed on $\lambda(\varepsilon)$. In Chapter II, we had considered that cosmic rays propagate along a tube of force of constant section, of length 2L, open at both ends, and we assumed that the diffusion rate and the density of cosmic ray sources are uniform along the tube. This model has to be modified, to account for the presence of two different regions along the tube. Region 1, of length $2L_1$, corresponds to the gas disk. Because most of the suspected cosmic ray sources -- pulsars, supernovae, galactic center -- lie inside the gas disk, we assume that it contains all of the cosmic ray sources, which are uniformly distributed with a density S. D_1, the diffusion coefficient of cosmic rays in region 1, is very large. Region 2 is the part of the tube that lies in the slabs; the corresponding diffusion coefficient is D_2. If z is the distance to the center of the tube, we may solve equation (II-1) for this case, to obtain the distribution function of the cosmic rays:

$$\mathcal{F}(z) = \frac{S L_1 L_2}{D_2} + \frac{S}{2 D_1} (L_1^2 - z^2)$$

in region 1

$$\mathcal{F}(z) = \frac{S L_1}{D_2} (L - z)$$

in region 2

(IV-34)

(where $L = L_1 + L_2$).

Since $D_1 >> D_2$, the density of cosmic rays is essentially uniform in the gas disk. The mean age is obtained by solving equation (II-2); in the gas disk, it is essentially independent of z, and equal to:

$$\langle \tau \rangle = \frac{L_2 (L_1 + L_2/3)}{D_2} \qquad \text{(IV-35)}$$

Also, in region 1, the anisotropy is given by:

$$\delta_1 = \frac{\lambda_2 z}{L_1 L_2} \qquad \text{(IV-36)}$$

where $\lambda_2 = 3D_2/c$ is the mean free path of cosmic rays in region 2. Let us further assume: $L_1 = 3/4\ L_2$; then:

$$\langle \tau_1 \rangle = \frac{13}{4} \frac{L_2^2}{\lambda_2 c} \qquad \text{(IV-37)}$$

Composition arguments indicate that cosmic rays at the earth have traversed $\sim$ 4g of matter, i.e. have spent about 3 million years in the gas disk (see Chapter II, section 2). Then, $\langle \tau_1 \rangle \simeq 6$ million years. Also, the cosmic ray anisotropy at 300 Bv is: δ_1 (300 Bv) $\leq 4 \times 10^{-4}$. Then, the distance of the sun to the center of the tube is:

$$z_\odot = \frac{3}{13} c\, \delta_1(300\,Bv) \langle \tau_1 \rangle \lesssim 165\ pc. \qquad \text{(IV-38)}$$

We have seen that, if $n_e \simeq 0.02$, $\omega = 1/10$ and $2\pi/k_0 \simeq 10$ pc, the mean free path in the slab of cosmic rays of rigidity $\varepsilon_{(Bv)}$ is:

$$\lambda_2(\varepsilon) \simeq 4.5\ \varepsilon^{0.11}\ pc\ ,\ for\ \ 10Bv \lesssim \varepsilon \lesssim 10^6 Bv \qquad \text{(IV-39)}$$

Consequently, $\lambda_2(300\ Bv) = 8.4$ pc, and $L_2 = 2150$ pc.

Let us assume that the cosmic ray source spectrum is a power law: $S \propto \varepsilon^{-n}$. Then, in the gas disk (equation IV-34), for $10 \lesssim \varepsilon \lesssim 10^6$ Bv,

$$\mathcal{F}(\varepsilon, z) \propto \frac{S}{D_2} \propto \varepsilon^{-n-0.11} \qquad \text{(IV-40)}$$

Comparing with the observed value of the cosmic rays spectral index, 2.6, we find that our theory fits the observation if $n = 2.48$. At rigidities above 10^6 Bv, diffusion perpendicular to the magnetic field becomes important; thus, a cosmic ray of rigidity 3×10^6 Bv would diffuse ~ 100 pc across the field in the time it travels a length L_2 along it. This would explain the break in the observed cosmic ray spectrum at these rigidities.

6) Summary of results.

We have shown that when cosmic ray streaming instabilities and charged-neutral collision damping are the only two processes that control the wave spectrum, relativistic cosmic rays of low rigidity ($\varepsilon \lesssim 10$ Bv) are easily detained in the gas disk of the galaxy, while higher rigidity cosmic rays can only be detained in regions of very low density of neutral particles. Even in such regions, the cosmic ray streaming velocity would be a strong function of the rigidity -- a result that is difficult to reconcile with the isotropy of the cosmic ray flux at rigidities in the range 10^2 - 10^6 Bv and possibly with the lack of break in the cosmic ray spectrum in the same range of rigidities. (Chapter I). A similar difficulty would arise if cosmic rays were to be confined exclusively by their interactions with waves much shorter than their Larmor radii. (Chapter II).

When the waves are produced by sources other than the cosmic rays and the interactions between waves are effective, we have a situation fundamentally different from that considered above. We assume that the long waves, of dimensions comparable to the size of interstellar clouds, are generated in the interstellar medium. A spectrum of waves develops if the rate at which they transfer their energy to smaller waves is faster than the dissipation rate. To estimate the potentialities of this approach, we adopted a rate

of energy transfer between waves that seems a reasonable upper limit: that suggested by Heisenberg for hydrodynamic turbulence. (Chapter II).

In a medium where the density of neutral particles is appreciable, the damping rate due to charged-neutral collisions is large. To develop a spectrum of waves, the rate of energy transfer must overcome the damping rate. The level of turbulence has to be very high, and the power necessary to maintain it exceeds by several orders of magnitude any acceptable value. (Chapter IV).

On the other hand, if the long waves are generated in a medium where the total density is low enough, the power input in waves, which is finally dissipated in heat, may be sufficient to keep the medium hot and highly ionized. Although at a high temperature magnetoacoustic waves are damped efficiently by thermal conduction effects, a spectrum of Alfvèn waves may still develop. The mean free path of ultrarelativistic cosmic rays of rigidity $10 \text{ Bv} < \varepsilon \lesssim 10^6 \text{ Bv}$ in the presence of the resulting spectrum of waves is either constant, or a power law in ε with a small exponent. (Chapters III, IV).

Finally, we propose a "sandwich" model for the propagation of cosmic rays in the galaxy, in which cosmic rays are contained in an enlarged galactic disk, consisting of the gas disk and of two slabs of diffuse and hot gas at each side. Individual cosmic rays traverse the gas disk at large speeds;

while in the slabs, where turbulence is stirred by supernovae explosions or high velocity clouds, they are efficiently scattered and they diffuse at a slow pace. Consequently, the anisotropy of the cosmic ray flux in the gas disk is low. Furthermore, if the source spectrum of the cosmic rays is a power law, then so is the spectrum of cosmic rays in the gas disk. This argument applies to cosmic rays of rigidity $\lesssim 3.10^6$ Bv; rays of higher energy escape from the slabs by diffusing across the field lines, rather than along them. Their time of residence in the galaxy is shorter than that of the less energetic particles, so that a break occurs in the cosmic ray spectrum.

This model is in agreement with the results of observations concerning the slabs -- pulsar dispersion and Faraday rotation measurements, non-thermal continuum radiation -- as well as with cosmic ray data -- spectrum, age and isotropy.

APPENDIX IV-1

Normal modes in a partially ionized medium with a magnetic field.

We consider, for simplicity, a pure hydrogen gas, partially ionized, immersed in a uniform magnetic field B_0. Let ρ_0 and n_0 be the mass and number density of neutral particles, and ρ^* and n^* that of charged particles. The degree of ionization is parametrized by $y = \rho^*/\rho_0$. ν_0 is the frequency of collisions of charged with neutral particles.

The dispersion relation, in the hydromagnetic case ($v_A >> v_s$ = sound velocity), was derived by Kulsrud and Pearce(1969):

$$(\bar{\omega}^2 - \omega_k^2)\,\bar{\omega} + i\,\nu_0\left[(1+y)\,\bar{\omega}^2 - \omega_k^2\, y\right] = 0 \qquad \text{(AIV-1)}$$

where ω_k is the natural frequency of the hydromagnetic modes in the charged medium only ($\omega_k = k v_A \cos\xi$ for α waves, and $\omega_k = k v_A$ for ms waves; $v_A = \dfrac{B_0}{\sqrt{4\pi\,\rho^*}}$.

Let ω and Γ be respectively the real and imaginary part of $\bar{\omega}$; in the following, we write all the frequencies in units of $\nu_0/2$. Then, equation (AIV-1) is equivalent to:

$$\Gamma^3 + 2(1+y)\,\Gamma^2 + 1/4\,\left(\omega_k^2 + 4\,(1+y^2)\right)\Gamma + \omega_k^2/4 = 0$$

$$\omega^2 = 3\,\Gamma^2 + 4\,(1+y)\,\Gamma + \omega_k^2 > 0 \qquad \text{(AIV-2)}$$

or

$$\Gamma^3 + 2(1+y)\Gamma^2 + \omega_k^2 \Gamma + 2\omega_k^2 y = 0$$

$$\omega = 0$$

(AIV-3)

solutions for these equations can be readily found. The solutions of (AIV-2) represent the hydromagnetic waves, for which:

$$\bar{\omega} = \pm \left[\omega_k^2 \frac{y}{1+y} - \frac{\omega_k^4}{8(1+y)^3} \left(1 - \frac{3y}{1+y} \right) \right] - i \frac{\omega_k^2}{4(1+y)^2}$$

(AIV-4)

for $\omega_k << (1 + y)$, and

$$\bar{\omega} = \pm \left(\omega_k^2 - 4(1+y) \right) - i \qquad \text{for } \omega_k >> y .$$

(AIV-5)

(When $y << 1$, these results are in agreement with those of Kulsrud and Pearce). Thus, waves whose frequency is above the effective collision frequency between charged and neutral particles (1 when $y << 1$; y when $y >> 1$) involve only the ionized part of the medium, and are damped at a rate $\Gamma = 1$. Longer waves, for which $\omega << 1$, carry the whole medium, and the damping rate is reduced to $\Gamma = \frac{\omega_k^2}{4(1+y)^2}$

The decoupling of the motions of the two components of the medium occurs at $\omega_k \approx$ maximum $(1,y)$. If $y < \frac{1}{8}$, we define:

$$\omega_{k_{1,2}} = \frac{1}{2} + 10y - 4y^2 \pm \left(\frac{1}{2} - 4y\right)\sqrt{1-8y} \qquad \text{(AIV-6)}$$

When $|\omega_{k_1}| < |\omega_k| < |\omega_{k_2}|$, the real part of the frequency is equal to zero and waves do not propagate; Γ is one of the solutions of equation (AIV-3).

The third solution to equation (AIV-1) is a purely damped mode: $\omega = 0$, and Γ satisfies equation (AIV-3). The properties of this mode are easily derived in two limits:

1)
$$\omega_k^2 << (1+y)^3 , \qquad \bar{\omega} = -i\,2(1+y) \qquad \text{(AIV-7)}$$

The kinetic energy of neutral particles, the kinetic energy of charged particles and the magnetic energy per unit volume associated to this mode are in the relation:

$$\frac{y}{1+y} : \frac{1}{1+y} : \frac{\omega_k^2}{4(1+y)^3} \qquad \text{(AIV-8)}$$

Thus, most of the energy is in the form of kinetic energy of the least abundant component; the magnetic energy is negligible.

2) $$\omega_k^2 >> y\,(1+y), \quad \bar{\omega} = -\,i\,2y\left(1 + \frac{4y}{\omega_k^2}\right) \qquad \text{(AIV-9)}$$

and the relations (AIV-8) are replaced by:

$$1 \quad : \quad \frac{16\,y^3}{\omega_H^4} \quad : \quad \frac{4y}{\omega_H^2} \qquad \text{(AIV-10)}$$

At high frequencies, this mode involves almost exclusively motions of the neutral particles; for this reason, we refer to it as the "neutral mode."

Plots of the solutions of equation (AIV-1), for $y = 0.1$ and $y = 10$ respectively, are given in Figures (IV-7) and (IV-8).

Figure (IV-7)

Γ and ω as a function of ω_k, when y=0.1.

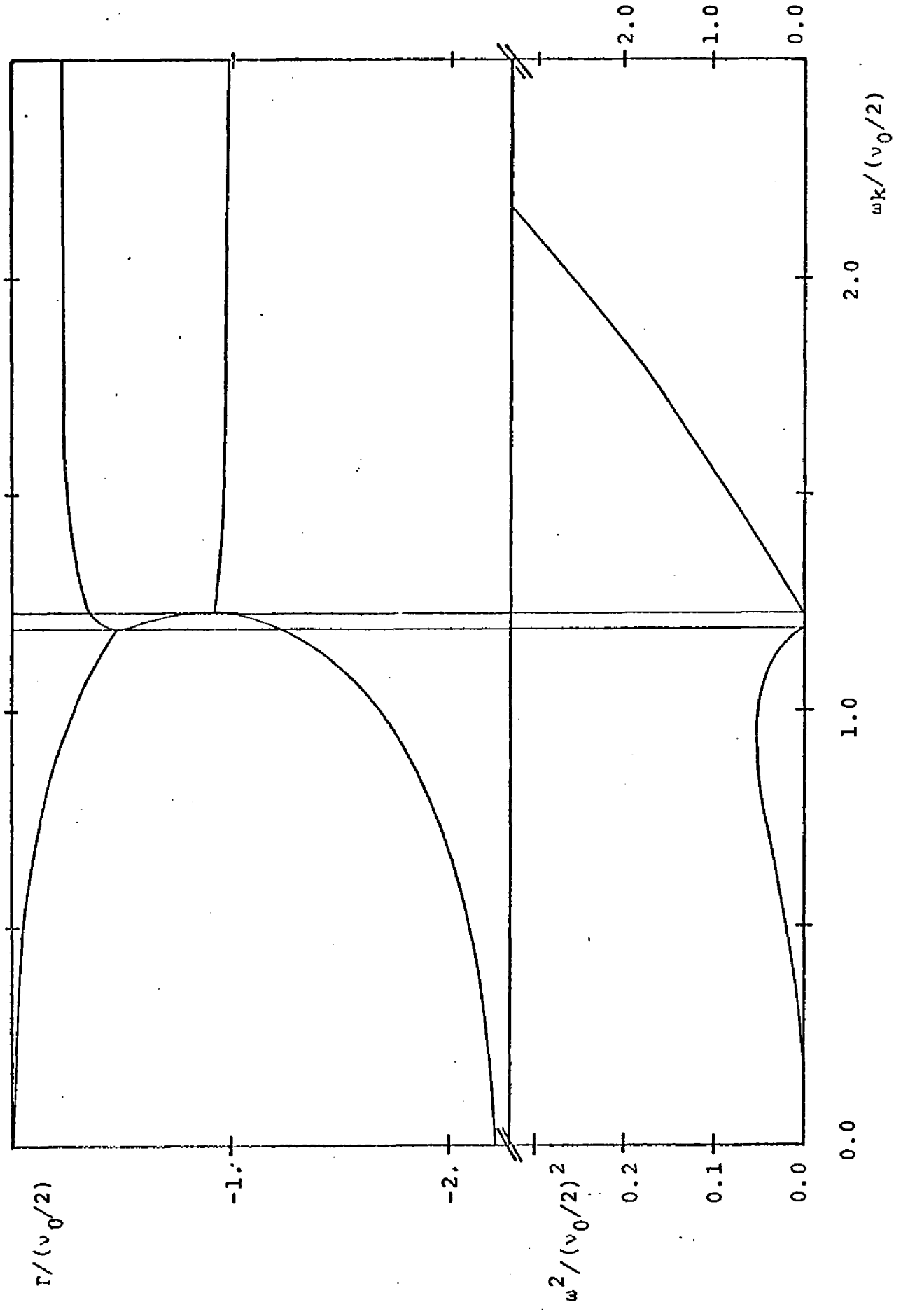
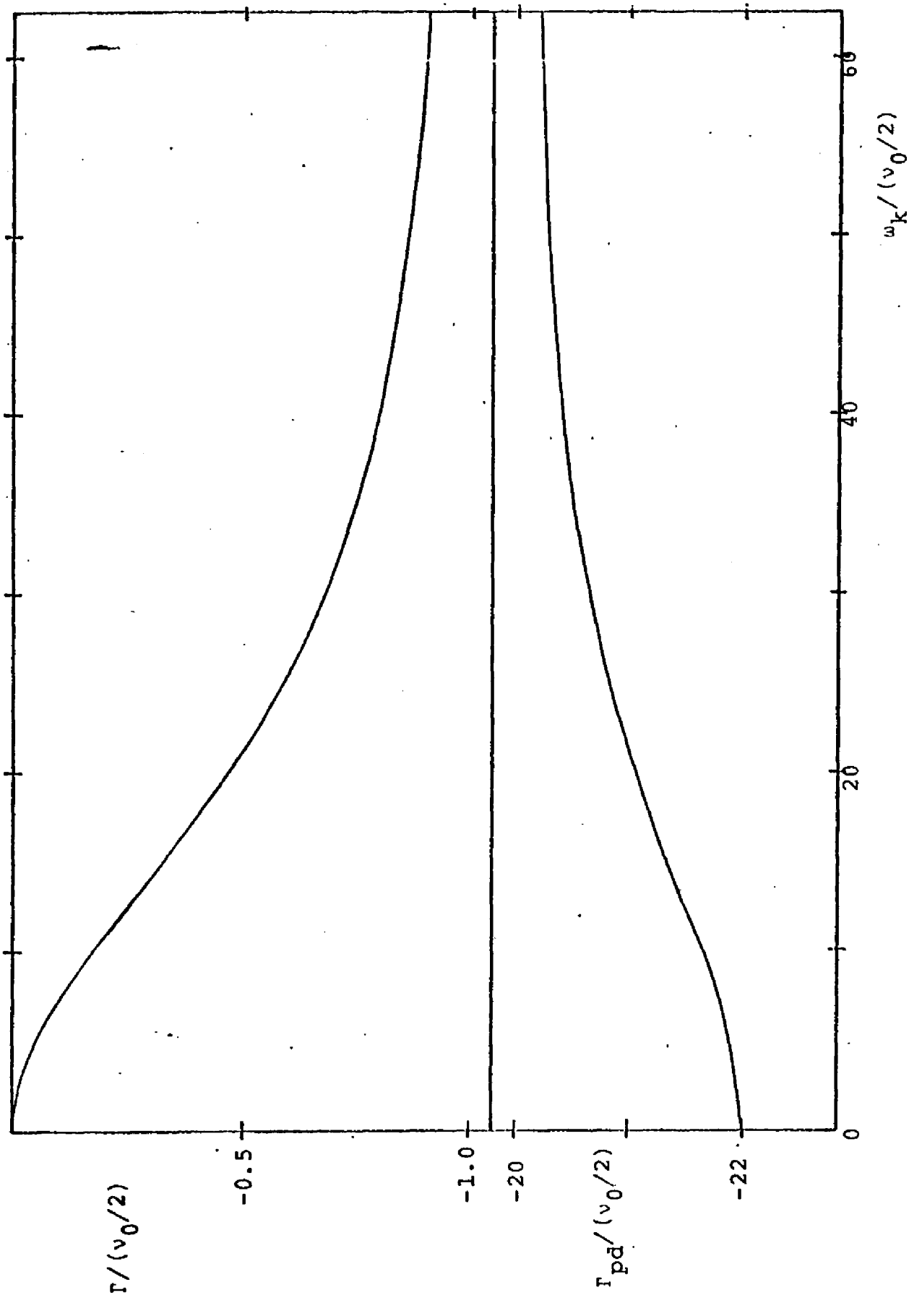

0.0
1.0
2.0
$\omega_k/(\nu_0/2)$
0.0
1.0
2.0
$\omega^2/(\nu_0/2)^2$
0.0
0.1
0.2
-2.
-1.
$\Gamma/(\nu_0/2)$

Figure (IV-8)

Γ as a function of ω_k, when y=10. Γ_{pd} is the damping rate of the purely damped mode.


-0.5
-1.0
-20
-22
0
20
40
60
$\Gamma/(\nu_0/2)$
$\Gamma_{pd}/(\nu_0/2)$
$\omega_k/(\nu_0/2)$

APPENDIX IV-2

Effect of ultrarelativistic cosmic rays on the waves.

The aim of this appendix is to justify our having neglected the effect of the cosmic rays on the waves, in the preceding discussions of the development of the wave spectrum.

Cosmic rays absorb or emit wave energy, and thus they may play a role in determining the shape of the wave spectrum. Alfvèn waves are excited or damped by cosmic rays of Larmor radii r_ℓ comparable to their wavelength; the damping rate of waves of wavenumber $k \simeq (1/10^{12}\ \epsilon_{Bv})\,cm^{-1}$ is given by equation (I-1):

$$\Gamma_{res} \simeq \Gamma_1 \left(-1 + \frac{|v_A|}{|v_A|} \right) \qquad \text{for} \quad k \lesssim 10^{-13}\,cm\,. \tag{BIV-1}$$

(A positive Γ_{res} means that cosmic rays give energy to the waves -- i.e. the waves are growing).

where

$$\Gamma_1 \simeq \frac{5 \times 10^{-12}}{n^*\ \varepsilon^{1.5}} \tag{BIV-2}$$

and

$$|v_R| \simeq -\frac{D}{\mathcal{F}} \frac{\partial \mathcal{F}}{\partial z} \simeq \frac{\lambda c}{3L} \tag{BIV-3}$$

Magnetosonic (and magnetoacoustic) waves, in addition to resonating with cosmic rays with $r_\ell \approx 1/k$, accelerate cosmic rays of shorter r_ℓ through the Fermi mechanism (Fermi 1949). A cosmic ray of momentum p and Larmor radius r_ℓ exchanges energy with waves of wavenumber $\vec{k}$ (where $|k| << 1/r_\ell$) at a rate (Kulsrud and Ferraro 1971):

$$\frac{\langle \Delta p^2 \rangle}{2t} = p^2 \gamma I(k_z) \qquad \text{(BIV-4)}$$

where

$$I(k_z) = \frac{\langle B_{1_z}^* (k_z) \; B_{1_z}(k_z) \rangle}{B_0^2} \qquad \text{(BIV-5)}$$

and

$$\gamma = \frac{\pi}{4} \frac{\omega^2}{k_z c} \qquad \text{(BIV-6)}$$

If W_{cR} is the total energy density of cosmic rays, the damping rate of magnetosonic waves by the Fermi mechanism is:

$$\Gamma_F \simeq -\frac{1}{2} \frac{W_{cR}}{B_0^2/8\pi} \gamma \qquad \text{(BIV-7)}$$

In the vicinity of the sun, $W_{cR} \approx 0.9$ ev/cm^3 (Ginzburg and Syrovatskii 1964). If $B_0 = 3\mu$ G,

$$\Gamma_F \simeq -\frac{\pi}{2}\frac{v_A^2}{c}\left(\frac{|k|}{|k_z|}\right)|k| \qquad \left(|k| < 10^{-13}\ \text{cm}\right)$$

(BIV-8)

The effect of cosmic rays on the spectrum of waves can be neglected if the ratio of Γ_{res} and Γ_F to the assumed rate of energy transfer between waves, $\Gamma_T \sim \sqrt{Fk^3}$, is small. We will show that this is the case for the situations considered in sections 2 and 3 of Chapter IV.

a) Neutral medium (section 2).

We have seen that, because of charged-neutral collisions, a spectrum of waves can develop only if $E \gtrsim 10\ E_m$, where $E_m \simeq 5 \times 10^4$ erg/g sec. If $E > 10\ E_m$, the spectrum of waves approaches the Kolmogorov form:

$$F \gtrsim 1.57\,(10\,E_m)^{2/3}\,k^{-5/3}$$

(BIV-9)

Thus,

$$\Gamma_T \gtrsim 100\,k^{2/3}\ \text{cm}$$

(BIV-10)

Since F is large, λ is small, $|v_R| \ll v_A$, and:

$$\Gamma_{res} \simeq -\Gamma_1\,(\varepsilon = 1/10^{12}k) = -2.5 \times 10^8\,k^{1.5}$$

(BIV-11)

while $\Gamma_F \simeq 1100$ k.

Thus, for the range of interest, namely:

$$10^{-18}\ \text{cm}^{-1} \lesssim k \lesssim 10^{-13}\ \text{cm}^{-1},$$

$$2 \times 10^{-9} \lesssim \frac{\Gamma_{res}}{\Gamma_T} \lesssim 5 \times 10^{-4} \qquad \text{(BIV-13)}$$

and

$$10^{-5} \lesssim \frac{\Gamma_F}{\Gamma_T} \lesssim 5 \times 10^{-4} \qquad \text{(BIV-14)}$$

Both Γ_{res} and Γ_F can be neglected.

For somewhat lower values of E, it is still possible to develop a spectrum of waves, although a large fraction of the power input at k would be dissipated into heat by charged neutral collisions, at $k \simeq k_c$. At $k >> k_c$, the spectrum would return to the Kolmogorov slope, (see Chapter II, section 5) but F would be lower than its value in (BIV-9); as a consequence, Γ_T would be lower than its value in (BIV-10) and the inequality $\Gamma_T >> \Gamma_F, \Gamma_{res}$ may not be preserved. Thus, the effects of cosmic rays on that part of the spectrum should be considered. The bulk of the power input would still go to the medium.

b) Ionized medium (section 3).

If the spectrum of waves is as given in equations (IV-21) and (IV-22), and displayed in Figure (IV-2),

$$\Gamma_{T_\alpha} \simeq k^{2/3} \quad sec^{-1}$$

(BIV-15)

over the whole range of k considered, while

$$\Gamma_{T_{ma}} \simeq k^{2/3} \quad sec^{-1} \quad \text{only for} \quad k \lesssim k_s \simeq 2 \times 10^{-17} cm .$$

In this case, (unless $z << L$), $\frac{|v_R|}{v_A} \gtrsim 1$, so that, in the resonant interactions, cosmic rays are transferring energy to the waves, and

$$\Gamma_{res} \simeq 2 \times 10^{48} \; k^{1.5} \quad sec^{-1} .$$

(BIV-16)

Then, for $10^{-18} cm^{-1} < k < 10^{-13} cm^{-1}$,

$$10^{-8} \lesssim \frac{\Gamma_{res}}{\Gamma_T} \lesssim 10^{-4}$$

(BIV-17)

Again, the effect of Γ_{res} is negligible.

The rate of damping of magnetoacoustic waves by the Fermi mechanism is slower than that of magnetosonic waves:

$$\Gamma_F \lesssim 600 k \quad sec^{-1}$$

(BIV-18)

Magnetoacoustic waves are supressed at $k \gtrsim k_s$ by thermal conduction effects; the fraction of the power input in these waves at $k \gtrsim k_0$ that goes to the cosmic rays is only of the order of:

$$\left.\frac{\Gamma_F + \Gamma_{res}}{\Gamma_T}\right]_{k=k_s} \simeq \quad 0.005.$$

REFERENCES

Aller, L. H., 1961, The Abundances of the Elements (Interscience, New York).

Baldwin, J. E., 1967, IAU Symposium n. 31, 337.

Bottcher, C., 1970, personal communication.

Bottcher, C., McCray, R., Jura, M., and Dalgarno, A., 1970, Astrophys. Letters, 6, 237.

Braginskii, S. I., 1965, Reviews of Plasma Physics, ed. M. A. Leontovich (New York: Consultants Bureau), 1, 205.

Bridle, A. H. and Venugopal, V. R., 1969, Nature, 224, 545.

Cox, D. F., and Tucker, W. L., 1969, Astrophys. J., 157, 1157.

Cox, D. P. and Daltabuit, E., 1971, Astrophys. J., 167, 113.

Dalgarno, A., 1958, Phil. Trans., A250, 426.

Dalgarno, A. and Dickinson, A. S., 1968, Planet. Space Sci., 16, 911.

Duprée, A. K., 1971, to appear in The Gum Nebulae and Related Problems, edited by S. P. Maran, J. C. Brandt, and T. P. Stecher, NASA.

Fermi, E., 1949, Phys. Rev., 75, 1169.

Field, G. B., 1965, Astrophys. J., 142, 531.

Gottesman, S. T. and Gordon, M. A., 1970, Astrophys. J., 162, L93.

Gould, R. J., 1971, Astrophys. Letters, 8, 129.

Jura, M. 1971, Ph.D. Dissertation, Harvard University (unpublished).

Kulsrud, R. M. and Pearce, W. P., 1969, Astrophys. J., 156, 445.

Kulsrud, R. M. and Ferrara, A., 1971, to be published in Astrophys. and Space Sci.

McCray, R. and Schwarz, J., 1971, to appear in The Gum Nebulae and Related Problems, edited by S. P. Maran, J. C. Brandt, and T. P. Stecher, NASA.

Morrison, P. and Sartori, L., 1969, Astrophys. J., 158, 541.

Oort, J. H., 1970, Astron. and Astrophys., 7, 381.

Peek, J. M., 1966, Phys. Rev., 143, 33.

Reynolds, R. J., Roesler, F., Scherb, F., and Boldt, E., 1971, preprint.

Spitzer, L., 1967, Physics of Fully Ionized Gases, (Interscience, New York).

Spitzer, L., 1968, Diffuse Matter in Space, (Interscience, New York).

Weyman, R., 1967, Astrophys. J., 147, 887.

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