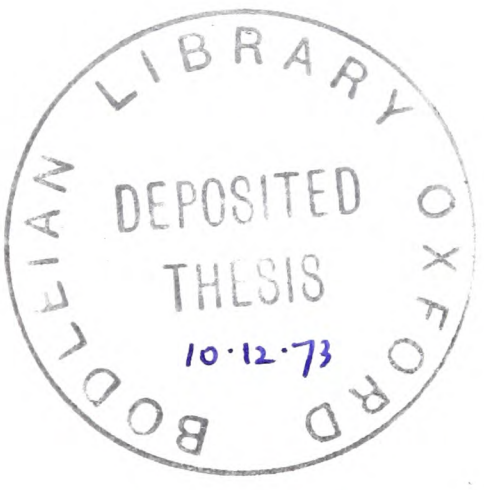
THE THEORY OF INTERNATIONAL TRADE IN CAPITAL GOODS

Murdo Alasdair Macdonald Smith
University College, Oxford



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The central concern of this thesis is to identify and analyse the circumstances in which international trade in second-hand machines will take place, and to describe the consequences of such trade. It turns out that this topic is not so esoteric as it may initially seem, and part of the thesis is devoted to exploring alternative models of trade in capital goods, and to showing the extent to which all such models exhibit common features.

The method of approach is theoretical and largely mathematical, although some empirical data from secondary sources are presented.

A survey of discussions of the desirability of underdeveloped countries importing second-hand machines reveals considerable differences of opinion, and the absence of a consistent theoretical treatment. The larger part of Chapter 1 is taken up by a theoretical analysis of international trade in vintage models of capital formation. Within a unified framework of perfect competition and perfect foresight, a wide range of technical assumptions can be treated, and their economic consequences analysed. Fairly weak assumptions lead to the conclusion that the existence of factor price differentials will cause countries with lower wage rates to specialise exclusively in the use of old machines. The rather meagre empirical evidence available, of which a major part is evidence of intranational trade in Japan, is consistent with the hypothesis that factor prices differentials are the main force underlying this trade, although the evidence is by no means conclusive. It seems a reasonable conclusion that it is a pervasive feature of vintage models with factor price differentials that trade in second-hand machines takes place and that there is a tendency for particular countries to specialise in the use of particular vintages.

At this level of generality, however, not much more may be said. In order to investigate more deeply the implications of trade in vintage models, it is necessary to concentrate on more rigidly specified cases. Chapters 2 and 3 analyse steady states in the model in which the technical specifications of the only type of machine available are exogenously determined and there is labour-augmenting embodied technical progress: the 'clay-clay' model. With two countries growing at the same steady rate, the country with the lower wage rate and higher profit rate uses only second-hand machines. To analyse the effects of trade, we need to make some assumption about saving behaviour so that comparisons between steady states with free trade and steady states in autarchy may be made. In the literature on dynamic trade models, one of two assumptions is normally chosen: that (gross) saving rates are kept fixed, or that profit rates are fixed. In vintage models there is a third potential candidate, the net saving rate, but it is here shown that it is unsuitable, not providing a well-defined description of saving behaviour.

Chapter 2 adopts the fixed gross saving rate assumption and establishes that if the two countries have saving rates sufficiently far apart for factor price equalisation not to occur and if there is convergence to steady state, then trade will in the long run raise the consumption level in the high saving country which specialises in new machines, and raise the wage rate and lower the profit rate in the low saving country which specialises in old machines. It may allow full employment in the low saving country even if in autarchy it was unable to sustain full employment. Examples show that consumption in the low saving country may be lowered by trade, and the factor price ratio in the high saving country may move in either direction.

The alternative assumption that profit rates are fixed ('classical saving') is analysed in Chapter 3, where trade is shown to raise wage rates in both countries, and to affect consumption through a combination of three effects: (a) static gains from trade tend to raise consumption in both countries, (b) the country with the higher profit rate specialises in old machines so tending to raise its immediate consumption and reduce its long run consumption, while the other country does the opposite, if each country has an efficient saving objective, (c) trade tends to reduce the consumption of the more inefficient country to the benefit of the one with the higher profit rate, if there is inefficient saving.

Chapter 4 analyses similarly the putty-clay model, in which there is the possibility of choice of technique. Remarkably, the fact that the low wage country now has the possibility of constructing machines more technically labour intensive than those in use elsewhere does not alter the pattern of trade: in this case also, the only machines it uses are second-hand machines imported from the high wage country.

A major point of interest in all three chapters is the effect labelled (b) above: the fact that trade in second-hand machines typically is associated with intertemporal substitution of consumption. This phenomenon has been noted in the literature on trade in the two-sector model, and Chapter 5 aims to show that it is a typical feature of models of trade in capital goods. The pattern of trade in the vintage models is shown to be analogous to the pattern in the two-sector model and in linear models. At first sight this aspect of trade may seem far removed from traditional trade theory, but in fact it is readily rationalised: countries with high profit rates and low saving rates are like impatient consumers, and trade allows them to reduce the capital intensity of their production and substitute consumption now for consumption later.

It emerges from some examples in Chapter 2 and from the analysis of Chapter 5 that the classical saving assumption that steady state saving programmes are characterised by fixed profit rates is in several respects more satisfactory and illuminating than the assumption of fixed saving rates.

There are many limitations to the methods used in the thesis: neither saving assumption is likely to be an accurate description of reality; the assumption throughout that both countries have the same steady growth rate is implausible; there are no transport costs; there is no real uncertainty; comparisons are made only between free trade and autarchy, with no discussion of tariffs; there is no discussion of the stability of steady states; the vintage models of Chapters 2 to 4 are all one-sector models; and producers are assumed to be perfectly competitive and perfectly prescient. But the most important limitation is the absence of the sort of empirical evidence that would permit one to reach detailed policy conclusions: evidence on the existence of significant externalities, on the input requirements of different machines (e.g. the skill requirements of maintenance), and on the hypothesis of ex-post absence of substitutability.

The thesis cannot therefore produce detailed practical recommendations, or blanket endorsement or condemnation of imports of used machines. Rather the aim is to clarify the nature of the issues involved and show what sort of considerations are relevant, to describe the pattern of trade that may usually, though not invariably, be expected to emerge, and to show that trade in models of capitalist production typically involves issues somewhat different from, though related to, the traditional concerns of trade theory.

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PREFACE

The title of this thesis appears to promise a work of considerable generality: a casual glance at its contents will reveal that the central issue discussed is apparently a very limited one, the question of under what circumstances trade in second-hand capital goods will or should take place. I offer two justifications for the title. The first is that the vintage approach to modelling capital is the natural approach to adopt when one's concern is with a topic like trade in machines. It is one of the main conclusions of this thesis that in vintage models, trade in machines quite typically takes the form of trade in second-hand machines. The second justification is that I do examine some non-vintage models, and find that the pattern of trade and its effects is strikingly consistent throughout three very different types of models.

Chapter 1 looks at past advocacy of and opposition to importing of second-hand machines by underdeveloped countries and at the meagre empirical data on the subject. A theoretical approach to trade in vintage technologies is presented. Chapters 2 and 3 focus on a particular model: the clay-clay model with labour-augmenting technical progress. In this model, a country with a lower wage rate specialises, if trade is permitted, in the use of second-hand machines. The consequences of trade for factor prices and consumption levels in steady state are examined, on the hypothesis that countries keep their gross saving rates fixed, and the alternative hypothesis that they keep their profit rates fixed. Chapter 4 extends this analysis to the putty-clay model, in which it emerges that trade in second-hand machines is as dominant a feature as it is in the clay-clay model. The last substantive

chapter, Chapter 5, looks at alternative models of trade in capital goods, and the effect in them of alternative saving assumptions. The brief concluding chapter summarises my conclusions.

With the exception of part of Chapter 1, this work is entirely theoretical, and the theory is presented mathematically, although no mathematics more high-brow than advanced calculus is used. My aim is not to provide conclusive answers to particular problems, but to show what are the relevant issues to be considered by policy-makers, and to add to our understanding of the processes involved in international trade in capital goods.

* * * * *

I have accumulated several debts in the process of planning and writing this thesis. Frank Hahn suggested that the problems surrounding trade in second-hand capital might be interesting. I am grateful to the Master and Fellows of University College for the opportunity to work and study in Oxford, and for keeping my teaching duties at a level which allowed me to undertake this research. Some aspects of the work were discussed in informal seminars in Oxford and at the London School of Economics, from which I received useful and stimulating feedback. My greatest debt is to my supervisor, James Mirrlees. I am extremely grateful for the time and energy he spent discussing this work with me, and for his many valuable suggestions.

There are three published articles to which my indebtedness is sufficiently great to warrant acknowledgement here: the articles referred to as Solow et al. (1966), Bliss (1968), and Stiglitz (1970a). I am also grateful for access to the unpublished thesis referred to as James (1970).

I should also like to thank Ronny Morse, Luba Mumford, Gerry Preece, and Barbara Silver for their joint efforts in producing this typescript in a very short time.

Alasdair Smith

London School of Economics

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CHAPTER 1

Second Hand Machines and Underdeveloped Countries

1.1. The existing literature

The discussion of the economic problems of underdeveloped countries has over the last two decades concentrated to a great extent on the choice of techniques of production. Those theoretical studies which emphasise the abundance of labour and shortage of capital typically conclude that the appropriate choice is of techniques more labour intensive than those in use in developed countries, even when allowance is made for the need to build up surpluses for investment. (See, for example, the introduction to Sen (1968).[1])

One of the main problems which tends to be avoided in this literature is the question of how the chosen techniques are to be implemented in practice. The implication of writing down a production function and representing the choice of technique as the costless choice of a set of input combinations is that appropriate blueprints are to be found in the local technical library; and further, the possibility that there may be strongly increasing returns to scale at low levels of output in individual capital goods industries is ignored. The literature on such problems as the transfer of technology (e.g. Chudson (1971)[2]),'intermediate technology'(e.g. Schumacher (1971)[3]), and related issues shows that real difficulties do exist.

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1 Sen, A.K., Choice of Techniques, third edition (Blackwell; Oxford, 1968).

2 Chudson, W.A., The International Transfer of Commercial Technology to Developing Countries, UNITAR Research Report 13 (United Nations; New York, 1971).

3 Schumacher, E.F., "Industrialisation through 'Intermediate Technology'", Chapter 7 of R. Robinson (ed.), Developing the Third World: The Experience of the Nineteen Sixties (Cambridge University Press; Cambridge, 1971).

Once it is recognised that there may be serious problems involved in bringing particular techniques into use in an underdeveloped country, an explicit account must be given of the source of the capital inputs to the production process: if the country is to produce its own capital goods, there may be problems about whether the scale of operation in the capital goods industry is economic; if it is to import capital goods, there may be difficulty in finding a source of supply of the appropriate kind of machine. One fairly frequently suggested solution is for underdeveloped countries to import second-hand machines from developed countries.

This is advocated in Shonfield (1960)[1], pp.164-5:

> '... much of the equipment required during the early stages of an industrialization programme could be readily provided second-hand ... It would be necessary first of all to break down the strong prejudice which exists... The main fear is that something will go wrong and that there will be no help to be obtained from the seller of the machine ... [there is] an assumed likelihood that they will be subject to more frequent faults and breakdowns ... [In India] the typical small workshop nowadays starts up with a well used machine, often in a fairly battered condition, and quickly learns the tricks of improvisation, which seem to keep it running and producing against all odds.'

Similar arguments are put forward in Bhagwati (1966)[2], p.195:

> 'Owing to rapid obsolescence, many advanced countries scrap machinery when its technical life is yet unspent. Underdeveloped countries could import such equipment and find it more economical than buying new machines. When machines have been scrapped by industries in countries where wages are relatively high, their use in the underdeveloped countries where labour is relatively cheaper, should also be economical. The possibility of such advantageous imports extends also to durable consumer goods where new models send the prices of the older models tumbling down in affluent countries such as the United States.

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[1] Shonfield, A. The Attack on World Poverty (Chatto and Windus; London, 1960).

[2] Bhagwati, I The Economics of Underdeveloped Countries (Weidenfeld and Nicholson, World University Library; London, 1966).

> There are only two difficulties which seem to reduce the attractiveness of such imports. First, there may be difficulties in getting spare parts for old machinery and durables. Second, the equipment is likely to have been roughly used, in view of its rapid obsolescence, and may demand considerable maintenance which may be a scarce and expensive resource in underdeveloped countries. These factors may well outweigh the advantages of employing used machines and durables, but not necessarily. To those who have seen the rapid growth of small, machine and metal-working units in the underdeveloped countries, and the facility with which old buses and equipment are kept going in these areas, there seems great room for optimism.
>
> In fact, it would be highly profitable for the developing countries to look systematically into possibilities of importing used equipment and to work out the relevant economics in each specific context. It may even be useful, in this connection, to explore the Army Surplus Disposal programmes, which are an immensely rich source of bargains in the United States, for example. An imaginative and careful approach to this unexplored question could pay very rich dividends.'

For deeper analyses of the issues involved we can turn to Netherlands Economic Institute (1958)[1] (referred to as N.E.I. below), Sen (1962)[2], Waterston (1964)[3], United Nations (1966)[4], and James (1970)[5].

It might appear that the obvious first step is to list the advantages and disadvantages associated with the use of second-hand machines, as is done in the reports of the U.N. and the N.E.I. Sen's analysis, however, shows that the classification of characteristics into advantages and disadvantages is dependent on the economic circumstances of the user, so we had better write all of the characteristics in one list. Not all the characteristics listed below

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1 Netherlands Economic Institute, Second-Hand Machines and Economic Development (Division of Balanced International Growth, publication no.15/58; Rotterdam, 1958).

2 Sen, A.K., "On the Usefulness of Used Machines", Review of Economics and Statistics, 44, 3, 1962.

3 Waterston, A., "Good Enough for Developing Countries?", Finance and Development, 1, 2, 1964.

4 United Nations, Centre for Industrial Development, Report of Expert Group on Second-Hand Equipment for Developing Countries, ST/CID/8, 66.II.B.9 (United Nations; New York, 1966).

5 James, D.D., The Economic Feasibility of Employing Used Machinery in Less Developed Countries, Ph.D. thesis, Michigan State University (unpublished, 1970).

are noted in each of the references above, but I indicate the source only when there are conflicting or partially conflicting statements.

Characteristics of Second-Hand Machines

1. Cheaper (and cheaper in foreign exchange).
2. More labour intensive.
3. Smaller in scale and more flexible in use.
4. Have shorter life
5. Less efficient, reliable, and precise.
6. Require less skill (N.E.I., U.N., James); require more skill (Waterston).
7. Lower operating costs (U.N.); raise operating costs (James).
8. Simpler to maintain (U.N.); require more maintenance and overhauls (N.E.I., U.N., James).
9. Easier to manufacture spare parts (U.N.); less easy to buy spare parts (N.E.I., Waterston, U.N., James).
10. Shorter delivery lag (N.E.I., Waterston, U.N., James); higher transport and transaction costs (N.E.I., James).
11. Enhance learning by doing (U.N., James); widen technological gap (U.N.).
12. More difficult to finance.

Most of the items on the list are fairly straightforward, and are plausible consequences of the fact that second-hand machines suffer from deterioration with age, or obsolescence as machines built to more modern specifications come into use. But some may require a little explanation.

Among the 'transaction' costs noted by the N.E.I. study are the costs of finding suppliers or buyers, collection of goods, grading and

pricing of goods, distribution, and storage. These problems are apparently considered serious enough for the Institute to recommend that trade concentrate on types of machines where the market is not thin, e.g. well known brands of which there are many machines in use.

Why the use of second-hand machines should widen the technological gap is perhaps not obvious. Paragraph 70 of the U.N. report reads:

> 'Proponents of the complete prohibition of second-hand equipment for use in developing countries affirm that used equipment will slow down economic development by saddling countries undergoing industrialization with an obsolete technology. Such technology would make impossible effective competition with industrialized countries, maintain the technological gap between developing and developed countries, slow down the training process of technical cadres, reduce possible productivity gains, and effectively relegate developing countries to a permanent second-class status as economic powers.' (p.12)

A rather more sophisticated version of this argument is advanced in Todaro (1970)[1], where it is argued that using second-hand machines ties underdeveloped countries to the labour-saving technical progress of the developed countries, where the introduction of capital saving domestic technologies would give a better prospect for the long run solution of the problems of underdeveloped countries.

It should be mentioned that one of the main reasons why there are problems in financing the use of second-hand machines is purely institutional: many countries have import restrictions (U.N. Report, para. 87), and many aid donors discriminate against second-hand machines (paras. 102, 103). (One of the reasons for this discrimination is to prevent currency smuggling by the device of 'buying' worthless imports at grossly inflated prices from a foreign collaborator, a

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[1] Todaro, M.P., "Some Thoughts on the Transfer of Technology from Developed to Less Developed Nations", Eastern Africa Economic Review, 2, 1, 1970, pp.53-64.

practice which is easier when the imports do not have published price lists.)

The final, and most important, comment which must be made is that the various characteristics attributed to second-hand machines are not necessarily independent of one another, nor of the circumstances in which the machine is used. In particular, the lower price of second-hand machines is a result of the impact of the other characteristics. Second-hand machines will be of particular interest to underdeveloped countries when the conditions prevailing in these countries are such that the prices at which developed countries supply second-hand machines are sufficiently low to make their use attractive.

Different authors have given different examples of such a set of circumstances. The fact that the cheapness of labour may reduce the effect of increased maintenance costs is noted by James, the U.N., and the N.E.I. That the lower price is especially important for countries short of capital and of foreign exchange is noted by James and by the N.E.I., and particularly by Sen, who considers the case of older machines being differentiated only by being nearer their physically determined scrapping date and therefore having a lower price. The N.E.I. observes that older technologies may be land-intensive and therefore appropriate for use where rents are low. The U.N., the N.E.I., and James all mention the fact that an underdeveloped economy may attach less weight to having high quality output.

On the other hand, Waterston points out that where old machinery requires skilled labour which is in short supply it may be inadvisable for an underdeveloped country to purchase it.

The U.N. and James both put a great deal of emphasis on the argument that it is possible to say little of much generality. The

U.N. Report cites examples of bad bargains, like first generation computers. James finds it necessary to look for a multiplicity of special factors, like flexibility and learning effects, to explain why trade takes place.

This is the basic issue that I shall discuss later in this chapter, after a brief survey of the empirical importance of the question in the next section. Is it indeed the case that little can be said in general? Or can the main features be captured in a unified theoretical discussion?

1.2. Some empirical data

It might seem that the obvious next step is to look at empirical data on the characteristics of actual machines. Unfortunately, data of this kind does not appear to be available: the example given in the N.E.I. study, the discussion of the economics of maintenance in Strassman (1968)[1], and the examples used by James all depend on arbitrary, if plausible, assumptions about the prices and the technical characteristics of second-hand machines. It is not clear what useful conclusions can be drawn from hypothetical examples. The price data from the second-hand market for some tractors given in Schwartz (1971)[2] does no more than show the imperfection of this particular market, since retail, wholesale, and auction prices are widely separated and do not move together over time.

We have,then, to turn to observation of trade flows rather than direct observation of the reasons for the flows. International trade in second-hand machines appears to have occurred at least as long ago

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[1] Strassman, W.P., "Maintenance, Durability, and Secondhand Equipment", chapter 6 of Technological Change and Economic Development (Cornell University Press; Ithaca, N.Y., 1968).

[2] Schwartz, S.L., "Second-Hand Machinery in Development, or how to recognize a bargain", University of British Columbia, Dept. of Economics, Discussion Paper 75, December 1971 (forthcoming in Journal of Development Studies).

as the end of the nineteenth century. Baranson (1969)[1] refers on p.9 to Japanese imports of second-hand textile machinery from England (although the most striking feature of the Japanese experience, described in Ranis (1957)[2],was their ability to adapt any machines to highly labour-intensive working methods).

International trade in used machines is a striking feature of the contemporary Japanese economy. Shinohara (1962)[3] gives the following information on the use of used machines (p.24), based on the Japanese Census of Manufactures:

TABLE 1

Proportion of Second-Hand Fixed Assets in Total Purchases of Fixed Assets in Manufacturing Firms in Japan

Size of firm by number of employees	1954 %	1955 %	1956 %	1957 %
4 - 9	48.8	40.2	34.3	41.0
10 - 19	44.1	40.8	29.9	35.0
20 - 29	39.5	34.3	28.7	30.5
30 - 49	35.0	28.9	26.1	26.4
50 - 99	31.5	22.0	22.3	22.1
100 - 199	23.0	16.3	16.8	15.2
200 - 299	15.2	9.1	9.9	9.3
300 - 499	13.9	10.1	9.1	7.4
500 - 999	11.2	5.2	4.2	4.6
1000 +	4.6	4.1	4.9	3.3
Total	18.3	14.3	12.3	10.9

This table is perhaps most illuminating when accompanied by data on factor prices and productivity. This information is given in

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[1]Baranson, J., Industrial Technologies for Developing Economies (Praeger; New York, 1969).

[2]Ranis, G., "Factor Proportions in Japanese Economic Development", American Economic Review, 47 (September 1957), pp.594-607.

[3]Shinohara, M., Growth and Cycles in the Japanese Economy (Hitotsubashi University; Tokyo, 1962).

Taira (1970)[1], pp.170-171, from which the table below is adapted.

TABLE 2

Wages, Factor Productivities, and Capital Intensity

by Size of Firm in Manufacturing, 1957[a]

Size of firm[b] (Persons)	Wages[c] (¥'000)	Factor[d] productivities Labor (¥'000)	Capital (Percent)	Rate of profit[e] (Percent)	Capital intensity of labor (¥'000)[f]
Total (or Average)	16	43	13.3	8.3	324
1 - 3	9	13	13.9	4.3	93
4 - 9	10	18	18.6	8.2	97
10 - 19	11	23	25.6	13.3	90
20 - 29	12	26	26.9	14.4	97
30 - 49	12	29	28.4	16.7	102
50 - 99	13	35	25.8	16.2	136
100 - 199	14	41	22.1	14.5	186
200 - 299	16	47	20.2	13.3	233
300 - 499	17	58	16.8	11.9	345
500 - 999	19	65	14.6	10.3	447
1000 +	25	77	10.0	6.8	769

a All data quoted or computed from A Statistical Handbook of Small and Medium Enterprises (Tokyo: Ministry of Finance Printing Office, 1959), p.38.

b Classified by the number of employed persons, including proprietors, family workers, and paid employees.

c Average monthly earnings per paid employee.

d Net value added divided by the respective factor. Labor refers to all the employed persons in the firm; capital to the value of fixed assets.

e The net value added less labor cost divided by the value of fixed assets.

f The value of fixed assets per employed person

Strassman's survey of the use of second-hand equipment in Mexico and Puerto Rico shows the same pattern, although most of the used

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[1] Taira, K., Economic Development and the Labor Market in Japan (Columbia University Press; New York, 1970).

machinery in these countries appeared to have been bought from the U.S.A. rather than from within the countries themselves, as in the case of Japan. Half of the Mexican firms, and three-fifths of the Puerto Rican firms surveyed used second-hand equipment in major processes. Use of second-hand machines was concentrated 1) in producers of durable goods, apparently because of the need for volume, speed, and quality in many production processes for nondurables; 2) in small firms; 3) in subsidiaries of foreign companies, some of whom bought used machines from their parent companies; 4) in companies located so as to have easy access to U.S. suppliers of machines and spare parts. The overall proportion of firms in Strassman's survey using second-hand equipment is, as we should expect, much greater than the proportion in Japan in the mid-1950's (although distance from suppliers may be as important a determinant of this difference as relative factor prices). In short, these observations are perfectly consistent with the issues discussed in the previous section.

Data on international trade is provided by the U.N. and by James. The U.N. study estimates that the annual turnover of the U.S. market in second-hand metal-working equipment was about $500 million in 1965, if which about 2.5% was exported to underdeveloped countries. Total annual demand by underdeveloped countries was estimated, it is not stated how, at about $1200 million. (Use of values rather than physical volume is rather unsatisfactory and should make one wary of attaching too much significance to these actual numbers.) Since the U.S. dominates this field it seems unlikely that the proportion of second-hand equipment of this type imported by underdeveloped countries could be of a higher order than, say, 5%, even when we take into account inter-governmental transfers channelled through the U.S. Agency for International Development.

James however arrives at considerably higher estimates from U.S. trade statistics in the four categories for which statistics of trade in second-hand equipment have been published. From January 1965 to July 1969, 34% of all exports of metalcutting, metalworking, sewing, and papermaking equipment by the U.S. to underdeveloped countries were of second-hand equipment. After allowing for the domestic construction of up to one third of their equipment by these countries, and for the fact that the U.S. may be atypical because of its proximity to Latin America, James suggests that if these categories of machine are typical (and they do constitute, it is known, a large proportion of the manufacturing capital stock of underdeveloped countries), then something like 10% to 20% of the manufacturing capital stock in underdeveloped countries is supplied by imports of second-hand machines.

Whatever weight we choose to give to such admittedly crude estimates, the trade statistics themselves give a useful indication of the relative importance of used machines for underdeveloped countries, in that the figure of 34% of imports by underdeveloped countries being of used machines compares with a corresponding figure of 16% for developed countries.

To this data on the importance of second-hand machines to underdeveloped countries I now wish to add some observations of a rather different kind: observations of the fluctuating attitudes of the agencies of the United Nations to this subject, evidence of a considerable degree of confusion.

The 1966 report cited above recommended lifting restrictions on trade, improving information flows in the market, and further studies. It got a fairly lukewarm reception in the Committee for Industrial Development, and James reports that by 1967 all work on this topic in

the U.N. Industrial Development Organisation (UNIDO) had stopped.

References to the subject, however, appear in several UNIDO publications. In UNIDO (1969b)[1], a monograph on the iron and steel industry, an example is given on p.52 of a semi-integrated steel mill (i.e. one using scrap metal as input) where the use of second-hand capital might reduce the total investment cost by 15 per cent although this reduction is said to 'be insufficient to make the plant much more competitive'.

The development of UNIDO's reported views on the use of second-hand equipment in the textile industry is instructive. The report of a symposium in Athens in 1967, UNIDO (1969a)[2], said that 'the use of second-hand equipment could seldom be justified and could not be advocated generally ... its use in individual cases might be feasible' (p.72), while the monograph based on the same symposium, UNIDO (1969c)[3], has rather changed the emphasis:

> 'The rapid pace of technical development in the industrialized countries during recent years has sometimes led to machinery being scrapped when only about five years old in order to make way for even more efficient models. The purchase of such used equipment, if in good condition, should be of particular interest to a development [sic] country, since it would save foreign exchange; in any case, the higher output that could be achieved with the most up-to-date machinery might not be marketable. The potential disadvantages, however, should also be recognized ...' (pp.47-48).

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[1] UNIDO, Iron and Steel Industry, Monographs on Industrial Development, no.5, ID/40/5, E.69.II.B.39, vol.5 (United Nations; New York, 1969).

[2] UNIDO, Report of the International Symposium on Industrial Development, Athens 1967, ID/11, E.69.II.B.7 (United Nations; New York, 1969).

[3] UNIDO, Textile Industry, Monographs on Industrial Development, no.7, ID/40/7, E.69.II.B.39, vol.7 (United Nations; New York, 1969).

The UNIDO manual on joint venture agreements, UNIDO (1971)[1], recommends that the developing country's share of the joint venture should insist on machinery and equipment '(a) of the latest design, ... (e) in accordance with the highest specifications of quality production.' (p.46).

In 1970, UNIDO reported its intention (in the unpublished document ID/B/44) to set up a project which would provide information to developing countries on the availability of used equipment. In a letter of 28 August 1972, the Industrial Information Section of UNIDO informed me that 'these plans were not implemented since after due consideration and discussions during different meetings, many countries receiving UNIDO assistance were not in favour of this project.'

Here then is a topic of considerable practical importance. The major international agency for development which has taken an interest in the topic appears to have difficulty arriving at a consistent approach.

The rest of this chapter is devoted to the description and analysis of a theoretical framework in which, I hope, the principal features of these problems are captured. It is, even in principle, impossible to set up a model whose assumptions cover every conceivable eventuality.

1.3. The theory of trade in used machines

Assumptions and definitions

Consider two countries, indexed $i = 1, 2$, whose wage rates at time t are respectively $w_1(t)$, $w_2(t)$, always strictly positive. A type of machine exists whose technical characteristics are defined by the fact that a machine constructed at time v ('of vintage v') and used at time t produces an output flow $f(t, v)$ and requires a labour input flow $g(t, v)$. These functions are independent of the country in which the machine is used, and of the history of the machine, and there are no

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[1] UNIDO, Manual on the Establishment of Industrial Joint Venture Agreements in Developing Countries, ID/68, E.71.II.B.23 (United Nations; New York, 1971).

transport costs.

The following assumptions are made:

(A1) $f(t, v)$ and $g(t, v)$ are continuous in v for every t;

(A2) for every t, $f(t, v)$ is nondecreasing in v and $g(t, v)$ nonincreasing in v.

A producer using the machine in country i gets a stream of net earnings, 'quasi-rent', given by

$$\rho_i(t, v) = f(t, v) - w_i(t)\, g(t, v) \ .$$

The assumptions above imply that for each i and for every t, $\rho_i(t, v)$ is nondecreasing and continuous in v. A third assumption is:

(A3) for every v, $w_i(t)$, $r_i(t)$ and $\rho_i(t, v)$ are all continuous in t.

These assumptions can be, and will later be, altered and relaxed; an innocuous extra assumption about f and g will be added below, where the context will make its meaning clear.

Among the technologies which satisfy these assumptions is the following set of cases:

$$f(t, v) = f_1(t - v)\, f_2(v), \quad g(t, v) = g_1(t - v)\, g_2(v),$$

where f_1 and g_2 are nonincreasing, f_2 and g_1 are nondecreasing. f_1 and g_1 represent deterioration with age, f_2 and g_2 represent embodied technical progress and obsolescence. If $f_1 = g_1 = 1$, then the assumptions about f_2 and g_2 are equivalent to the assumption that the rate of labour augmenting progress equals or exceeds the rate of capital augmenting progress.

Particular examples of this technology are:

$f(t, v) = e^{-\mu(t-v)}$, $g(t, v) = e^{\lambda(t-v)}$, pure exponential deterioration with age;

$f(t, v) = e^{\mu v}$, $g(t, v) = e^{-\lambda v}$, pure exponential embodied technical progress, capital augmenting at rate μ and labour augmenting at rate $\lambda + \mu$.

Competitive allocation of machines with perfect foresight

Let S_i be the set of dates on which a particular machine will be used in country i and let T_i be the set of dates on which it will be owned but not used in country i. For the sake of definiteness and without loss of generality, let T_1 include all dates beyond the scrapping date of the machine.

Assume perfect foresight. Then competitive allocation requires that the price of this particular machine, of vintage v, should at time t be the discounted value of future quasi-rents, where the quasi-rent earned by an unused machine is zero, and the discount rate is that appropriate to the owner of the machine, i.e. price is, at time t,

$$P(t, v) = \int_t^{\infty} q(u, v)\, e^{-R(u,t)}\, du \qquad (1)$$

where $q(u, v) = \rho_i(u, v)$ $\quad u \in S_i$,

$q(u, v) = 0$ $\quad u \in T_i$,

$$R(u, t) = \int_t^u r(s)\, ds,$$

$r(s) = r_i(s)$ $\quad s \in S_i \cup T_i$.

$r_i(s)$ is the discount rate or profit rate in country i at time s. (The implication of wage and profit differentials is that international flows of labour and investment are assumed not to occur. The implication of only w and r being different between countries and of $f(t, v)$ being independent of i is that there are only two nontraded factors.)

Competitive equilibrium further requires that any other future pattern of use and ownership should have a present value no greater

than the competitive present value, i.e.

$$\int_t^\infty q'(u, v)\, e^{-R'(u,t)} du \leq P(t, v) \tag{2}$$

for all other partitions of (t, ∞) into S_i, T_i, and their corresponding q', R'.

The continuity of $w_i(t)$, $\rho_i(t, v)$ means that the fundamental theorem of calculus may be applied to give

$$\begin{aligned} \frac{\partial P(t, v)}{\partial t} &= r_i(t)\, P(t, v) - \rho_i(t, v) && t \in S_i, \\ \frac{\partial P(t, v)}{\partial t} &= r_i(t)\, P(t, v) && t \in T_i, \end{aligned} \tag{3}$$

well-known equilibrium conditions.

Theorem 1.1. Let $q_i(t, v) = \max \{\rho_i(t, v), 0\}$. Then

$$\begin{aligned} \frac{\partial P(t, v)}{\partial t} &= \min_{i=1,2} \{r_i(t)\, P(t, v) - \rho_i(t, v),\ r_i(t)\, P(t, v)\} \\ &= \min_{i=1,2} \{r_i(t)\, P(t, v) - q_i(t, v)\} . \end{aligned}$$

Proof. Because of the equalities (3) proved above for $t \in S_i$ and $t \in T_i$, all we need show is that it is never the case that

$$\frac{\partial P(t, v)}{\partial t} > \min_i \{r_i(t)\, P(t, v) - q_i(t, v)\} .$$

Suppose that this inequality holds at some time t_o. By continuity there exists $\delta > 0$ and $t_1 = t_o + \delta$ such that the inequality holds on (t_o, t_1).

Now $P(t, v)$ is the competitive price, by hypothesis. Consider the following alternative future life for the machine. From t_o to t_1 the machine is allocated to use and ownership so that the price $Q(t, v)$ satisfies

$$\frac{\partial Q(t, v)}{\partial t} = \min \{r_i(t)\, P(t, v) - q_i(t, v)\} .$$

From t_1 onwards the machine is allocated competitively, so

$$Q(t_1, v) = P(t_1, v) .$$

The fact that P(t, v) is the competitive price implies that

$$Q(t, v) \leq P(t, v) \qquad \text{for } t \in [t_o, t_1).$$

Therefore when $t \in [t_o, t_1)$

$$\frac{\partial Q(t, v)}{\partial t} = \min \{r_i(t)\, P(t, v) - q_i(t, v)\}$$

$$< \frac{\partial P(t, v)}{\partial t} ,$$

and $$Q(t_1, v) = Q(t_o, v) + \int_{t_o}^{t_1} \frac{\partial Q(t, v)}{\partial t}\, dt$$

$$< P(t_o, v) + \int_{t_o}^{t_1} \frac{\partial P(t, v)}{\partial t}\, dt = P(t_1, v) .$$

Contradiction. The assumed inequality may not hold at any time. Q.E.D.

The theorem, loosely stated, demonstrates that competitive forces at every point in time direct the machine into that situation in which capital gains are minimised or losses maximised because that is the situation in which factor costs are minimised.

Now if machines of this type are in use in each country (as is the case in any economic model where machines of a particular type are essential to the economy, for example the one sector fixed proportions vintage model discussed in Chapter 2) then for each i there exists at least one v such that

$$\frac{\partial P(t, v)}{\partial t} = r_i(t)\, P(t, v) - \rho_i(t, v) \ .$$

Let
$$k(t, v) = r_1(t)\, P(t, v) - \rho_1(t, v) - r_2(t)\, P(t, v) + \rho_2(t, v)$$
$$= (r_1(t) - r_2(t))\, P(t, v) + (w_1(t) - w_2(t))\, g(t, v) \ . \qquad (5)$$

The fact that for each i there is at least one v for which a machine is in use in country i means that there exists a value of v such that $k(t, v) \geq 0$ and a value such that $k(t, v) \leq 0$. Now f and g are continuous in v, so that P is continuous in v and therefore k(t, v) is continuous in v. There exists for each t at least one v_o such that $k(t, v_o) = 0$.

If at time t, $w_1(t) > w_2(t)$, then

$$0 = k(t, v_o) = (r_1(t) - r_2(t))\, P(t, v_o) + (w_1(t) - w_2(t))\, g(t, v_o) \ ,$$

and
$$r_1(t) < r_2(t) \ .$$

Similarly, $r_1(t) < r_2(t)$ implies $w_1(t) > w_2(t)$.

Now g(t, v) is nonincreasing in v, and since $\rho_i(t, v)$ is nondecreasing in v, P(t, v) is nondecreasing in v. $\frac{\partial P(t, v)}{\partial v} = 0$ and $P(t, v) > 0$ only when a range of vintages exists such that the technical characteristics of the range of vintages are independent of v for all future times for which the machine is valuable. I assume that this is not true, i.e. that for all v in the neighbourhood of v_o there exists some $\tau > t$ such that $g(\tau, v_o) \neq g(\tau, v)$ or $f(\tau, v_o) \neq f(\tau, v)$ and $P(\tau, v_o) > 0$. I refer to this above as an innocuous assumption because if a range of vintages is effectively identical at all points in the future there is no loss of generality in redefining them all to have the same v.

Therefore, P(t, v) is strictly increasing and k(t, v) is strictly decreasing in v at v_o, and for this t, v_o is unique. $k(t, v) > 0$ for

$v < v_o$, and $k(t, v) < 0$ for $v > v_o$.

Now replace assumption (A2) above with the following assumption:

(A2') for every t, $\frac{f(t, v)}{g(t, v)}$ is nondecreasing in v, and either

(i) $\frac{g(u, v)}{g(t, v)}$ or (ii) $\frac{f(u, v)}{f(t, v)}$ is nondecreasing in v, for all $u \geq t$.

If this assumption is satisfied with (i) being true, let

$$Q(t, v) = \frac{P(t, v)}{g(t, v)} ,$$

so
$$\frac{1}{g(t, v)} \frac{\partial}{\partial t} (g(t, v)\, Q(t, v)) = \frac{1}{g(t, v)} \frac{\partial P(t, v)}{\partial t}$$

$$= \frac{1}{g(t, v)} \min \{r_i(t)\, P(t, v) - q_i(t, v)\}$$

$$= \min \{r_i(t)\, Q(t, v) - \frac{q_i(t, v)}{g(t, v)}\} .$$

Now $Q(t, v) = \int_t^\infty \frac{q(u, v)}{g(u, v)} \frac{g(u, v)}{g(t, v)} e^{-R(u,t)} du$, and since

$\frac{\rho_i(u, v)}{g(u, v)} = \frac{f(u, v)}{g(u, v)} - w_i(u)$, the assumption (A2') implies that P(t, v) is nondecreasing in v, and strictly increasing if we again make the additional assumption that no two vintages are identical. This case now has exactly the same form as the previous case, with $\frac{1}{g} \frac{\partial}{\partial t} g.Q$ replacing $\frac{\partial P}{\partial t}$ and again there is a unique v_o such that $k(t, v_o) = 0$.

If (i) is not true but (ii) is then let

$$Q(t, v) = \frac{P(t, v)}{f(t, v)} = \int_t^\infty \frac{q(u, v)}{f(u, v)} \frac{f(u, v)}{f(t, v)} e^{-R(u,t)} du ,$$

and since $\frac{\rho_i(u, v)}{f(u, v)} = 1 - w_i(u) \frac{g(u, v)}{f(u, v)}$, the same type of argument applies and again v_o is unique.

All of this may be collected together as a theorem.

Theorem 1.2. If assumptions (A1), (A2), (A3) or (A1), (A2'), (A3) hold, and no two vintages are technically identical (as defined above), and the type of machine described is in use in both countries at time t, then $w_1(t) > w_2(t)$ if and only if $r_1(t) < r_2(t)$; and there then exists a unique vintage v_o such that

$$r_1(t)\ P(t, v) - \rho_1(t, v) \gtreqless r_2(t)\ P(t, v) - \rho_2(t, v)$$

according as $v \lesseqgtr v_o$. Country 2 uses only machines of vintages earlier than v_o; country 1 uses only machines of vintages later than v_o.

It should be noted that this theorem states that at any time the vintages in use in the low wage country will all be older than the vintages in use in the high wage country. It does not require that 'the' high wage country be the same at different times. It says nothing about the pattern of use over time of a particular machine. It is also worth noting that in the case of pure technical progress (i.e. no deterioration with age) assumption (A2') is weaker than assumption (A2). For assumption (A2) requires the output-capital ratio to rise and the labour-capital ratio to fall as v increases; whereas since g(t, v) and f(t, v) are independent of t, (A2') requires only that $\frac{f(t,v)}{g(t, v)}$ increase, i.e. the output-labour ratio to rise. (A2') allows a negative rate of capital augmenting progress, and states that newer machines are less labour intensive only in the sense of having lower labour-output ratios.

Ownership of machines

Theorem 1.3. Machines which are not being used but are still valuable are owned only in the country with the lower profit rate.

Proof. If $r_1(t) < r_2(t)$, $r_1(t)\ P(t, v) < r_2(t)\ P(t, v)$ and $\min \{r_i(t)\ P(t, v) - q_i(t, v)\} < r_2(t)\ P(t, v)$. Ownership in country 2

of an idle machine would imply $\frac{\partial P(t, v)}{\partial t} = r_2(t)\ P(t, v)$ which would contradict Theorem 1.1. Q.E.D.

We now know that at any time machines of the type we have been considering fall into at most four classes: machines owned and in use in country 1, machines owned and in use in country 2, idle machines owned in country 1, valueless machines. (As a matter of definition, country 1 has the lower profit rate and higher wage rate.)

If $\rho_1(t, v) > 0$, $r_1(t)\ P(t, v) - \rho_1(t, v) < r_1(t)\ P(t, v)$, and the machine may not be idle.

It may well be, however, that for sufficiently small $\rho_2(t, v)$, $r_1(t)\ P(t, v) < r_2(t)\ P(t, v) - \rho_2(t, v)$.

If $\rho_2(t, v) \leq 0$, $\rho_1(t, v) < 0$ since $w_1(t) > w_2(t)$ and $\frac{\partial P(t, v)}{\partial t} = r_1(t)\ P(t, v)$.

$P(t, v)$ increases with v, so all valueless machines are of earlier vintage than machines with positive prices.

Therefore the following relationships hold at time t between the different classes of machine:

1. Machines used in country 1 have positive $\rho_1(t, v)$ and are all newer than all machines in the other classes.
2. Machines used in country 2 have positive $\rho_2(t, v)$.
3. Machines with positive value but currently idle have nonpositive $\rho_1(t, v)$, and are owned in country 1 in the knowledge that future factor price changes will make them economically useful. Some of these machines may have positive $\rho_2(t, v)$, and be newer than some machines in class 2. Others may have negative $\rho_2(t, v)$; they are all older than the machines in class 2.
4. Machines with negative $\rho_i(t, v)$, $i = 1, 2$, and zero price will never again be used. They are all older than all machines in the other classes.

Again, it should be emphasised that all of this refers to the distribution of vintages at a given time t, not to the history of particular machines as t changes. As time passes, a machine may switch between classes 1, 2, and 3 many times. We do know, though, that a machine has a strictly positive price until it passes out of use for the last time; valueless machines, i.e. class 4, are never resurrected. They are scrapped.

All of these results have a fairly straightforward interpretation. The cost of producing the output stream $f(t, v)$ is the sum of imputed capital cost, the labour cost, and the capital losses, i.e. $r_i(t)\, P(t, v) + w_i(t)\, g(t, v) - \frac{\partial P(t, v)}{\partial t}$. Theorem 1.1 states that the machine indexed (t, v) is used in that country where the cost of producing output is minimised and is equal to the value of the output. This is the standard competitive result translated into the vintage model. The assumptions made about the technology imply that, in one sense or another, machines of earlier vintage are more labour intensive. If labour is relatively more expensive in country 1 and machines are used by both countries this can only be because country 1 has a lower profit rate. Then older machines cost less and (sometimes) require more labour: country 2 has a comparative advantage in economising on capital and using more labour, i.e. using older machines. Holding unused machines is an activity with a zero labour input, and is undertaken only by country 1, if at all.

The results can be generalised in several ways.

Generalisation to several countries

If there are n countries, all with different wage rates, let them be numbered so that

$$w_1(t) > w_2(t) > w_3(t) > \ldots > w_n(t) .$$

Theorem 1.1 extends immediately to give

$$\frac{\partial P(t, v)}{\partial t} = \min_{i=1,\ldots,n} \{r_i(t)\, P(t, v) - q_i(t, v)\} . \qquad (6)$$

Theorems 1.2, 1.3 extend to give the results that if the type of machine is in use in countries i and j at time t, and $i < j$, then

$$r_i(t) < r_j(t)$$

and all vintages in use in country j are older than all those in use in country i, and only the country with the lowest profit rate holds unused machines.

Several nontraded inputs

Suppose that there are m nontraded inputs. Then, in the obvious notation, quasi-rent in country i is

$$\rho_i(t, v) = f(t, v) - \sum_{k=1}^{m} w_{ik}(t)\, g_k(t, v) .$$

Let $g_k(t, v)$ be nonincreasing in v for $k = 1, \ldots, m_1$ and nondecreasing in v for $k = m_1+1, \ldots, m$, and assume similar continuity properties to those assumed above.

If $m_1 = m$, so all inputs are smaller for newer machines, then $P(t, v)$ is nondecreasing in v, and may be assumed to be strictly increasing without loss of generality. Then $w_{ik}(t) \geq w_{jk}(t)$, $k = 1, \ldots, m_1$, with at least one strict inequality implies that if this type of machine is used in countries i and j at time t then

$$r_i(t) < r_j(t)$$

and all vintages in use in country j are older than those used in country j.

If $m_1 < m$, older machines are not physically less efficient

in all respects. We have to assume that conditions in all countries are such that $P(t, v)$ is increasing in v, that $w_{ik}(t) \leq w_{jk}(t)$, $k = m_1+1, \ldots, m$, and that $r_i(t) < r_j(t)$ in order to ensure that country j uses only machines older than all machines used in country i. This is the obvious generalisation of assumption (A2). Analogous generalisations of (A2') are possible, but the multiplication of further cases seems unnecessary. In each case the result turns on the monotonicity in v of the expression analogous to $k(t, v)$.

Uncertainty

If expectations of future prices of factors and of machines change over time, as expectations of current prices turn out to be more or less mistaken, allocation of machines will be quite different from the allocation in the case of perfect foresight. I therefore do not consider the possibility of uncertainty about the course of future prices.

Some types of uncertainty about technology can be considered. Suppose that a machine built at time v has a probability $p(t, v)$ of surviving to time t. If adequate insurance markets exist, or there is no risk aversion, the price of a machine will be its present expected value:

$$P(t, v) = \int_t^\infty \frac{p(u, v)}{p(t, v)} q(u, v)\, e^{-R(u,t)}\, du$$

since $\frac{p(u, v)}{p(t, v)}$ is the probability of a machine which has survived to time t surviving to time u.

Now assume in addition to (A1, 2, 3) that $\frac{p(u, v)}{p(t, v)}$ is nondecreasing in v for all $u \geq t$, i.e. older machines are less reliable. Among the decay functions which satisfy this assumption are the exponential decay function $p(t, v) = e^{-\lambda(t-v)}$, the linear decay function $p(t, v) = \frac{T + v - t}{T}$, and the function $p(t, v) = e^{-\lambda(t-v)^2}$ in which the

rate of decay increases linearly with the age of the machine.

Then $P(t, v)$ is increasing in v, and

$$\frac{1}{p(t, v)} \frac{\partial}{\partial t} (p(t, v) P(t, v)) = \min_i \{r_i(t) P(t, v) - q_i(t, v)\} .$$

The rest of the analysis is unchanged, and again we have a unique vintage v_o, machines older than which are used only in the low wage country, newer machines being used in the high wage country.

If (A2') replaces (A2), the same modification used in the proof of Theorem 1.2 may be used to extend the result to this case also.

This result that unreliability is an argument in <u>favour</u> of the use of old machines by low wage countries may be thought to be counter-intuitive; it certainly does not appear in the literature. The interpretation of the result would appear to be that the increased unreliability is fully reflected in the decreased price, while the decreased price leads to the choice of this technique by the country with the high discount rate.

<u>Discontinuities</u>

If assumption (A1) is dropped, so that $f(t, v)$ and $g(t, v)$ are no longer continuous in v, the proof of Theorem 1.2 must be modified, although the proofs of Theorems 1.1 and 1.3 are unchanged.

$P(t, v)$ is, in general, not continuous in v. When assumption (A2) is satisfied, $k(t, v)$ is strictly decreasing in v, is nonnegative at some v and nonpositive at other v. But there need not exist a v_o such that $k(t, v_o) = 0$. However, there does exist a unique v_o such that $k(t, v) > 0$ for all $v < v_o$, and $k(t, v) < 0$ for all $v > v_o$. A similar result can be proved when (A2') replaces (A2). The only modification to the statement of Theorem 1.2 is that the equality when $v = v_o$ need not now hold.

The next step is to replace assumption (A3) by: (A3') for every v, $w_i(t)$, $r_i(t)$ and $\rho_i(t, v)$ are piecewise continuous in t. Theorem 1.1 now requires modification. The equality need not hold at points of discontinuity of $\rho_i(t, v)$, $w_i(t)$, or $r_i(t)$, for the proof of Theorem 1.1 requires continuity in the neighbourhood of t_o.

Theorems 1.1, 1.2, 1.3 all require to be modified to make it clear that the results hold almost everywhere, but not necessarily at points of discontinuity.

The assumptions could be further weakened so as to require only that P(t, v) and R(u, t) be integrable, but it is not clear that such a modification would add anything worthwhile to our understanding of the economics of these issues.

The most important example of a technology which satisfies these weaker assumptions is the following:

$$g(t, v) = a, \text{ for all } t, v; \quad f(t, v) = b, \; t - v \leq T,$$
$$f(t, v) = 0, \; t - v > T;$$

the 'one-hoss shay' machine which dies suddently at age T.

In this case P(t, v) is strictly increasing in v, for $v \geq t - T$, and P(t, t-T) = 0. k(t, t-T) is positive, and k(t, v) is continuous and strictly decreasing in v for all $v \geq t - T$, i.e. the one point of discontinuity is the scrapping vintage, and the analysis is essentially unchanged from the analysis in the continuous case. This is the model discussed in Sen (1962)[1].

Interpretation of the model

That many aspects of the model are fairly loosely defined is

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[1] Sen, A.K., "On the Usefulness of Used Machines", Review of Economics and Statistics, 44, 3, 1962.

evidence of the generality of its applications. I have made no assumptions about the structure of the world in which the machines are used.

The possible application of the analysis to the one sector fixed proportions 'clay-clay' vintage model has already been noted. Another possible interpretation is that the type of machine described comes from an ex-ante variable proportions, ex-post fixed proportions 'putty-clay' vintage model. In such a model there is a range of types of machine which may be constructed at time v. A machine is defined not by (v, t) but by (k, v, t) where k indexes the type of machine. It is implausible that assumptions (A2) or (A2') could apply to all machines independently of k. If we assume that the assumptions do apply to each type k separately, then the analysis above shows that, at each t, every machine of type k will be used exclusively in one country, or will have a unique 'vintage switchover point' v_o, as defined above. Out of steady state this is rather a weak result. In steady state, however, it implies that either machines constructed in the high wage country are infinitely long lived or they are sold to the low wage country before their scrapping date is reached. (For on the day their quasi-rent falls to zero in the high wage economy it is profitable to use them in the low wage economy.) This is the model to which I shall return when I tackle the question in Chapter 4 of whether a low wage country should use only second-hand machines, second-hand machines and domestically produced machines, or only home-grown machines.

The essential unity of all the examples discussed in this section should be stressed. The same general argument has been used, with slight modifications in some cases, to prove Sen's result that low wage

countries will use older machines, even if the only difference is that they are nearer the date of their death, and to prove the same result for a wide range of assumptions about the technical characteristics of older machines - that they have higher labour/machine ratios, or lower output/machine ratios, or higher labour/output ratios. In general there are, in the one nontraded input case, two connected forces leading to specialisation in old machines by the low wage country:
1) older machines are, in one sense or another, relatively labour intensive and therefore comparatively suitable for a low wage user;
2) older machines have, in one sense or another, lower capital cost, and are comparatively suitable for a user with high profit rates.
Force 2) has been a factor in all the cases I have discussed; in some cases, however, force 1) is absent, e.g. the 'one-hoss shay' case analysed by Sen.

It is a straightforward matter to represent the choice of technique in this model in an isoquant diagram. The labour cost of producing a unit flow of output on a machine with characteristics (t, v) in country i is

$$w_i(t)\,\frac{g(t,\,v)}{f(t,\,v)}\,.$$

The capital cost is

$$r_i(t)\,\frac{P(t,\,v)}{f(t,\,v)} - \frac{\partial P(t,\,v)}{\partial t}\,\frac{1}{f(t,\,v)} = r_i(t)\left(\frac{P(t,\,v)}{f(t,\,v)} - \frac{1}{r_i(t)}\,\frac{\partial P(t,\,v)}{\partial t}\,\frac{1}{f(t,\,v)}\right)$$

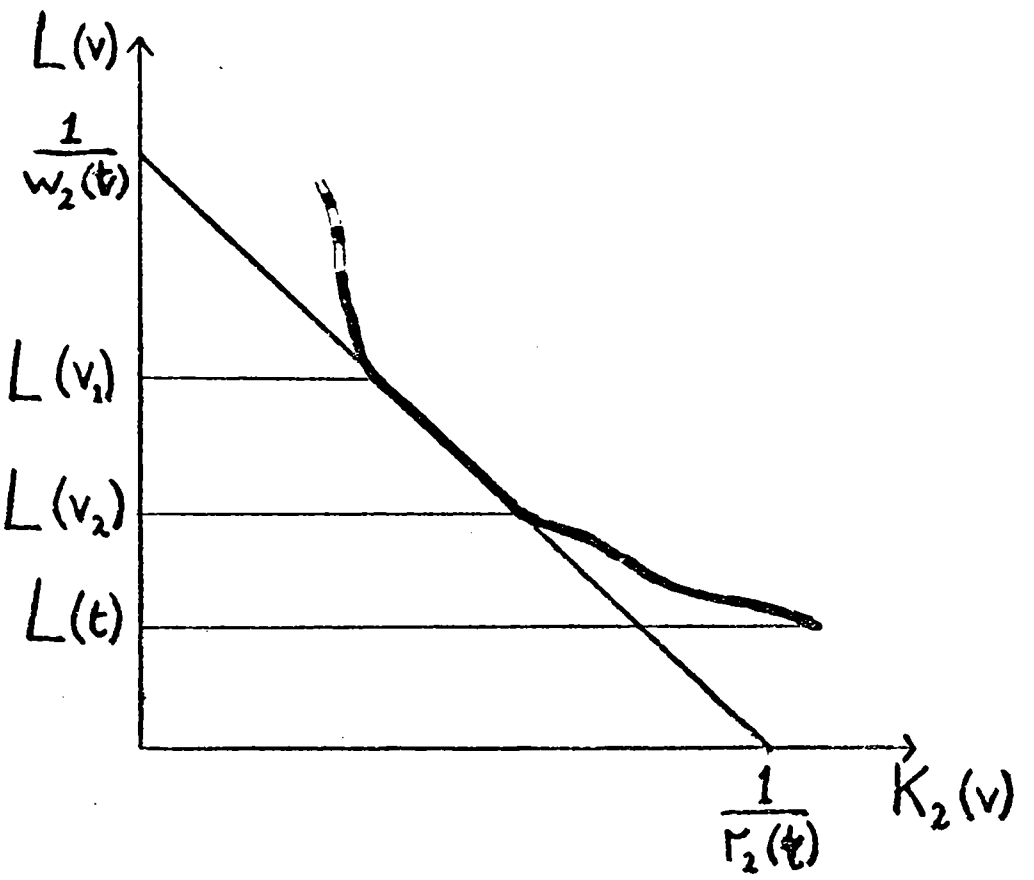
If vintage v is used in country i at time t, (6) implies

$$\frac{\partial P(t,\,v)}{\partial t} = r_i(t)\,P(t,\,v) - f(t,\,v) + w_i(t)\,g(t,\,v)$$

so the labour cost and capital cost sum to 1. If it is used elsewhere,

the sum of the labour and capital cost exceeds 1.

Therefore, if at a given time t we write $\frac{g(t, v)}{f(t, v)}$ as $L(v)$ and $\frac{P(t, v)}{f(t, v)} - \frac{1}{r_i(t)} \frac{\partial P(t, v)}{\partial t} \frac{1}{f(t, v)}$ as $K_i(v)$, and if there are three countries in the model, so that machines built before v_1 are used in country 3, and those built after v_2 are used in country 1, then the unit isoquant for country 2 is:



Finally, it should be noted that everything in this section applies to the analogous model defined in discrete time.

1.4. The model with transport costs

In the final section of this chapter I consider the question of how much light is cast by the model analysed in section 1.3 on the issues discussed and the data presented in the first two sections. First, however, I wish to point out the importance of the assumption that there are no transport costs by analysing briefly a simple model with transport costs.

The only difference between this model and the model of the previous section is that here it is assumed that it costs the fixed amount T to transport a machine from one country to another. For simplicity, assumptions (A1), (A2), and (A3) are made.

Now there are two prices for a given machine, $P_1(t, v)$ if it is in country 1, $P_2(t, v)$ in country 2. If vintage v is used in country 1,

$$\frac{\partial P_1(t, v)}{\partial t} = r_1(t)\, P_1(t, v) - \rho_1(t, v) \; ; \tag{7}$$

if it is used in country 2,

$$\frac{\partial P_2(t, v)}{\partial t} = r_2(t)\, P_2(t, v) - \rho_2(t, v) \; . \tag{8}$$

For all t, v,

$$- T \leq P_1(t, v) - P_2(t, v) \leq T \; . \tag{9}$$

Let $k_1(t, v) = (r_2(t) - r_1(t))\, P_1(t, v) + (w_2(t) - w_1(t))\, g(t, v)$.

Assume that $r_2(t) > r_1(t)$ and $w_2(t) < w_1(t)$. (We can no longer prove that one inequality implies the other.) $P_1(t, v)$ is strictly increasing in v, so $k_1(t, v)$ is strictly increasing in v.

If (7), (8) hold, and $\frac{\partial P_1(t, v)}{\partial t} = \frac{\partial P_2(t, v)}{\partial t}$, and $P_1(t, v) = P_2(t, v) + T$, then

$$r_2(t)\, (P_1(t, v) - T) - \rho_2(t, v) = r_1(t)\, P_1(t, v) - \rho_1(t, v)$$

i.e. $$k_1(t, v) = r_2(t)\, T \; ,$$

which, at a given t, defines a unique vintage v_2.

If (7), (8) hold and $\frac{\partial P_1(t, v)}{\partial t} = \frac{\partial P_2(t, v)}{\partial t}$, and $P_2(t, v) = P_1(t, v) + T$, then $k_1(t, v) = - r_2(t)\, T$, which defines a unique $v_1 < v_2$.

If country 1 buys machines of vintage v at time t from country 2,

$$P_1(t, v) = P_2(t, v) + T ,$$

$$\frac{\partial P_1(t, v)}{\partial t} = r_1(t)\, P_1(t, v) - \rho_1(t, v) ,$$

and $$\frac{\partial P_2(t, v)}{\partial t} \leq r_2(t)\, P_2(t, v) - \rho_2(t, v) .$$

Further, since $P_1(t, v) - P_2(t, v)$ is at its upper bound,

$$\frac{\partial P_1(t, v)}{\partial t} \leq \frac{\partial P_2(t, v)}{\partial t}$$

i.e. $$r_1(t)\, P_1(t, v) - \rho_1(t, v) \leq r_2(t)\, P_2(t, v) - \rho_2(t, v)$$

$$= r_2(t)\, P_1(t, v) - r_2(t)\, T - \rho_2(t, v)$$

$$\therefore \quad k_1(t, v) \geq r_2(t)\, T ,$$

$$\therefore \quad v \geq v_2 .$$

Similarly, if country 2 buys machines of vintage v at time t from country 1,

$$r_2(t)\, (P_1(t, v) + T) - \rho_2(t, v) \leq r_1(t)\, P_1(t, v) - \rho_1(t, v)$$

$$\therefore \quad k_1(t, v) \leq - r_2(t)\, T$$

$$\therefore \quad v \leq v_1 .$$

Therefore, machines of vintage between v_1 and v_2 stay where they are. A machine being transported from 2 to 1 was built after v_2, and is older than all machines being transported from 1 to 2, for they were all constructed before v_1. Unfortunately little more can be said. The intermediate range of machines which are definitely not traded may encompass all machines (and will in some cases, for example the

extreme case where T exceeds the cost of constructing a new machine in either country). Nor is there any assurance that machines outside this range will actually be traded. If country 2 holds machines of vintage v,

$$\frac{\partial P_2(t, v)}{\partial t} = r_2(t)\, P_2(t, v) - \rho_2(t, v)$$

$$\geq r_2(t)\, P_1(t, v) - \rho_2(t, v) - r_2(t)\, T$$

$$\geq k_1(t, v) + \frac{\partial P_1(t, v)}{\partial t} - r_2(t)\, T$$

$$\frac{\partial P_2(t, v)}{\partial t} - \frac{\partial P_1(t, v)}{\partial t} \geq k_1(t, v) - r_2(t)\, T$$

so that for sufficiently large $v > v_2$, P_2 grows so much faster than P_1 that (9) will be violated <u>if</u> factor prices remain unchanged for sufficient time (e.g. in steady state) and country 2 continues to hold these machines. Similarly, there may exist sufficiently small $v < v_1$ so that in steady state country 1 will eventually have to sell any machines of this vintage. It is difficult to make this argument more precise.

The most important point has, however, been made. The results of the previous section implying complete specialisation by particular countries in particular vintages of machine in a very wide range of technologies depends crucially on the absence of transport costs. If transport costs are not negligible, only a pale shadow of complete specialisation remains. One consolation is available. Kindleberger (1962)[1], pp.14-15, quotes 2 per cent as a typical proportion of transport cost in the delivered value of manufactures, 'including even bulky and heavy manufactures such as machine tools', and adds that their transport costs 'can safely be ignored by economists'.

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[1] Kindleberger, C.P., <u>Foreign Trade and the National Economy</u> (Yale University Press; New Haven, 1962).

1.5. Application of the theory to underdeveloped countries

There are several levels on which one may tackle the question of whether the theory presented in section 1.3 is an adequate representation of the main issues involved in international trade in second-hand capital goods and the connected question of whether it provides an appropriate guide to policy.

We could be rigidly falsificationist, seek falsifiable predictions of the theory, and reject the theory if any of the predictions are not fulfilled. On this criterion we should certainly reject the theory of section 1.3. The U.S. trade statistics which formed the basis of James' estimates (p. 11 above) show that 66% of imports of four categories of machine by underdeveloped countries from the U.S. were of new machines. In other words, some countries with lower wage rates than the U.S. were using machines of more recent vintage than similar machines in the U.S. (on the reasonable assumption that not all machines of these types were exported). We have to conclude that it is not the case that transport costs are negligible for all machines or, perhaps the more important explanation, that all older machines do not require more of those inputs which are relatively cheap in underdeveloped countries compared with the U.S. and less of relatively expensive inputs. Even at the most casually empirical level this is not a surprising conclusion.

We have to retreat to a less exacting criterion. Is it the case that the model, in a reasonably simple form focusing on a small number of nontraded inputs, does in fact describe the main forces underlying the international trade which does take place in second-hand machines, even if other forces mean that trade and specialisation do not go so far as the theory predicts? If this criterion is satisfied then the theory provides a foundation on which further analysis of the economic

implications of trade can be based.

If we look back at the list of characteristics of used machines on p.4, we see that the majority can be incorporated in the type of model I have described. In the model, older machines were cheaper (characteristic 1), were more labour intensive (2), had shorter physical and/or economic life (4), and were less reliable and produced a less valuable output (5). Further, the difference between old and new machines in requirements of skilled labour for maintenance of other reasons (6, 8) can be included by treating skilled and unskilled labour as separate inputs.

The difference in opinions about the direction of differences in operating costs (7), and to a lesser extent spare parts availability (9), illustrates what I believe to be the main benefit of treating these questions in a formal theoretical manner rather than in an informal descriptive way. The United Nations report states (para.36) that lower operating costs are sometimes cited as an advantage of used machines. One must ask, however, why such an advantage, if it exists, is not exactly compensated for by a price higher than it would otherwise have been. And indeed we have seen that in the model presented, within a country there is a whole range of vintages in use, the price and depreciation on each machine adjusting so that the total input cost of producing a unit of output is the same on every machine. It is international market 'imperfections', in the form of the absence of international investment and labour flows that make certain vintages attractive to certain countries. If the lower operating costs of old machines are due to lower requirements of unskilled labour, then I have shown above that this is an argument <u>against</u> the use of old machines by countries with low unskilled wage rates. If the lower operating costs are due to lower requirements of freely traded inputs, no producer will

find it advantageous or disadvantageous to use older rather than newer machines.

We have seen that the theory cannot say anything very interesting about transport costs (which could be widened in definition to include other transactions costs), and a fortiori cannot say anything about differences between old and new machines in transport and transactions costs (10, 12). Nor does the theory take into account the supposed advantage of used machines in being of smaller scale and having greater flexibility in use (3).

Perhaps the most fundamental objection is the fact that the theory takes no account of externalities and learning effects. James states that his observations suggest that learning effects internal to the firm resulting from the experience of maintaining old machines are an important incentive to the use of these machines in economies where such skills are scarce. We should expect there to be substantial external benefits also, for typically learning effects are not completely internal to the producer. Todaro's objection also, referred to above on p.5, is a statement about externalities. Essentially he is saying that the market underestimates the product of indigenous technology to a poor country in that it fails to take account of the effect that the development of such a technology has on future production functions.

Formally this type of point can be expressed within the terms of the model. Just as the model was extended to include several nontraded inputs so we can consider joint production replacing $f(t, v)$ by, say, $f_a(t, v) + p_i(t)\, f_b(t, v)$, where $f_a(t, v)$ is the physical product, which behaves as $f(t, v)$ did, and $f_b(t, v)$ is the flow of learning or other externality, with $p_i(t)$ the corresponding price (or shadow price). If $f_b(t, v)$ decreases with v, and $p_2(t) > p_1(t)$, as James suggests,

specialisation is reinforced by this factor. If, however, $f_b(t, v)$ increases with v as Todaro suggests, then the argument no longer goes through (and if private producers do specialise, ignoring the externality, there is a case for government intervention).

Is it then the case that the phenomenon which we observe of underdeveloped countries importing a significant proportion of their capital stock in the form of second-hand machines is the result of factor price differentials favouring cheapness and labour intensity (moderated, possibly, by the effect of skilled maintenance requirements and transport costs), or is it the case that it is the result of the benefits of small scale, flexibility, and (possibly mistaken) expectations of future effects on technical possibilities? Unfortunately the observations available do not give a conclusive answer. The Japanese census data in Table 1 (p. 8) and Strassman's data (p.10) display strikingly similar features: second-hand machinery is concentrated in small firms. Apart from pointing out the importance of transport costs (including transactions costs, which are surely the reason for the higher concentration in subsidiaries of foreign companies), Strassman also states that second-hand machines were used mainly by producers of durable goods, who placed less weight on the need for speed and quality in production. There is a classic identification problem here: do producers use old machines because they do not need to worry about speed and quality, or do they choose to produce products which can be produced on second-hand machines because factor prices lead them to choose these machines? It is well known that small firms find that capital is relatively expensive compared to labour, and this is very strikingly confirmed in Table 2 above, where there is a strong positive correlation between wage rates and firm size. (Casual observation of the

normal behaviour of lending institutions suggests that this would be reinforced by a negative correlation between firm size and the cost of borrowing funds.) The impact of these figures is somewhat dulled by the evidence that small firms are uniformly less efficient than large firms. No doubt it is logically possible to argue that these small, labour intensive, inefficient firms using low-wage labour use second-hand machines for reasons independent of their factor prices. On the whole, however, the strength of the correlation between factor prices and the use of used machines in Japanese manufacturing industry in 1957 suggests a strong causal link.

It seems plausible therefore to argue that since factor price differentials do exist between developed and underdeveloped countries, it is likely that they play an important role in encouraging trade in second-hand machines. There are other forces also which may encourage trade; there are non-price barriers to trade; and there may be reasons to discourage some privately profitable trade; but factor price differentials appear to have the leading role.

The final issue which has to be faced before we proceed is raised by Schwartz's data. Prices in a particular second-hand market in the U.S. vary in ways incompatible with a perfect market: for example, the relation between wholesale and retail price cannot reasonably be explained by the value of dealers' services. If this is a general phenomenon, what value is to be attached to an analysis founded on the assumptions of perfect foresight and perfect competition?

The perfect foresight assumption is certainly untenable as an exact description of reality. In practice, entrepreneurs are bound to have expectations which turn out, from time to time, to be mistaken and have to be revised. Continual and systematic revision of expectations

destroys the analysis. Random errors, even if frequent, should do no more than weaken the predictive power of the theory. It is not clear what assumptions are available as credible alternatives. In some ways the assumption of perfect competition is more difficult to defend. In a truly competitive world a high wage producer would be forced to get rid of his old machines because of their negative net contribution to profits. This does not happen in the real world. In a competitive world, a low wage producer would be forced out of business if vanity led him to use new machines. This does not happen in the real world either. The weakness of competitive forces may be a stronger reason than high transport costs or failure of foresight for the failure of the world to specialise as much as the theory predicts. Again, however, it must be emphasised that it is, to borrow econometric terms, the efficiency rather than the unbiasedness of the theory that is called into question.

There is a second important defence of the two assumptions: that they are the appropriate assumptions to make in a prescriptive analysis. If we interpret the $r_i(t)$, $w_i(t)$ as shadow prices, rational and prescient planners in each country will wish to maximise the present value of each machine, and equivalently minimise and equalise the unit cost of production in all processes. The theory is a description of the rational world even if it is only an approximate description of the real world.

1.6. Summary; directions for further work

I have presented a theory of international trade in machines in the context of a model whose assumptions in their simplest forms capture the features of differing factor intensity, expected lifetime, and reliability which are most commonly presented in the literature

as the main forces underlying trade in second-hand machines. The model can be extended to cover a wider range of possibilities, including the possibility of intangible joint products. In the absence of high transport and transaction costs it appears to have reasonable descriptive power. As a prescriptive model, it makes the assumptions about relative shadow prices which are generally believed to be appropriate in the analysis of relations between developed and underdeveloped countries, and can be extended to analyse the effects of intangible externalities.

Does it then provide an adequate basis for ending UNIDO's uncertainty about how to approach these issues? Unfortunately it does not: three directions are indicated for further work without which it is not possible to arrive at satisfactory answers to the practical problems.

First it would be desirable to have some estimate of the quantitative significance of the alleged intangible effects: learning effects, and effects on the availability and viability of future capital saving technologies. It is not possible to carry out this evaluation from a desk; indeed it may be that there is little to be said in general about it and that it is necessary to approach this question at the level of the individual project analysis. Neither categorical dismissal of the possible existence of important externalities nor sweeping statements about their pervasiveness in underdeveloped countries are of value if unsupported by hard evidence. The investigation of the possible existence of these phenomena in the case of second-hand technology is an empirical problem which ought to be tackled, but which I do not propose to tackle here.

The next question is raised by the model considered as a prescriptive guide to economic planners. In choosing an economy's saving programme, a government is effectively choosing a set of shadow

prices in terms of which investment projects should be evaluated. If the shadow wage is low and the discount rate high so that investment in used machines is favoured, what are the effects on the planners' objectives? In particular, how do choice of saving rate, international trade, and levels of domestic consumption interact in this type of model? This type of question cannot be answered in the very general kind of model presented in this chapter, and in the next two chapters I attempt to answer such questions in a very simple economy which is a special case of the model discussed above and which is in steady state growth.

Finally there is the question raised above, on p.27, of whether a country should use particular vintages of a type of machine used in other countries rather than use machines of a type peculiar to the country itself. Again this is a question that cannot be answered by the theory of this chapter. We require a more detailed discussion of the outcome of investment decisions such as is only possible in a model of steady growth. This problem is approached in Chapter 4.

CHAPTER 2

A Model of Steady Growth with Trade in Machines

2.1. Introduction

In this chapter I examine the effects of international trade in a world in which the 'clay-clay' vintage technology describes production possibilities. I abstract from almost all the issues discussed in Chapter 1, and from the possibility of choice of input coefficients for machines, and focus on the effects of differing prices and labour requirements on differing vintages of machines.

A homogeneous output is produced which is either consumed or costlessly transformed into a machine of which the future output and labour input are determined by the date of the machine's construction alone, i.e. the only technical choice to be made is the number of machines to construct, and once a machine is constructed, the only choice to be made is whether to use it or not. The definitive description of this technology is Solow et al. (1966).[1]

They establish that in an economy with this technology and labour force growth at rate n, investment may grow at rate g only if technical progress is purely labour-augmenting at the rate λ where $g = n + \lambda$. Assume that there is no physical deterioration of machines.

Let 'a machine' be defined as that amount of capital clay which incorporates one unit of the homogeneous output. In the notation of Chapter 1, f(t,v) is constant and, without loss of generality, we can

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[1] Solow, R. M., J. Tobin, C. C. von Weizsäcker, and M. Yaari, "Neoclassical Growth with Fixed Factor Proportions", Review of Economic Studies 32, 2 (April 1966), pp. 79-115.

choose the time unit so that $f(t,v) = 1$. $g(t,v)$ is a function only of v and is proportional to $e^{-\lambda v}$. By choice of the labour unit we may let $g(t,o) = 1$, so that $g(t,v) = e^{-\lambda v}$. I have already pointed out (p.15) that this technology satisfies (A1) and (A2).

Then in an economy with labour force $L(t) = L_o e^{nt}$, in steady state Solow et al.'s section II.9 establishes that investment is given by $I(t) = I_o e^{gt}$ where $g = n + \lambda$, and

$$L_o = \frac{I_o}{g - \lambda}(1 - e^{-(g-\lambda)m}) \tag{1}$$

$$w(t) = w_o e^{\lambda t} = (e^{-\lambda m})e^{\lambda t} \tag{2}$$

$$Y(t) = Y_o e^{gt} = \frac{I_o}{g}(1 - e^{-gm})e^{gt} \tag{3}$$

$$s = \frac{I(t)}{Y(t)} = \frac{I_o}{Y_o} = \frac{g}{1 - e^{-gm}} \tag{4}$$

where m is the constant age at which machines are scrapped.

A machine built at time v has quasi-rent

$$\rho(t,v) = 1 - e^{-\lambda m} e^{\lambda(t-v)}$$

i.e. $$\rho(n) = 1 - e^{-\lambda(m-n)} \tag{5}$$

where $n = t - v$, the age of the machine. When the machine reaches age m, the wage cost equals 1, the output of the machine, and the quasi-rent is zero. At no future date will it ever have nonnegative quasi-rent, its price is zero, and it is scrapped.

The competitive price of a machine built at time v, given constant discount rate r, is

$$P(t,v) = \int_t^{v+m} \rho(u,v)e^{-r(u-t)}du$$

$$= \int_o^{m-t+v} \rho(u+t,v)e^{-ru}du \qquad (6)$$

$$= \int_o^{m-t+v} (1 - e^{-\lambda(m-u-t+v)})e^{-ru}du,$$

i.e. $$P(n) = \int_o^{m-n} (1 - e^{-\lambda(m-u-n)})e^{-ru}du$$

$$= \int_o^{m-n} \rho(u+n)e^{-ru}du \qquad (7)$$

$$= \int_n^{m} \rho(u)e^{-r(u-n)}du$$

$$= \frac{1}{r}(1 - e^{-r(m-n)}) - \frac{e^{-\lambda(m-n)}}{\lambda-r}(e^{(\lambda-r)(m-n)} - 1).$$

The price of a new machine must in equilibrium equal the cost of production. With no loss of generality, the price numéraire is chosen as one unit of output, so the cost of producing a machine is 1. Therefore,

$$1 = P(o,o) = P(o)$$

$$= \int_o^{m} (1 - e^{-\lambda(m-u)})e^{-ru}du \qquad (8)$$

$$= \frac{1}{r}(1 - e^{-rm}) - \frac{e^{-\lambda m}}{r-\lambda}(1 - e^{-(r-\lambda)m}).$$

Equations (1-5) and (8) correspond respectively to Solow et al.'s equations (14), (16), (17), (18), (31) and (33). I have used the fact, which they prove, that the discount rate must be constant in steady state.

Differentiation of the first version of (6) gives

$$\frac{\partial P(t,v)}{\partial t} = rP(t,v) - \rho(t,v) \qquad (9)$$

or, equivalently, by differentiating the third and fourth versions of (7),

$$P'(n) = rP(n) - \rho(n)$$

$$= -\frac{\lambda}{r-\lambda}\left(e^{-\lambda(m-n)} - e^{-r(m-n)}\right). \qquad (10)$$

One result which Solow et al. do not prove is of some use to my argument later, and it is more convenient to state and prove it here.

Lemma 2.1 In the economy described by equations (1 - 10), a steady state with a low positive r and high w_o has a lower price for a machine of age $n > 0$ than a steady state with a higher r and lower w_o.

Proof Differentiating the first version of (10) with respect to m and then with respect to n:

$$\frac{d}{dm}P'(n) = \frac{dr}{dm}P(n) + r\frac{dP(n)}{dm} - \lambda e^{-\lambda(m-n)}$$

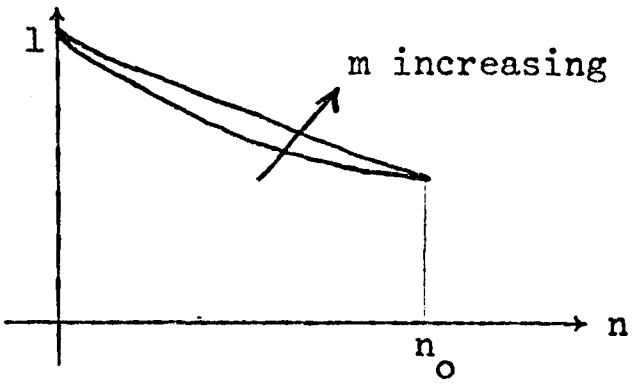
$$\frac{d}{dn}\frac{d}{dm}P'(n) = \frac{dr}{dm}P'(n) + r\frac{d}{dm}P'(n) - \lambda^2 e^{-\lambda(m-n)}$$

Since $\frac{dr}{dm}$ is defined by (8) and is clearly independent of n, and since, P(n) being suitably continuous in m and n, we can reverse the order of differentiation of the second term. Now $\frac{dr}{dm}$ is positive, P'(n) is negative and $-\lambda^2 e^{-\lambda(m-n)}$ is negative, so

$$\frac{d}{dn}\frac{d}{dm}P'(n) < r\frac{d}{dm}P'(n).$$

Therefore if $\frac{d}{dm}P'(n) \leq 0$ at $n = n_1$, since $r > 0$, $\frac{d}{dm}P'(n) < 0$ for all $n > n_1$. Since $P(o) = 1$ for all m, $n_1 = 0$ would imply $\frac{d}{dm}P(n) < 0$ for all $n > 0$. But $\frac{d}{dm}P(n) > 0$ at $n = m$ since raising m raises the price from zero. Hence $\frac{d}{dm}P'(o) > 0$. Let n_o be the smallest value of n such that $\frac{d}{dm}P(n) = 0$. Clearly (see Diagram 1), $\frac{d}{dm}P'(n) \leq 0$. Hence $\frac{d}{dm}P'(n) < 0$ for $n > n_o$ and $\frac{d}{dm}P(n) < 0$ for all $n > n_o$, which is impossible if $n_o \leq m$, since $\frac{d}{dm}P(n) > 0$ at $n = m$.

Diagram 1



Hence $n_o \leq m$ is impossible, and $\frac{d}{dm}P(n) > 0$ for all $n \in (0,m]$. Q.E.D.

2.2. Gross and net saving rates in the clay-clay technology

The previous section summarises the principal features of the technology with which I shall be concerned in this and the next chapter. There is, however, one further issue which must be settled. In discussing the effects of international trade in a growth model, one wishes to compare steady states in the absence of trade ('autarchy') with steady states in which trade takes place.

Now it is evident that equations (1 - 10) above could describe a wide range of possible steady states. For example, when $s = g$, we see that $m = \infty$, $I_o = (g-\lambda)L_o$, $w_o = 0$, $Y_o = I_o/g$, $r = 1$, and $P(n) = \rho(n) = 1$ for all n. As s rises, m falls, I_o rises, w_o rises, Y_o rises, r falls, and $\rho(n)$ and $P(n)$ become strictly decreasing. Solow *et al.* point out in section IV.6 that it is possible to parameterise these steady states in alternative ways.

Instead of parameterising by the gross saving rate s, as is done above, one could adopt the discount rate as the parameter. This procedure would be particularly appropriate if one believed that saving was determined by a proportional relationship between net saving and net profit, for then the discount rate would be determined by the ratio of net saving to net profit σ_r through Solow *et al.*'s

equation (37): $r\sigma_r = g$. An alternative rationalisation would be to suppose that the steady growth path was the outcome of an optimal saving programme which had equated the rate of profit with the social discount rate. Such a parameterisation gives straightforward results since (8) determines m as a strictly increasing function of r.

So far as the model of section 2.1 is concerned, therefore, either s or r is a suitable parameter to index different steady states. No substantive matters are involved in the choice of one rather than another. We therefore have two natural ways of dealing with the long-run effects of trade: we can compare steady states across which countries hold their gross saving rates constant, or across which they hold their profit rates constant. It will emerge that there are significant differences between the two procedures.

A third possibility is considered inconclusively in IV.5,6 by Solow et al.: the use of the net saving rate, i.e. the ratio of net savings to net income, as a parameter. The relationships relevant to the behaviour of net saving rates in the model are (8) above, which relates r and m, the equation (36) in Solow et al. which relates ω, the ratio of depreciation to gross investment, to r,m:

$$\omega = \lambda \left\{ \frac{e^{-rm}}{(r-\lambda)(r-g)} + \frac{e^{-\lambda m}}{(r-\lambda)(g-\lambda)} - \frac{e^{-gm}}{(r-g)(g-\lambda)} \right\}, \qquad (11)$$

and the definition of the net saving rate:

$$\sigma = \frac{s(1-\omega)}{1-s\omega}. \qquad (12)$$

They point out that when $s = g$, since $m = \infty$, $\omega = 0$ and $\sigma = s$; while if $s \in (g,1)$, $\omega > 0$ and $\sigma < s$; and when $s = 1$, $\sigma = s = 1$. (The implicit assumption that $g < 1$ is required to ensure the existence

of a steady state with full employment). Now in order for a particular σ to define a unique steady state it is necessary that σ be a monotonically increasing function of s (or equivalently a monotonically decreasing function of m) since each s (or m) uniquely defines a steady state. Solow et al. state:

> 'We have not settled this question; so far as we know, it may be that the higher gross saving ratio, associated with a shorter lifetime, may so accelerate depreciation as to result in a smaller net saving rate. It is clear, however, that "on the average" a higher s is associated with a higher σ.' (p.104).

As I have pointed out briefly elsewhere (Smith (1973)[1]), it is possible to demonstrate that σ is never a monotonically increasing function of s. For (12) implies that

$$\frac{d\sigma}{dm} = \frac{1}{(1 - s\omega)^2}\left[(1 - s\omega)\left((1 - \omega)\frac{ds}{dm} - s\frac{d\omega}{dm}\right) + s(1 - \omega)\left(s\frac{d\omega}{dm} + \omega\frac{ds}{dm}\right)\right]$$

Now as $m \to \infty$, $\omega \to 0$ and $s \to g$ so that this implies that $\frac{d\sigma}{dm}$ is of the order of $\frac{ds}{dm} - s(1 - s)\frac{d\omega}{dm}$. (4) shows that

$$\frac{ds}{dm} = \frac{-\, g^2 e^{-gm}}{(1 - e^{-gm})^2}$$

which implies that $\frac{ds}{dm}$ is of the order of $-\, g^2 e^{-gm}$ for large m. (8) implies that

$$\frac{dr}{dm} = \frac{\frac{\lambda}{r-\lambda}\, e^{-\lambda m}(1 - e^{-(r-\lambda)m})}{\frac{1}{r^2}(1 - (1 + rm)e^{-rm}) - \frac{e^{-\lambda m}}{(r-\lambda)^2}(1 - (1 + (r-\lambda)m)e^{-(r-\lambda)m})}$$

and for large m, since r tends to 1 and $\lambda < g < 1$, $r-\lambda$ is positive, and $\frac{dr}{dm}$ is of the order of $r^2\lambda e^{-\lambda m}/(r-\lambda)$. (11) implies that

$$\frac{\partial\omega}{\partial r} = -\, \lambda^2 e^{-\lambda m}\left(\frac{1}{(r-\lambda)^2(g-\lambda)} + \frac{e^{-(r-\lambda)m}}{(r-\lambda)^2(r-g)^2}((r-\lambda)(r-g)m + 2r - g - \lambda)\right.$$

$$\left. - \frac{e^{-(g-\lambda)m}}{(r-g)^2(g-\lambda)}\right),$$

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[1] Smith, M.A.M., "A Note on Fixed Factor Proportions and Net Saving Rates", Review of Economic Studies 40, 2 (April 1973), pp. 297-8.

which, for large m, is of the order of $-\lambda e^{-\lambda m}/(r-\lambda)^2(g-\lambda)$, and that

$$\frac{\partial\omega}{\partial m} = -\lambda^2 e^{-\lambda m}\left\{\frac{1}{(r-\lambda)(g-\lambda)} + \frac{re^{-(r-\lambda)m}}{(r-\lambda)(r-g)} - \frac{ge^{-(g-\lambda)m}}{(r-g)(g-\lambda)}\right\}$$

which is of the order of $-\lambda^2 e^{-\lambda m}/((r-\lambda)(g-\lambda))$. Therefore, since $\frac{d\omega}{dm} = \frac{\partial\omega}{\partial m} + \frac{\partial\omega}{\partial r}\cdot\frac{dr}{dm}$, for large m $\frac{d\omega}{dm}$ is of the order of $-\lambda^2 e^{-\lambda m}/((r-\lambda)(g-\lambda))$.

Now $\frac{d\sigma}{dm}$ is of the order of $\frac{ds}{dm} - s(1-s)\frac{d\omega}{dm}$ which is of the order of $-g^2 e^{-gm} + s(1-s)\lambda^2 e^{-\lambda m}/((r-\lambda)(g-\lambda))$, which is of the order of its second term and is therefore positive. Therefore for values of s sufficiently close to g, $\frac{d\sigma}{ds}$ is negative. But as s rises from g to 1, σ rises from g to 1, so that $\frac{d\sigma}{ds}$ is positive for some values of s. Therefore σ is never a monotonic function of s or of m.

(8) may be solved for r as a function of m by a simple Newton-Raphson procedure. Then (4), (11) and (12) can be evaluated to give numerical examples of the relationship between σ, s, and m. In the tables below I set out some of the examples which I have computed.

TABLE 1 $\lambda = 0.02$, $g = 0.04$

r	m	ω	s	σ
- 7.89	1.0205	0.9950	1.0000	1.0000
- 7.86	1.0230	0.9950	0.9977	0.6806
- 3.0	2.0437	0.9874	0.5096	0.0130
- 1.5	3.1896	0.9765	0.3339	0.0116
0.0	10.3447	0.8732	0.1180	0.0167
0.5	36.6985	0.4991	0.0520	0.0267
1.0	∞	0.0	0.04	0.04

TABLE 2 $\lambda = 0.04$, $g = 0.06$

r	m	ω	s	σ
- 6.87	1.0313	0.9915	1.0000	1.0000
- 6.28	1.1000	0.9908	0.9394	0.1254
- 3.0	1.8188	0.9816	0.5804	0.0248
- 1.3	2.9811	0.9630	0.3663	0.0210
0.0	7.4207	0.8637	0.1670	0.0266
0.5	19.4130	0.5819	0.0872	0.0384
1.0	∞	0.0	0.06	0.06

TABLE 3 $\lambda = 0.4$, $g = 0.5$

r	m	ω	s	σ
- 1.90	1.3863	0.8453	1.0000	1.0000
- 0.5	2.0815	0.7382	0.7730	0.4714
0.0	2.6252	0.6539	0.6841	0.4284
0.33	3.2528	0.5612	0.6224	0.4197
0.5	3.7660	0.4912	0.5897	0.4224
1.0	∞	0.0	0.5	0.5

In each of these examples the function $\sigma(s)$ has only one turning point, a feature shared with all the examples I have computed. In the first two examples the turning point occurs at a negative profit rate, so that σ is a decreasing function of s for all values of s consistent with a positive profit rate. In the third example the turning point is at a positive profit rate. This example, however, because of the high growth rate, has a rather high minimum saving rate. (Because of the normalisation which made the physical capital-output ratio equal to 1, we could interpret the third example as a case with low growth rates and high capital-output ratio, e.g. $\lambda = 0.04$, $g = 0.05$, and capital-output ratio = 10.)

The implication of all this is that the use of net saving rates is not an appropriate way to parameterise steady states. For in every clay-clay technology there is a range of net saving rates each of which is associated with two values of m, r, w_o, I_o and Y_o. It appears that typically at least one of these two values of r is negative, in which case the other steady state has rather perverse comparative dynamics, in that an increase in 'saving', in the sense of an increase in the net saving rate, leads to a steady state with lower gross saving, higher profit rates, lower wage rates, and longer life of machines, i.e. a less 'capital intensive' state.

Now it is at least logically possible to hold the belief that individual agents may use a fixed gross saving rate or a fixed discount rate as a rough and ready guide to their consumption plans. Therefore to suppose that the economy as a whole is on a path exhibiting such features may be a reasonable first approximation to reality, although such a belief is open to many obvious objections such as aggregation problems, and the absence of an explanation of convergence to steady state. But the belief that rational agents use net saving rates as a guide to behaviour does not satisfy the first requirement of logical consistency: the consequences of the supposed behaviour are not well defined.

From now on, therefore, I shall assume that a country's saving programme is determined either by a fixed gross saving rate or a fixed discount rate.

2.3. Income identities in vintage models

It is convenient at this point to set out some definitions, conventions and results to which I shall refer at several points later.

Consider a general vintage technology in which output per machine is fixed at μ_o, and output per man on a machine of vintage v is $\lambda_o e^{\lambda v}$.

(In the technology of this chapter I have chosen units so that $\mu_o = \lambda_o = 1$; in the putty-clay technology $\mu_o = f(k)/k$, $\lambda_o = f(k)$.)

Suppose that on the 'cohort' of vintages aged between n_1 and n_2 the wage rate is

$$w(t) = (\lambda_o e^{\lambda t})w$$

and the discount rate is r. (I make no assumption about factor prices on other vintages). Investment in the world as a whole grows at rate g:

$$I(t) = Ie^{gt}.$$

Output from the cohort of vintages is:

$$Y_n(t) = \int_{n_1}^{n_2} \mu_o I(t-n)dn = Ie^{gt} \int_{n_1}^{n_2} \mu_o e^{-gn}dn.$$

The value of the capital stock in the cohort is

$$K_n(t) = \int_{n_1}^{n_2} P(n)I(t-n)dn = Ie^{gt} \int_{n_1}^{n_2} P(n)e^{-gn}dn$$

Net investment is

$$K_n'(t) = gK_n(t) = Ie^{gt} \int_{n_1}^{n_2} gP(n)e^{-gn}dn$$

Depreciation is

$$D_n(t) = \int_{n_1}^{n_2} (-P'(n))I(t-n)dn = Ie^{gt} \int_{n_1}^{n_2} (-P'(n))e^{-gn}dn$$

Gross investment is

$$\begin{aligned} I_n(t) &= K_n'(t) + D_n(t) \\ &= Ie^{gt} \int_{n_1}^{n_2} (gP(n) - P'(n))e^{-gn}dn \\ &= Ie^{g(t-n_1)}P(n_1) - Ie^{g(t-n_2)}P(n_2) \end{aligned}$$

Consumption is , if the cohort is self-supporting,

$$C_n(t) = Y_n(t) - I_n(t)$$
$$= Ie^{gt} \int_{n_1}^{n_2} (\mu_o - gP(n) + P'(n))e^{-gn}dn$$

Net income is

$$Z_n(t) = Y_n(t) - D_n(t)$$
$$= Ie^{gt} \int_{n_1}^{n_2} (\mu_o + P'(n))e^{-gn}dn$$
$$= C_n(t) + K_n'(t)$$

Employment is

$$L_n(t) = \int_{n_1}^{n_2} \frac{\mu_o}{\lambda_o} e^{-\lambda(t-n)} Ie^{g(t-n)}dn$$

and the wage bill is

$$W_n(t) = w(t)L_n(t) = Ie^{gt} \int_{n_1}^{n_2} \mu_o we^{\lambda n} e^{-gn}dn$$

Therefore

$$W_n(t) + rK_n(t)$$
$$= Ie^{gt} \int_{n_1}^{n_2} (\mu_o we^{\lambda n} + rP(n))e^{-gn}dn$$

Now if there is competitive pricing within the cohort,

$$P'(n) = rP(n) - \rho(n)$$
$$= rP(n) - \mu_o + \mu_o we^{\lambda n}$$

Therefore

$$W_n(t) + rK_n(t) = Ie^{gt} \int_{n_1}^{n_2} (\mu_o + P'(n))e^{-gn}dn$$
$$= Z_n(t) = C_n(t) + K_n'(t).$$

If the use of these machines is not competitive

$$P'(n) < rP(n) - \mu_o + \mu_o we^{\lambda n}$$

Therefore

$$W_n(t) + rK_n(t) > Z_n(t) = C_n(t) + K'_n(t)$$

As Bliss (1968)[1] points out (p.127), it is easy to prove the golden rule result in this framework. For given r, w, L_n, if C_n, K_n are competitive,

$$C_n = W_n + (r-g)K_n$$

and

$$C^o_n \leq W_n + (r-g)K^o_n \quad \text{for all other steady state paths.}$$

Therefore

$$C^o_n - C_n \leq (r-g)(K^o_n - K_n) = 0 \quad \text{if } r = g.$$

Therefore the competitive path with r = g is superior to all other steady growth paths, irrespective of the prices ruling in other cohorts.

2.4. Trade in the clay-clay model: fixed saving rates

Consider two countries, with labour forces $L_1(t)$, $L_2(t)$, and with identical technologies of the form described in section 2.1. Suppose that there is free trade in machines, but no international labour or investment flows, so that the analysis of Chapter 1 applies, in the presence of factor price differentials.

I define a steady state in such a model as follows: wage rates in both countries grow at rate λ, total gross investment grows at rate g, and $L_1(t)$, $L_2(t)$ grow steadily. Without loss of generality, I assume that the

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[1] Bliss, C.J., "On Putty-Clay", Review of Economic Studies 35, 2 (April 1968), pp.105-132.

wage level in the country indexed 2 is lower than in the country indexed 1. (There is no loss of generality in that I am, implicitly, defining 'different countries' as 'groups of workers with different wage rates'.) Formally,

$$I(t) = Ie^{gt}, \tag{13}$$

$$w_1(t) = w_1e^{\lambda t},\ w_2(t) = w_2e^{\lambda t}, \tag{14}$$

$$w_1 > w_2.$$

Now given that $f(t,v) = 1$ and $g(t,v) = e^{-\lambda v}$,

$$\rho_i(t,v) = 1 - w_ie^{\lambda(t-v)},$$

$$\text{i.e.}\rho_i(n) = 1 - w_ie^{\lambda n} \tag{15}$$

Now (A1, 2, 3) of Chapter 1 hold, and the existence of the wage differential allows us to apply the analysis of that chapter. The first point to note is that quasi-rent in each country is a strictly decreasing function of n, the age of the machine. Therefore the price of a machine decreases strictly with age. The ownership of unused but valuable machines by country 1 requires that

$$\frac{\partial P(t,v)}{\partial t} = r_1(t)P(t,v)$$

so if $r_i(t) \gtrless 0$ this implies machines appreciate with age, which is impossible. In steady state all valuable machines are used.

If $r_i(t) < 0$, the above argument no longer holds and it is possible, though not necessary, that machines pass through three phases: use in country 1 while $\rho_1 \gtrless 0$, ownership unused in country 1 depreciating at rate $r_1(t)$, and use in country 2. A machine may pass through either of the last two phases more than once. I shall say a little more about this case below, but for the most part I shall assume that all valuable machines are used.

Define m_2 by

$$w_2 = e^{-\lambda m_2}. \tag{16}$$

(15) implies that $\rho_2(m_2) = 0$, so that m_2 is the age at which competitive producers scrap machines. If, therefore, we assume that there is full employment in both economies, competitive equilibrium requires,

$$L_1(t) + L_2(t) = \int_{t-m_2}^{t} Ie^{gv}e^{-\lambda v}dv$$

$$= \frac{I}{g-\lambda}(1 - e^{-(g-\lambda)m_2})e^{(g-\lambda)t} \tag{17}$$

which is possible only if the sum of the labour forces grows at a constant rate n, and $g = n + \lambda$. But each of L_1, L_2 grow steadily and their sum grows at rate n. This is possible only if each grows at rate n:

$$L_1(t) = L_1e^{nt}, \quad L_2(t) = L_2e^{nt} \tag{18}$$

Then the analysis of Chapter 1 tells us that at a given time, machines of ages from 0 to m_1 are used in country 1, and those of ages from m_1 to m_2 are used in country 2 so that the following equations hold:

$$L_1(t) = \int_{t-m_1}^{t} Ie^{(g-\lambda)v}dv,$$

$$L_1 = \frac{I}{g-\lambda}(1 - e^{-(g-\lambda)m_1}); \tag{19}$$

$$L_2(t) = \int_{t-m_2}^{t-m_1} Ie^{(g-\lambda)v}dv,$$

$$L_2 = \frac{I}{g-\lambda}(e^{-(g-\lambda)m_1} - e^{-(g-\lambda)m_2}), \tag{20}$$

and m_1 is a constant.

Output in each country is given by:

$$Y_1(t) = \int_{t-m_1}^{t} Ie^{gv}dv = Y_1 e^{gt},$$

$$Y_1 = \frac{I}{g}(1 - e^{-gm_1}); \tag{21}$$

$$Y_2(t) = \int_{t-m_2}^{t-m_1} Ie^{gv}dv = Y_2 e^{gt},$$

$$Y_2 = \frac{I}{g}(g^{-gm_1} - e^{-gm_2}). \tag{22}$$

In steady state therefore, the fact proved in section 1.3 that there is at every instant a unique vintage, machines older than which are used, and used only, in country 1, now implies that each machine spends the first part of its life in country 1, and the rest in country 2, after which it is scrapped: i.e. in steady state what is true of the vintage dimension is true of the age dimension.

The price of a new machine is at time t, in equilibrium,

$$1 = P(t,t)$$

$$= \int_{t}^{t+m_1} (1 - w_1 e^{\lambda(u-t)})e^{-R_1(u,t)}du + e^{-R_1(t+m_1,t)}P(t+m_1,t),$$

and the price of a machine of age m_1 is

$$P(t,t-m_1) = \int_{t}^{t+m_2-m_1} (1 - w_2 e^{\lambda(u-t+m_1)})e^{-R_2(u,t)}du,$$

where

$$R_i(u,t) = \int_{t}^{u} r_i(s)ds$$

Now I assume that each country is to have a fixed gross saving rate. By 'gross saving' I refer to the value (rather than the volume) of the net physical addition to the capital stock. In country 1 at time

t, Ie^{gt} machines of value 1 are added, and $Ie^{g(t-m_1)}$ machines of value $P(t,t-m_1)$ are subtracted, and added to the capital stock of country 2, from which are subtracted only valueless machines. Therefore the saving rates are:

$$s_1 = \frac{Ie^{gt}(1 - P(t,t-m_1)e^{-gm_1})}{Y_1 e^{gt}}$$

$$= \frac{g(1 - P(t,t-m_1)e^{-gm_1})}{1 - e^{-gm_1}},$$

$$s_2 = \frac{Ie^{gt}e^{-gm_1}P(t,t-m_1)}{Y_2 e^{gt}}$$

$$= \frac{ge^{-gm_1}P(t,t-m_1)}{e^{-gm_1} - e^{-gm_2}}$$

At first sight it may seem odd that 'gross' investment should include prices in its definition, but it should be clear from section 2.2 that this is indeed the appropriate definition.[1]

Clearly s_1 and s_2 are constant if and only if $P(t,t-m_1) = p$, a constant. The steady state is defined therefore by the equations (19), (20) and

$$s_1 = \frac{g(1 - pe^{-gm_1})}{1 - e^{-gm_1}}, \qquad (23)$$

$$s_2 = \frac{gp}{1 - e^{-g(m_2-m_1)}}. \qquad (24)$$

Consumption levels in the two economies are $C_1(t) = C_1 e^{gt}$, $C_2(t) = C_2 e^{gt}$ where

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[1] See, however, the appendix to Chapter 2, p.171.

$$C_1 = Y_1 - I + pe^{-gm_1}I = (1 - s_1)Y_1, \tag{25}$$

$$C_2 = Y_2 - pe^{-gm_1}I \quad = (1 - s_2)Y_2. \tag{26}$$

Theorem 2.1. Equations (19), (20), (23), (24) determine for given s_1, s_2, L_1, L_2, a unique set of values of m_1, m_2, p, I, if s_1, $s_2 \geq g$.

Proof

Consider what happens as p increases in the interval $[0,1]$. When $p = 0$, (23) becomes identical to (4), the saving equation in an isolated country, and m_1 takes the value m_1' it would have in an isolated economy with saving rate s_1. As p rises towards 1, m_1 falls monotonically towards 0. (19) implies that as p rises from 0 to 1, I rises and therefore $Ie^{-(g-\lambda)m_1}$ rises, from $I_1'e^{-(g-\lambda)m_1'}$, the autarchy value corresponding to s_1, L_1, to infinity.

From (24) it can be deduced that when $p = 0$, $m_2 - m_1 = 0$, and as p rises $m_2 - m_1$ rises monotonically to m_2', the autarchy value. (20) shows then that, since $Ie^{-(g-\lambda)m_1}(1 - e^{-(g-\lambda)(m_2-m_1)})$ is constant, $Ie^{-(g-\lambda)m_1}$ decreases monotonically from infinity to I_2'.

(19), (23) therefore require $Ie^{-(g-\lambda)m_1}$ to be a strictly increasing function of p, going to infinity as p goes to 1. (20), (24) require it to be a strictly decreasing function of p, going to infinity as p goes to 0. There is therefore a unique value of p at which all four equations may be satisfied and this value of p determines m_1, m_2 from (23), (24) and I from (19) or (20).

Q.E.D.

It is worth noting that although $s \geq g$ is necessary and sufficient for the existence of steady state in the Solow et al. model, the condition $s_1 \geq g$, $s_2 \geq g$ is shown by the above theorem to be sufficient but not

necessary. $s_1 \geq g$ is necessary, but if $s_2 < g$, (24) shows that $m_2 - m_1$ tends to infinity when p tends to s_2/g, and $Ie^{-(g-\lambda)m_1} = (g-\lambda)L_2$ when $m_2 - m_1 = \infty$. Whereas Diagram 2 shows how existence of a solution is guaranteed if $s_2 \geq g$, Diagrams 3(a) and (b) show that if $s_2 < g$ a solution may or may not exist.

Diagram 2 $s_1 \geq g$, $s_2 \geq g$

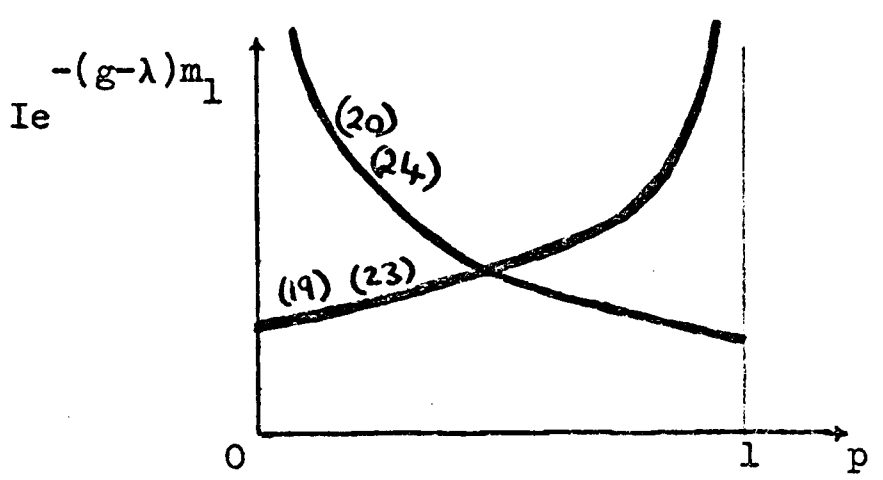
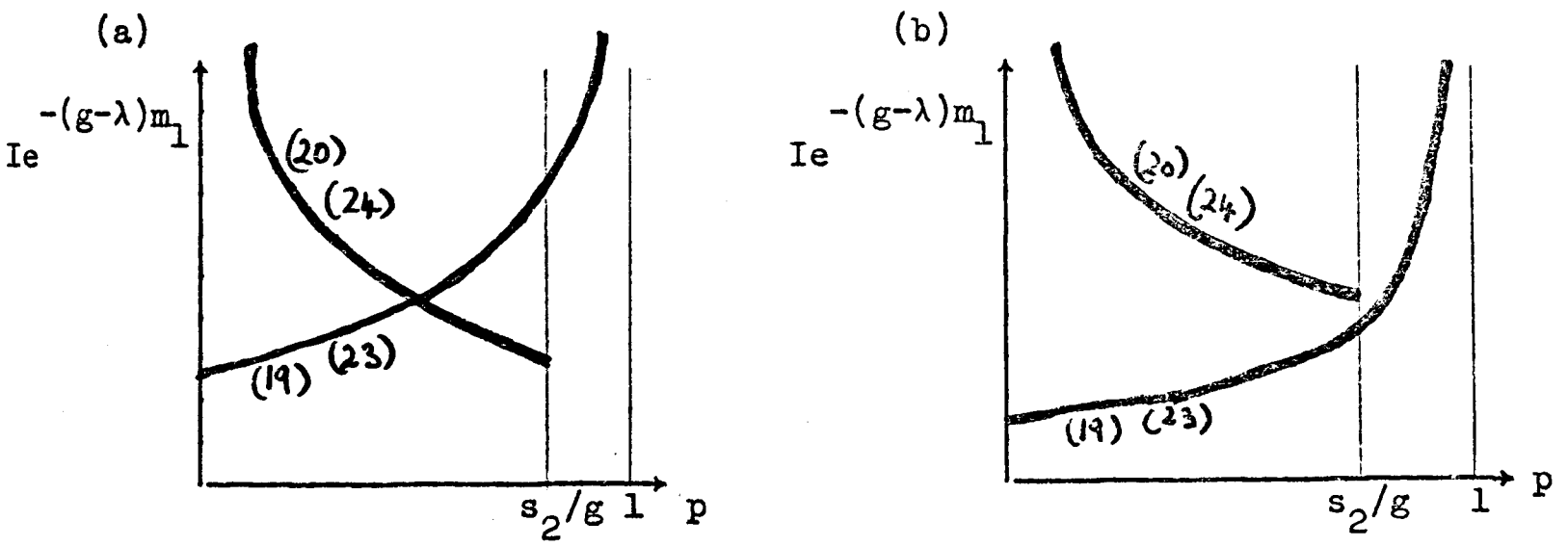


Diagram 3 $s_1 \geq g$, $s_2 < g$



We therefore have the interesting possibility that trade may allow full employment to be maintained in country 2 although it has such a low saving rate that full employment is impossible in autarchy.

We can now turn to consider prices in steady state. Since $P(t,t-m_1) = p$,

$$p = \int_t^{t+m_2-m_1} (1 - w_2 e^{\lambda(u-t+m_1)}) e^{-\int_t^u r_2(s)ds} du$$

$$= \int_o^{m_2-m_1} (1 - w_2 e^{\lambda(u+m_1-m_2)}) e^{-\int_t^{t+u} r_2(s)ds} du$$

which has exactly the form of the integral equation on p.101 of Solow et al. Their proof of the constancy of $r(t)$ therefore implies that $r_2(t)$ is a constant, say r_2, and

$$p = \int_o^{m_2-m_1} (1 - w_2 e^{\lambda(u+m_1-m_2)}) e^{-r_2 u} du$$

$$= \frac{1}{r_2}(1 - e^{-r_2(m_2-m_1)}) - \frac{1}{r_2-\lambda}(e^{-\lambda(m_2-m_1)} - e^{-r_2(m_2-m_1)}). \qquad (27)$$

The right hand side of (27) is a decreasing function of r_2, so (27) determines r_2 uniquely, and r_2 is a decreasing function of p.

Now Theorem 1.2 implies that

$$w_1 = w_2 + (r_2 - r_1(t))P(t,t-m_1)e^{-\lambda m_1}$$

and since $P(t,t-m_1) = p$, $r_1(t)$ is a constant, say r_1. This equation is the analogue for country 1 of equation (16), being the condition which must be satisfied if m_1 is chosen optimally. It is not obvious that w_1, r_1 are uniquely determined, but this is in fact true. Consider the behaviour of prices of machines of age less than m_1.

$$1 = P(t,t) = P(0)$$

$$= \int_o^{m_1} (1 - w_1 e^{\lambda u}) e^{-r_1 u} du + e^{-r_1 m_1} p,$$

and if $m \in (0, m_1)$,

$$P(t,t-m) = P(m)$$
$$= \int_m^{m_1} (1 - w_1 e^{\lambda u}) e^{-r_1(u-m)} du + e^{-r_1(m_1-m)} p,$$

which implies that $P'(m) = r_1 P(m) - 1 + w_1 e^{\lambda m}$. Also, it is known that $P(m_1) = p$.

Lemma 2.2. If $r_1 > 0$, $P(0) = 1$, $P'(m) = r_1 P(m) - 1 + w_1 e^{\lambda m}$, and $P(m_1) = p < 1$, then $\frac{d}{dw_1} P'(m_1) > 0$.

Proof.

$P(0) = 1$ clearly implies that r_1 is a continuous and decreasing function of w_1.

Differentiate $P'(m)$ with respect to w_1 and m:

$$\frac{d}{dw_1} P'(m) = \frac{dr_1}{dw_1} P(m) + r_1 \frac{d}{dw_1} P(m) + e^{\lambda m}$$

$$\frac{d}{dm} \frac{d}{dw_1} P'(m) = \frac{dr_1}{dw_1} P'(m) + r_1 \frac{d}{dw_1} P'(m) + \lambda e^{\lambda m}$$

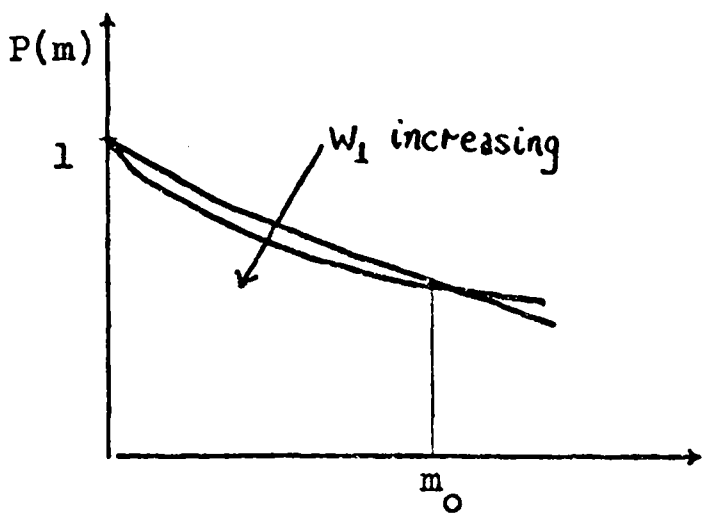
$$> r_1 \frac{d}{dw_1} P'(m)$$

since $P(m)$ satisfies the continuity conditions which allow the reversal of the order of differentiation, and $P'(m) < 0$.

Clearly, $r_1 \frac{d}{dw_1} P'(0) < 0$ for if not $\frac{d}{dm} \frac{d}{dw_1} P'(m) > 0$ for all $m > 0$, $\frac{d}{dw_1} P'(m) > 0$ and $\frac{d}{dw_1} P(m) > 0$, contradicting the fact that $P(m_1) = p$ for all w_1.

Now let m_o be the least positive m such that $\frac{d}{dw_1} P(m) = 0$, and suppose $m_o < m_1$. (See Diagram 4). Clearly $\frac{d}{dw_1} P'(m_o) \geq 0$, and therefore $\frac{d}{dm} \frac{d}{dw_1} P'(m) > 0$ for all $m > m_o$, $\frac{d}{dw_1} P'(m) > 0$ and $\frac{d}{dw_1} P(m) > 0$ for $m > m_o$, contradicting $P(m_1) = p$.

Diagram 4



Therefore m_1 is the least positive m such that $\frac{d}{dw_1}P(m) = 0$. Since for all $m \in (0, m_1)$, $\frac{d}{dw_1}P(m) < 0$ there exists an $\bar{m} < m_1$ such that $\frac{d}{dw_1}P'(\bar{m}) = 0$ so $\frac{d}{dw_1}P'(m) > 0$ for all $m > \bar{m}$. Q.E.D.

The close relationship between Lemmas 2.1 and 2.2 need scarcely be pointed out.

The implication of Lemma 2.2 is that since

$$P'(m_1) = r_1P(m_1) - 1 + w_1e^{\lambda m_1},$$

Theorem 1.2 therefore implies that

$$P'(m_1) = r_2p - 1 + w_2e^{\lambda m_1},$$

and only one value of w_1 will satisfy this requirement as well as the requirements of competitive pricing within country 1. Hence w_1, r_1 are uniquely determined by the equations:

$$1 = \int_0^{m_1} (1 - w_1e^{\lambda u})e^{-r_1u}\,du + e^{-r_1m_1}p \tag{28}$$

$$= \frac{1}{r_1}(1 - e^{-r_1m_1}) - \frac{w_1}{r_1-\lambda}(1 - e^{-(r_1-\lambda)m_1}) + pe^{-r_1m_1},$$

$$w_1 = w_2 + (r_2 - r_1)pe^{-\lambda m_1}. \tag{29}$$

One more problem remains. Initially it was assumed that $w_1 > w_2$. Now we have equations which determine values of w_1 and w_2 as a result of the specialisation in vintages. What extra restrictions on s_1 and s_2 are needed to ensure that $w_1 > w_2$?

Consider the isolated economy described by equations (1 - 10) as being made up of two 'cohorts' of vintages: 0 to m_1 and m_1 to m_2, where m has been chosen equal to m_2. Denote the price of a machine of age m_1 by p_o and the profit rate by r_o. The wage rate $w_o = w_2 = e^{-\lambda m_2}$. If each cohort did no investment in the other cohort then the expression for gross investment and gross output in section 2.3 shows that the gross saving rates for the cohorts would be respectively,

$$s_A = \frac{\int_o^{m_1} (gP(n) - P'(n))e^{-gn}dn}{\int_o^{m_1} e^{-gm}dm} = \frac{g(1 - p_o e^{-gm_1})}{1 - e^{-gm_1}}$$

$$s_B = \frac{\int_{m_1}^{m_2} (gP(n) - P'(n))e^{-gn}dn}{\int_o^{m_1} e^{-gm}dm} = \frac{gp_o e^{-gm_1}}{e^{-gm_1} - e^{-gm_2}}$$

Using both versions of (10) we have

$$gP(n) - P'(n) = (g-r)P(n) + 1 - e^{-\lambda(m-n)}$$

$$\frac{d}{dn}(gP(n) - P'(n)) = (g-r)P'(n) - \lambda e^{-\lambda(m-n)}$$

$$= -\lambda e^{-r(m-n)}\left(1 + \frac{g-\lambda}{\lambda-r}\left(1 - e^{-(\lambda-r)(m-n)}\right)\right)$$

which is always negative. Therefore

$$s_A > gP(m_1) - P'(m_1) > s_B.$$

Now compare these two cohorts within one isolated economy with the two trading countries with saving rates s_1, s_2. Suppose that $p = p_o$. Then clearly $w_1 = w_2 = w_o$ and $r_1 = r_2 = r_o$, for (2), (8), and (10) imply trivially

that (16), (27), (28) and (29) hold. Now as p falls, with m_1 and m_2 held constant, (16) implies that w_2 remains constant, but (27) shows that r_2 rises above r_o.

In order to find out what happens to w_1, r_1 we require a rather more complicated argument.

Lemma 2.3. In the economy described by (16), (27), competitive pricing implies that if p, m_1 are constant

$$\frac{d}{dm_2}P'(m_1) > 0.$$

Proof.

(27) shows that $\frac{dr_2}{dm_2} > 0$ for fixed m_1. Competitive pricing implies that for $n \in [m_1, m_2]$

$$P'(n) = r_2P(n) - \rho_2(n).$$

Now apply the method used in the proof of Lemma 2.1. The argument used there to prove $\frac{d}{dm}P'(0) > 0$ here proves $\frac{d}{dm_2}P'(m_1) > 0$. Q.E.D.

Compare now the following three economies:

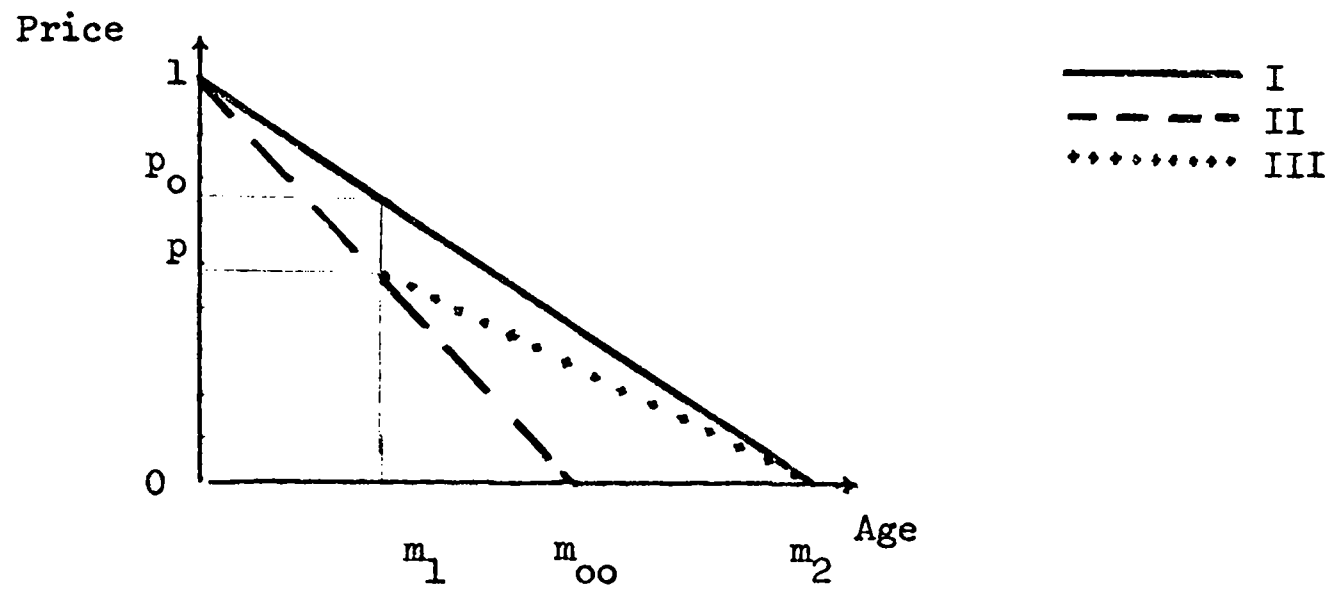
I: the isolated economy with factor prices w_o, r_o, scrapping age m_2 and $P(m_1) = p_o$;

II: the isolated economy with factor prices w_{oo}, r_{oo}, scrapping age m_{oo}, and $P(m_1) = p$;

III: the economy described by (16), (27), i.e. country 2 above, with factor prices w_2, r_2, scrapping age m_2, which buys machines of age m_1 at price $P(m_1) = p$.

Prices of machines in these three economies are represented in Diagram 5, for the case $p < p_o$. (The use of straight lines is for clarity only and is not realistic.)

Diagram 5



Lemma 2.1 implies that $w_{oo} > w_o$, $r_{oo} < r_o$ and $m_{oo} < m_2$. Now consider the division of economy II into two cohorts $[0,m_1]$, $[m_1,m_{oo}]$, and compare the second cohort with economy III i.e. country 2. Just as the second cohort in economy I had saving rate s_B less than the saving rate in the first cohort and therefore less than the saving rate in the economy as a whole, so the second cohort in economy II has a saving rate less than that of economy II as a whole. The only difference between economy III and the second cohort of economy II is that economy III keeps for a longer time the machines it buys at price p. Therefore, by Lemma 2.3, $P'(m_1)$ is larger in III than in II.

(29) requires that $P'(m_1)$ in III be the same as it is in country 1. Therefore $P'(m_1)$ is larger in country 1 than in economy II. But both country 1 and the first cohort of economy II satisfy the assumptions of Lemma 2.2. Therefore $w_1 > w_{oo}$ and $r_1 < r_{oo}$. But $w_{oo} > w_o = w_2$ and $r_{oo} < r_o < r_2$. Therefore $w_1 > w_2$ and $r_1 < r_2$.

Symmetrical arguments show that if $p > p_o$, $w_1 < w_{oo} < w_o = w_2$, $r_1 > r_{oo} > r_o > r_2$ which is incompatible with competitive equilibrium.

Finally from the definition of s_A and s_B, it is clear that $s_1 \geq s_A > s_B \geq s_2$ if and only if $p \leq p_o$.

If $p > p_o$ so $s_A > s_1 \gtrless s_2 > s_B$, then a steady state with each country specialising in vintages is not possible. In this case, trade will lead to factor price equalisation without specialisation. I am unable to say much more than that about this case, primarily because it appears that equilibrium is not unique. From now on therefore I consider only the case of nonequalised factor prices.

The results of this section are summarised in the following theorem.

Theorem 2.2. The steady state described by (13 - 29) is a competitive equilibrium if and only if $s_1 \gtrless s_A > s_B \gtrless s_2$. If $p < p_o$, $s_1 > s_A > s_B > s_2$ and $w_1 > w_2$, $r_1 < r_2$. If $p = p_o$, $s_1 = s_A > s_B = s_2$, and $w_1 = w_2$, $r_1 = r_2$.

When $m \in [0,m_1]$,

$$P(m) = \int_m^{m_1} (1 - w_1 e^{\lambda u}) e^{-r_1(u-m)} du + e^{-r_1(m_1-m)} p, \qquad (30)$$

$$\text{and } P'(m) = r_1 P(m) - 1 + w_1 e^{\lambda m}; \qquad (31)$$

and when $m \in [m_1,m_2]$

$$P(m) = \int_m^{m_2} (1 - w_2 e^{\lambda u}) e^{-r_2(u-m)} du, \qquad (32)$$

$$\text{and } P'(m) = r_2 P(m) - 1 + w_2 e^{\lambda m}. \qquad (33)$$

2.5. Comparisons between trade and autarchy: fixed saving rates

In section 2.1 I have summarised the behaviour of an isolated economy with the clay-clay technology. In section 2.4 I have analysed the behaviour of a freely trading two-country world with the same technology. We can now compare the two types of steady state, autarchy and free trade, on the assumption of constant gross saving rates. The notation used briefly in the previous section will be adopted: the superscript ' refers to the value of a variable in autarchy; the absence

of a superscript indicates a free-trade value. Alternative notation is adopted in the one case where confusion with the use of the same superscript to indicate differentiation might result.

For country 1,

$$s_1 = \frac{g}{1 - e^{-gm_1'}} = \frac{g(1 - pe^{-gm_1})}{1 - e^{-gm_1}},$$

$$(g-\lambda)L_1 = I_1'(1 - e^{-(g-\lambda)m_1'}) = I(1 - e^{-(g-\lambda)m_1}),$$

$$Y_1' = \frac{I_1'}{g}(1 - e^{-gm_1'}), \quad Y_1 = \frac{I}{g}(1 - e^{-gm_1}).$$

Since $p > 0$, $m_1' > m_1$ and $I > I_1'$. It is easy to prove that $Y_1 > Y_1'$. Country 1 sells machines at a positive price at an earlier age than the age they would be scrapped at in autarchy. Production is concentrated on newer, more productive vintages. Trade causes a rise in steady state production levels. Since s_1 is assumed to be the same in both states, $C_1 > C_1'$.

In country 2, if $s_2 \geq g$,

$$s_2 = \frac{g}{1 - e^{-gm_2'}} = \frac{gp}{1 - e^{-g(m_2-m_1)}}$$

$$(g-\lambda)L_2 = I_2'(1 - e^{-(g-\lambda)m_2'}) = Ie^{-(g-\lambda)m_1}(1 - e^{-(g-\lambda)(m_2-m_1)}),$$

$$Y_2' = \frac{I_2'}{g}(1 - e^{-gm_2'}), \quad Y_2 = \frac{I}{g}e^{-gm_1}(1 - e^{-g(m_2-m_1)}).$$

Since $p \leq 1$, $m_2 - m_1 \leq m_2'$. This implies $Ie^{-(g-\lambda)m_1} \geq I_2'$ and $Y_2e^{\lambda m_1} \geq Y_2'$. But this need not imply $Y_2 > Y_2'$. Trade concentrates production on a narrower range of vintages, but the tendency for this to raise steady state production may be offset by the fact that machine of age m_1 are

less productive than new machines. The possibility is raised that $Y_2 < Y_2'$ and $C_2 < C_2'$, and numerical examples below (p.75) show that this is indeed possible, although does not always happen.

For the world as a whole

$$L_1 + L_2 = \frac{I}{g-\lambda}\left(1 - e^{-(g-\lambda)m_2}\right)$$

$$Y_1 + Y_2 = \frac{I}{g}\left(1 - e^{-gm_2}\right)$$

$$s = \frac{I}{Y_1 + Y_2} = \frac{s_1Y_1 + s_2Y_2}{Y_1 + Y_2} = \frac{g}{1 - e^{-gm_2}}$$

so the world looks like an isolated economy with lifetime m_2, so far as physical quantities are concerned.

Obviously if $Y_2 \geq Y_2'$, $Y_1 + Y_2 > Y_1' + Y_2'$. In fact, this is true even if $Y_2 < Y_2'$. For existence of fully specialised equilibrium requires $s_1 > s_2$, hence $m_1' < m_2'$. Now consider the hypothetical world created from autarchy by taking recently scrapped machines from country 1 to country 2 and transferring workers from the oldest vintages in country 2 to these machines in sufficient numbers as to make the lifetime of machines equal in both countries. This hypothetical world has a production level Y', say. Clearly equalising scrapping ages while affecting nothing else raises production: $Y' > Y_1' + Y_2'$. The level of investment is kept unchanged at $I_1' + I_2'$, so the overall gross saving rate is

$$\bar{s} = \frac{I_1' + I_2'}{Y'}$$

Therefore

$$\bar{s} < \frac{I_1' + I_2'}{Y_1' + Y_2'} = \frac{s_1Y_1' + s_2Y_2'}{Y_1' + Y_2'} = s', \text{ say.}$$

But $s = \frac{s_1 Y_1 + s_2 Y_2}{Y_1 + Y_2}$, $Y_1 > Y_1'$, and $Y_2' > Y_2$ by hypothesis. Therefore s and s' are weighted averages of s_1 and s_2, $s_1 > s_2$, and s_1 has the heavier weight in s.

Therefore

$$s > s' > \bar{s}.$$

The state of the world in which saving is at rate s is physically identical to an isolated economy with saving rate s. The (hypothetical) state with saving rate $\bar{s}$ is also physically identical to an isolated economy. But the comparative dynamics of the one-country clay-clay model ensures higher production in the economy with higher saving rate, because of the shorter lifetime.

Therefore

$$Y_1 + Y_2 > Y'$$

Therefore

$$Y_1 + Y_2 > Y_1' + Y_2'.$$

If s and $\bar{s}$ are below the golden rule,

$$C_1 + C_2 > C' > C_1' + C_2'.$$

The comparison of factor prices in trade and autarchy is a little more complicated. Compare country 2 in trade with the isolated economy I described above (p.64). Each has the same scrapping age m_2, therefore the same wage rate. The saving rate in economy I is s, the same as the saving rate for the trading world as a whole. s is a weighted average of s_1, s_2, and $s_1 \gtrless s_A > s_B \gtrless s_2$, so that $s > s_2$. Comparing country 2 in autarchy with economy I, we see that each is an isolated economy, and country 2 has the lower saving rate, so the longer lifetime, higher profit rate and lower wage rate. Therefore $w_2' < w_2$.

Consider what happens to prices in country 2 in autarchy. It scraps at age m_2', so P(m) falls from 1 at m = 0 to 0 at m = m_2'. There exists an $m_1' \in (0,m_2')$ such that $P(m_1') = p$. Consider the two cohorts $[0,m_1']$, $[m_1',m_2']$ in this economy. The saving rate of the second cohort is

$$s_{2B} = \frac{gp}{1 - e^{-g(m_2'-m_1')}}$$

But it is proved above (p.63) that for any economy divided thus, $s_B < s_A$; so $s_{2B} < s_2 < s_{2A}$. Now

$$s_2 = \frac{gp}{1 - e^{-g(m_2-m_1)}}$$

and $s_2 > s_{2B}$ implies $m_2 - m_1 < m_2' - m_1'$.

In autarchy, therefore, a machine bought for price p is held for $m_2' - m_1'$ years and earns a profit rate r_2'. In trade, a machine bought for price p is held for a shorter time and earns r_2. The shorter lifetime means smaller quasi-rent and a shorter period in which to earn it. Therefore $r_2 < r_2'$.

It is not possible to arrive at definite conclusions on the movement of factor prices in country 1. If $s_1 = s_A > s_B = s_2$, then $w_1 = w_2$, $r_1 = r_2$, and $s_1 > s$ implies $w_1' > w_1$, $r_1' < r_1$. But when p falls below p_o, w_1 rises above w_2 and $w_1' > w_2$ no longer implies $w_1' > w_1$. It is possible that $w_1 > w_1'$, $r_1 > r_1'$, or that $w_1 > w_1'$, $r_1 < r_1'$. That $w_1' \geq w_1$ and $r_1' \geq r_1$ is impossible is clear, and can be formally shown as follows. Let p_1 be the price of a machine of age m_1 in the country 1 in autarchy. The, in a further obvious extension of notation,

$$\frac{g(1 - pe^{-gm_1})}{1 - e^{-gm_1}} = s_1 < s_{1A} = \frac{g(1 - p_1 e^{-gm_1})}{1 - e^{-gm_1}}$$

Therefore, $p > p_1$. $w_1' \geq w_1$ therefore implies that holding a machine from age 0 to age m_1 in economy 1 in autarchy earns lower quasi-rent, and at m_1 the machine is less valuable then would be the case in trade. Therefore $r_1' < r_1$.

If the two countries enter trade with widely separated saving rates, so that factor price equalisation is not possible, factor prices in country 1 may move further away from equalisation. A similar phenomenon is noted in a rather different model (with a different long-run savings objective) in Stiglitz (1970a)[1], p.463.

Numerical examples are given in the next section to confirm the possibility of this type of behaviour.

Finally, it should be noted that in discussing conditions for existence and uniqueness of steady states in this model, I have not discussed the issue of stability of steady state paths.

2.6. Numerical Examples

Several of the results of the previous section were of a rather negative nature: e.g. the possibility that $C_2' > C_2$ was mentioned. The only satisfactory way of showing that such a phenomenon is possible but not necessary is to produce both an example and a counter-example. I have computed numerical examples of economies with fixed gross saving rates in trade and in autarchy. In each case computation was straight forward apart from the relationship between profit rate and lifetime of of machines. In the case of autarchy, (8) was solved by a simple Newton-Raphson procedure which typically converged in three or four iterations. Examples of the two country world in trade were produced for given g, by arbitrarily choosing a profit rate and lifetime for an isolated economy.

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[1] Stiglitz, J.E., "Factor Price Equalization in a Dynamic Economy", Journal of Political Economy 78, 3(June 1970), pp.456-488.

This lifetime was taken as m_2. Prices in such an isolated economy were calculated for various m and an m_1 and corresponding p_o were chosen. m_1, m_2 then fixed the L_1/L_2 ratio from (19), (20). Arbitrary choice of $p < p_o$ then fixed s_1, s_2 from (23), (24). Output and consumption per capita were calculated from (19), (20), (21), (22), (25), (26).

m_2 fixed w_2, by (16). (27) then gave r_2, in a Newton-Raphson procedure. Substituting (29) in (28) gave an equation in r_1, m_1, solved in the same way; and then (29) gave w_1.

The s_1, s_2 were then substituted in the equations for an isolated economy, to give the corresponding figures for autarchy steady state.

The first seven examples set out below illustrate the various possibilities discussed above. Examples (2 - 7) are all given in order to show that the behaviour of C_2 is independent of the behaviour of w_1, r_1. (The fact that in some of the examples $g = \lambda$ and $n = 0$ causes no difficulty, for I have in no argument above assumed that n is strictly positive. Of course when $n = 0$, many of the formulae above have to be changed by the use of de l'Hôpital's rule).

In the tables, lower-case c refers to consumption per head.

Example 1 $s_2 < g$

$\lambda = 0.05$ $\quad g = 0.10$

$L_1 = 0.5370$ $\quad L_2 = 0.4630$ $\quad s_1 = 0.1469$ $\quad s_2 = 0.0887$

Trade $\quad p = 0.5250 \quad (p_o = 0.5833)$

$m_1 = 7.0000$ $\quad w_1 = 0.5009$ $\quad r_1 = 0.4357$ $\quad c_1 = 0.7272$

$m_2 = 8.9686$ $\quad w_2 = 0.4500$ $\quad r_2 = 0.5731$ $\quad c_2 = 0.5262$

Autarchy

$m_1' = 11.4238$ $\quad w_1' = 0.5649$ $\quad r_1' = 0.3418$ $\quad c_1' = 0.6675$

Full employment unattainable in 2 even if $m_2' = \infty$.

Example 2 $r_1 > r_1'$, $w_1 < w_1'$, $C_2 > C_2'$

$\lambda = 0.05$ $\quad g = 0.10$

$L_1 = 0.3296$ $\quad L_2 = 0.6704$ $\quad s_1 = 0.1608$ $\quad s_2 = 0.1005$

Trade $\quad p = 0.7012 \quad (p_o = 0.7791)$

$m_1 = 4.0000$ $\quad w_1 = 0.5873$ $\quad r_1 = 0.3274$ $\quad c_1 = 0.7632$

$m_2 = 11.9686$ $\quad w_2 = 0.4500$ $\quad r_2 = 0.5665$ $\quad c_2 = 0.5706$

Autarchy

$m_1' = 9.7296$ $\quad w_1' = 0.6148$ $\quad r_1' = 0.2562$ $\quad c_1' = 0.6776$

$m_2' = 53.5938$ $\quad w_2' = 0.0686$ $\quad r_2' = 0.9275$ $\quad c_2' = 0.4806$

Example 3 $r_1 > r_1'$, $w_1 > w_1'$, $C_2 > C_2'$

$\lambda = 0.05$ $g = 0.10$

$L_1 = 0.2533$ $L_2 = 0.7467$ $s_1 = 0.1701$ $s_2 = 0.1038$

Trade $p = 0.7546$ $(p_o = 0.8384)$

$m_1 = 3.0000$ $w_1 = 0.6714$ $r_1 = 0.2244$ $c_1 = 0.7721$

$m_2 = 12.9686$ $w_2 = 0.4500$ $r_2 = 0.5652$ $c_2 = 0.5873$

Autarchy

$m_1' = 8.8605$ $w_1' = 0.6421$ $r_1' = 0.2012$ $c_1 = 0.6813$

$m_2' = 32.9526$ $w_2' = 0.1925$ $r_2' = 0.7946$ $c_2' = 0.5343$

Example 4 $r_1 < r_1'$, $w_1 > w_1'$, $C_2 > C_2'$

$\lambda = 0.05$ $g = 0.05$

$L_1 = 0.2505$ $L_2 = 0.7495$ $s_1 = 0.1351$ $s_2 = 0.0692$

Trade $p = 0.6233$ $(p_o = 0.7791)$

$m_1 = 4.0000$ $w_1 = 0.6860$ $r_1 = 0.1865$ $c_1 = 0.7839$

$m_2 = 11.9686$ $w_2 = 0.4500$ $r_2 = 0.6490$ $c_2 = 0.5735$

Autarchy

$m_1' = 9.2453$ $w_1' = 0.6299$ $r_1' = 0.2267$ $c_1 = 0.6926$

$m_2' = 25.6419$ $w_2' = 0.2775$ $r_2' = 0.7012$ $c_2 = 0.5246$

Example 5 $r_1 > r_1'$, $w_1 < w_1'$, $C_2 < C_2'$

$\lambda = 0.05$ $\quad g = 0.05$

$L_1 = 0.3131$ $\quad L_2 = 0.6869$ $\quad s_1 = 0.1125$ $\quad s_2 = 0.0764$

Trade $\quad p = 0.6451$ $\quad (p_o = 0.7168)$

$m_1 = 5.0000$ $\quad w_1 = 0.5439$ $\quad r_1 = 0.3815$ $\quad c_1 = 0.7853$

$m_2 = 10.9686$ $\quad w_2 = 0.4500$ $\quad r_2 = 0.5682$ $\quad c_2 = 0.5537$

Autarchy

$m_1' = 11.7595$ $\quad w_1' = 0.5555$ $\quad r_1' = 0.3564$ $\quad c_1' = 0.6710$

$m_2' = 21.2475$ $\quad w_2' = 0.3456$ $\quad r_2' = 0.6243$ $\quad c_2' = 0.5689$

Example 6 $r_1 > r_1'$, $w_1 > w_1'$, $C_2 < C_2'$

$\lambda = 0.05$ $\quad g = 0.05$

$L_1 = 0.2505$ $\quad L_2 = 0.7495$ $\quad s_1 = 0.1175$ $\quad s_2 = 0.0778$

Trade $\quad p = 0.7012$ $\quad (p_o = 0.7791)$

$m_1 = 4.0000$ $\quad w_1 = 0.5873$ $\quad r_1 = 0.3274$ $\quad c_1 = 0.7999$

$m_2 = 11.9686$ $\quad w_2 = 0.4500$ $\quad r_2 = 0.5665$ $\quad c_2 = 0.5681$

Autarchy

$m_1' = 11.0877$ $\quad W_1' = 0.5744$ $\quad r_1' = 0.3266$ $\quad c_1' = 0.6774$

$m_2' = 20.5591$ $\quad w_2' = 0.3577$ $\quad r_2' = 0.6103$ $\quad c_2' = 0.5762$

Example 7 $r_1 < r_1'$, $w_1 > w_1'$, $C_2 < C_2'$

$\lambda = 0.01$ $g = 0.01$

$L_1 = 0.1542$ $L_2 = 0.8458$ $s_1 = 0.0276$ $s_2 = 0.0176$

Trade $p = 0.7953$ $(p_o = 0.8837)$

$m_1 = 11.0000$ $w_1 = 0.6579$ $r_1 = 0.3214$ $c_1 = 0.9208$

$m_2 = 60.3350$ $w_2 = 0.4900$ $r_2 = 0.5570$ $c_2 = 0.6608$

Autarchy

$m_1' = 44.9915$ $w_1' = 0.6377$ $r_1' = 0.3432$ $c_1' = 0.7831$

$m_2' = 84.3018$ $w_2' = 0.4304$ $r_2' = 0.5618$ $c_2' = 0.6638$

Finally, two examples to show how trade may cause extreme changes in factor prices in country 1, raising w_1 above 1, the maximum attainable level in autarchy, and reducing r_1 from $r_1' > g$ to $r_1 \leq 0$.

Example 8 $w_1 > 1$

$\lambda = 0.05$ $g = 0.10$

$L_1 = 0.0887$ $L_2 = 0.9113$ $s_1 = 0.2389$ $s_2 = 0.1100$

Trade $p = 0.8539$ $(p_o = 0.9488)$

$m_1 = 1.0000$ $w_1 = 1.7432$ $r_1 = -1.0289$ $c_1 = 0.7425$

$m_2 = 14.9686$ $w_2 = 0.4500$ $r_2 = 0.5631$ $c_2 = 0.6236$

Autarchy

$m_1' = 5.4223$ $w_1' = 0.7625$ $r_1' = -0.1964$ $c_1' = 0.6707$

$m_2' = 23.9660$ $w_2' = 0.3017$ $r_2' = 0.6741$ $c_2' = 0.5792$

Now it is clear that Example 8 cannot be taken seriously as an example of the operation of the model, even though it does satisfy the six equations defining equilibrium. For the fact that $w_1 > 1$ implies that $\rho_1 < 0$ for all m, so that this is not a competitive equilibrium. The key to understanding what is going on here is the fact that $r_1 < 0$, which raises the possibility of some machines being held unused while they still have useful life ahead of them. Clearly, Example 8 shows that if $s_1 = 0.2389$, $s_2 = 0.1100$, $L_1 = 0.0887$, and $L_2 = 0.9113$, such 'speculative' holding of machines must be a feature of the competitive steady state. I do not discuss this case in detail because the possibility that profit rates be negative seems not very interesting, except as an indication of pathological over-saving.

It is easily checked that in Example 9, $\rho_1(m) > 0$ for all $m \in [0,m_1]$. The interesting feature of this example is the demonstration that a value of s_1 which in autarchy gives rise to a high profit rate, in trade may give rise to a zero or negative profit rate. I have not checked that in Example 9, $r_2P(m) - \rho_2(m) \leq r_1P(m)$ for all $m \in [m_1,m_2]$ which would be necessary to prove conclusively that there is no speculative holding of machines in this case. Even if this were not so, Example 9 would show, from continuity considerations, the possibility of constructing an example in which $r_1' > g$ and $r_1 = 0$, and in such an example there is no speculation.

Example 9 $r_1' > g$, $r_1 \lesseqgtr 0$

$\lambda = 0.05$ $g = 0.05$

$L_1 = 0.1252$ $L_2 = 0.8748$ $s_1 = 0.1425$ $s_2 = 0.0801$

Trade $p = 0.8055$ $(p_o = 0.8950)$

$m_1 = 2.0000$ $w_1 = 0.8787$ $r_1 = -0.0241$ $c_1 = 0.8160$

$m_2 = 13.9686$ $w_2 = 0.4500$ $r_2 = 0.5640$ $c_2 = 0.5990$

Autarchy

$m_1' = 8.6435$ $w_1' = 0.6491$ $r_1' = 0.1858$ $c_1' = 0.6963$

$m_2' = 19.5654$ $w_2' = 0.3760$ $r_2' = 0.5892$ $c_2' = 0.5868$

2.7. Summary and conclusions

In the concluding section of Chapter 1, I justified the application of the theory of that chapter to the detailed analysis of steady states in a specific and simple model on the grounds that it is necessary to look at the consequences of the choice of a saving programme or a set of shadow prices in order to understand the implications of the choice.

If then two countries with equal growth rates, the same one-sector clay-clay technology, and fairly widely separated fixed gross saving rates allow trade to take place in machines and in the output good, but prevent international flows of labour and investment, and if the resulting state tends towards a steady state (for which proposition Solow *et al.*'s proof of convergence in the one country model gives *prima facie* plausibility but no more), then the following statements may be made about the 'long-run' effects of trade:

1. Production and consumption in the high saving country are raised.
2. The wage rate in the high saving country, which is in autarchy

higher than in the low saving country, may rise or fall. The profit rate may also rise or fall. Both rates may rise, or only one, but both may not fall. The greater the difference in saving rates between the two countries, the more likely it is that the wage rate rises. The profit rate may fall from above the growth rate to zero or below. The wage rate may rise so high and the profit rate fall far enough below zero so that entrepreneurs in country 1 hold machines unused for a time before selling them to country 2.

3. Production and consumption in the low saving economy may rise or fall.

4. The wage rate in the low saving country rises and the profit rate falls.

5. Total world production rises. (I have been unable to produce an example where aggregate consumption falls because of saving inefficiency).

Of these phenomena, by far the most interesting is the fact that trade may lead to a fall in the long run consumption in the low-saving country, as a result of it using old and physically less efficient machines. In the next chapter and in Chapter 5 the economic logic underlying this will become clearer. For the moment I wish only to observe that although perhaps slightly surprising there is nothing paradoxical about the result. What happens is that the low saving country takes advantage of the opportunity of trading in machines to sell its new machines and consume the proceeds, so raising short-run consumption at the expense of, in the long-run, having low consumption and a capital stock of low value.

The unpredictability of the behaviour of factor prices in the high saving country shows the unsatisfactory nature of the assumption of fixed saving rates. This is most clear in Example 9, where in autarchy

the saving rate of 0.1425 corresponds to a profit rate above the growth rate which could quite plausibly be a modified golden rule path, i.e. the saving behaviour is consistent with rational intertemporal behaviour. To maintain this saving rate in the trade equilibrium brings about a nonpositive profit rate indicating grossly inefficient intertemporal behaviour.

It does seem therefore that the alternative assumption that the countries hold their profit rates constant may in certain respects be more plausible, and may also give a clearer indication of the nature of the choices opened up by trade in machines. I investigate this possibility in the next chapter.

CHAPTER 3

The Clay-Clay Model with Fixed Profit Rates

3.1. Introduction: existence of long-run equilibrium

The model considered in this chapter is formally the same as the model considered in the previous chapter. The difference is that in comparing steady states in trade and in autarchy, the assumption that gross saving rates are unchanged is now replaced by the assumption that profit rates are unchanged. The equations describing trade equilibrium are, of course, independent of which assumption is made and equations (2.13) to (2.33) therefore still apply (although, for convenience, I shall reproduce and renumber some of them below).

One issue which it is desirable to settle is the question of under what conditions r_1 and r_2 define a unique steady state in the trading world. The question has intrinsic interest, and in answering it the question of how trade affects factor prices is answered also. (Further, the method of proof is a simple version of the main proof of the next chapter). The relevant equations are (2.19), (2.20), (2.27), (2.16), (2.28) and (2.29) but with, initially, no assumption that m_1, m_2 and $P(m_1)$ are constant. Assume that $r_2 > r_1 \geq 0$, so that there is no speculation in machines in steady state.

First, ignore the full employment condition and consider the implications of fixing r_1, r_2, w_1, w_2. We have:

$$w_2 = e^{-\lambda m_2} \tag{1}$$

$$P(m_1) = \frac{1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right) - \frac{1}{r_2-\lambda}\left(e^{-\lambda(m_2-m_1)} - e^{-r_2(m_2-m_1)}\right) \tag{2}$$

$$w_1 = w_2 + (r_2 - r_1)P(m_1)e^{-\lambda m_1} \tag{3}$$

$$1 = \frac{1}{r_1}(1 - e^{-r_1 m_1}) - \frac{w_1}{r_1 - \lambda}(1 - e^{-(r_1-\lambda)m_1}) + P(m_1)e^{-r_1 m_1} \tag{4}$$

From (1), w_2 fixes m_2 uniquely. (3) implies that $P(m_1)e^{-\lambda m_1}$ is constant. $P(m_1)$ is clearly decreasing in m_1, so $P(m_1)e^{-\lambda m_1}$ is decreasing in m_1, which proves that m_1 is constant. Therefore for fixed r_i, w_i, (1), (2) and (3) determine fixed m_2, m_1 and $P(m_1)$.

Now the right hand side of (4) may be rewritten:

$$P(0) = \int_0^{m_1} (1 - w_1 e^{\lambda m})e^{-r_1 m}\,dm + e^{-r_1 m_1}\int_0^{m_2 - m_1} (1 - w_2 e^{\lambda(m+m_1)})e^{-r_2 m}\,dm.$$

A decrease in w_1 or w_2, keeping m_1 and m_2 constant, clearly raises $P(0)$. But (1) and (3) are the conditions for optimal choice of m_1, m_2 to maximise $P(0)$. Therefore a decrease in w_1 or w_2, accompanied by consequent changes in m_1, m_2 must *a fortiori*, raise $P(0)$. If $P(0)$ is a decreasing function of both w_1 and w_2, then the requirement that $P(0) = 1$ implies that in equilibrium w_1 is a decreasing function of w_2, for fixed r_1 and r_2.

If $w_1 = w_2$, (3) shows that $P(m_1) = 0$, so $m_1 = m_2$. The equations become

$$w_1 = e^{-\lambda m_1}$$

$$1 = \frac{1}{r_1}(1 - e^{-r_1 m_1}) - \frac{e^{-\lambda m_1}}{r_1 - \lambda}(1 - e^{-(r_1-\lambda)m_1})$$

so that w_1 takes the value corresponding to r_1 in autarchy.

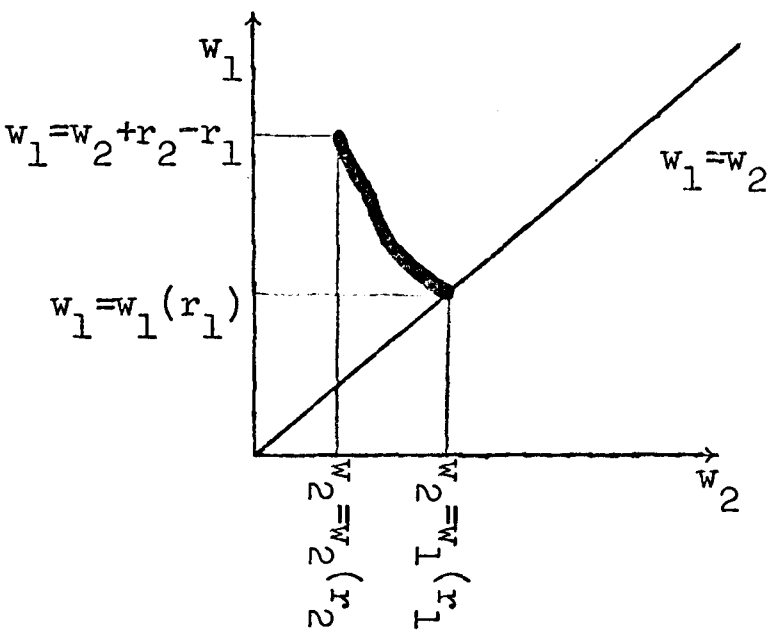
If $w_1 = w_2 + r_2 - r_1$, $P(m_1)e^{-\lambda m_1} = 1$ so $m_1 = 0$ and the value of m_2 given by

$$1 = \frac{1}{r_2}(1 - e^{-r_2 m_2}) - \frac{1}{r_2 - \lambda}(e^{-\lambda m_2} - e^{-r_2 m_2})$$

satisfy the equations. This implies that w_2 takes the value corresponding to r_2 in autarchy.

Therefore the fixed r_1, r_2 give rise to a range of possible values of w_1, w_2 satisfying (1 - 4) which may be represented in Diagram 1.

Diagram 1



Now when $w_1 = w_2$, $m_2 = m_1$ and m_1 is given by $w_1 = e^{-\lambda m_1}$. As w_1 rises, w_2 falls and (1) shows that m_2 rises. (3) shows that $P(m)e^{-\lambda m_1}$ rises so m_1 falls. When $w_1 = w_2 + r_2 - r_1$, $m_1 = 0$.

Full employment requires that

$$L_1 = \frac{I}{g-\lambda}\left(1 - e^{-(g-\lambda)m_1}\right), \tag{5}$$

$$L_2 = \frac{I}{g-\lambda}\left(e^{-(g-\lambda)m_1} - e^{-(g-\lambda)m_2}\right), \tag{6}$$

which imply that L_1/L_2 is a function of m_1 and m_2, strictly increasing in m_1 and strictly decreasing in m_2, i.e. L_1/L_2 decreases strictly from infinity at $w_1 = w_2$ to zero at $w_1 = w_2 + r_2 - r_1$. Hence (5) and (6) define a unique point on the interior of $w_1 = f(w_2)$ line in Diagram 1.

Before stating all of this formally, all that has to be noted is the implicit assumption above that r_1 and r_2 were each compatible with equilibrium in autarchy, i.e. $r_i \lesseqgtr 1$.

Theorem 3.1. Equations (1 - 6) define a unique set of values of m_1, m_2, w_1, w_2, I and $P(m_1) = p$, if $0 \lesseqgtr r_1 < r_2 \lesseqgtr 1$. Both w_1 and w_2 are greater than the values they take in autarchic steady states with profit rates r_1, r_2, but w_2 is less than w_1 and less than the value w_1 takes in autarchy.

The proof of this theorem follows immediately from the fact that (1 - 6) define a unique point on the interior of the $w_1 = f(w_2)$ line.

Clearly in the event that $r_1 = r_2$ the line shrinks to the point $w_1 = w_2 = w(r_1)$.

Note that the larger is the labour force of one country relative to the other, the smaller is the gain from trade of its wage earners, because the smaller is the divergence between its wage rates in trade and in autarchy.

Since $r_1 \gtreqless 0$ there is no speculative holding of machines, which implies $\rho_1(m_1) > 0$, i.e. $m_3 > m_1$ where m_3 is defined by $w_1 = e^{-\lambda m_3}$.

What has been proved about wage rates implies the following about scrapping and trading ages (where, as before, a prime indicates the autarchy value):

$$m_1 < m_3 < m_1' < m_2 < m_2'.$$

(In the degenerate case of $r_1 = r_2$, $m_1 < m_3 = m_1' = m_2 = m_2'$).

3.2. The effect of trade on consumption

In section 2.3 it was demonstrated that the contribution to consumption coming from the cohort $[n_1, n_2]$ is

$$C_n(t) = Ie^{gt} \int_{n_1}^{n_2} (1 - gP(n) + P'(n))e^{-gn}dn$$

since $\mu_o = 1$ in the model discussed here. Now if the factor prices ruling in this cohort are w, r and if there is competitive pricing,

$$1 - gP(n) + P'(n) = 1 - gP(n) + rP(n) - 1 + we^{\lambda n}$$

$$= (r-g)P(n) + we^{\lambda n}.$$

Hence consumption per worker in the cohort, if it is self-sufficient, is

$$c_n(t) = \frac{C_n(t)}{L_n(t)} = c_n e^{\lambda t}$$

where

$$c_n = \frac{\int_{n_1}^{n_2} ((r-g)P(n)e^{-\lambda n} + w)e^{-(g-\lambda)n}dn}{\int_{n_1}^{n_2} e^{-(g-\lambda)n}dn}$$

If $r = g$, $c_n = w$. In particular, if every cohort has the same factor prices and $r = g$, consumption per man in each cohort is the same and equal to the common wage rate.

If $r > g$, since $P(n)$ and $e^{-\lambda n}$ are both positive decreasing functions of n, $(r-g)P(n)e^{-\lambda n} + w$ is a decreasing function of n and

$$(r-g)P(n_1)e^{-\lambda n_1} + w > c_n > (r-g)P(n_2)e^{-\lambda n_2} + w$$

and consumption per head is higher on the cohorts of newer machines.

In fact this is true even if every cohort no longer has the same factor prices so long as each has a profit rate greater than the growth rate. For if c_i and c_{i+1} are the consumption levels on the adjacent cohorts $[n_{i-1}, n_i]$, $[n_i, n_{i+1}]$ and $g \leq r_i < r_{i+1}$ then

$$c_i \geq (r_i - g)P(n_i)e^{-\lambda n_i} + w_i$$

$$= (r_{i+1} - g)P(n_i)e^{-\lambda n_i} + w_{i+1} > c_{i+1}$$

since competitive pricing implies as usual that

$$w_{i+1} = w_i + (r_i - r_{i+1})P(n_i)e^{-\lambda n_i}$$

By completely symmetrical arguments it is easy to prove that when the profit rate is below the discount rate consumption per head is on increasing function of the age of the machines in each cohort, again irrespective of whether there is factor price equalisation between cohorts.

The underlying logic should be clear: when saving is below the golden rule level machines are being kept too long, and increased investment in new machines would raise long run consumption, while the reverse is true above the golden rule.

With this equipment we can now analyse the effect of trade on consumption when $r_1 < r_2$ and r_1 and r_2 are the same in trade and autarchy.

Theorem 3.2. If $r_1 \geq g$, $C_1 > C_1'$, i.e. steady state consumption is higher in trade than in autarchy if r_1 is the same in both cases and $r_1 < r_2$. If $r_2 \leq g$, $C_2 > C_2'$. If $r_1 < g$, C_1 may be greater or smaller than C_1'; and if $r_2 > g$, C_2 may be greater or smaller than C_2'.

Proof

Consider first the case of country 1 when $r_1 \geq g$. In autarchy $w_1' = e^{-\lambda m_1'}$. In trade, $e^{-\lambda m_1} \geq w_1 > w_1'$ so that $m_1 < m_1'$. In trade, competitive pricing implies

$$1 = P(0) = \int_0^m (1 - w_1 e^{\lambda n}) e^{-r_1 n} dn + e^{-r_1 m} P(m) \qquad \text{for } m \leqslant m_1$$

while in autarchy, using the subscript a to indicate autarchy prices in order to avoid confusion with differentiation,

$$1 = P_a(0) = \int_0^m (1 - w_1' e^{\lambda n}) e^{-r_1 n} dn + e^{-r_1 m} P_a(m).$$

Clearly $w_1' < w_1$ implies that $P_a(m) < P(m)$ for $m > 0$.

Denoting the prices of machines age m_1 by p, p_a in the two regimes, we have the result that $p_a < p$.

Now consider the two cohorts, $[0,m_1]$, $[m_1,m_1']$ in economy 1 in autarchy, indexed A, B respectively. From the discussion above we know that $r_1 \gtrless g$ implies

$$c_{1A}' \gtrless c_1' \gtrless c_{1B}'$$

with strict inequalities if $r_1 > g$. But

$$c_{1A}' = \frac{\int_0^{m_1} ((r_1-g)P_a(m)e^{-\lambda m} + w_1')e^{-(g-\lambda)m} dm}{\int_0^{m_1} e^{-(g-\lambda)m} dm}$$

$$< \frac{\int_0^{m_1} ((r_1-g)P(m)e^{-\lambda m} + w_1)e^{-(g-\lambda)m} dm}{\int_0^{m_1} e^{-(g-\lambda)m} dm} = c_1$$

Therefore

$$c_1 > c_1', \text{ and } C_1 > C_1'$$

(Note that even if $r_1 = g$, the fact that $r_2 > r_1$ makes $w_1' \lessgtr w_1$, so leading to a rise in C_1.)

Proof of the fact that when $r_1 < g$ the change in consumption may be positive or negative is provided by examples in the next section. It

should be clear from the argument above why there is indeterminacy. For now, $c_1' > c_{1A}'$ but there are opposing forces involved in the comparison of c_{1A}' and c_1. Clearly if w_1 is very close to w_1', $P(m)$ is very close to $P_a(m)$ and c_{1A}' is close to c_1, but this apparently need not always be the case.

A rather different type of argument has to be used for country 2. Suppose $r_2 \lesseqgtr g$. Consider the cohort $[m_2' - m_2 + m_1, m_2']$ of country 2 in autarchy. (Since $m_2' - m_2 > 0$, $m_1 > 0$, and $m_2 - m_1 > 0$, this is a strict subset of the whole economy). The price of a machine of age $m_2' - m_2 + m_1$ is

$$p = \frac{1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right) - \frac{1}{r_2-\lambda}\left(e^{-\lambda(m_2-m_1)} - e^{-r_2(m_2-m_1)}\right)$$

the same as the price of a machine aged m_1 in the trading economy with factor prices r_2, w_2. Thus

$$s_{2B}' = \frac{gp}{1 - e^{-g(m_2-m_1)}} = s_2.$$

But $m_2' > m_2$, so although this cohort with saving rate s_{2B}' holds machines for the same number of years as the economy with saving rate s_2, the latter uses machines which are on the average newer, and hence has higher steady state output per man at every instant: $y_2 > y_{2B}'$.

Therefore

$$c_2 = (1 - s_2)y_2 > (1 - s_{2B}')y_{2B}' = c_{2B}'$$

But $r_2 \lesseqgtr g$ implies $c_{2B}' \gtreqless c_2' \gtreqless c_{2A}'$

Therefore

$$c_2 > c_2' \text{ and } C_2 > C_2'.$$

If $r_2 > g$, $c'_{2B} < c'_2$ and the total effect is unpredictable. Examples of both $C_2 > C'_2$ and $C_2 < C'_2$ are given in the next section.

Q.E.D.

It is important to the understanding of the effects of trade to distinguish between why $C'_1 > C_1$ in some cases when $r_1 > g$, and why $C'_2 > C_2$ in some cases when $r_2 > g$.

Consider the value of the capital stock in country 1 in autarchy, at autarchy prices:

$$K_1^a(t) = \int_{t-m'_1}^{t} P_a(t,v)I'_1(v)dv$$

$$= \int_{o}^{m'_1} P_a(m)I'_1 e^{g(t-m)}dm = K_1^a e^{gt}$$

where $K_1^a = I'_1 \int_{o}^{m'_1} P_a(m)e^{-gm}dm$.

The value of the same capital stock at trade prices is

$$K'_1(t) = K'_1 e^{gt}$$

where

$$K'_1 = I'_1 \int_{o}^{m'_1} P(m)e^{-gm}dm > I'_1 \int_{o}^{m'_1} P_a(m)e^{-gm}dm = K_1^a$$

Therefore

$$k_1^a = \frac{K_1^a}{L_1} < k'_1 = \frac{K'_1}{L_1} = \frac{\int_{o}^{m'_1} (P(m)e^{-\lambda m})e^{-(g-\lambda)m}dm}{\int_{o}^{m'_1} e^{-(g-\lambda)m}dm}$$

$P(m)e^{-\lambda m}$ is a decreasing function of m, and $m'_1 > m_1$,

Therefore

$$k_1^a < k'_1 < \frac{\int_{o}^{m_1} P(m)e^{-\lambda m}e^{-(g-\lambda)m}dm}{\int_{o}^{m_1} e^{-(g-\lambda)m}dm} = k_1$$

The value of the capital stock per worker is increased in the shift from autarchy steady state to trade steady state.

It is not possible to establish a definite relationship between K_2 and K_2': again there are two conflicting forces, the use of only old machines tends to depress the value of the capital stock, while the higher wage and earlier scrapping date tends to raise it. It is, however, easy to establish that if $r_2 > g$, then $C_2' > C_2$ only if $K_2' > K_2$. For in trade equilibrium we have (see section 2.3)

$$r_2K_2 + w_2L_2 = gK_2 + C_2,$$

while the autarchy equilibrium is uncompetitive at trade prices so

$$r_2K_2' + w_2L_2 \gtrless gK_2' + C_2'$$

Therefore

$$C_2' - C_2 \lesseqgtr (r_2-g)(K_2'-K_2)$$

and $C_2' > C_2$, $r_2 > g$ implies $K_2' > K_2$.

Therefore, if, when $r_1 < g$, $C_1' > C_1$, what is observed is the effect of inefficiency. If the country were initially on the steady growth path with consumption C_1' and capital stock K_1', and the opening of trade leads to C_1, K_1; in the movement to this new path, country 1 must initially reduce consumption below C_1' in order to raise K_1' towards K_1, and the inefficiency of its savings programme means that the rise in K itself is associated with a fall in consumption. There <u>are</u> gains from trade, shown by the fact that $r_1 < g$ is not incompatible with $C_1 > C_1'$ (see Example 3 below), but the effect of trade may be to increase the ability of the economy to postpone consumption, and the losses of increased inefficiency may outweigh the gains from trade.

A quite different process takes place in cases involving $C_2' \triangleright C_2$. In such cases, $K_2' > K_2$ so that along the path from autarchy steady state to trade steady state, country 2 is able to reduce its net investment below gK and so raise its consumption initially above C_2'. (It should be remembered that K_2' is the value of the autarchy capital stock at trade prices.) The country with high discount rate r_2 values current consumption relatively highly, and may take the gains from trade in the form of immediate consumption even to the extent of reducing its long-run consumption level.

These processes are seen most clearly in the borderline case which arises as $r_2 - r_1 \to 0$. In the limit we have specialisation and factor price equalisation. If $r_1 = r_2 < g$, then $m_1' = m_2' = m_2$ and comparing the cohorts $[0,m_1]$, $[m_1,m_2]$ we see that $c_1 < c < c_2$ where $c = (C_1 + C_2)/(L_1 + L_2)$, by the argument presented at the beginning of this section. But $c = c_1' = c_2'$. Therefore, $c_1 < c_1'$ and $C_1 < C_1'$.

Similarly, when $r_1 = r_2 > g$, $c_1 > c_1' = c = c_2' > c_2$, and $C_2 < C_2'$.

When $r_1 < r_2$, $m_1' < m_2'$ and the opening up of trade leads to very straightforward gains resulting from the establishment of a common scrapping age throughout both countries. Such gains, which it seems reasonable to call the 'static' gains from trade, may sometimes outweigh the 'dynamic' effects of the reallocation of country 1's workers to the cohort of newer machines and country 2's workers to the older cohort, in these cases where the dynamic effect tends to cause $C_i < C_i'$. Only when $r_1 = r_2$ do we observe the pure effects of specialisation.

I shall have more to say on this question in Chapter 5 where analogous dynamic effects of specialisation are observed in other models of trade in capital gains.

Finally, it must be recorded that I am unable to exclude the possibility that $C_1 + C_2 < C_1' + C_2'$ even if $r_2 > r_1 > g$. For,

$$C_1 = w_1L_1 + (r_1-g)K_1$$

$$C_2 = w_2L_2 + (r_2-g)K_2$$

$$C_1' < w_1L_1 + (r_1-g)K_1'$$

$$C_2' < w_2L_2 + (r_2-g)K_2'$$

Therefore

$$C_1' + C_2' < C_1 + C_2 + (r_1-g)(K_1'-K_1) + (r_2-g)(K_2'-K_2)$$

$$< C_1 + C_2 + (r_2-g)(K_2'-K_2)$$

The first inequality arises from what I have called the static gains from trade: for it is easily confirmed that if $r_1 = r_2$, since $w_i = w_i'$ and $K_1 + K_2 = K_1' + K_2'$,

$$C_1' + C_2' = C_1 + C_2 + (r_1-g)(K_1'-K_1) + (r_2-g)(K_2'-K_2)$$

$$= C_1 + C_2.$$

The second inequality arises from the reallocation of machines because of the differences in apparent time preference of the two countries.

There seems no reason why, if country 2 is sufficiently impatient, $K_2' - K_2$ should not be large enough to outweigh the effects of these two inequalities: i.e. why country 2's impatience should not outweigh the effects of country 1's patience and the static gains from trade. I have, however, been unable to produce an example in which this occurs. (Compare Meade (1955)[1], p.468:

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[1] Meade, J.E., The Theory of International Economic Policy, volume 2. Trade and Welfare (Oxford University Press; London, 1955).

'Consider a change in international economic policy which in itself represents a movement towards greater economic efficiency and so makes a contribution to economic welfare. World income is thereby raised. Now if at the same time the change causes a redistribution of income from rich countries to poor countries or from rich classes to poor classes within countries, its direct positive contribution to economic efficiency will be reinforced by an indirect positive contribution to economic equity. But unfortunately the greater the redistribution from rich countries to poor countries, or from rich citizens to poor citizens within a country, the smaller the part of the increased world income which will be added to world savings. What is desirable on distributional grounds may well be undesirable in its effects upon the world supply of savings, and vice versa.'

3.3. Numerical examples

The following examples were computed in the same way as the examples of section 2.5 except that the autarchy examples were solved for fixed r_i rather than for fixed s_i.

Example 1 $r_2 > r_1 > g$, $C_1 > C_1'$, $C_2 > C_2'$

$\lambda = 0.05$ $\quad g = 0.10$

$L_1 = 0.4022$ $\quad L_2 = 0.5978$ $\quad r_1 = 0.1921$ $\quad r_2 = 0.7603$

Trade $\quad p = 0.5018$ $\quad (p_o = 0.7168)$

$m_1 = 5.0000$ $\quad s_1 = 0.1768$ $\quad w_1 = 0.6721$ $\quad c_1 = 0.7322$

$m_2 = 10.9686$ $\quad s_2 = 0.0753$ $\quad w_2 = 0.4500$ $\quad c_2 = 0.5681$

Autarchy

$m_1' = 8.7309$ $\quad s_1' = 0.1717$ $\quad w_1' = 0.6463$ $\quad c_1' = 0.6818$

$m_2' = 29.9273$ $\quad s_2' = 0.1053$ $\quad w_2' = 0.2239$ $\quad c_2' = 0.5475$

Example 2 $r_2 > r_1 > g$, $C_1 > C_1'$, $C_2 < C_2'$

$\lambda = 0.05$ $g = 0.10$

$L_1 = 0.3296$ $L_2 = 0.6704$ $r_1 = 0.3274$ $r_2 = 0.5665$

Trade $p = 0.7012$ $(p_o = 0.7791)$

$m_1 = 4.0000$ $s_1 = 0.1608$ $w_1 = 0.5873$ $c_1 = 0.7632$

$m_2 = 11.9686$ $s_2 = 0.1005$ $w_2 = 0.4500$ $c_2 = 0.5706$

Autarchy

$m_1' = 11.1055$ $s_1' = 0.1491$ $w_1' = 0.5739$ $c_1' = 0.6696$

$m_2' = 18.5666$ $s_2' = 0.1185$ $w_2' = 0.3952$ $c_2' = 0.6149$

Example 3 $r_2 > g > r_1$, $C_1 > C_1'$, $C_2 > C_2'$

$\lambda = 0.05$ $g = 0.10$

$L_1 = 0.3296$ $L_2 = 0.6704$ $r_1 = 0.0660$ $r_2 = 0.7542$

Trade $p = 0.5453$ $(p_o = 0.7791)$

$m_1 = 4.0000$ $s_1 = 0.1924$ $w_1 = 0.7573$ $c_1 = 0.7344$

$m_2 = 11.9686$ $s_2 = 0.0781$ $w_2 = 0.4500$ $c_2 = 0.5848$

Autarchy

$m_1' = 7.2581$ $s_1' = 0.1938$ $w_1' = 0.6957$ $c_1' = 0.6835$

$m_2' = 29.4380$ $s_2' = 0.1056$ $w_2' = 0.2295$ $c_2' = 0.5499$

Example 4 $r_2 > g > r_1$, $C_1 < C_1'$, $C_2 > C_2'$

$\lambda = 0.05$ $g = 0.10$

$L_1 = 0.7396$ $L_2 = 0.2604$ $r_1 = 0.0140$ $r_2 = 0.2055$

Trade $p = 0.0932$ $(p_o = 0.1035)$

$m_1 = 5.0000$ $s_1 = 0.2398$ $w_1 = 0.7148$ $c_1 = 0.6761$

$m_2 = 2.1072$ $s_2 = 0.0490$ $w_2 = 0.7009$ $c_2 = 0.7036$

Autarchy

$m_1' = 6.7908$ $s_1' = 0.2029$ $w_1' = 0.7121$ $c_1' = 0.6824$

$m_2' = 8.9238$ $s_2' = 0.1694$ $w_2' = 0.6401$ $c_2' = 0.6811$

These examples shows that it is indeed the case that when $r_2 > g$, $C_2 > C_2'$ and $C_2 < C_2'$ are both possible, and that when $r_1 < g$, $C_1 > C_1'$ and $C_1 < C_1'$ are both possible.

3.4. Summary and conclusions

Chapters 2 and 3 taken together answer the second question raised in the conclusion of Chapter 1. The implications of the choice of a particular saving programme which lead to a country specialising in the use of old machines should now be fairly clear.

If the planners aim to bring the economy on to a path on which the profit rate is equal to some target rate (or if the saving programme is determined by a proportional relationship between net saving and net profits and if paths on which this relationship holds tend towards steady state), then the following results hold for the case where the target rate is higher than the profit rate in its trading partner.

1. The wage rate is raised by trade.

2. If the target discount rate is less than the growth rate, i.e. if the saving objective is inefficiently ambitious, consumption is raised by trade. Trade with a more inefficient partner reduces the effects of inefficiency.

3. If the target discount rate is greater than the growth rate, consumption may, but need not, be raised in the long run. If, however, steady state consumption falls, short run consumption is raised by trade. Trade with a more patient partner may allow more scope for impatience.

These results are closely related to the results of trade in a fixed saving rate regime derived in Chapter 2. Suppose that $r_2 > g$ and r_2 is held constant in trade and autarchy. Comparing the two states, we observe the result of two opposing forces: a) the static gains from trade tend to raise consumption in all periods, but arise only when there is a significant difference between the country's discount rate and the (lower) discount rate ruling in the 'rest of the world'; b) the dynamic gains from trade, arising from the enhanced possibilities of substituting immediate consumption for postponed consumption, lead to an increase in the former at the expense of steady state consumption. If a constant gross saving rate defines the saving programme, there is an additional factor to be taken into account: c) a fixed saving rate causes r_2 to fall as a result of trade, implying a further rise in w_2, a shortening of the life of machines, and a rise in production and consumption.

(As a by-product of the derivation of the results above, parallel results have been derived for the high-saving, low discount rate country.)

What then does all of this tell us about desirable policies for underdeveloped countries faced with the choice of whether or not to allow trade in old machines? If the low shadow wage and capital scarcity of such countries represents a genuine preference for current consumption against future consumption, so that the trade patterns described could be the result of a full-scale intertemporal optimisation exercise, then obviously the possibility of reduced long-run consumption ought to be viewed with equanimity and, indeed, with pleasure.

Further, even if a poor country is not impatient in the long-run sense of having a high pure discount rate in its objective function, it will nonetheless typically be the case that initial shortage of capital leads to it choosing a path on which the discount rate applicable to investment projects is initially high, and only gradually drops towards a long-run value $\delta + g$, say, where δ is a pure discount rate. Initially it will specialise in the use of used machines but in the long run may well not. In this case also we need not be concerned about the fall in long run consumption, for the relevant 'long run' never arrives. Consider in outline what happens in two simple cases.

1) Profit rate in the rest of the world is $r_1 = r_2$, the long run objective of the poor country. The poor country follows a Ramsey path, equating the instantaneous profit rate to the rate of decrease of the marginal utility of consumption per head. The opening up of trade gives a momentary boost to consumption, as the country sells its newest machines. Then it follows a path of gradually increasing consumption per head and decreasing discount rate, asymptotically tending to the state where there is factor price equalisation, although it still owns the oldest portion

of the world's capital stock. Assuming $r_1 = r_2 > g$, this gives lower consumption per head than the autarchy steady state with $r_2 = g$, but it never pays the country to raise its long run consumption by buying new machines from the rest of the world, even given price equalisation. For this would require, at some point, a rise in consumption per head, and therefore an increase in the rate of decrease of marginal utility and an increase in the discount rate above r_1. The country gained an initial increase in consumption at the expense of consumption later, but the marginal value of consumption later is less than of the initial increase.

2) $r_2 < r_1$: in the long run the poor country is less impatient than the world. Then, again, initially the poor country sells its newest machines, then gradually moves towards factor price equalisation. When factor price equalisation is attained, it continues to increase consumption so as to keep its discount rate equal to r_1, gradually exchanging old machines for new machines at the equalised prices, until it specialises in new machines. Then it continues to accumulate capital, raising the discount rate above r_1 and raising consumption as it tends towards the discount rate r_2. In this case, the initially boosted consumption is followed by a period of lowered consumption and then by a final phase in which consumption is higher than it could be in autarchy.

Now all of this assumes that given optimal choice of its instantaneous discount rate a country does better (given that its producers are all price takers) to follow the path indicated by competitive prices, i.e. to allow trade when its producers wish to trade. This follows from the usual argument connecting competitive equilibrium

and Pareto efficiency: that an activity is unprofitable indicates that alternative uses of its inputs may be found which produce more of some outputs and at least as much of all of them. Only for inefficiently low discount rates can alternatives to competitive activities be found which give more of everything.

But this whole series of arguments assumes a rather idealised view of the world, in which a country is somehow able accurately to specify its objectives and, what is more, control its behaviour so as to ensure they are carried out. It is at least plausible that a country with an inegalitarian social structure and a poorly developed governmental and fiscal machine may well find itself unable to save as high a proportion of its production as the preferences of its citizens indicate is desirable. (Obviously I am here brushing aside the difficulties of aggregating preferences, which seem tangential to my concerns.) It may be that a fixed ratio of gross investment to gross income, or of net investment to net profits is a reasonable expression of such a saving constraint. In either case, we have seen, such a constraint makes the country behave as if it were impatient and substitute consumption now for consumption later.

It is not a startlingly new proposition that failure to fulfil one optimality condition casts doubt on the desirability of fulfilling others: the theory of 'second-best' has an extensive literature. It might, however, seem plausible at first sight to suppose that a failure to make correct intertemporal choices should not affect the desirability of choosing efficient policies at every instant of time. What I have attempted to show here is that the question of whether at any time to use old rather than new machines is a question which involves issues of intertemporal consumption allocation as well as static efficiency.

Finally a comment on the appropriateness of this clay-clay model as the illustration of these issues. It is, after all, a very special case of the model of Chapter 1, and carries not a great deal of conviction as an accurate idealisation of the real world. But my concern here has not been to provide a well-specified set of policy prescriptions for the real world, it has rather been to give examples of the nature of the issues involved in the choice of policies in such models. For such a task it is appropriate, and desirable, to use the simplest possible model. The same issues might arise in, say, a putty-clay model, which is a defensible idealisation of the world, but might be in danger of being obscured by other problems.

If, therefore, the argument of Chapter 1 is accepted that changing factor proportions and factor price differentials are the central features of the problem, the particular model discussed in this and the previous chapter shows that while the prima facie case for underdeveloped countries to use second-hand machines remains, it has to be recognised that the issues involved are not simply a matter of static efficiency but rather raise problems of dynamic as well as static efficiency. There always exist better policies than interference with trade, but it is not impossible that institutional constraints on such policies may make interference with trade in second-hand machines desirable.

CHAPTER 4

Trade in Machines in the Putty-Clay Model

4.1. Introduction

Even if the basic framework of perfect competition, perfect foresight, absence of transport costs, and labour-augmenting embodied technical progress is accepted as an appropriate framework in which to attempt answers to the questions raised at the start of this work, there is a striking inadequacy in the analysis so far. In Chapter 1 the fact was noted that the theoretical analysis of section 1.3 could have only limited application to models in which there was the possibility of constructing a range of different types of machine. The model of Chapters 2 and 3 explicitly ruled out the possibility of any choice of technique: the technical coefficients were given exogenously.

Such a model is necessarily unsuitable for answering the final question posed at the end of Chapter 1: will a poor country specialise in used machines even if it has the option of constructing machines to its own specifications? In this chapter I shall introduce this possibility in a form which seems to me probably to overestimate the attractiveness of new machines to poor countries. The assumption is that techniques in both countries are chosen from a range of possibilities represented by a smooth differentiable production function with constant returns to scale in the inputs of labour and capital. Once a machine is constructed it operates for the rest of its life with the input-output coefficients chosen at the time of construction. This set of assumptions implies that a poor country producing small output is as efficient as an advanced country operating the same techniques on a larger scale.

It may be illuminating to make some a priori guesses about the pattern of specialisation and trade which will emerge in a model like this. When I began to think about this question the following appeared to me to be reasonable conjectures:

1. If σ, the elasticity of substitution of the ex-ante production function is sufficiently small, so that the real range of choice of technique is fairly restrictive, the model is effectively very like the clay-clay model, and there will be full specialisation by the poorer country in second-hand machines.

2. When σ is very large, so that a small wage differential makes a very great difference to the optimal choice of technique in an isolated economy, the use of second-hand machines built according to blueprints appropriate to high wage economies will be relatively less attractive than indigenous techniques, and the use of indigenous techniques is both possible and likely.

3. A low wage country will always use some second-hand machines. If it did not, the high wage country would scrap machines when $P_1 = \rho_1 = 0$. But such a machine has $\rho_2 > 0$ because of the lower wage, and not to buy a useful machine whose price is zero is clearly suboptimal. Therefore the possibility that is raised by high σ is the possibility of the use of both second-hand machines and new machines of a different type. It is known that in an isolated economy with the putty-clay technology, high values of σ can lead to the use of more than one technique at one time. It seems likely therefore that the conditions required to exclude the possibility of a low wage country building some of its own machines will be closely related to the conditions required to prove uniqueness of technique choice in one-country models.

4. When the low wage country is large relative to the high wage country, i.e. when L_2/L_1 is large, we should expect two conflicting forces. (a) Large demand for second-hand machines will tend to raise prices so that used machines are relatively less attractive. (b) The importance of the second use of machines will have a greater influence on the machine producers so that the machines will be relatively closer to the type of technique appropriate to low-wage economies. There seems no prior basis for judging which of these two effects should be the stronger.

The reader should perhaps pause before embarking on the formal analysis, to consider whether he finds it plausible on general grounds that the introduction of choice of technique should reduce the amount of trade in second-hand machines and whether the conjectures above, in particular conjecture 3, seem reasonable.

4.2. Factor prices in the putty-clay model with trade in machines

The putty-clay technology

The technology which I discuss in this chapter is the putty-clay technology described and analysed in Bliss (1969).[1] The methods used in several proofs below follow fairly closely the strategies adopted in Bliss's proofs.

There is, as in the clay-clay model, a uniform output good which may be consumed or irreversibly invested. A 'machine of type k' has the following characteristics: one unit of output good may be costlessly transformed into a machine which produces an output stream f(k)/k, and

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[1] Bliss, C.J., "On Putty-Clay", Review of Economic Studies 35, 2 (April 1968), pp.105-132.

requires labour input $e^{-\lambda v}/k$ where v is the date of construction of the machine. f(k) is a 'conventional neoclassical' production function: i.e. $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$. It is assumed to be regularly strictly concave (Bliss, p.107), i.e. for each $h > 0$, there exists k_h such that $f(k_h) < hk_h$. (Unlike Bliss, I do not normalise λ to be equal to 1, as this would make comparison with the earlier chapters more difficult.) There are constant returns to scale, so that the investment of k units in machines of type k gives rise to output f(k) from labour input $e^{-\lambda v}$. The output-labour ratio is $f(k)e^{\lambda v}$. Below, I refer to such an investment, k machines of type k, as 'an investment of type k'. A machine, once constructed, may either be used with these input-output coefficients or may be idle, but the coefficients can not be changed.

Bliss proves that a steady state in an isolated economy in this model exists, has constant scrapping age T for machines of a particular type, has wage rate $w(t) = we^{\lambda t}$ and constant profit rate r, but several different types of machine may be associated with one value of r. If k is unique, then higher values of k are associated with lower values of r.

The precise result which Bliss proves about the relation between w and r is the following (Theorem 3, p.115):

> 'Either, for each positive w less than a given w_o, or for each positive w, there exists a unique positive r, and positive k and T (not necessarily unique), such that the [net present value of an investment of type k] achieves its maximum value for positive k and T, and this maximum value is zero.'

In addition, Bliss proves that r is a strictly decreasing function of w. This means that the model has a downward sloping factor price frontier

which may or may not cut the w axis, and either cuts or is asymptotic to the r axis. By a slight extension of Bliss's proof it is possible to make this result a little more precise, showing under what circumstances the frontier cuts the r axis, and proving that it is asymptotic to the w axis if it does not cut it.

Theorem 4.1. Let $r_o = \lim_{k \to o} f(k)/k$, if the limit exists. To each positive $r < r_o$, or if r_o does not exist, to each positive r, there corresponds a unique positive w, such that the net present value of an investment of type k achieves its maximum value for positive k and T, and this maximum is zero.

Proof

The net present value of an investment of type k is

$$P_r(k) = -k + \int_o^T (f(k) - we^{\lambda t})e^{-rt}at$$

where lifetime T is chosen so that $f(k) = we^{\lambda T}$, for given k, w.

Therefore

$$P_r(k) = -k + \frac{f(k)}{r}(1 - e^{-rT}) - \frac{w}{r-\lambda}(1 - e^{-(r-\lambda)T})$$

$$= \frac{f(k)-rk}{r} - \frac{f(k)}{r}e^{-rT} - \frac{w}{r-\lambda}(1 - e^{-(r-\lambda)T})$$

$$< \frac{f(k)-rk}{r}$$

Now for any given positive r, it is possible because of the regular strict concavity property of f(k) to choose k' such that $f(k) - rk < 0$ for all $k \geq k'$. Let $w = f(k')$. Then only when $k \geq k'$ is quasi-rent positive so could $P_r(k)$ be positive. But since $f(k) - rk < 0$, $P_r(k) < 0$ for all $k \geq k'$. Hence for all k, $P_r(k) < 0$

Therefore

$$Q(r,w) = \max_k P_r(k) < 0.$$

(Bliss proves that Q always exists.)

Therefore for every positive r, there exists a positive w such that $Q(r,w) < 0$.

Suppose that there exists r such that $\lim_{w\to 0} Q(r,w) < 0$. Choose a positive k such that $f(k) - rk > 0$. This is possible if and only if $r < r_o$ if r_o exists, since f(k)/k is a decreasing function of k, by concavity.

$$w = e^{-\lambda T} f(k) ;$$

therefore as $w \to 0$, $T \to \infty$ for this fixed k.

Therefore

$$\lim_{w\to 0} P_r(k) = -k + \frac{f(k)}{r} > 0$$

Therefore there exists w such that $P_r(k) > 0$

Therefore $Q(r,w) = \max_k P_r(k) \geq 0$, contradicting the hypothesis. Therefore for all r, or for all $r < r_o$ if r_o exists, there exists a positive w such that $Q(r,w) \geq 0$.

Q(r,w) is continuous in r,w. Therefore for every positive $r(< r_o)$ there exists a positive w such that $Q(r,w) = 0$. Q.E.D.

The effect of factor price differentials

Consider now what happens in a world of two countries with different factor prices.

Let the wage rates be respectively $w_1(t) = w_1 e^{\lambda t}$, $w_2(t) = w_2 e^{\lambda t}$. An investment of type k employs $e^{-\lambda v}$ men and produces output f(k). The output good is the price numéraire, so that the quasi-rent of such an investment of age m in country i is

$$\rho_i(m) = f(k) - w_i(v+m)e^{-\lambda v}$$

$$= f(k) - w_i e^{\lambda m}$$

Suppose that $0 \leq r_1 < r_2$. The general theory of Chapter 1 shows that there is a unique switchover age m_1 such that country 1 holds the $[0,m_1]$ cohort of machines and country 2 the $[m_1,m_2]$ cohort. It is possible that $m_1 = 0$ or $m_2 = m_1$. No assumption is made that the m_i are constant. (Note the small change of notation from the proof above. I use m_2 instead of T for the sake of compatibility with the notation of earlier chapters.)

If the present value of an investment of type k at age m is P(k,m), then when $m \in [0,m_1]$,

$$P(k,m) = \int_m^{m_1} \rho_1(n)e^{-r_1(n-m)}\,dn + P(k,m_1)e^{-r_1(m_1-m)}$$

$$= \int_m^{m_1} (f(k) - w_1 e^{\lambda n})e^{-r_1(n-m)}\,dn + P(k,m_1)e^{-r_1(m_1-m)}$$

$$= \frac{f(k)}{r_1}(1 - e^{-r_1(m_1-m)}) - \frac{w_1 e^{\lambda m}}{r_1-\lambda}(1 - e^{-(r_1-\lambda)(m_1-m)})$$

$$+ P(k,m_1)e^{-r_1(m_1-m)}$$

and when $m \in [m_1,m_2]$,

$$P(k,m) = \int_m^{m_2} \rho_2(n)e^{-r_2(n-m)}\,dn = \int_m^{m_2} (f(k) - w_2 e^{\lambda n})e^{-r_2(n-m)}\,dn$$

$$= \frac{f(k)}{r_2}(1 - e^{-r_2(m_2-m)}) - \frac{w_2 e^{\lambda m}}{r_2-\lambda}(1 - e^{-(r_2-\lambda)(m_2-m)})$$

Profit maximisation with perfect foresight implies that k, m_1, m_2 are chosen for given r_1, r_2, w_1, w_2 so that in perfect competition:

$$\max_{k,m_1,m_2} \{ - k + P(k,0)\} = 0.$$

The first order condition for m_2 is straightforward. With no loss of generality I confine my attention to values of k which can give rise to positive quasi-rents i.e. $k \geq \underline{k}$ where $f(\underline{k}) = \min (w_1, w_2)$. $P(k, 0)$ increases with m_2 when and only when quasi-rent is positive. If $w_2 \leq w_1$, m_2 is chosen so that $\rho_2(m_2) = 0$:

$$w_2 = f(k)e^{-\lambda m_2}; \tag{1}$$

and if $w_1 < w_2$, m_2 is chosen so that $\rho_1(m_2) = 0$ and $m_2 = m_1$:

$$w_1 = f(k)e^{-\lambda m_1} = f(k)e^{-\lambda m_2}. \tag{1a}$$

Theorem 1.1 implies (since $r_i \geq 0$) that m_1 is chosen so that

$$\frac{\partial P(k,m)}{\partial m} = \min_{i=1,2} \{r_i P(k,m) - f(k) + w_i e^{\lambda m}\},$$

i.e. if $0 < m_1 < m_2$

$$r_1 P(k,m_1) + w_1 e^{\lambda m_1} = r_2 P(k,m_1) + w_2 e^{\lambda m_1}; \tag{2}$$

while if $m_1 = 0$

$$r_1 P(k,0) + w_1 \geq r_2 P(k,0) + w_2 \tag{2a}$$

and if $m_1 = m_2$,

$$r_1 P(k,m_2) + w_1 e^{\lambda m_2} \leq r_2 P(k,m_2) + w_2 e^{\lambda m_2}, \text{ i.e. } w_1 \leq w_2. \tag{2b}$$

For given k, these relations define unique globally optimal m_1, m_2 and the first order conditions are sufficient.

The first order conditions for maximisation with respect to k, given optimal choice of m_1, m_2 for each k, is

$$0 = \frac{d}{dk}(-k + P(k,0))$$

$$= \frac{\partial}{\partial k}(-k + P(k,0)) + \frac{\partial}{\partial m_1}(-k + P(k,0))\frac{dm_1}{dk}$$

$$+ \frac{\partial}{\partial m_2}(-k + P(k,0))\frac{dm_2}{dk}.$$

Using the second versions of $P(k,0)$, $P(k,m_1)$ above:

$$\frac{\partial}{\partial m_1}(-k + P(k,0)) = f(k)e^{-r_1 m_1} - w_1 e^{(\lambda - r_1)m_1} - r_1 e^{-r_1 m_1} P(k,m_1)$$

$$+ e^{-r_1 m_1}(-f(k) + w_2 e^{\lambda m_1} + r_2 P(k,m_1))$$

$$= 0, \quad \text{if (2) is satisfied.}$$

If (2) is not satisfied but (2a) is, then $\frac{\partial}{\partial m_1}(-k + P(k,0)) < 0$ and $dm_1/dk = 0$. If (2) is not satisfied but (2b) is, $w_2 > w_1$ and (1a) holds.

$$\frac{\partial}{\partial m_2}(-k + P(k,0)) = e^{-r_1 m_1}(f(k) - w_2 e^{\lambda m_2})e^{-r_2(m_2 - m_1)} = 0$$

if (1) is satisfied. If (1) is not satisfied, $m_2 = m_1$ and (1a) and (2b) hold. Then

$$\frac{\partial}{\partial m_2}(-k + P(k,0)) + \frac{\partial}{\partial m_1}(-k + P(k,0)) = (f(k) - w_1 e^{\lambda m_2})e^{-\lambda m_1}$$

$$= 0, \quad \text{by (1a),}$$

and

$$\frac{dm_1}{dk} = \frac{dm_2}{dk}.$$

Therefore

$$0 = \frac{d}{dk}(-k + P(k,0)) = \frac{\partial}{\partial k}(-k + P(k,0))$$

$$= -1 + \frac{f'(k)}{r_1}(1 - e^{-r_1 m_1}) + e^{-r_1 m_1}\frac{f'(k)}{r_2}(1 - e^{-r_2(m_2 - m_1)})$$

Therefore

$$1 = f'(k)\{\frac{1}{r_1}(1 - e^{-r_1 m_1}) + \frac{e^{-r_1 m_1}}{r_2}(1 - e^{-r_2(m_2-m_1)})\} \quad (3)$$

I now wish to show that

$$Q(k, m_1, m_2, r_1, r_2, w_1, w_2) = -k + P(k,0)$$

does have a maximum on the set $k \geq \underline{k}$, $m_2 \geq 0$, $m_1 \in [0,m_2]$, for given r_i, w_i. Assume that (1), (2) hold.

The first step is to prove that $m_2 - m_1$ is a constant, independent of k, if (1) and (2) are satisfied.

(1) implies $w_2 = e^{-\lambda m_2} f(k)$

Therefore

$$\frac{dm_2}{dk} = \frac{f'(k)}{\lambda f(k)}, \quad \text{since } w_2 \text{ is constant.}$$

(2) implies $w_1 = w_2 + (r_2-r_1)e^{-\lambda m_1} P(k,m_1)$

Therefore

$e^{-\lambda m_1} P(k,m_1)$ is constant.

From (2), (1),

$$e^{-\lambda m_1} P(k,m_1) = \int_{m_1}^{m_2} (f(k)e^{-\lambda m_1} - w_2 e^{\lambda(n-m_1)})e^{-r_2(n-m_1)}\,dn$$

$$= \int_0^{m_2-m_1} (f(k)e^{-\lambda m_1} - w_2 e^{\lambda n})e^{-r_2 n}\,dn$$

$$= w_2 \int_0^{m_2-m_1} (e^{\lambda(m_2-m_1)} - e^{\lambda n})e^{-r_2 n}\,dn.$$

Differentiate with respect to k:

$$0 = w_2\{0 + \int_0^{m_2-m_1} \lambda e^{\lambda(m_2-m_1)} e^{-r_2 n}\, dn\}\frac{d(m_2-m_1)}{dk}$$

$$= (\lambda w_2 e^{\lambda(m_2-m_1)} \int_0^{m_2-m_1} e^{-r_2 n}\, dn)\frac{d(m_2-m_1)}{dk}$$

The term in brackets is positive if $m_2 - m_1 > 0$

Therefore

$$\frac{d(m_2-m_1)}{dk} = 0$$

and

$$\frac{dm_1}{dk} = \frac{dm_2}{dk} = \frac{f'(k)}{\lambda f(k)} > 0.$$

Next, the aim is to restrict the domain of the function to a compact set of (k, m_1, m_2). Let

$$Q(k, r_1, r_2, w_1, w_2) = \max_{m_1, m_2} \{-k + P(k,0)\}$$

which we know exists and is unique.

$$Q(k, m_1, m_2, r_1, r_2, w_1, w_2) \leq Q(k, r_1, r_2, w_1, w_2)$$

If the right hand side is maximised on $k \geq \underline{k}$, the left hand side is maximised.

$$Q(k, r_1, r_2, w_1, w_2) = -k + \frac{f(k)}{r_1} - e^{-r_1 m_1}\frac{f(k)}{r_1}$$

$$+ \frac{w_1}{r_1-\lambda} e^{-(r_1-\lambda)m_1} - \frac{w_1}{r_1-\lambda} + e^{-r_1 m_1}\frac{f(k)}{r_2}$$

$$- e^{-r_1 m_1}\frac{f(k)}{r_2} e^{-r_2(m_2-m_1)}$$

$$+ \frac{w_2}{r_2-\lambda} e^{-(r_1-\lambda)m_1-(r_2-\lambda)(m_2-m_1)}$$

$$- \frac{w_2}{r_2-\lambda} e^{-(r_1-\lambda)m_1}$$

$$= \frac{f(k)-r_1k}{r_1} - e^{-r_1m_1} f(k) \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$+ \frac{w_1}{r_1-\lambda} e^{-(r_1-\lambda)m_1} - \frac{w_1}{r_1-\lambda}$$

$$- e^{-r_1m_1} f(k) \frac{1}{r_2} e^{-r_2(m_2-m_1)}$$

$$- \frac{w_2}{r_2-\lambda} e^{-(r_1-\lambda)m_1} \left(1 - e^{-(r_2-\lambda)(m_2-m_1)}\right)$$

Since f(k) is regularly strictly concave, there exists k_{r1} such that $f(k) - r_1k$ is negative and strictly decreasing for $k > k_{r1}$.

$$e^{-r_1m_1} f(k) = e^{-(r_1-\lambda)m_1} \left(f(k)e^{-\lambda m_1}\right)$$

and $$f(k)e^{-\lambda m_1} = \left(f(k)e^{-\lambda m_2}\right)e^{\lambda(m_2-m_1)} = w_2 e^{\lambda(m_2-m_1)}$$

which is a constant, since w_2 is given, and $m_2 - m_1$ is constant when (1), (2) are satisfied.

Therefore as k increases, m_1 increases and

$$e^{-(r_1-\lambda)m_1} \to \infty \qquad \text{if } \lambda > r_1$$

$$\to 0 \qquad \text{if } r_1 \gg \lambda$$

Therefore

$$- e^{-r_1m_1} f(k) \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \to 0 \text{ or } -\infty \text{ since } r_2 > r_1.$$

$$\frac{w_1}{r_1-\lambda} e^{-(r_1-\lambda)m_1} \to 0 \text{ or } -\infty$$

$$\frac{w_1}{r_1-\lambda} \text{ is constant}$$

$$- e^{-r_1 m_1} f(k) \frac{1}{r_2} e^{-r_2(m_2-m_1)} \to 0 \text{ or } -\infty \text{ since } m_2 - m_1 \text{ is constant}$$

and $- w_2 e^{-(r_1-\lambda)m_1} \frac{1}{r_2-\lambda} (1 - e^{-(r_2-\lambda)(m_2-m_1)}) \to 0$ or $-\infty$.

Therefore, for sufficiently large k, say $k \geq \bar{k}$, $Q(k, r_1, r_2, w_1, w_2)$ is negative and monotonically decreasing. It therefore attains a global maximum on $k \geq \underline{k}$ if it attains a maximum on $k \in [\underline{k}, \bar{k}]$. But Q is continuous in k, so it must attain a maximum on this compact set.

Therefore if (1), (2) are satisfied, $Q(k, m_1, m_2, r_1, r_2, w_1, w_2)$ does attain a maximum on the set $k \geq \underline{k}$, $m_2 \geq 0$, $m_1 \in [0, m_2]$.

If (1), (2) are not satisfied either $m_1 = 0$ or $m_2 - m_1 = 0$, in which case the equations become identical to the equations for an isolated economy and the argument used by Bliss in the first part of the proof of his Theorem 3 (on which the above argument is modelled) applies directly to prove the existence of a maximum.

Theorem 4.2. Let

$$Q(r_1, r_2, w_1, w_2) = \max_{k, m_1, m_2} \{- k + P(k,0)\}.$$

If $r_1 < r_2 (< r_o$, if r_o exists), there exists a range of positive values of w_1, w_2 such that $Q(r_1, r_2, w_1, w_2) = 0$, and on this range w_2 is a continuously strictly decreasing function of w_1.

Proof

The fact that $Q(r_1, r_2, w_1, w_2)$ exists was established above. First, I show that Q is continuous and decreasing in both w_1 and w_2. If $w_1^o < w_1$ and (2) or (2b) are satisfied at w_1, then clearly, since $m_1 > 0$,

$$Q(k, m_1, m_2, r_1, r_2, w_1^o, w_2) > Q(r_1, r_2, w_1, w_2)$$

where k, m_1, m_2 take maximising values associated with w_1, for then all that has happened is that the quasi-rent on $[0,m_1]$ has risen. Therefore

$$Q(r_1, r_2, w_1^o, w_2) > Q(r_1, r_2, w_1, w_2).$$

$$Q(r_1, r_2, w_1^o, w_2) - Q(r_1, r_2, w_1, w_2)$$

$$= Q(k^o, r_1, r_2, w_1^o, w_2) - Q(k^o, r_1, r_2, w_1, w_2)$$

$$+ Q(k^o, r_1, r_2, w_1, w_2) - Q(k, r_1, r_2, w_1, w_2)$$

where k^o is the value (or one of the values) of k which maximises Q at w_1^o, and k similarly corresponds to w_1.

Therefore

$$Q(k^o, r_1, r_2, w_1, w_2) \leqslant Q(k, r_1, r_2, w_1, w_2).$$

Consider w_1^o in the neighbourhood $|w_1^o - w_1| \leqslant \delta$ of w_1. For each w_1^o in this neighbourhood there is a $\bar{k}$ such that $Q(k, r_1, r_2, w_1, w_2)$ is negative and monotonically decreasing in k for all $k \geqslant \bar{k}$. So we can find $\bar{\bar{k}}$ such that this is true for all w_1^o in the neighbourhood, by continuity of $\bar{k}(w_1^o)$. Therefore $k^o \in [\underline{k},\bar{k}]$. $Q(k^o, r_1, r_2, w_1^o, w_2)$ is continuous in w_1^o and k^o. Therefore since k^o is a member of a compact set there is a uniform δ such that for all k^o, $|w_1^o - w_1| \leqslant \delta \Rightarrow$

$$|Q(k^o, r_1, r_2, w_1^o, w_2) - Q(k^o, r_1, r_2, w_1, w_2)| < \varepsilon$$

Therefore

$$0 < Q(r_1, r_2, w_1^o, w_2) - Q(r_1, r_2, w_1, w_2) < \varepsilon$$

i.e. Q is continuously strictly decreasing in w_1.

Identical arguments show that if (2) or (2a) and (1) are satisfied, but (1a) is not satisfied, so that $m_2 - m_1 > 0$, Q is continuously strictly decreasing in w_1.

By assumption, r_1 and r_2 satisfy the conditions of Theorem 4.1. Therefore each is compatible with equilibrium in an isolated economy and defines a unique positive wage rate. Let these rates be denoted w_1^A, w_2^A so that

$$Q(r_1, w_1^A) = Q(r_2, w_2^A) = 0.$$

They are the wage rates that would exist in autarchic steady states in the respective countries with profit rates r_1, r_2. $r_1 < r_2$ implies that $w_1^A > w_2^A$.

Now $r_1 < r_2$, $w_1 = w_2$ imply

$$r_1 P(k, m_1) + w_1 e^{\lambda m_1} < r_2 P(k, m_1) + w_2 e^{\lambda m_1}$$

for all $m_1 < m_2$, where (1) fixes m_2.

Therefore if $w_1 = w_2$, $m_2 = m_1$, and all the equations reduce to the equations describing autarchic steady state with factor prices w_1, r_1.

Therefore

$$Q(r_1, r_2, w_1, w_2) = Q(r_1, w_1)$$

Therefore

$$Q(r_1, r_2, w_1^A, w_1^A) = Q(r_1, w_1^A) = 0$$

(A fortiori, this is true for $w_2 > w_1$.)

When $w_1 > w_2$

$$r_2 P(k, m_2) + w_2 e^{\lambda m_2} = w_2 e^{\lambda m_2} < w_1 e^{\lambda m_2} = r_1 P(k, m_2) + w_1 e^{\lambda m_2}$$

Therefore

$$m_2 - m_1 > 0 \quad \text{by (2b)},$$

and Q is strictly increasing in w_2.

Now recall that in an isolated economy with factor prices w_2^A, r_2 there may be many values of k associated with equilibrium. Let k_2^A be the largest such value, and define

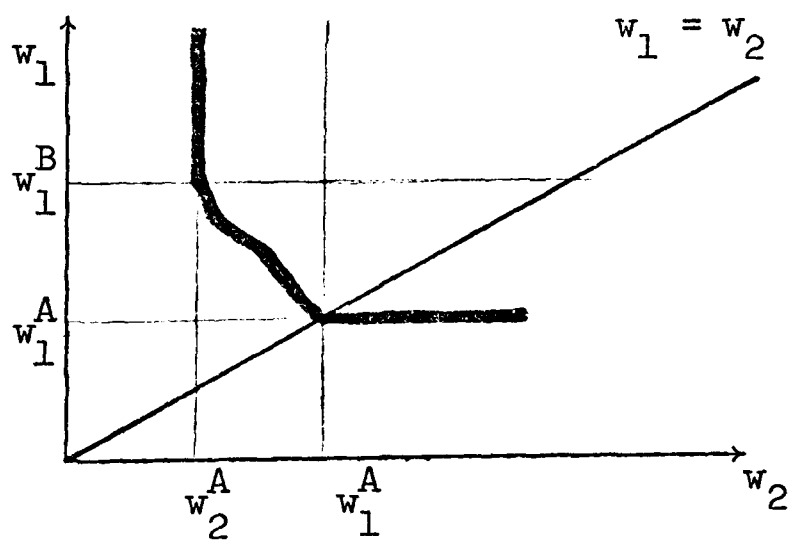
$$w_1^B = w_2^A + (r_2 - r_1)k_2^A,$$

so that (1), (2) hold with $m_1 = 0$ and $k = k_2^A$.

When $w_1 > w_2$ and $m_1 > 0$, since Q is continuously strictly decreasing in both w_1 and w_2, $Q = 0$ defines w_2 as a continuously decreasing function of w_1. When $w_1 > w_2$ and $m_1 = 0$, $Q(r_1, r_2, w_1, w_2) = Q(r_2, w_2) = 0$ if and only if $w_2 = w_2^A$. If $w_1 \geq w_1^B$, (1), (2) or (2a) and (3) are satisfied. If $w_1 < w_1^B$, we can find $k = k_2^A$ such that (2a) is violated when $m_1 = 0$. Therefore, if $w_1^B > w_1 > w_2$, $m_1 > 0$ and $m_2 - m_1 > 0$ so that w_2 is a continuously strictly decreasing function of w_1. As w_1 increases on $[w_1^A, w_1^B]$, w_2 decreases on $[w_2^A, w_1^A]$. Q.E.D.

What the theorem shows is that for given r_1, r_2 there is a range of possible w_1, w_2, compatible with competitive pricing, which are illustrated in Diagram 1.

Diagram 1



When $r_1 = r_2$, $w_1^A = w_2^A = w_1^B = w$, say. $w_1 > w$ implies $m_1 = 0$ and

$w_2 > w$ implies $m_2 - m_1 = 0$. $w_1 = w_2 = w$ means that m_1 is indeterminate.

When $w_1 \gtrless w_1^B$, $m_1 = 0$, $w_2 = w_2^A$, and $m_2 = m_2^A$, the value of lifetime in economy 2 in autarchy. When $w_2 \gtrless w_1^A$, $m_2 - m_1 = 0$, $w_1 = w_1^A$ and $m_1 = m_1^A$. Clearly full employment in both countries is impossible in either situation, for in each case one country has no machines. It is not obvious that there need exist a full employment equilibrium on the $0 < m_1 < m_2$ part of the $w_2 = f(w_1)$ line. It is to the question of the existence and uniqueness of such an equilibrium that I now turn my attention.

4.3. Existence and uniqueness of equilibrium

Steady state is defined in the same way as in the clay-clay model in section 2.4. Each wage rate rises at the rate of labour-augmenting technical progress (as described in the previous section). Total gross investment rises at the rate g: $I(t) = Ie^{gt}$.

Now one unit of investment made at time v employs labour input $e^{-\lambda v}/k$. Therefore, employment at time t on machines of ages $[0,m_1]$ is

$$\int_{t-m_1}^{t} \frac{I}{k} e^{(g-\lambda)v} dv = \frac{I}{k} (1 - e^{-(g-\lambda)m_1}) e^{nt},$$

and employment on the $[m_1,m_2]$ cohort is

$$\int_{t-m_2}^{t-m_1} \frac{I}{k} e^{(g-\lambda)v} dv = \frac{I}{k} (e^{-(g-\lambda)m_1} - e^{-(g-\lambda)m_2}) e^{nt}.$$

If $L_1(t) = L_1 e^{nt}$, $L_2(t) = L_2 e^{nt}$, full employment requires

$$L_1 = \frac{I}{k} (1 - e^{-(g-\lambda)m_1}) \tag{4}$$

$$L_2 = \frac{I}{k} (e^{-(g-\lambda)m_1} - e^{-(g-\lambda)m_2}), \tag{5}$$

i.e. $L_2/L_1 = (1 - e^{-(g-\lambda)(m_2-m_1)})/(e^{(g-\lambda)m_1} - 1)$.

If $w_1 = w_1^A$, $m_1 = m_2$ and $L_2/L_1 = 0$. If $w_2 = w_2^A$, $m_1 = 0$ and $L_1/L_2 = 0$. It is necessary to study the effect of movement along the 'wage frontier' of Diagram 1 on k, m_1, and m_2 in order to prove the existence of a point where L_1/L_2 takes its full employment value.

No assumption has been made, or conclusion reached, that k is uniquely determined by a given quadruple of factor prices. A sufficient condition is that

$$\frac{d^2}{dk^2}(-k + P(k, 0)) < 0$$

when (1), (2), (3) hold, for then they define a unique maximum with respect to k.

$$\frac{d}{dk}(-k + P(k, 0))$$

$$= \frac{\partial}{\partial k}(-k + P(k, 0)) + \frac{\partial}{\partial m_1}(-k + P(k, 0))\frac{dm_1}{dk} + \frac{\partial}{\partial m_2}(-k + P(k, 0))\frac{dm_2}{dk}$$

$$= -1 + f'(k)\{\frac{1}{r_1}(1 - e^{-r_1 m_1}) + \frac{e^{-r_1 m_1}}{r_2}(1 - e^{-r_2(m_2-m_1)})\}$$

$$+ \frac{dm_1}{dk} e^{(\lambda-r_1)m_1}(w_2 - w_1 + (r_2 - r_1)P(k, m_1)e^{-\lambda m_1})$$

$$+ \frac{dm_2}{dk} e^{-r_1 m_1 - r_2(m_2-m_1)}(f(k) - w_2 e^{\lambda m_2})$$

Therefore

$$\frac{d^2}{dk^2}(-k + P(k,0)) = f''(k)\{\frac{1}{r_1}(1 - e^{-r_1 m_1}) + \frac{e^{-r_1 m_1}}{r_2}(1 - e^{-r_2(m_2-m_1)})\}$$

$$+ f'(k)e^{-r_1 m_1}\{(1 - \frac{r_1}{r_2}(1 - e^{-r_2(m_2-m_1)}))\frac{dm_1}{dk} + e^{-r_2(m_2-m_1)}\frac{d(m_2-m_1)}{dk}\}$$

$$+ \left(\frac{d^2m_1}{dk^2} + (\lambda-r_1)\left(\frac{dm_1}{dk}\right)^2\right)e^{(\lambda-r_1)m_1}(w_2 - w_1 + (r_2-r_1)P(k,m_1)e^{-\lambda m_1})$$

$$+ \frac{dm_1}{dk} e^{(\lambda-r_1)m_1}(r_2-r_1) \frac{d}{dk}(P(k,m_1)e^{-\lambda m_1})$$

$$+ \left(\frac{d^2m_2}{dk^2} - r_1 \frac{dm_1}{dk}\frac{dm_2}{dk} - r_2 \frac{dm_2}{dk}\frac{d(m_2-m_1)}{dk} e^{-r_1m_1-r_2(m_2-m_1)}(f(k) - w_2e^{\lambda m_2}\right)$$

$$+ \frac{dm_2}{dk} e^{-r_1m_1-r_2(m_2-m_1)}(f'(k) - w_2\lambda e^{\lambda m_2}\frac{dm_2}{dk})$$

$$= \frac{f''(k)}{f'(k)} + \frac{(f'(k))^2}{\lambda f(k)} e^{-r_1m_1}\left(1 - \frac{r_1}{r_2}(1 - e^{-r_2(m_2-m_1)})\right)$$

using (1), (2), (3) and the fact proved above that (2) implies that $\frac{dm_1}{dk} = \frac{dm_2}{dk}$ for fixed w_i, r_i.

The condition that this be negative is analogous to Bliss's concavity condition (46), and reduces to Bliss's condition when $r_1 = r_2 = r$. Britto (1969)[1] has shown that if the elasticity of substitution σ of the ex ante production function $f(k)$ is less than 1, then uniqueness is assured. Unfortunately, the same result apparently need not hold in this model for the following reason. The strategy of Britto's proof is to show that in Bliss's model:

$$\frac{f(k)-kf'(k)}{f'(k)} = \frac{\frac{w}{f(k)}\frac{1}{r-\lambda}(1 - e^{-(r-\lambda)T})}{\frac{1}{r}(1 - e^{-rT})} = \frac{\frac{1}{r-\lambda}(e^{(r-\lambda)T} - 1)}{\frac{1}{r}(e^{rT} - 1)}$$

(I use my notation and normalisation rather than Bliss's.) The left hand side is the share of wages in an economy with malleable capital and production function $f(k)$, and is an increasing function of k if $\sigma \leq 1$. The right hand side is a strictly decreasing function of T. Since $w = f(k)e^{-\lambda T}$, for fixed w, T is a strictly increasing function of k.

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[1] Britto, R., "On Putty-Clay: a Comment", Review of Economic Studies 37, 3 (July 1969), pp.395-398.

Therefore if $\sigma \lesssim 1$ each pair (w,r) is associated with a unique k, T.

The corresponding relation in my model is

$$\frac{f(k)-kf'(k)}{f'(k)} = \frac{\frac{w_1}{f(k)}\frac{1}{r_1-\lambda}(1 - e^{-(r_1-\lambda)m_1}) + \frac{w_2}{f(k)}\frac{e^{(\lambda-r_1)m_1}}{r_2-\lambda}(1 - e^{-(r_1-\lambda)/(m_2-m_1)}}{\frac{1}{r_1}(1 - e^{-r_1m_1}) + \frac{e^{-r_1m_1}}{r_2}(1 - e^{-r_2(m_2-m_1)})}$$

$$= \frac{e^{-\lambda(m_2-m_1)}(\frac{w_1}{w_2}\frac{1}{r_1-\lambda}(e^{(r_1-\lambda)m_1} - 1) + \frac{1}{r_2-\lambda}(1 - e^{-(r_2-\lambda)(m_2-m_1)}))}{\frac{1}{r_1}(e^{r_1m_1} - 1) + \frac{1}{r_2}(1 - e^{-r_2(m_2-m_1)})}$$

Now for fixed w_i, r_i, $m_2 - m_1$ is constant, so only m_1 varies in the right hand side of this equation. As k increases since $w_2 = f(k)e^{-\lambda m_2}$, m_2 and therefore m_1 increase. There is, however, no assurance that the right hand side is a monotonic function of m_1.

I am therefore forced to make the assumption:

$$\frac{f''(k)}{f'(k)} + \frac{(f'(k))^2}{f(k)} e^{-r_1m_1}(1 - \frac{r_1}{r_2}(1 - e^{-r_2(m_2-m_1)})) < 0 \qquad (A)$$

Now consider the effect of a change in w_1, w_2 for fixed r_1, r_2.

In equilibrium

$$0 = -k + P(k, 0)$$

$$= -k + \int_0^{m_1} (f(k) - w_1e^{\lambda m})e^{-r_1m}dm + e^{-r_1m_1}\int_{m_1}^{m_2} (f(k) - w_2e^{\lambda m})e^{-r_2(m-m_1)}dm$$

As w_1, w_2, k, m_1, m_2 change:

$$0 = \{-1 + f'(k)(\frac{1}{r_1}(1 - e^{-r_1m_1}) + \frac{e^{-r_1m_1}}{r_2}(1 - e^{-r_2(m_2-m_1)})\}\frac{dk}{dw_1}$$

$$+ \{e^{-r_1m_1}(f(k) - w_1e^{\lambda m_1}) - r_1P(k,m_1)e^{-r_1m_1} - e^{-r_1m_1}(f(k) - w_2e^{\lambda m_1})$$

$$+ r_2 P(k,m_1) e^{-r_1 m_1}\} \frac{dm_1}{dw_1} + \{e^{-r_1 m_1}(f(k) - w_2 e^{\lambda m_1}) e^{-r_2(m_2-m_1)}\} \frac{dm_2}{dw_1}$$

$$- \int_o^{m_1} e^{(\lambda-r_1)m}\, dm + e^{(r_2-r_1)m_1} \frac{dw_2}{dw_1} \int_{m_1}^{m_2} e^{(\lambda-r_2)m}\, dm.$$

(1), (2), (3) imply that the coefficients of $\frac{dk}{dw_1}$, $\frac{dm_1}{dw_1}$, and $\frac{dm_2}{dw_1}$ are all zero so that

$$\frac{dw_2}{dw_1} \frac{1}{\lambda-r_2} (e^{(\lambda-r_2)(m_2-m_1)} - 1) = - \frac{1}{\lambda-r_1} (1 - e^{-(\lambda-r_1)m_1}) \qquad (4)$$

Further, (2) implies

$$w_1 = w_2 (1 + (r_2 - r_1) g(m_2 - m_1))$$

where

$$g(m_2-m_1) = \int_o^{m_2-m_1} (e^{\lambda(m_2-m_1)} - e^{\lambda m}) e^{-r_2 m}\, dm$$

(see p.110 above), so that

$$g' = \lambda e^{\lambda(m_2-m_1)} \frac{1}{r_2} (1 - e^{-r_2(m_2-m_1)})$$

Therefore

$$1 = \frac{dw_2 w_1}{dw_1 w_2} w_2 (r_2 - r_1) g' \frac{d(m_2-m_1)}{dw_1} \qquad (5)$$

(1) gives

$$\frac{dw_2}{dw_1} = f'(k) e^{-\lambda m_2} \frac{dk}{dw_1} - \lambda w_2 \frac{dm_2}{dw_1} \qquad (6)$$

(3) implies, since $\frac{dm_1}{dk} = \frac{dm_2}{dk} - \frac{d(m_2-m_1)}{dk}$

$$\frac{f''(k)}{f'(k)}\frac{dk}{dw_1} + f'(k)\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\right)e^{-r_1 m_1}\frac{dm_2}{dw_1} \tag{7}$$

$$= f'(k)e^{-r_1 m_1}\left(1 - \frac{r_1}{r_2}\right)\left(1 - e^{r_2(m_2-m_1)}\right)\frac{d(m_2-m_1)}{dw_1}$$

I now use (4a- 7) to establish that $\frac{dk}{dw_1} < 0$ when (A) holds. (6) and (7) with $\frac{dm_2}{dw_1}$ eliminated imply that

$$\left(f'^2 e^{-\lambda m_2}\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\right)e^{-r_1 m_1} + \lambda w_2 \frac{f''}{f'}\right)\frac{dk}{dw_1}$$

$$= f'\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\right)e^{-r_1 m_1}\frac{dw_2}{dw_1}$$

$$+ \lambda w_2 f' e^{-r_1 m_1}\frac{r_2-r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\frac{d(m_2-m_1)}{dw_1}$$

The factor on the left hand side is

$$\lambda w_2\left\{\frac{f'^2}{f}e^{-r_1 m_1}\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\right) + \frac{f''}{f'}\right\} < 0$$

if assumption (A) holds. The factor on the right hand side is

$$f' e^{-r_1 m_1}\left\{\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\right)\frac{dw_2}{dw_1}\right.$$

$$\left. + w_2(r_2-r_1)e^{-\lambda(m_2-m_1)}g'\frac{d(m_2-m_1)}{dw_1}\right\}$$

$$= f' e^{-r_1 m_1}\left\{\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)\right)\frac{dw_2}{dw_1}\right.$$

$$\left. + e^{-\lambda(m_2-m_1)}\left(1 - \frac{dw_2}{dw_1}\frac{w_1}{w_2}\right)\right\} \qquad \text{by (5a)}$$

$$= f'e^{-r_1 m_1}\{\frac{dw_2}{dw_1}\left(1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right) - \frac{w_1}{w_2} e^{-\lambda(m_2-m_1)}\right) + e^{-\lambda(m_2-m_1)}\}$$

Now

$$\frac{w_1}{w_2} e^{-\lambda(m_2-m_1)} = \frac{w_1}{f(k)} e^{\lambda m_1} = \frac{w_2}{f(k)} e^{\lambda m_1} + (r_2-r_1)\frac{P(k,m_1)}{f(k)} \quad \text{by (1), (2)}$$

$$= e^{-\lambda(m_2-m_1)} + (r_2-r_1)\frac{1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right)$$

$$- \frac{r_2-r_1}{r_2^-} e^{-\lambda(m_2-m_1)}\left(1 - e^{-(r_2-\lambda)(m_2-m_1)}\right)$$

Therefore

$$1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right) - \frac{w_1}{w_2} e^{-\lambda(m_2-m_1)}$$

$$= 1 - \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right) - e^{-\lambda(m_2-m_1)} - 1 + e^{-r_2(m_2-m_1)}$$

$$+ \frac{r_1}{r_2}\left(1 - e^{-r_2(m_2-m_1)}\right) + \frac{r_2-r_1}{r_2-\lambda} e^{-\lambda(m_2-m_1)}\left(1 - e^{-(r_2-\lambda)(m_2-m_1)}\right)$$

$$= e^{-\lambda(m_2-m_1)}\left(1 - e^{-(r_2-\lambda)(m_2-m_1)}\right)\left(\frac{r_2-r_1}{r_2-\lambda} - 1\right)$$

$$= e^{-\lambda(m_2-m_1)}\left(1 - e^{-(r_2-\lambda)(m_2-m_1)}\right)\frac{\lambda-r_1}{r_2-\lambda}$$

Therefore the right hand side of equation in $\frac{dk}{dw_1}$ is

$$f'e^{-r_1 m_1 - \lambda(m_2-m_1)}\{\frac{dw_2}{dw_1}\frac{\lambda-r_1}{r_2-\lambda}\left(1 - e^{-(r_2-\lambda)(m_2-m_1)}\right) + 1\}$$

$$= f'e^{-r_1 m_1} e^{-\lambda(m_2-m_1)}\{-\left(1 - e^{-(\lambda-r_1)m_1}\right) + 1\} \qquad \text{by (4)}$$

$$= f'e^{-\lambda m_2} > 0.$$

Hence the result:

Theorem 4.3. For all the values of w_1, w_2 for which w_2 is a strictly decreasing function of w_1, if assumption (A) holds in equilibrium so that k is unique, then k is a strictly decreasing function of w_1.

Intuitively, what is going on here is: 1) as Bliss has shown, as the wage rises in the putty-clay model in one country, if the technique k chosen is unique, then k rises; 2) when (A) is satisfied, Bliss's uniqueness assumption is satisfied so that k is lower when $m_1 = m_2$ than when $m_1 = 0$, since in the first case the world is the low wage country, while in the second case it is the high wage country; 3) w_1 rises as country 1 becomes 'less important' in the world as a whole, i.e. tends towards being a price taker, so country 2's preference for a lower k has more weight.

Unfortunately, it has not proved possible for me to establish what signs are taken by dm_2/dw_1 and dm_1/dw_1, although (5a) shows that $d(m_2-m_1)/dw_1 > 0$. That dm_2/dw_1 is indeterminate in the absence of further information is unsurprising, since dm/dw is of indeterminate sign in the one country putty-clay model without some further restrictions. (Bardhan (1973)[1] shows that $\sigma \leq \frac{2}{3}$ is sufficient to ensure $dm/dw < 0$.) At the point $w_1 = w_2 = w_1^A$, we have $m_2 = m_1 = m_1^A$, in obvious notation. At the point $w_1 = w_1^B$, $w_2 = w_2^A$, we have $m_2 = m_2 - m_1 = m_2^A$. Since we cannot predict the sign of $m_2^A - m_1^A$ from the fact that $w_2^A < w_1^A$ we clearly cannot predict the sign of dm_2/dw_1. It seems plausible that for small enough σ, $dm_2/dw_1 > 0$.

That the sign of dm_1/dw_1 has not been determined is unsatisfactory. For $m_1 = 0$ when $w_1 = w_1^B$, and $m_1 = m_2 = m_1^A$ when $w_1 = w_1^A < w_1^B$. Clearly,

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[1] Bardhan, P., "More on Putty-Clay", International Economic Review 14, 1 (February 1973), pp.211-222.

$dm_1/dw_1 < 0$ 'on the average' when $m_1 \in (0,m_2)$.

If $\frac{dm_1}{dw_1} < 0$, $\frac{d}{dw_1}(\frac{L_2}{L_1}) > 0$ and there is a unique point on the wage frontier at which (4), (5) are satisfied, i.e. a unique trade equilibrium. If $\frac{dm_1}{dw_1} > 0$ at some w_1, but (A) is still satisfied, equilibrium still exists but uniqueness is not guaranteed. If (A) is not satisfied, so that k is not necessarily uniquely determined, the possibility arises that at some (w_1,w_2) there are two competitive values of k, say k_B, k_C with associated m_1, m_2 such that, in the obvious notation, $(L_2/L_1)_B < (L_2/L_1)_E < (L_2/L_1)_C$ where $(L_2/L_1)_E$ is the full employment value. Clearly all convex combinations of k_B, k_C are competitive, and there exists one which gives rise to the value $(L_2/L_1)_E$. Hence equilibrium still exists. It is also clear that if (A) is not satisfied so that there is a multiplicity of k at some values of (w_1,w_2), it is very likely that multiple equilibria exist.

Therefore, although there is always at least one pair (w_1,w_2) with $w_1 > w_2$ associated with full employment equilibrium for given r_1, r_2, with $r_1 < r_2$, there may be more than one such pair. All equilibria are in the interior of the set $[w_1^A, w_1^B]$ and therefore satisfy $w_2^A < w_2 < w_1^A$. This means that the question raised at the start of this chapter has now been answered: at the factor prices r_2, w_2 which are observed in the trade equilibrium, all techniques available in autarchy are uncompetitive since $w_2^A < w_2$. Country 2 will use only old machines, even given the option of constructing more labour intensive new machines than are in use elsewhere.

Looking back at the list of 'reasonable conjectures', we see that conjectures 2 - 4 have all proved to be mistaken. In particular, conjecture 3, that this question was somehow bound up with the question

of uniqueness of k, is wrong. The market <u>always</u> reduces the price of old machines sufficiently for them to be sufficiently labour intensive and cheap, in some loose sense, to render uncompetitive the use of new machines of whatever specification.

This result and its implications for the effect of trade on factor prices are summed up as follows:

<u>Theorem 4.4.</u> In steady state in the putty-clay technology in which two trading countries have profit rates r_1, r_2, such that $r_2 > r_1 \geq 0$ <u>all</u> machines spend the first part of their life in country 1 and the remainder in country 2, and the wage rates in the respective countries are such that $w_1 > w_2$.

<u>Corollary 4.4.</u> In the free-trade steady state in the putty-clay model, each country has a higher wage rate than in the autarchic steady state, and $w_1 > w_1' > w_2 > w_2'$.

<u>4.4. Consumption in trade and autarchy</u>

Recall that the relationships discussed in section 2.3 apply to any vintage model. In particular, it is the case that in a putty-clay model the cohort n_1, n_2 has consumption

$$C_n(t) = Ie^{gt} \int_{n_1}^{n_2} (\mu_o - gP(n) + P'(n))e^{-gn}dn$$

but in the notation of this chapter, $\mu_o = f(k)/k$ and $P(n) = P(k,n)/k$ so that

$$C_n(k,t) = \frac{I}{k} e^{gt} \int_{n_1}^{n_2} (f(k) - gP(k,n) + \frac{\partial P(k,n)}{\partial n})e^{-gn}dn$$

if Ie^{gt} machines of type k are built at time t.

Now

$$\frac{\partial P(k,n)}{\partial n} = rP(k,n) - f(k) + we^{\lambda n}$$

Therefore

$$C_n(k,t) = \frac{I}{k} e^{gt} \int_{n_1}^{n_2} ((r-g)P(k,n) + we^{\lambda n})e^{-gn} dn$$

$$= \frac{I}{k} e^{gt} \int_{n_1}^{n_2} ((r-g)P(k,n)e^{-\lambda n} + w)e^{-(g-\lambda)n} dn$$

Therefore the contribution to consumption per head made by the $[n_1, n_2]$ cohort of machines of type k is

$$c_n(k,t) = c_n(k)e^{\lambda t}$$

where

$$c_n(k) = \frac{\int_{n_1}^{n_2} ((r-g)P(k,n)e^{-\lambda n} + w)e^{-(g-\lambda)n} dn}{\int_{n_1}^{n_2} e^{-(g-\lambda)n} dn}$$

and if $r > g$, $(r-g)P(k,n)e^{-\lambda n}$ is a decreasing function of n so that

$$(r-g)P(k,n_1)e^{-\lambda n_1} + w > c_n(k) > (r-g)P(k,n_2)e^{-\lambda n_2} + w.$$

In the trade steady state, each country looks like a self-supporting cohort. It is, of course, necessary to make a uniqueness assumption in order to make comparative dynamics statements. Suppose therefore that (A) holds for all relevant r_1, r_2.

As we allow $r_2 - r_1$ and, consequently, $w_1 - w_2$ to tend to zero, we arrive at a limiting case in which (A) is Bliss's uniqueness assumption, and $r_2 = r_1 = r$, $w_2 = w_1 = w$, with complete specialisation by country 2 in older machines. (Compare the limiting case in the clay-clay model, p.91 .) The inequalities above imply that

$$c_1 > c_1' = c = c_2' > c_2$$

if $r > g$; with equalities everywhere if $r = g$, and the reverse inequalities if $r < g$.

But whereas in the clay-clay model the result in the limiting case can be extended to give some definite results when $r_1 > r_2$, this does not appear to be the case in the putty-clay model. All that we know is that

$$c_1 > (r_1-g)P(k,m_1)e^{-\lambda m_1} + w_1$$

$$= (r_2-g)P(k,m_1)e^{-\lambda m_1} + w_2 > c_2,$$

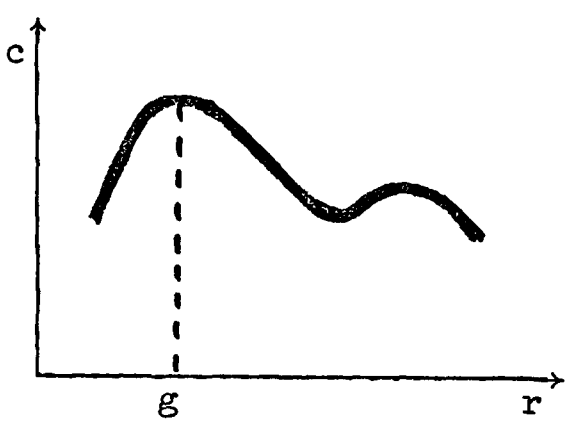
from (2) and the inequalities above. But there seems to be no reason for c_1' and c_2' to have any particular relation to c_1 and c_2. It is easily seen that the proof of Theorem 3.2 will not extend to the case when k is variable. The economic reason for the absence of comparative dynamics results when $r_2 > r_1$ seems to be the following.

Bliss states (p.128) that the relationship between consumption and gross saving rate in the putty-clay model is described by the upper envelope of the class of similar curves for the clay-clay models each of whose set of factor proportions is given by a point on the putty-clay model's ex ante production function. (I have to admit that I am unable to see why this should be so. No justification is given by Bliss.) Further, Bliss shows (p.122) that when the lifetime of machines is a decreasing function of r, the saving rate may be a non-monotonic function of r.

The fact that c(s) is an envelope of single-peaked functions of s implies that it is a single-peaked function. The fact that s may

not be monotonic in r means that c(r) need not be a single-peaked function, and may have the form illustrated in Diagram 2.

Diagram 2



It is this absence (apart from the golden rule result) of any comparative dynamics results about steady state consumption in the one-country case that seems to be the basis of the apparent impossibility of extending the 'local' result that $c_1 > c_1' = c_2' > c_2$ when $r_1 = r_2 > g$ to a 'global' result in the case $r_1 < r_2$, analogously to Theorem 3.2. In the clay-clay model, all of the additional effects arising from the inequality of r_1 and r_2 seemed to raise long-run consumption in both countries. In the putty-clay model, the additional effects include changes in k which apparently need not necessarily raise long-run consumption.

4.5. Conclusions

The main conclusion is simple, if somewhat surprising: the possibility of ex-ante substitutability in the choice of production techniques does not weaken the presumption that poor countries use old machines. (I find this result surprising because it runs counter to my original intuitions on this matter, and because I am unable to think up a convincing verbal explanation of the result.)

If the condition (A) holds, then the type of machine constructed in the trade steady state is more labour intensive than the (unique) type of machine constructed in country 1 in autarchy steady state, and less labour intensive than the type constructed in country 2 in autarchy steady state. Only one type of machine is constructed, and it is associated with steady state values of w_1 and w_2, but the particular w_1, w_2, k which rule in steady state need not be uniquely determined by L_1, L_2, r_1, r_2. But in any case, trade raises the steady state value of w in each country.

So far as consumption is concerned, we still have the phenomenon observed in Chapters 2 and 3 of substitution of 'consumption-now' and 'consumption-later' via trade in second-hand machines. It is not here possible to derive results as clear-cut as the results of Chapter 3, because of the extra complications introduced by ex-ante substitutability. There is still the same tendency for trade to lead to a rise in C_1 and a fall in C_2, when $r_2 > r_1 \geq g$, but this is now clearly seen to be a 'local' result which will hold for values of r_2 greater than but close to r_1. When the divergence between r_2 and r_1 is greater, the inter-temporal substitution still occurs, but may be obscured by the other effects associated with the change from autarchy to trade.

In short, the putty-clay model in this context behaves in roughly the same way as the clay-clay model. The discussion at the end of Chapter 3 of the policy implications of the model carries over unchanged to the putty-clay model.

CHAPTER 5

Trade and Consumption in Alternative Models of Capital Accumulation

5.1 Introduction

In the preface to this thesis, two justifications are offered for the generality of its title. I hope that the preceding chapters show the validity of the first justification : that trade in second-hand is a pervasive feature of vintage models involving countries with factor price differentials.

The objection could, however, be made that the vintage approach is by no means universally accepted as the most appropriate way to model capital accumulation. It is the aim of this chapter, not to counter this objection directly, but to attempt to demonstrate the validity of my second justification, by examining some alternative models of trade in capital goods. There are, of course, important differences between these models and the vintage models but there are also striking similarities.

In particular, I am here concerned with considering in these alternative models the question of how trade affects consumption on steady state paths. I am also concerned with the question of whether the appropriate comparison is of paths with the same saving rate or with the same interest rate.

5.2 The two-sector model with fixed profit rates

This section is devoted to expounding and clarifying the analysis in Stiglitz (1970a)[1] of the effects of trade on consumption in the two-sector model with malleable, shiftable capital.

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[1] Stiglitz, J.E., "Factor Price Equalization in a Dynamic Economy", Journal of Political Economy, 78, 3 (June 1970), pp. 456-488.

Consumption and investment goods are produced by techniques described by conventional constant returns concave production functions, identical in both countries :

$$C = C\ (K_c,\ L_c) = L_c\ c(k_c)$$
$$Z = Z\ (K_z,\ L_z) = L_z\ z(k_z) \qquad (1)$$

where C is the output of consumption goods, Z investment goods, K_i, L_i are inputs of capital and labour to the respective sectors, and $k_i = K_i/L_i$.

The investment good is chosen as the numéraire, and prices are assumed to be competitively determined :

$$r = \max\ [pc'\ (k_c),\ z'\ (k_z)]$$
$$w = \max\ [p\ (c - k_c\ c'(k_c)),\ z - k_z\ z'(k_z)] \qquad (2)$$

where r is the profit rate, w the wage rate, and p the price of consumption goods.

It is assumed that the two countries A and B have time preference rates δ^A, δ^B respectively, with $\delta^A > \delta^B$, and that each country aims towards an optimal steady state in which the profit rate is equated to this time preference rate. (I have here chosen to use Stiglitz's notation. When comparisons are made between this model and the model of Chapter 3, the reader will require to recall that in that chapter, it was country 1 which had the lower long run profit rate, whereas here it is country B.)

In autarchy steady state, both sectors produce positive levels of output and (2) becomes

$$r = pc'(k_c) = z'(k_z)$$

$$w = p\left(c - k_c\, c'(k_c)\right) = z - k_z\, z'(k_z)$$

which imply that

$$\omega = \frac{w}{r} = \frac{c - k_c\, c'(k_c)}{c'(k_c)} = \frac{z - k_z\, z'(k_z)}{z'(k_z)} \quad .$$

Differentiation of these equations shows that they define k_c and k_z as strictly increasing functions of ω. Stiglitz assumes that either $k_z(\omega) < k_c(\omega)$ for all ω, or that $k_c(\omega) < k_z(\omega)$ for all ω, i.e. that one sector is less capital intensive at all factor price ratios. $pc'(k_c) = z'(k_z)$ fixes p as a function of ω, which can be shown to be a decreasing function when $k_c > k_z$ and an increasing function when $k_z > k_c$. (This conforms with intuition : the consumption good becomes relatively more expensive as labour becomes relatively more expensive if and only if it is relatively labour intensive in production.)

The assumption of non-reversal of factor intensities, i.e. that one sector is unambiguously the more capital intensive, thus ensures that the factor price equalisation theorem holds. Since in steady state $r^A > r^B$ by assumption, at least one country must specialise. It is clear intuitively, and easy to prove formally, that either country A specialises in the labour intensive sector or country B specialises in the capital intensive sector.

(In common with Stiglitz, I have sketched the main outlines above rather than give proofs. A full treatment of the model may be found in the seminal article of Oniki and Uzawa (1965)[1].)

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[1] Oniki, H. and H. Uzawa (1965), "Patterns of Trade and Investment in a Dynamic Model of International Trade", Review of Economic Studies 32, 1 (January 1965), pp. 15-38.

Consider a country with this technology in autarchy. Denote by λ the share of the total labour force which is employed in the investment goods sector. In steady state, with rate of growth n,

$$\lambda z\ (k_z) = nk = n\ \left(\lambda k_z + (1 - \lambda)\ k_c\right)$$

$$\therefore \qquad nk_z = \lambda z\ (k_z) - n\ (1 - \lambda)\ (k_c - k_z)\ , \qquad (3)$$

and consumption per man is

$$c^o = (1 - \lambda)\ c\ (k_c) \qquad (4)$$

(the superscript ^{o} being used to indicate the autarchy value). Differentiation of these relationships shows that c^o is an increasing function of r when $r < n$ and a decreasing function when $r > n$.

Stiglitz then considers the case where this country trades with the 'rest of the world', the country being so small that it is a price taker. Then unless the country has the same factor prices as the world it is specialised in the production of one good. If it is specialised in consumption goods then its consumption level in trade steady state is

$$c^T = c(k) - nk/\hat{p} \qquad (5)$$

where $\hat{p}$ is fixed by world prices and

$$r = \hat{p}c'\ (k)\ . \qquad (6)$$

(6) implies that $\frac{dk}{dr} < 0$, and (5) that

$$\frac{dc^T}{dk} = c'\ (k) - \frac{n}{\hat{p}} = \frac{r - n}{\hat{p}}\ .$$

If it is specialised in investment goods,

$$c^T = \frac{z(k) - nk}{\hat{p}} \qquad (7)$$

where

$$r = z'(k) \quad , \tag{8}$$

which imply that again $\frac{dk}{dr} < 0$ and

$$\frac{dc^T}{dk} = \frac{r - n}{\hat{p}} \quad ,$$

so that in both cases $\frac{dc^T}{dr}$ has the sign of $n - r$.

Finally, when factor prices are identical in the country to world factor prices, the level of consumption depends on the relative size of the two sectors.

$$c^T = \lambda \frac{z(k_z) - nk_z}{\hat{p}} + (1 - \lambda) \frac{\hat{p}c(k_c) - nk_c}{\hat{p}}$$

and, k_z, k_c, $\hat{p}$ being fixed,

$$\frac{dc^T}{d\lambda} = \frac{z(k_z) - \hat{p}c(k_c) - n(k_z - k_c)}{\hat{p}}$$

$$= \frac{(\tilde{r} - n)(k_z - k_c)}{\hat{p}}$$

since $z(k_z) = w + rk_z = pc(k_c) + r(k_z - k_c)$ from (2), where $\tilde{r}$ is the world profit rate.

Therefore, if $\tilde{r} > n$ and $k_z > k_c$, when $r < \tilde{r}$, the country specialises in investment goods, and c^T is a function of r whose derivative takes the sign of $n - r$. When r reaches $\tilde{r}$, λ begins to fall from 1 to 0, and therefore c^T falls since $dc^T/d\lambda > 0$. For $r > \tilde{r}$, $\lambda = 0$, the country is specialised in consumption goods and $\frac{dc^T}{dr} < 0$ since $r > n$.

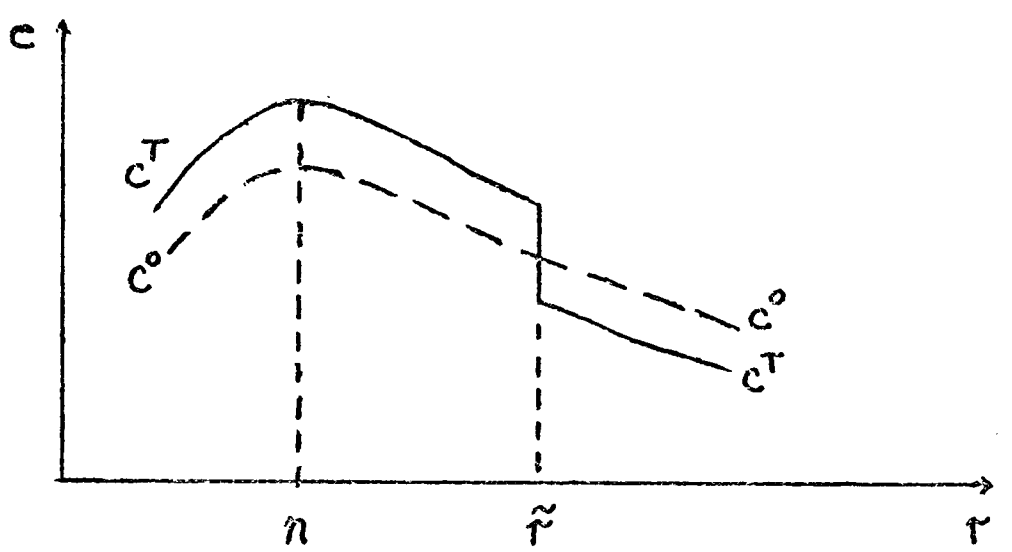
Similarly, if $\tilde{r} > n$ and $k_z < k_c$, for small r the country specialises in consumer goods; when r passes through $\tilde{r}$, λ goes from 0 to 1 and c^T falls; and when $r > \tilde{r}$ it specialises in investment goods.

Finally, note that there must exist some value of $\lambda \in (0, 1)$ at

which it happens that no trade takes place. This state is identical to the autarchy state in which $r = \tilde{r}$.

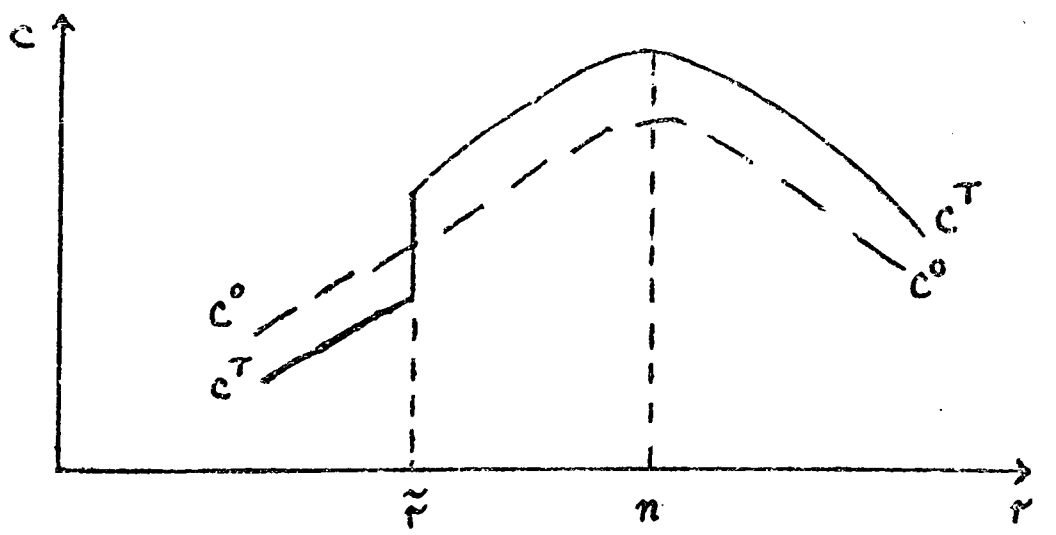
Therefore, graphing c^T and C^O against r, whether $k_z > k_c$ or $k_c > k_z$, we will get a picture something like Diagram 1, which is Stiglitz's Figure 2.

Diagram 1 $c^O(r)$ and $c^T(r)$, $\tilde{r} > n$



A similar argument (since all that changes is the sign of $dc^T/d\lambda$) establishes that if $\tilde{r} < n$, the situation becomes that illustrated in Diagram 2.

Diagram 2 $c^O(r)$ and $c^T(r)$, $\tilde{r} < n$



It is not clear whether Stiglitz believes these two cases to be exhaustive. Although it is clear that in the neighbourhood of $\tilde{r}$ the relationship between c^O and c^T is as shown, there is no apparent reason why c^O and c^T should not also intersect at some other value of r.

Consider, for example, the case of a country which specialises in investment goods in trade steady state. This would arise if $r > \tilde{r} > n$ and $k_c > k_z$, for then $p^T < p^o$, where p^T is the world price and p^o the price in the country in autarchy.

Denote by λ^T the proportion of investment goods not exported.

$$\lambda^T z(k_z) = nk_z$$

$$= \lambda z(k_z) - n(1 - \lambda)(k_c - k_z), \text{ from (3), since}$$

k_z is the same in trade and autarchy.

$$\therefore \quad c^T = \frac{1}{p^T}(1 - \lambda^T) z(k_z)$$

$$= \frac{1}{p^T}(1 - \lambda) z(k_z) + \frac{n}{p^T}(1 - \lambda)(k_c - k_z)$$

$$= \frac{1 - \lambda}{p^T}\left[p^o c(k_c) - (r - n)(k_c - k_z)\right] ,$$

since $$z(k_z) = w^o + rk_z$$

$$= p^o c(k_c) - r(k_c - k_z)$$

$$\therefore c^T < \frac{p^o}{p^T}(1 - \lambda) c(k_c) = \frac{p^o}{p^T} c^o, \text{ from (4), since } r > n \text{ and}$$

$k_c > k_z$. If r is close to $\tilde{r}$ then p^o is close to p^T and the fact that $k_c > k_z$ implies that $c^T < c^o$, as shown in Diagram 1. However, if k_c is very close to k_z at the factor-price ratio ruling in autarchy, then c^T is very close to $p^o c^o / p^T$. It is easily shown that in this model,

$$\frac{1}{p}\frac{dp}{d\omega} = \frac{1}{k_c + \omega} - \frac{1}{k_z + \omega} ,$$

so that if $z(\cdot)$ and $c(\cdot)$ are such that $k_c - k_z$ becomes large as ω moves from ω^o, the value ruling in this country in autarchy, to $\tilde{\omega}$ the value ruling in the rest of the world, then p^o is much greater than p^T. In

this case, $c^T > c^o$, a counterexample to the proposition that Diagram 1 always holds.

If $\tilde{r} > r > n$ and $k_z > k_c$, again $p^T < p^o$ and the country specialises in investment goods.

$$c^T = \frac{1 - \lambda}{p^T} \left(p^o\, c(k_c) - (r - n)\,(k_c - k_z)\right)$$

$$> \frac{p^o}{p^T} c^o > c^o \quad .$$

If $\tilde{r} > n > r$, $c^T < \frac{p^o}{p^T}\, c^o$, and it is possible that either $c^T < c^o$ or $c^T > c^o$, although when r is close to n, $c^T > c^o$.

There are similar results in those cases in which the country specialises in consumption goods. If $r > \tilde{r} > n$ and $k_z > k_c$, $p^o < p^T$ and

$$c^T = c(k_c^T) - \frac{nk_c^T}{p^T}$$

where k_c^T is different from k_c, the autarchy value, since

$$r = p^o\, c'\,(k_c) = p^T\, c'\,(k_c^T)$$

which implies that $k_c^T > k_c$.

$$\frac{d}{dk}\left(c(k) - \frac{nk}{p^T}\right) = \frac{p^T\, c'\,(k) - n}{p^T}$$

$$> \frac{r - n}{p^T} \quad \text{for} \quad k < k_c^T$$

$$\therefore \qquad c^T > c(k_c) - \frac{nk_c}{p^T} > c(k_c) - \frac{nk_c}{p^o}$$

with equalities replacing inequalities only when $p^o = p^T$. (3) implies that

$$nk_c = \lambda \left(z(k_z) - n\,(k_z - k_c)\right)$$

$$= \lambda \left(p^o\, c(k_c) + (r - n)\,(k_z - k_c)\right)$$

$$\therefore \qquad c^T > (1 - \lambda)\, c(k_c) - (r - n)\,(k_z - k_c)/p^o$$

$$= c^o - (r - n)\,(k_z - k_c)/p^o < c^o \quad .$$

When p^o is close to p^T and $k_z - k_c$ is large, the second inequality dominates and $c^T < c^o$. When r is much larger than $\tilde{r}$ and p^o is much less than p^T and $k_z - k_c$ is small, $c^T > c^o$.

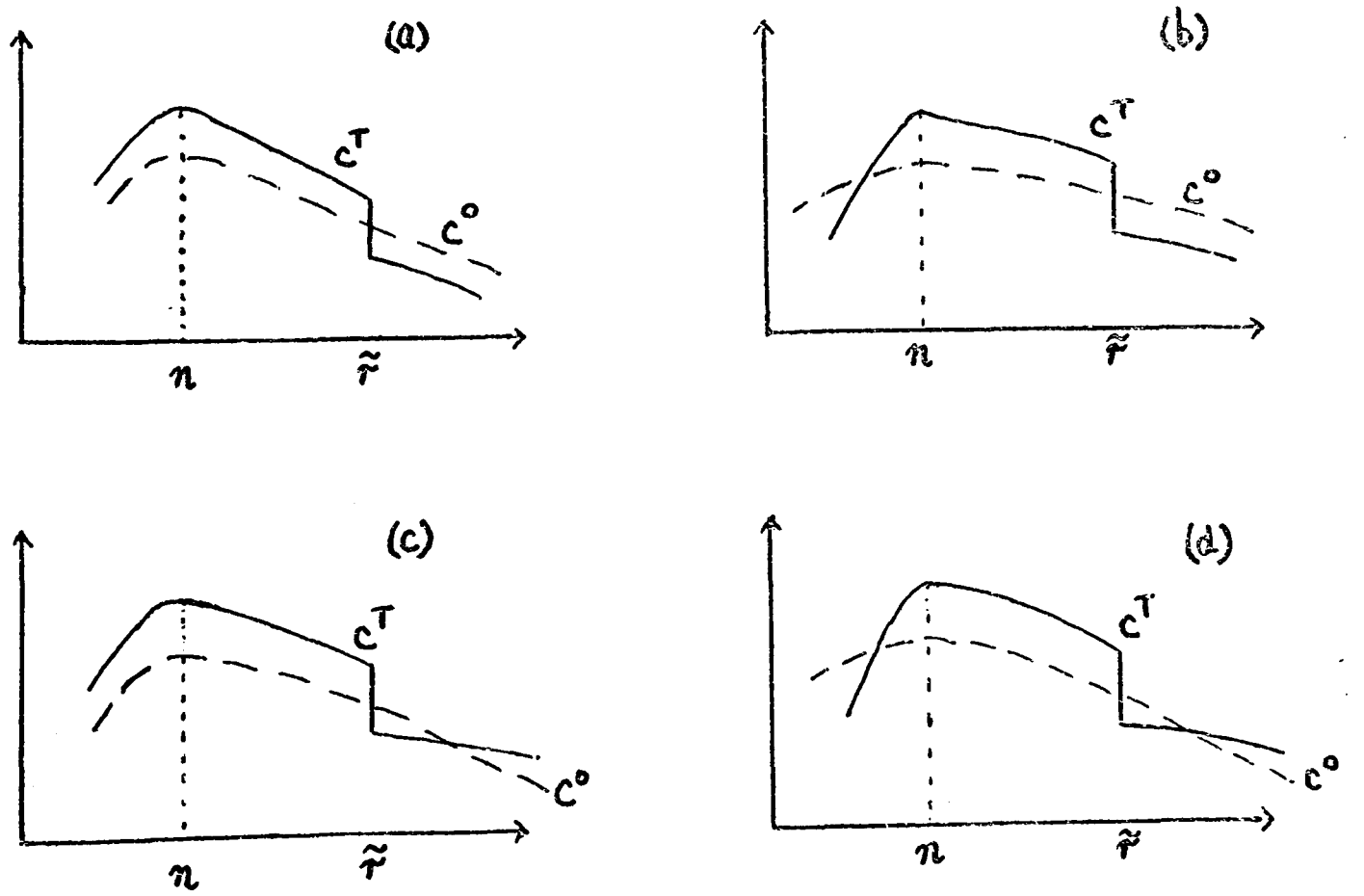
If $\tilde{r} > r > n$ and $k_c > k_z$, again the country specialises in consumption goods and

$$c^T > c^o - (r - n)\,(k_z - k_c) > c^o \quad .$$

If $\tilde{r} > n > r$, it is possible that $c^T < c^o$ or $c^T > c^o$, although $c^T > c^o$ when r is close to n.

All of this may be summarised by illustrating in Diagram 3 the four possibilities of which Diagram 1 is only the first.

Diagram 3 $c^o(r)$ and $c^T(r)$, $\tilde{r} > n$



What is going on here is exactly analogous to the behaviour observed in the vintage model in Chapter 3 (see especially pp. 86-9). 1. The divergence between r and $\tilde{r}$ leads to a difference between p^o and p^T which always tends to raise c^T above c^o. This I have called the 'static' gain from trade. 2. When $r > n$, the specialisation by the country in one sector tends to raise c^T above c^o if $r < \tilde{r}$ and reduce it below c^o if $r > \tilde{r}$. The country enjoys a 'dynamic' gain from trade which takes the form of intertemporal consumption substitution, towards the present if it is relatively impatient, the future it it is relatively patient. 3. When $r < n$, the 'dynamic' gain disappears, and is replaced by a tendency for c^T to fall below c^o because of inefficiency.

As Stiglitz points out, it is incorrect to describe a reduction in steady state consumption as a loss, if the profit rate exceeds n, precisely because such a reduction is always associated with a reduction in the capital intensity of production. For in the respective steady states,

$$c^o = w^o + (r - n)\, k^o$$

$$c^T = w^T + (r - n)\, k^T$$

and $w^T \geq w^o$ since the autarchy techniques are available when the trade techniques are used.

$$\therefore \qquad c^T < c^o \Rightarrow (r - n)\, k^T < (r - n)\, k^o$$

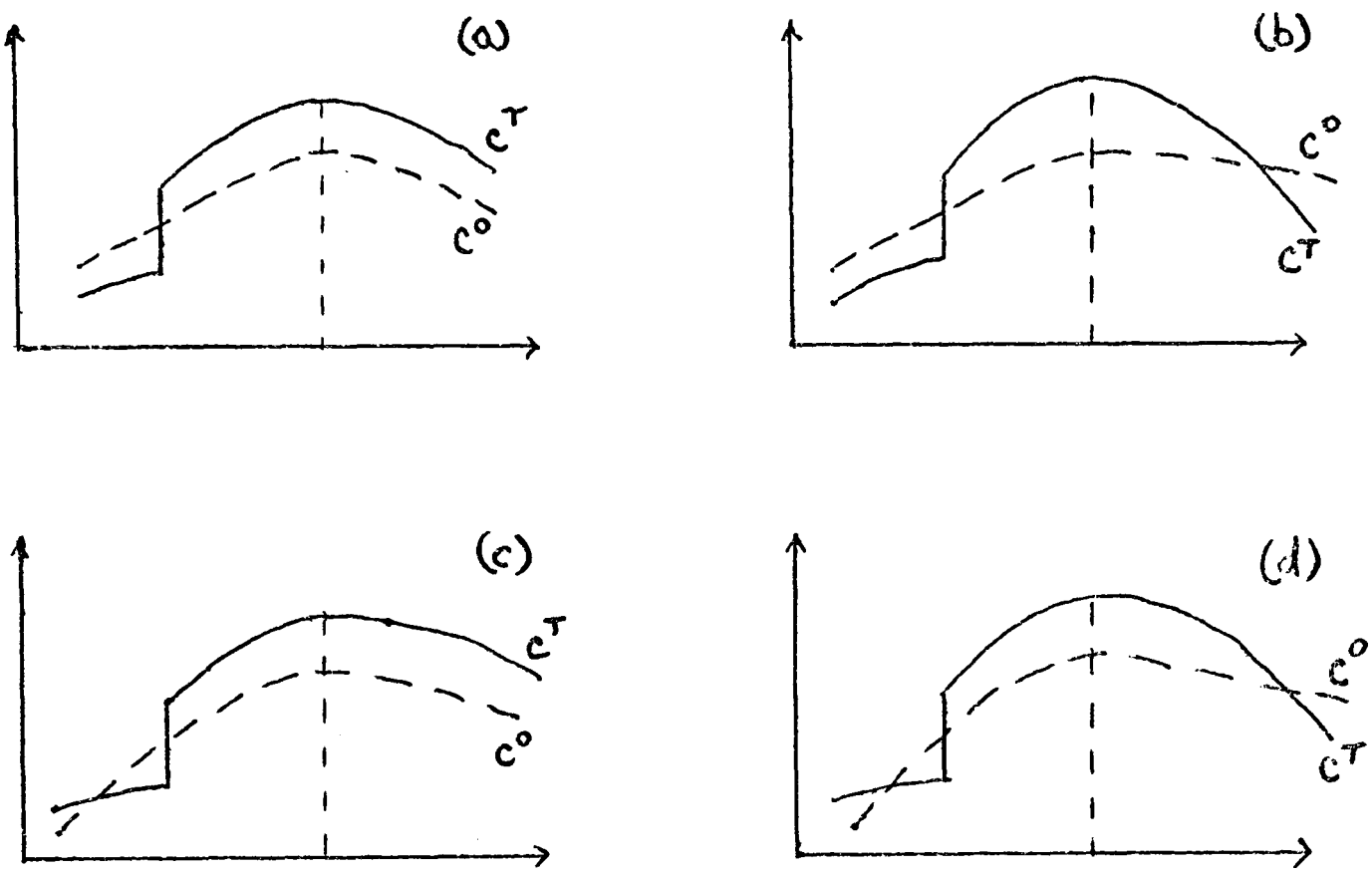
$$\Rightarrow k^T < k^o \quad \text{if} \quad r > n \,.$$

This confirms my statement above that when trade lowers steady state consumption, if $r > n$ this is because it raises consumption in the short run, by allowing the acountry to sell off some of its capital

stock (cf. p. 91 above).

The reader can confirm that when $\tilde{r} < n$, the same arguments give rise to the cases illustrated in Diagram 4, of which Diagram 2 is the special case.

Diagram 4 $c^{o}(r)$ and $c^{T}(r)$, $\tilde{r} < n$



The difference between this case and the previous one is that the 'dynamic' gain from trade takes the form, when $r \in [\tilde{r}, n]$ and the less capital intensive technique is chosen, of higher short-run consumption (because of the reduction in the value of the capital stock) and higher long-run consumption, as a result of the country being less inefficient than the rest of the world.

By examining what happens to the 'rest of the world', we are able to generalise the analysis to cover the case where the country is not a price-taker. The rest of the world has k_z, k_c and $\hat{p}$ fixed by $\tilde{r}$ in both trade and autarchy and

$$c = \lambda \frac{z(k_z) - nk_z}{\hat{p}} + (1 - \lambda) \frac{\hat{p}c(k_c) - nk_c}{\hat{p}}$$

which implies (see above, p. 135) that

$$\frac{dc}{d\lambda} = \frac{(\tilde{r} - n)\ (k_z - k_c)}{\hat{p}}$$

Trade raises λ if the small country specialises in consumption goods, reduces it if it specialises in investment goods. For example, if $r > \tilde{r} > n$ and $k_z > k_c$, the small country specialises in consumption goods, the rest of the world has a higher λ and a higher c since $dc/d\lambda > 0$. By examining each case in turn, it is easy to confirm that the two 'dynamic' forces acting in the case of the small country are present but the 'static' gain from trade is absent. Trade with an impatient small country raises steady state consumption, with a patient small country reduces it, if $\tilde{r} > n$. If $\tilde{r} < n$, inefficiency reverses these effects : consumption is raised if and only if the small country is more inefficient.

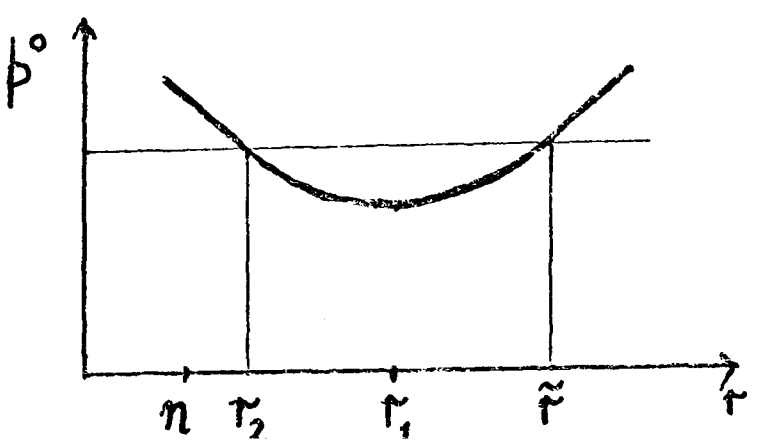
It is now a simple matter to describe the general case of trade between two countries of the same order of magnitude. Typically, one country will not specialise. Then it acts like the price-maker, the 'rest of the world' above, and the effect of trade on its steady-state consumption is as described in the previous paragraph. The other must specialise and it acts like a price-taker, and its consumption is affected by the additional tendency for the 'static' gain from trade to raise steady-state consumption. It may be that the relative size of the two countries is precisely such that each specialises. Then the world price settles at a level intermediate between their autarchy prices, and both enjoy some 'static' gains.

Thus Stiglitz's statement (p. 466) that 'the country with the high rate of time preference has an equilibrium consumption per capita lower

after trade than before, and conversely for the other country, independent of assumptions about capital intensity' is not strictly true since it ignores the fact that one or both countries may have inefficient profit rates, which tends to raise the consumption of the relatively efficient country, and that one country, and possibly the other also, will be specialised in one sector and this tends to raise its consumption.

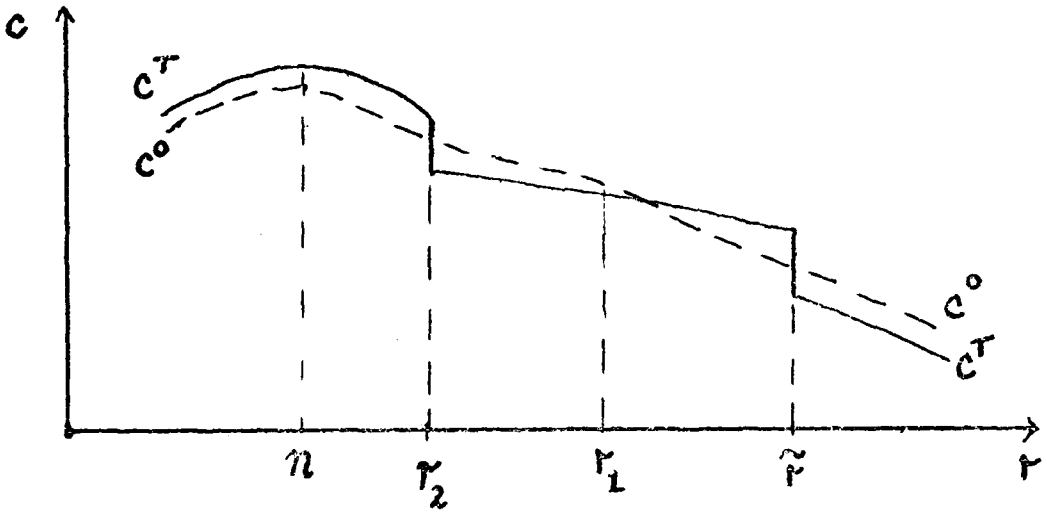
Finally the ambuiguity of the expression 'independent of assumptions about capital intensity' must be dealt with. Earlier (p. 459) he makes the assumption that one sector is more capital intensive than the other at all factor price ratios. This would imply that the phrase quoted refers to the fact that the analysis hold irrespective of which sector it is that is more capital intensive. In fact it can be shown that even if there are factor intensity reversals the analysis still holds : but only in a very modified form. That some modification is required is seen in the following example. Suppose $n < r < \tilde{r}$ and the 'rest of the world' is not specialised. Then Diagram 3 shows that we have the unambiguous result that trade raises steady state consumption in the small country and reduces it in the rest of the world, if either $k_z > k_c$ or $k_c > k_z$ for all relevant factor price ratios. But now suppose that $k_z > k_c$ for $r < r_1$, and $k_c > k_z$ for $r > r_1$, where $r_1 < \tilde{r}$. Then p^o is a decreasing function for $r < r_1$, and an increasing function for $r > r_1$. Define r_2 by $p^o(r_2) = p^o(\tilde{r})$ (see Diagram 5), and suppose that $r_2 > n$.

Diagram 5 $p^o(r)$



Then a small country with $r \in (r_2, \tilde{r})$ finds $p^o < p^T = p^o(\tilde{r})$ and specialises in consumption goods, while if $r < r_2$ it specialises in investment goods. Note that when $r \in (r_2, r_1)$ the country specialises in the less capital intensive good even though $r < \tilde{r}$. Therefore as r rises towards r_2 c^T falls gradually. At r_2, λ falls from 1 to 0 and since $dc^T/d\lambda = (r_2 - n)(k_z - k_c)/p^T > 0$, c^T falls sharply at r_2. It then falls gradually on $(r_2, \tilde{r})$ and at $\tilde{r}$, λ rises from 0 to 1 and since $dc^T/d\lambda = (\tilde{r} - n)(k_z - k_c)/p^T < 0$, c^T falls sharply. At r_1, $c^T > c^o$ since $dc^T/d\lambda = 0$ which implies $c^o = c(k_c) - nk_c/p^o$, and it is proved above that $c^T > c(k_c) - nk_c/p^o$. Hence Diagram 6.

Diagram 6 c^T and c^o with factor intensity reversal



This shows that if we allow factor intensity reversals, the one global result available previously, that the country with the lower profit rate always has $c^T > c^o$ if its profit rate exceeds n, does not now hold.

Further, it was shown above that consumption in the 'rest of the world' goes up if and only if it exports the good that is more capital intensive at profit rate $\tilde{r}$, i.e. the consumption good in this example. But this is so if and only if $r < r_2$ or $r > \tilde{r}$. Thus if $r < r_2$ consumption is increased by trade in both the small country and the 'rest of the world', even though the rest of the world is more impatient. (This

may seem a little curious, but the reason is simple : when $r < r_2$ the small country specialises in what it sees as the capital intensive product, i.e. investment goods, so forcing the rest of the world in the direction of increased production of consumption goods, which it sees as more capital intensive.)

To avoid this discussion becoming even more taxonomic I do not discuss any other cases of factor intensity reversal. The reader should be able to work out quite easily what happens in other cases.

I sum up by listing what happens to steady state consumption when two countries with respective profit rates r^A, r^B, such that $r^A > r^B > n$, open up trade (again the reader should be able to see what happens if one or both countries have $r < n$). When there are factor intensity reversals, the phrase '$r^A - r^B$ is small' is to be understood as meaning that it is at least small enough so that there are no reversals in the interval $[r^A, r^B]$. If there is a reversal in the interval, nothing may be said.

(a) In the absence of capital intensity reversals, trade raises c^B.

(b) If there are reversals of capital intensity, trade necessarily raises c^B only if $r^A - r^B$ is small.

(c) Trade necessarily reduces c^A in the absence of capital intensity reversals only if $r^A - r^B$ is small or if country A does not specialise.

(d) Trade necessarily reduces c^A when there are capital intensity reversals only if $r^A - r^B$ is small.

The analogy with consumption behaviour in the vintage model should now be clear. The use of old machines is unambiguously more labour intensive than the use of new machines, and is therefore favoured by the country with the higher profit rate. In trade equilibrium neither country uses machines of all vintages, so both look like price-takers

(although they are not). When the two profit rates are close, the effect of intertemporal substitution of consumption is the strongest force affecting consumption, and the country with the high profit rate has steady-state consumption reduced by trade (assuming all profit rates exceed the growth rate) while the other country has increased consumption. When the two profit rates diverge, then trade means not only specialisation in different vintages by different countries, but also changes in the lifetime of machines and, in the putty-clay model, changes in the type of machines constructed. If the divergence is great enough, the effects of these forces may obscure the effects of specialisation per se.

Although the discussion in this section may seem somewhat removed from discussions more explicitly oriented towards integrating optimal growth theory and trade theory (e.g. Ryder (1967)[1]), it should be clear that there is no conflict. One describes the features of an optimal path if one is interested in how, for example, the pattern of specialisation or the optimum tariff alters over time. Apart from the concluding section of Chapter 3 where a sketchy account was presented of how trade should evolve on an optimum path, I have not here been concerned with these matters. Rather my concern has been, by comparing steady states with the same profit rate, which could be interpreted as optimal steady states, to show the basic nature of the effect of trade on the optimal timing of consumption.

5.3 The two-sector model with fixed saving rates

For three reasons, it appears to me that the assumption adopted in the previous section that a country keeps its long run profit rate constant

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[1] Ryder, H.E., "Optimal Accumulation and Trade in an Open Economy of Moderate Size", Essay V of K. Shell (ed.) Essays on the Theory of Optimal Economic Growth (M.I.T. Press; Cambridge, Mass., 1967), pp. 87-116.

is more satisfactory than the assumption of constant saving rate which is the commonly adopted alternative. In the first place, the fixed profit rate assumption can be justified as being the policy that would be chosen as the long run objective of a rational planner (see Stiglitz, p. 40), whereas this is not true of the fixed saving rate assumption except in special cases. Secondly, we have seen in Chapter 2 that in the vintage model at least, the fixed saving rate assumption can lead to implausible conclusions : Example 9 in that chapter showed a country changing from an efficient path in autarchy to an extremely inefficient path in trade. Thirdly, and more subjectively, it seems to me that for the clay-clay model Chapter 3 above gives a clearer picture of the economic forces underlying changes in consumption due to trade than does Chapter 2.

The assumption of fixed saving rates is adopted by Johnson (1971)[1] (on which see also Johnson (1972)[2], and Bertrand (1973)[3]), Vanek (1971)[4], and Deardorff (1973)[5].

Vanek deals with the issue briefly. An increase in the price of the consumption good to a trading economy raises the value of its income measured in investment goods, and hence leads to capital accumulation, given the fixed saving rate. The total effect on long-run consumption is the sum of this positive effect and the negative direct effect of the higher price. He states, without proof, that the terms of trade which yield the minimum consumption in steady state are, in general,

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[1] Johnson, H.G., "Trade and Growth : a Geometrical Exposition", Journal of International Economics 1, 1 (February 1971), pp. 83-102.

[2] Johnson, H.G., "Trade and Growth : a Correction", Journal of International Economics 2, 1 (February 1972), pp. 87-88.

[3] Bertrand, T.J., "Trade and Growth : a Comment", Journal of International Economics 3, 2 (May 1973), pp. 193-196.

[4] Vanek, J., "Economic Growth and International Trade in Pure Theory", Quarterly Journal of Economics 85, 3 (August 1971), pp. 377-390.

[5] Deardorff, A.V., "The Gains from Trade in and out of Steady-State Growth", Oxford Economic Papers N.S. 25, 2 (July 1973), pp. 173-191.

different from the terms of trade yielding the autarchy level of consumption. This implies that in some cases, trade can reduce long-run consumption.

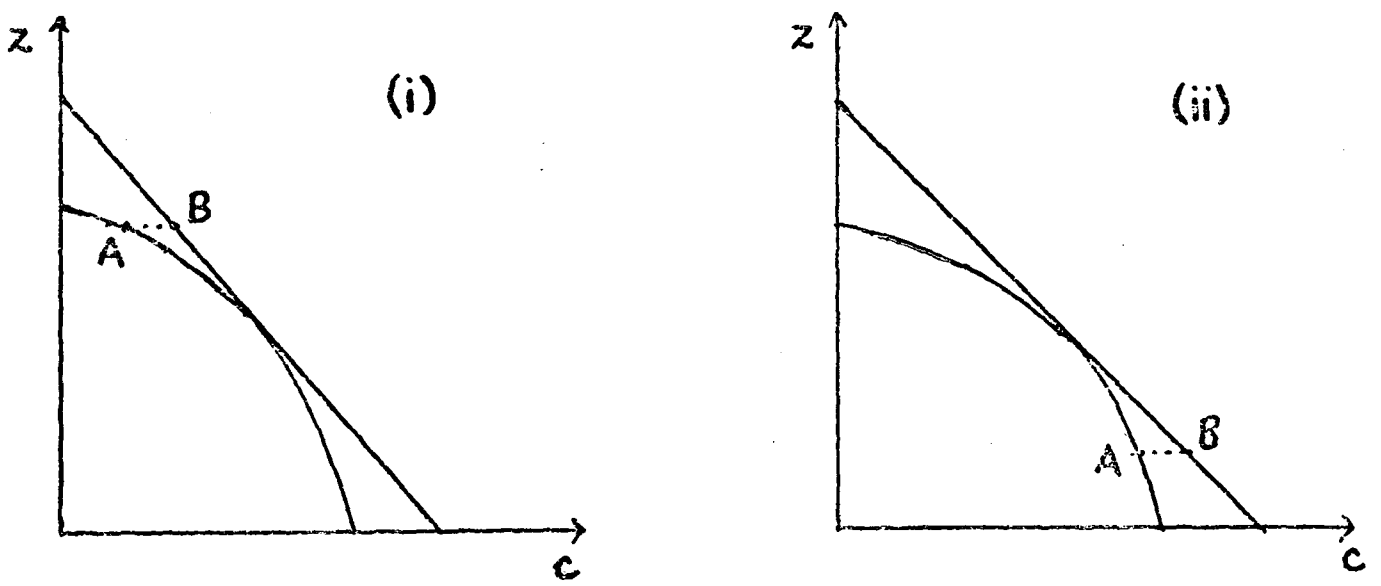
Johnson is more explicit about the circumstances in which this would happen. His analysis relates only to efficient saving programmes, and he derives the following results :

(i) If a country has an initial comparative advantage in the consumption good, trade leads to capital accumulation, and the combined effect of the price change and of accumulation is to raise steady state consumption per man.

(ii) If a country has an initial comparative advantage in the investment good, there will be decumulation. The combined effect may or may not be to reduce steady state consumption per man.

The basic reason why these results hold is as follows. The shift in the price ratio when the economy is opened up always means that the same quantity of investment goods may be obtained as in autarchy, with an increased quantity of consumption goods (compare A and B in Diagram 7).

Diagram 7



But if the initial position was the autarchy steady state, this is the quantity of investment goods required to maintain the capital stock,

and so the transformation curve, fixed. The price change in (i), however, raises the value of production at investment goods prices and so raises the rate of accumulation, given the fixed saving rate. If the economy is below the golden rule, this accumulation leads to a new steady state with even higher consumption than the (hypothetical) steady state (at B in the diagram) than would have been maintained by a lower saving rate. In (ii), however, there is decumulation.

Deardorff proves the following results for the case of $k_c > k_z$ (p. 181) :

> 'If an economy's savings ratio is equal to the golden rule savings ratio s* which maximises steady-state per capita consumption when the economy is closed, then any trade will increase its steady-state per capita consumption. If $s < s^*$, then a little trade is worse than no trade at all if it involves exporting the investment good. If $s > s^*$, a little trade is worse than no trade at all if it involves exporting the consumption good.'

The implication of the phrase 'a little trade' is that in Johnson's case (ii) the direct effect of a small price change is outweighed by the decumulation it causes, while a large price change may cause relatively less decumulation. Deardorff moves on to consider the implications of this for trade between two countries with saving propensities $s_a < s_b$. If $s_a < s_b < s^*$, then since $k_c > k_z$, county A will export the investment good and will tend to have reduced consumption in the long run, while country B will have increased steady-state consumption.

If $k_c < k_z$, it is shown that when $s_a < s_b < s^*$, it is country B which tends to have its consumption reduced by trade.

(Results are also derived for saving rates in excess of s* : e.g. if $s^* < s_a < s_b$ and $k_c > k_z$, consumption may fall in B and must rise in A.)

These results are not incompatible with the results in the model with fixed profit rates. However, when we examine how they are to be

reconciled we see that their oddest feature is the fact that when $k_c < k_z$, it is the country with the higher saving rate which faces the possibility of reduced steady-state consumption. In autarchy, $s_a < s_b$ implies $r_a > r_b$, so that if the countries kept their profit rates constant after trade opens up, it would be country A which would tend to have reduced steady-state consumption. This must imply that keeping profit rates constant causes such a decrease in s_a and/or increase in s_b that restoring them to their former values greatly increases c^A and reduces c^B.

Note, however, that there seems to be no relationship between these results and the results of Chapter 2. In the clay-clay technology, $s_a < s_b$ implies that B will tend to specialise in new machines and A in old machines. The steady-state consumption level will rise in B and may fall in A. This is analogous to what happens in the above model when $k_c > k_z$ and $s_a < s_b < s^*$. It is difficult to see any obvious analogy between the use of old machines in the clay-clay model, and the export of investment goods in the two sector model. Further, the results of Chapter 2 held independently of whether or not saving rates were efficient : this is not the case here.

I have already given some reasons to support my preference for the constant-profit-rates assumption, and conclude this section by offering another. Johnson, in the main article cited, writes (p. 99) :

> 'Specifically, the results suggest that protection is a bad policy, in the sense of inhibiting the growth of output and consumption per head, in countries which export consumption goods and import investment goods, and a good policy for achieving the same objectives in countries which export investment goods and import consumption goods. ... Since the less developed countries typically or "stylistically" belong in the former category and the developed countries in the latter category, the happy conclusion suggests itself that freer trade on both sides would tend to reduce inequalities of consumption per head ... between the two groups of countries.'

I leave aside the implicit assumption here that it is desirable to increase current inequality in order to reduce long-run inequality, and concentrate on the issue of whether one can justify the stylised interpretation of the world as a two-sector two-homogeneous-factor model, in which the consumption goods sector is relatively labour intensive. Of what use are stylised models? Since, necessarily, they are not meant to be accurate descriptions, they can be of use only if there is reason to believe that the phenomena observed in the models are not attributable to some special unrealistic feature of the model and would be observed in other models.

On this criterion I believe Stiglitz's analysis to be more illuminating than Johnson's. Johnson's results turn crucially on the direction of relative factor intensities. Further, his results do not appear to carry over to the simplest model in which quasi realistic machines replace mythical capital jelly : for in such a model the 'less-developed' country does tend to have consumption reduced by trade in the long run. In addition, it seems inconceivable that his analysis could be extended to the putty-clay model, given the fact that saving rates in the putty-clay model are, in general, not monotonically related to profit rates. By contrast, the Stiglitz analysis applies no matter which sector is capital intensive, and applies in a modified form even when there are factor intensity reversals. Exactly analogous behaviour was found in the clay-clay model in Chapter 3; and in section 4.4 the same phenomena were observed in a modified way in the putty-clay model. In short, the view of the world offered by the Stiglitz model - of poorer countries behaving like impatient countries, substituting immediate consumption for future consumption - can plausibly be argued to be a view which identifies an effect of trade which we should expect to have some impact in the real world, even if its impact is obscured

by the effects of other forces and of changing circumstances over time. The results in the Johnson model seem so model-specific, and the model is so far removed from reality that it would appear to be much more difficult to make an equally plausible case for this model.

5.4 Input-output models

Consider now the model with n goods produced in a fixed coefficient production process whose inputs are homogeneous labour and the goods themselves. Initially, I concentrate on the case where there is only one process for each good.

Let a_{ij} be the input of good i required to produce one unit of good j and let a_{oj} be the labour input required. Let A be the n × n matrix with (i, j)th entry a_{ij}, and let a_o be the 1 × n vector with jth entry a_{oj}.

If x_{it} is the output of good i in time period t and c_{it} is the amount of the good consumed, the difference is the amount required as inputs to the next period's production :

$$x_{it} = c_{it} + \sum_{j=1}^{n} a_{ij}\, x_{jt+1} \ .$$

In vector notation,

$$x_t = c_t + Ax_{t+1} \ .$$

The labour requirement is

$$\ell_t = \sum_{j=1}^{n} a_{oj}\, x_{jt} = a_o\, x_t \ .$$

Now consider steady state in this model. For simplicity, suppose that the labour force is constant and equal to L. Then

$$L = a_o\, x \qquad (9)$$

and
$$x = c + Ax$$
$$= [I - A]^{-1} c \quad . \tag{10}$$

In competitive conditions, the price of x_j equals its unit cost of production if x_j is produced :

$$p_j = \left(\sum_{i=1}^{n} p_i \, a_{ij} + w \, a_{oj} \right) (1 + r) \quad ,$$

where w is the wage rate, which is paid at the start of the production period, and r is the interest rate. If p is the 1 × n vector of prices,

$$p = (pA + wa_o) (1 + r)$$
$$= (1 + r) \, wa_o \, [I - (1 + r) A]^{-1} \quad . \tag{11}$$

Gross output is

$$Y = p.x \tag{12}$$

and the value of consumption is

$$C = p.c \tag{13}$$

(which is the same as net output since there is no net investment in a stationary state).

(13), (10), (9) imply

$$C = px - pAx$$
$$= (1 + r) \, w \, a_o \, x + rpAx$$
$$= wL + r \, (wL + pAx)$$

Consider two stationary countries with factor prices (r_1, w_1), (r_2, w_2), where $r_2 > r_1$. If country 1 produces good j,

$$p_j = \left(\sum_{i=1}^{n} p_i \, a_{ij} + w_1 \, a_{oj} \right) (1 + r_1),$$

and competitiveness implies

$$p_j \leq \left(\sum_{i=1}^{n} p_i \, a_{ij} + w_2 \, a_{oj} \right) (1 + r_2),$$

strict inequality implying that country 2 does not use that technique, and so does not produce any of good j. Similar equations and inequalities hold for goods produced in country 2. Full employment in both countries requires that each have some production process in operation, which is possible only if $w_1 > w_2$.

$$(1 + r_2) \left(\sum_{i=1}^{n} p_i \, a_{ij} + w_2 \, a_{oj} \right)$$

$$- (1 + r_1) \left(\sum_{i=1}^{n} p_i \, a_{ij} + w_1 \, a_{oj} \right)$$

$$= (r_2 - r_1) \sum_{i=1}^{n} p_i \, a_{ij} - \left(w_1 (1 + r_1) - w_2 (1 + r_2) \right) a_{oj} .$$

Therefore, in trade equilibrium, if

$$R = \frac{w_1(1 + r_1) - w_2(1 + r_2)}{r_2 - r_1}$$

all processes with

$$\sum_{i=1}^{n} p_i \, a_{ij}/a_{oj} > R \qquad (14)$$

have

$$(1 + r_2) \left(\sum_{i=1}^{n} p_i \, a_{ij} + w_2 \, a_{oj} \right)$$

$$> (1 + r_1) \left(\sum_{i=1}^{n} p_i \, a_{ij} + w_1 \, a_{oj} \right)$$

and may be used only in country 1. Similarly, when the inequality is reversed, the process may be used only in country 2. A process may be used in both countries only if $\sum_{i=1}^{n} p_i \, a_{ij}/a_{oj} = R$.

The rationale behind this should be clear. The ratio to be compared with R is the ratio of the value of material inputs to the amount of labour input. Since the only capital in the model is circulating capital, processes for which this ratio is high are relatively capital intensive and are therefore used in country 1 which has a low interest rate. (Note that I have made no assumptions about the unique determination of the price vector or of R by r_1 and r_2. All of my argument is intended to apply to any long-run equilibrium, not only unique equilibria.)

Equations (9-13) describe the world consisting of two countries, with $L_1 + L_2$ replacing L in (9). If, however, country 1 uses only techniques satisfying (14), where for given p,R, w_1, and w_2 are fixed by the requirement that the two labour forces be fully employed, then for all techniques used in economy 1,

$$\sum_{i=1}^{n} p_i \, a_{ij} \, x_j \geq R \, a_o \, x_j$$

and, in economy 2,

$$\sum_{i=1}^{n} p_i \, a_{ij} \, x_j \leq R \, a_o \, x_j \quad .$$

Summing over all activities in each economy, and denoting the respective activity vectors by x_1, x_2 :

$$pA \, x_1 \geq R \, a_o \, x_1 = RL_1 \tag{15}$$

$$pA \, x_2 \leq R \, a_o \, x_2 = RL_2 \, , \tag{16}$$

where one of the inequalities is strict, given the weak assumption that not all techniques are equally capital intensive at the ruling prices.

Suppose now that only one good, say the good indexed 1, is a consumption good, and the rest are pure intermediate goods. Let this good be the numéraire, so $p_1 = 1$. Then (13) shows that C is the first element

of the c vector. Since there is no consumer choice, (9), (10) fix the same c_1/L, x/L in both countries in autarchy, and in the world as a whole in trade. Indicating values of variables in autarchy by primes :

$$\frac{C_1'}{L_1} = \frac{C_2'}{L_2} = \frac{C_1 + C_2}{L_1 + L_2} = w_1' + r_1 (w_1' + p_1' Az)$$
$$= w_2' + r_2 (w_2' + p_2' Az) \qquad (17)$$

where $z = x_1'/L_1 = x_2'/L_2 = (x_1 + x_2)/(L_1 + L_2)$.

Now, in trade,

$$C_1 = p.x_1 - pAx_1,$$
$$= w_1 L_1 + r_1 (w_1 L_1 + pAx_1)$$

so $$C_1/L_1 = w_1 + r_1 (w_1 + pAz_1) , \qquad (18)$$

and similarly,

$$C_2/L_2 = w_2 + r_2 (w_2 + pAz_2) , \qquad (19)$$

where $z_i = x_i/L_i$.

Therefore,

$$C_1/L_1 - C_2/L_2 = w_1 (1 + r_1) - w_2 (1 + r_2) + r_1 pA z_1 - r_2 pA z_2$$
$$= R (r_2 - r_1) + r_1 pA z_1 - r_2 pA z_2 .$$

But (15), (16) imply that $pAz_1 \gtrless R \gtrless pAz_2$ with one strict inequality, so if $r_2 > r_1 > 0$.

$$C_1/L_1 - C_2/L_2 > R (r_2 - r_1) + r_1 R - r_2 R = 0$$

$$\therefore \quad \frac{C_1}{L_1} > \frac{C_1 + C_2}{L_1 + L_2} > \frac{C_2}{L_2}$$

$$\therefore \quad \frac{C_1}{L_1} > \frac{C_1'}{L_1} \text{ and } \frac{C_2}{L_2} < \frac{C_2'}{L_2} .$$

The inequalities are reversed if $r_1 < r_2 < 0$. Nothing may be said if $r_1 < 0 < r_2$.

Thus we have behaviour analogous to that observed in the Stiglitz model and in the clay-clay model of Chapter 3. Here we have a definite result that $C_2 < C_2'$ if $r_2 > r_1 > 0$, because there are no 'static' gains from trade, since the model lacks any substitutability, but otherwise the behaviour of consumption is identical.

However, just as when we relax the rigidity of the clay-clay model to allow ex ante substitutability the results become less sharply determined, so in this case once we introduce consumer choice or choice of techniques in production we are able to obtain definite results only in a limiting case.

For if we introduce more than one consumer good, we introduce the possibility that $C_1'/L_1 \neq C_2'/L_2 \neq (C_1 + C_2)/(L_1 + L_2)$ because, in general, the price vectors p_1', p_2' and p are different, and even with identical tastes this typically would lead to different choices in different situations.

Similarly, once we allow choice of technique, we should expect a_o, A to be different in each country in autarchy, and different again in the world in trade equilibrium. If a_o, A are chosen from a discrete spectrum of techniques, consumption need not be a well-behaved function of the interest rate even if there is only one consumption good. (See section V.B. of Bruno et al. (1966)[1].) Therefore, so far as analysing the effects of trade on consumption in this model is concerned, we seem to be in the same situation as with the putty-clay model (section 4.4 above).

In neither case does there appear to be any possibility of deriving

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[1] Bruno, M., E. Burmeister, and E. Sheshinski, "The Nature and Implications of the Reswitching of Techniques", Quarterly Journal of Economics 80, 4 (November 1966), pp. 526-553.

globally applicable results relating autarchy steady states to free trade steady states. The shift from autarchy to trade will normally involve changes in technique whose effects on consumption will be unpredictable. The most we can hope for is the sort of local result obtained in the putty-clay model and in the Stiglitz model with factor intensity reversals : a result for the limiting case where $r_1 = r_2 = r$ but each country specialises, obtained as $r_2 - r_1 \to 0$. (Compare pp.127 and 145.)

Even in this case it is desirable to make some restrictive assumptions. To avoid introducing extraneous considerations, I assume tastes are identical in both countries, i.e. faced with the same price vector, each country chooses the same consumption vector. Further I confine discussion to models in which there is only, as above, circulating capital. In such models we know that the dynamic nonsubstitution theorem holds (see Stiglitz (1970b),[1] Theorem 2), so that p is uniquely determined by r.

Suppose then that $r_1 = r_2 = r$ and $w_1 = w_2 = w$, and at these factor prices, the chosen set of techniques is unique and is a_o, A. Further, since it is the limiting case where there is specialisation that we are considering, there will exist some R such that, in trade equilibrium, (15) and (16) hold. Then (17-19) above hold, since at the unique p, the same consumption bundle is chosen in all cases. But because of factor price equalisation, (17) now is

$$C_1'/L_1 = C_2'/L_2 = (C_1 + C_2)/(L_1 + L_2)$$

$$= w + r\,(w + pAx)\ ,$$

and (18) and (19) are

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[1] Stiglitz, J.E., "Non-Substitution Theorems with Durable Capital Goods", Review of Economic Studies 37, 4 (October 1970), pp. 543-553.

$$C_1/L_1 = w + r\ (w + pAz_1)$$

$$C_2/L_2 = w + r\ (w + pAz_2)\ .$$

As before, $pAz_1 > pAz_2$, so if $r > 0$

$$\frac{C_1}{L_1} > \frac{C_1'}{L_1} = \frac{C_1 + C_2}{L_1 + L_2} = \frac{C_2'}{L_2} > \frac{C_2}{L_2}\ ,$$

and if $r < 0$ the inequalities are reversed.

Therefore again we have the result that if two countries enter trade, and aim towards a steady state with the same profit rate as the autarchy steady state, the one with the higher profit rate will tend to have reduced steady state consumption while the other will tend to have increased steady state consumption, if their target profit rates are sufficiently close, where the precise meaning of 'sufficiently close' is not clearly defined.

Only the most modest claims may be made for the analysis of this section. We can see the same process at work as was observed in the two sector model and the vintage models : the country with the higher interest rate chooses, in trade, to specialise in less capital intensive techniques which tends to reduce steady state consumption, while the converse process happens in the other country. But once we move away from models in which the area of choice is limited there appears to be a high probability that this phenomenon is accompanied and obscured by other phenomena.

Further, although the phenomenon has been observed in different types of models, and may therefore plausibly be supposed to be a feature of many other models also, it cannot be the case that it occurs in all models. The reason for this is simply that in a model in which the non-substitution theorem does not apply, not very much can be said about the pattern of trade, since the relation between product prices and factor

prices is not, in general, well defined. It does, however, seem a reasonable conjecture that in any model in which product prices are uniquely determined by factor prices, the pattern of intertemporal substitution observed in the models discussed above will again emerge.

5.5 Other aspects of trade, capital, and growth

One of the aims of this chapter is to demonstrate the existence of important common features between the vintage models of trade with which I have been concerned in earlier chapters and other models of trade in capital goods. It is equally important to identify the differences between the models discussed here and other models involving trade and capital.

First, the vintage models discussed in Chapters 1 to 4 should be distinguished from the models of Bardhan (1966, 1970)[1,2] and Petith (1972)[3]. Bardhan's model is a two-sector model in which the production functions are clay-clay. There is no trade in used machines. The model turns out to resemble very closely the two-sector model with differentiable production functions. Petith's model has no trade at all in capital goods. The vintage technology is used purely as an illustration of the effects of joint production.

Further, my concern throughout has been with the effects of trade on accumulation. There is a considerable literature on the effects of growth on trade. (See Hanson (1970)[4] for a survey of this literature,

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[1] Bardhan, P.K., "International Trade Theory in a Vintage Capital Model", Econometrica 34, 4 (October 1966), pp. 756-767.

[2] Bardhan, P.K., "Embodied Technical Progress, Economic Life of Machines and Comparative Advantage", Chapter 5 of Economic Growth, Development, and Foreign Trade (John Wiley; New York, 1970).

[3] Petith, H., "Vintage Capital, Joint Production, and the Theory of International Trade", International Economic Review 13, 1 (February 1972), pp. 148-159.

[4] Hanson, J.A., Growth in Open Economies Lecture Notes in Operations Research and Mathematical Systems, 59 (Springer-Verlag; Berlin, 1970).

and an attempt to consider both sets of effects together.) In contrast, I have considered models with, usually, only one consumption good, and therefore have paid no attention to such important questions as the likely effects of expansion on the relative prices of, say, agricultural and manufactured products.

Nor have I considered the effects of flows of investment. It has been assumed throughout that no foreign investment takes place (and that there are no international labour movements either).

Finally, it should be noted that throughout I have assumed that when two countries trade they have the same growth rates. This is an undesirable assumption for obvious reasons. There is some recent work (Kemp (1970)[1], Khang (1971)[2]) showing that in the two-sector model unequal growth rates do not affect the stability of convergence to steady state, although in the steady state the slower growing country is insignificantly small compared with the faster growing country. Clearly close to steady state the properties of such a model would be virtually identical to the properties of a model of two countries of unequal size but equal growth rates. The use of steady states as models of a world which is never actually in steady state requires something of an act of faith, and it is not clear that the assumption of equal growth rates greatly adds to the burden of unrealism already borne.

5.6 Conclusions

In correcting and extending Stiglitz's analysis, I have shown the very close analogy between the two-sector model and the vintage models

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[1] Kemp, M.C., "International Trade between Countries with Different Natural Rates of Growth", Economic Record 40, 116 (December 1970), pp. 467-481.

[2] Khang, C., "Equilibrium Growth in the International Economy : the Case of Unequal Natural Rates of Growth", International Economic Review 12, 2 (June 1971), pp. 239-249.

of earlier chapters so far as the long run effects of trade are concerned. A similar analogy exists with trade in general linear technologies. It seems therefore likely that in any model of capitalist production we should expect to observe a pattern of specialisation in which the country with the higher interest rate chooses the least capital-intensive techniques and tends thereby to reduce its long-run consumption.

By contrast, there are some differences between the two-sector model and the 'clay-clay' vintage model if the assumption of fixed saving rates is adopted; for example, whether $r > g$ seems to be of significance in the two-sector model and of no significance in the 'clay-clay' model. When results are model-specific they are of doubtful value unless, incredibly, one has discovered the definitive description of the real world. It seems therefore that the fixed-interest-rates model adds to our understanding of the real world in a way in which the fixed-saving-rates model does not, even if the assumption of fixed interest rates is not itself wholly plausible.

CHAPTER 6

Policy Implications

Throughout the previous five chapters there are several places where a reader fastidious about the distinction between positive and normative economics may feel that I have been carelessly interchanging 'ought' and 'is'. Further, it may seem that the balance between theory and empirical observation is weighted rather heavily towards theory.

The two objections are not independent. If producers behave competitively and with perfect foresight, if externalities and increasing returns to scale are absent, and if the input prices they face properly reflect the social value of the alternative uses of the inputs, the competitive equilibrium is an optimum. I can therefore justify confusing 'ought' and 'is' if these conditions are satisfied and if there are no objections to the competitive optimum on distributional grounds.

When there are significant externalities, policy makers will not necessarily wish to allow the market to guide decisions. If, for example, the use of second-hand machines reduces the likelihood of a poor country developing an appropriate indigenous technology at some future time, then the authorities may wish to restrict imports of second-hand machines. If, on the other hand, the imperfection of competitive forces or lack of foresight allows firms to use new machines where second-hand machines are more appropriate, the authorities may wish to intervene to encourage firms to behave as if they were perfect competitors. These are empirical issues on which I have nothing to offer. (It might seem fair to say that it is up to those who object to my analysis to produce evidence of externalities, but I think this would be unfair. The welfare implications of my analysis depend on

the absence of externalities. If externalities are prevalent, my analysis is useless; if not, it is useful. There is no reason to place the onus of producing evidence on one side only.)

One also must recognise that a government may legitimately intervene if it believes that producers value inputs incorrectly. Indeed it is one of the aims of this study to show the consequences of an inappropriate discount rate. (See the conclusion to Chapter 3, p.99 .) Equally, it must be recognised that income redistribution may be a legitimate reason for intervention (although here it is perhaps fair to put the onus of proof on those who advocate trade restriction rather than directly redistributive policies.)

Although theory cannot be a guide to detailed policy in the absence of empirical data, some general policy guidelines do emerge from the theory.

In the first place, we now have an indication of what kind of empirical data is of interest and what is not. I have already pointed out (p.34) that supposed disadvantages of second-hand machines, such as increased operating costs, are only disadvantages in so far as their effects are not fully reflected in the market price. It is comparative advantage which matters, so that only differing requirements of non-traded inputs are relevant. The fact that an old machine may require increasing amounts of semi-skilled attention is unimportant if semi-skilled labour is available. If by contrast, it requires highly skilled maintenance, which may have a very high opportunity cost in a poor country, then this is a relevant objection. Chapter 4 gives a further, and important, example of the distinction between relevant and irrelevant objections. Thus although Todaro's objection

(p.5) on the grounds of hampering the future viability of indigenous technology may be a relevant objection, Chapter 4 shows that, by contrast, if currently available technologies are representable as points on a production function common to all countries, the present existence of labour intensive techniques is not a relevant objection to the use of second-hand machines.

Further, if legitimate grounds are found for interference with trade, the theory still describes policy in a modified form. Replace prices, wages, and profit rates throughout by shadow prices, shadow wages, and discount rates and we have our description of policy. Now it is certainly the case that in modelling underdeveloped countries simply as countries with low wage rates and (sometimes) constrained saving rates, I have ignored many important aspects of the problems of these countries. Nonetheless, it seems scarcely credible that the shadow wage in a poor country, even when all relevant features of the economy are considered, should be as high as the wage rate in a developed country, or that the discount rate should be as low. If this is granted, and if one accepts the picture I have presented of the difference between old and new machines being a difference in labour requirements, then the optimal policy will still be to import used machines. (Recall that the data of Chapter 1 were consistent with this hypothesis of labour intensity changing with age and vintage.)

Finally, theory has value in so far as it helps us to understand the logic, if any, of events. In this case, Chapters 3 and 5 show that the use of second-hand machines by countries with high discount rates is only one manifestation of a phenomenon which appears to be fairly general when trade in capital goods is involved. Apparently

not much attention has been paid in the literature to this aspect of international trade, whereby a country short of capital and of current consumption specialises in those techniques which are least capital intensive, and thus is able to reduce the immediate disparity between its current consumption and the consumption of the rich, at the expense of future inequality. Armed with this explanation of trade (and, therefore, with a logically consistent account of why trade may sometimes be undesirable), we are able to see why the idea that trade may be 'saddling countries undergoing industrialization with an obsolete technology' is, on the whole, an unsatisfactory idea. (We also, incidentally, see that when one examines the effects of trade in the long run, the choice of one's assumption about saving behaviour may be of some significance.)

In addition, the many qualifications which properly must accompany the conclusions of the theoretical analysis should not be allowed to obscure the existence of some definite conclusions. There <u>is</u> a strong presumption that underdeveloped countries in many circumstances will, and ought to, import second-hand machines. Chapter 1 suggests that all machines of the same type and vintage should at a given time be used in only one country: observations of contrary behaviour require explanation. This does not exclude the possibility of different countries using different types of machines rather than different vintages. Chapter 4 shows, however, that this objection does not carry as much weight as one might at first expect, but in multi-sector vintage models one probably should expect specialisation according to sector as well as according to vintage.

It is worth emphasising that it should not be understood that I am advocating indiscriminate encouragement of trade in second-hand machines. (If forced to guess whether an optimising world subject to factor price differentials would have more or less trade in used machines than the present world, I should guess more, but that is beside the point.) Indeed, it would be positively harmful to give the impression that economic theory in some sense gives blanket endorsement to the use of second-hand machines by developing countries. A planner in such a country may have bitter experience of instances of unsuccessful investment in used machines which turned out, for example, to require frequent, expensive, highly skilled maintenance. For such a person to get the impression that economists advocate a policy which he knows to be inadvisable is to discredit economics and economists in his eyes. Rather it is the job of theory to delimit the extent of our knowledge, to add to our understanding of economic processes, and to indicate what kind of empirical information is required in order to make further progress. These have been my aims in this work.

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Appendix to Chapter 2

One of the examiners of this thesis, Christopher Bliss, has suggested that the definitions of saving rates adopted on p.57 are not obviously the most appropriate definitions. Compare the following two economies: (1) the isolated economy which scraps machines at age m_1, which has output

$$Y_1 = \frac{I}{g}(1 - e^{-gm_1})$$

where $$L_1 = \frac{I}{g - \lambda}(1 - e^{-(g-\lambda)m_1}) ,$$

and whose saving rate is therefore

$$s = \frac{g}{1 - e^{-gm_1}} ;$$

(2) the trading economy with the same L_1 which sells machines at age m_1 at price $p > 0$, which therefore has the same Y_1 and I. Clearly the second economy is better off, for it has an additional source of income, the sale of machines, worth $pe^{-gm_1} I$.

The definition of saving rate that I have adopted involves counting this extra income as a reduction in gross investment so that

$$s_1 = \frac{I(1 - pe^{-gm_1})}{Y_1} .$$

It might seem that a more appropriate procedure would be to treat the exported machines as a joint product, let income be

$$Y_1 + pe^{-gm_1} I ,$$

and define

$$s_1 = \frac{I}{Y + pe^{-gm_1 I}}$$

There are several ways of looking at the differences between these two definitions.

1. Section 2.2 shows that, given competitive pricing,

$$Y_1 = W_1 + r_1 K_1 + D_1$$

so output equals the sum of factor incomes and depreciation. Therefore,

$$Y_1 + pe^{-gm_1 I} = W_1 + rK_1 + pe^{-gm_1 I} + D_1 ,$$

which shows that the second definition of income implies that the revenue from the sales of machines should be treated as a factor income, accruing to the owners of machines.

2. In the model described by equations (16-26), total world production is $Y_1 + Y_2$. If the second definition of income is adopted then either the sum of the two countries' incomes exceeds $Y_1 + Y_2$ by $pe^{-gm_1 I}$, or the income of country 2 is to be calculated as $Y_2 - pe^{-gm_1 I} = C_2$. This implies that country 2 does no saving.

3. It is conventional in national income accounting (see, for example, D.C. Rowan, Output, Inflation and Growth (Macmillan; London, 1968), p.63) to take account of the value of the physical change in stocks. The approach I have adopted could be interpreted (see equation (25) for example) as treating the sale of machines as a reduction in stocks rather than a subtraction from gross investment.

The alternative approach involves taking no account of the sale of machines in calculating gross national expenditure.

Of course, no rules may be made about choice of definitions: one can choose whichever set of definitions seems most appropriate, so long as it is internally consistent. I have chosen the set which seemed to me to be the more appealing.

The most important point to make is that this discussion further demonstrates the essentially arbitrary nature of the assumption of fixed saving rates.