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City Size, Land Use and Transportation Costs

by

Eliyahu Borukhov

A dissertation submitted to The Johns Hopkins University in conformity with the requirements for the degree of Doctor of Philosophy.

Baltimore, Maryland

1972

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Abstract

One of the major advances in the understanding of the internal structure of cities and metropolitan areas in the last decade, was the publication of several studies which established a family of urban models in the tradition of Von Thünen.

The pioneers of this approach were Lowdon Wingo, (1961) and William Alonso (1964). The main hypothesis, which lies at the center of these models, is that the degree of accessibility from every location to the center of the city is a major determinant of the internal spatial structure of a city; and that the degree of accessibility could be measured by the distance from the center. The theory was refined in the hands of Mills (1967 and 1969) and Muth (1969). They have also tested the implications of their models against data on land values and density of land use. Their tests tend, by and large, to confirm the theory as an approximation to reality.

The purpose of my work is to continue the theoretical analysis of these writers. One purpose of the extension is to look for more testable implications of the theory. For that purpose I have tried several versions and modifications of the basic models that were suggested by those writers.

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In Chapter II, I present one such model. I interpret that model as a model of a city where time (and other variable costs) are only costs of transportation. For that model I calculated total transportation costs as a function of city size. The results of these calculations are then tested against data on the amount of time that people spend on travel in 34 cities (of various sizes). The results tend - in my judgement - to add credibility to the theory which is represented in this work.

Chapter III is devoted to an effort to derive from another version of the model implications about the proportion of land allocated to transportation. Two propositions are derived: One is that the proportion of land devoted to transportation should decline with distance from the center to the edge of the city. This is confirmed by data on the proportion of land used for streets and roads in a number of cities. The second proposition is that the proportion of land devoted to transportation should increase with city size. The empirical data give weak evidence in favor of this proposition.

Chapter IV and V are devoted exclusively to theoretical analysis. Chapter IV surveys the theory of

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local public goods and extends it with the help of the models that were developed in previous chapters. The literature on local public goods seems to have ignored the role that land rents can play in the efficient use of space. Attention is called to this aspect of the problem. The results of this chapter can also be interpreted as a theoretic definition of the concept of an optimal city size.

Chapter V is devoted to an analysis of the effects of the property tax. Again these effects are analysed within the context of a model that was developed in previous chapters. The conclusions of this chapter confirm some of the conclusions of the literature on the property tax (such as Netzer (1966)). But in addition the spatial effects of the property tax (on land values and density in different parts of cities) are derived. It is shown that even with uniform rates, the property tax tends to encourage the intensive use of land in the suburbs and to raise its price, at the expense of land values near the center of a city.

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Acknowledgement

I would like to express my gratitude to professors Edwin S. Mills and William H. Oakland. Without their patient supervision, and continuous encouragement throughout the writing of this work, this study would not have been completed.

My wife patiently forebore my preocupation with this study, and encouraged me to pursue it over a long period of time.

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Chapter I

Introduction

Alvin H. Hansen stated in 1960, in assessing the economic issues and problems of the 1960's and 1970's: "Inflation I think is not the most important of these. We may experience an upward drift in prices in these two decades, but it is not likely to become a serious problem. Nor is the highly important problem of adequate aggregate demand likely to be the most important problem.

What then is the most important economic problem which will confront the United States in the next 20 years? It is, I believe, the problem created by the sweeping increase in urbanization."[1)]

From the point of view of 1972 it seems that Hansen exaggerated in playing down the importance of inflation and unemployment as economic problems. However he certainly did not exaggerate in pointing at the growing urgency of the economic problems in the cities. As a matter of fact the persistence of inflation and unemployment adds

1) Alvin H. Hansen, "Economic Issues of the 1960's" McGraw Hill 1960, pp 181-182 as quoted by Meyer, Kain and Wohl "The Urban Transportation Problem", Harvard University Press, 1965 p.1.

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complexity to some urban problems, or at least, frustrates the efforts to solve them.

This work is based on the fundamental belief that better understanding of the basic economic structure of cities will become helpful in finding solutions to some urban problems. No such solutions are suggested in this work, though I hope that the reader will find that the discussion, in the latter sections of this work, is relevant to some practical problems.

One of the major advances in the understanding of the internal structure of cities and metropolitan areas in the last decade, was the publication of several studies which established a family of urban models in the tradition of Von Thünen[2]. The pioneers of this approach were Lowdon Wingo and William Alonso..

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2) 1. William Alonso "Location and Land Use" Harvard University Press 1964.
2. Martin Beckmann "On the Distribution of Urban Rent and Residential Density" Journal of Economic Theory. June 1969.
3. Emilio Casetti "Equilibrium Land Values and Population Density in Urban Setting" Economic Geography, (Vol 47 No. 1), January 1971 pp 16-20.
4. E. Casetti and G. Papageorgiou "A Spatial Equilibrium Model of Urban Structure" The Canadian Geographer (Vol 15 No.1), Spring 1971, p. 30-37.
5. O. Hochman and D. Pines "Competitive Equilibrium of Transportation and Housing in the Residential Ring of an Urban Area" Environment and Planning(Vol 2 No.1) 1971 pp 51-61.

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Writing, as late as 1959 Roland Artle made the following comment on the state of knowledge in this area: "A major limitation of the whole analysis and description (of the economy of Stockholm) so far is that all the findings derived or derivable relate to

2.(Continued)

6. John F. Kain "The Journey to Work as a Determinant of Residential Location" Papers of the Regional Science Association , 1962 p 137-160.
7. Lester Lave "Congestion and Urban Location", Papers of the Regional Science Association, 1970, pp 133-150. This is anabbreviated version of: Lave Lester "Transportation, City Size and Congestion Tolls" , Rand Memorandum, April 1969.
8. Edwin S. Mills,"An Aggregative Model of Resource Allocation in a Metropolitan Area", American Economic Review, May 1967, pp. 197-210.
9. Edwin S. Mills, "The Value of Urban Land" in Perloff H. (editor) The Quality of the Urban Environment, Johns Hopkins University Press for Resources for the Future, Baltimore 1969, pp. 231-253.
10. Edwin S. Mills, "The Efficiency of Spatial Competition", The Regional Science Association Papers, Vol.XXV, 1970, pp. 71-82.
11. Edwin S. Mills,and de Ferranti,David M., "Market Choice and Optimum City Size", American Economic Review, May 1971, pp. 340-345.
12. Richard F. Muth, "Cities and Housing", Chicago, University of Chicago Press 1969.
13.David Pines "The Effects of Variations in some Determinants of a Mono-Center Urban Form under Competitive Equilibrium", Tel-Aviv University, Center for Urban and Regional Studies, Tel-Aviv 1971.
14. Lowdon Wingo, "Transportation and Urban Land", Resources for the Future, Washington D.C. 1961.

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the Stockholm area as a whole. Therefore, a worthwhile extension of the work seems to be to explore whether such findings can be linked-up with findings that relate to smaller zones, such as the central business district or a part thereof within Stockholm. The ultimate goal of this work will be to establish invariant or predictable locational relationships or patterns of urban land use, such that analysis and descriptions of the present type can be pushed one step further: to comprehend also the spatial arrangement of economic activities within the commumity under study"[3].

Since that time a new body of theory has been developed with the explicit purpose of explaining "intra-metropolitan" structure, in terms of distance from city center[4].

---

3) Roland Artle "Studies in the Structure of the Stockholm Economy" Stockholm. 1959 p 113.

4) Using the words of Lowdon Wingo: "The term intra-metropolitan model is used to describe a genus of mathematical descriptions specifiying at some level, the manner in which one or more sectors of the urban economy are spatially distributed within the boundaries of the urban region". Lowdon Wingo, "An Economic Model of the Utilization of Urban Land for Residential Purposes", Papers of the Regional Science Association, 1961. p.191.

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The major breakthrough was the hypothesis that both the value of urban land and the intensity of its use are determined by the degree of accessibility at every location, and that the degree of accessibility could be measured by the distance from the center of the city.

To use again the words of Lowdon Wingo: "In most empirical studies distance from the center has been measured "as the crow flies". At a finer grain, however, urban phenomena related to the inter-accessibility of the various parts of the region may be obscured by this too simple spatial framework. A reasonable concern for the context of the locational decision requires more a sense of accessibility than simple spatial extension that expresses the shortest distance between two points as measured along feasible transportation routes"[5].

A recent empirical study[6] of house prices, concluded that : "nearness, in time sense, to the central area is considered to be an important determinant of house price, because of the fact that the whole employment situation in the London area is dominated by the central area.

---

5) Wingo, Ibid, p.194.

6) J. Stuart Wabe "A study of House Prices as a Means of Establishing the Value of Journey Time, The Rate of Time Preference and the Valuation of Some Aspects of the Enviornment in the London Metropolitan Region", Applied Economics, December 1971, pp. 247-255.

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Some six million jobs are located within the metropolitan region and, of these, approximately twenty per cent are located within the geographically small area of central London. Thus an overriding aspect of the demand for houses will be the desire of central area workers to have a convenient location, as expressed in terms of journey time to the center. One minute decrease in the journey time to the center is reflected in house prices to be worth £ 20.38 . Even if a person does not work in the center, he still has to pay a premium for the benefit of being closer, in time sense, to the central area. Thus his value on nearness to the center is at least equal to that of a person who works in the central area".

This basic hypothesis was consciously inspired by the Von Thünen theory of concentric rings of agricultural location[7]. In Von Thünen's theory the value of land, and its use, are determined by its distance from the city, which is a point at the center of the plain, and which is the only potential market for agricultural output - rent being savings in transportation costs.

Von Thünen's model consists of a single city at the center of a uniform plain. The city is the only potential market for the agricultural output produced

7) See: Wingo,L. "Transportation and Urban Land", 1961 p. 24. Alonso, Ibid, pp 3,5, and 37.

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on this plain. Distance from this city (which is a point at the center of the endless plain) determines in his theory both the value of land and its use. He showed that in these circumstances products such as milk and fruits for which transport costs are highest would be produced nearer to the city and that other products would be produced in concentric rings in order of decreasing transport costs. Von Thünen also concluded that the price of a product in the city must be high enough to cover the costs of the most distant producer, including his transport costs, and that less distant land would earn a rent attributable to its location, equal to the sum of production and transport costs to the city at the most distant producing location minus the sum of these costs at the less distant location[8].

Similarly a city can also be thought of as consisting of several rings centered around some central point. The center could be thought of as a central business district in which employment activities are carried, or as a point to where all the output (or the export output)

---

8) This summary of Von Thünen's theory is drawn from Muth (1969) Op.Cit. p.6

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of the city had to be shipped to.[9].

The second ring around the center is a residential ring. The whole city is surrounded by agricultural areas.

The focus of the analysis of my work as well as that of most work in this area is on the residential ring.

The determination of the locations and width of these rings is discussed by Muth[10] and Mills[11]. Muth examined the urban rural competition for land. Mills examined the central business district demand for land and its competition with the residential ring.

An interesting example of the application of a Von Thünen model to urban structure is Lester Lave's[12].

---

9) Both Alonso and Wingo make extremely simplifying assumptions on this point: When Alonso considers the equilibrium of the individual, he states simply that: "The city in which the individual arrives is a simplified city, it lies on a featureless plain, and transportation is possible in all directions. All employment and all goods and services are available only at the center of the city".(Location and Land Use" p.18). When he considers the equilibrium of firms he mentions at least four factors that influence the relative desir ability of different locations in the city for firms. Then he goes on to state that he will consider only one factor - distance from the center. "In our completely centralized city this implies.. accessibility of the site to potential customers will decrease with distance from the center". The other factors which relate to the interdependence of business locations "are too complex for analysis here" (Location and Land Use" p.44).
Wingo states: "It is generally characteristic of larger cities that the preponderance of employment does take place in or adjacent to the central business district", (Wingo, "Transportation and Urban Land" 1961,p.102).

10) Richard Muth "Economic Change and Rural Urban Land Conversion" Econometrica, January 1961, pp 1-23

11) Mills(1967),Op.Cit. 12) Lester Lave,Op.Cit.

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By distinguishing between different modes of transportation, and different social classes (which differ by their transportation costs) he was able to generate in his model several rings within the residential area. As in Von Thünen's theory, land rents - which are determined by transportation costs - serve a crucial role in allocating land in every ring to a particular group, and to exclude others from using it.

The basic hypothesis - that distance from the center of the city is a "bad" (to be understood as a disutility or as a cost), and common economic arguments about utility maximizing consumers, are sufficient for the construction of a theory that will explain the intensity of land use in the "residential" areas of a city and the determination of the value of land in that area. This was essentially done by Alonso. Alonso's theory can be presented in brief in the following way[13]:

Consider a city inhabited by N identical individuals. Let each of them have the following utility function:

$$W = W(z,q,u) \qquad (1)$$

13) This interpretation, or formulation of Alonso's theory draws heavily on Emilio Casetti (1971), Op.Cit., D. Pines (1971), Op.Cit. and Richard Muth (1969), Op.Cit. Some pitfalls in Alonso's analysis were pointed out and corrected by Casetti and Pines. Works cited in note No. 2.

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Where: u is the distance from the center of the city,

q is the quantity of residential land that they occupy,

z is the quantity of all other composite consumption goods.

Let us also assume that W is at least twice differentiable, and that the level of W increases (at a decreasing rate) when z increases, that the level of W increases (at a decreasing rate) when q increases, but that the level of W decreases when u increases[14]. Assume also that every individual makes some fixed number of trips to the center of the city at a constant cost of t per mile per year. Each individual is confronted by the following budget constraint: The sum of expenditures on consumption, rent and transportation costs should add up to income:

---

14) Strictly speaking these are overly strong assumptions It is not necessary to assume that the second order derivatives of the function W are negative (which means that utility is measurable). It is sufficient to assume that the indifference curves in the commodities plane are convex from below. (See the following footnote). However I feel that this is a minor relaxation of the assumptions. The assumption that all the inhabitants of the city have the same utility function W seems to me to be much more serious.

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$$y = pz + R(u)q + tu \qquad (2)$$

Where: R(u) is rent of a unit of land u miles from the center per year, p is the price of z , and t is transportation cost per mile.

The individuals are assumed to maximize (1) subject to (2). The necessary conditions for that maximum are equilibrium conditions for the individuals.

For any given values of y,p,R(u) and u, say y,p*,R*(u) and u*, these necessary conditions are:

$$\frac{W_q}{W_z} = \frac{R^*(u)}{p^*} \qquad (3)$$

and:

$$y = p^*z + R^*(u)q + tu^* \qquad (4)$$

Where W_q and W_z are partial derivatives of the function (1) with respect to these arguments. These two conditions imply a unique combination of z and q that will yield a maximum attainable level of utility[15].

---

15) If the sufficient second order conditions hold, which are that at the neighbourhood of the optimum the following inequality holds:

$$W_{qq}W_z^2 - 2W_{qz}W_qW_z + W_{zz}W_q^2 < 0$$

(This simply means that the indifference curve in the q and z plane is convex from below), and that the solution is not a corner solution. (See Casetti p.17). We will assume that these conditions are satisfied, and will ignore the possibility that a solution does not exist, or that it is a corner solution.

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Let us denote these optimal values of z and q as z* and q*. These values of z and q are themselves functions of all the constants appearing in the utility function and the budget constraint and of R*(u) and u*.

Let us write these solutions:

$$z^* = f(R^*(u), u^*, p^*, t, y) \tag{5}$$

$$q^* = g(R^*(u), u^*, p^*, t, y) \tag{6}$$

The maximum level of utility which is attainable (for given R(u) and u) can be written:

$$W^* = W(z^*, q^*, u^*) = W(f, g, u^*) \tag{7}$$

If we assume that W* is a constant and that R(u) is a variable instead, this last equation becomes another implicit function that defines, for a given value of W (and all the constants appearing in the utility function and the budget constraint) and of u*, a maximum rent that the individual will pay for a unit of land (at distance u*) and still maintain that level of utility denoted as W*.

$$R(u^*) = h(W^*, u^*, p^*, t, y) \tag{8}$$

This function is what Alonso called a "bid price function".

"A bid price curve of a resident is the set of prices for land the individual could pay at various distances while deriving a constant level of satisfaction; that is to say, if price of land were to vary with

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distance in the manner described by the bid price curve, the individual would be indifferent among locations"[16].

Let us further assume that in equilibrium (market equilibrium) no individual can increase his utility by relocating within the city. This means that the maximum level of utility attainable by each individual throughout the city is identical (remember that our city is inhabited by identical individuals). If the uniform maximum level of utility which is attainable everywhere in our city is some arbitrary value: $\bar{W}$, then substituting this value for W^* in (8) will define a market equilibrium rent function for our city.

$$R(u) = h(u,p^*,t,y,\bar{W}) \qquad (9)$$

Substituting this rent function into (6) will give us the quantity of residential land that will be occupied by an individual living u miles from the center - which is the reciprocal of the "net residential density"[17]

This is a pretty general theory and it does not predict much about the specific form of the rent or the density function. Some more specific predictions about these functions can be deduced from assumptions about the signs of the derivatives of the typical utility

16) Alonso, Op.Cit. , p.59.

17) A major shortcoming of this theory is that it does not explain how $\bar{W}$ is determined!

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function W. These avenues were investigated by Casetti and Pines. This crude theory was refined and modified in the hands of Mills and Muth who sought to derive from this theory more specific (and testable) predictions about the structure of cities.

More specific (and testable) predictions can be deduced if specific functional forms are assumed.

Beckmann (1969)[18], for instance suggested that the utility function (1) is of the Cobb-Douglas form. He assumed also a Pareto distribution of incomes in the city, and derived a specific functional form for the density, and land values functions.

Mills (1967 and 1969)[19] also used Cobb-Douglas function. However instead of assuming that all people in the city have the same utility levels, he assumed that they all consume the same quantity of housing, which could be produced by various proportions of capital and land according to a Cobb-Douglas production function for housing.

It should be noted that if we interpret "housing" as: "The bundle of services yielded both by structures and also by the land or sites on which they are built"[21],

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18) Op. Cit.
19) Op. Cit.
20) Muth (1969) Op.Cit. p.18

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then the substitution of capital for land is done partly by the producers of houses, and partly by the individuals who are willing to live in more crowded areas.

Mills's formulation has the merit that the utility maximization problem is converted into a cost minimization problem which may seem more natural, perhaps, in the eyes of people who shy at the notion of maximizing total utility ("social welfare"). Minimizing total production costs of a city is a much more agreable notion.

Both Mills and Muth have directed great efforts toward developing models that will yield a theoretical explanation of the negative exponential rent and density functions. The empirical fact that both rents and densities decline exponentialy with distance from city centers, was publicized by Colin Clark as early as 1951[22]. Theoretic models that explained this phenomenon were published by Muth (1961[23] and 1969[24]) and by Mills (1967[25] and 1969[26]).

The purpose of this work is to extend the theoretical analysis of these writers. One purpose of the extension

22) Colin Clark "Urban Population Densities" Journal of the Royal Statistical Society, Series A, 1951 pp 490-496.
Colin Clark "Population Growth and Land Use" McMillan 1967
23) Richard Muth "The spatial Structure of the Housing Market" Papers of the Regional Science Association 1961 pp 207 220.
24) Op. Cit.
25) Op. Cit.
26) Op. Cit.

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is to look for more testable implications of the theory. For that purpose I have tried several versions and modifications of the basic models that were suggested by those writers.

In chapter II, I present one such model of a city with constant marginal costs of transportation, which could be interpreted as a model of a city with constant speed transportation system.

The model is solved in two alternative ways: First it is solved as a "Technocratic" efficiency problem. An optimal land use pattern is the intensity pattern which minimizes total costs of the city (construction + transportation). Based on these assumptions a cost minimization problem is set and solved, as a problem in the calculus of variations.
Second it is solved as a "market simulation" problem. The implications of assuming that perfect competition prevails in the factor and output markets are worked out.

It turns out that the same density pattern (intensity of land use) is derived in the two cases, and thus the efficiency of competition - in this model - is established.

Once the density function for the model is found, it

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is possible to calculate how total costs will vary with changes in total population. It is found that total transportation costs T are a function of population size N of the form: $T = A N^{\delta}$

This function turns out to fit reasonably well (R^2=0.75) data on time spent on travel in 34 cities which vary in size from 30 thousand to 6.5 millions.

Chapter III is devoted to an effort to derive from another version of the model implications about the proportion of land devoted to transportation. This is a fixed coefficients model. The main feature of this model is that it allows explicitly for the use of land in transportation. Specifically it assumes that some fixed amount of land is required in order to transport a person a given distance. Two propositions are derived: One is that the proportion of land devoted to streets and roads should decline with distance from the center to the edge of the city. This is confirmed by data on the proportion of land that is used for streets and roads in a number of American cities.

The second proposition is that the proportion of land devoted to streets and roads increases with city size. The empirical data give weak evidence in favor of this proposition. These propositions are derived under the rigid assumptions of fixed coefficients in

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both the housing and transportation industries. Substitution of other factors for land in transportation should be able to keep down the proportion of land allocated to transportation. However it cannot prevent the inevitable rise of transportation costs, when the size of a city grows.

The last section of this chapter is devoted to theoretic discussion of the implications of imperfect pricing of land used for transportation. It is argued that - under certain conditions - lack of pricing will cause cities to become more spread out than otherwise.

Chapter IV and V are devoted exclusively to theoretical analysis. Chapter IV surveys the theory of local public goods and extends it with the help of the models that were developed in previous chapters. The literature on local public goods seems to have ignored the role that land site rents can play in the efficient use of space, and that locational preferences are reflected in rents. Attention is called to this aspect of the problem. A model which allows for changes in density (intensity of land use) is presented, and the optimal size of a service area (or a city) is determined within the context of such a model. The results of this chapter can also be interpreted as a theoretic

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definition of the concept of an optimal city size.

Chapter V is devoted to an anlysis of the effects of the property tax. Again these effects are analysed within the context of a model that was developed in previous chapters. The conclusions of this chapter confirm some of the conclusions of the literature on the property tax (such as Netzer[27]). But in addition the spatial effects of the property tax (on land values and density in different parts of cities) are derived. It is shown that even with uniform rates, the property tax tends to encourage the intensive use of land in the suburbs and to raise its price, at the expense of land values near the center of a city.

27) Dick Netzer "The Economics of the Property Tax" The Brookings Institution, Washington D.C. 1966.

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Chapter II

City Size and Transportation Costs in a Model of a City with one Mode Transportation System

1. The Basic Assuptions

The purpose of this chapter is first to construct a simple model of a city (a metropolitan area) and to calculate in this model how transportation costs vary with city size. Second: to try and examine whether the available data conform to what is expected according to that model.

The model is based on assumptions which are similar to those used in recent literature in this area which was cited in the previous chapter[1]. Three of these assumptions are worthy of special comment:

(a) Along with many of these studies, we do not ask in this chapter why does a city exist. We take it as given that a city exists and that it is centered around some point. This assumption can be interpreted in several ways: According to one interpretation the point is the Central Business District (C.B.D.), and everybody has to commute to work there. Since this is

1) See note 2 in the previous chapter p.2.

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perhaps the most natural metaphor I will use its terminology, however there is nothing in the model which contradicts any of the other interpretations. According to another interpretation all the output that is produced in the city has to be delivered to this point (say a port). This assumption can be relaxed somewhat: it is sufficient that some fraction of the labor force - or of output has to be shipped to the center, and this is the raison d'etre of the city[2]. A third possible interpretation is to assume that the city is centered around some indivisible public facility (such as a temple, a school or a market) which some fraction of the population visits very often[3].

In the formal model that follows we shall not be concerned with what actually happens in the city center, neither shall we be concerned with the area of land which is taken by the center. Actually we assume that the center is a point (that does not take any space).

(B) The second element which is a necessary condition for the concentration of people and economic activity around the "central point" is transportation costs. Let us adopt the simplest possible assumption about the

2) See Mills (1967) Op.Cit. p. 204.

3) This topic will be taken up in chapter IV again.

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technology of transportation, that marginal costs are constant - for instance if transportation costs are proportional to the distance travelled.

The assumption that marginal costs of transportation are constant is quite restrictive. Strictly speaking if transportation requires land, and if the price of land varies from place to place, so should also the price of transportation. So that it is valid only if no land is used in the production of transportation[4]. However, the assumption can be valid when the cost of land is not included in transport prices. Or when the cost of transportation reflects only time (and other variable costs). Therefore the model could be called a model of a "one-input-transportation" city, and it might be able to predict variations in travel time, in situations where time is the only cost, or the main cost of transportation.

We can think of this model as a description of some "future city", where transportation is by helicopters, or of a city where people walk on foot, or whenever the capacity of the streets never becomes an effective limitation. This assumption is made also explicitly by Muth[5],

4) See Mochman and Pines (1971) Op.Cit. p.60.
5) Muth (1969) Op.Cit p. 72.

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and also by Beckmann[6]. Kuth quotes also an unpublished study by Pendleton that used data from Washington,D.C. and suggested that marginal costs of transportation are constant (in Washington, D.C. at least).

In paranthesis I would like to indicate at an implication of departing from this assumption: If the subjective cost of time spent travelling is increasing - (due to increasing marginal disutility, or convexity of indifference curves), this might explain why only very few people live in the immediate proximity of their working place. Two implications of this latter assumption could be pointed to:The first is the (subjective) cost of a short trip is low. The second is that, if the market prices of housing at every distance are determined by the marginal cost of travelling from this distance to the center, then there is an opportunity for people whose subjective marginal cost of travelling is low to take advantage of the decline in the price of houses.

In particular this opportunity is open to that fraction of the labor force whose working place is in the residential districts (say service workers, but also

6) Beckmann (1969) Op.Cit.p. 60.

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grocery shopkeepers, teachers etc..). They will have an incentive to live further out (from the center), for as long as their marginal cost of time spent travelling is less than the rate of decline in market prices for housing, they will - so to speak - sell their "space" in the ring where they work to some of the people who commute to the center, and will buy their house somewhere further out. This change will be beneficial to both parties, and will result in a net decrease in subjective cost of time spent travelling. It will come to end when the marginal cost of time spent travelling is equal to the rate of decline in the price of housing in every distance from the center.

(6) The model is aimed mainly at analysing the residential sector of the city, and it allows for the possibility of substitution of land for other factors of production in that sector. Following Mills[7] let us assume that the substitution of land for other factors of production takes place only in the production of houses (apartments). The residents of our city who have identical incomes and tastes live in houses (or apartments)

7) Mills, Op.Cit.

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of equal and fixed size. However these houses are produced at variable proportions of land and other factors (mainly capital). Specifically we shall assume that houses are produced according to a Cobb-Douglas production function.

2. Optimal Pattern of Land Use:

The total costs of transportation in a city depend on the pattern, or spatial distribution, of the population in the city. If a lot of people live near the center transportation costs will be less than if more people lived further out.

By changing the density of population in various parts of the city it is possible to greatly affect transportation costs. This can be achieved by changing the amount of other factors of production that are employed with every unit of land - in different locations.

From the economic point of view, cities can be thought of, as systems to economize on transportation costs. This is achieved mainly, by increasing the amount of capital and other factors which are concentrated on each unit of land near the center. In the jargon of city planners this is called increasing the intensity of land use. Obviously increasing the "intensity" of land use has its price.

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Structures are becoming increasingly expensive as more capital and less land are used for their construction. In an efficient city these costs should be balanced against the possible savings in transportation costs, so that the marginal construction and other costs, of increasing the intensity of the use of given piece of land are equal to the alternative costs of transportation that could be saved at that point. This argument will (hopefully) become more rigorous and clear by the model which is presented in the rest of this section[8].

Suppose that a city consists of N households. They live in houses (apartments) of equal size which can be produced by variable proportions of land and capital. The cost of capital is fixed at w, and land is free, and is available in all directions from the center. All the residents of this city (or one from every household) have to come every day to the center of the city. The costs of transportation are assumed to be a linear function of the distance travelled, that is for household i living u_i miles from the center transportation costs are:

8) Lave (1969) Op.Cit. has calculated total transportation costs in a model which does not allow for the possibility of factor substitution in cities, and therefore for the possibility to save on transportation costs through this substitution.

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$$Ti = bu_i \qquad (1)$$

If the number of people (households)who live on every ring at distance u from the center is n(u), and is equal to the number of houses at that ring h(u), then total transportation costs in the city are:

$$T^* = \int_0^{\bar{u}} bh(u)udu \qquad (2)$$

Where $\bar{u}$ is the radius of the city.

Let us assume that houses are produced according to:

$$h = AL^{\alpha} K^{\beta} \qquad \alpha+\beta=1 \qquad (3)$$

Where : h = quantity of houses (equal to the number of households N)

L = amount of land

K = amount of capital.

By choosing proper units of measurement A can always be made unity. Irving Hoch[9] estimated a similar "Cobb-Douglas" production function. However he seems to interpret it as that of the construction industry (whose output is structures), while in this work the concept output should be understood as the services that are provided by the owners of houses to the consumers.

---

9) Irving Hoch "The three Dimensional City" in Perloff Harvey (editor) "The Quality of the Urban Environment" Johns Hopkins University Press for Resources for the Future ,Baltimore 1969. pp 75-135.

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Since the amount of land on every ring at distance u from the center is $2\pi u$, the number of houses (apartments) that can be produced on that ring can be changed (efficiently) only by changing the input of capital (at an increasing marginal rate). The costs of the houses that are built on every ring will be an increasing function of the number of people (households) that are to be housed on that ring. It is:

$$wK(u) = w(2\pi u)^{-\alpha/\beta} n(u)^{1/\beta} \qquad (4)$$

Total construction costs in the city are:

$$wK^* = \int_0^{\bar{u}} w(2\pi u)^{-\alpha/\beta} n(u)^{1/\beta} du \qquad (5)$$

We can now formulate the following cost minimization problem:

$$\text{Min: } b\int_0^{\bar{u}} n(u)u\,du + w\int_0^{\bar{u}} (2\pi u)^{-\alpha/\beta} n(u)^{1/\beta}\,du \qquad (6)$$

$$\text{Subject to : } \int_0^{\bar{u}} n(u)du = N \qquad (7)$$

This is a problem in the calculus of variations.

The problem is to find the function n(u) which minimizes the sum of (6) subject to (7).

A necessary condition for the solution is:

$$bu + \frac{w}{\beta}(2\pi u)^{-\alpha/\beta} n(u)^{\alpha/\beta} = \lambda \qquad (8)$$

Where λ is the Lagrangian multiplier of restriction (7).

Solving for n(u) this gives:

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$$n(u) = \left(\frac{\beta}{w}(\lambda - bu)\right)^{\beta/\alpha} 2\pi u \qquad (9)$$

In order to evaluate λ we have to consider its definition. From its definition it follows that its economic interpretation is the marginal cost of additional household in the city. In an efficient city this should be the same everywhere; i.e.: it should not matter whether the additional resident settles at the edge of the city (at $\bar{u}$) on free land, where his costs will be only transportation costs; or whether he settles in the center where he does not have any transportation costs but he only adds to construction costs; or anywhere in between.

According to this argument:

For any two rings at distance u and u' from the center the following condition should hold:

$$bu + \frac{w\bar{x}(u)}{n(u)} = bu' + \frac{w\bar{x}(u')}{n(u')} \qquad (10)$$

If we choose u' to be $\bar{u}$, and rearrange (10) we get:

$$b(\bar{u}-u) = \frac{w}{\beta}(2\pi u)^{-\alpha/\beta} n(u)^{\alpha/\beta} \qquad (11)$$

which expresses the condition that it is worthwhile to increase the number of people who are housed at every ring until the costs of adding another family (marginal construction costs) will be equal to the savings from transportation costs that family would have incurred

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if it were to live on free land at the edge of the city. Equation (11) gives another way to solve for n(u) (which is an alternative to solving the calculus of variations problem in (6) and (7)).It gives :

$$n(u) = \left(\frac{\beta b}{w}(\bar{u}-u)\right)^{\beta/d} 2\pi u \qquad (12)$$

This is equivalent to (9) once we substitute $\lambda = b\bar{u}$. The function n(u) is the density at u times the area of land available at u (in other words density = $n(u)/2\pi u$).

3. The Implications of Perfect Competition

An interesting question to ask at this point is whether the same pattern of density which is determined by the function n(u) which minimizes costs in this model will be achieved under assumptions of perfect competition. The answer is positive. Under the same technological conditions (in the housing and transportation industries), if we assume that perfect competition prevails in the output and factor markets of the housing industry, the same density pattern will result. The cost minimizing efforts of every consumer and producer - trying to minimize its own costs - will result in an efficient pattern of land use.

Let us also assume that both output and factor

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markets (of the housing industry) are competitive, and that the price of K is given (and is a constant w). Then for the Cobb-Douglas production function (3) it is true that the price of houses P at distance u from the center of the city is :

$$P(u) = DR(u)^{\alpha} \qquad (21)$$

where R(u) is the rent of land at distance u and D a constant[10].

If the consumption of housing per capita is constant, the price of houses at distance u should decline with distance at a rate equal to the additional transportation costs.

If the price of houses were not declining at a rate equal to additional transportation costs there would be an incentive to some people to move and to take advantage of this discrepency. Thus the desire of people to minimize the sum of their housing plus transportation costs, is the force that equates the value of this sum over the whole city.

---

10) $D = (\alpha^{\alpha}\beta^{\beta})^{-1}w^{\beta}$

See: Marc Nerlove "Estimation and Identification of Cobb-Douglas Production Functions" Chicago, Rand McNally, 1965.

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If we assume constant marginal costs of transportation, we get as a condition of equilibrium in the housing market:

$$P'(u) = -b \qquad (22)$$

(where b = the constant marginal costs of transportation[11]).

Taking a derivative of (21) with respect to u and substituting into (22) gives:

$$D\, R(u)^{-\beta} R'(u) = -b \qquad (23)$$

This is a differential equation in R(u). Using as an initial condition the assumption that rent at the edge of the city $\bar{u}$ is equal to some minimum level (agricultural rent - Ra).

i.e.: $R(\bar{u}) = Ra$, we get:

$$R(u) = \left(Ra^{\alpha} + \frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\gamma} \qquad \gamma = \frac{1}{\alpha} \qquad (24)$$

Also for a Cobb-Douglas production function under perfect competition we know that the value of marginal product of a factor equals its price:

$$R(u) = \alpha \frac{h(u)}{L(u)} P(u) \qquad (25)$$

Assuming that $L(u) = 2\pi u$ (this can be easily relaxed to

11) A comment about the dimensions of these prices might be in order. Obviously, both the prices of housing P(u), and transportation b should be expressed at a common time denominator. That is to say: both should be prices of a flow per the same unit of time, say a year.

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a constant fraction of 2π), and substituting (24) into (25) we get:

$$h(u) = \frac{2\pi u}{\alpha D}\left(Ra^{\alpha} + \frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\beta/\alpha} \qquad (26)$$

Substituting Ra = 0 this is the same function as (12) above[12].

4. Total Transportation Costs

Having solved for n(u) (or h(u)) we can calculate the integral in (5) - (8), for the optimal distribution of population within the city.

The total number of households in this city is:

$$N = H^* = \int_0^{\bar{u}} n(u)\,du \qquad (27)$$

Substituting (26) into (27) :

$$H^* = \int_0^{\bar{u}} \frac{2\pi u}{\alpha D}\left(Ra^{\alpha} + \frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\beta/\alpha} du \qquad (28)$$

$$H^* = 2\pi\left\{\frac{D}{b^2(\gamma+1)}\left(Ra^{\alpha} + \frac{b\bar{u}}{D}\right)^{\gamma+1} - \frac{D}{b^2(\gamma+1)}Ra^{\alpha+1} - \frac{\bar{u}}{b}Ra\right\} \qquad (29)$$

This is a relation between the number of households in the city H^* and its radius $\bar{u}$. The solution of this equation gives $\bar{u}$ as a function of H^*, or the population of the city N if the number of persons per household is constant.

---

12) $D = (\alpha^{\alpha}\rho^{\beta})^{-1} w^{\beta}$.

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If the agricultural rent Ra is zero:

$$\bar{u} = \frac{D}{b} G^{\frac{1}{\gamma+1}} H^{* \frac{1}{\gamma+1}} \qquad (30)$$

Where: $G = \frac{b^2(\gamma+1)}{D2\pi}$

If Ra is only approximately zero then (30) is only an approximation.

Transportation costs for people who live at distance u is:

$$T(u) = h(u)bu = \frac{2\pi bu^2}{\alpha D}\left(\frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\gamma-1} \qquad (31)$$

Total transportation costs for all people in the city is:

$$T^* = \frac{2\pi b}{\alpha D}\int_0^{\bar{u}} u^2\left(\frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\gamma-1} du \qquad (32)$$

$$T^* = \frac{D^2 4\pi}{b^2(\gamma+1)(\gamma+2)}\left(\frac{b\bar{u}}{D}\right)^{\gamma+2} \qquad (33)$$

Substituting for $\bar{u}$ from (30) gives total transportation costs as a function of the number of households in the city:

$$T^* = \frac{2D}{(\gamma+2)} G^{\frac{1}{\gamma+1}} H^{* \frac{\gamma+2}{\gamma+1}} \qquad (34)$$

Now, the addition to total transportation costs in the city with an increment of one additional household (after all the possible savings by internal rearrangement have taken place), is the derivative of T* with respect to H* :

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$$\frac{dT^*}{dH^*} = \frac{2D}{(\gamma+1)} G^{\frac{1}{\gamma+1}} H^{*\frac{1}{\gamma+1}} \qquad (35)$$

On the other hand average transportation costs are lower:

$$\frac{T^*}{H^*} = \frac{2D}{(\gamma+2)} G^{\frac{1}{\gamma+1}} H^{*\frac{1}{\gamma+1}} \qquad (36)$$

Dividing (35) by (36) it is obvious that marginal costs are always higher than average costs by a factor of $\frac{\gamma+2}{\gamma+1}$.

The upper limit of this expression occurs when $\alpha = 1$ and is $\frac{3}{2}$, and the lower limit is 1 and it occurs when α approaches zero. For any reasonable value of α (between zero and 1) marginal costs are always higher than average costs.

In addition to the increase in transportation costs, the cost of capital and other factors of production used in the construction of houses will increase also when they are substituted for land which becomes more expensive when population increases.

Using again the condition that the value of marginal product of a factor equals its price, we have for the "other factor" :

$$k(u)w = \beta h(u)P(u) \qquad (37)$$

Solving this time for k(u)

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$$k(u)w = \frac{\beta 2\pi}{\alpha} u\left(\frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\gamma} \tag{38}$$

The total costs for the "other factors" is:

$$w\bar{K}^* = \int_0^{\bar{u}} \frac{\beta 2\pi}{\alpha} u\left(\frac{b\bar{u}}{D} - \frac{bu}{D}\right)^{\gamma} du \tag{39}$$

$$w\bar{K}^* = \frac{\beta}{\alpha} \frac{D}{(\gamma+2)G}\left(\frac{b\bar{u}}{D}\right)^{\gamma+2}$$

Substituting for $\bar{u}$ from (30) we get total expenditure on the "other factor" as a function of the number of households in the city:

$$w\bar{K}^* = \frac{\beta}{\alpha} \frac{D}{(\gamma+2)} G^{\frac{1}{\gamma+1}} H^{*\frac{\gamma+2}{\gamma+1}} \tag{40}$$

Total costs (construction + transportation) are:

$$T^* + w\bar{K}^* = (2+\frac{\beta}{\alpha})\frac{D}{(\gamma+2)} G^{\frac{1}{\gamma+1}} H^{*\frac{\gamma+2}{\gamma+1}} \tag{41}$$

The addition to total costs of the city with an increment of one additional household is the derivative of (41) with respect to H* (the number of households in the city).

$$\frac{d(T^* + w\bar{K}^*)}{dH^*} = DG^{\frac{1}{\gamma+1}} H^{*\frac{1}{\gamma+1}} \tag{42}$$

This is exactly equal to the price that an additional household will have to pay for his house plus transportation costs which is equal everywhere in the city to $b\bar{u}$.

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$$P(u) + bu = b\bar{u} \qquad (43)$$

Substituting from (30) for $\bar{u}$ we get :

$$b\bar{u} = D G^{\frac{1}{\gamma+1}} H^{* \frac{1}{\gamma+1}}$$

which is identical with (42).

However before concluding that the price system guarantees that every household pays a price which is equal to the social cost of its decision to live in the city, we should point out [illegible] important distributional effects. The price of land rises with the increase of population in the city, which brings profits to the landowners at the expense of all the inhabitants of the city, new and old. This would not be a problem if and only if everybody owned the land on which his house is built, otherwise this will be an external cost to the old residents of the city.

5. Empirical Estimates

Generally the increase in requirements for inputs in urban transportation should manifest itself in either (or in a combination) of the following three ways:

1) A rise in the per capita amount of time spent in travel.

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2) A rise in the per capita amount of capital invested in transportation.

3) A rise in the proportion of land used for transportation.

However our model does not allow for the possibility of substitution between various inputs in the transportation industry. Strictly speaking the model could be interpreted as a description of a city with a transportation sector which uses only one input, or a combination, in fixed proportions, of several inputs. The most important input in urban transportation is time spent travelling.

Data on the time that people spend travelling in various cities are available from surveys and studies of traffic in various cities. Alan M. Voorhees and Associates[13] published data on average trip length (in miles) and trip duration (in minutes) in 34 American cities ranging in size from 30,000 (Beloit, Wis.) to 6,500,000 (Los Angeles, Ca.) (See Table 1). Though the data are from different sources, "attempts were made (by Alan M. Voorhees, et al.) to keep them as compatible as possible". They tried to fit to these data regressions

13) Alan M. Voorhees and Associates "Factors and Trends in Trip Lengths" ,Highway Research Board, National Cooperative Highway Research Program Report No.48. Washington D.C. 1958.

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Work Trip Length Characteristics and Major Influencing Factors

	City	Population (1,000's)	Average Trip Duration (Min)	Average Trip Length (Mi)	Average Network Speed (MPH)
1.	Los Angeles,Cal	6,489	16.0	8.8	31.0
2.	Philadelphia,Pa	3,635	20.1	7.2	21.5
3.	Washington,D.C.	1,808	14.1	5.9	24.7
4.	Pittsburgh,Pa.	1,804	12.6	4.2	20.7
5.	Baltimore,Md.	1,419	16.7	7.0	24.6
6.	Minneapolis-St. Paul	1,377	12.5	5.1	24.5
7.	New Orleans,La.	845	9.1	3.0	20.2
8.	Providence, R.I.	685	14.6	--	--
9.	Fort Worth, Tex.	503	15.7	8.1	30.9
10.	Lackawanna-Luzern	453	19.2	8.7	27.1
11.	Broward County,Fla	440	13.7	--	--
12.	Ottawa-Hull,Ont.	406	12.6	5.3	25.2
13.	Nashville,Tenn.	347	10.8	5.4	30.0
14.	Edmonton,Alta.	336	11.6	5.8	30.0
15.	Worcester,Mass.	281	10.9	4.9	27.3
16.	Virginia Peninsula	277	11.5	6.3	33.0
17.	Knoxville,Tenn.	258	9.4	--	--
18.	Davenport,Iowa	227	7.7	3.2	24.9
19.	Charlotte,N.C.	210	11.0	5.5	30.0
20.	Chattanooga,Tenn.	205	10.8	5.4	30.0

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Work Trip Length Characteristics and Major Influencing Factors (cont.)

City	Population (1,000's)	Average Trip Duration (min)	Average Trip Length (mi)	Average Network Speed (MPH)
21. Erie,Pa	177	9.4	3.4	21.7
22. Waterbury,Conn.	142	10.1	5.9	35.0
23. Springfield,Ill	134	7.5	3.6	29.2
24. Pensacola,Fla.	128	8.7	4.4	30.3
25. Regina,Sask.	127	8.0	3.3	24.5
26. Greensboro,N.C.	123	8.9	4.3	29.0
27. Lexington, Ky.	112	9.1	5.7	35.0
28. Springfield,Mo.	110	8.4	---	-----
29. Altoona, Pa.	103	11.1	3.1	27.2
30. St.Catharines, Ont.	99	13.6	---	-----
31. Sioux Falls, S.Dak.	67	7.0	2.9	24.8
32. Tallahassee,Fla.	48	7.3	3.7	30.4
33. Hutchson,Kans.	38	6.1	2.0	19.2
34. Beloit,Wis.	33	6.7	2.9	25.9

Source: Alan M. Voorhees and Associates, p.8

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of the following form:

$$y = N^a S^c \qquad (51)$$

where:

y = the average trip duration (t) or length (d)

N = the city population

S = the average network speed in m.p.h.

a, c = constants.

Trips were classified according to purpose, the most interesting for us are home to work trips.

Even without allowance for the influence of other factors, both trip length and trip duration were found to be significantly correlated with city size. And a logarithmic relationship was found to fit better, which is in agreement with the form of our equations: (34) - (36).

Estimating an equation for average home to work trip duration (in minutes) gave the following result:

$$\log t = -0.02 + 0.19 \log N \qquad (52)$$

with $r^2 = 0.71$ (and the standard error of a = 0.03). Estimating an equation for average work trip length (in miles) gave the following result:

$$\log d = -0.77 + 0.19 \log N \qquad (53)$$

with $r^2 = 0.75$ (and the standard error of a = 0.02).

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Obviously the time spent travelling depends not only on the distance travelled but also on the amount of other inputs used in travelling besides time (such as capital invested in roads, vehicles etc..). Alan M. Voorhees did not use any direct data on these inputs, but used the "average network speed" in every city as an indication of the general condition of traffic in that city. Including average speed as another variable in the regression improved the results.

It should be noted that the cost of transportation, in terms of time spent on travelling a given distance, is inversely related to speed. An increase in speed implies that the cost (in terms of time) of travelling one mile, goes down proportionately. Thus an increase in speed is equivalent in our notation to a reduction in b[14].

Including speed in the equation for average work trip (in miles) increased the value of r^2 to 0.85. The estimated equation was:

$$\text{Log } d = -5.86 + 0.20 \log N + 1.49 \log S \qquad (54)$$

(the standard error of a = 0.02, and of c = 0.20).

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14) I am indebted to Professor Carl F. Christ for pointing this out.

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It is interesting to note that the value of a in these estimates (around 0.2) implies according to equation (36) a value of 0.25 for α in the production function for houses (3). This is not out of line with the direct estimate of α by Irving Hoch[15].

15) See Irving Hoch, Ibid p.80.

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Chapter III

The Allocation of Land to Transportation

A second important input in urban transportation is land. The purpose of this chapter is first to construct theoretical models that will explain the amount of land devoted to transportation in various parts of a city, and in cities of different sizes. Second to try and examine whether the available data on the amount of land used for transportation in various cities, and in various parts of a same city conform to what is expected according to these models.

The last section of the chapter is devoted to the question under what conditions will the cost of land be reflected in private costs of transportation, as the users of the transportation system see them; and what are the effects of deviations between the two.

Another purpose of this chapter is to explain why the fact that land is used for transportation, i.e.: that transportation competes with other uses for space, is a cause for dis-economies of scale in cities. I will argue that as a city grows its transportation system requires a higher proportion of the city's land area, unless some other inputs are substituted for land in the production of transportation. This implies that

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when a city grows its transportation costs are rising by more than in proportion to the rise in size. This should still be correct even if other inputs are substituted for land in the transportation industry. This substitution can mitigate the rise in costs but it cannot eliminate it altogether.

To see why we should expect these dis-economies consider the following argument.: "Suppose we consider the possibility of doubling the population of a city by doubling the height of every building. If this were feasible and if twice as many people now traveled between each pair of points as before, then it would lead to just twice the demand for transportation as before. But if transportation requires land as an input, it must use more land after the doubling of population than before. Thus some land previously used for buildings must now be used for transportation, thus requiring new buildaings at the edge of the city. But the edge of the city has now moved out, and some people must make longer trips than before, requiring more transportation inputs, thus a doubling of the city's population requires more than doubling transportation inputs"[1].

1) Mills. Op.Cit. 1967 p.199

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Clearly doubling the height of every building is not the most efficient way of increasing a city, so let us consider another possibility and that is, doubling the area of land that the city occupies: Suppose that a town is divided into two parts:

1. A central business district where everybody works.
2. A residential periphery.

Suppose now that the population of the town doubles also. The question is whether it will suffice to double the area of the town. Assuming the same degree of intensity in the use of land, doubling of the area will be sufficient to residential and industrial uses. However just doubling of the area of the land used for transportation will not do as it will be necessary for the people that will live in the new outer residential quarters to pass through the "old" residen--tial quarters. Likewise some people will have to pass through "new" employment areas to reach to the "old" center. This implies that the area of land required for transportation will increase by more than in proportion to the increase in population (unless other inputs will be substituted for land in the transportation industry) and also that the inputs of

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other factors such as capital, time etc. will rise by more than population. The inputs of other factors used in the transportation industry will rise even more, if they are used also as substitutes for land.

The determination of the proportion of land devoted to transportation will be further explained in the more detailed models that are presented in the next section of this chapter.

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1. Models of Land Allocation

This section is devoted to the presentation of two theoretical models that explain the determination of the proportion of land allocated to transportation. The models are based on certain rigid assumptions, and their conclusions are naturally restricted to the validity of these assumptions. The analysis of these models turned out to be a fairly complicated task. I have therefore conducted it in two stages going from simpler case to the more complicated:

A. The first and most simple model, is of a rectangular (or rather linear) city à la Solow and Vickery[2] with fixed coefficients in both the housing and the transport activities. Suppose that our city stretches along some strip with a width w, and length u. Transportation takes place only along the u direction (if w is small we can neglect the cost of moving in that direction - this is why this is actually a model of a linear city). Suppose that everybody has to come (say once a day) to the "center" of the city (u = 0), and that the city stretches from u = 0 only in one direction. Let the

2) Robert M. Solow and William S. Vickery "Land Use in a Long Narrow City" Journal of Economic Theory, December 1971 pp 430-447.

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transfer of a man through a band of infinitissimal width require b square feet of land. Let the housing of a person (or a family) require s square feet of land. Then at every band at distance u from the "center", total land used for transportation will be b times the total number of people who pass through that point, which is equal to the total number of people who live further away, which is the sum of the number of people who live at distances from u to $\bar{u}$. The rest of the land at u can be used for housing, and we have (at every u):

$$b\int_{u}^{\bar{u}} n(u)du + sn(u) = w \qquad (1)$$

Where: n(u) is the number of people who live at distance u.

This is an identity which holds at every u. Differentiating with respect to u, gives a differential equation in n(u).

$$sn'(u) - b\, n(u) = 0 \qquad (2)$$

The solution of this equation is:

$$n(u) = A\, e^{\frac{b}{s}u} \qquad (3)$$

Where A is an arbitrary constant. Using as an initial condition the assumption that at the edge of the city (at $u = \bar{u}$) all land is used for housing, that is to say:

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$$n(\bar{u}) = \frac{W}{s} \tag{4}$$

We can solve for A, and the solution for n(u) becomes:

$$n(u) = \frac{W}{s} e^{\frac{b}{s}(u-\bar{u})} \tag{5}$$

The total number of people who live in this city N is:

$$\int_0^{\bar{u}} n(u)du = N$$

$$N = \int_0^{\bar{u}} \frac{W}{s} e^{\frac{b}{s}(u-\bar{u})} du \tag{6}$$

or:

$$N = \frac{W}{b} - \frac{W}{b} e^{-\frac{b}{s}\bar{u}} \tag{7}$$

The total number of people N can never exceed $\frac{W}{b}$ which is the capacity limitation if all land is used for transportation at any given point.

The proportion of land used for housing (out of total city area) is:

$$\phi = \frac{s\bar{N}}{W\bar{u}} = (\frac{W}{b} - \frac{W}{b}e^{-\frac{b}{s}\bar{u}})\frac{s}{W\bar{u}} \tag{8}$$

$$\phi = \frac{s}{b\bar{u}} - \frac{se^{-\frac{b}{s}\bar{u}}}{b\bar{u}}$$

And the proportion of land used for transportation θ is $1-\phi$.

Indeed land used for transportation at distance u is:

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$$b\int_{0}^{\bar{u}} n(u)du = b\int_{0}^{\bar{u}} \frac{w}{s} e^{\frac{b}{s}(u-\bar{u})} du = w - we^{\frac{b}{s}(u-\bar{u})} \qquad (9)$$

and total land used for transportation from 0 to $\bar{u}$ is:

$$T = \int_{0}^{\bar{u}} (w - we^{\frac{b}{s}(u-\bar{u})})du = w\bar{u} - \frac{ws}{b} + \frac{ws}{b}e^{-\frac{b}{s}\bar{u}} \qquad (10)$$

The proportion of land used for transportation θ is:

$$\frac{w\bar{u}}{w\bar{u}} - \frac{ws}{w\bar{u}b} + \frac{ws}{w\bar{u}b} e^{-\frac{b}{s}\bar{u}} = 1 - \frac{s}{b\bar{u}} + \frac{s}{b\bar{u}} e^{-\frac{b}{s}\bar{u}} \qquad (11)$$

The change in the proportion of land used for housing, as city size increases is the derivative of ϕ with respect to $\bar{u}$:

$$\frac{d\phi}{d\bar{u}} = -\frac{s}{b\bar{u}^2}\left[1-(1+\frac{b\bar{u}}{s})e^{-\frac{b}{s}\bar{u}}\right] \qquad (12)$$

Since the proportion of land used for transportation θ is : $1-\phi$, its derivative is the same as (12) with the opposite sign, i.e.:

$$\frac{d\theta}{d\bar{u}} = \frac{s}{b\bar{u}^2}\left[1 -(1+\frac{b\bar{u}}{s})e^{-\frac{b}{s}\bar{u}}\right] \qquad (13)$$

(12) is always negative, and (13) always positive. $((1+g)e^{-g}$ is always less than 1, according to Taylor series: $e^{g} = 1+g+\frac{1}{2}g^2+..)$.

Taking a derivative of (7) with respect to $\bar{u}$ we can find the change of population which is associated

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with a change in city length ($\bar{u}$). The inverse of this derivative will be the change in length $\bar{u}$, associated with a change in population N. Substituting this into (12) or (13), will give the change in the proportion of land used for either purpose as the population increases. The change in the proportion of land devoted to transportation is:

$$\frac{dq}{dN} = \frac{s}{w\bar{u}} \left(\frac{s}{b\bar{u}} e^{\frac{b}{s}\bar{u}} - \left(\frac{s}{b\bar{u}} + 1 \right) \right) \qquad (14)$$

This will also always be positive.

We have thus seen that in this simple model the proportion of land that has to be devoted to transportation rises when the city size increases.

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B. Let us next consider the case of fixed coefficients in a circular city. Suppose that a city is composed of people who occupy each a lot of fixed and equal size s, and who go every day to the center of the city. Suppose that the technology of transportation is also such that it takes a certain fixed amount (a) of land to transfer a man through a point. The amount of land that is required for roads at any given point at distance u from the center of the city is equal to the number of people who pass through that point multiplied by the coefficient a.

$$L_1(u) = a\ t\ (u) = a\int_u^{\bar{u}} n(u')du' \qquad (21)$$

Where : $L_1(u)$ is the amount of land demanded for streets at distance u from the center.

t(u), (the output of transportation produced at distance u) is the number of people who pass through a ring at distance u from the center.

n(u) is the number of people who live at each ring, and $\bar{u}$ is the edge of the city.

The amount of land devoted to housing at every ring is:

$$L_2(u) = s\ n(u) \qquad (22)$$

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Now if total land at each ring is equal to $2\pi u$, we have (as an identity) that at every ring:

$$a\int_{u}^{\bar{u}} n(u')du' + s\, n(u) = 2\pi u \qquad (23)$$

The variable that adjusts to guarantee that this identity is satisfied is the number of people who live at each ring $n(u)$, and therefore (23) is an implicit differential equation in $n(u)$.

Differentiating (23) with respect to u we get the differential equation:

$$s\, n'(u) - a\, n(u) - 2\pi = 0 \qquad (24)$$

The solution of this equation is:

$$n(u) = A\, e^{\frac{a}{s}u} - \frac{2\pi}{a} \qquad (25)$$

where A is an arbitrary constant that can be evaluated according to the initial condition that at the edge of the city (at $\bar{u}$) all land is devoted to housing, that is:

$$n(\bar{u}) = \frac{2\pi\bar{u}}{s} = A\, e^{\frac{a}{s}\bar{u}} - \frac{2\pi}{a} \qquad (26)$$

and

$$A = \left(\frac{2\pi\,\bar{u}}{s} + \frac{2\pi}{a}\right) e^{-\frac{a}{s}\bar{u}} \qquad (27)$$

Substituting this into (25) we get:

$$n = \left(\frac{2\pi\,\bar{u}}{s} + \frac{2\pi}{a}\right) e^{\frac{a}{s}(u-\bar{u})} - \frac{2\pi}{a}$$

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$$\text{or:}\quad n = \frac{2\pi}{a}\left(\left(\frac{a\bar{u}}{s} + 1\right) e^{\frac{a}{s}(u-\bar{u})} - 1\right) \tag{28}$$

If n is positive at some small value of u it will be positive at all greater values of u since it is a monotonically rising function.

For n to be positive say at u = 1, the following condition should hold:

$$\frac{2\pi}{a}\left(\frac{a\bar{u}}{s} + 1\right) e^{-\frac{a}{s}(\bar{u}-1)} > \frac{2\pi}{a}$$

$$\left(\frac{a\bar{u}}{s} + 1\right) > e^{\frac{a}{s}(\bar{u}-1)}$$

$$\ln\left(\frac{a\bar{u}}{s} + 1\right) > \frac{a}{s}(\bar{u}-1) \tag{30}$$

Strictly speaking the number of people that can be accomodated in such a city is limited by the capacity of the transportation system at the center. Let us imagine that the center of city is occupied by a circular market place with a radius of 1, then the number of people who can enter the market place is limited by the capacity of the "gates" of that market, which is according to our assumptions $\frac{2\pi}{a}$.

Thus this is an upper limit on the number of people who can pass through the "gates", and an upper limit on total population of our city.

The total number of people who live in the city N is:

$$\int_1^{\bar{u}} n(u)\,du = N \tag{31}$$

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$$\int_{1}^{\bar{u}} \frac{2\pi}{a}\left[\left(\frac{a\bar{u}}{s}+1\right)e^{\frac{a}{s}(u-\bar{u})}-1\right]du = \left\{\frac{2\pi}{a}\left(\frac{s}{a}\left(\frac{a\bar{u}}{s}+1\right)e^{\frac{a}{s}(u-\bar{u})}-u\right)\right\}_{1}^{\bar{u}}$$

$$N = \frac{2\pi}{a}\frac{s}{a}\left(\frac{a\bar{u}}{s}+1\right) - \frac{2\pi\bar{u}}{a} - \frac{2\pi}{a}\frac{s}{a}\left(\frac{a\bar{u}}{s}+1\right)e^{-\frac{a}{s}(\bar{u}-1)} + \frac{2\pi}{a}$$

$$N = \frac{2\pi}{a}\frac{s}{a} - \frac{2\pi}{a}\frac{s}{a}\left(\frac{a\bar{u}}{s}+1\right)e^{-\frac{a}{s}(\bar{u}-1)} + \frac{2\pi}{a} \qquad (32)$$

If condition (30) is satisfied N will be smaller than $\frac{2\pi}{a}$ for all permissable values of $\bar{u}$.

If : $\left(\frac{a\bar{u}}{s}+1\right) > e^{\frac{a}{s}(\bar{u}-1)}$ condition (30)

then: $\left(\frac{a\bar{u}}{s}+1\right)e^{-\frac{a}{s}(\bar{u}-1)} > 1$

and: $\frac{s}{a}\left(\frac{a\bar{u}}{s}+1\right)e^{-\frac{a}{s}(\bar{u}-1)} > \frac{s}{a}$

Having solved for the function n(u) we can go back to equation (23) and calculate the amount of land that is allocated to housing and to transportation at every ring.

The amount of land devoted to housing is:

$$L_2(u) = s\,n(u) = 2\pi\left[\bar{u} + \frac{s}{a}\right]e^{-\frac{a}{s}(\bar{u}-u)} - \frac{s}{a}\Big] \qquad (34)$$

The amount of land devoted to transportation (at every ring) is :

$$L_1(u) = a\int_{u}^{\bar{u}} n(u)\,du = \frac{2\pi s}{a}\left(\frac{a\bar{u}}{s}+1\right)(1-e^{-\frac{a}{s}(\bar{u}-u)}) - 2\pi(\bar{u}-u)$$

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$$L_1(\omega) = 2\pi\left[(\bar{u} + \frac{s}{a})(1-e^{-\frac{a}{s}(\bar{u}-u)}) - (\bar{u}-u)\right] \qquad (35)$$

These two expressions (34) and (35) can be verified as they should add up to $2\pi u$ from (23).

The proportion of land devoted to housing, out of every ring σ' is:

$$\sigma' = \frac{sn(u)}{2\pi u} = \frac{1}{u}\left[(\bar{u} + \frac{s}{a})e^{-\frac{a}{s}(\bar{u}-u)} - \frac{s}{a}\right] \qquad (36)$$

$$\frac{d\sigma'}{du} = \frac{s}{au^2}\left[1 + (\frac{a\bar{u}}{s}+1)(\frac{au}{s}-1)e^{-\frac{a}{s}(\bar{u}-u)}\right]$$

This proportion is rising with distance from the center. By assumption it reaches 1 at $\bar{u}$ (the edge of the city).

$$\frac{d\sigma'}{du} = \frac{s}{au^2}\left[1-(\frac{a\bar{u}}{s}+1)(1-\frac{au}{s})e^{-\frac{a}{s}(\bar{u}-u)}\right] \qquad (37)$$

This derivative is positive if the following inequality holds:

$$1 > (\frac{a\bar{u}}{s}+1)(1-\frac{au}{s})e^{-\frac{a}{s}(\bar{u}-u)} \qquad (38)$$

or:

$$e^{\frac{a}{s}(\bar{u}-u)} > (\frac{a\bar{u}}{s}+1)(1-\frac{au}{s}) \qquad (39)$$

and this inequality holds always!

The proportion of land devoted to transportation (out of every ring) is $1-\sigma'$. It is declining when σ' is rising, and by assumption it reaches zero at the city edge (at $\bar{u}$). Empirical evidence on this matter will be cited in the subsequent section.

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Total land used for housing $L_2^* = s\,N$

The proportion of land used for housing (out of total city area) is:

$$\psi = \frac{s\,N}{\pi(u^2-1)} \tag{40}$$

Substituting from (32):

$$\psi = \frac{2\,s^2}{(\bar{u}^2-1)a^2}\left[1-(\frac{a\bar{u}}{s}+1)e^{-\frac{a}{s}(\bar{u}-1)}+\frac{a}{s}\right] \tag{41}$$

$$\frac{d\psi}{d\bar{u}} = \frac{4s^2\bar{u}}{a^2(u^2-1)^2}\left\{\left[\frac{a\bar{u}}{s}+1+\frac{a^2(u^2-1)}{2s^2}\right]e^{-\frac{a}{s}(\bar{u}-1)}-1-\frac{a}{s}\right\} \tag{42}$$

This is negative if the following inequality is satisfied:

$$\bar{u}+\frac{s}{a}+\frac{a(\bar{u}^2-1)}{2s} \quad < \quad (1+\frac{s}{a})e^{\frac{a}{s}(\bar{u}-1)} \tag{43}$$

This inequality is satisfied for all permissable values of $\bar{u}$ (greater than 1).

In order to see that this is so, substitute $x = \bar{u}-1$ and $\frac{s}{a} = g$ in (43), and expand according to Taylor series:

$$x+1+g+\frac{x^2}{2g}+\frac{x}{g} \quad < \quad (1+g)(1+\frac{x}{g}+\frac{1}{2}(\frac{x}{g})^2+\ldots) \tag{44}$$

When the proportion of land devoted to housing is declining, its complement, the proportion of land devoted to transportation is increasing, and the rate of change is the same (with an opposite sign).

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$$\frac{d\theta}{d\bar{u}} = -\frac{d\Psi}{d\bar{u}} \qquad (45)$$

We have thus seen that in this model also, the proportion of land that has to be devoted to transportation rises with an increase in city size.

The rent-distance function for this model is declining at an exponential rate with distance from the center of the city.

Let us first assume that the only input in transportation is land, and that it is priced in a perfectly competitive way (so that its price is changing with every change in u). Then if people minimize the sum of their transportation costs plus their payment for rent, competition for land will insure, that this sum is equal everywhere, i.e.:

$$a\int_0^{\bar{u}} R(u)du + s\,R(u) = C \qquad (46)$$

or: $$a\,R(u) + s\,R'(u) = 0 \qquad (47)$$

This is a differential equation in R(u) whose solution is:

$$R(u) = A\,e^{-\frac{a}{s}u} \qquad (48)$$

where A is an arbitrary constant. If we assume as an initial condition that at the edge of the city $\bar{u}$ rent is equal to the agricultural rent Ra, i.e.: $R(\bar{u}) = Ra$ (49)

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$$A = Ra\, e^{\frac{a}{s}\bar{u}} \tag{50}$$

and: $$R(u) = Ra\, e^{\frac{a}{s}(\bar{u}-u)} \tag{51}$$

Next let us assume that transportation costs consist of the price of land used for transportation (at every u) plus a "time" cost which is proportional to distance,

i.e.: $$T(u) = a\int_{u}^{\bar{u}} R(u) + bu \tag{52}$$

The efforts of the cost minimizing people will now result in the following condition (equivalent to (46)):

$$a\int_{u}^{\bar{u}} R(u)du + bu + sR(u) = C \tag{53}$$

Differentiating we get the following differential equation:

$$aR(u) + s\,R'(u) + b = 0 \tag{54}$$

Whose solution (using the same initial condition) is:

$$R(u) = \left(Ra + \frac{b}{a}\right) e^{\frac{a}{s}(\bar{u}-u)} - \frac{b}{a} \tag{55}$$

This type of rent function is a special case of Mills eq.(22). Mills observes there that "exponential decline is a special case where there is no factor substitution possible in housing, land is the only input"[3].

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3) Mills (1967) Op.Cit. p.208

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We have seen that in a model with fixed coefficients in transportation, the proportion of land out of total city area that has to be devoted to transportation rises with an increase in city size. In these models this also implies that per-capita amount of land used for transportation rises with an increase in population. If land is a scarce resource this also implies that per-capita costs will rise.

It is evident that, in general, the relation between the proportion of land used for transportation and city size depends in a critical way on factor substitution in the transportation industry. Substitution of other factors for land can keep the proportion of land used for transportation from rising . However this will involve rising costs. Substitution can mitigate the rise in costs but it cannot eliminate it completely.

Substitution can slow down the rate at which costs rise with city size, but it cannot stop this rise completely. To see that this is so:
Suppose that at some initial fixed prices for all factors, some proportion of factors is the "least cost combination". If this proportion will be kept constant, and in particular; if the land-output ratio will be constant, when the city grows, this will imply - as we have seen -

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that a greater proportion of its land area will have to be devoted to transportation.

If the proportion of land devoted to transportation is to be kept from rising, the average ratio of "capital and other factors" to land has to rise in the transportation industry. This can be achieved only if this ratio will increase in some, or in all, points along the way from the edge to the center of the city.

If the marginal rate of substitution of capital (and other factors) for land in the production of transportation is rising when more capital (and other factors) is substituted for land, then unit costs of transportation output (the cost of passenger mile) will increase as more capital will be used in that industry. (The price of capital "and other factors" is assumed to remain constant throughout).

The easier it is to substitute other factors for land the slower will be the rise in unit costs. At the limit: if land is completely dispensable in transportation (i.e.: transportation can be produced without land), transportation can be produced at constant unit costs. In that case we go back to the model of chapter II (constant marginal transportation costs), and even in that model per capita transportation costs were shown to rise with an increase in population (because of increasing distances).

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Another possibility to save on transportation costs and on land is to increase the density of land use. i.e.: to substitute capital for land in the housing industry.

As we have seen in the previous chapter, increasing the density of land use is a way to save on transportation costs. Increasing density means that travel distances become shorter, less transportation is used and less inputs are used in the transportation industry. This is achieved at a price of increasing the quantity of other inputs used in the housing industry.

It might be interesting to investigate in detail a model with factor substitution in the housing industry which allows explicitly for the use of land in transportation, and to see how is the quantity of land that is used in transportation determined. Mills's 1967 model is such a model. It assumes a Cobb-Douglas production function for housing and a fixed requirement of land in transportation.

One result that was shown above for the fixed coefficients model was that the proportion of land devoted to transportation at every ring declines with distance from the center. This proposition can be

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shown to hold under the assumptions of Mills's model[4].

The exact functional form of the proportion of land devoted to transportation at every ring is different under the assumptions of Mills's model, from that of equations (34)-(36) above, but the general principle holds.

On the other hand, since the model is much more complicated it is difficult to prove or disprove the second proposition: that the proportion of land used for transportation, out of total city area, is a rising function of city population. The model becomes unmanageable and I could not establish whether this proposition holds or not in the context of that model.

However I should point out that if we are interested in the behaviour of total city costs and how they rise with city population then, when factor substitution is allowed for, it is not land inputs alone which matter, but total costs.

Total city costs will rise when city population increases even when substitution is allowed for. Factor substitution can slow down the rate at which costs rise but it cannot halt this rise completely. Unless capital is a perfect substitute for land in the housing industry also. That is to say: unless land is a dispensable factor altogether in

4) Mills E.S. "An Aggregative Model of Resource Allocation in a Metropolitan Area" American Economic Review, May 1967, p 208.

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the construction of cities. In other words: rising costs with size are inevitable if space is an indispensable factor.

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2. The Empirical Evidence

It is a common implication of the models that were discussed above that the proportion of land devoted to transportation should increase as we move from the edge of the city towards its center. In terms of our notation in the above models if u is the distance from the center of the city and θ the proportion of land devoted to transportation - at every distance - then θ is a decreasing function of u.

There are several pieces of evidence which conform to that expectation. It is necessary to point out that the evidence that is presented relates only to land used for "streets and alleys". It does not include land used for railroads, for parking and airports[4]. Even if we could obtain data about these categories there is an additional difficulty of distinguishing between land used for intra-urban transportation and inter urban transportation. The latter is persumably much more important in the case of railroads and airports then it is in streets.

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4) The Chicago survey had special category for parking, but that included data only about parking lots of 10.000 square feet or more and only in part of the survey area for which floor area measures were obtained (the city of Chicago and some suburbs). Beyond that area parking was included with the use served. The Pittsburgh study does not include separate data about parking. Land used for railroad rights of way, airports and communications was included in one category with utilities.

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1. The most detailed information is available from the Chicago Area Transportation Survey.

The data are presented in Table 1.

Table 1

Ring	Land area used for streets and alleys (in acres)	Total land area (in acres)	Proportion of land used for streets
1.	210.1	733.4	28.6
2.	2530.9	7936.7	31.7
3.	4786.4	16687.3	28.7
4.	7297.5	26382.8	27.6
5.	14362.2	54396.5	26.4
6.	16532.4	82709.6	20.0
7.	22216.7	187985.8	11.8
8.	25615.4	422511.1	6.1

Source : "Chicago Area Transportation Survey" Vol 1 Table 21: (Generalized Land Use by District) pp. 110-111. (For a map of the area, and definitions of the districts see there).

It indicates that the proportion of land used for streets rises as we go from the outer ring of the metropolitan area toward the center. It reaches a maximum at the second ring, and is slightly lower at the center which is the Central Business District.

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It should be pointed out that the picture which is revealed by these data is different from the picture which is described by the authors of the Report of the Chicago Area Transportation Survey in page 20 of Vol 1. This is because their analysis was based on proportions of land uses out of total land in use. In the outer rings a considerable amount of land is vacant. Exclusion of the vacant land from the calculation obviously changes the picture.

2. A similar picture is given also by data derived from the Pittsburgh Area Transportation Study which are presented in Table 2.

Table 2

Ring	Land Area Used for Streets (in acres)	Total Land area (in acres)	Proportion of area used for streets.
1.	122.4	472.1	25.9
2.	718.1	2 820.0	25.4
3.	1 610.5	8 838.7	18.2
4.	2 908.8	16 461.2	17.7
5.	3 731.2	24 323.7	15.3
6.	7 222.1	59 097.3	12.2
7.	8 320.4	92 799.6	8.9
8.	4 502.2	63 755.8	7.1

Source: "Pittsburgh Area Transportation Study" Vol 1. Final Report Table 41 pp 98-99 (For a map of the area and definitions of the districts see figure 67 there).

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3. Additional evidence to support this expectation is given by the following observations:

In New York City the proportion of land used for streets and roads is 30.1% (see table 3) . However it is as high as 35.5% in Manhattan; and 29.8% in Bronx, 31.8% in Brooklyn, 29% in Queens and only 28% in Richmond Borough[5].

4. In Rochester N.Y. (which is a city of approximately 300 thousands) streets and roads take about 17.2% of the city area. However John F Curtin [6] reports that "More than one half of the land in downtown Rochester is devoted to transportation".

B. Data on the proportion of total land used for streets and roads in various American cities are available from a survey published by Niedercorn and Hearle[7]. The data are deficient in many ways. The source of the data are questionaires that were sent to city planning commissions. The definitions of the different uses of land that were applied "are not precisely uniform for all cities". However Niedercorn and Hearle expressed

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5) Niedercorn J. and Hearle "Recent Land Use Trends in 48 Large American Cities" Rand 1963. p. 27

6) John F. Curtin "Traffic Transit - Parking in Downtown Rochester" Highway Research Board, Bulletin 293,1961, p. 62-102.

7) Niedercorn and Hearle, Ibid.

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their hope that :"they are reasonably comparable and thus suitable for many types of land use analysis"[8].

Nevertheless it seemed to me to be interesting to examine whether these data conform to what is anticipated according to the theoretic considerations that were presented above.

However before proceeding to the empirical analysis of these data it is necessary to include in the analysis additional variables. The theoretic models that were developed above are in terms of a city with a given economic structure. When we make a cross-section comparison of cities of different sizes, we can expect, for instance, that the proportion of the labor force that travels every day to the C.B.D. is different. In a city like Baltimore, for instance, which is a "manufacturing type" city, less people commute to the C.B.D. than in Washington which is a government-administration center.

The differences in the fraction of the labor force that travels to the center in various cities could be the result of the differences in the "industrial structure" cities, and differences in the degree of economies of scale in the various industries. Different cities specialize in different industries, and it could well be that

8) Niedercorn and Hearle Ibid , p.3

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different industries require different degree of spatial concentration, and therefore "explain" or "justify" differences in the concentration of employment (in the same way as differences in the economies of scale in various industries "explain" or "justify" different plant size for efficient operation).

We know very little on the subject of economies of scale in a spatial context (i.e.: what is the nature of the economies which depend on spatial proximity - or continuity) though this is persumably the basic reason for the existence of cities[9]. Anyway whether this is the cause of differences between cities, it would be desirable to separate the effects of different degrees of concentration on transportation costs from the effects of size.

Unfortunately I do not have data on employment in Central Business Districts. However as Meyer Kain and Wohl[10] give data on the land area of central business districts in many cities, I used as a proxy of the degree of concentration in a given city the proportion of the

9) John F. Kain has recently expressed the opinion that "present unaerstanding of the determinants of the location of basic industry is too limited to dignify with an explicit representation" in their model. In a paper presented at the ASSA convention in Detroit December 1970. (John F. Kain "The N.B.E.R. Urban Simulation Model and Urban Economics in the 1970's" pp 11-12).

10) Meyer Kain and Wohl, "The Urban Transportation Problem", Harvard 1965.

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land area of the C.B.D. from total city area.

Another variable that might affect the demand for transportation, and therefore the input of resources in the transportation industry, is income. It can be expected that if the level of income is different between cities there might also be differences in proportion of resources devoted to transportation (if the demand for transportation is income elastic).

I can now restate the hypothesis to be tested :
The proportion of land used for transportation T in city i is directly related to :

1) The size of the population in that city Ni.

2) The proportion of the area of the C.B.D as a fraction of total city area Bi.

3) The level of income in that city Yi.

Generally this can be written in the following form:

$$T_i = f(N_i, B_i, Y_i) \quad f_n > 0, \quad f_b > 0, \quad f_y > 0 \qquad (101)$$

a linear approximation of (35) is:

$$T_i = a_0 + a_1 N_i + a_2 B_i + a_3 Y_i \qquad (102)$$

The hypothesis is that : $a_1, a_2, a_3 > 0$

The form of equation (41) above suggests that a logarithmic regression should do better but the linear version was found to be as good.

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At a preliminary stage of the analysis I found that for the smaller of the cities no pattern could be established. Colin Clark commenting on these same data said that :"Outstanding in the smaller towns is the very large amount of space wasted (or at any rate prematurely dedicated) to roads"[11]. This may indicate that there are increasing returns to scale in the use of land for roads in smaller cities, resulting, perhaps, from the fact that streets of some minimum width are necessary even with little traffic. This contention would imply that the proportion of land devoted to streets should decline with size in small cities. However this relationship is not detectable in the data either. Therefore I decided to limit the regression analysis only to the 15 largest cities with population of more than a million. The data on these 15 cities are presented in table 3.

The data on land use refer to the cities in their legal boundaries, which persumably contain only part of the city as an economic unit, consequently I tried two alternative measures of population size: the population of the city proper and the population in the metropolitan area. The former measure: population in the city proper did better in the regressions (see table 4). Perhaps this

11) Colin Clark "Population Growth and Land Use" Mcmillan 1967p. 353.

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Table 3 : Data on Land Use in 15 Cities

City	Area (acres)	Percentage of area used for streets	Percentage of area of C.B.D.	Population in city (thousands)	Population in S.M.S.A (thousands)	Income (1950)
New York	204681	30.1	2.8	7793	10695	$2080
Chicago	144064	23.9	0.4	3540	6221	2085
Los Angeles	293034	14.4	0.3	2479	6743	1929
Philadelphia	86975	19.3	1.6	2003	4343	1732
Detroit	89343	27.9	0.7	1775	3762	1990
Baltimore	51355	19.5	1.0	943	1727	1692
Cleveland	49230	16.3	2.6	884	1797	1950
St. Louis	39836	22.1	1.9	857	2060	1787
Washington	39362	31.1	9.9	827	2002	2015
San Francisco	30850	24.7	2.7	781	2783	2007
Boston	30359	18.9	2.9	717	2589	1655
Dallas	182704	12.7	0.2	680	1084	1838
Pittsburgh	36889	17.4	1.2	612	2405	1662
Seattle	55057	25.3	0.8	534	1107	1831
Cincinnati	49736	13.7	0.6	502	1072	1712

Sources : Columns (1) (2) and (4) are from Niedercorn and Hearle (1963), Tables 6 and 12.
The data for city population are their estimates for the same dates as the land use surveys (ranging from 1948 to 1962 but mainly around 1960).
The data for Philadelphia are from Manvel (1968) pp. 46-47.
Column (5) U.S. Bureau of the Census "County and City Data Book 1962" Table 3 pp 432-455.
Column (6) "Metropolitan area Incomes 1929-66" Survey of Current Business, August 1968 Table 1 pp 32-37.
Column (3) Actual area from Meyer Kain and Wohl "The Urban Transportation Problem " (1965) p. 86 divided by column (1).

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Table 4 : Results of Regressions

A. Linear Regressions

Equation	R^2	F	Degrees of Freedom	Coefficients: (1) Population	(2) Share of C.B.D	(3) Income	Variables used: Population	Income
(1)	0.49	5.7	(2,12)	1.3 per million	0.88	----	city	---
t				(2.10)	(2.71)			
(2)	0.55	4.5	(3,11)	0.8 "	0.73	0.012	city	average
t				(1.01)	(2.17)	(1.24)		
(3)	0.54	4.3	(3,11)	1.2 "	1.05	0.003	city	median
t				(1.91)	(1.78)	(1.10)		
(4)	0.45	4.8	(2,12)	0.8 "	0.89	------	S.M.S.A	------
t				(1.78)	(2.66)			
(5)	0.53	4.1	(3,11)	0.4 "	0.72	0.014	S.M.S.A	average
t				(0.68)	(2.06)	(1.39)		
(6)	0.48	3.4	(3,11)	0.7 "	1.11	0.003	S.M.S.A	median
t				(1.43)	(1.76)	(0.87)		
B. Logarithmic Regressions								
(7)	0.57	5.0	(3,11)	0.06 "	0.16	1.3	city	average
t				(0.68)	(2.74)	(1.52)		
(8)	0.54	4.3	(3,11)	0.11 "	0.14	1.0	city	median
t				(1.48)	(2.27)	(1.16)		
(9)	0.57	4.9	(3,11)	0.05 "	0.15	1.4	S.M.S.A	average
t				(0.53)	(2.64)	(1.81)		
(10)	0.50	3.7	(3,11)	0.10 "	0.13	1.2	S.M.S.A	median
t				(1.08)	(2.04)	(1.18)		

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is the result of the differences in the time of the estimates. The estimates of city population are synchronized with the land use surveys, while the S.M.S.A. population estimates are as of 1960.

I experimented with a number of measures of income: average per capita income, and median family income. Because of the very long time that is necessary to achieve adjustment in land use, I tried also to use data on income in 1950 (as well as 1960). The results of the regressions are quite sensitive to the kind of measure that is used for income. This is perhaps the result of the correlation between per capita income and city population (which is 0.56) relative to median family income (which is only 0.13).

Results of the regressions of the proportion of land used for streets on: (1) the population in each city, (2) the share of C.B.D. in city area and (3) income are presented in table 4.

The coefficients have the right signs, but admittedly the levels of significance are rather low. In particular the inclusion of income in the regressions lowers the value of the t statistic of the population coefficient from more than 2 (equation(1) to about 1 (equation 2).

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In terms of the partial correlation coefficients of the second equation in table 4 the differences in population "explain"only about 1/12 of the variation of the percentage of land used for streets while variation in the share of C.B.D. "explains" about 1/3 of that variation, and income "explains" about 1/6. 45 per cent of the variation remain "unexplained".

Since Los Angeles is considered some times to be an exception to the general pattern of other cities I tried to exclude it from the regression. This has indeed raised the values of R^2 (and of the partial coefficients). For instance: in equation 3: R^2 becomes 0.65 (F= 6.1) and the t statistic for the coefficient of population becomes 2.31. In equation 8: R^2 becomes 0.63 (F= 5.7) and the t statistic of the population coefficient - 1.99.

Niedercorn and Hearle collected also data on the amount of land that is "undeveloped" in each city, that is to say "vacant". They have also presented data for land uses as percentages of only developed land. There are reasons to think that the amount of land left vacant in a city is determined within the same framework of allocating land to all other purposes. Therefore I do not find it surprising that when I used as a dependent variable in the regressions the proportion that land used for streets takes out of developed land (instead of total city land

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area), the results were at even lower level of significance than those in table 4. Philadelphia was left out of these regressions, since these data are not available from Manvel[12].

I tried also to capture the effect of the difference in the share that the "legal" cities (for which we have data) each have in the "true" city. The smaller is that share - that is to say the more "central" is the central city, it is expected, as explained above, that the density of land use would be higher, and that streets and roads will take greater share of land.

I tried to capture this effect by including in the regressions a measure of the share of the central city in the metropolitan area. As a measure of this share I used the fraction of the population of the city from the population of the S.M.S.A. However the inclusion of this ratio - as a variable in the regression did not work too well. It did not improve the correlation coefficients in a significant way.

In summary I have to admit that the results are disappointing, although they do not contradict the basic hypothesis, the support that they give it is rather weak. The low levels of significance could be interpreted to

12) Manvel A.D. "Land Use in 106 Large Cities" Research Report No. 12 Prepared for the Consideration of the National Commission on Urban Problems 1968.

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mean that the proportion of land devoted to transportation fails to rise with city size either because factor substitution keeps it from rising, or because other simplifications assumed in the model are too unrealistic. It might be worthwhile to mention some of the causes of variation which could not be captured in this analysis.

On the one side there are differences between the ideal cities of nice circular shape assumed in the model, and the real cities which we observe. Cities are not circular. Their shpaes are "disturbed" by topographic conditions and by historical developments which cannot be easily changed.

The period of time necessary for adjustment in land use is very long, so that at any given time we can expect to find considerable deviations from the long run equilibrium positions to which our theoretical expectations apply.

On the other hand the data is far from perfect: The definitions of the categories of land are not specific enough - and perhaps not uniformly applied in all cities. The surveys in the various cities were carried in different years. The data refer to cities in their legal boundaries which leaves out in each case a different (and unknown) portion of the city. Perhaps it is surprising that in spite of these difficulties such a simple model can explain even partially differences in the pattern of land use.

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5. Deviations from Optimality

At this point it is necessary to point out that all of the above models, designed to analyse the allocation of land for urban transportation, assume that roads are perfectly divisible, and that they are perfectly priced, at prices which reflect - at each point - the opportunity cost of using land - at that point - for other purposes.

In a market economy efficiency in the allocation of resources requires that resources be priced at marginal costs. Consumers ought to pay the marginal costs of their decisions to use additional unit of any resource. In the case of land used for transportation this means that people ought to pay for the use of each unit of land a price that reflects its opportunity cost. Since the price that ought to be charged for land used in transportation should also vary from place to place. In particular, if the value of land is a function of its distance from the center of the city, then the cost of land used in transportation should also vary with distance from the center.

When a person decides to buy a house at some distance from his place of work, he also decides to use the land used for roads leading from his house to his place of work. If roads were completely divisible this would not

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pose a problem, and he would have to buy together with his new house a lane of land from his house to his place of work. The width of the lande to be determined by the technology of transportation to be used, and the price of land. The price of land will depend on its exact location, i.e.: does it pass through more or less expensive areas of the city.

If this scheme were possible then it would have presented to the residents of the city prices which would have reflected opportunity costs of land, and would have given them incentives to economize on the use of roads in general, and on the use of expensive land for roads in particular.

However this scheme is not practiced because the fact is that most roads are provided by the public sector, and are not priced in a way which remotely resembles our perfect scheme. This has some important implications both to the allocation of land for roads and for the size of cities.

For our purpose it does not matter whether roads are indeed pure public goods according to Musgrave's or Samuelson's definition. It suffices that exclusion is generally not practiced; "either because of technical difficulties or because it is considered inefficient or

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undesirable"[13]. The result is zero pricing for land used for roads.

Now if the public sector does not price the use of land for roads this is likely to introduce a downward bias to the price of transportation as perceived by the consumers . I will argue that this in turn might, under certain circumstances cause cities to become more spread out than otherwise.

Whenever a good or a service is provided at zero price we should expect its demand to increase to the point that the marginal utility that the consumers derive from it is driven to zero. If the consumers have to incur some cost in order to be able to enjoy the service they will demand that quantity, at which their marginal utility from that service will equal that cost. For instance if the beach is free, people would like to enjoy it to the point where their marginal utility from additional unit of sun bathing will go to zero. On the other hand if there is some cost to staying there longer (the margin being an additional hour) or to getting to the beach (the margin being going to the beach another time), then the quantity of sun bathing that people demand, will be the quantity at which their utility from a marginal unit equals to its cost.

13) Julius Margolis "The Demand for Urban Public Services" in Harvey S. Perloff and Lowdon Wingo (editores), "Issues in Urban Economics " Johns Hopkins University Press. 1968 p. 545.

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As Julius Margolis has emphasized, "if a good is supplied at zero price we can expect an excess demand at that price". "The response to excess demand is that some sort of rationing (other than price) is adopted, such as congestion"[14].

In the short run, when the capacity of the roads can not be changed, congestion can regulate the traffic and serve as a "rationing" mechanism. Most of the literature in this area was devoted to the investigation of the implications of this mechanism and its shortcomings.[15]

On the other hand in the long run the capacity of the roads can be changed, as more land can be allocated from other uses to roads.

The "long run" effects of imperfect pricing of roads have received relatively little attention. I would like to analyse in this section the "long run" implications of zero pricing of roads. The analysis suggests that at least in certain circumstances this will lead to cities becoming more spread out than otherwise. The analysis which is presented in the rest of this section is devoted to this purpose.

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14) Margolis, **Ibid**, p. 545.

15) For instance Mills (in a forthcoming book) analyses the implications of having a fixed roads network on the structure of cities. Specifically, he analyses the effects of having a road system whose "design capacity" is fixed at some proportion to total land area at each ring, and analyses the effects of this restriction on commuting costs and housing location.

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congestion is not agreable to most people. It creates an outcry from the public, or from those who find themselves particularly hurt by this "mechanism", and the authorities are called to correct the situation by providing more (free) roads.

At the risk of repeating the obvious, I might repeat here that the existence of excess demand for roads when roads are privided at zero pricing cannot indicate whether there was any excess, or whether there was actually excess supply of roads, if roads were priced at full cost.

Actually if roads are provided under zero pricing.- there is reason to suspect that a situation of over supply exists, that is to say: the quantity of roads that is provided is more than the quantity that would be called for if the utility from a marginal unit or road space were to equal its opportunity cost in other uses.

For instance: Suppose that the public consider delays beyond some level of congestion to which they were used in the past "intolerable", and that they call on the authorities to provide more road space. Economic efficiency would have required that at this stage a comparison be made between the cost of providing more road space,

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(including the value of the land in alternative uses) and the value of the benefits in terms of the marginal cost of the delays imposed on the users of the roads.

But suppose that instead, the practice of the traffic authorities is to aim at providing road space so that the traffic will flow at speeds that should not fall below some conventional or existing standard (say 30 mile per hour), or suppose that when they observe or project an increase in traffic they will accomodate the additional traffic by providing more roads that will enable it to flow at the existing convention speed. If these rules are followed we should expect that the resulting allocation of land to roads is biased and that there is an excessive supply of road spce. More generally the system may produce and consume too much transportation services.

This is because as the demand for transportation increase over time the authorities do not let its price to the public, to go up (to reflect its increased cost), but rather attempted to keep its price effectively constant, by providing more road space

One can stop at this point with the conclusion that zero pricing of roads introduces a distortion in the use of factors of production in the transportation industry:

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too much land is used relative to other factors(time and capital).

This same conclusion was articulated by W. Vickery[16] though he arrived at it through a different reasoning.

However there is another possibility. If we take a general equilibrium view of the city, this is not the end of the story. In a general equilibrium model of a city the volume of traffic should be linked to the pattern of land use both being simultaniously determined. Within such a system we should ask what are the implication on the pattern of land use in the city of zero pricing of roads.

Generally if transportation is under priced - i.e.: people perceive of the price of transportation as being lower than what it is really, we should expect cities to tend to grow excessively and to become more spread out than otherwise. This statement can be made both more rigorous and clear, I hope with help of the model that was presented in the second section of this chapter.

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16) William Vickery: "General and Specific Financing of Urban Services" in Howard G. Schaller (editor), "Public Expenditure Decisions in the Urban Community" Resources for the Future, Washington D.C. 1962 p.62-90.
(-)"Pricing as a tool in coordination of local transportation in "Transportation Economics" N.B.E.R., New York 1965 p. 275-291.
(-)"Optimization of Traffic and Facilities" Journal of Transport Economics and Policy, May 1967 pp 125-136.

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If the practice that was described above - that the traffic authorities aim at providing road space so that the traffic will flow at speeds that should not fall below some standard (say 30 mile per hour) - is an approximation of actual practices, it implies that in the long run transportation behaves as if it is a fixed coefficients activity : both speed and the amount of land that is used to accomodate a given flow of traffic (say 1500 cars per hour) will appear to be rigid[17].

Therefore we can investigate the implications of these practices in models which assume fixed coefficients in the transportation industry - similar to the models that were presented in the previous sections of this chapter.

Let us assume for instance that the transportation of a person through a band of an infinitissimal width requires a units of land and b units of time (or other variable inputs whose price is constant). The use of these coefficients assumes away the possibility of factors substitution in the production of transportation. transportation costs ofcourse will depend on the price of the land which is used for it. The costs of going u miles will be:

17) Whether this is a tolerable approximation of reality is a question of judgement. A description of relevant facts can be found in Meyer Kain and Wohl (1965)/Op.Cit. pp 69-81 and also pp 557-558.

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$$T(u) = b\,u + \int_0^u aR(u')du' \qquad (1)$$

where R(u') is the price of land (rent) at all points from 0 to u.

Then we can ask what happens if people are not charged for the use of land for roads, so that they perceive of the cost of transportation as being only the cost of the time (and other variable inputs), that is in our notation. They think that:

$$T(u) = b\,u \qquad (2)$$

In order to investigate this question we can incorporate these two alternative cost functions in alternative models of city and find out what are their implication.

It should be pointed out that incorporating the cost function (1) is more complicated than any of the models that were presented above, when we have considered separately the cost of time and the cost of land each at a time.

Let us assume that we have a city of N people. All the residents of our city commute daily to the center. They live in houses which are located in a circle around the center in all directions. If houses are produced according to a Cobb-Douglas production function:

$$H = AL^{\alpha} K^{\beta} \qquad \alpha+\beta=1 \qquad (3)$$

and if we assume perfect competition in the housing industry then the price of houses at every distance from

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the center will relate to the price of land R(u) in the following way:

$$P(u)=B\ R(u)^{\alpha} \qquad (4)$$

If we assume in addition that the residents of our city have equal incomes and tastes, and that therefore they live in houses (apartments) of equal size, then we shall have as an equilibrium condition in the housing market the condition that the price of housing should decline with distance from the center at a rate equal to additional transportation costs. i.e.:

$$P'(u) = -T'(u) \qquad (5)$$

Differentiating (1) and (4) and substituting into (5) we get:

$$b + a\ R(u) + \alpha B R(u)^{\alpha-1} R'(u) = 0 \qquad (6)$$

This is a non linear differential equation in R(u) and unfortunately I do not know how to obtain an anlytic solution to this equation.

If I could obtain an analytic solution to this equation, it would have been possible to compare the resulting rent function, and the density function which is consistent with it, with the rent and density functions which will result if the residents of our city consider only time costs (and ignore the land costs of transportation), i.e.: they behave as if (2) is the transportation cost function. The rent function and density function which will

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result in that case were derived and presented as the first model in a previous chapter.

My conjecture is that the a model with the cost of transportation function (1) will show for cities of equal size higher rents and densities near the center, and lower at the edge of the city.

This conjecture is strengthened by the analysis of the simpler model which is presented in the following section.

Let us consequently make a further simplification, and assume that the residents of our city live in houses which occupy constant size lots, that is to say that housing is also a fixed coefficients industry.

By making this assumption we assume away the possibility of adjustments in the intensity of land use, however we retain the possibility to calculate the differences in the patterns of rents.

Let us assume that each person occupies a lot of size s (i.e.: he occupies s square feet of land). If the costs of transportation are represented by equation (1) then equilibrium in the housing (land) market implies that

$$a\int_0^u R(u)du + bu + sR(u) = C \qquad (7)$$

differentiating we get this time:

$$aR(u) + b + sR'(u) = 0 \qquad (8)$$

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which is a differential equation in R(u).

The solution of this equation is:

$$R(u) = (Ra + \frac{b}{a})e^{\frac{a}{s}(\bar{u}-u)} - \frac{b}{a} \qquad (9)$$

where: $\bar{u}$ = the radius of the city.

Ra = agricultural rent, and when we use as an initial condition the assumption that rent at the edge of a city (at $\bar{u}$) equals agricultural rent (Ra).

We can now compare this rent funtion with the one that will result from assuming cost function (2). The equilibrium condition in the housing (land) market will now be:

$$b + s R'(u) = o \qquad (10)$$

which implies (using the same initial condition):

$$R(u) = Ra + \frac{b}{s}(\bar{u}-u) \qquad (11)$$

Let us now compare the rent that is implied by these two rent function say at u = 1, which is say the boundary of the C.B.D. So that $\bar{u}$ is much larger than 1 . $\bar{u}$ is kept constant throughout this comparison.

The value of (9) at u=1 is:

$$R(1) = (Ra + \frac{b}{a})e^{\frac{a}{s}(\bar{u}-1)} - \frac{b}{a} \qquad (12)$$

The value of (11) at u = 1 is:

$$R(1) = Ra + \frac{b}{s}(\bar{u}-1) \qquad (13)$$

The difference (12) - (13) is:

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$$Ra(e^{\frac{a}{s}(\bar{u}-1)} - 1) + \frac{b}{a}(e^{\frac{a}{s}(\bar{u}-1)} - 1) - \frac{b}{s}(\bar{u}-1) \qquad (14)$$

If this expression is positive than the value of (9) at u=1 is higher than that of (11).

According to Taylor's series:

$$e^{\frac{a}{s}(\bar{u}-1)} = 1 + \frac{a}{s}(\bar{u}-1) + \frac{1}{2}(\frac{a}{s}(\bar{u}-1)^2 + \ldots \qquad (15)$$

Substituting this into (14) it is evident that (14) is positive for sufficiently large values of $\bar{u}$.

Thus we have seen that - in this model - if people consider only time (and other variable) costs of transportation and ignore the land costs of transportation which are persumed to be taken care of by the public sector, then rents near the center of the city will tend to be lower.

In this model this cannot lead to any change in the intensity of land use, because this is assumed to be fixed. The difference in the pricing regime cannot have any "real" effect in this model. The density is constant. The radius of the city is determined by the requirements as determined by the coefficients a and s.

However if we were to allow variability in the proportion of land that is used for housing (like in equation (3)); and is we assume that this proportion is determined by relative price of land; then we would

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expect that the difference in the pricing regime will have a "real" effect. Specifically if rents are lower at a given point according to one pricing regime, we would expect that to lead to lower density of land use at that point. So that the "under pricing" of transportation will lead to both lower rents and lower densities near the center of the city,

Once we allow variable proportions, the radius of the city ($\bar{u}$) can change. A given population can now be housed at a higher or lower density, and the radius of the city is not independent of the pricing regime. It should be expected that with higher rents and densities near the center, the city may shrink in area (for a given population) relative to the area it would occupy under "zero" pricing for roads.

The idea that distortions in the pricing of transportation lead to too low rents near the center of cities was articulated several times by Vickery[18]. However he arrives at this conclusion through a completely different reasoning. He uses this conclusion to argue that land market values are too low for calculating benefits and costs of road improvements, and that the usage of these too low values, causes land to be used lavishly in transportation (and persumably in other uses also).

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18) W. Vickery, Op.Cit.

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Chapter IV
Optimal Service Areas for Provision and Financing of Local Public Goods

As noted in Chapter II one of the possible attractions of the centers of cities is the location there of public facilities. The last section of this chapter describes a model of a city which has some public facility at its center, and discusses the efficient organization of land use around such a facility.

A "local public good" is a public good which can be enjoyed only by people in a particular area. The classic bridge is located at a particular place (across a particular river), and it is enjoyable only at that place, so that only people in that area, who might want to cross that river, can ever use it.

A local public good is thus a public good whose benefits are available only to people within a limited area, and has no effects outside that area.

The benefits can be available in a uniform way within this area, or they can decline with the distance from the point of production. To use Musgrave's words: the benefits can be "homogeneous within a given radius around the

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service center and zero on the outside". Or there could be "a tapering off of benefit intensity with increasing distance from the service center". "A streetlight gives more protection close by than further away and so forth"[1].

The implications of these spatial limitations on the incidence of benefits from local public goods were first investigated by Charles M. Tiebout and also by R.A. Musgrave, in his recent book[2]. The purpose of this chapter is first to review briefly the analysis of Tiebout and Musgrave and then to criticize some of their conclusions and suggest some extensions in the case of declining benefits[3].

The first two sections of this chapter are devoted to a summary of the analysis of the case of uniform benefits. The third section surveys the analysis of

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1) Richard A. Musgrave, "Fiscal Systems", Yale University Press, 1969. p. 296.

2) Charles M. Tiebout, "A Pure Theory of Local Expenditures", Journal of Political Economy. October 1956. pp 416-424. Charles M. Tiebout, "An Economic Theory of Fiscal Decentralization" in National Bureau of Economic Research "Public Finances Needs Sources and Utilization" 1961. pp 78-96.
Richard A. Musgrave, op.cit.

3) It is perhaps worth emphasizing that this is a different problem from the much more often discussed problem of the external spillovers of benefits from one local authority to another.

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Tiebout of the determination of the optimal size of service areas in the case of declining benefits. It is pointed out that this problem is very similar to the problem of finding an optimal market area for firms with fixed costs. The fourth section considers the problem of the taxation which is necessary to finance local public goods. The schemes that were suggested by Tiebout and Musgrave are rejected as they ignore the implications of the fact that people can move, and that locational preferences are reflected in site rents. Furthermore, the literature on local public goods seems to have ignored the role that land site rents can play in the efficient use of space. Attention is called to this aspect of the problem.

The last section of the chapter presents a model in which the optimal size of a service area (a city) is determined together with the pattern of land use and rents. The model used is the one which was developed in chapter II, and it allows for greater efficiency to be achieved through changes in the intensity of land use at various distances from the service center.

Following Musgrave we can start by considering the question of the optimal provision of local public

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goods from the point of view of the "Allocation Branch"; assuming that questions of distribution are being dealt with on the national level.

Distributional measures if not accepted voluntarily by the people affected can be applied only by compulsion. If applied by compulsion it should be applied by such political entities which have sufficient control over their citizens to be able to enforce their policies, and to avoid mass avoidance. Local governments are obviously lacking the power to enforce strong distributional measures. As in the other chapters of this work, we will mainly assume, in the formal analysis that will follow, that the inhabitants of our "cities" have equal incomes and tastes. Only occasionally we will indicate the kind of implications of relaxing these assumptions.

Let us now follow Charles M. Tiebout, and ask what is the spatial structure that will result in efficient provision of local public goods.

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1. Uniform Benefits

Let us first consider the case of a local public good where benefits are limited to a specific area but which are available over this area in an even way.

As examples for this case Tiebout suggests that: "Police patrol cars provide, more or less, uniform protection for all residents throughout the precinct covered. Trucks which spray against mosquitoes are likely to spray uniformly throughout the Municipality"[4].

In addition to the usual problems of deciding on the quantity of the public good (the level of the service) and how to share its cost, the optimal production of a local public good involves another dimension, and that is the number and the size of the producing units.

The area that benefits from each service center (each police station) is not technically constant. But is itself subject to some choice, and the choice of the size of the service area has a bearing on the cost at which the service is provided.

Tiebout's formulation of the problem is illustrated by the following example:

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4) Tiebout (1961) p. 80.

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"Assume a city of 100 square miles in which the population is evenly distributed, there are no differences in income within the population, and further, a uniform demand for police protection. Assume the demand is known. Further, suppose that police protection is a pure public good within a patrolled precinct. That is to say, the patrol car which protects your house also protects mine. Thus, total output $X_p = X_1 = X_2 \ldots = X_n$ where n is the number of consumers who all consume in common. A unit of output is some number indicating a certain amount of protection spread evenly throughout a police precinct. Thus, to say a five-square-mile precinct has 600 units of output implies that each resident receives 600 units of protection. (We grant that it is difficult to define units of output - units of production - in operational terms. If a patrol car passes everybody's house three times a day instead of twice, cet.par. output has gone up by some amount.)

The problem is to set up an optimum number of precincts within the city and provide uniform police protection. (Whether these units are independent police forces or precincts is not an issue. The same type of analysis applies to both cases. It is analogous to firm and plant economies)"[5].

5) Tiebout (1961) p.82 .

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Among the variables that affect the cost of providing a particular service are the level of the service to be provided L (measured by units of output), the population density D; and the size of the service area R (measured by the geographic area)[6].

As the number of people who benefit from the availability of a given public good increases, the cost per capita falls (in this sense we can say that there are always economies of scale in the production of public goods). For a given density of the population we can increase the number of people who benefit from a given public good only by increasing the size of the service area. However, for many local public goods, after a certain size is reached, per capita costs stop falling and eventually start to go up, so that the per capita cost curve becomes U shaped.

Tiebout simply assumes that the average costs curve of providing any given level of service is U shaped, with respect to the size of the area covered (area measured in square miles). Now Tiebout's notion that the geographic size of the service area is by itself an important variable affecting costs, suggests that somehow increasing distances within the service area either

6) This notation is taken from Musgrave Op.Cit p 297.

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increase costs (say by increasing the distance that patrol cars have to drive from the station to their assignments etc) or decrease benefits (say by the longer time it will take for the police or the fire department to answer a call). This calls for explicit consideration of distances in determining the optimal size of a service station. This will be done in the next section (on the case of declining benefits).

Another approach to the problem is suggested by Werner Z. Hirsch[7]. In his formulation the relevant variable which determines the costs of providing a great many local services is the size of the population served (the number of the "customers" of each station)[8].

According to Hirsch the cost structure of a number of so called "municipal services" such as police protection, libraries, fire protection and schools is characterized by the significance of fixed costs, on the one hand, and of a limited capacity on the other hand. These together account for a U-shaped average cost curve for these services[9].

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7) Werner Z. Hirsch "Expenditure Implications of Metropolitan Growth and Consolidation" Review of Economics and Statistic, August 1959 pp 232-241.

8) In his earlier 1956 paper Tiebout follows this formulation, when he defines "an optimal community size in terms of the number of residents for which this bundle of services can be produced at the lowest average cost"Op.Cit p 419.

9) Hirsch is aware that increasing territory and distances adds costs. He states for instance "The police department may seldom operate in the rising expenditure phase. Location considerations produce diseconomies of scale and in turn lea to the opening of branch stations." Ibid p.235

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Now, if we accept Hirsch's formulation, that population size is the relevant variable that determines costs, then these "local public services" are not proper public goods in the classic Samuelson's sense, and we cannot support Tiebout's insistence to carry the analysis on the assumption that they are strict "public goods".

The "public" nature of these services changes as we move from the falling section of their cost curve to its rising section. When the number of people served is small there might not be any marginal positive cost associated with serving an additional customer (marginal costs are zero, average costs are falling and there is no rivalry in consumption)[10].

On the other hand when these services are provided at the rising section of their cost curve, they do not conform anymore to Samuelson's classic definition of a public good. Since adding additional beneficiary - at this range - involves additional costs. However, since these services are typically provided by the public

10) William S. Vickrey "Decreasing Costs, Publicly Administered Prices and Economic Efficiency" in "The Analysis and Evaluation of Public Expenditures: the PPB System" Joint Economic Committee, Congress of the U.S. Vol I pp 119-148.

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sector, and since at smaller scales of operation there might not be any additional costs, we might still want to call these services public services [11].

If the most efficient scale of operation for a producing unit (minimum per capita cost) is reached at a size smaller than is necessary to supply the whole market area (say a city) (at a given level of service), then it is efficient to have a number of producting units (plants) each operating at minimum per capita costs. This is the case of the multi-plant firm discussed by Patinkin[12].

A very difficult question arises because of the indivisibility of a service unit (the number of units must be an integer). For example what if the optimal size of a police precinct is 5 square miles and the city contains 27 square miles? There is no answer short of comparing total costs, in this case, of 5 and 6 police stations. Patinkin discusses this problem also.

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11) Buchanan suggests another formulation of the problem. The size of the group is entered as an argument in the individual consumers utility functions. When the size of the group increases he assumes that their utility goes down.
J.M. Buchanan "An Economic Theory of Clubs" Economica February 1965 pp 1-14.

12) Don Patinkin, "Multiple Plant Firms Cartels and Imperfect Competition", Quarterly Journal of Economics, February 1947, pp 173-205. Also a "Note" August 1947 pp 651-657.

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We will ignore this difficulty, and assume that an integer number of units, each operating at minimum average costs, exactly exausts the total area to be served. Obviously, if it takes only one and a half units operating at minimum average costs to serve the whole area this is a serious problem. However, if the service in question is provided in relatively large number of centers, say more than 10 or 20, in a given area, then perhaps this is not such a serious simplification to ignore this problem.

Given the minimized costs of producing any quantity (level) of services, and given that people bear this cost, they can decide what is the level of service they wish to have (and to pay for).

Let us assume that all the people who benefit from our service have identical tastes and incomes. The marginal benefit that they derive from the service will also be equal. Let us suppose that through some Musgravian political process they manage to decide on the right level of the service -when their marginal benefits are exactly equal to the marginal cost of producing another unit of the service in question. Let us also assume that benefit taxes are set at the level of average per capita costs.

Setting benefit taxes at levels that will cover costs

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has many attractions, not the least of which is that the service will pay its own way. If taxes are set at a level that will cover costs (i.e.: at average costs) then only when the scale of the operation is at minimum average cost, they will coincide with marginal production costs.

2. A Digression on Differences in Tastes

Paranthetically, to the discussion of this chapter we can mention here that differences in tastes are easier to handle in this context than in the general case of a public good. Generally it is very difficult to reach an optimum decision because people will not reveal their true preferences and therefore it is impossible to assess people according to their marginal benefits.

First if a certain public good is produced at many locations, and if there are no major spillovers from one location to the other, i.e.: if it is a local public good according to our definition, then it is possible to separate the decisions on the production of this service at the different locations. To let the residents of each locality decide on the level of services that they wish to have (and pay for). Furthermore to the extent that there are no external spillovers from one district to another it is possible to subdivide a given locality to smaller

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districts, and to allow each district to reach autonomous decisions on the level of services that its residents wish to have, and thus reduce the discrepancy between peoples' marginal benefits from the given service.

Second, since people can migrate between localities, their movements can serve as a mechanism to reach better allocations. People who are not happy with the level of services provided in one locality, or the level of taxes, can migrate to another locality where the services provided suit them better. People can reveal their preferences and vote with their feet.

Migration will tend to cluster together people of similar demand for public services who will then be able to have more nearly that level of public services at which their marginal benefit equals the cost to them (and the marginal cost of production).

Thus to the extent that the benefits of some public goods accrue only (or mainly) within a limited area there is a possibility to obtain greater efficiency in the provision of these public goods through the grouping together of people with similar demand for public services, and letting these groups reach autonomous decisions.

However, this analysis has some serious limitations. It is important to emphasize that it is conditional on the

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assumption that the benefits do not spill over to outside areas. If there are external effects (if either benefits or costs accrue to residents in other areas) then efficient decision cannot usually be reached without the intervention of a higher level of government that embraces all the areas affected by the decision[13].

This gives one criterion for the delineation of local governments. Economic efficiency can be achieved if each unit of local government and its responsibilities will be defined so that there will be little or no external effects of its dicisions. To the extent that cities or metropolitan areas are natural benefit areas this justifies their autonomy[14].

Another serious limitation of this analysis is that it ignores considerations of income distribution. When these considerations are included in the analysis the problem becomes significantly more complicated[15].

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13) R.A. Musgrave Ibid p 299-302. Alan Williams, "The Optimal Provision of Public Goods in the Theory of Local Government" Journal of Political Economy, February 1966. pp 18-33. also: W.C. Brainard, February 1967 pp 86-90.

14) Werner Z. Hirsch, "Local vs. Areawide Government Services" National Tax Journal, December 1964 pp 331-339.

15) For an effort to include distributional considerations in this context see: Jerome Rothenberg "Local Decentralization and the Theory of Optimal Government" in M.J.E.R. "The Analysis of Public Output" 1970 pp 31-64. In that paper Rothenberg suggests four types of criteria with which to evaluate the optimality of local political jurisdictions:
(1) Minimization of political externalities (which can be achieved in small homogenious groups)
(2) Minimization of political externalities across jurisdictior
(3) Minimization of the resource cost of providing public output.
(4) Maximization of the achievement of social redistributive goals.

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3. Declining Benefits

The second case considered by Tiebout is that of a local public good whose benefits decline with the distance from the point of production :As before, if the number of people who benefit from the public good in question increases: the cost per capita falls. However, as we increase the size of the area to be served by each producing unit, the distance between the marginal beneficiaries (at the margin of the area) and the producing unit increases, and the benefits that they derive decline. By way of examples Tiebout suggests that fire protection, air raid sirens, emergency hospital treatment and so forth provide examples of benefits declining with distance.

This complicates the situation considerably. We have to balance one against the other, the savings from greater units of production against the decline in benefits due to the increase in the average distance between the point of production and the consumers.

A similar problem is a familiar problem in the theory of location of production in space. Generally if there are transportation costs as the market area of a plant is expanded, the fixed cost per unit of output falls, but the average transportation cost per unit of output rises, "regardless of any economies of scale in production"[16].

16) Martin Beckmann, Location Theory, Random House, N.Y. 1968.

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There will be some size at which average transportation costs will exactly counter balance the reduction in average production costs, and at this point the sum of average production and transportation costs will reach its minimum. The size at which this minimum is reached determines the most efficient number of plants to operate (and the size of the market to be served by each plant). Therefore "economies of scale in production are always consistent with an optimum scale of plant".

Tiebout applied an analogous criterion to our problem and suggested to choose that size which maximizes per capita surplus of benefits over costs. If the density of people in the area is uniform this will mean maximizing the average surplus of benefits over costs per area.

The application of this criterion means maximizing the sum of peoples' utilities. It assumes additivity of peopls' utilities. There are well known objections to the use of this criterion. It raises serious questions of distribution. In other words: its application means that some people may profit at the expense of others. In spite of these well known objections to the application of this criterion, it is widely used in applied welfare economics[17].

17) For a recent statement of the argument for its continued use see: A.C.Harberger "Three Basic Postulates for Applied Welfare Economics an Interpretive Essay" Journal of Economic Literature, September 1971 pp 785-797.

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We shall return to this problem later when we consider the problem of pricing (sharing of cost).

The decline in benefits (utility) with distance can be interpreted in several ways: it could mean some physical phenomenon such as the decline in the intensity of light with the distance from a lamp, or the fading of radio signals with distance from the station. It could also mean increase in transportation costs or in time that is required to get from the beneficiaries' homes to the center; such as the additional time of getting to a school or to a hospital. If these costs are negligible then we return to the case of uniform benefits within the benefit area. In practice it might be extremely difficult to evaluate this decline in benefits (increase in costs), particularly in the case of services which provide protection against an uncertain danger such as a fire. It is probably very difficult to evaluate, how much is it worth to live ten minutes from a fire station, rather then eleven minutes? Nevertheless following Tiebout let us assume that the rate of decline of the utility derived from fire protection with distance from the fire station is known and is translated into monetary terms.

In order to see how the principle of maximum average surplus is applied, let us first assume that the

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level of the services to be provided is fixed, and that the type and capacity of the production facilities (the type of the fire station in Tiebout's example) is also given. (These assumptions will be relaxed later).

Since we wish to concentrate on the problems of a public good, let us assume that at a given location there is no loss in utility if more people enjoy our public good. That is to say, if the density of the population within a given radius from the point of production (station) increases, the addition to the number of the beneficiaries does not diminish the utility of each of the individuals involved. In other words there is no rivalry in the consumption of this service. For instance if the service in question is fire protection, we assume that the number of people within the service area is such that the probability of two fires occurring at the same time is negligible. If the service in question is television, we assume that at the range of numbers of inhabitants in the area there is no interference in reception, and that therefore the benefit that one consumer derives from receiving the broadcast is independent of the number of other people who receive the same broadcast. It is sufficient if this no-rivalry holds at the range up to size of each service area. Also if the type and therefore the costs per station are given, then the greater is the area served by each station, the lower are per capita costs. On the other hand as the size of the

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area to be served from each center increases, the distance to the furthest beneficiaries rises and the benefits that they derive decline.

Suppose that the benefits per capita B decline in a linear way with the distance u from the center, that is: $B = a - bu$ (1)

Where B is expressed in monetary terms.

Suppose also that each service area is circular, and that by proper choice of units of measurement we define density = 1.

Then total benefits over a service area are:

$$\int_0^{\bar{u}} 2\pi u(a-bu)du = a\pi\bar{u}^2 - \frac{2}{3}b\pi\bar{u}^3 \qquad (2)$$

Where $\bar{u}$ is the radius of a service area.

If the only costs that we consider are fixed (per service center) then the surplus is:

$$S = a\pi\bar{u}^2 - \frac{2}{3}b\pi\bar{u}^3 - C \qquad (3)$$

Average surplus is:

$$\frac{S}{\pi\bar{u}^2} = a - \frac{2}{3}b\bar{u} - \frac{C}{\pi\bar{u}^2} \qquad (4)$$

Taking a partial derivative of average surplus with respect to $\bar{u}$ and equating to zero this gives:

$$\frac{2}{3}b - \frac{C}{\pi\bar{u}^3} = 0 \qquad (5)$$

The solution of this equation:

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$$\bar{u} = \left(\frac{3\,c}{2\pi\, b}\right)^{\frac{1}{3}} \qquad (6)$$

Gives the value of $\bar{u}$ that maximizes the value of the average surplus. This determines the optimal size of the area to be served by each service center (plant), and the number of such centers.

Assuming that each service area is circular, leaves out some areas unserved, in the interstices between the circles. These occupy approximately 9 per cent of the area[18]. We could assume that nobody lives in these interstices. This assumption will become pretty reasonable in the context of the last section of this chapter, where we allow for changes in density. If we required that all the area be served then the service area would have to be hexagonal[19].

Tiebout did not consider the implications of the possible variations in the capacity of the producing unit, and thus increasing the benefit area.

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18) See M. Beckmann "location Theory" p 46.

19) For the calculation of the size of the optimal hexagonal service area see:
M. Beckmann "Equilibrium Versus Optimum Spacing of Firms and Patterns of Market Areas" (mimeographed) Brown University, April 1970.
Beckmann presents an analogous calculation for the market area of firms with fixed costs.
See also : Dennis Capozza "Transportation Costs and the Provision of Urban Governmental Services" (mimeographed) University of Southern California, 1972.

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There are certainly cases when this is technically impossible or almost impossible. (Such as the case of TV stations). However, generally, varying the capacity of the service station is technically possible but involves costs. It is these costs thich limit the size of the benefit area of each producing unit, it is worthwhile to increase the scale of the producing unit, and the size of the benefit area until the additional costs of increasing the capacity of the producing unit outweigh the additional benefits that accrue by increasing the benefit area.

Since the number of beneficiaries (at a given density) increase at a higher power than the radius of the benefit area, if costs do not increase at a faster rate there will be no finite optimal size. On the other hand, if costs increase (at some size) at a faster rate, this will result in a finite optimal size.

In other words the size of the service area and the scale of the production will be finite only if the costs of extending the service area increase - at some size - at a faster rate than benefits. Alternatively, if - at some size - the benefits derived from the service decline at a faster rate, this will also result in a finite size of the service area. Actually these two formulations could reflect the same real phenomenon, that there are some objective difficulties in extending the service beyond

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certain limits. These difficulties can be reflected either in increased costs or in reduced benefits. It is these difficulties which effectively determine the size of the service area.

It should be emphasized that this conclusion results from our assumption that by increasing the "capacity" of the producing unit the benefits derived from it will increase over all the area and not only by people who live at the additional area that will be added to the service area. If we assume instead that the people who live near the center of the area do not get more benefits from the increased capacity, but that only people at the margin of the area, who are brought inside the service area benefit from the increased capacity, the situation changes. Indeed in many cases, one can argue that actually there is no greater benefit from being located near to the point of production; as the only thing that matters is whether one is within the service area or not; so that we return to the uniform benefits case.

4. Sharing of Costs

Now, let us assume that the optimum size and the distance between the operating units is given. The next question is to decide how to share the cost of the service between the beneficiaries.

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Tiebout suggested two schemes, both based on a declining rates schedule. "One scheme would tax each taxpayer in proportion to his share in total benefits." People who live nearer to the point of production get a greater share of total benefits, and they will have to pay a greater shre of the total cost. The surplus received by each individual will not be equal but proportional to the tax paid.

The second scheme would be to tax people so that each person's surplus is equal. Each person's tax will be his total benefits minus the average "surplus".[20]

This view that tax rates should decline was endorsed by Musgrave who states: "If benefit intensity declines in successive rings around the center, so should cost assessments. Different rings, locations, will then contribute at varying rates corresponding to their declining benefit shares. Translated into voting mechanism, residents of both the outer and inner rings should participate in the supply determination, but residents of the inner ring with a given taste and income will be called upon to contribute more than similar residents in the outer ring".[21]

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20) Tiebout 1961 p.90

21) Musgrave Ibid, p. 296.

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The difficulty with the rule of declining tax rates is that it does not recognize that one way for people to reveal their preferences is through their locational decisions. Furthermore it ignores the possibilities to achieve greater efficiency through migration.

Both Tiebout and Musgrave seem to have ignored the possibilities implied by the fact that people can move, and that their location is a result of their choice. Generally the way to evaluate people's preferences is through their behaviour. In this case through their locational decisions. The only way to evaluate the decline in benefits is through their impact on the relative desireability of nearer locations, and the decline in the value of land with distance from the fire station. If the benefits from a fire station decline with distance, people will prefer to live nearer to the fire station. Locations near the fire station will become relatively more attractive and their price will rise. The greater benefits will be reflected in higher rents that will be paid for land in nearer locations and these higher rents can in turn be taxed through a tax on the value of land.

The inefficiency that can be caused by the application of the principle of equal (or proportional) "surplus" can perhaps be seen by analogy with the case of transportation or delivery costs. If people are charged the

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same price for a good (say water) at their homes, without regard to the actual cost of delivering the good at different distances from the source of supply, then people will not have incentive to locate themselves nearer to the source. On the other hand if people were to be charged according to the actual cost of delivering the good to them they will have an incentive to economize on delivery costs and to locate nearer to the source (or at least to include this consideration in their locational decisions).

If people will desire to locate nearer to the source of supply, locations near the source will become relatively more attractive than far locations, and this will be reflected in higher prices (rents) for these locations. These rents which will accrue to land at nearer locations will reflect savings in delivery costs. Indeed I think that the availability of public services is one of the determinants, which is often neglected, of land values.

The ability of people to move and the resulting differences in rents can serve as an allocational device if there are differences in tastes. Allocative efficiency will be achieved when people who consume a lot of the good in question (and incur a lot of delivery costs) will find it profitable to pay higher rents and locate nearer to the source of supply, while people who consume less will locate further away and take advantage of the lower prices for land.

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In the case of declining benefits from Tiebout's fire station this means that people who put greater value on fire protection will choose to live nearer to the station; while people who put less value on it will take advantage of lower rents at further locations.

Thus it seems that greater efficiency can be achieved by uniform pricing (benefit taxation) within each benefit area. Benefit taxes should be levied at a level equal to the minimum average per capita cost of providing the service. Application of this rule may create higher land values near the centers of production of the service, which can in turn be partially taxed by land value taxation without any loss in efficiency.

5. A Model of Optimal Service Area with Factor Substitution

On another dimension of the problem higher rents will encourage movement toward greater density of people nearer to the source of supply. Assuming that average costs decline with density, and that density causes disutility, efficiency requires that density should be increased untill the disutility from increased density equals the possible savings in costs[22].

22) Musgrave Ibid, p. 298.

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Higher rents will indeed encourage greater economy in the utilization of land nearer to the source of supply, (by substitution of capital for land, in housing, for instance).

The following section is intended to illustrate the effects of the changes in the intensity of land use on the structure of the service areas and their size. It will also illustrate the interrelationships between density, land rents, and the optimum size of a service area.

If the reader is willing to abstract from other reasons for the existence of cities, and to assume, for argument sake, that cities exist only in order to enjoy indivisible local public goods; if the reader is willing to think about cities as agglomerations of people around some public facilities, then the model that will be presented in the following section is a model of optimal city size.

Imagine a country where the only reason for the existence of cities is the availability of some public facility (say a TV station) which is indivisible. This public facility is produced at a constant cost C with respect to number of people served. There is no limitation on the number of people who can enjoy the service (or alternatively the limitation is not effective at the optimum);

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so that as the number of beneficiaries grows, the average cost per person goes down.

Imagine also that the land area of the country is big, so that the "cities" are surrounded by reserves of open space, the value of which is negligible, so that the cities can expand freely in all directions.

The benefits that people derive from the public service decline with distance from it, and therefore - other things being equal - people prefer to live nearer to the center of service. This can be done at a cost. There are costs to crowding more people into a given area. Increased density may involve loss of utility - especially if we speak about increasing densities in urbanized areas. The costs are more visible if we look at construction costs. Housing more people on a given area, means that more houses should be built per unit of land, more capital is to be used (relative to land) in the production of housing. In urbanized areas that could be achieved by building high apartment houses instead of one level homes. However producing houses with more capital and less land is costly, because scarce resources (capital) is substituted for a free one (land).

The first problem, then, is to find the right degree of intensity of land use. The intensity of land use which

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will be optimal is that which maximizes the excess of social benefits over construction costs. In general the optimal intensity will vary with distance from the center of the city. It will be highest at the center, and will gradually decline.

Only then can we proceed to the next problem which is to find the city population size N that will maximize the average - per capita - surplus of benefits from the public service over the costs of providing it plus the costs of constructing the city.

Instead of assuming that each person (family) occupies a fixed size piece of land (which is what we assumed earlier when we assumed that density is constant, let us now assume that the inhabitants of our city each occupies a house (apartment) of equal size).

Suppose that the elasticity of substitution between land and other factores of production used in producing housing is one, and that this is a constant returns to scale industry. Then we can write the production function for housing in the Cobb-Douglas form:

$$H = A L^{\alpha} K^{\beta} \qquad \alpha+\beta = 1 \qquad (11)$$

Where: H = the quantity of housing produced.

L = the quantity of land.

K = the aggregate quantity of other factors of production (capital).

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We will continue to assume that benefits from the public good which is provided at some central point decline in a linear way (as in (1) above):

$$B = a - bu \qquad (12)$$

Equilibrium conditions for the individual household in our "city" will require now that the price of houses decline with distance from the center at the same rate as the value of the benefits from the public good which is provided at the center.

$$P'(u) = -b \qquad (13)$$

where $P(u)$ is the price of housing at distance u from the center.

If the price of housing were not declining at a rate equal to the decline (with distance) in the value of the benefits from the public good, there would be an incentive for some people to move and take advantage of this discrepency. Either to buy some benefits for less than it is worth (by moving in) or to save on the price of housing (by moving out).

The conditions described by equations (11) - (13) are formally the same conditions which characterized the model of chpter II (compare equations (5) and (22) there). Under these conditions we have seen there (see equation (24) there) that the price of land will decline with distance from the center according to:

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$$R(u) = \frac{b}{D}(\bar{u}-u)^{\gamma} \qquad \gamma = \frac{1}{\alpha} \qquad (14)$$

Where R(u) is the rent of land at distance u from the center and D is a constant.

$\bar{u}$ is the radius of the service area (the "city") as before.

It an also be shown that the number of houses that will be built at every ring at distance u from the center (and therefore also the number of people who live at each ring) is :

$$H(u) = \frac{2\pi u}{\alpha D}\left[\frac{b}{D}(\bar{u}-u)\right]^{\frac{\beta}{\alpha}} \qquad (15)$$

The total number of people who live in our city is:

$$N = \int_0^{\bar{u}} H(u)du = \int_0^{\bar{u}} \frac{2\pi u}{\alpha D}\left[\frac{b}{D}(\bar{u}-u)\right]^{\frac{\beta}{\alpha}} du \qquad (16)$$

Where $\bar{u}$ is the radius of the city.

$$N = E\left(\frac{b}{D}\bar{u}\right)^{\gamma+1} \qquad (17)$$

Where $E = \frac{2\pi D}{b^2(\gamma+1)}$

The total value of the benefits (from the public good) in the "city" (market area) will be :

$$B^* = \int_0^{\bar{u}} H(u)(a-bu)du \qquad (18)$$

$$B^* = \frac{2\pi u}{\alpha D}\left[\frac{b}{D}(\bar{u}-u)\right]^{\frac{\beta}{\alpha}}(a-bu)du \qquad (19)$$

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$$B^* = a\, b\left(\frac{b}{D}\,\bar{u}\right)^{\gamma+1} - \frac{2Db}{(\gamma+2)}\left(\frac{b}{D}\,\bar{u}\right)^{\gamma+2} \qquad (20)$$

The first term equal to a N, the second which is a measure of total reduction of benefits due to distance is equivalent to total "transportation costs" in this model. It can also be expressed as a function of N by substituting for $\bar{u}$ from (17) into (20).

The surplus of benefits over costs will now be (20) less the fixed cost of the public facility C and also construction costs of the houses - the cost of other factors of production K (besides land) used in the production of housing : $\int_0^{\bar{u}} w\, K(u)\, du$.

Using the condition that (under perfect competition) the value of marginal product of a factor equals its price we can find what is the quantity of K that will be used at every distance u from the center. (If the price of K is assumed to be a constant w, and the price of land is known from (14), and the quantity of land at each ring is given - this is sufficient to determine what quantity of K will be used at that ring).

$$w\, K(u) = \frac{\beta}{q}\, 2\pi u\left[\frac{b}{D}(\bar{u}-u)\right]^{\gamma} \qquad (21)$$

The total cost of K will be :

$$w\, K^* = \frac{\beta}{q}\int_0^{\bar{u}} 2\pi u\left[\frac{b}{D}(\bar{u}-u)\right]^{\gamma} du$$

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$$W_N^* = \frac{\beta}{\alpha} \frac{\mu}{(\gamma+2)} \left(\frac{b}{L} \bar{u}\right)^{\gamma+2} \qquad (22)$$

which is seen to be very similar to the second term of (20) (that which I called total "transportation cost"). It can also be expressed as a function of N by substitution from (17).

The surplus of benefits over costs will thus be:

$$S = a N \left(\frac{b}{L} \bar{u}\right)^{\gamma+1} - (2 + \frac{\beta}{\alpha}) \frac{\mu}{(\gamma+2)} \left(\frac{b}{L} \bar{u}\right)^{\gamma+2} - C \qquad (23)$$

We need to substitute for $\bar{u}$ in the expression because maximizing average surplus per unit of area is not anymore equivalent to maximizing average surplus per capita. Substituting for $\bar{u}$ from (17) into (23) gives:

$$S = a N - (2 + \frac{\beta}{\alpha}) \frac{b}{(\gamma+2)} G^{\frac{1}{\gamma+1}} N^{\frac{\gamma+2}{\gamma+1}} - C \qquad (24)$$

Where: $G = \frac{1}{L}$.

Average surplus (per capita) is :

$$\frac{S}{N} = a - (2 + \frac{\beta}{\alpha}) \frac{\mu}{(\gamma+2)} G^{\frac{1}{\gamma+1}} N^{\frac{1}{\gamma+1}} - \frac{C}{N} \qquad (25)$$

Taking a partial derivative of average surplus with respect to N and equating to zero this gives:

$$\frac{C}{N^2} = \frac{\mu}{(\gamma+2)} G^{\frac{1}{\gamma+1}} N^{\frac{1}{1+\alpha}} \qquad (26)$$

The solution of this equation gives the optimal population:

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$$N^* = \left(\frac{C(\gamma+2)}{DG^{1/\gamma+1}}\right)^{\frac{\gamma+1}{\gamma+2}} \qquad (27)$$

Where N^* is the optimal population size of each service area (or a city) which can in turn give us, by substitution from (17) the radius $\bar{u}$ that corresponds to this optimal population size, it is:

$$\bar{u}^* = \frac{1}{b}\left[C(\gamma+2)D^{(\gamma+1)}G\right]^{\frac{1}{\gamma+2}} \qquad (28)$$

Where $\bar{u}^*$ is the radius of an optimal service area (or a city).

A comparison of equations (27) and (28) with equation (6) indicates what are the differences between the Tiebout - Beckmann model described by equation (1) - (6) - when density was assumed to be fixed and uniform, and the system described by equation (11) - (28) which allows for variations in density to be determined endogenously. For instance, a rise in C (which is a measure of the size, or the degree of economies of scale of the public facility) will tend to increase the optimal size of each service area (or a city) in both cases. But while in the first model the increases in area and population are always, by assumption, proportional, the second model indicates a much quicker rate of increase of population than area (if γ is more than 1 which is always the case in this model, since α must be less than 1)[23].

23) Chapter II cites evidence which suggests that $\gamma = 4$.

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A rise in b which is the marginal cost of distance (interpreted either as transportation costs or as a rate of decline in per capita benefits), reduces the radius (and the area) of the optimal service area in the two models. But while in the Tiebout-Beckmann model it cannot affect the density, in the model of equations (11) - (28) it affects density, causes changes in the whole pattern of densities around the service center, and lowers overall density, (see equation (15).

A rise in b will therefore cause, in the second model, a much greater decrease in land area of each service area than the decrease in the population in each service area.

Indeed according to equation (27) the number of people in an optimal sized service area will decline with a rise in b at a power of a $\frac{1}{3}$ (when $\gamma=4$). According to (28) the area of land in each service area will decline with a rise in b at a power of $\frac{2}{3}$ (when $\gamma=4$), (remember that $G = \frac{b^2(\gamma+1)}{2\pi\rho}$).

On the other hand, according to equation (6) a rise in b will reduce both the area and population in the Tiebout-Beckmann model, at a power of $\frac{2}{3}$.

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Chapter V

The Effects of the Property Tax

Another problem that can be analysed in the framework of the models that were presented in the privious chapters is the effects of the property tax.

The term "The Property Tax" is used in the sense in which it is used in the U.S. i.e.: An ad valoren tax on real property (buildings and land)[1]. The tax applies in principle also to other types of property, chiefly business equipment and inventories. However, the effective rates of tax on these other types of property are significantly lower than those that apply to real property, "in part because much of it is exempted from tax legally or extralegally, and because some personal property in classes not exempted is simply not discovered by the assessor. Industries with a substantial personal property component are likely to enjoy lower overall effective rates than industries with a much larger proportion of assets in the form of real property"[2].

1) See Dick Netzer "The Economics of the Property Tax", The Brookings Institution, Washington D.C. 1966 p.11.

2) Ibid p. 25

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Moreover, "In nearly all states, the overall effective rates on farm real estate are well below the rates on non-agricultural real property"[3]. Thus in the U.S. the property tax is now mainly a tax on the value of urban real estate.

Because of the greater capital intensity of housing services relative to other industries, the property tax is especially heavy on housing. A tax of a modest rate of, say, 3 per cent on the capitalized value of a house can easily turn out to be a tax at a rate of 25 per cent of the annual rental of the same house. Actually, Netzer presents data which show that "housing property taxes in 1957 were equal to nearly 25 per cent of national income originating in private non-farm housing, and nearly one-sixth of personal consumption expenditure (Department of Commerce concept) for private non-farm housing. Property taxes are also sizeable proportions of money outlays for housing - monthly expenditures of homeowners and rental receipts of tenant-occupied properties. In most large metropolitan areas outside the South, property tax payments in 1959 - 60 averaged over 20 per cent of money expenditures for housing by owner occupiers of single-

3) Ibid, p. 28.

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family houses, and nearly 30 per cent in the New York, Boston and Buffalo areas"[4]. As Netzer points out, this is a non trivial tax. "It is noteworthy that the property tax on housing is higher in rate than any other generally used American consumption tax, except taxes on liquor, tobacco and gasoline"[5].

A change of this magnitude in the relative prices which face the consumer is bound to have considerable effects on the allocation of resources.

Throughout this chapter, we will discuss the effects of the property tax "while holding public expenditures constant in real terms"[6]. The analysis is a comparison of the effects of a particular method to finance a given level of expenditures, relative to an alternative, a neutral method of raising tax revenue, such as a poll tax for instance. In general, changes in the level of public expenditure will have effects which will interact with the effects of the tax. We abstract from these effects , and assume that expenditures are kept constant because we want to isolate the effects of a particular method of raising tax revenue - imposing property taxes.

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4) Ibid ,p.30. 5) Ibid.

6) The quotation is from Musgrave, who suggests that this procedure should be applied to the analysis of tax policy in general. See Richard Musgrave "The Theory of Public Finance" (1959) pp 211-213.

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The effects of the property tax may be conveniently separated into three different effects: The first effect is the substitution effect on the allocation of resources between housing and other industries; the second is due to the fragmentation of the taxing authorities; and the third is the effect on the internal structure of cities.

The first two effects have been discussed extensively in the literature on the property tax. On the other hand the third effect seems to have remained unnoticed[7]. The main purpose of this chapter is to discuss the third effect and to explain its mechanism.

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7) Actually I have been able to trace only one writer who seems to hint that it exists (Neutze 1969). Max Neutze "Property Taxation and Multiple-Family Housing" in Becker (1969) p. 115-128. Arthur P. Becker (editor) "Land and Building Taxes, their Effect on Economic Development" proceedings of a Symposium sponsored by the Committee on Taxation Resources and Economic Development , University of Wisconsin Press 1969, which contains 11 papers on the effect of the property tax.

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1. The Substitution Effect

The substitution effect of the property tax is perhaps the most extensively discussed effect of the property tax. With the price of housing being raised people will consume less housing and will substitute for it other consumption goods which are not taxed that heavily. This statement is correct unless either the demand for housing or the supply of housing are completerly inelastic (and there is no reason to believe that either is perfectly inelastic).

If housing services are taxed more heavily than other industries, it is inevitable that the price, to the consumer, of housing, will rise relative to the price to the consumer of other goods. Consumers faced with the higher relative price for housing are likely to choose to consume less housing than they would without the tax. Since resources used in producing housing services (particularly capital) have alternative uses in other industries, part of their quantity which would have been used in producing housing without the tax, will be shifted to some other industries.

Thus the property tax is likely to have a negative effect on the quantity of housing produced and consumed.

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It tends to divert resources away from housing into other uses. For lack of better term I call this change in the quantity of housing - the "substitution effect".

The literature on the "incidence" of the property tax, is concerned mainly with another aspect of the effects of the property tax: price adjustments. "The focus of the analysis of tax incidence is on how various tax regimes affect factor returns and commodity prices"[8]. With regard to the property tax specifically, the question is to what extent is the tax borne by the consumers (through higher prices) or by the owners of capital (through reduced returns).

In his recent summary of the theory of the incidence of the property tax, Mieszkowski[9] emphasizes the distinction between the effects of the property tax viewed as a global (nation wide) tax and the effects of property taxes imposed at varying rates in different places.

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8) Peter Mieszkowski:"Tax Incidence Theory", The Journal of Economic Literature, December 1964 p.1103. See also R. Musgrave "The Theory of Public Finance", (1959) pp 227-231.

9) Peter Mieszkowski, "The Property Tax An Exise Tax or a Profit Tax?",Cowles Foundation, Discussion Paper No. 304, November 1970.

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According to his reconciliation of the conflicting views in the literature on the question of the incidence of the property tax, the burden of the property tax is borne by the owners of land and capital at the national level, and by the consumers at the local level. "The basic distinction that needs to be emphasized is between the global effects of the property tax with its effects at the city or state level".

"At the national level the excise effects of the property tax may be of secondary importance, and overall profits are decreased by the average rate of tax. However, for a single city which may impose or increase the property tax the effects are quite different"[10].

This analysis of the incidence of the property tax, in the global, holds only if certain assumptions hold; two of which are worthy of special mention: a). The supply of capital in the economy is fixed, or at least is independent of changes in the rate of return (after tax) that owners of capital receive[11].

10) Mieszkowski, Ibid , p.13.

11) This assumption is made also by A.C. Harberger, "The Incidence of Corporate Income Tax", Journal of Political Economy, June 1962, pp 215-240.

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b). The property tax applies at the same rate to all industries. However this is not the case. As Mieszkowski admits : "Housing services may be taxed more heavily than industrial capital. If this is true the price of housing in general will rise relative to the price of industrial goods. There will be reallocation of resources and since housing is very capital intensive the price of capital will fall relative to the price of labor"[12]. The final result will thus depend not only on the rate applied to each industry but also on the elasticity of supply of each factor, and the elasticity of substitution between the factors in each industry.

The incidence issue is not essential for the rest of this chapter.

While it is interesting to find out who is the bearer of the "global" burden of the property tax, it is also important to call attention to other undesirable effects of the property tax which fall into the category of excess burden; that is to say: undesirable distortions in the allocation of resources, which could be avoided if another tax were used.

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12) Mieszkowski ,Ibid,p. 17.

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2. The Fragementation of Tax Authorities

According to Netzer there were more than 80,000 taxing units in the U.S. each with its own powers to impose its own rates !

As already noted there is a difference whether we examine the property tax as a global - nation wide - tax, or as a fragmented tax imposed at every location by a particular tax authority.

"For a town acting independently of other towns, the burden of the property tax will fall on its residents"[13].

"An increase in the town's rate of tax will increase the cost of capital (to that town) by the amount of tax "[14]. "Higher housing rents (and higher costs of other locally produced goods and services) are due to increases in the cost of capital resulting from property tax rates which are above the average for the country as a whole"[15].

These differentials might have condiserable undesirable effects on the spatial distribution of

13) Mieszkowski, Ibid, p.14
14) Ibid, p.13
15) Ibid, p.8

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economic activity. Netzer, for instance concludes that although the tax does not seem to have induced inefficient shifts in economic activity among regions of the country, it has significant effects on the spatial distribution of economic activity within metropolitan areas.

"The adverse consequences of the tax for the spatial distribution of economic activity within urban areas, in contrast, is readily apparent, heavy taxation of real property is a deterrent to rebuilding of the big cities, especially when it is high in relation to the prevailing rates in the suburbs"[16].

Netzer presents a considerable amount of data to support his conclusion that : "Older large cities - of the Northeast and Midwest - generally operate with higher effective property tax rates than their suburbs. Moreover, the level of effective rates in these cities is high in absolute terms"[17].

"Other things being equal, it clearly provides an incentive to locate investment in suburban areas in most large metropolitan areas, thus marginally

16) Netzer Ibid, p. 166.
17) Ibid, p.75.

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deterring renewal of the big cities physical plant on a private unsubsidized basis"[18].

These differentials are seen to create undesirable consequences: "These relatively high tax rates in central cities have consequences which reinforce the central cities plight. Relatively high tax rates on a tax base in which business property is a major component - can help speed the dispersal of economic activity from central cities to outlying areas. Relatively high tax rates on the limited proportion of the tax base consisting of high value housing can help speed this dispersal as well"[19].

He is certainly not the only one to hold this view, it has become conventional wisdom to blame the high property tax rates in central cities for a lot of their fiscal hardships.

It is important however to keep these effects in proper perspective. To use again Netzer's words: "This does not mean that property tax differentials have been major contributors to the decentralization of metropolitan areas in the past few decades, or

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18) Netzer, Ibid p.75.
19) Ibid, p. 123

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that they are currently of major consequence. The forces making for decentralization are so potent that it can be confidently asserted that the tide would not have been stemmed even has there been large property tax differentials in favor of central cities. In fact the decentralization process seems to have proceeded no less rapidly in those few areas in which the central city does appear to have an advantage in effective tax rates. Nevertheless, if tax differentials against the central city are widening, which may be the case, the chances become greater that local fiscal arrangement will have an unneutral locational effect: that businesses and residents who otherwise might have preferred a central city location, will choose instead a location in outer portions of the metropolitan area. And if, on balance, the migrants contribute more to tax revenue than the public service costs they occasion, the situation of the central cities is further worsened".[20]

To the extent that support of central cities and their rebuilding is a desirable social goal, "property tax differentials which further limit these possibilities

20) Netzer, Op.Cit.,p. 123.
This last point is discussed by Albert Teplin, "Fiscal Incentives of Metropolitan Location", Unpublished paper, Johns Hopkins University 1970.

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are surely worthy of concern, if for no other reason than the fact that governments can revise their own fiscal systems more readily than they can reverse economic and sociological forces. While local tax differentials may be of small consequence for inter-regional and inter-state location decisions, they surely have far more scope for influence within a single urban area where so many other factors affecting location are subject to rather modest differentiation"[21].

It is important to notice that in this analysis a central role is played by the differentials in tax rates between central cities and suburbs. It will be the purpose of the next section to show that the property tax, even if it is imposed at an equal rate, will still have a decentralizing influence.

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21) Metzer Ibid, p.124.

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3. Effects on City Structure.

If we accept the hypothesis that the price of housing depends on its location, and that when people buy or rent housing they buy not only shelter but also accessibility[22], then a tax on the price of housing will have more complex effects on the structure of the city.

Accessibility to any point can generally be bought in two ways: Either by having a home near the desired point or by greater expenditure on transportation to that point. A tax on the price of housing will change the relative price of one of these ways as compared to the other. Since the tax is not imposed on transportation it makes it relatively cheaper to travel to the center rather than to rent housing near the center.

This distortion in the locational pattern in the city can be demonstrated by the following simple model: Suppose that a tax at a rate t is imposed on the price of housing in our city. It does not matter whether under the law, the tax is the liability of the consumers (the tenants)

22) This hypothesis is of course central to this work and to a lot of the recent literature on urban structure. Empirical evidence in support of this hypothesis is presented by Colin Clark, Muth, Kain, Mills and others. For references see chapter I p. 2,5, and 15.

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or the manufacturers (the landlords); Then the price of housing to the consumers $P_c(u)$ will be:

$$P_c(u) = (1+t)P_h(u) \qquad (1)$$

Where $P_h(u)$ is the "manufacturers" price of housing at distance u from the center of the city.

It should be emphasized that the term housing as used here means the whole bundle of services that land and buildings give. The price of housing means here the price for this flow of services and thus is more closely related to the rents paid for rented housing than to the cost of constructing a house. In particular the "manufacturers" price is the price - net of tax - that the landlords realize, to be distinguished from the price that a builder will get for constructing such a house, even though in equilibrium these two should be equal.

For sake of consistency of the model we can assume that the property tax is a substitute for an income tax or a poll tax of the same yield. Since the citizens of our city have equal incomes and if we ignore the incentive effects of an income tax, the income tax and the poll tax are equivalent. The analysis is thus a comparison of the effects of a particular method of financing

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the government expenditures which are assumed to be given throughout.

We will also assume that the inhabitants of our city live in homes (apartments) of equal size, and that this size does not change as a result of the imposition of the property tax. This is of course an unrealistic assumption. With a higher price for housing people will consume less housing, and will substitute for it other goods, which are not taxed that heavily. However if we make this assumption we can isolate the effect of the property tax on the internal structure of urban areas from its general substitution effect.

Let us assume as before that houses are produced according to the Cobb-Douglas production function:

$$h = A L^{\alpha} K^{\beta} \qquad \alpha+\beta = 1 \qquad (2)$$

Where: h = is quantity of housing

L = is quantity of land

K = is quantity of capital

Let us also assume that capital and other factors are supplied to our city at constant prices[23]. Then we

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23) This is an overly strong assumption. It is sufficient to assume that the price of the other factors is uniform over the whole city. However we can assume that the overall supply of capital to the city is not infinitely elastic, and that if the city buys more of it, it has to pay a higher price.

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know that under perfect competition in the housing market the manufacturers price of housing is equal to :

$$P_h(u) = D\, R(u)^{\alpha} \qquad (3)$$

Where R(u) is the rent of land at distance u from the center of the city, and D a constant.

Now, if the inhabitants of our city are all interested only in accesibility to one central point, and if they all live in houses of the same size then equilibrium in the market for houses implies that the rate of decline in the price of houses with distance from the center should be equal to transportation costs.

$$P'_c(u) = -b \qquad (4)$$

If marginal transportation costs are a constant b.
Substituting from (1) and (3) into (4) gives:

$$P'_c(u) = (1+t)\alpha D R(u)^{\alpha-1} R'(u) = -b \qquad (5)$$

This is a differential equation in R(u).
Its solution is:

$$R(u) = \left[\frac{b}{(1+t)D}(\bar{u}-u)\right]^{\gamma} \qquad \gamma = \frac{1}{\alpha} \qquad (6)$$

Where $\bar{u}$ is the radius of the city (the edge of the city), using as an initial condition the assumption that at $\bar{u}$ rent is zero.

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Examination of this expression shows that, ceteris paribus, (if the radius of the city $\bar{u}$ remains constant), a rise in the tax rate will lower rents:

$$\frac{dR}{dt} = -\gamma(1+t)^{-(\gamma+1)}\left[\frac{b}{D}(\bar{u}-u)\right]^{\gamma} \qquad (7)$$

This reduction in land rents is the absorption of part of the burden of the tax by the owners of land.

It should be noticed that the impact of the tax on the rent depends on the distance from the center of the city. The greater is the distance the smaller is the reduction in land rents.

However this is not the end of the story. The radius of the city $\bar{u}$ will also change with a change in the tax rate.

$$\bar{u} = (1+t)^{\frac{\gamma-1}{\gamma+1}} \left(\frac{D}{b}\right)^{\frac{1}{\gamma+1}} G^{\frac{1}{\gamma+1}} H^{\frac{1}{\gamma+1}} \qquad (8)$$

Where H is the total number of households in the city and G is a constant difined by:

$$G = \frac{b^2(\gamma+1)}{D2\pi}$$

From (8) we can see that $\bar{u}$ is a positive function of the tax rate. The radius of city grows, for a given population size H, when the tax rate is being raised:

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$$\frac{d\bar{u}}{dt} = \frac{(t-1)}{(\gamma+1)} \frac{D}{b} G^* H^* (1+t)^{\frac{-2}{\gamma+1}} \qquad (9)$$

$$H^* = H^{\frac{1}{\gamma+1}} \qquad G^* = G^{\frac{1}{\gamma+1}}$$

We see that a rise in the tax on housing has the effect of spreading out or stretching the city, pushing people to move out into land which was not built up before the tax rise, and thus distorting the optimal balance between transportation costs and intensity of land use; causing the city to produce more transportation and to use its land less intensive than would be optimal.

When we substitute (8) into (6) :

$$R(u) = \left[(1+t)^{\frac{-2}{\gamma+1}} G^* H^* - \frac{b\,u}{D(1+t)}\right]^{\gamma} \qquad (10)$$

$$\frac{dR}{dt} = \gamma\left[(1+t)^{\frac{-2}{\gamma+1}} G^* H^* - \frac{b\,u}{D(1+t)}\right]^{\gamma-1} \left[\frac{b\,u}{D(1+t)^2} - \frac{2}{\gamma+1}(1+t)^{\frac{-\gamma+3}{\gamma+1}} G^* H^*\right] \qquad (11)$$

It should be noticed that the impact of a rise in t is still negative when u is zero or near zero. i.e.: near the center of the city. However it is not so when we move out from the center of the city.

Actually rent must go up at the old edge of the city ($\bar{u}$). Before the rise in the tax the rent at $\bar{u}$ was zero, and if the edge moves out as is indicated by (9),

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rent has to be positive at the old edge, which is now inside the city limits.

This analysis also implies that density of land use will decline near the center of the city, and will grow at the margin of the city.

For any homogeneous production function like (2) the intensity of use of the factors of production is proportional to their prices. In our case that means that as the price of land goes down, less capital will be used with any given quantity of land, and that also less output will be produced on any given quantity of land.

Indeed the output of housing, at every ring at distance u from the center is:

$$h(u) = \frac{2\pi u}{\alpha D}\left[\frac{b}{(1+t)D}(\bar{u}-u)\right]^{\frac{\beta}{\alpha}} \qquad (12)$$

The density at every distance is:

$$\frac{h(u)}{2\pi u} = (\alpha D)^{-1}\left[\frac{b}{(1+t)D}(\bar{u}-u)\right]^{\frac{\beta}{\alpha}} \qquad (13)$$

This is a negative function of the tax rate.

$$\frac{dh}{dt} = -\frac{\beta}{\alpha}(1+t)^{-\gamma}\frac{2\pi u}{\alpha D}\frac{b}{D}(\bar{u}-u)^{\frac{\beta}{\alpha}} \qquad (14)$$

For given radius $\bar{u}$ of the city this is seen to be negative, which means the density of population will decline everywhere - and that the total population for a given radius

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will decline also.

Another way to see the same thing is to observe that for a given radius $\bar{u}$ (and given marginal transportation costs b) the price the consumers will pay for housing at distance u from the center is invariant to the tax rate. It is:

$$P_c(u) = b(\bar{u}-u) \qquad (15)$$

With a rise in the tax rate, the net price of housing that the landlords will realize will go down and they will find it profitable to reduce the amount of housing produced at every location.

On the other hand if the total size of the population H is to be kept constant, and if the amount of housing produced at every ring at distance u from the center goes down with a rise in the tax, the radius of the city $\bar{u}$ must go up.

Indeed since $H = \int_0^{\bar{u}} h(u)du \qquad (16)$

we can calculate from (16) what is the radius of the city $\bar{u}$ as a function of the tax rate t and the total population size H.

By substitution from (12) into (16) this gives

$$H = \int_0^{\bar{u}} \frac{2\pi u}{\alpha D}\left[\frac{b}{(1+t)D}(\bar{u}-u)\right]^{\frac{\beta}{\alpha}}du = \frac{(1+t)^2}{G}\left[\frac{b}{D}\frac{\bar{u}}{(1+t)}\right]^{\gamma+1} \qquad (17)$$

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Equation (8) is the solution of this equation for $\bar{u}$.

If we consider the changes in $\bar{u}$ that are necessary to accomodate a constant population H with a higher tax rate t, the resulting changes in the pattern of density in the city are more complex.

$$h(u) = \frac{2\pi u}{\alpha D}\left[(1+t)^{\frac{-2}{1+\gamma}} G^{\frac{1}{\gamma+1}} H^{\frac{1}{\gamma+1}} - \frac{bu}{D(1+t)}\right]^{\frac{\beta}{\alpha}} \qquad (18)$$

This is seen to be very similar to equation (10), and the same arguments that apply to the changes in land rents apply to changes in density.

Density will decline near the center of the city and will go up near the old edge of the city, as the edge of the city is pushed outward, and the built up area extends into land not used before.

Since for a given population size H, the total area of the city ($\pi\bar{u}^2$) will go up, overall density will go down. In other words generally housing will be produced using a higher proportion of land and less capital relative to the quantities that would have been chosen without (or with a lower) tax.

If the supply of capital used in the production of housing is not infinitely elastic (as was assumed above), but is elastic, its price will go down, and its owners will bear part of the burden of the tax.

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4. Differentials between City and Suburb

We can also describe in this framewoek the effects of a differential between the tax rate in a central city and its suburbs.

Suppose that our city (metropolitan area) with a population of H, is devided into two parts : A centeral city with radius u^* ($0 < u^* < \bar{u}$), and suburbs which extend from u^* as far out into the country as necessary to accomodate (to house) all the remainder of the population. The only difference between the central city and the suburbs that we allow is a difference in the property tax rate. The differences in the tax revenue are compensated for by differences in the income tax or the poll tax. In particular the residents of the suburbs have the same "need" for accessibility to the center as the residents of the central city. Therefore the price the consumers are willing to pay for housing at any distance from the center will still be:

$$P_c(u) = b(\bar{u}-u) \qquad (20)$$

Suppose now that the central city imposes a tax at a rate t' which is higher then the rate t which the suburbs impose, ($t < t'$).

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Let us now examine what happens to the price of housing and of land near the city limit (u*). The price of housing to the consumers, for a given $\bar{u}$, is invariant to the rate of tax.

Within a small distance du, the price of housing to the consumers should not vary by more than :

$$d P_c(u) = -b\, du \qquad (21)$$

If the price of housing to the consumers just inside the city limits $P_c(u^*)$ is the same as the price of housing to the consumers just outside the city limits, and if the rates of tax are different, then the net price of housing that the landlords realize $P_h(u)$ must be different, and it must be lower inside the city limits.

If: $(1+t)P_h(u^*) = (1+t')P'_h(u^*) = P_c(u^*)$ (22)

and $t' > t$, then the net price of housing to the landlords just inside the city limit must be lower:

$$P'_h(u^*) < P_h(u^*) \qquad (23)$$

Where $P'_h(u)$ denotes the price of housing inside the city limit.

This implies that they will offer for land lower rents, since from (3) it is obvious that R(u) is:

$$R(u) = P_h(u)D^{-1} \qquad (24)$$

and also that the intensity of land use (density will be lower inside the city limit than it is outside.

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5. Conclusion

The conclusion of this chapter is that - for a given level of expenditures of local governments - an increase in the property tax, substituting an alternative source of revenue, will tend to lower densities and land values near the center of the city and to encourage the spreading out of the built up area. Within given limits of a central city, an increase in the property tax will tend to lower land values and intensity of land use and to divert development from the central city to the suburbs.

It is important to keep these conclusions in proper perspective. Other very strong forces (rising incomes, technological change in the transportation and communications industry, induced changes in manufacturing wholesaling and other industries) are at work, and they have tended during the past few decades to favor decentralization of cities. Netzer might be right in holding the view that the tide would not have been stemmed even had there been large property tax differential in favour of central cities. However I believe that the property tax is still playing a significant role in the decentralization of cities.

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Bibliography

1. Alonso William, "Location and Land Use", Harvard University Press, Cambridge, Mass. 1964.
2. Artle Roland, "Studies in the Structure of the Stockholm Economy", Stockholm 1959.
3. Beckmann Martin, "Location Theory", Random Houses, New York, 1968.
4. ————, "On the Distribution of Urban Rent and Residential Density", Journal of Economic Theory, June 1969. pp.60-67.
5. ————, "Equilibrium versus Optimum Spacing of Firms and Patterns of Market Areas", (mimeographed) Brown University, April 1970.
6. Buchanan,J.M., "An Economic Theory of Clubs" Economica, February 1965, pp.1-14.
7. Capoza Dennis, "Transportation Costs and the Provision of Urban Governmental Services" (mimeographed), University of Southern California,1972.
8. Casetti Emilio, "Equilibrium Land Values and Population Density in Urban Setting" Economic Geography, (Vol 47 No 1), January 1971 pp 16-20.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

9. Casetti,E. and Papageorgiou,G., "A Spatial Equilibrium Model of Urban Structure", The Canadian Geographer (Vol 15 No. 1), Spring 1971,pp.30-37.

10. Clark Colin, "Urban Population Densities", Journal of the Royal Statistical Society, Series A, pp. 490-496.

11. ————,"Population Growth and Land Use", Mcmillan 1967.

12. Hansen Alvin H., "Economic Issues of the 1960's", McGraw Hill 1960.

13. Harberger A.C., "The Incidence of the Corporate Income Tax", Journal of Political Economy, June 1962, pp. 215-240.

14. ————, "Three Basic Postulates for Apllied Welfare Economics", Journal of Economic Literature, September 1971, pp. 785-797.

15. Hoch Irving, "The Three Dimensional City", In Perloff Harvey (ed.) "The Quality of the Urban Environment", Johns Hopkins University Press for Resources for the Future, Baltimore, 1969,pp 75-135.

16. Hochman O. and Pines D., "Competitive Equilibrium of Transportation and Housing in the Residential Ring of an Urban Area", Environment and Planning, Vol No. 1, 1971 pp. 51-61.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

17. Hirsch Werner Z., "Expenditure Implications of Metropolitan Growth and Consolidation" Review of Economics and Statistic, August 1959, pp. 232-241.

18. ————, "Local vs, Areawide Government Services", National Tax Journal, December 1964, pp.331-339.

19. Kain John F. "The Jorney to Work as a Determinant of Residential Location" Papers of the Regional Science Association, 1962, pp. 137-160.

20. Lave Lester, "Congestion and Urban Location", Papers of the Regional Science Association, 1970, pp.133-150. This is an abreviated version of: Lave Lester, "Transportation, City Size and Congestion Tolls" Rand Memorandum, April 1969.

21. Manvel,A.D. "Land Use in 106 Large Cities" Research No. 12 Prepared for the Consideration of the National Commission on Urban Problems 1968.

22. Margolis Julius, "The Demand for Urban Public Services" in Perloff Harvey S. and Wingo Lowdon (ed.) "Issues in Urban Economics" Johns Hopkins University Press 1968.

23. Meyer ,J.,Kain,J. and Wohl,M., "The Urban Transportation Problem", Harvard 1965.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

24. Mieszkowski Peter, "Tax Incidence Theory" Journal of Economic Literature, December 1969 pp 1103-1124.

25. ————, "The Property Tax: An Excise Tax or a Profit Tax?" Cowles Foundation Discussion Paper No. 304, November 1970.

26. Mills Edwin,S., "An Aggregative Model of Resource Allocation in a Metropolitan Area", American Economic Review, May 1967, pp. 197-210.

27. ————, "The Value of Urban Land" in Perloff H. (ed.) "The Quality of the Urban Environment", Johns Hopkins University Press for Resources for the Future, Baltimore 1969, pp.231-253.

28. ————, "The Efficiency of Spatial Competition", The Regional Science Association Papers, Vol XXV, 1970, pp.71-82.

29. ————, and de Ferranti,David M., "Market Choice and Optimum City Size", American Economic Review, May 1971, pp. 340-345.

30. Musgrave Richard, "The Theory of Public Finance", McGraw Hill 1959, p.

31. ————, "Fiscal Systems" Yale University Press, 1969.

32. Muth Richard, "The Spatial Structure of the Housing Market", Papers of the Regional Science Association, 1961 pp. 207-220.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

33. Muth Richard, "Economic Change and Rural Urban Land Conversion", Econometrica, January 1961 pp.1-23.

34. ————, "Cities and Housing", Chicago, University of Chicago Press 1969.

35. Nerlove Marc, "Estimation and Identification of Cobb-Douglas Production Functions", Chicago, Rand McNally 1965.

36. Netzer Dick, "The Economics of the Property Tax", The Brookings Insitution, Washington D.C. 1966 p.11.

37. Neutze Max, "Economic Policy and the Size of Cities" Australian National University, Canberra 1965.

38. ————, "Property Taxation and Multiple-Family Housing" in Becker Arthur (ed.) "Land and Building Taxes their Effect on Economic Development, University of Wisconsin Press 1969, pp. 115-128

39. Niedercorn J. and Hearle,E."Recent Land Use Trends in 48 Large American Cities", Rand 1963.

40. Patinkin Don, "Multiple Plant Firms Cartels and Imperfect Competition", Quarterly Journal of Economics, Ferbruary 1947, pp. 173-203.

41. ————, A "Note", Quarterly Journal of Economics, August 1947, pp 651-657.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.

42. Pines David, "The Effects of Variations in some Determinants of a Mono-Center Urban Form under Competitive Equilibrium", Tel-Aviv University, Center for Urban and Regional Studies, Tel-Aviv 1971.

43. Rothenberg Jerome, "Local Decentralization and the Theory of Optimal Government" in N.B.E.R., "The Analysis of Public Output", 1970, pp.31-64.

44. Solow Robert M. and Vickrey William S. "Land Use in a Long Narrow City", Journal of Economic Theory, December 1971 pp. 430-447.

45. Teplin Albert, "Fiscal Incentives of Metropolitan Location", Unpublished paper, Johns Hopkins University 1970.

46. Tiebout Charles M., "A Pure Theory of Local Expenditures", Journal of Political Economy. October 1956, pp. 416-424.

47. ———, "An Economic Theory of Fiscal Decentralization" in National Bureau of Economic Research "Public Finances Needs Sources and Utilization", 1961 ,pp 78-96.

48. Vickrey William, "General and Specific Financing of Urban Services" in Schaller Howard, G. (ed.) "Public Expenditure Decisions in the Urban Community", Resources for the Future, Washington D.C. 1962 p. 62-90.

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49. Vickery William, "Pricing as a tool in coordination of Local Transportation", in "Transportation Economics", N.B.E.R, New York 1965,p.275-291.

50. ————, "Optimization of Traffic and Facilities", Journal of Transport Economics and Policy, May 1967, pp. 123-136.

51. Voorhees Alan,M. and Associates, "Factors and Trends in Trip Lengths" Highway Research Board ,Report No. 48, 1968.

52. Wabe Stuart,J., "A Study of House Prices as a Means of Establishing the Value of Journey Time, The Rate of Time Preference and the Valuation of some Aspects of the Enviornment in the London Metropolitan Region" Applied Economics, December 1971, pp. 247-255.

53. Wingo Lowdon, "Transportation and Urban Land", Resources for the Future, Washington D.C. 1961.

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Curriculum Vitae

Eliyahu Borukhov was born in Jerusalem in 1933. He was graduated from the Hebrew University of Jerusalem with a B.A. in 1961.

During the years 1963 to 1968 he was a member of the staff of the Economic Planning Authority of Israel. He attended the Johns Hopkins University from 1968 to 1971 on fellowships granted by the University.

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